Using 1-Jettiness to Measure 2 Jets in DIS in 3 Ways

Christopher Lee
LANL Theoretical Division

in collaboration with **Daekyoung Kang** and **Iain Stewart** *(MIT)*

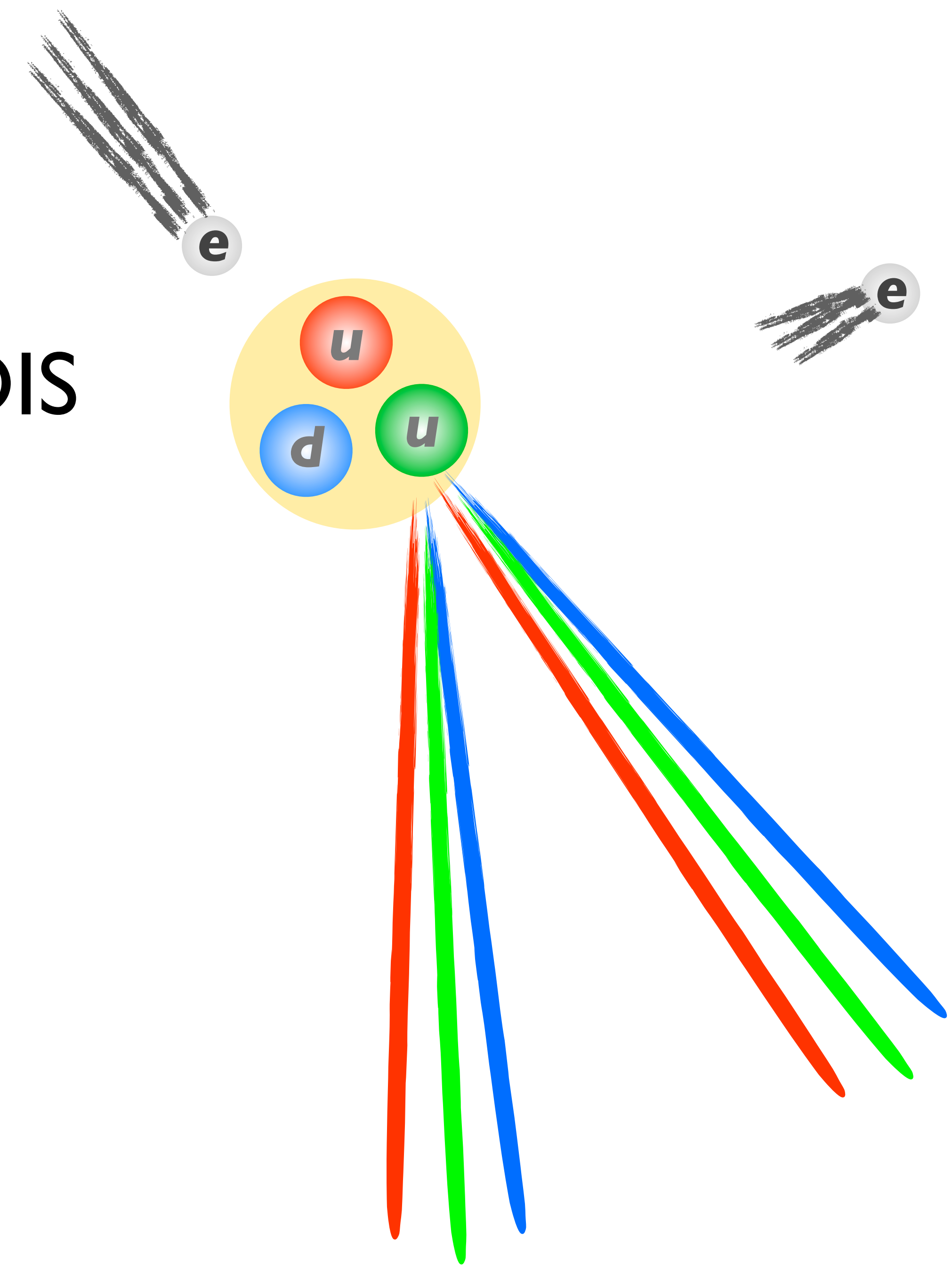
Los Alamos NATIONAL LABORATORY

Massachusetts Institute of Technology

Duke UNIVERSITY March 7, 2013 SCET 2013

Outline

- History and Motivation
- 3 Ways to Define 1-Jettiness in DIS
- Factorization for 2 Jets:
  Beam and Jet Functions
- NNLL Resummed Predictions
  for 3 Versions of DIS 1-Jettiness
- Future



Brief History and Motivation

- In SCET, DIS near the endpoint $x = 1$ has been studied extensively.

Manohar;
Becher, Neubert, Pecjak;
Chay, Kim; Chen, Idilbi, Ji, Yuan;
Fleming, Zhang

- Away from $x = 1$, the final state can have more than one jet.
- DIS event shapes:
  - In QCD, thrust to NLO, resummed to NLL. Antonelli, Dasgupta, Salam (1999)
  - In SCET, (one version of) 1-jettiness at NLL [now NNLL] Kang, Mantry, Qiu (2012) [Kang, Liu, Mantry, Qiu (yesterday)]
- e^+e^- event shapes have been resummed in SCET to N^3LL accuracy matched to N^3LO, leading to precise extraction of α_s

Becher, Schwartz; Chien, Schwartz;
Abbate, Fickinger, Hoang, Mateu, Stewart

- Extractions of α_s from DIS jet cross sections not yet at same precision.
- Theoretical tools now available for significant improvement of DIS event shape predictions and α_s extraction

Strong Coupling

Process	Collab.	Value	Exp.	Th.	Total	(%)
(1) Inc. jets at low Q^2	H1	0.1180	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.6}_{-8.1}$
(2) Dijets at low Q^2	H1	0.1155	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.8}_{-8.2}$
(3) Trijets at low Q^2	H1	0.1170	0.0017	$^{+0.0091}_{-0.0073}$	$^{+0.0093}_{-0.0075}$	$^{+7.9}_{-6.4}$
(4) Combined low Q^2	H1	0.1160	0.0014	$^{+0.0094}_{-0.0079}$	$^{+0.0095}_{-0.0080}$	$^{+8.2}_{-6.9}$
(5) Trijet/dijet at low Q^2	H1	0.1215	0.0032	$^{+0.0067}_{-0.0059}$	$^{+0.0074}_{-0.0067}$	$^{+6.1}_{-5.5}$
(6) Inc. jets at medium Q^2	H1	0.1195	0.0010	$^{+0.0052}_{-0.0040}$	$^{+0.0053}_{-0.0041}$	$^{+4.4}_{-3.4}$
(7) Dijets at medium Q^2	H1	0.1155	0.0009	$^{+0.0045}_{-0.0035}$	$^{+0.0046}_{-0.0036}$	$^{+4.0}_{-3.1}$
(8) Trijets at medium Q^2	H1	0.1172	0.0013	$^{+0.0053}_{-0.0032}$	$^{+0.0055}_{-0.0035}$	$^{+4.7}_{-3.0}$
(9) Combined medium Q^2	H1	0.1168	0.0007	$^{+0.0049}_{-0.0034}$	$^{+0.0049}_{-0.0035}$	$^{+4.2}_{-3.0}$
(10) Inc. jets at high Q^2 (anti-k_T)	ZEUS	0.1188	$^{+0.0036}_{-0.0035}$	$^{+0.0022}_{-0.0022}$	$^{+0.0042}_{-0.0041}$	$^{+3.5}_{-3.5}$
(11) Inc. jets at high Q^2 (SIScone)	ZEUS	0.1186	$^{+0.0036}_{-0.0035}$	$^{+0.0025}_{-0.0025}$	$^{+0.0044}_{-0.0043}$	$^{+3.7}_{-3.6}$
(12) Inc. jets at high Q^2 (k_T; HERA I)	ZEUS	0.1207	$^{+0.0038}_{-0.0036}$	$^{+0.0022}_{-0.0023}$	$^{+0.0044}_{-0.0043}$	$^{+3.6}_{-3.6}$
(13) Inc. jets at high Q^2 (k_T; HERA II)	ZEUS	0.1208	$^{+0.0037}_{-0.0032}$	$^{+0.0022}_{-0.0022}$	$^{+0.0043}_{-0.0039}$	$^{+3.6}_{-3.2}$
(14) Inc. jets in γp (anti-k_T)	ZEUS	0.1200	$^{+0.0024}_{-0.0023}$	$^{+0.0043}_{-0.0032}$	$^{+0.0049}_{-0.0039}$	$^{+4.1}_{-3.3}$
(15) Inc. jets in γp (SIScone)	ZEUS	0.1199	$^{+0.0022}_{-0.0022}$	$^{+0.0047}_{-0.0042}$	$^{+0.0052}_{-0.0047}$	$^{+4.3}_{-3.9}$
(16) Inc. jets in γp (k_T)	ZEUS	0.1208	$^{+0.0024}_{-0.0023}$	$^{+0.0044}_{-0.0033}$	$^{+0.0050}_{-0.0040}$	$^{+4.1}_{-3.3}$
(17) Jet shape	ZEUS	0.1176	$^{+0.0013}_{-0.0028}$	$^{+0.0091}_{-0.0072}$	$^{+0.0092}_{-0.0077}$	$^{+7.8}_{-6.5}$
(18) Subjet multiplicity	ZEUS	0.1187	$^{+0.0029}_{-0.0019}$	$^{+0.0093}_{-0.0076}$	$^{+0.0097}_{-0.0078}$	$^{+8.2}_{-6.6}$
HERA average 2004		0.1186	± 0.0011	± 0.0050	± 0.0051	± 4.3
HERA average 2007		0.1198	± 0.0019	± 0.0026	± 0.0032	± 2.7

Table 1: Values of $\alpha_s(M_Z)$ extracted from jet observables at HERA together with their uncertainties (rows 1 to 18). The 2004 [10] and 2007 [11] HERA averages are shown in the last two rows.

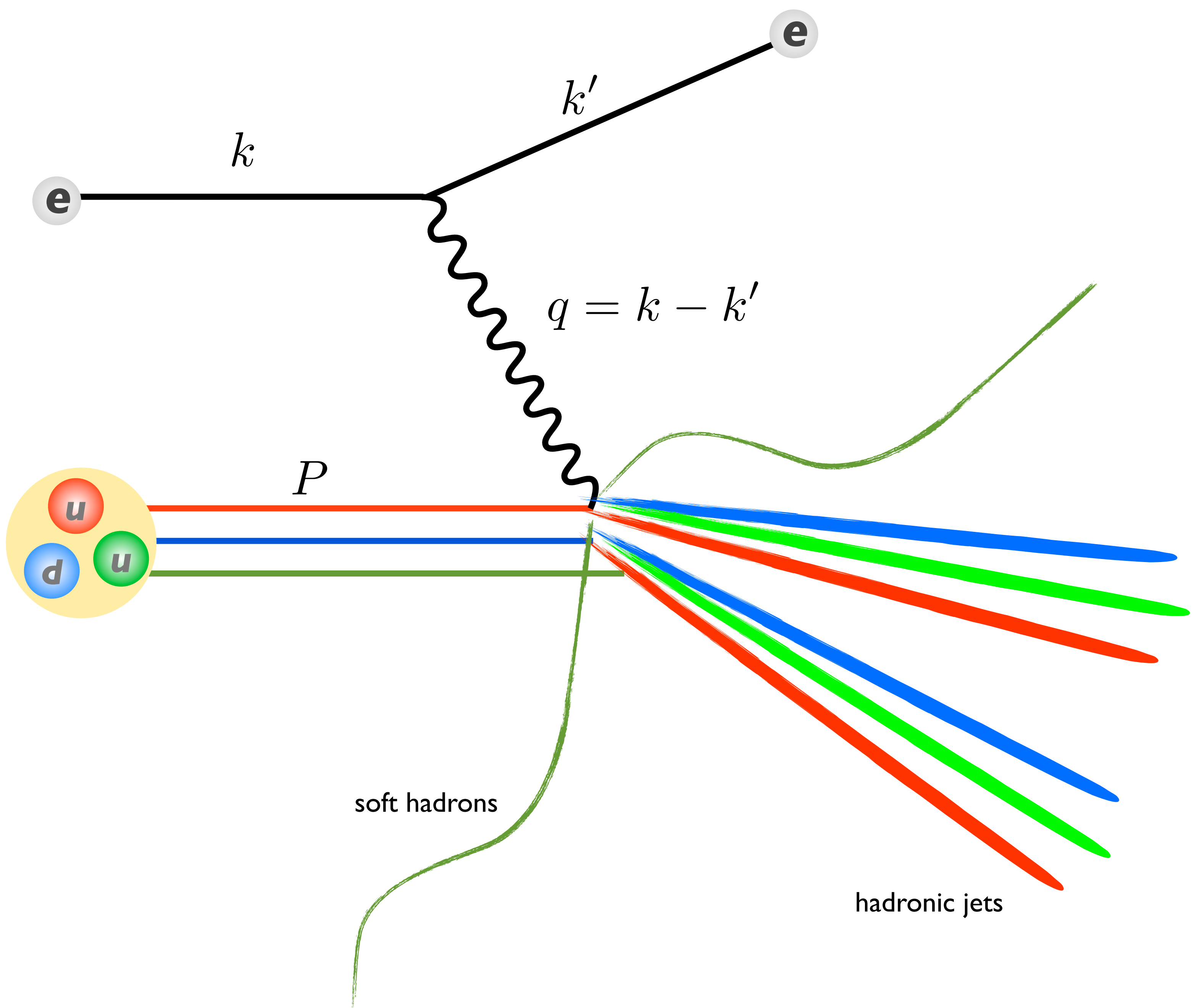
Extractions from exclusive jet cross sections have order 10-20% uncertainty, dominated by theory

Improve to level of e^+e^-?

C. Glasman, in the Proceedings of the Workshop on Precision Measurements of α_s [1110.0016]

I. Three Ways to Define 1-Jettiness in DIS

DIS Kinematics



$$s = (k+P)^2$$ squared center-of-mass energy

$$Q^2 = -q^2$$ momentum transfer

$$x = \frac{Q^2}{2P \cdot q}$$ Björken scaling variable

$$y = \frac{P \cdot q}{P \cdot k}$$ lepton energy loss in proton rest frame

$$Q^2 = xys$$

$$p_X = q + P$$ total momentum of final hadronic state

$$p_X^2 = \frac{1-x}{x} Q^2$$ invariant mass of final hadronic state

Limit $x \to 1$ corresponds to single collimated jet in final state

We will look away from *x* = *1* at *two*-jet like final states

Problems with Jet Cross Sections

- Exclusive jet cross sections (fixed number of jets) typically depend on
  - choice of jet algorithm
  - jet sizes
  - jet vetoes
- These parameters generate a number of logarithms (NGLs, clustering logs, log R) in perturbation theory which we do not yet know how to resum
- ***N*-Jettiness**: an *inclusive* observable picking out *N*-jet final states by measurement of a single parameter, logs of which *can* be resummed in perturbation theory

N-jettiness

An *inclusive* event shape over all final state hadrons *excluding* more than *N* jets:

$$\tau_N = \frac{2}{Q^2}\sum_i \min\{q_B\cdot p_i, q_1\cdot p_i, \ldots, q_N\cdot p_i\}$$

Stewart, Tackmann, Waalewijn (2010)

Vector q_B is aligned with the incoming proton beam and $q_{1,\ldots,N}$ with final state jets.
Final state hadrons *i* are grouped with the axis "closest" to it.

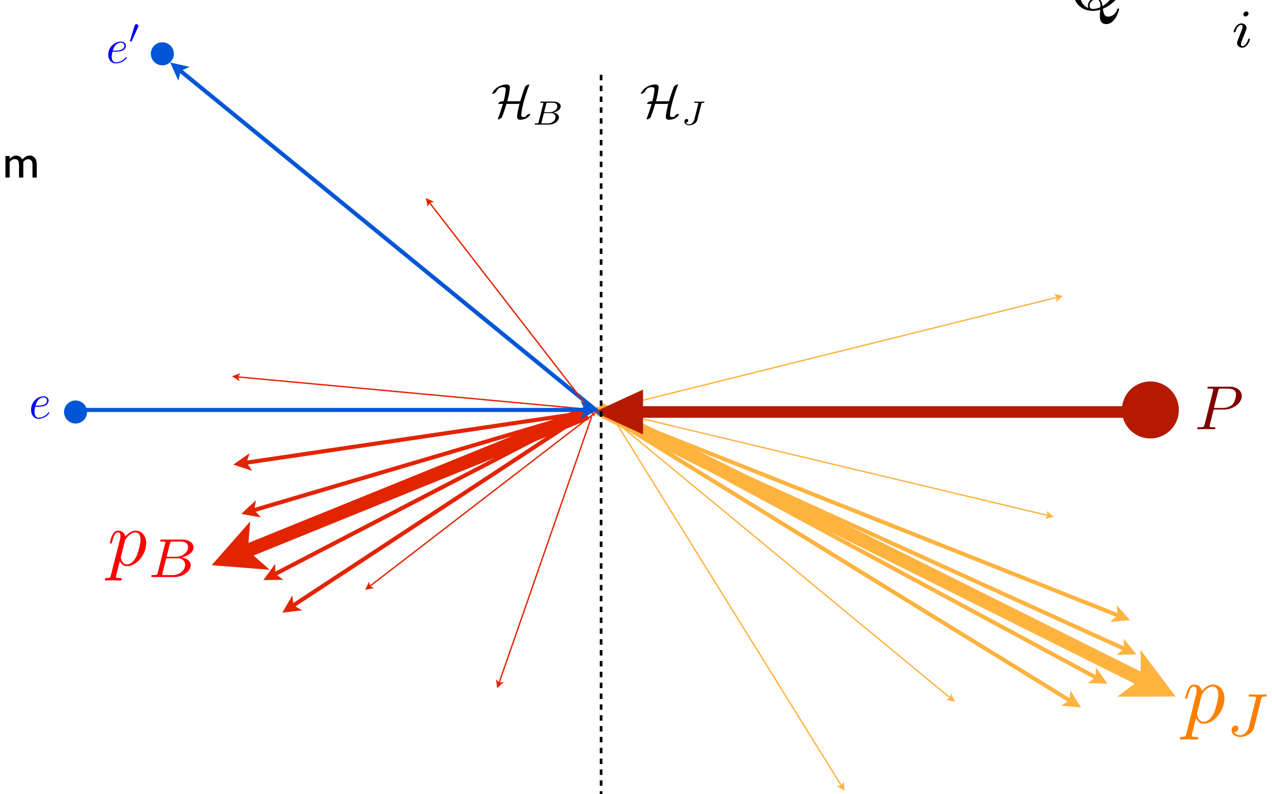
As $\tau_N \to 0$, final state contains exactly *N+1* pencil-like jets (one from beam radiation).

We will look at "1-jettiness" in DIS.

$$\tau_1 = \frac{2}{Q^2}\sum_i \min\{q_B\cdot p_i, q_J\cdot p_i\}$$

Center-of-momentum frame:

$k = \sqrt{s}\frac{n_z}{2}$



$P = \sqrt{s}\frac{\bar{n}_z}{2}$

N-jettiness

An *inclusive* event shape over all final state hadrons *excluding* more than *N* jets:

$$\tau_N = \frac{2}{Q^2}\sum_i \min\{q_B\cdot p_i, q_1\cdot p_i, \ldots, q_N\cdot p_i\}$$

Stewart, Tackmann, Waalewijn (2010)

Vector q_B is aligned with the incoming proton beam and $q_{1,\ldots,N}$ with final state jets.
Final state hadrons *i* are grouped with the axis "closest" to it.

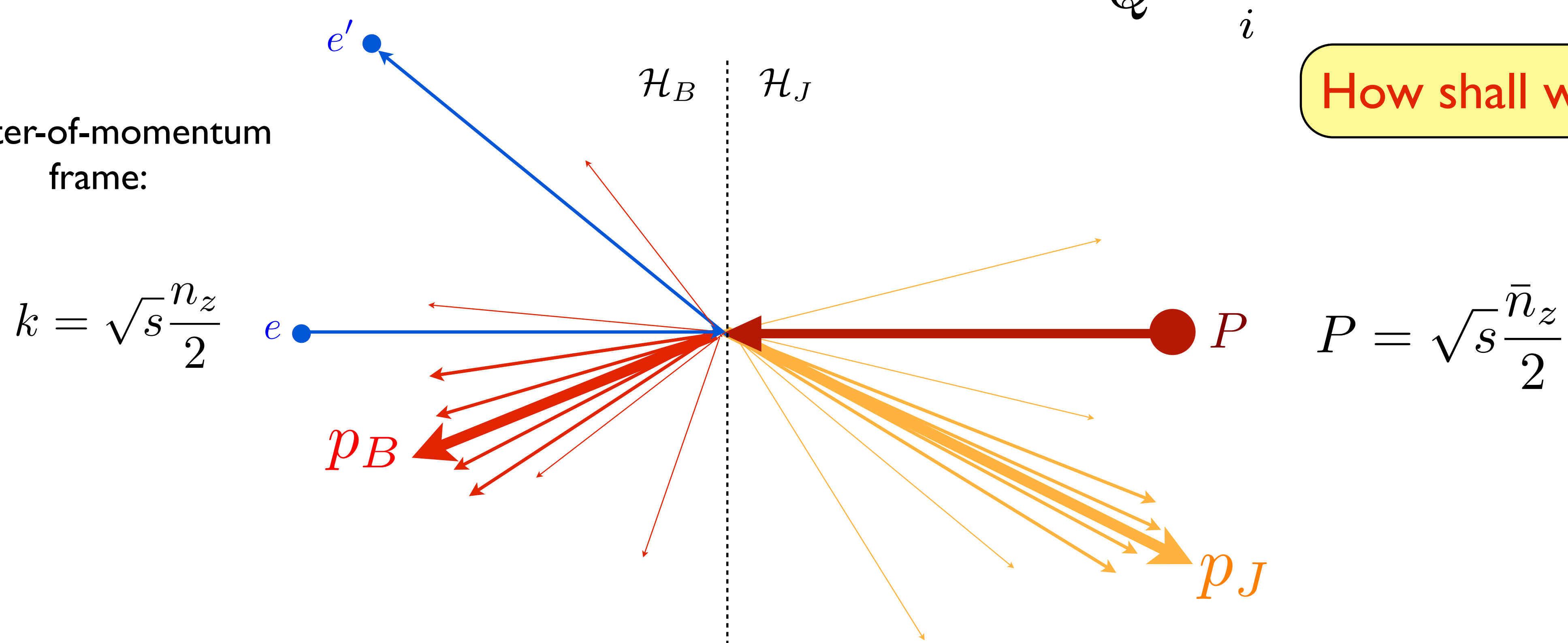
As $\tau_N \to 0$, final state contains exactly *N+1* pencil-like jets (one from beam radiation).

We will look at "1-jettiness" in DIS.

$$\tau_1 = \frac{2}{Q^2}\sum_i \min\{q_B\cdot p_i, q_J\cdot p_i\}$$

How shall we pick q_B and q_J?

Center-of-momentum frame:



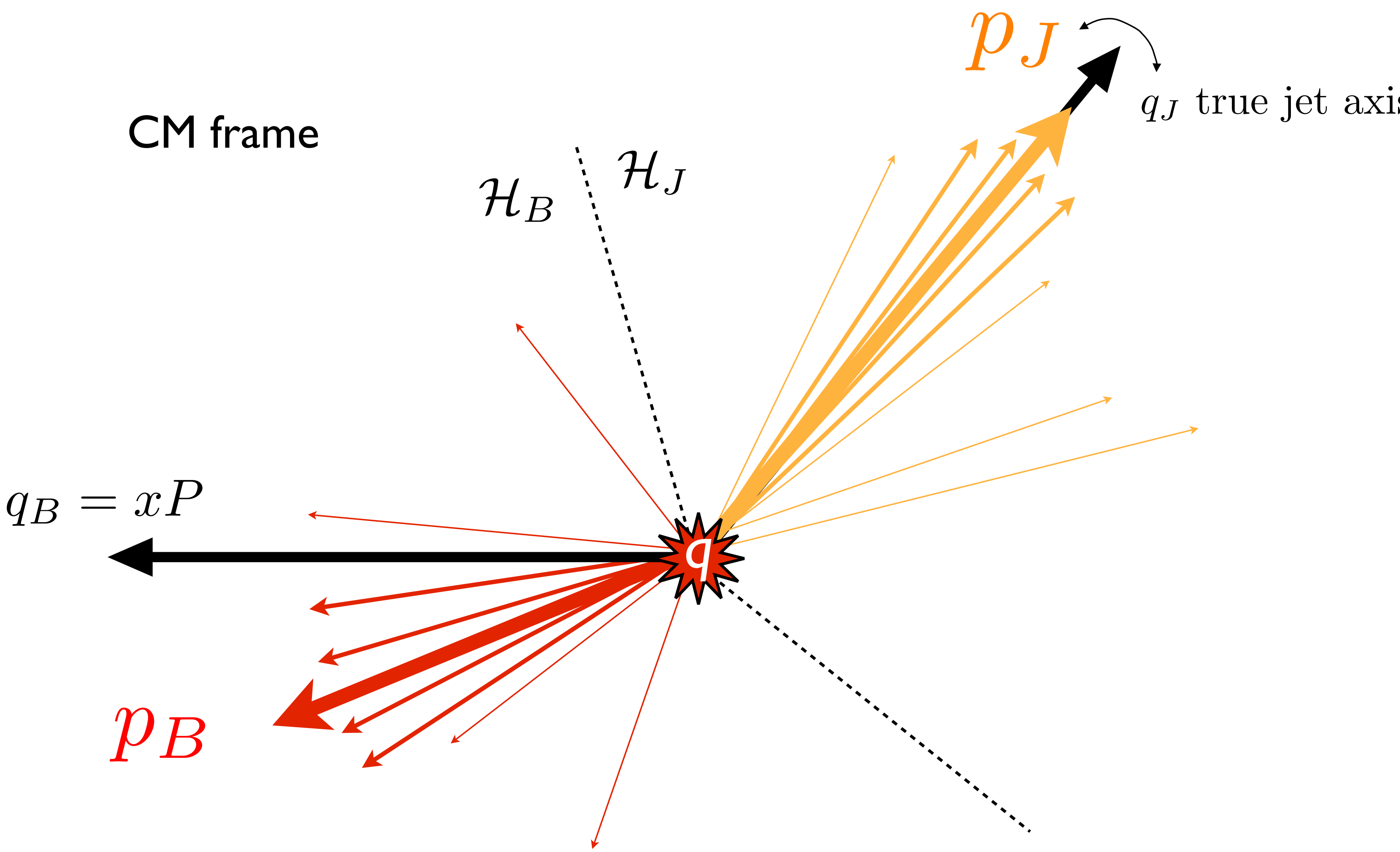
Three choices for DIS 1-jettiness

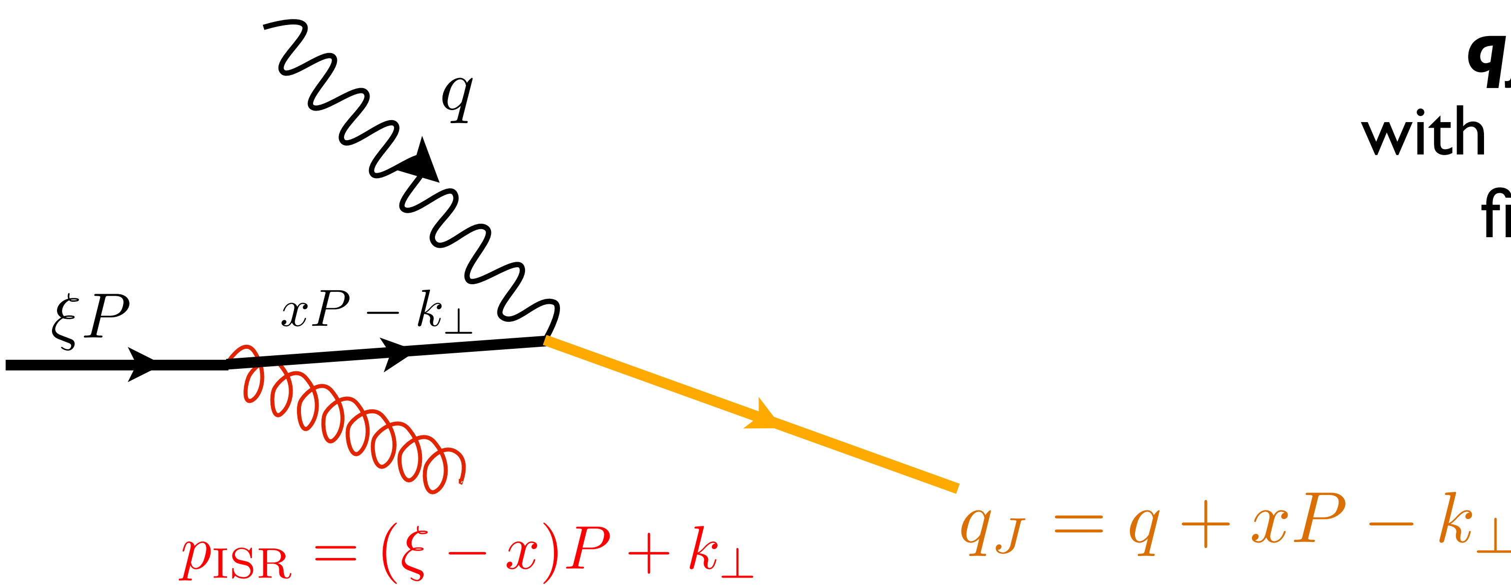
τ_1^a

$$q_B = xP$$

$$q_J = \text{true jet axis}$$

Kang, Mantry, Qiu (2012)



