Approximate NNLO stop-pair production cross section at LHC

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