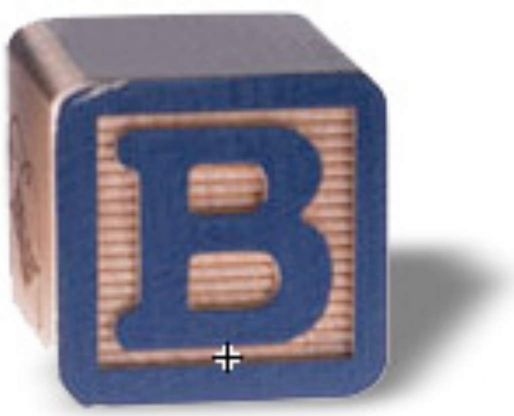


$k_\perp \sim Q\lambda$

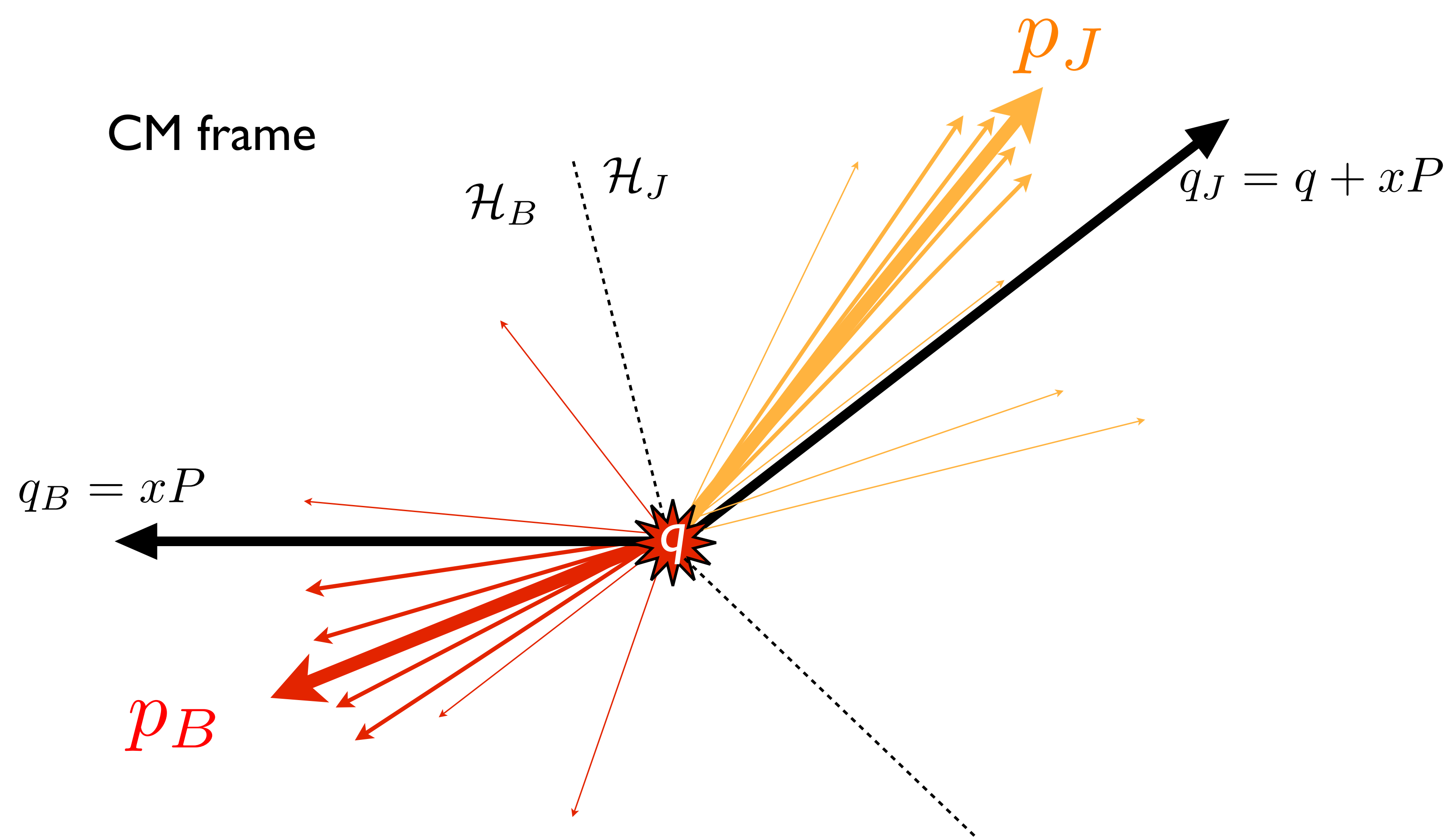
$\boldsymbol{q_J}$ is **A**ligned with the jet momentum, with no relative label transverse momentum: find by jet algorithm or minimization

⇨ depends on momenta of final-state hadrons

Three choices for DIS 1-jettiness



$$\tau_1^b$$

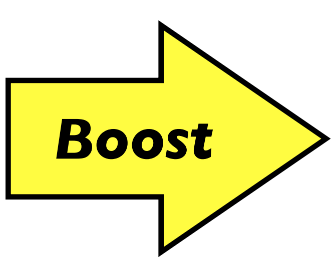


$$q_B = xP$$

$$q_J = q + xP$$

same as DIS thrust
by Antonelli, Dasgupta, Salam
(1999)

q_J no longer exactly aligned with jet, but simpler in that $q+xP$ is given only by lepton and initial-state proton momenta

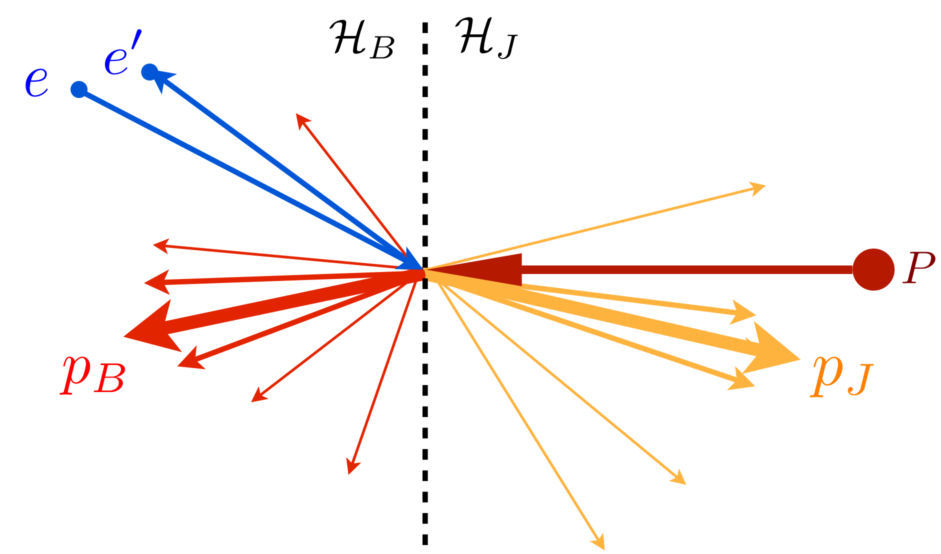


Breit frame:

$$q = (Q, 0, 0, Q)$$

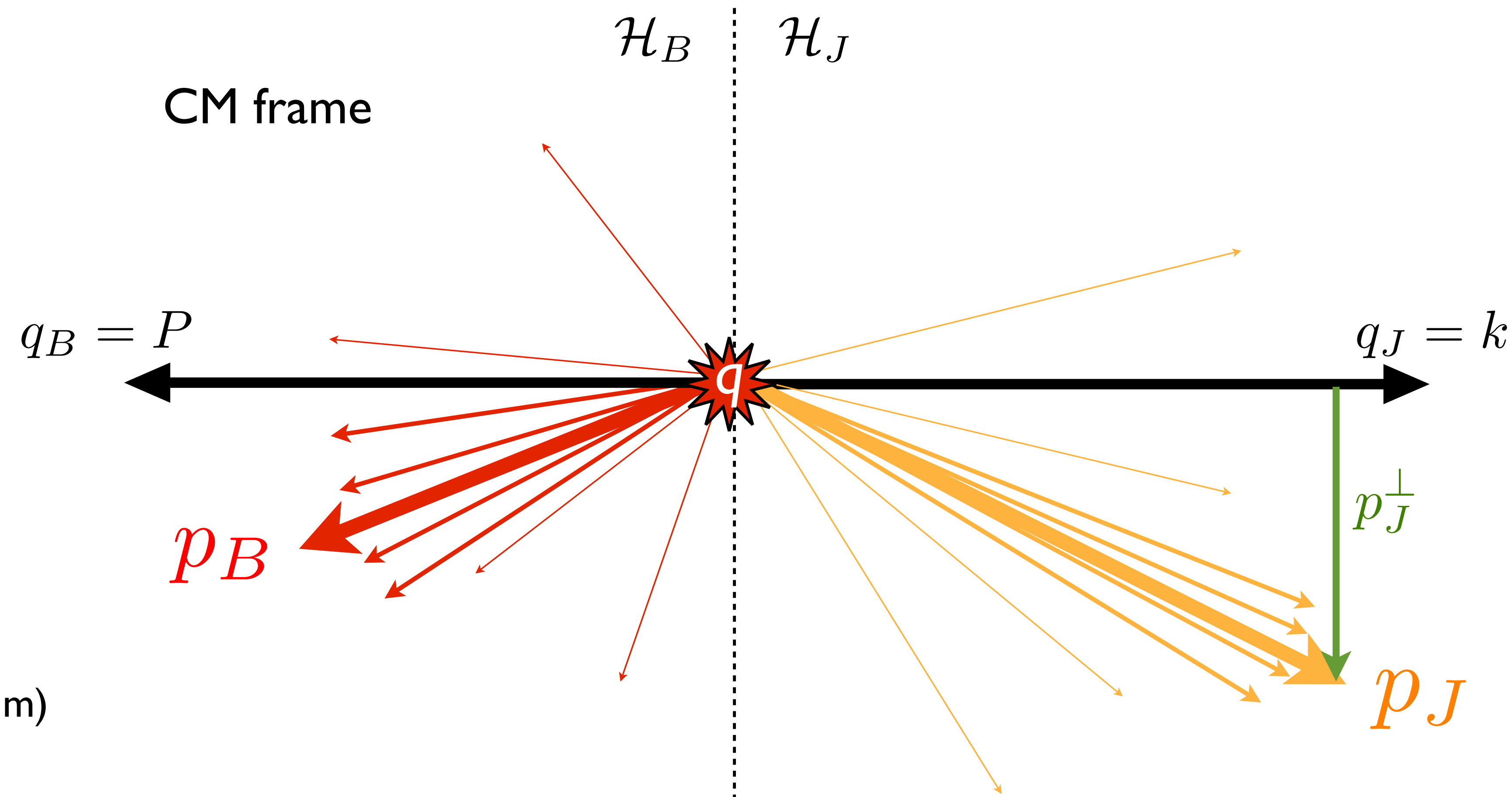
$$q_B = Q\bar{n}_z \quad q_J = Qn_z$$

1-jettiness regions are hemispheres in Breit frame



Three choices for DIS 1-jettiness

τ_1^c



$$q_B = P$$
$$q_J = k$$

(electron momentum)

measures thrust in back-to-back hemispheres in **C**enter-of-momentum frame

momentum transfer ***q*** itself has a nonzero transverse component:

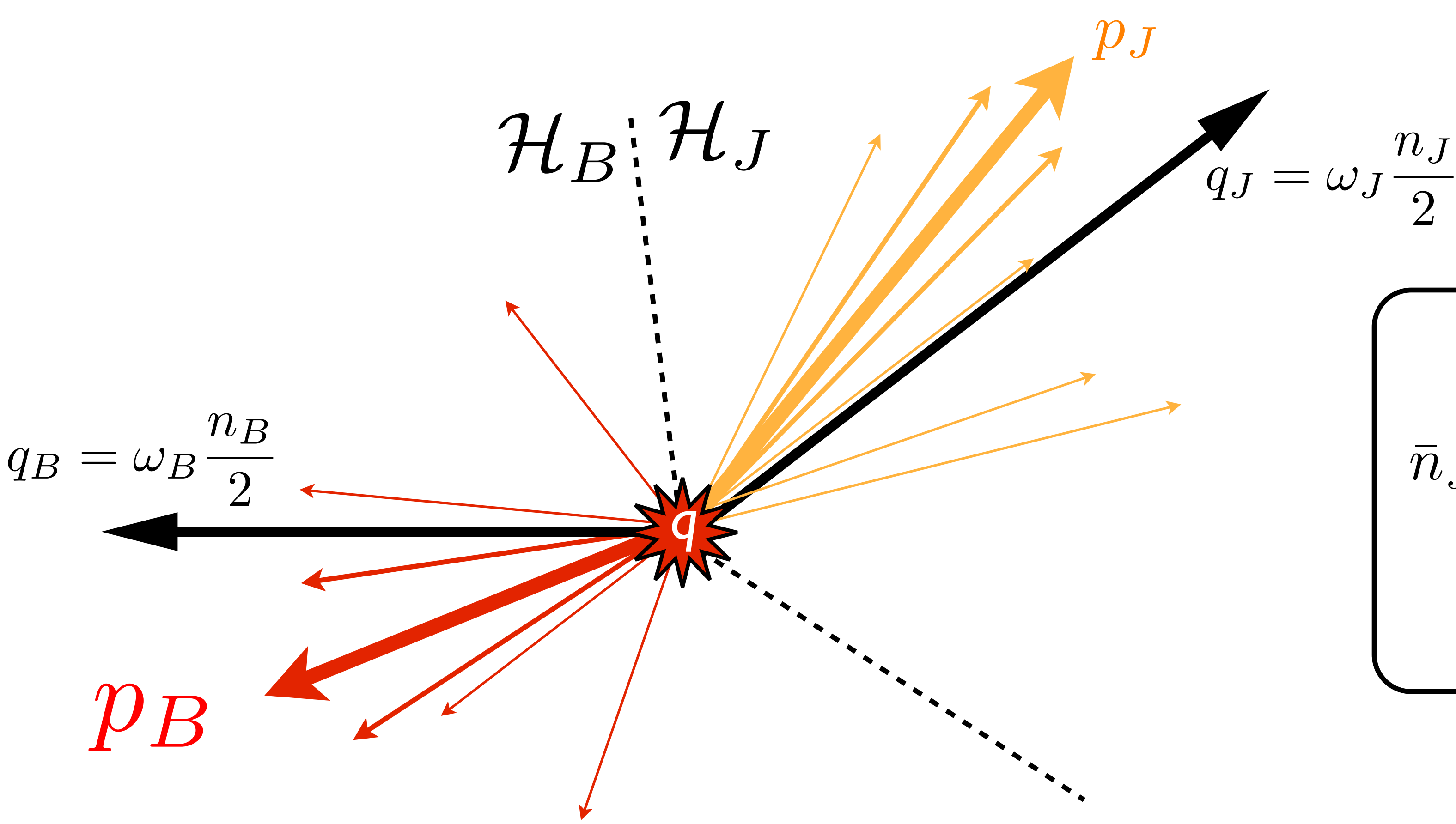
$$q = y\sqrt{s}\frac{n_z}{2} - xy\sqrt{s}\frac{\bar{n}_z}{2} + \sqrt{1-y}\,Q\hat{n}_\perp$$

seemingly simplest definition: *in practice* hardest to calculate!

Restriction: $p_J^\perp$ has to be small for 1-jettiness τ_1^c to be small $\Rightarrow 1 - y \sim \lambda^2$

II. Factorization for Two Jets: Beam and Jet Functions

Light-Cone Directions



Choose conjugate directions:

$$\bar{n}_J = \frac{2}{n_J \cdot n_B} n_B \qquad \bar{n}_B = \frac{2}{n_J \cdot n_B} n_J$$

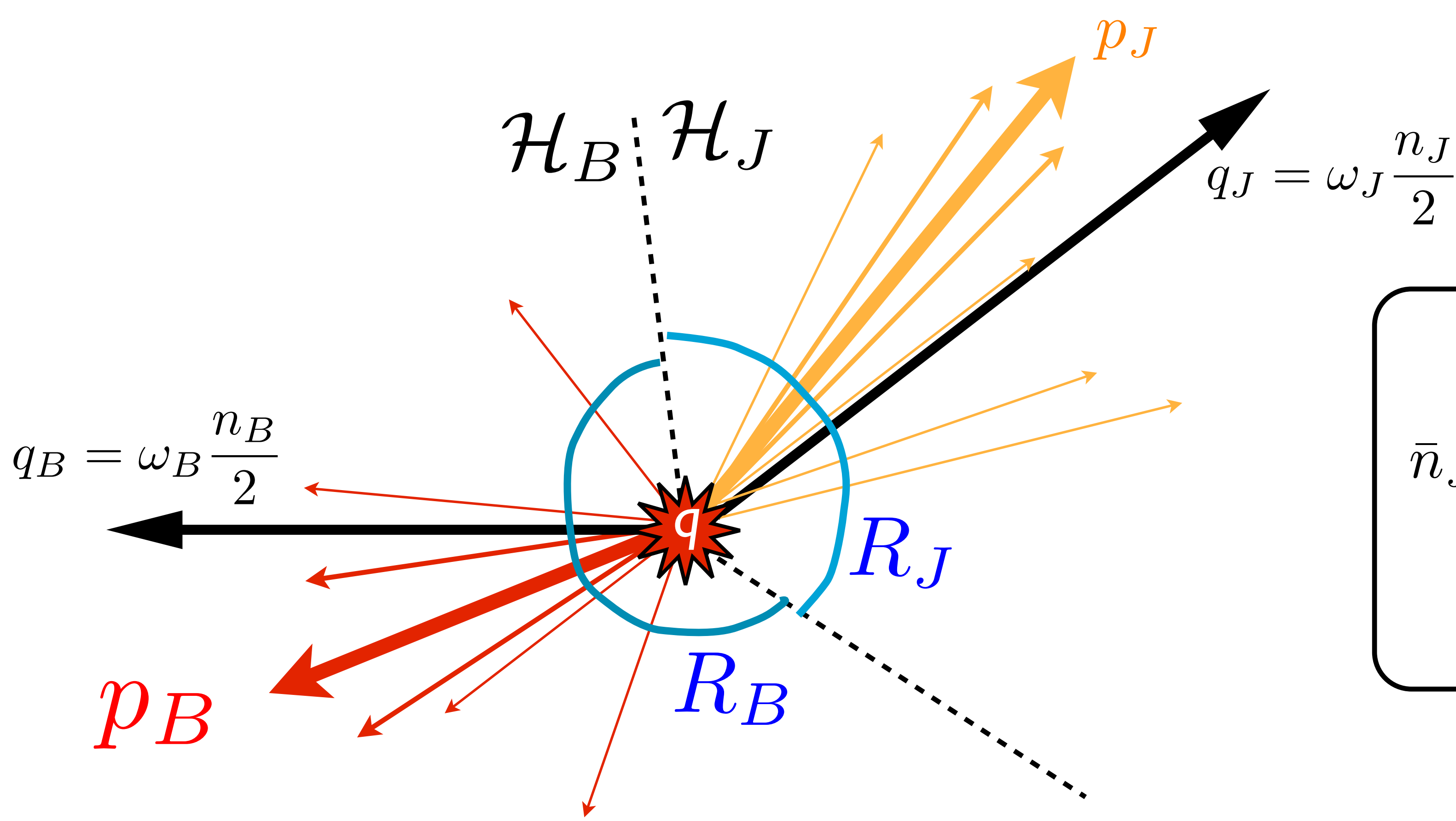
$$\Rightarrow n_J \cdot \bar{n}_J = n_B \cdot \bar{n}_B = 2$$

Sizes of beam and jet regions:

$$\frac{n_J \cdot p}{\bar{n}_J \cdot p} < \frac{\omega_B}{\omega_J} \frac{n_J \cdot n_B}{2} \equiv R_J^2$$

$$\frac{n_B \cdot p}{\bar{n}_B \cdot p} < \frac{\omega_J}{\omega_B} \frac{n_J \cdot n_B}{2} \equiv R_B^2$$

Light-Cone Directions



Choose conjugate directions:

$$\bar{n}_J = \frac{2}{n_J \cdot n_B} n_B \qquad \bar{n}_B = \frac{2}{n_J \cdot n_B} n_J$$

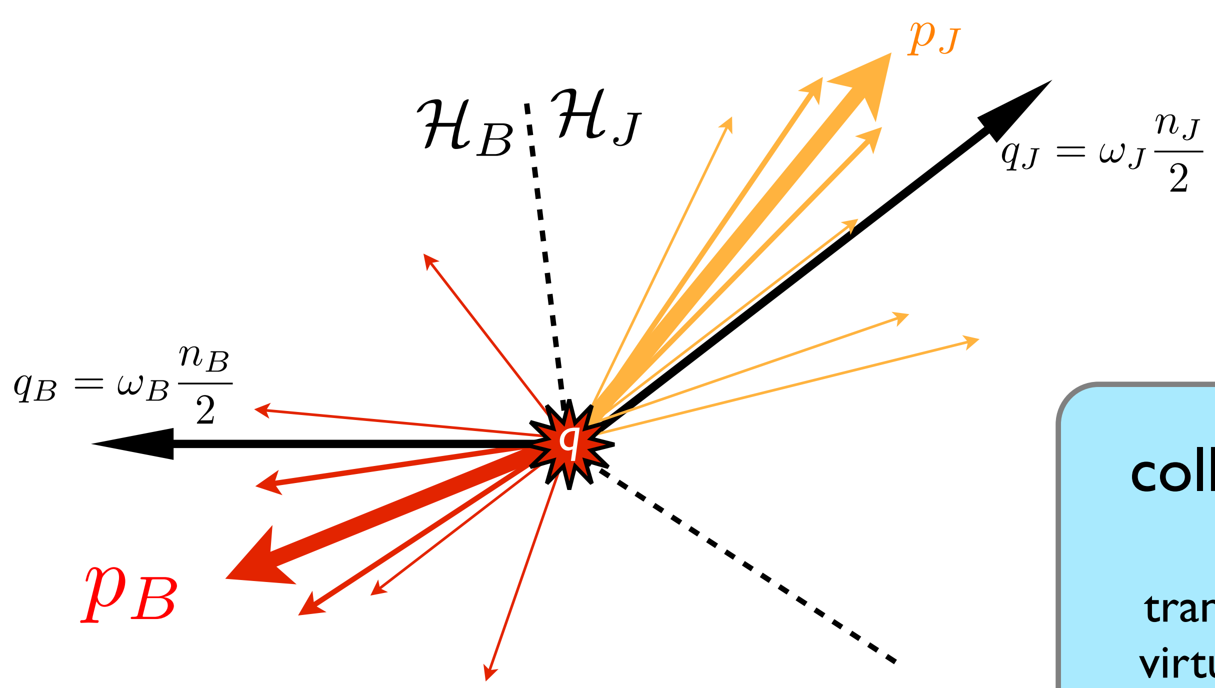
$$\Rightarrow n_J \cdot \bar{n}_J = n_B \cdot \bar{n}_B = 2$$

Sizes of beam and jet regions:

$$\frac{n_J \cdot p}{\bar{n}_J \cdot p} < \frac{\omega_B}{\omega_J} \frac{n_J \cdot n_B}{2} \equiv R_J^2$$

$$\frac{n_B \cdot p}{\bar{n}_B \cdot p} < \frac{\omega_J}{\omega_B} \frac{n_J \cdot n_B}{2} \equiv R_B^2$$

Beam, Jet, and Soft Contributions



In each case of 1-jettiness, $\tau_1 = \frac{n_J \cdot p_J}{Q_J} + \frac{n_B \cdot p_B}{Q_B}$

contributions from different modes: $\tau_1 = \tau_J^{n_J} + \tau_B^{n_B} + \tau_S$

collinear contributions: $\tau_J^{n_J} = \frac{t_J}{s_J}\,, \quad \tau_B^{n_B} = \frac{t_B}{s_B}$

transverse virtualities: $t_J = \bar{n}_J \cdot p_J \, n_J \cdot p_J^{n_J}$ $\quad t_B = \bar{n}_B \cdot p_B \, n_B \cdot p_B^{n_B}$

soft contribution: $\tau_S = \frac{n_J \cdot k_J}{Q_J} + \frac{n_B \cdot k_B}{Q_B} \longrightarrow \frac{n'_J \cdot k_J + n'_B \cdot k_B}{Q_R}$

soft boost invariance: $n_{J,B} \longrightarrow n'_{J,B} = \frac{n_{J,B}}{R_{J,B}}$ $\quad Q_R \equiv \frac{Q_J}{R_J} = \frac{Q_B}{R_B}$

Lorentz invariant constants:

	s_J	s_B	Q_R
τ_1^a	Q^2	Q^2	Q
τ_1^b	Q^2	Q^2	Q
τ_1^c	Q^2	xQ^2	$\sqrt{x}\,Q$

Factorization Theorem for 1-Jettiness

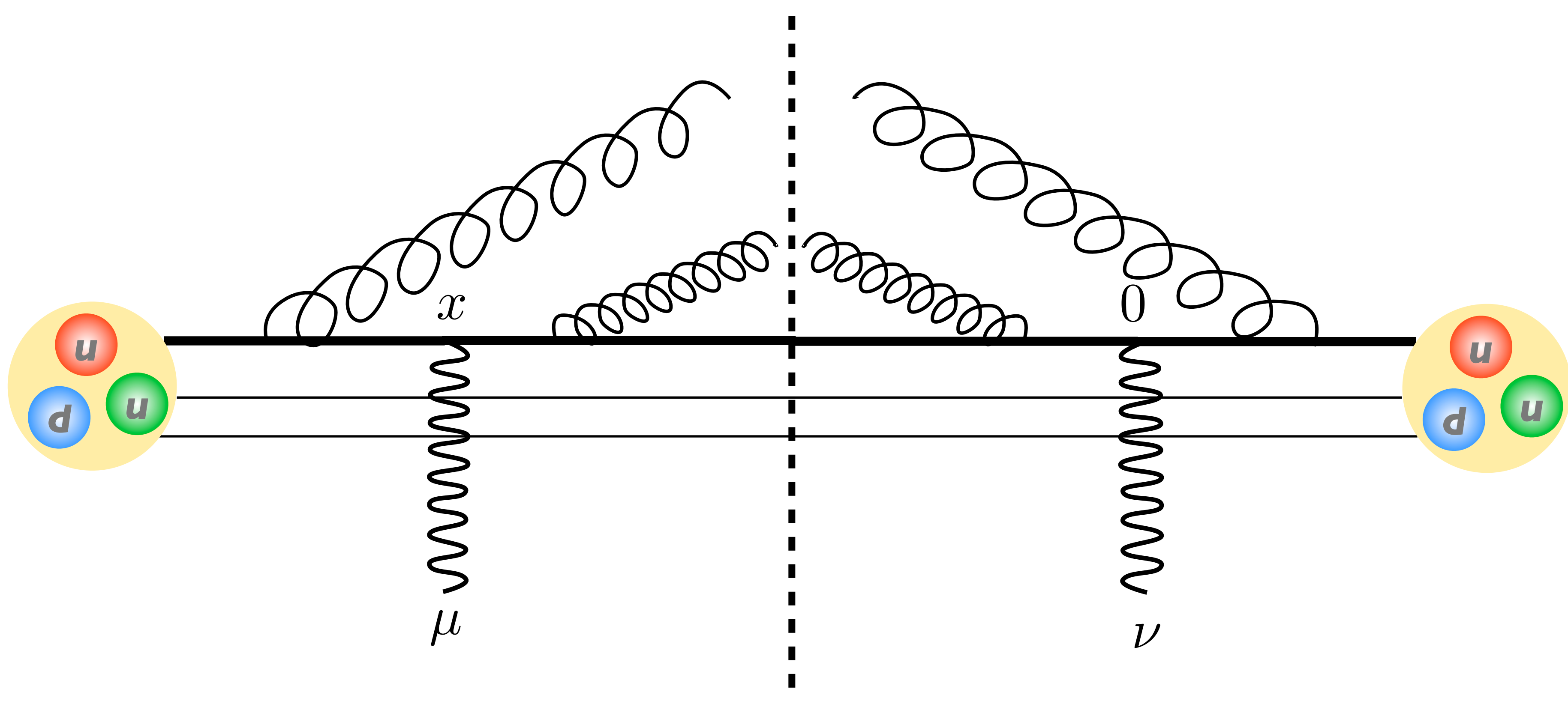
$$\frac{d\sigma(x,Q^2)}{d\tau_1} = L_{\mu\nu}(x,Q^2) W^{\mu\nu}(x,Q^2,\tau_1)$$

leptonic tensor   hadronic tensor

Start in QCD:

$$W^{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x} \langle P|\bar{q}\gamma^\mu q(x)\delta(\tau_1 - \hat{\tau}_1)\bar{q}\gamma^\nu q(0)|P\rangle$$

$$\hat{\tau}_1|X\rangle = \tau_1(X)|X\rangle$$



Measure τ_1 of particles crossing the cut

Factorization Theorem for 1-Jettiness

$$\frac{d\sigma(x,Q^2)}{d\tau_1} = L_{\mu\nu}(x,Q^2)W^{\mu\nu}(x,Q^2,\tau_1)$$

Match onto 2-jet operators in SCET:

$$W_{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x} \sum_{n_1,n_2} \int d^3\tilde{p}_1 d^3\tilde{p}_2 e^{i(\tilde{p}_2-\tilde{p}_1)\cdot x} C_\mu^*(\tilde{p}_1,\tilde{p}_2) C_\mu(\tilde{p}_1,\tilde{p}_2)$$
$$\times \langle P_{n_B}|\bar{\chi}_{n_2,\tilde{p}_2}(x)\overline{T}[Y_{n_2}^\dagger(x)Y_{n_1}(x)]\chi_{n_1,\tilde{p}_1}(x)$$
$$\times\, \delta(\tau_1 - \hat{\tau}_1^{n_1} - \hat{\tau}_1^{n_2} - \tau_1^s)$$
$$\times\, \bar{\chi}_{n_1,\tilde{p}_1}(0)T[Y_{n_1}^\dagger(0)Y_{n_2}(0)]\chi_{n_2,\tilde{p}_2}(0)|P_{n_B}\rangle$$

collinear jet operators in SCET

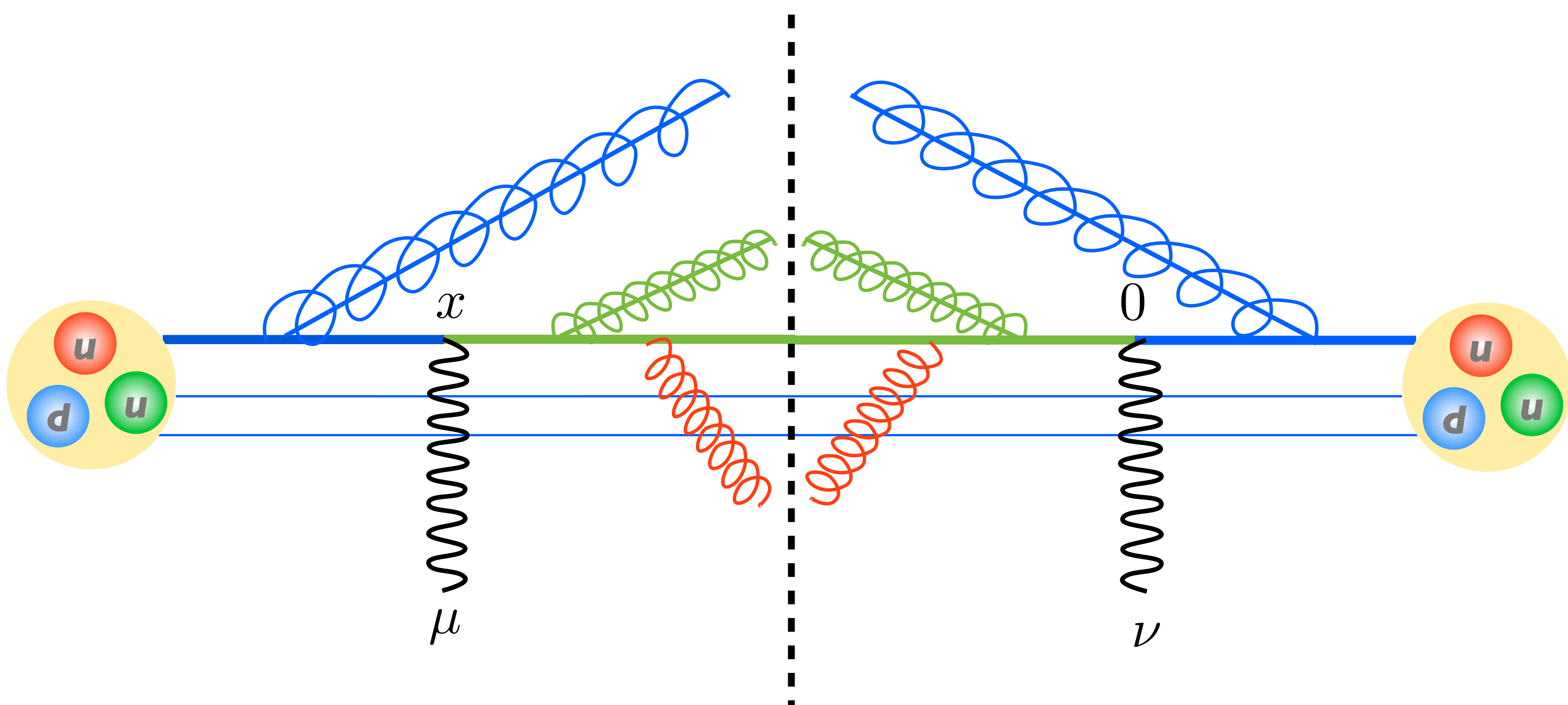
$$\chi_n = [W_n \xi_n]$$

collinear Wilson line

collinear quark field

soft gluon Wilson lines

$$Y_{n_{1,2}}$$



Factorization Theorem for 1-Jettiness

$$\frac{d\sigma(x,Q^2)}{d\tau_1} = L_{\mu\nu}(x,Q^2)W^{\mu\nu}(x,Q^2,\tau_1)$$

Match onto 2-jet operators in SCET:

$$W_{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x} \sum_{n_1,n_2} \int d^3\tilde{p}_1 d^3\tilde{p}_2 e^{i(\tilde{p}_2-\tilde{p}_1)\cdot x} C^*_\mu(\tilde{p}_1,\tilde{p}_2) C_\mu(\tilde{p}_1,\tilde{p}_2)$$
$$\times \langle P_{n_B}|\bar{\chi}_{n_2,\tilde{p}_2}(x)\overline{T}[Y^\dagger_{n_2}(x)Y_{n_1}(x)]\chi_{n_1,\tilde{p}_1}(x)$$
$$\times\, \delta(\tau_1 - \hat{\tau}_1^{n_1} - \hat{\tau}_1^{n_2} - \tau_1^s)$$
$$\times\, \bar{\chi}_{n_1,\tilde{p}_1}(0)T[Y^\dagger_{n_1}(0)Y_{n_2}(0)]\chi_{n_2,\tilde{p}_2}(0)|P_{n_B}\rangle$$

collinear jet operators in SCET

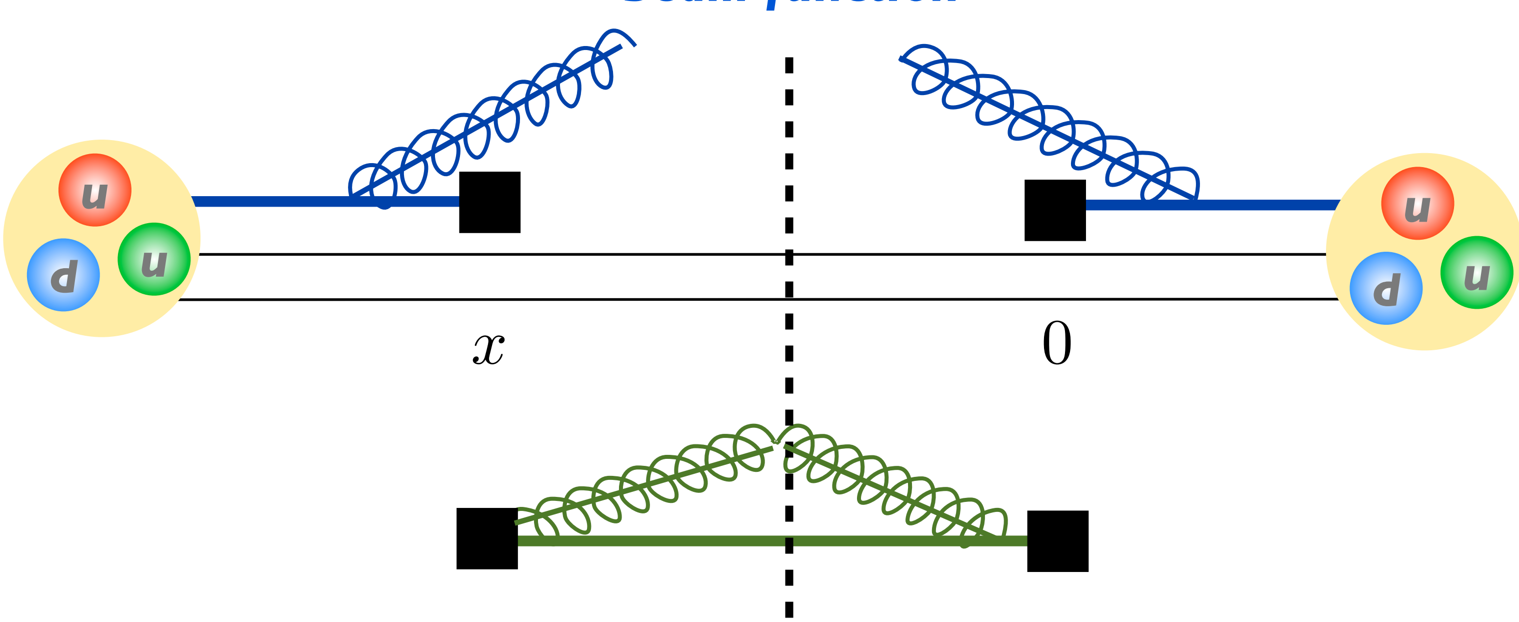
$$\chi_n = [W_n \xi_n]$$

collinear Wilson line   collinear quark field

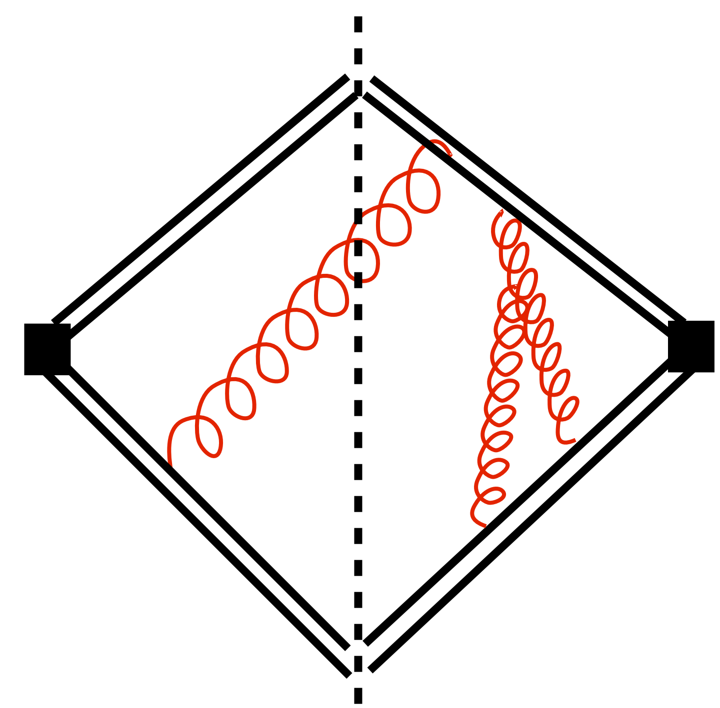
soft gluon Wilson lines

$$Y_{n_{1,2}}$$

“beam function”



“jet function”



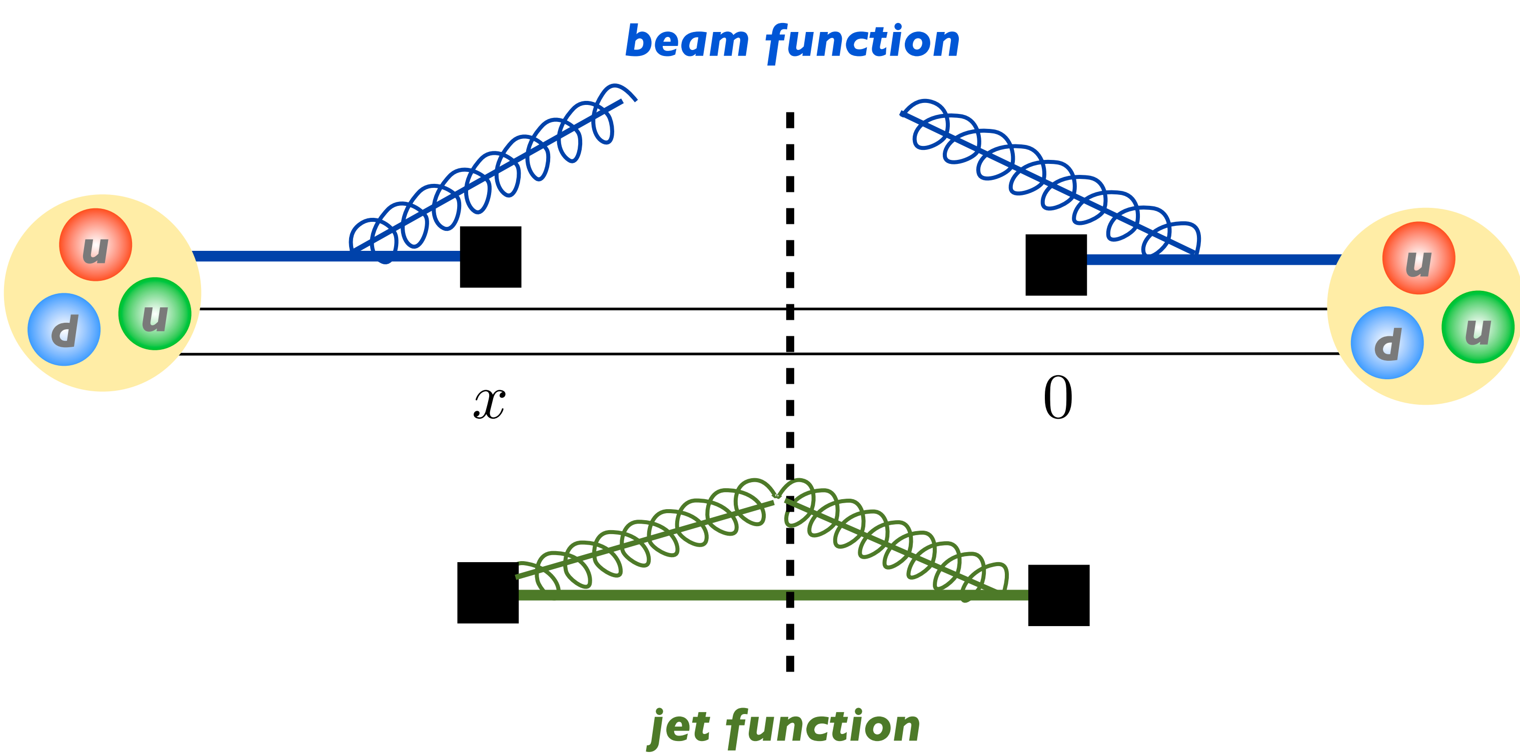
“soft function”

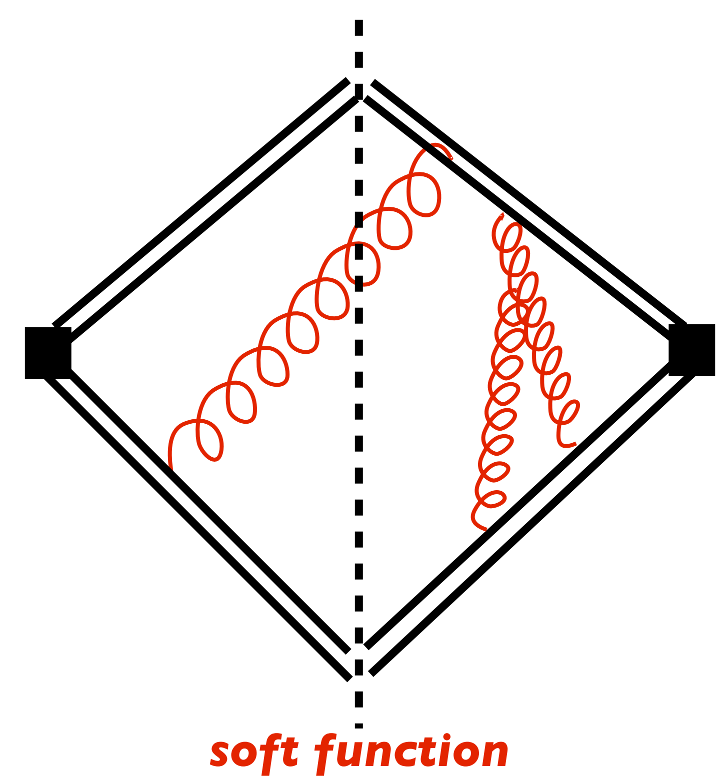
Factorization Theorem for 1-Jettiness

Factor collinear and soft matrix elements:

$$
\begin{aligned}
W_{\mu\nu}(x,Q^2,\tau_1) = \int d^2\tilde{p}_\perp \int d\tau_J d\tau_B d\tau_S\; & C^*(Q^2,\mu)C(Q^2,\mu)\;\delta\Big(\tau_1 - \frac{t_J}{s_J} - \frac{t_B}{s_B} - \frac{k_S}{Q_R}\Big) \\
& \times \langle 0|[Y^\dagger_{n'_J} Y^\dagger_{n'_B}](0)\delta(k_S - n'_J\cdot\hat{p}_{J'} - n'_B\cdot\hat{p}_{B'})[Y_{n'_B} Y_{n'_J}](0)|0\rangle \\
& \times \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\chi_{n_B}](0)|P_{n_B}\rangle \\
& \times \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle
\end{aligned}
$$

(+ permutations)





Factorization Theorem for 1-Jettiness

Factor collinear and soft matrix elements:

hard function

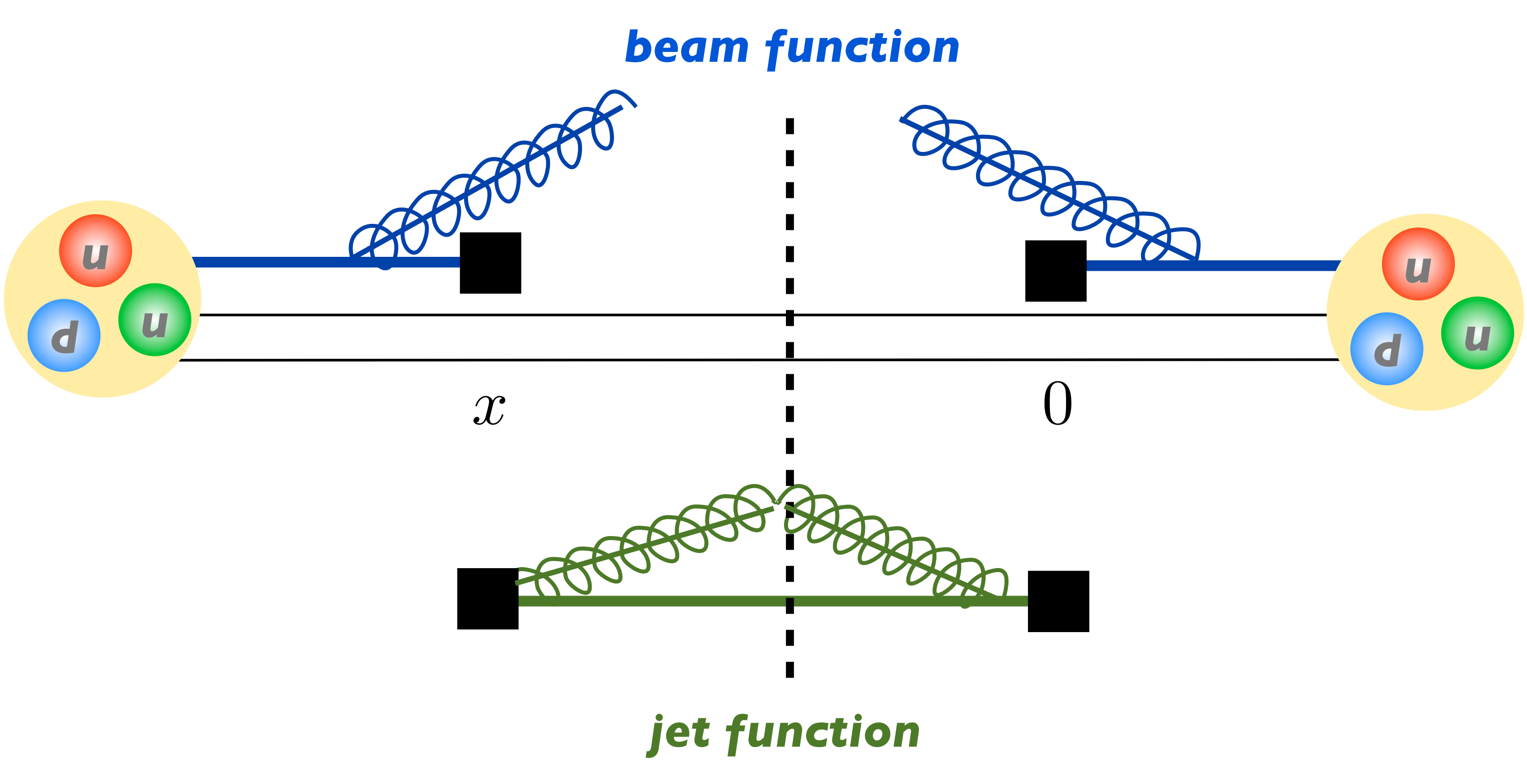
$$W_{\mu\nu}(x, Q^2, \tau_1) = \int d^2\tilde{p}_\perp \int d\tau_J d\tau_B d\tau_S \, \boxed{C^*(Q^2,\mu) C(Q^2,\mu)} \, \delta\Big(\tau_1 - \frac{t_J}{s_J} - \frac{t_B}{s_B} - \frac{k_S}{Q_R}\Big)$$

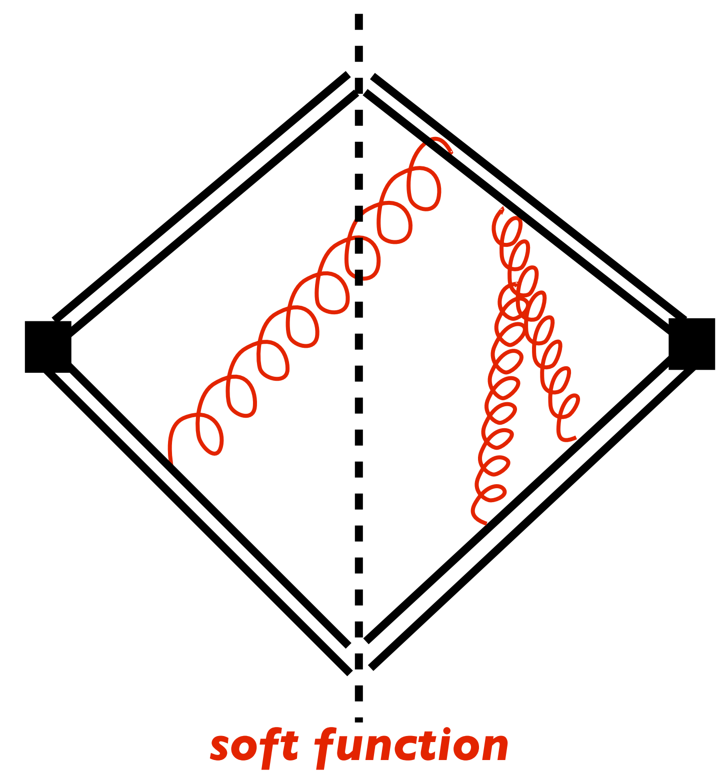
soft function $\quad \times \langle 0 | [Y^\dagger_{n'_J} Y^\dagger_{n'_B}](0) \delta(k_S - n'_J \cdot \hat{p}_{J'} - n'_B \cdot \hat{p}_{B'}) [Y_{n'_B} Y_{n'_J}](0) | 0 \rangle$

beam function $\quad \times \langle P_{n_B} | \bar{\chi}_{n_B}(0) \delta(Q_B \tau_B - n_B \cdot \hat{p}^{n_B}) [\delta(\bar{n}_B \cdot q + \bar{n}_B \cdot \mathcal{P}) \delta^2(\tilde{p}_\perp - \mathcal{P}_\perp) \chi_{n_B}](0) | P_{n_B} \rangle$

jet function $\quad \times \langle 0 | \chi_{n_J}(0) \delta(Q_J \tau_J - n_J \cdot \hat{p}^{n_J}) \delta(\bar{n}_J \cdot q + \bar{n}_J \cdot \mathcal{P}) \delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp) \bar{\chi}_{n_J}(0) | 0 \rangle$

(+ permutations)

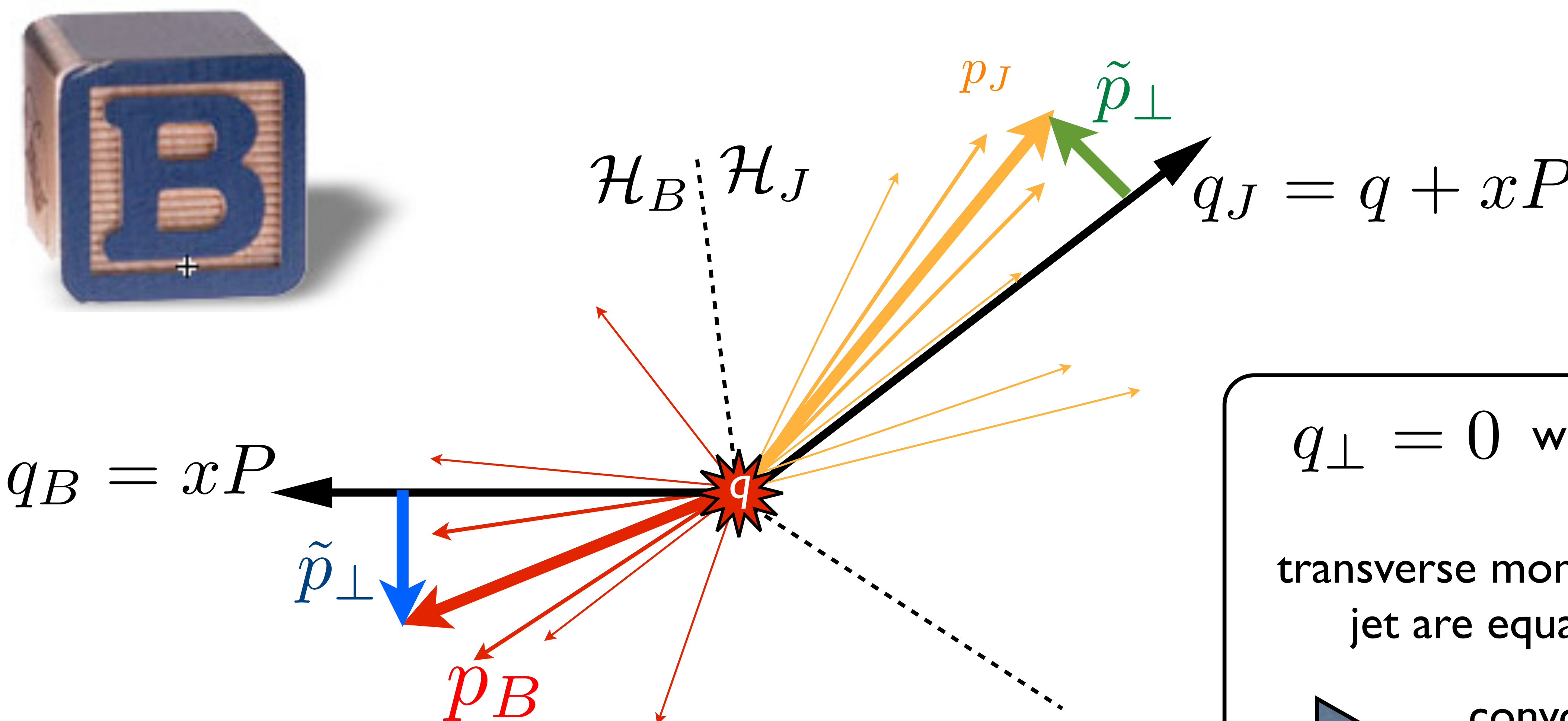




Transverse jet and beam momenta

Convolution between jet and beam transverse momenta:

$$W \supset \int d^2\tilde{p}_\perp \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\chi_{n_B}](0)|P_{n_B}\rangle$$
$$\times \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle$$



$q_\perp = 0$ with respect to this $\boldsymbol{q_J}$ and $\boldsymbol{q_B}$

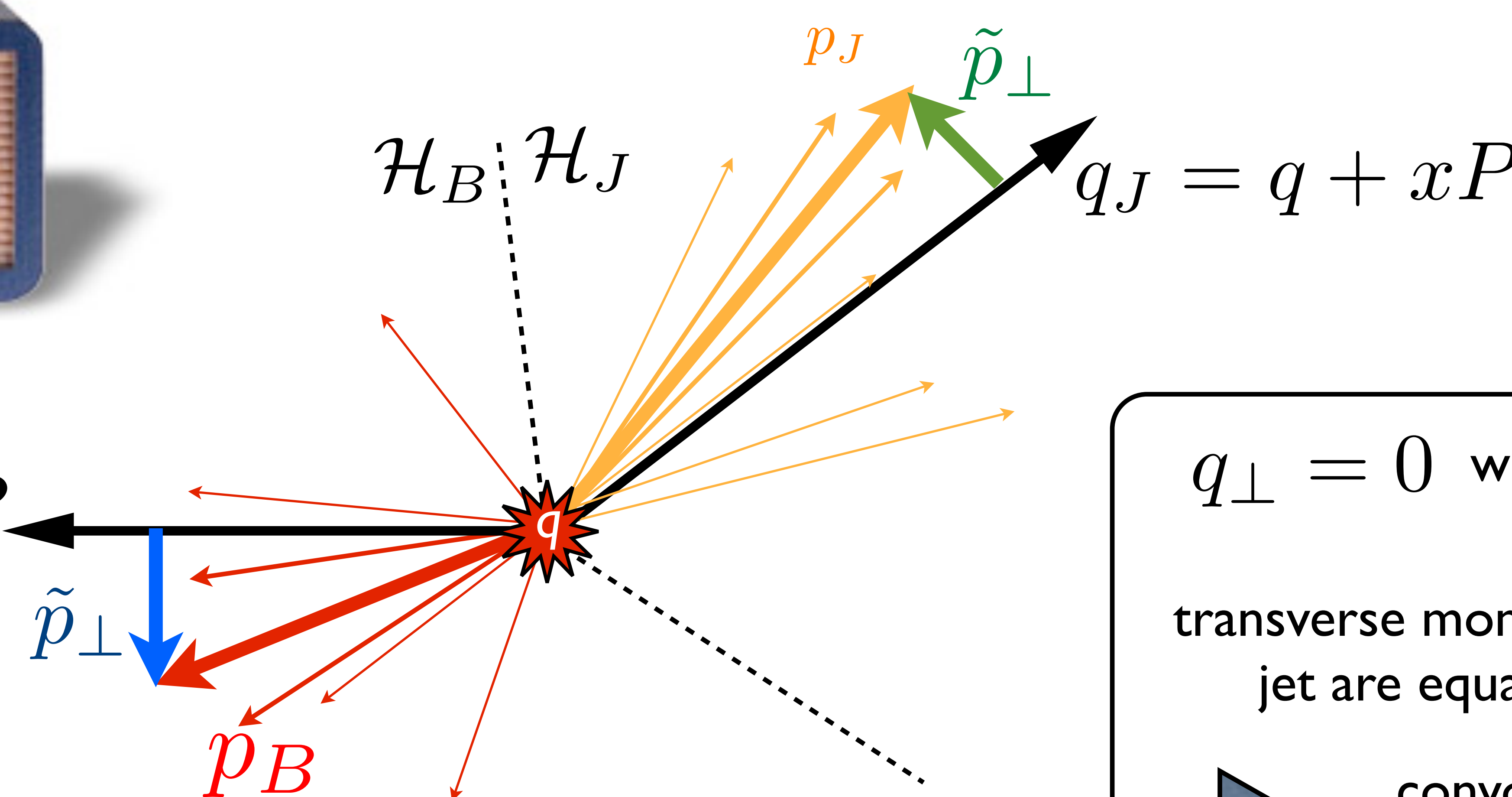
transverse momenta of beam and jet are equal and balanced

→ convolution over $\tilde{p}_\perp$ remains, depends on kT-dependent beam function (perturbative kT)

Transverse jet and beam momenta

Convolution between jet and beam transverse momenta:

$$W \supset \int d^2\tilde{p}_\perp \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\chi_{n_B}](0)|P_{n_B}\rangle$$
$$\times\,\langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(\cancel{q_\perp} + \tilde{p}_\perp + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle$$



$q_\perp = 0$ with respect to this $\boldsymbol{q_J}$ and $\boldsymbol{q_B}$

transverse momenta of beam and jet are equal and balanced

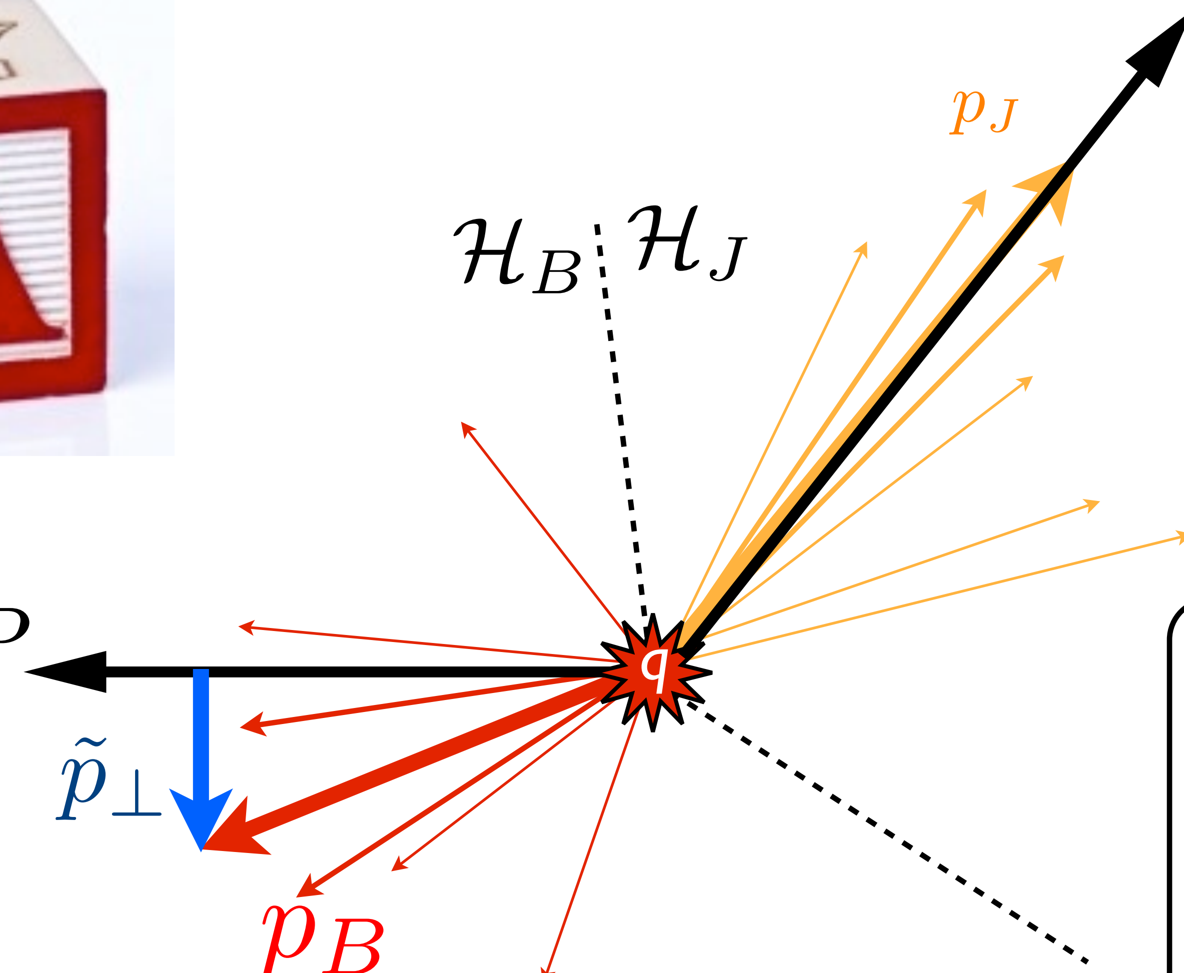
convolution over $\tilde{p}_\perp$ remains, depends on kT-dependent beam function (perturbative kT)

Transverse jet and beam momenta

Convolution between jet and beam transverse momenta:

$$W \supset \boxed{\int d^2\tilde{p}_\perp} \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\boxed{\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)}\chi_{n_B}](0)|P_{n_B}\rangle$$

$$\times\, \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\boxed{\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)}\bar{\chi}_{n_J}(0)|0\rangle$$



$q_J = p_J + \mathcal{O}(Q\lambda^2)$

$\Rightarrow n_J$ aligned so that transverse label momentum on χ_{n_J} is zero

jet function independent of beam $\tilde{p}_\perp$

integral averages over beam $\tilde{p}_\perp$

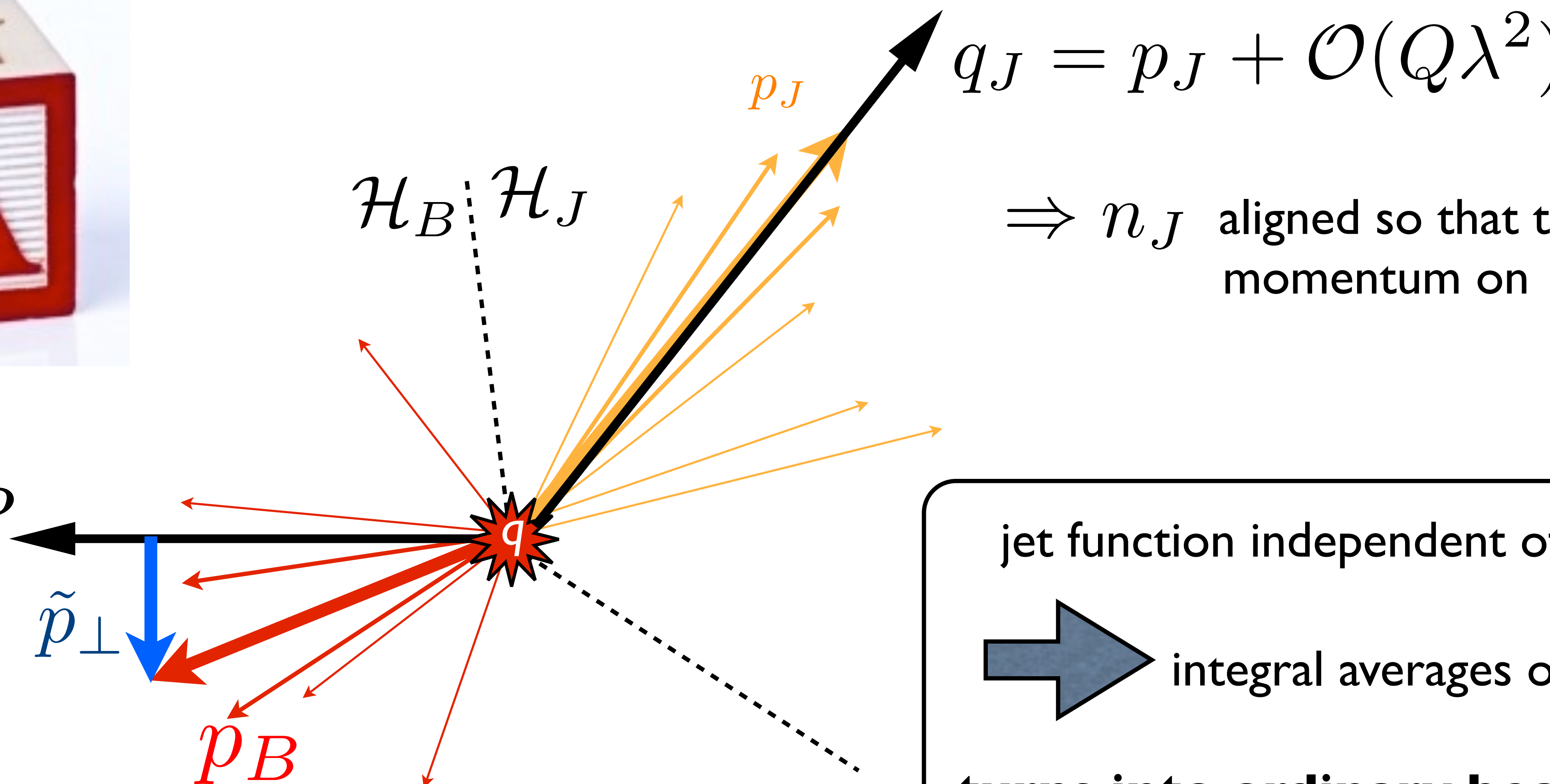
turns into ordinary beam function

difference between q_J axes for case A and B is a leading-order effect on the argument of beam and jet functions

Transverse jet and beam momenta

Convolution between jet and beam transverse momenta:

$$W \supset \int d^2\tilde{p}_\perp \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\chi_{n_B}](0)|P_{n_B}\rangle$$
$$\times \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(\cancel{q_\perp + \tilde{p}_\perp} + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle$$



$\Rightarrow n_J$ aligned so that transverse label momentum on χ_{n_J} is zero

jet function independent of beam $\tilde{p}_\perp$

integral averages over beam $\tilde{p}_\perp$

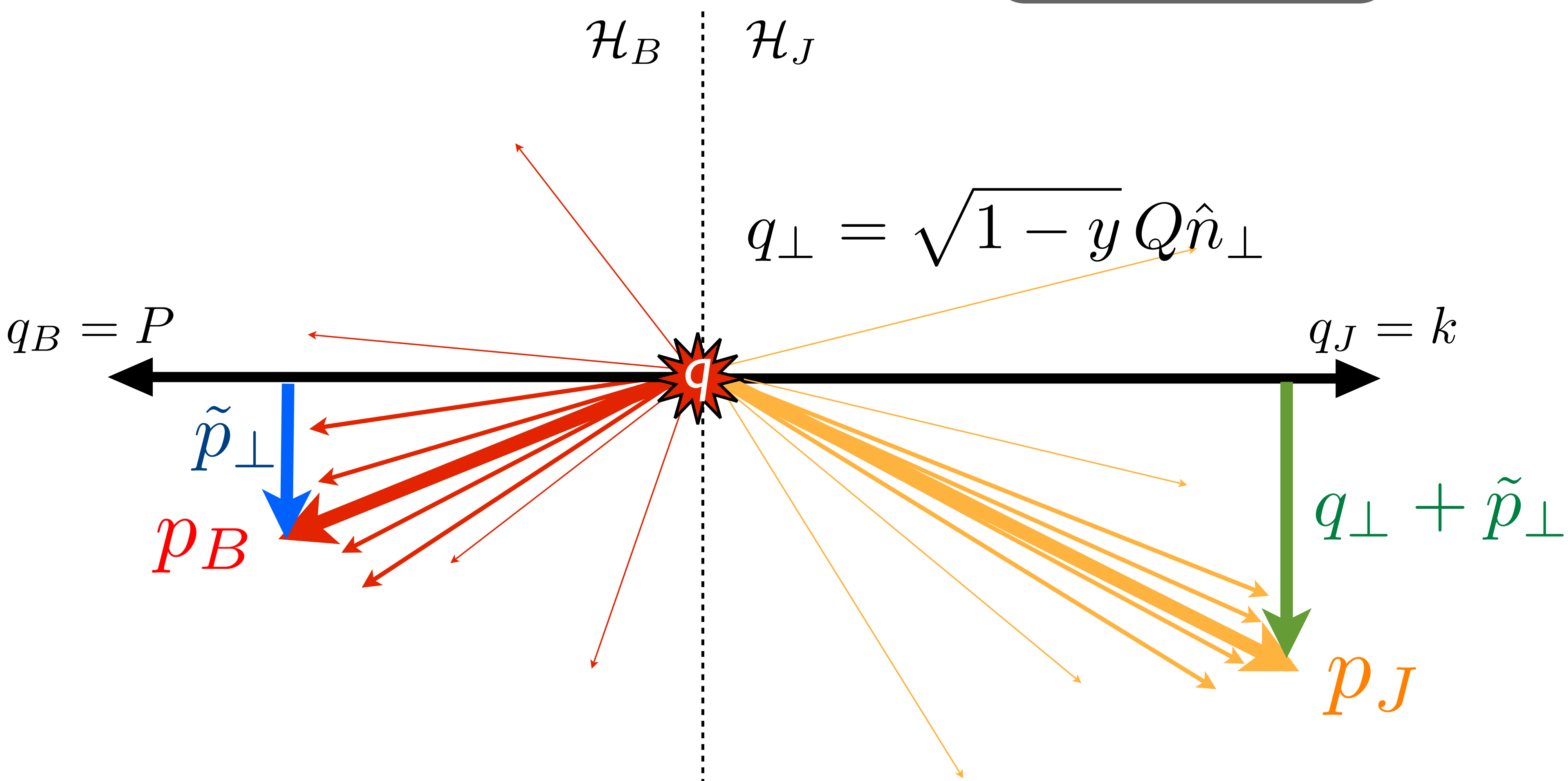
turns into ordinary beam function

difference between q_J axes for case A and B is a leading-order effect on the argument of beam and jet functions

Transverse jet and beam momenta

Convolution between jet and beam transverse momenta:

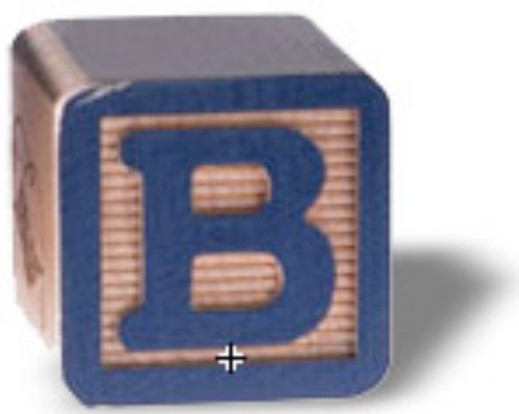
$$W \supset \boxed{\int d^2\tilde{p}_\perp} \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\boxed{\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)}\chi_{n_B}](0)|P_{n_B}\rangle$$
$$\times \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\boxed{\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)}\bar{\chi}_{n_J}(0)|0\rangle$$



momentum transfer ***q*** itself has nonzero transverse component relative to ***P, k*** → nontrivial convolution between jet function and pT-dependent beam function

Factorization Theorems for 1-Jettiness

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\Big(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)$$
$$\times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$

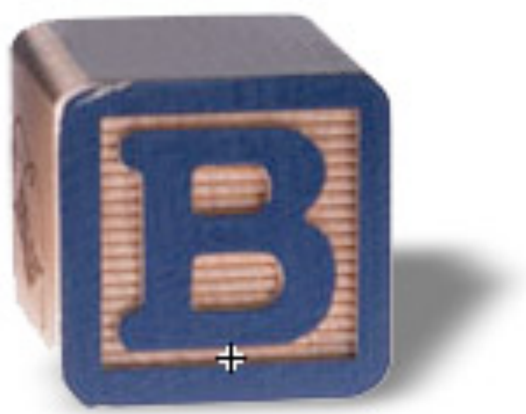


$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\Big(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)$$
$$\times J_q(t_J - \mathbf{p}_\perp^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^c} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\Big(\tau_1^c - \frac{t_J}{Q^2} - \frac{t_B}{xQ^2} - \frac{k_S}{\sqrt{x}Q}\Big)$$
$$\times J_q(t_J - (\mathbf{q}_\perp + \mathbf{p}_\perp)^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

Factorization Theorems for 1-Jettiness

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\Big(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big) \times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$

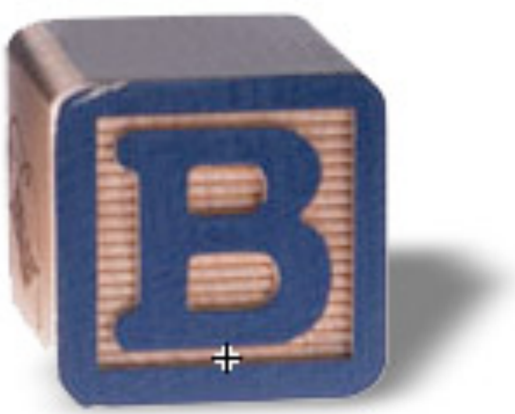


$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\Big(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big) \times J_q(t_J - \mathbf{p}_\perp^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^c} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\Big(\tau_1^c - \frac{t_J}{Q^2} - \frac{t_B}{xQ^2} - \frac{k_S}{\sqrt{x}Q}\Big) \times J_q(t_J - (\mathbf{q}_\perp + \mathbf{p}_\perp)^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

Factorization Theorems for 1-Jettiness

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\Big(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big) \times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$

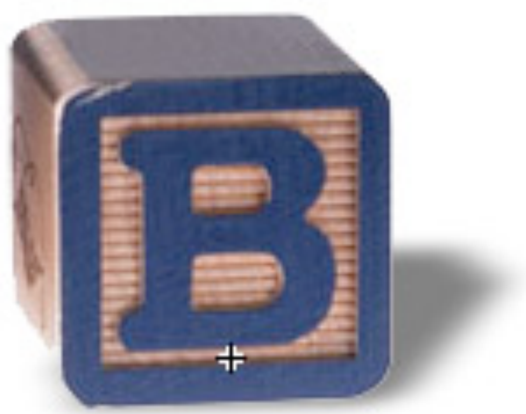


$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\Big(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big) \times J_q(t_J - \mathbf{p}_\perp^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^c} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\Big(\tau_1^c - \frac{t_J}{Q^2} - \frac{t_B}{xQ^2} - \frac{k_S}{\sqrt{x}Q}\Big) \times J_q(t_J - (\mathbf{q}_\perp + \mathbf{p}_\perp)^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

Factorization Theorems for 1-Jettiness

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\left(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right) \times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$



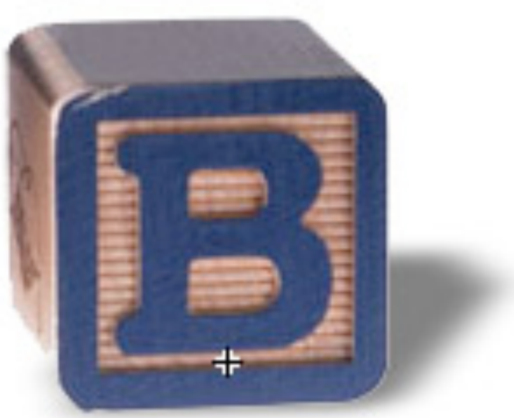
$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\left(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right) \times J_q(t_J - \mathbf{p}_\perp^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^c} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\left(\tau_1^c - \frac{t_J}{Q^2} - \frac{t_B}{xQ^2} - \frac{k_S}{\sqrt{x}Q}\right) \times J_q(t_J - (\mathbf{q}_\perp + \mathbf{p}_\perp)^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

Differences between versions A and B

$$\boxed{n_J^A = n_J^B + \mathcal{O}(\lambda)}$$

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\Big(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big) \times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$



$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\Big(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big) \times J_q(t_J - \mathbf{p}_\perp^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

Differences between versions A and B

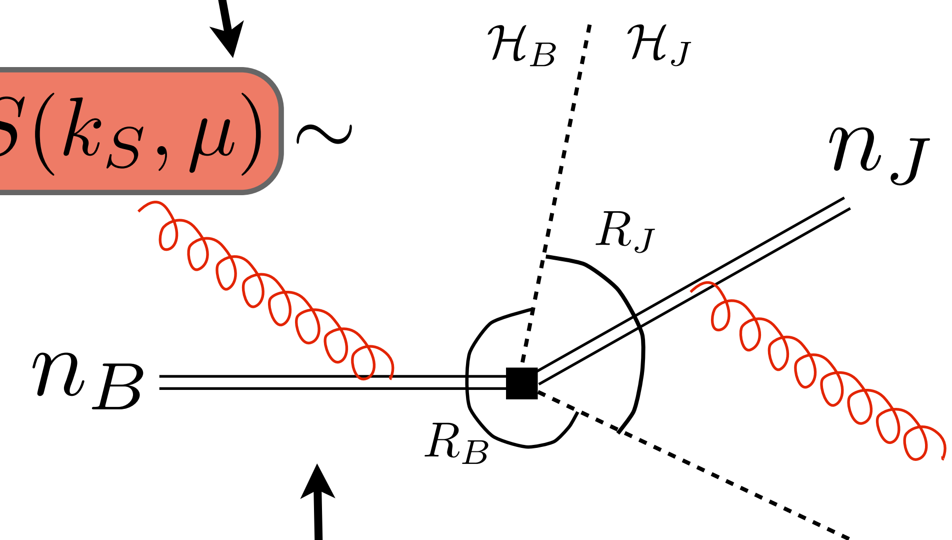
$$\boxed{n_J^A = n_J^B + \mathcal{O}(\lambda)}$$

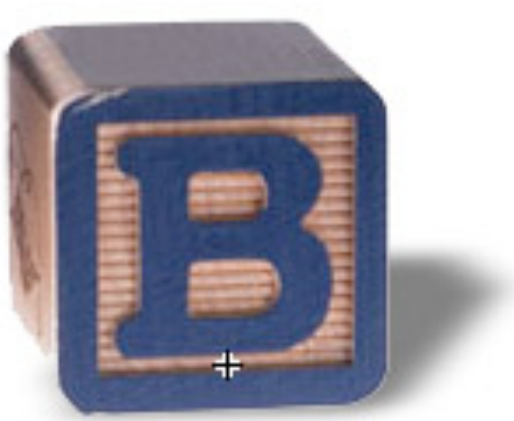
$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J\, dt_B\, dk_S\, \delta\Big(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)$$
$$\times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$

Differences **power suppressed:**

$$H(Q^2,\mu) = |C(Q^2,\mu)|^2\, \mathrm{Tr}\Big(\Gamma \frac{\not n_J}{4}\Gamma' \frac{\not n_B}{4}\Big)$$

$$S(k_S,\mu) \sim$$





$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp\, dt_J\, dt_B\, dk_S\, \delta\Big(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)$$
$$\times J_q(t_J - \mathbf{p}_\perp^2,\mu)\mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

Differences between versions A and B

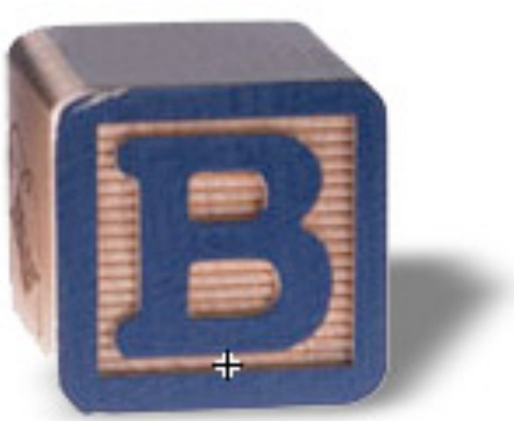
$$\boxed{n_J^A = n_J^B + \mathcal{O}(\lambda)}$$

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\left(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right)$$
$$\times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$

Differences
leading order:

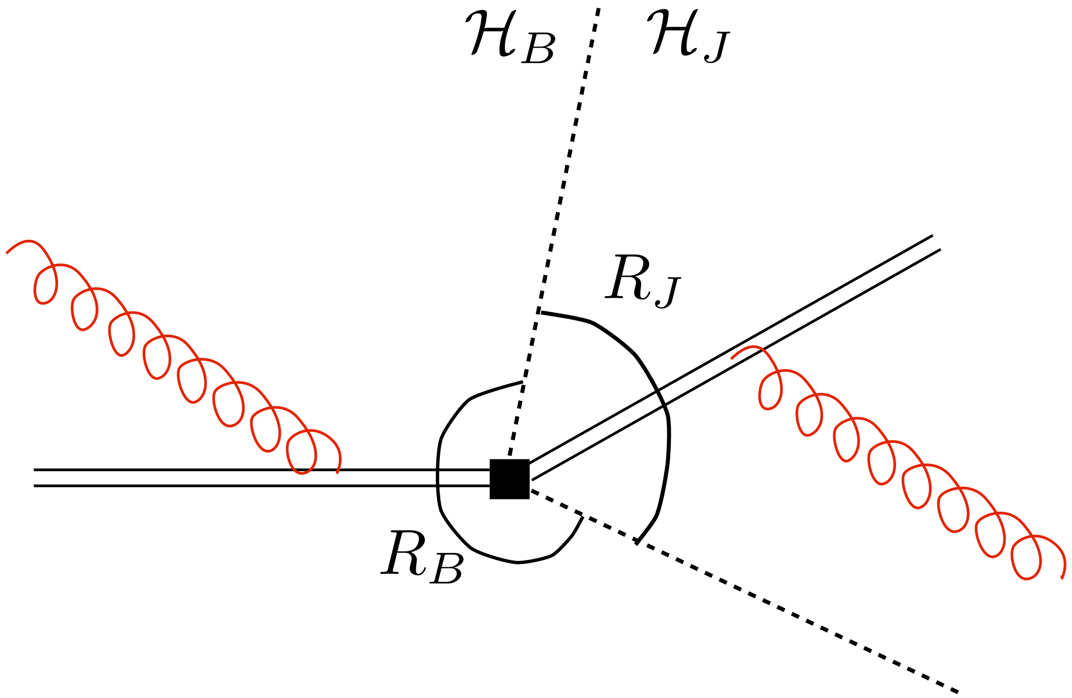
argument of jet function

type of beam function

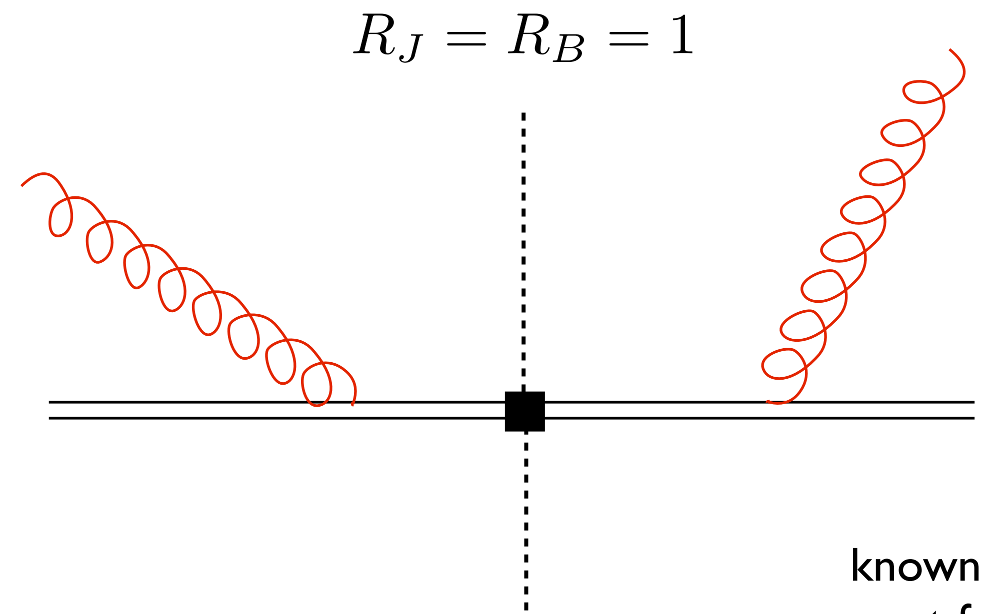


$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\left(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right)$$
$$\times J_q(t_J - \mathbf{p}_\perp^2,\mu) \mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

Boost to Hemisphere Soft Function



1-jettiness soft function



hemisphere soft function

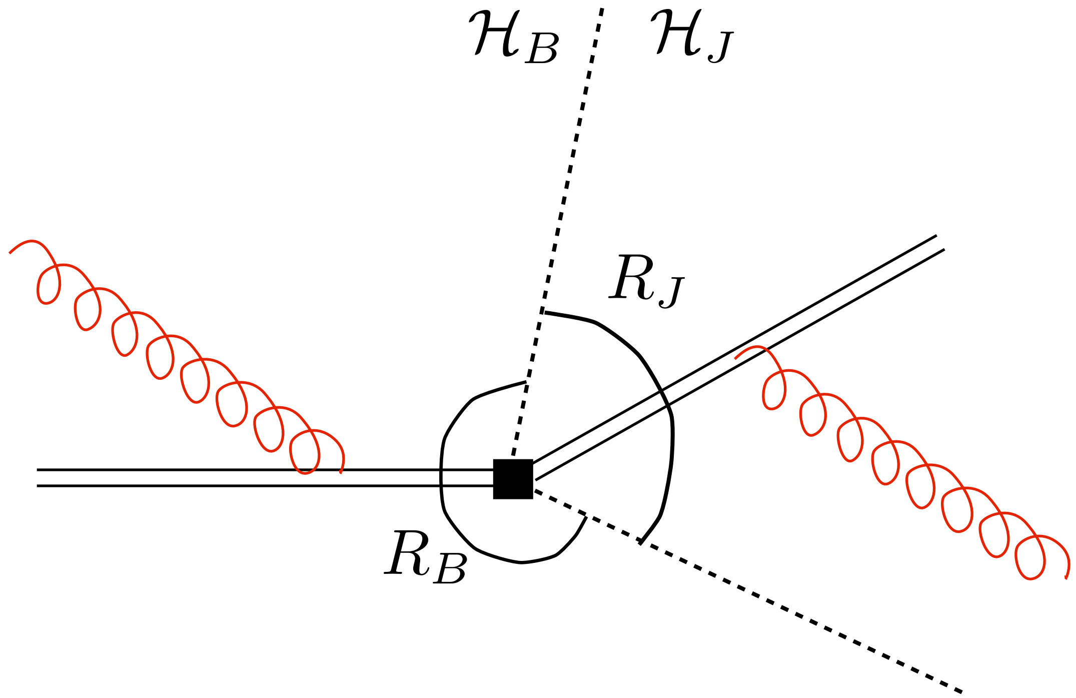
known to 2 loops except for constant

anomalous dimension known to 3 loops

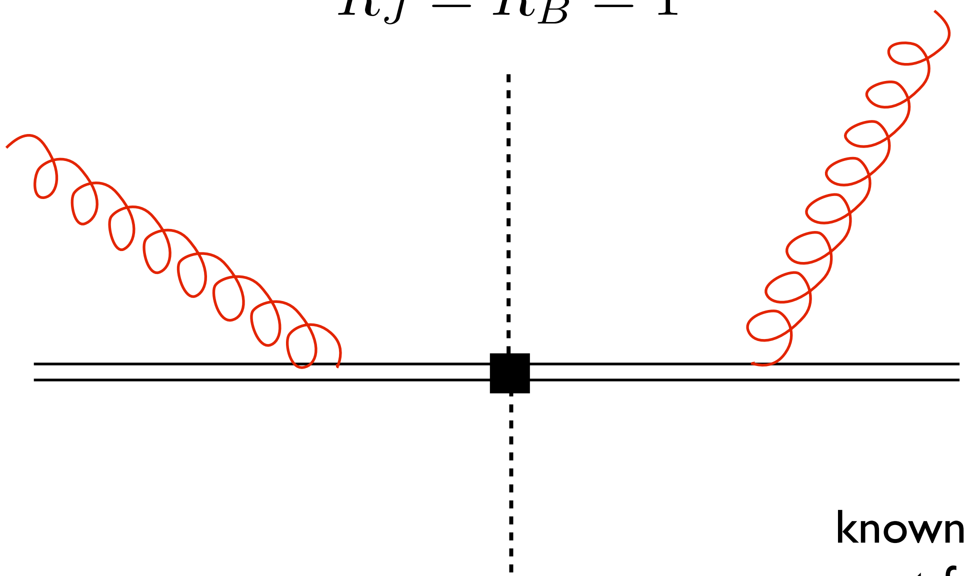
$$S(k_J, k_B, q_J, q_B, \mu) = \frac{1}{N_C}\,\mathrm{tr}\sum_{X_s}\left|\langle X_s|\,T[Y_{n_B}^{\dagger}Y_{n_J}](0)\,|0\rangle\right|^2 \delta\Big(k_J - \sum_{i\in X_s}\theta(q_B\cdot k_i - q_J\cdot k_i)n_J\cdot k_i\Big)$$

$$\times\,\delta\Big(k_B - \sum_{i\in X_s}\theta(q_J\cdot k_i - q_B\cdot k_i)n_B\cdot k_i\Big)$$

Boost to Hemisphere Soft Function



1-jettiness soft function



hemisphere soft function

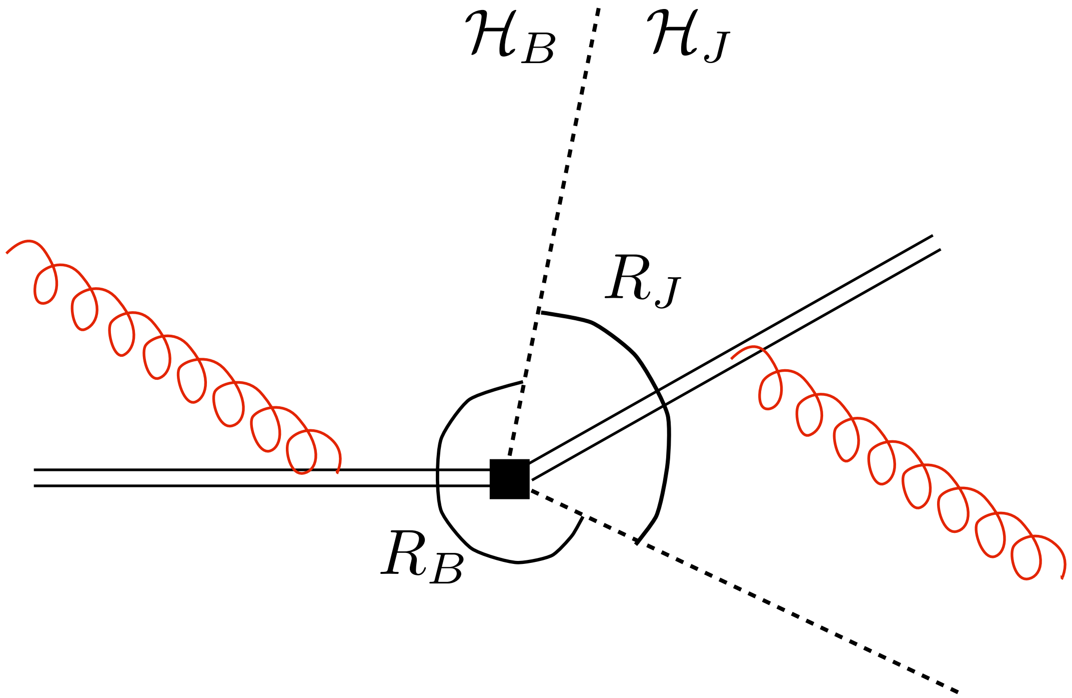
known to 2 loops except for constant

anomalous dimension known to 3 loops

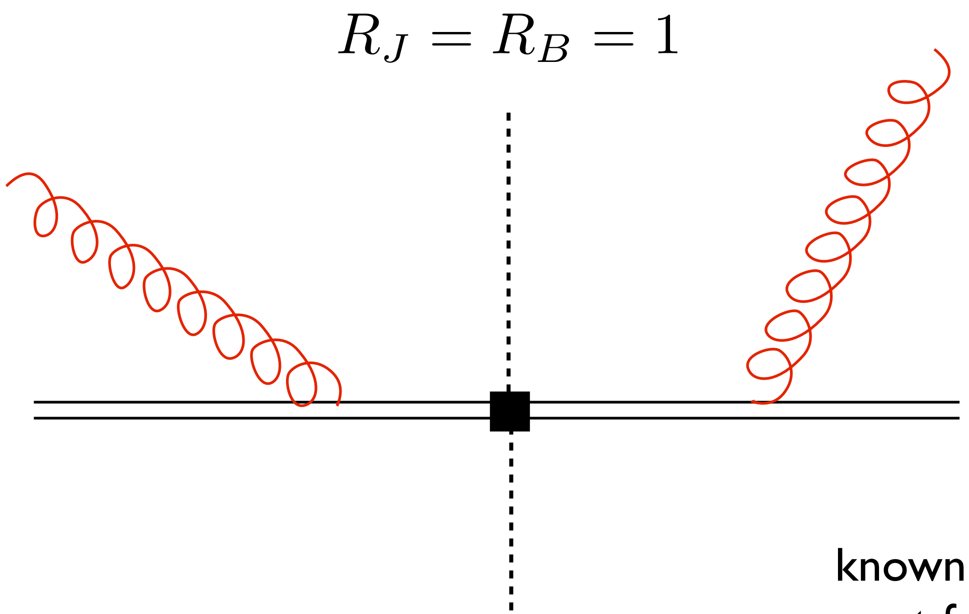
$$S(k_J, k_B, q_J, q_B, \mu) = \frac{1}{N_C}\,\mathrm{tr}\sum_{X_s}\left|\langle X_s|\,T[Y^\dagger_{n_B} Y_{n_J}](0)\,|0\rangle\right|^2 \delta\Big(k_J - \sum_{i\in X_s}\theta(q_B\cdot k_i - q_J\cdot k_i)\,n_J\cdot k_i\Big)$$
$$\times\,\delta\Big(k_B - \sum_{i\in X_s}\theta(q_J\cdot k_i - q_B\cdot k_i)\,n_B\cdot k_i\Big)$$

$$n_J \to n'_J = \frac{n_J}{R_J} \qquad R_J = \sqrt{\frac{\omega_B n_J\cdot n_B}{2\omega_J}}$$
$$n_B \to n'_B = \frac{n_B}{R_B} \qquad R_B = \sqrt{\frac{\omega_J n_J\cdot n_B}{2\omega_B}}$$

Boost to Hemisphere Soft Function



1-jettiness soft function



hemisphere soft function

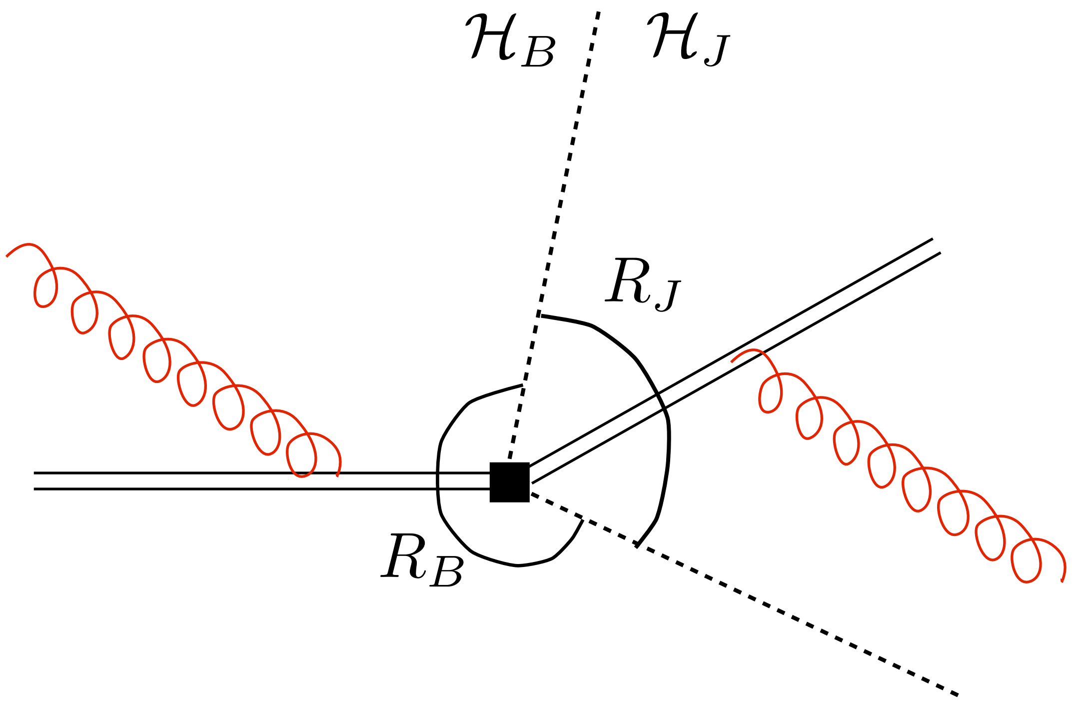
known to 2 loops except for constant

anomalous dimension known to 3 loops

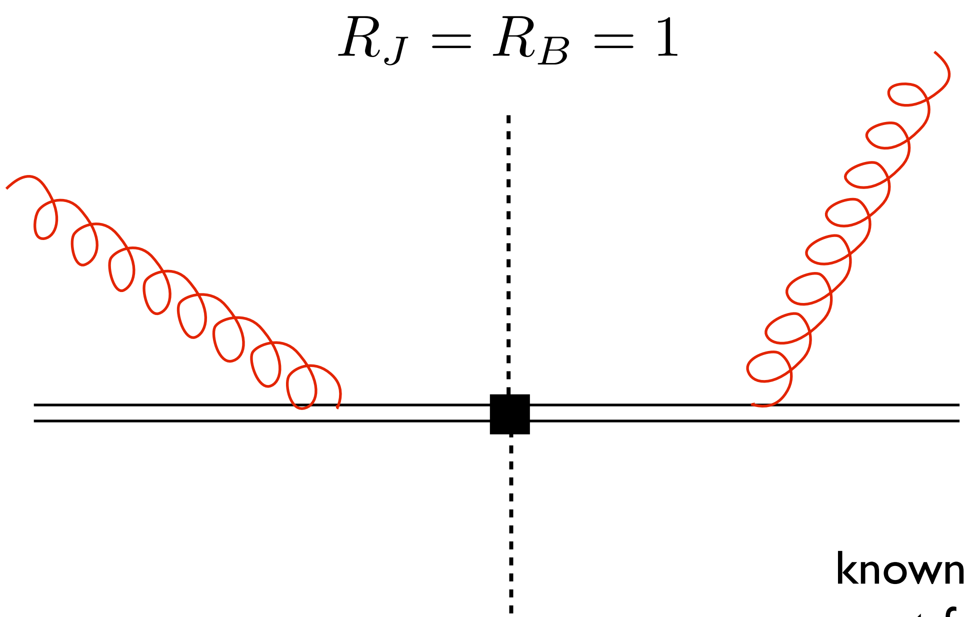
$$S(k_J, k_B, q_J, q_B, \mu) = \frac{1}{N_C R_J R_B} \operatorname{tr} \sum_{X_s} \left| \langle X_s | \, T[Y^\dagger_{n'_B} Y_{n'_J}](0) \, |0\rangle \right|^2 \delta\Big(\frac{k_J}{R_J} - \sum_{i \in X_s} \theta(n'_B \cdot k_i - n'_J \cdot k_i) n'_J \cdot k_i \Big)$$
$$\times \, \delta\Big(\frac{k_B}{R_B} - \sum_{i \in X_s} \theta(n'_J \cdot k_i - n'_B \cdot k_i) n'_B \cdot k_i \Big) = \frac{1}{R_J R_B} S_{\text{hemi}}\Big(\frac{k_J}{R_J}, \frac{k_B}{R_B}, \mu \Big) \, .$$

$$n_J \to n'_J = \frac{n_J}{R_J} \qquad R_J = \sqrt{\frac{\omega_B n_J \cdot n_B}{2\omega_J}}$$
$$n_B \to n'_B = \frac{n_B}{R_B} \qquad R_B = \sqrt{\frac{\omega_J n_J \cdot n_B}{2\omega_B}}$$

Boost to Hemisphere Soft Function



1-jettiness soft function



hemisphere soft function

known to 2 loops except for constant

anomalous dimension known to 3 loops

$$S(k_J, k_B, q_J, q_B, \mu) = \frac{1}{N_C R_J R_B} \operatorname{tr} \sum_{X_s} \Big| \langle X_s | \, T[Y^\dagger_{n'_B} Y_{n'_J}](0) \, |0\rangle \Big|^2 \, \delta\Big(\frac{k_J}{R_J} - \sum_{i\in X_s} \theta(n'_B\cdot k_i - n'_J\cdot k_i) n'_J\cdot k_i\Big)$$
$$\times \, \delta\Big(\frac{k_B}{R_B} - \sum_{i\in X_s} \theta(n'_J\cdot k_i - n'_B\cdot k_i) n'_B\cdot k_i\Big) = \frac{1}{R_J R_B} S_{\rm hemi}\Big(\frac{k_J}{R_J}, \frac{k_B}{R_B}, \mu\Big) \,.$$

$$n_J \to n'_J = \frac{n_J}{R_J} \qquad R_J = \sqrt{\frac{\omega_B n_J\cdot n_B}{2\omega_J}}$$
$$n_B \to n'_B = \frac{n_B}{R_B} \qquad R_B = \sqrt{\frac{\omega_J n_J\cdot n_B}{2\omega_B}}$$

$$S(k_J, k_B, R_J, R_B, \mu) = \frac{1}{R_J R_B} S_{\rm hemi}\Big(\frac{k_J}{R_J}, \frac{k_B}{R_B}, \mu\Big)$$

cf. Feige, Schwartz, Stewart, Thaler (2012)


III. Predictions for DIS 1-Jettiness
at Next-to-
Next-to-
Leading-Log

Resummation of Logs

- Solution of RG Equation automatically resums logs to all orders in α_s
- Order of logarithmic accuracy (LL, NLL, etc.) depends on accuracy to which anomalous dimensions and fixed-order matrix elements are known:

	Γ_F	γ_F	c_F	$\beta[\alpha_s]$
LL	α_s	1	1	α_s
NLL	α_s^2	α_s	1	α_s^2
NNLL	α_s^3	α_s^2	α_s	α_s^3

Resummation of Logs

- Solution of RG Equation automatically resums logs to all orders in α_s
- Order of logarithmic accuracy (LL, NLL, etc.) depends on accuracy to which anomalous dimensions and fixed-order matrix elements are known:

	Γ_F	γ_F	c_F	$\beta[\alpha_s]$
LL	α_s	1	1	α_s
NLL	α_s^2	α_s	1	α_s^2
NNLL	α_s^3	α_s^2	α_s	α_s^3

+ 1 order for primed counting

Resummation of Logs

- Solution of RG Equation automatically resums logs to all orders in α_s
- Order of logarithmic accuracy (LL, NLL, etc.) depends on accuracy to which anomalous dimensions and fixed-order matrix elements are known:

	Γ_F	γ_F	c_F	$\beta[\alpha_s]$
LL	α_s	1	1	α_s
NLL	α_s^2	α_s	1	α_s^2
NNLL	α_s^3	α_s^2	α_s	α_s^3

All pieces known for DIS 1-jettiness

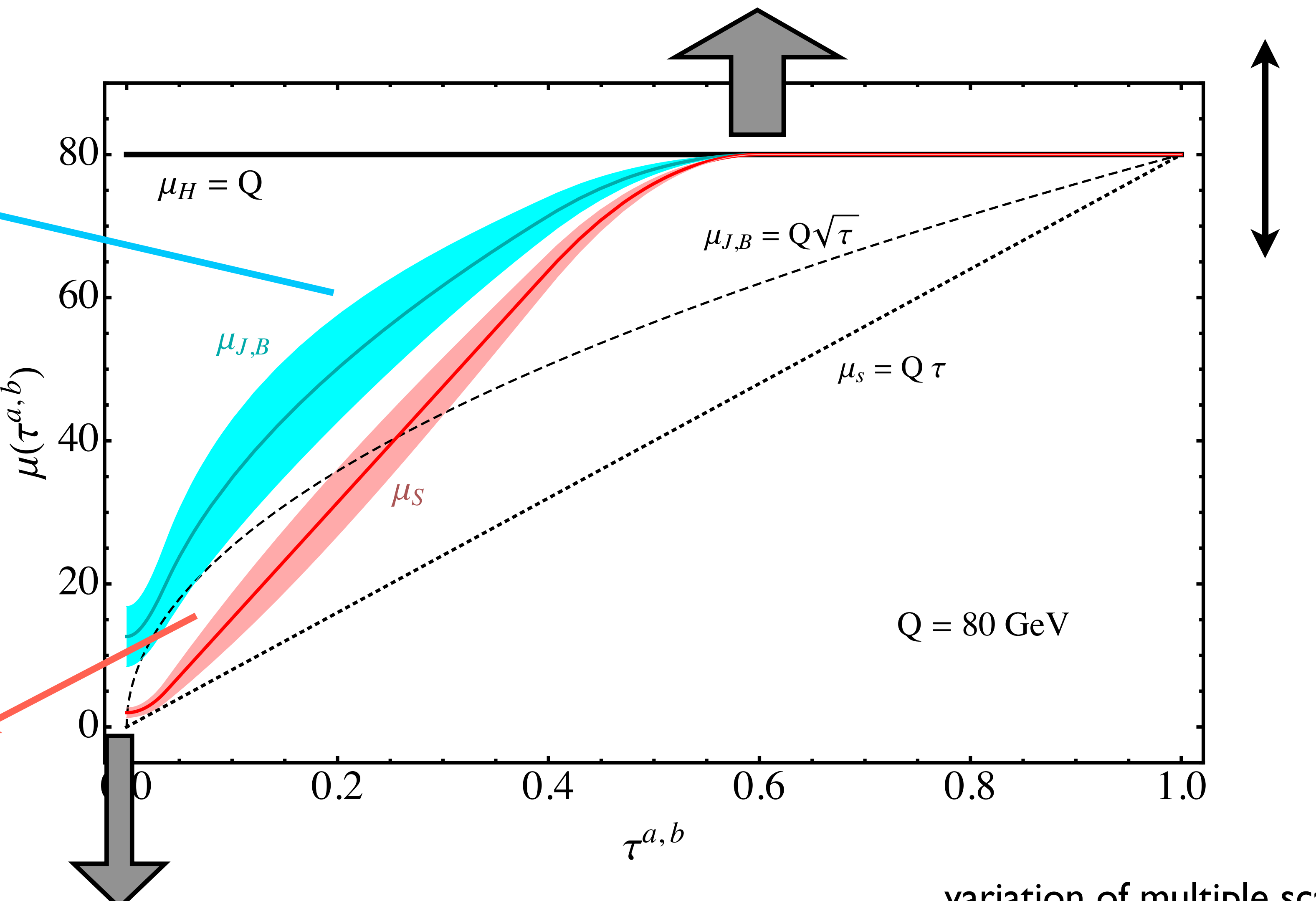
+ 1 order for primed counting

Theoretical Uncertainty

Uncertainty from missing terms of higher order in perturbation theory estimated by varying the scales in three ways:

Stewart, Tackmann, Waalewijn (2010)

need to turn off resummation for matching to fixed order at large τ



1. overall scale between Q/2 and 2Q

2. jet/beam scale within cyan band

3. soft scale within red band

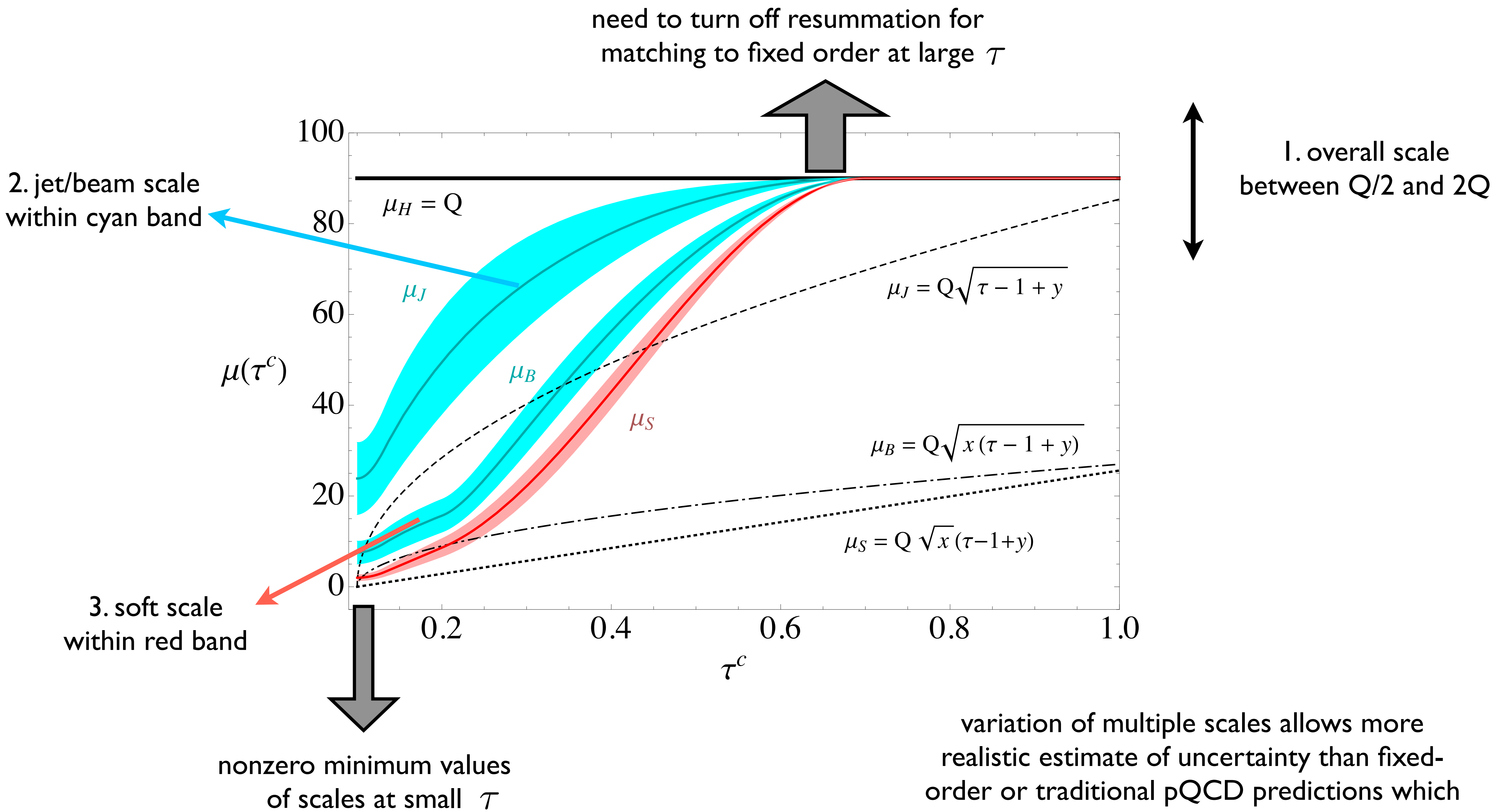
nonzero minimum values of scales at small τ

variation of multiple scales allows more realistic estimate of uncertainty than fixed-order or traditional pQCD predictions which only allow one (overall) scale variation

Theoretical Uncertainty

Uncertainty from missing terms of higher order in perturbation theory estimated by varying the scales in three ways:

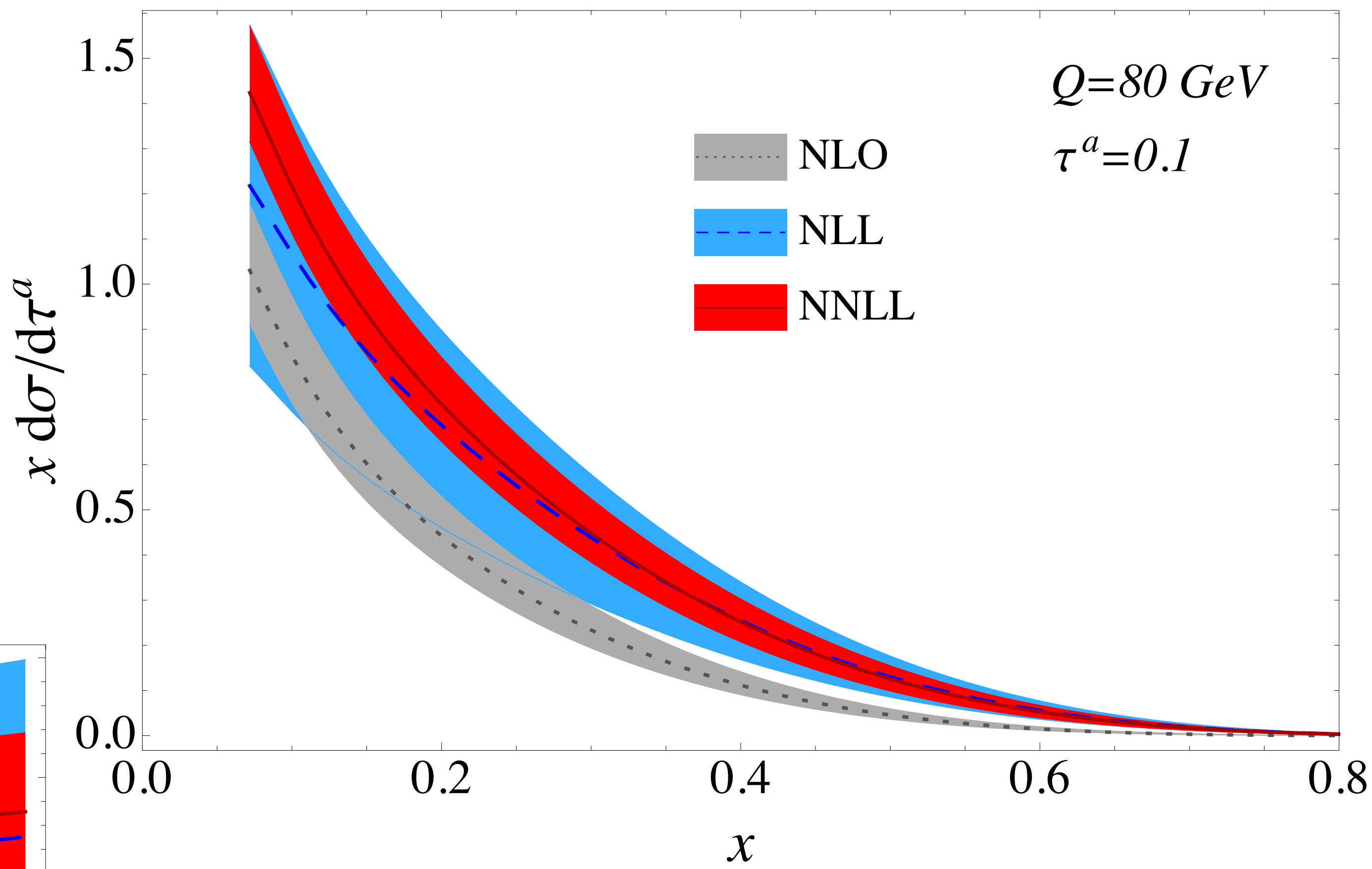
Stewart, Tackmann, Waalewijn (2010)



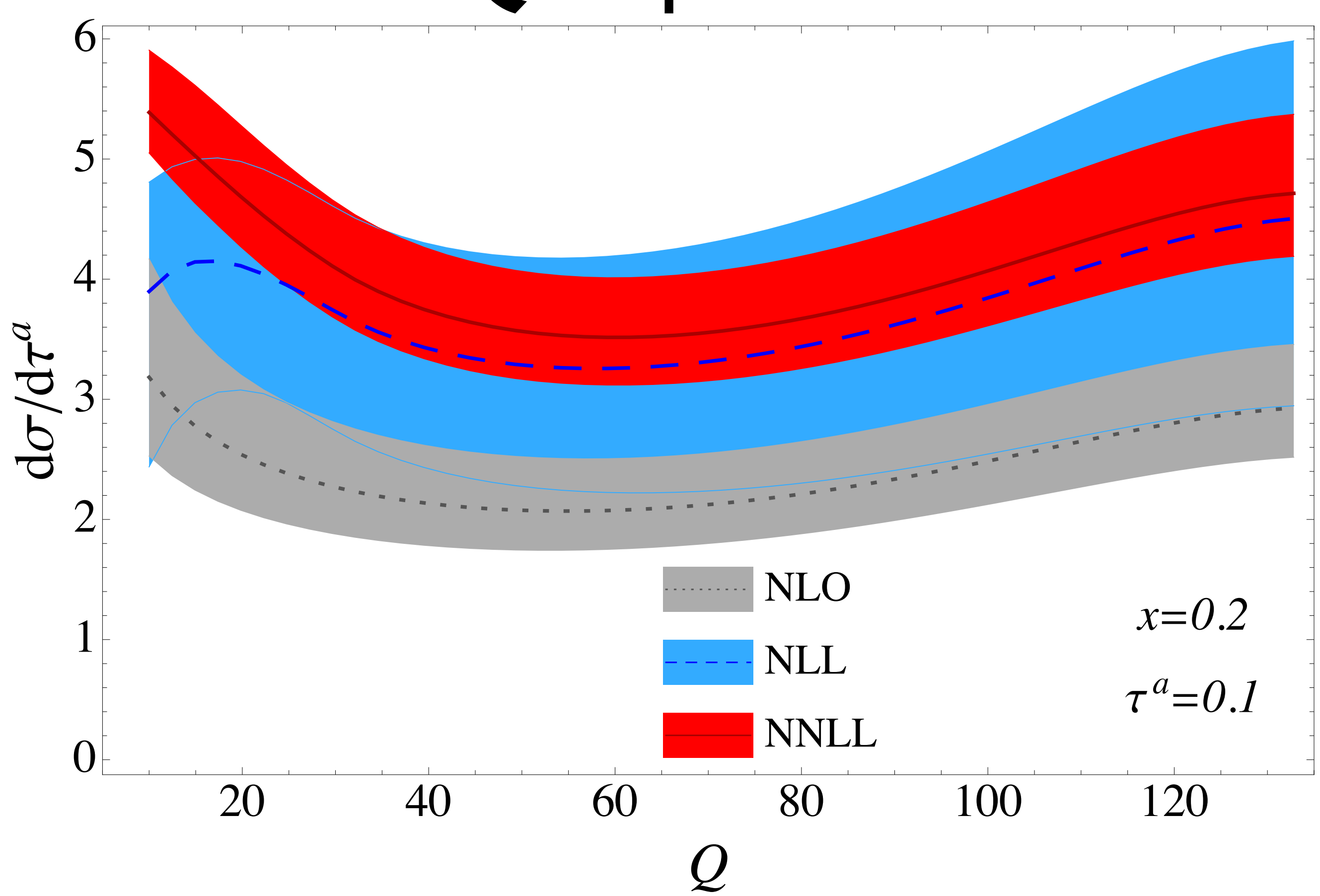
variation of multiple scales allows more realistic estimate of uncertainty than fixed-order or traditional pQCD predictions which only allow one (overall) scale variation

Predictions for DIS 1-jettiness

x dependence:

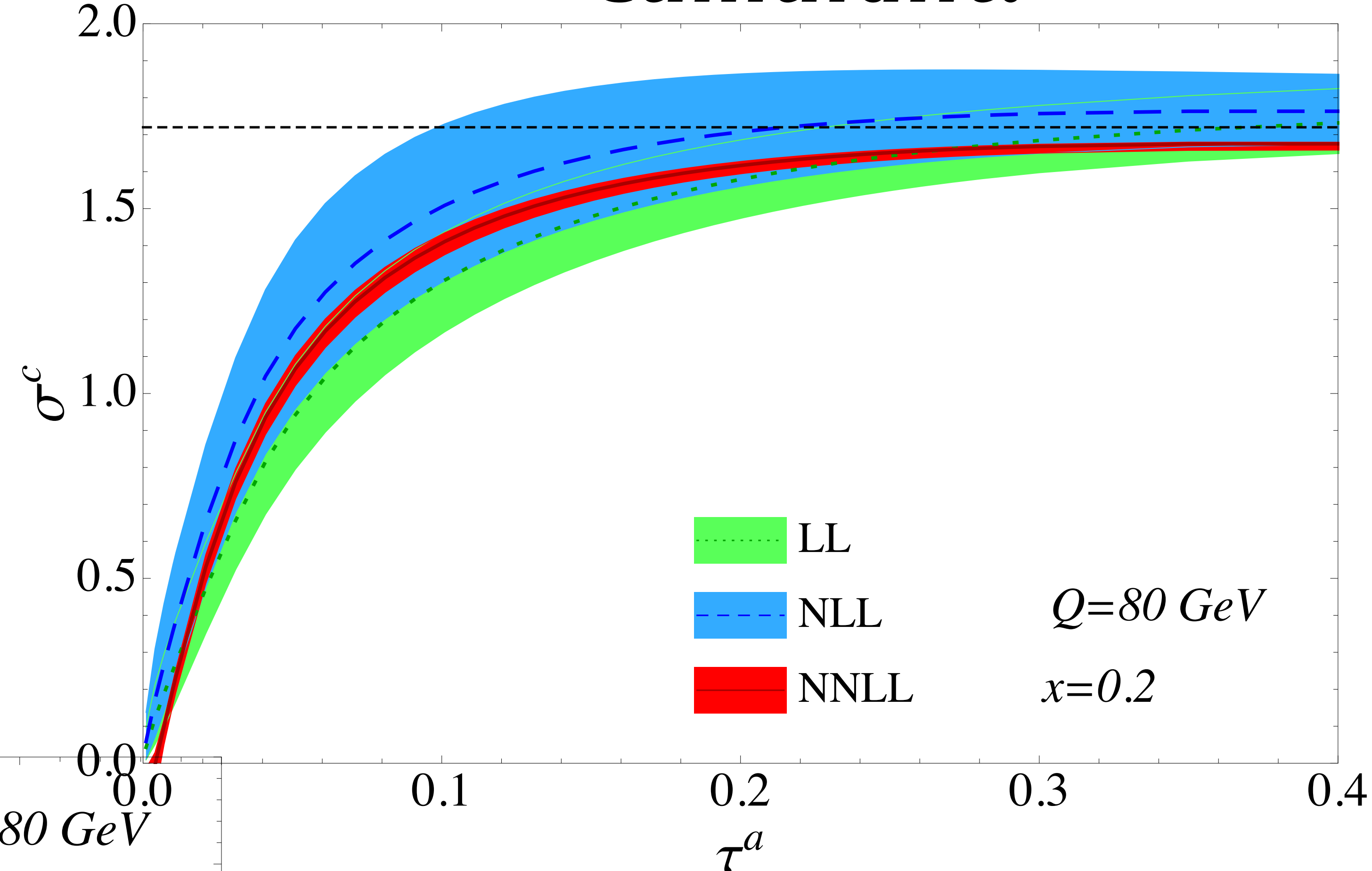


Q dependence:

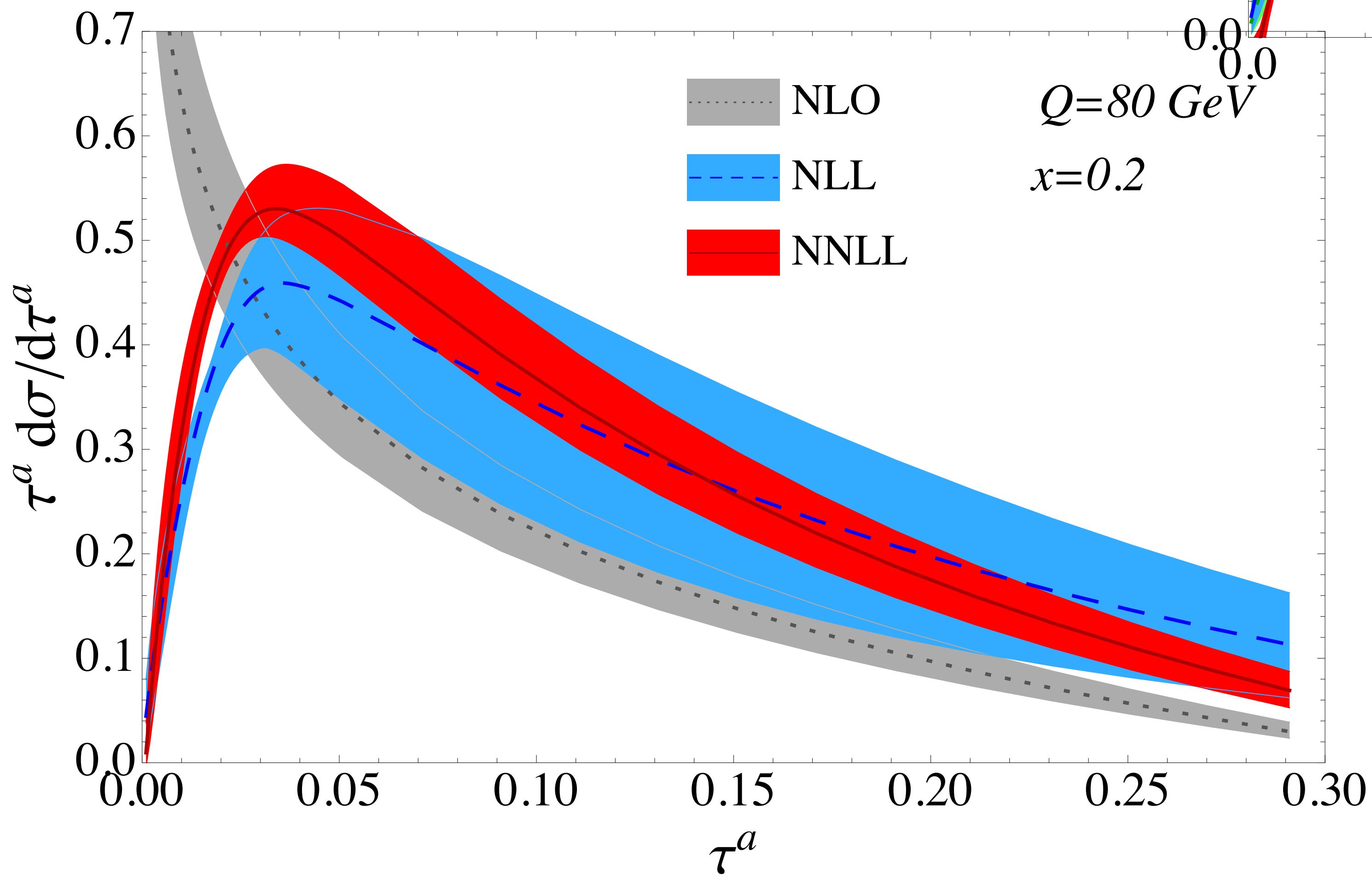


Predictions for DIS 1-jettiness

cumulant:

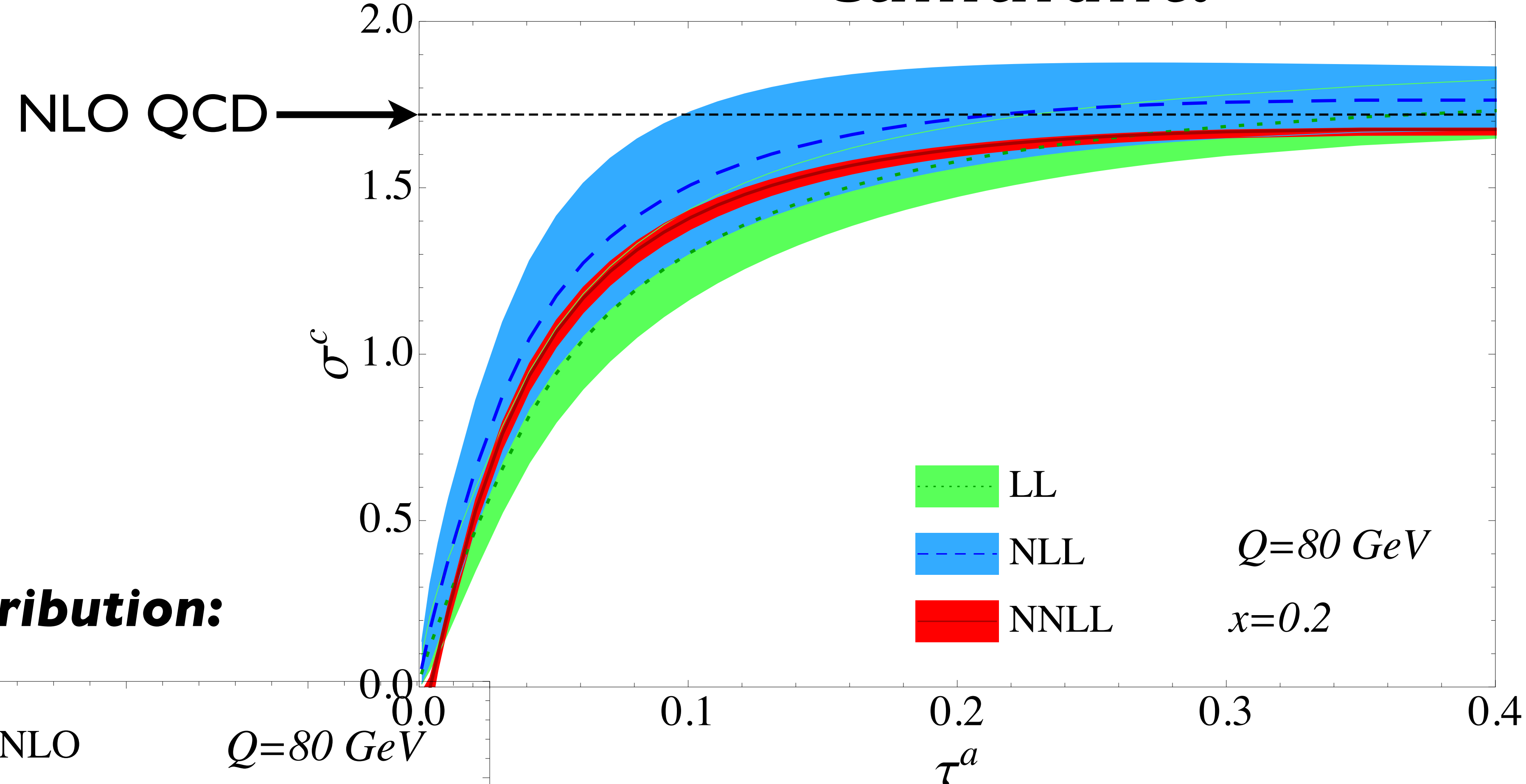


differential distribution:

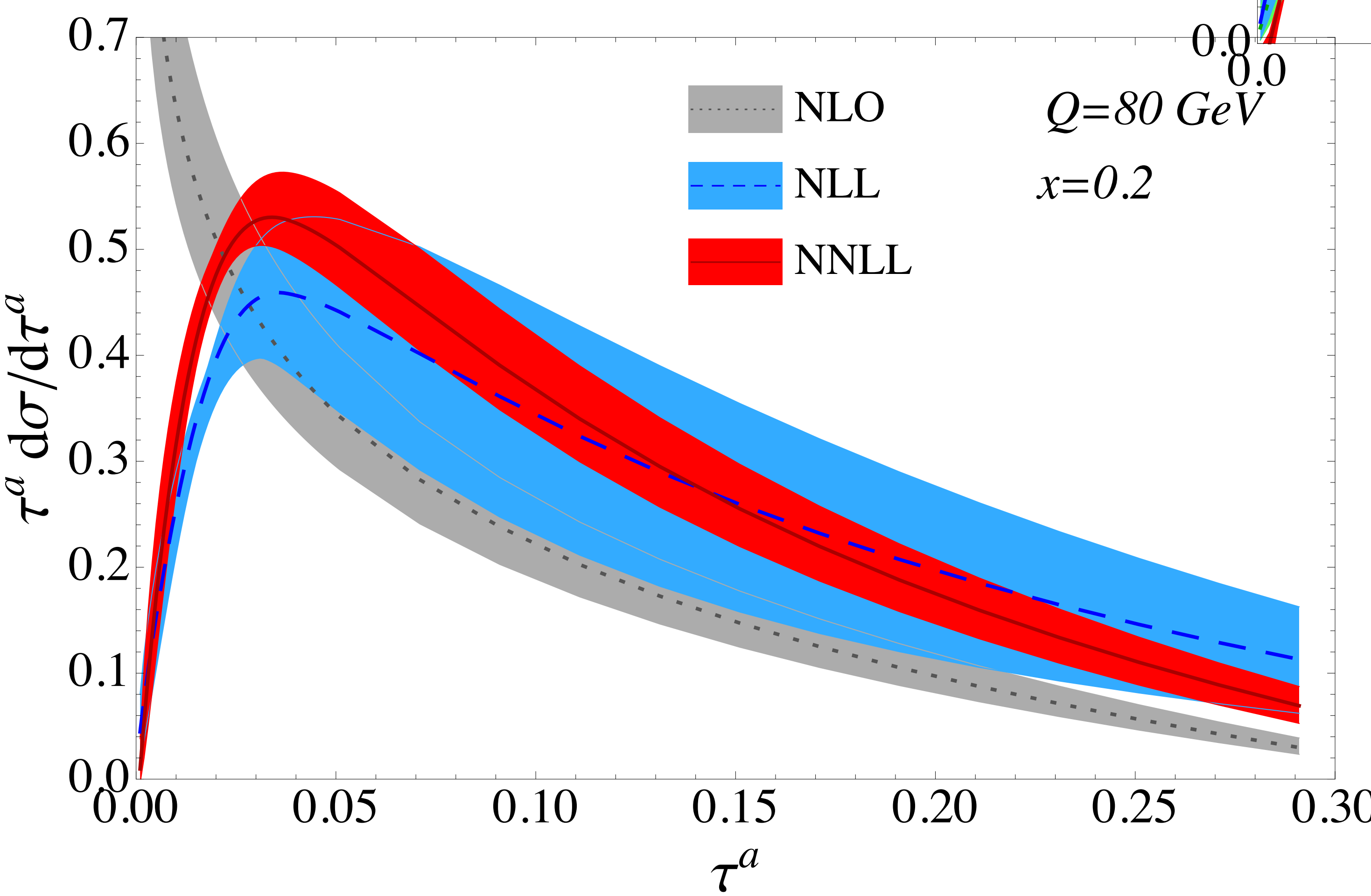


Predictions for DIS 1-jettiness

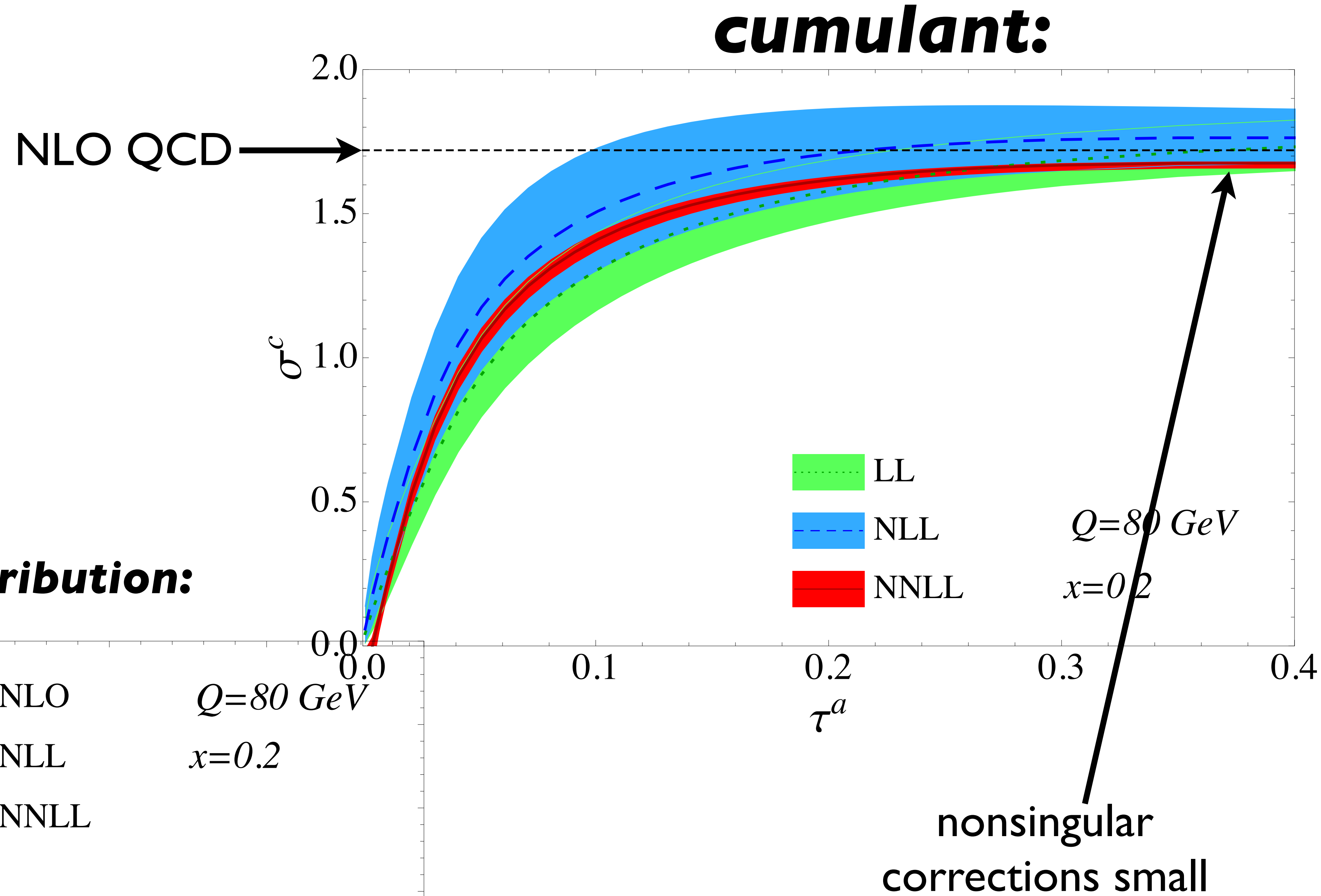
cumulant:

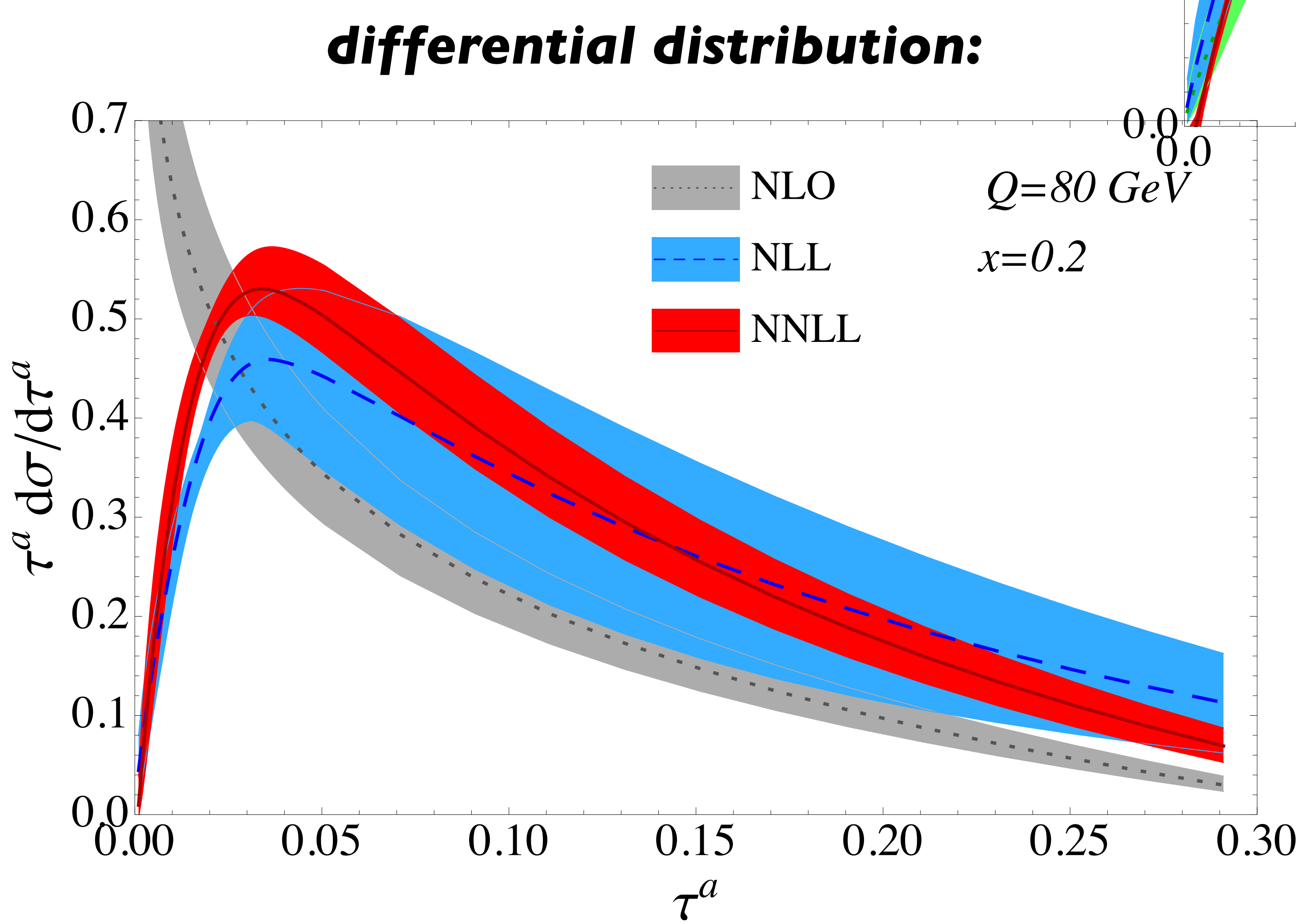


differential distribution:



Predictions for DIS 1-jettiness



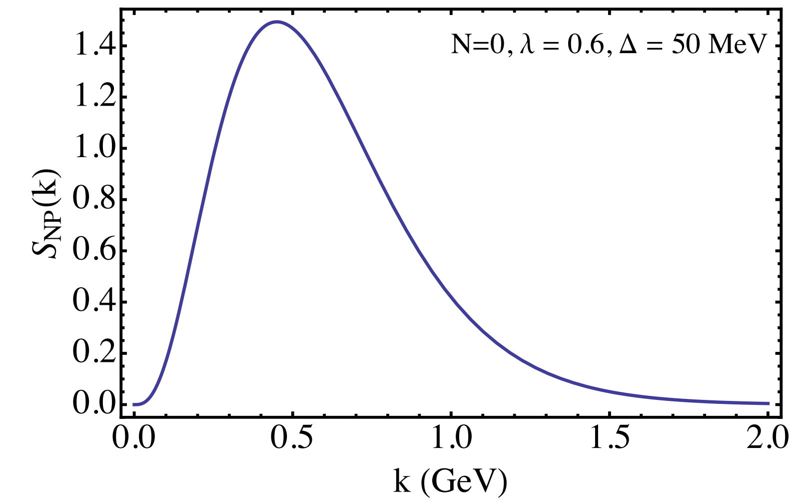


Predictions for DIS 1-jettiness

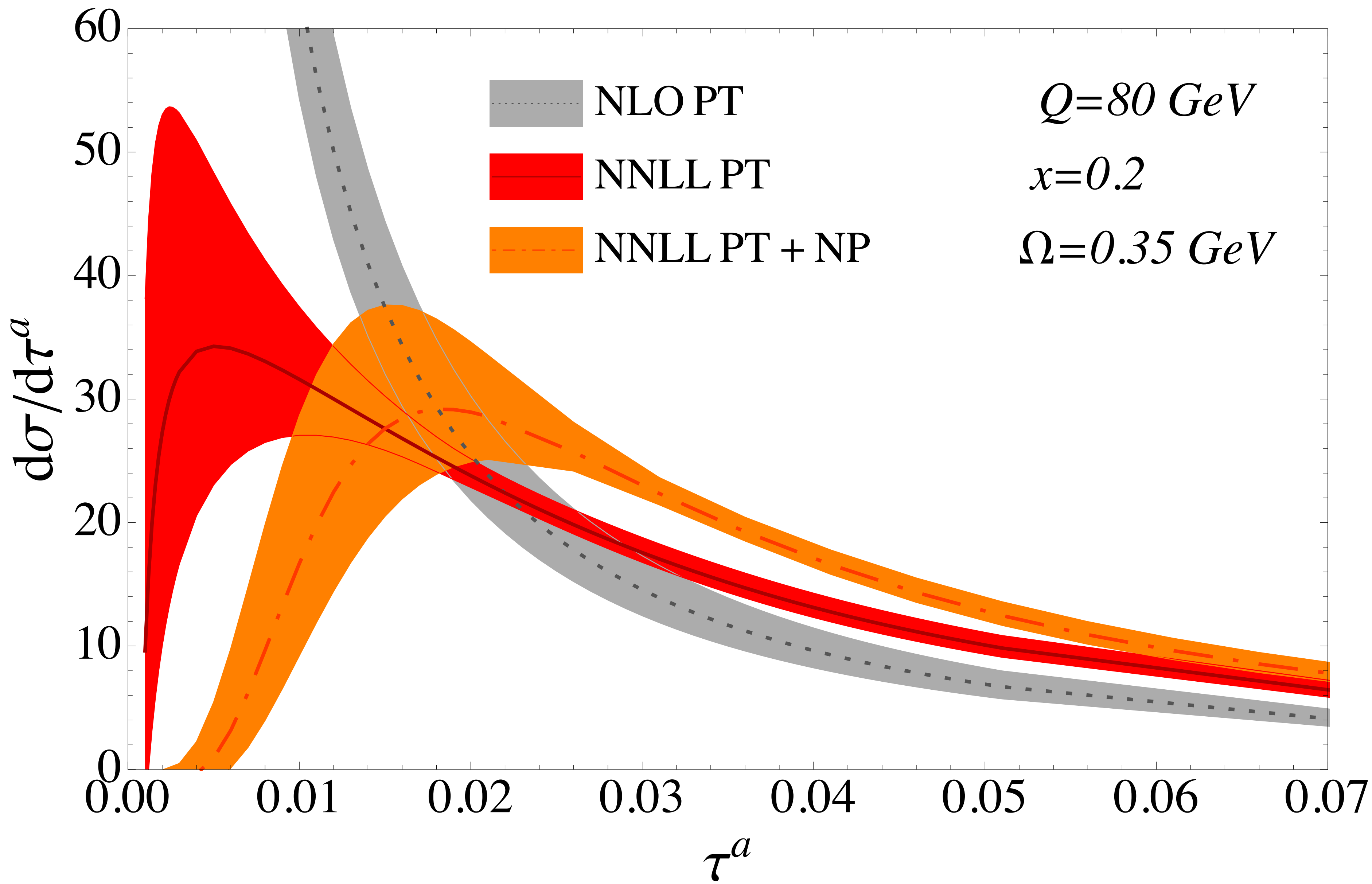
$$S_{NP}(l) = f(l - \Delta)$$

$$f(l) = \frac{1}{\lambda}\sum_{n=0}^{N} c_n f_n\Big(\frac{l}{\lambda}\Big)$$

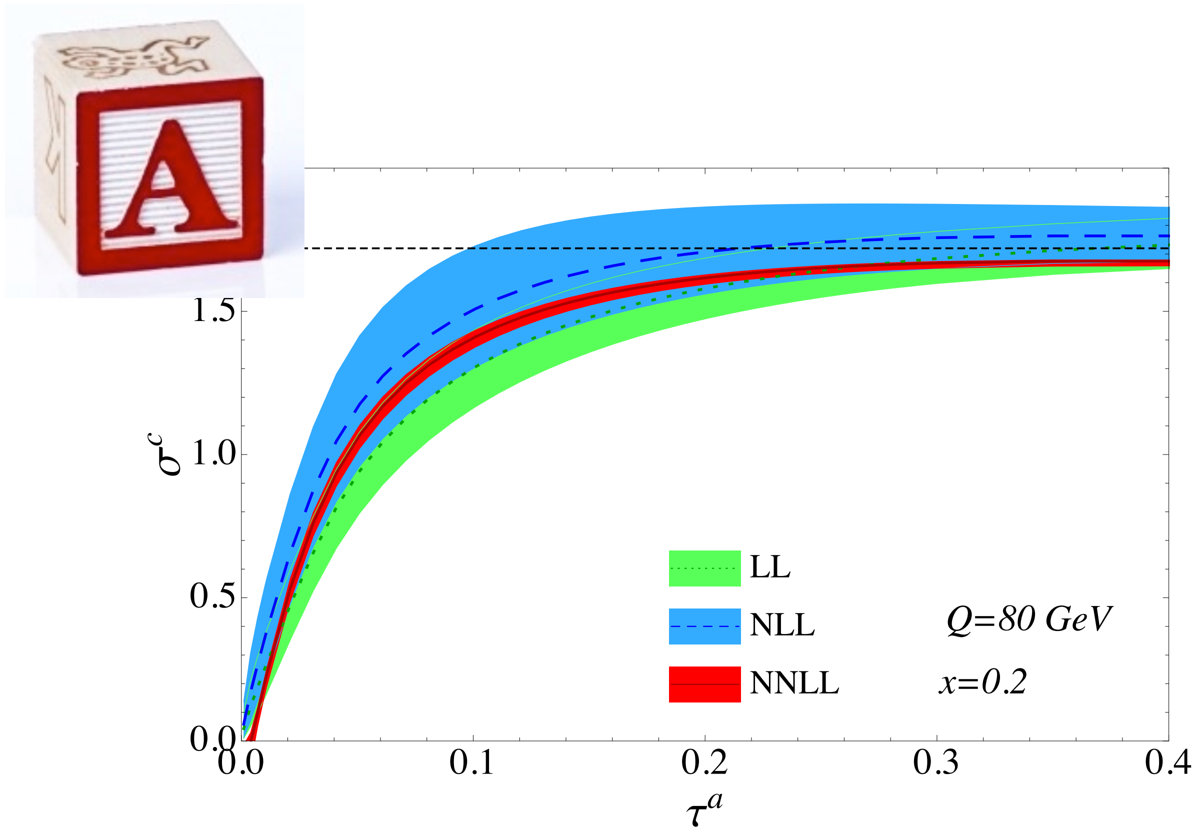
Ligeti, Stewart, Tackmann (2008)

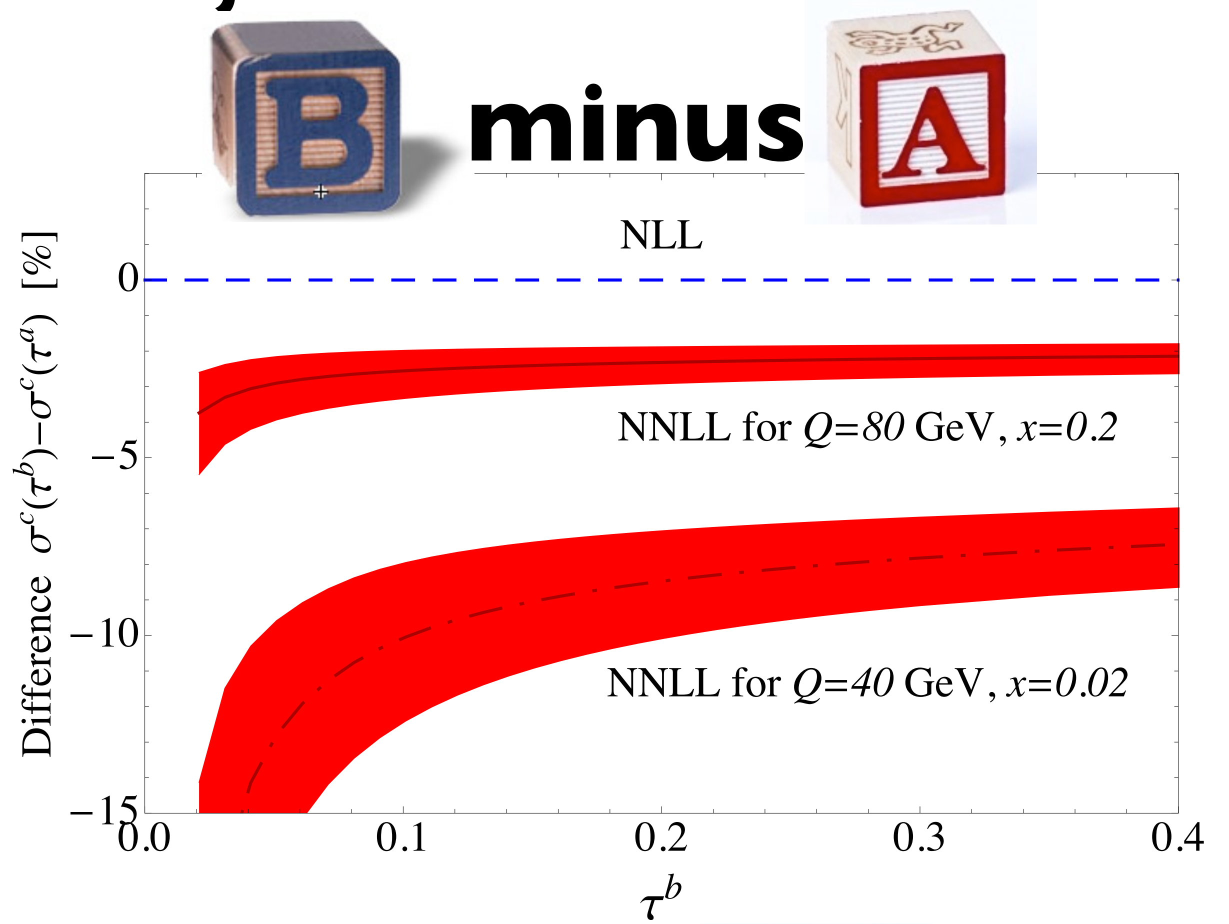


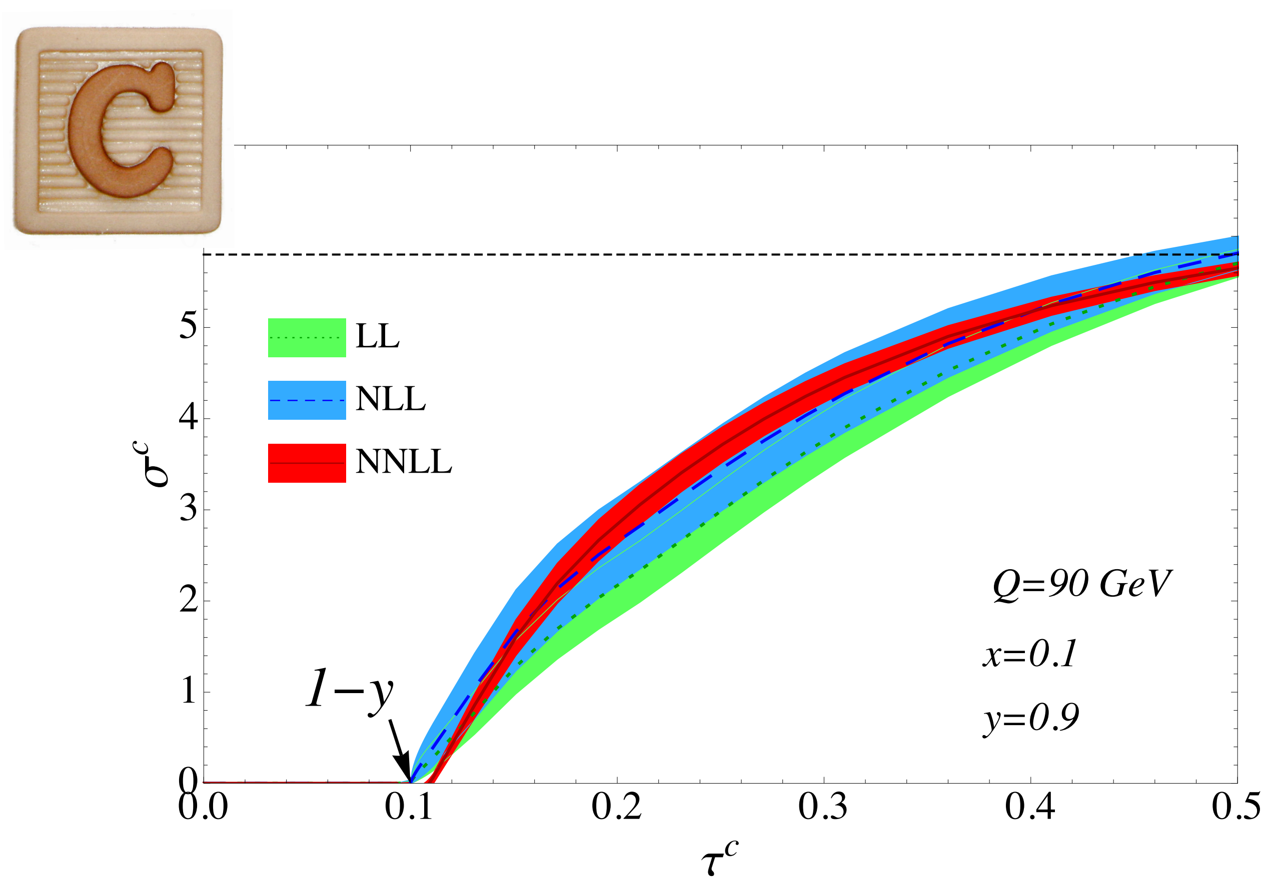
convolution with NP shape function:

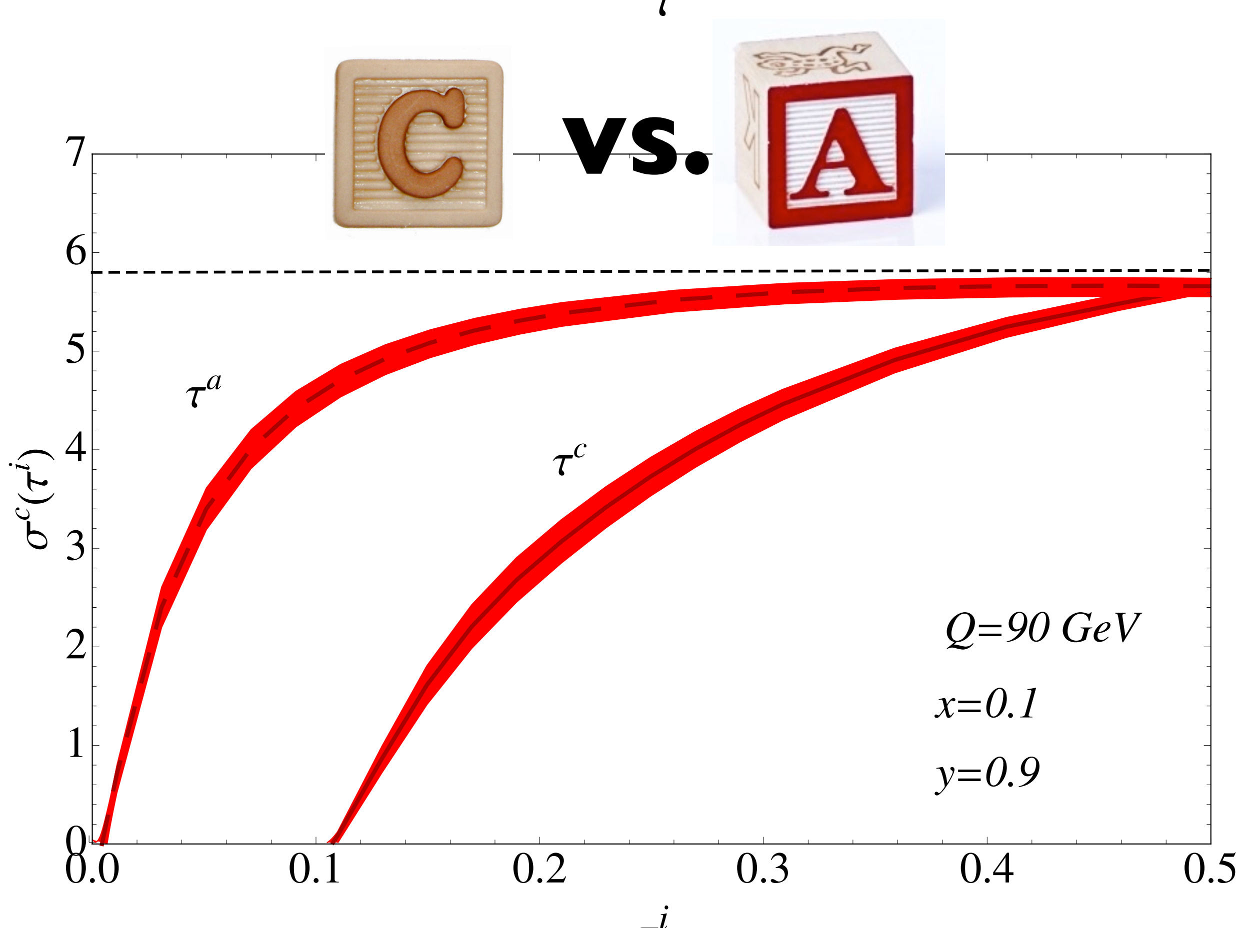


Predictions for DIS 1-jettiness

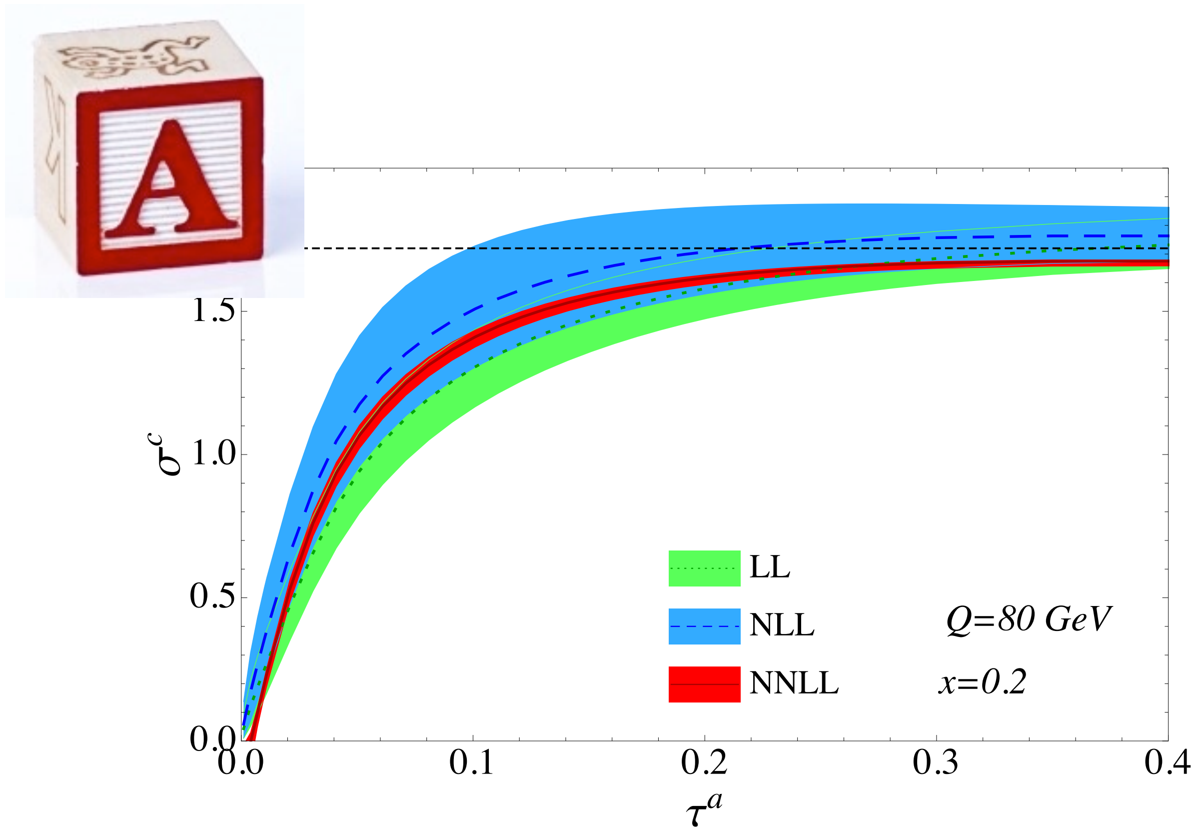


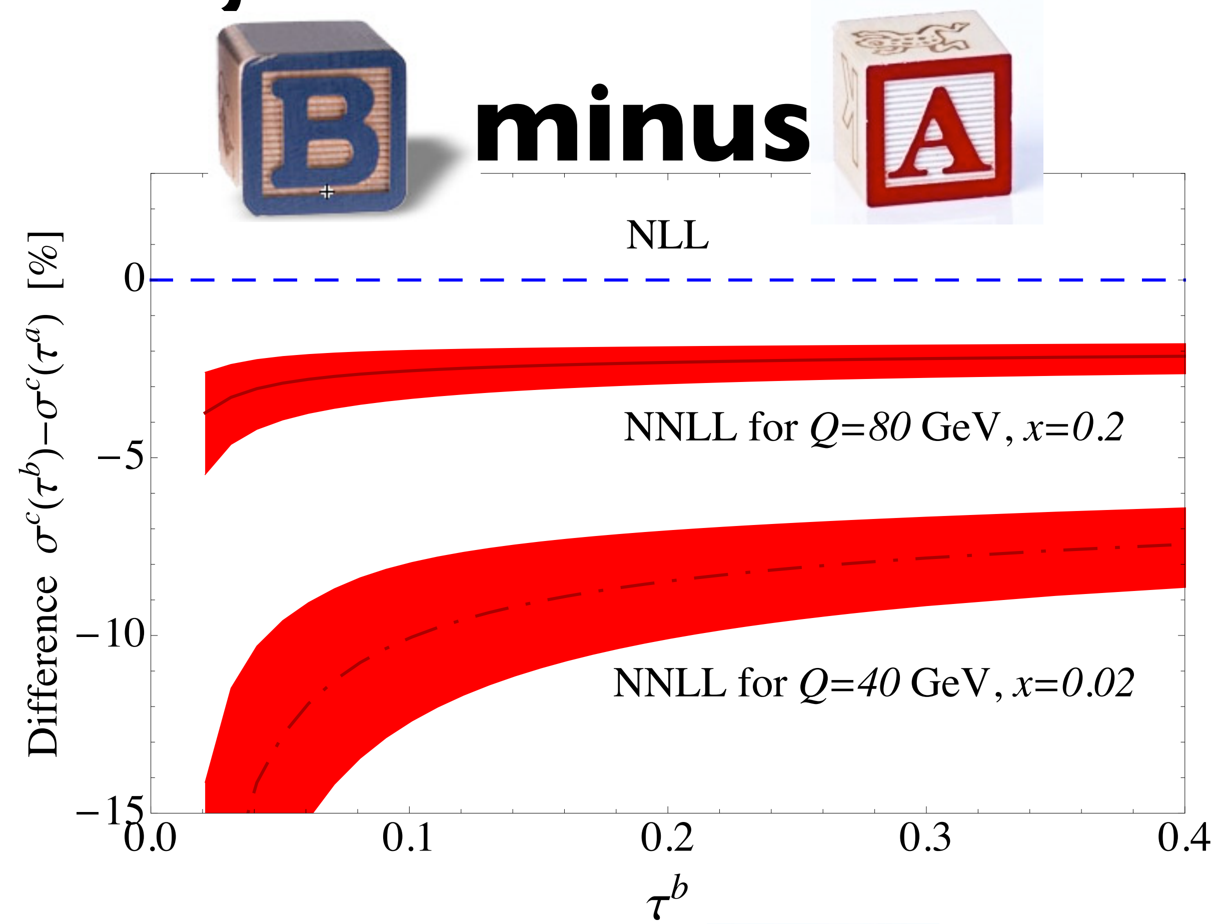


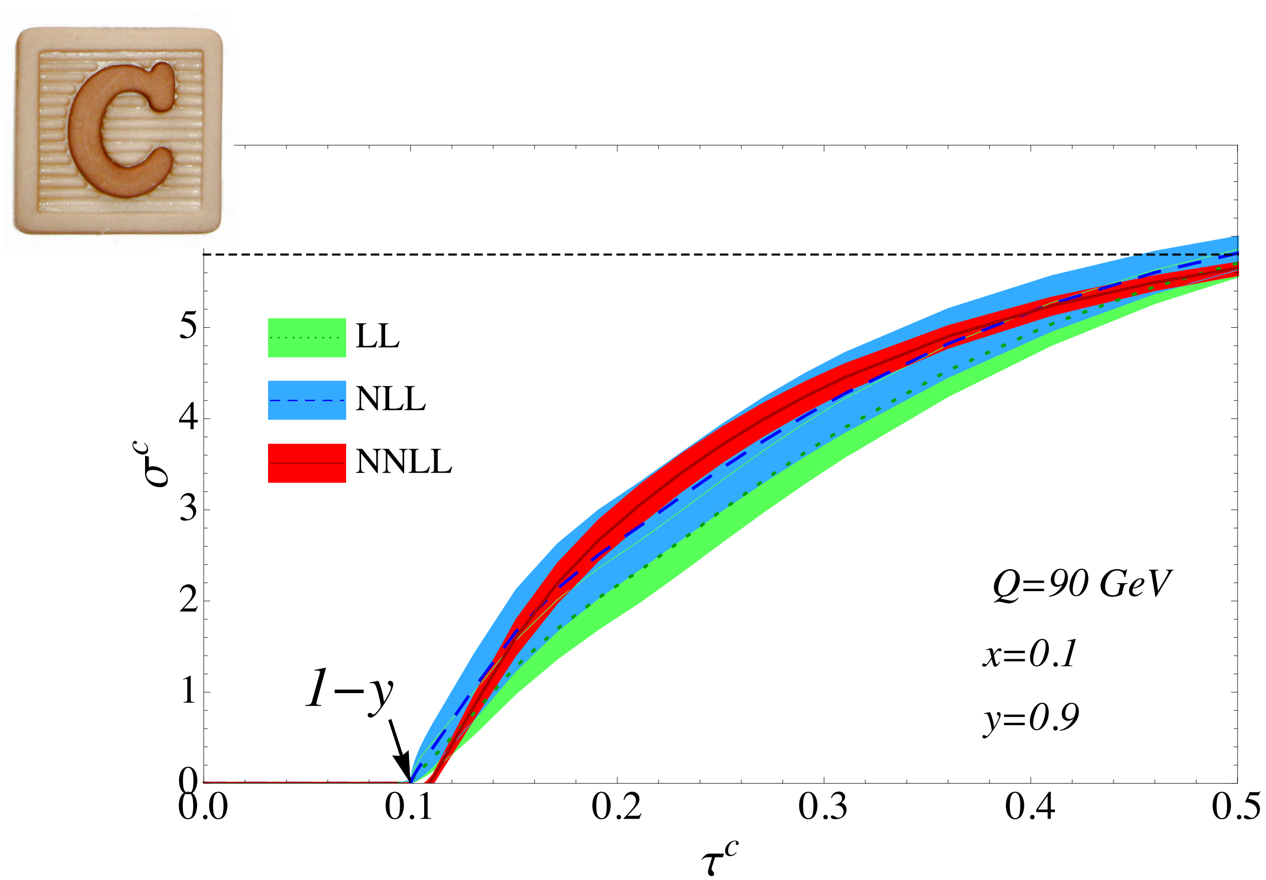


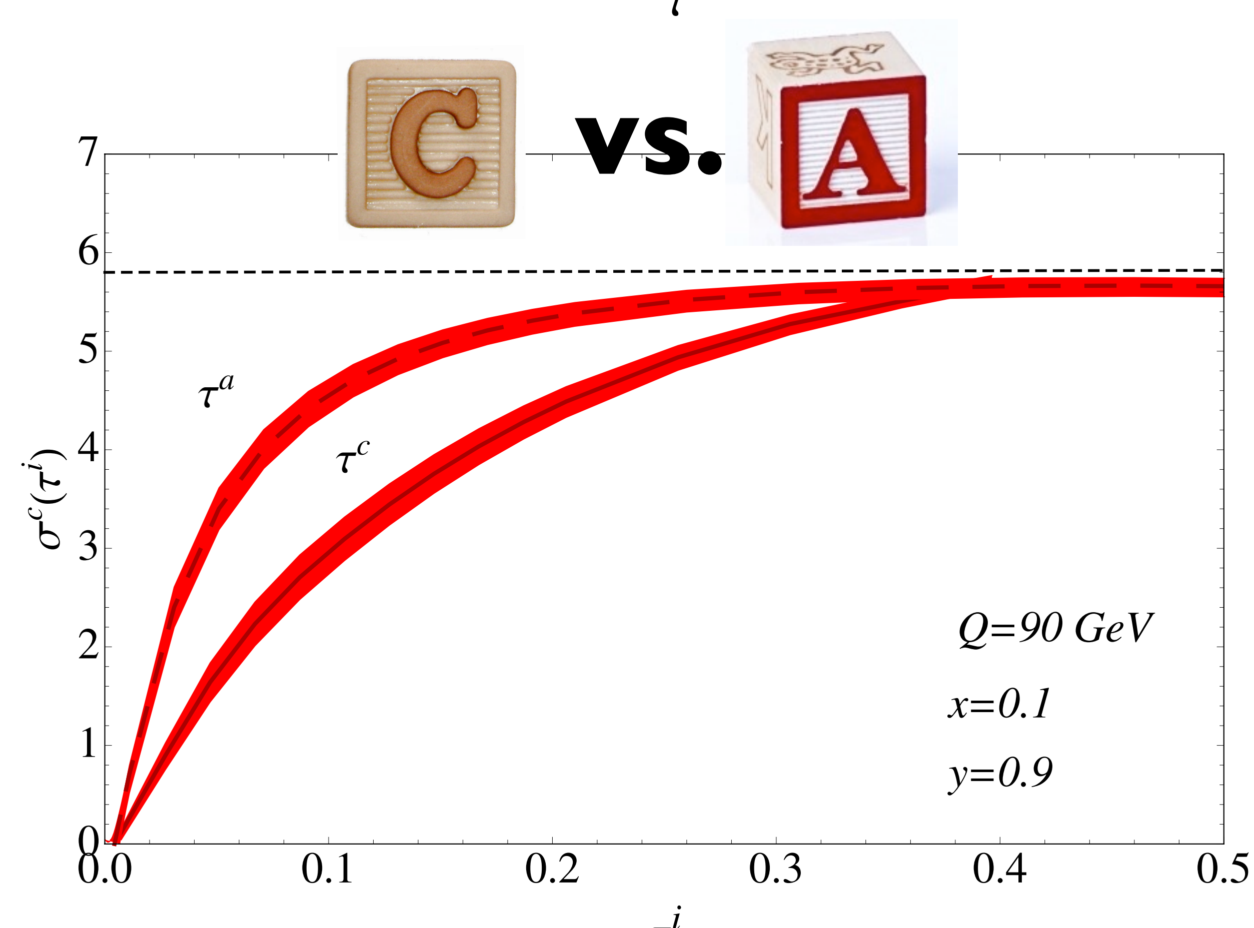


Predictions for DIS 1-jettiness









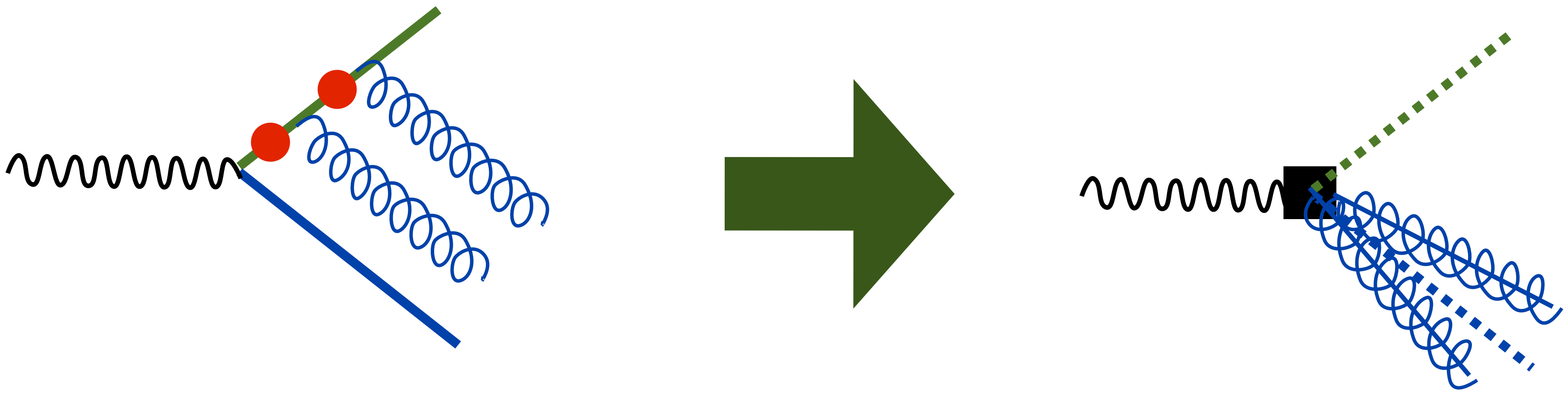
The Future

- Calculate rapidity gap distribution
- Two-loop beam function and constant in two-loop soft funciton can be used to achieve N^3LL accuracy
    - Heard about a 2-loop quark beam function here [Lübbert, Gehrmann, Yang]
- Calculate fixed order 1-loop 1-jettiness cross sections in QCD to accurately predict far tail (large τ_1) region
- Comparison to data for improved extraction of strong coupling, nonperturbative soft function, and PDFs

Conclusions

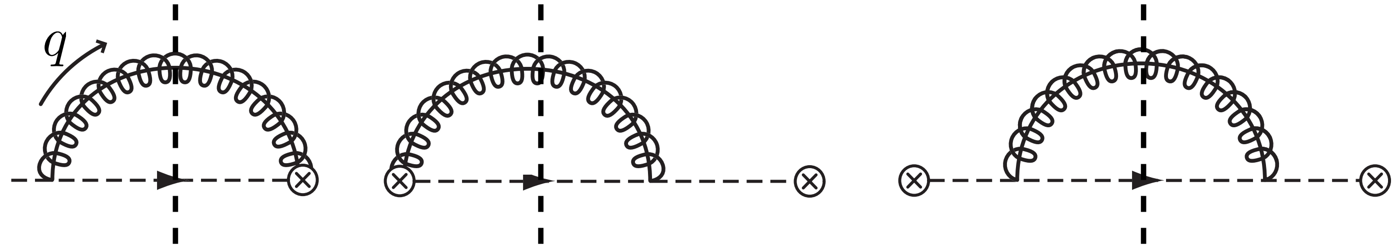
- We have computed 1-jettiness cross sections probing 2 jets in DIS in 3 different ways
- We used SCET to factorize and resum the cross sections to NNLL accuracy, the highest achieved to date for DIS jet cross sections
- SCET provides the tools:
  - to vastly improve the perturbative accuracy of large classes of cross sections in medium and high-energy nuclear and particle physics
  - to improve the extraction of universal nonperturbative functions and the strong coupling

Hard and Jet Functions



$$H(Q^2,\mu) = 1 + \frac{\alpha_s(\mu) C_F}{2\pi}\left(-\ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{\pi^2}{6}\right) + \dots$$

known to 3 loops



$$J(t,\mu) = \delta(t) + \frac{\alpha_s(\mu) C_F}{4\pi}\left\{(7-\pi^2)\delta(t) - \frac{3}{\mu^2}\left[\frac{\mu^2\theta(t)}{t}\right]_+ + \frac{4}{\mu^2}\left[\frac{\theta(t)\ln(t/\mu^2)}{t/\mu^2}\right]_+\right\} + \dots$$

known to 2 loops

anomalous dimension known to 3 loops

Beam Function and PDFs

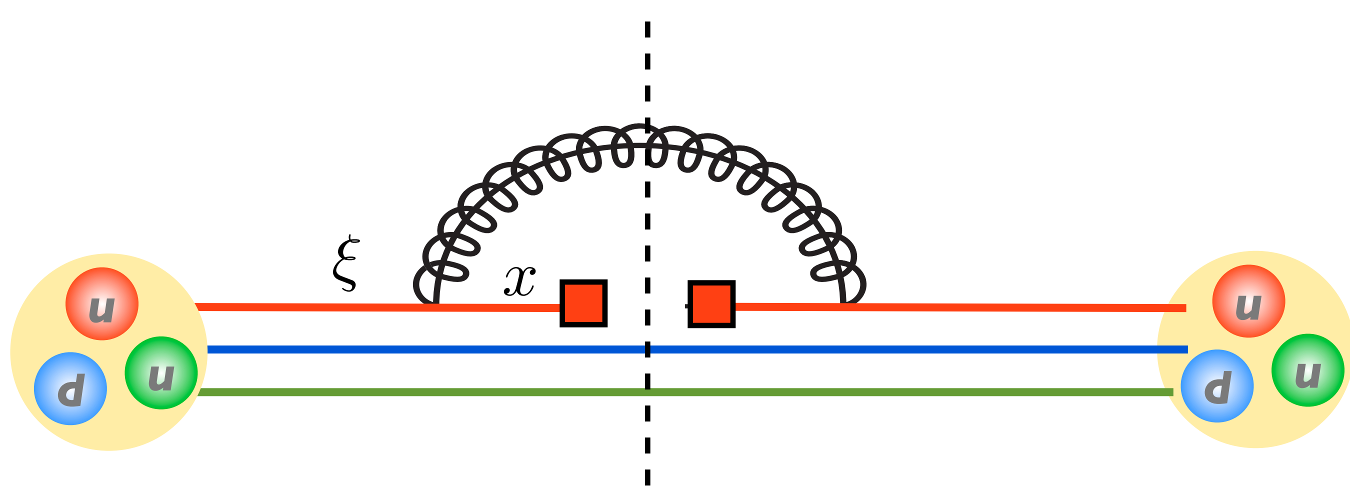
transverse momentum dependent beam function:

$$B(\omega k^+, x, k_\perp^2, \mu) = \frac{\theta(\omega)}{\omega} \int \frac{dy^-}{4\pi} e^{ik^+ y^-/2} \langle P_n(P^-)|\bar{\chi}_n\Big(y^- \frac{n}{2}\Big)\delta(xP^- - \bar{n}\cdot\mathcal{P})\delta(k_\perp^2 - \mathcal{P}_\perp^2)\chi_n(0)|P_n(P^-)\rangle$$

match onto PDF

$$f(x,\mu) = \theta(\omega)\langle P_n(P^-)|\bar{\chi}_n(0)\delta(xP^- - \bar{n}\cdot\mathcal{P})\chi_n(0)|P_n(P^-)\rangle$$

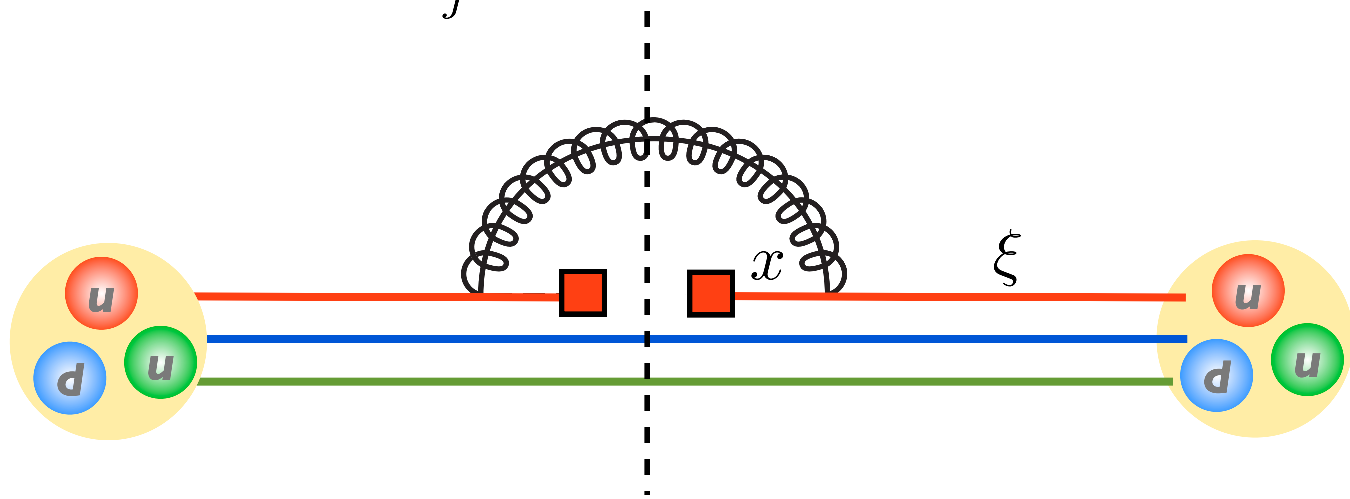
$$\mathcal{B}_q(t, x, \mathbf{k}_\perp^2, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\Big(t, \frac{x}{\xi}, \mathbf{k}_\perp^2, \mu\Big) f_j(\xi, \mu)$$



Measure small light-cone momentum $k^+ = t/P^-$
and transverse momentum $\mathbf{k}_\perp$
of initial state radiation

Generalized Beam Function at 1-loop

$$\mathcal{B}_q(t, x, \mathbf{k}_\perp^2, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\Big(t, \frac{x}{\xi}, \mathbf{k}_\perp^2, \mu\Big) f_j(x, \mu)$$



$$\mathcal{I}_{qq}(t, z, \mathbf{k}_\perp^2, \mu) = \frac{1}{\pi}\delta(t)\delta(1-z)\delta(\mathbf{k}_\perp^2) + \frac{\alpha_s(\mu)C_F}{2\pi^2}\theta(z)\bigg\{\frac{2}{\mu^2}\bigg[\frac{\theta(t)\ln(t/\mu^2)}{t/\mu^2}\bigg]_+ \delta(1-z)\delta(\mathbf{k}_\perp^2)$$
$$+ \frac{1}{\mu^2}\bigg[\frac{\theta(t)}{t/\mu^2}\bigg]_+ \bigg[P_{qq}(z) - \frac{3}{2}\delta(1-z)\bigg]\delta\Big(\mathbf{k}_\perp^2 - \frac{(1-z)t}{z}\Big)$$
$$+ \delta(t)\delta(\mathbf{k}_\perp^2)\bigg[\bigg[\frac{\theta(1-z)\ln(1-z)}{1-z}\bigg]_+ (1+z^2) - \frac{\pi^2}{6}\delta(1-z) + \theta(1-z)\Big(1 - z - \frac{1+z^2}{1-z}\ln z\Big)\bigg]\bigg\} \quad (162a)$$

Jain, Procura, Waalewijn (2009)

anomalous dimension known to 3 loops

$$\mathcal{I}_{qg}(t, z, \mathbf{k}_\perp^2, \mu) = \frac{\alpha_s(\mu)T_F}{2\pi^2}\theta(z)\bigg\{\frac{1}{\mu^2}\bigg[\frac{\theta(t)}{t/\mu^2}\bigg]_+ P_{qg}(z)\delta\Big(\mathbf{k}_\perp^2 - \frac{(1-z)t}{z}\Big) + \delta(t)\delta(\mathbf{k}_\perp^2)\bigg[P_{qg}(z)\ln\frac{1-z}{z} + 2\theta(1-z)z(1-z)\bigg]\bigg\}, \quad (162b)$$

Tells us that PDFs should be evaluated at the beam radiation scale *t*

ordinary beam function: $B(t, x, \mu) = \int d^2k_\perp \mathcal{B}(t, x, \mathbf{k}_\perp^2, \mu)$

Stewart, Tackmann, Waalewijn (2009)

Nonperturbative Soft Model Function

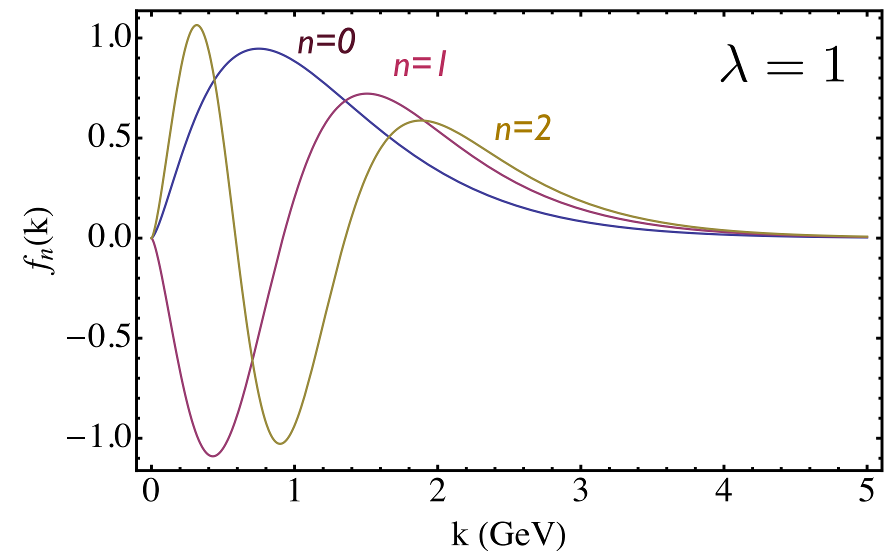
Convolution of perturbative soft function (soft radiation) with nonperturbative model function (hadronization):

$$S(k_S,\mu) = \int dl\, S_{\mathrm{PT}}(k_S - l,\mu) S_{\mathrm{NP}}(l)$$

$$S_{NP}(l) = f(l-\Delta)$$

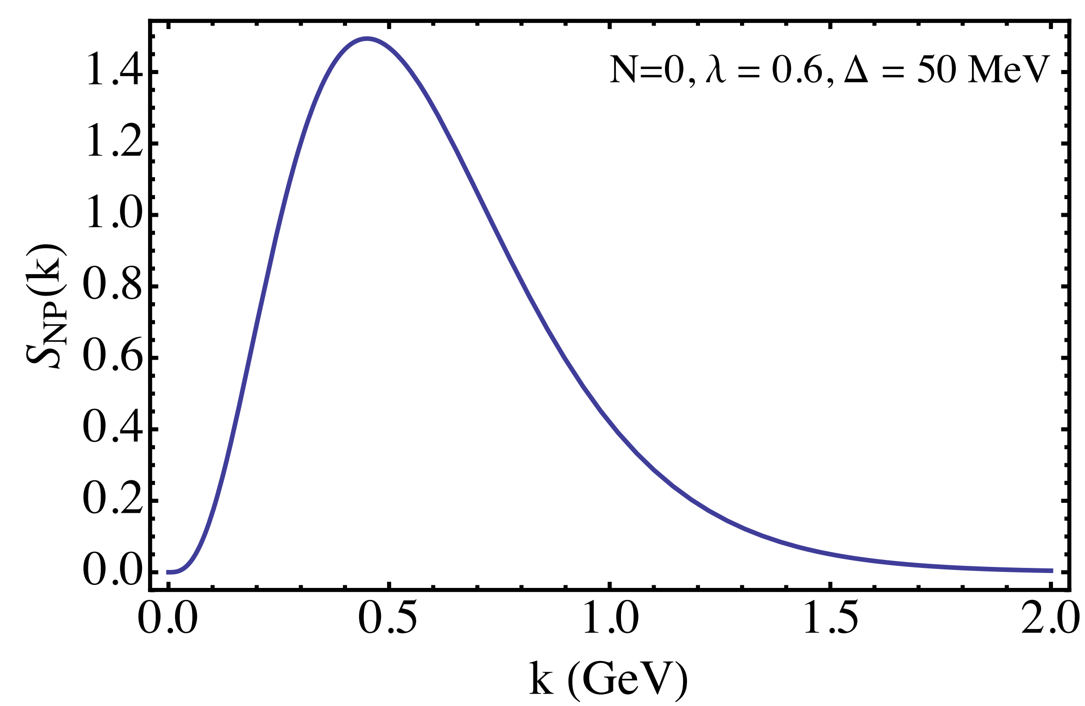
$$f(l) = \frac{1}{\lambda}\sum_{n=0}^{N} c_n f_n\Big(\frac{l}{\lambda}\Big)$$

Ligeti, Stewart, Tackmann (2008)



Basis coefficients, width and gap should be fit to data for one event shape and value of *Q*. Universality allows predictions for other event shapes and values of *Q*.

In following results, the following model function will be used:



The Beam Thrust Cross Section for Drell-Yan at NNLL Order

Iain W. Stewart, Frank J. Tackmann, and Wouter J. Waalewijn

Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA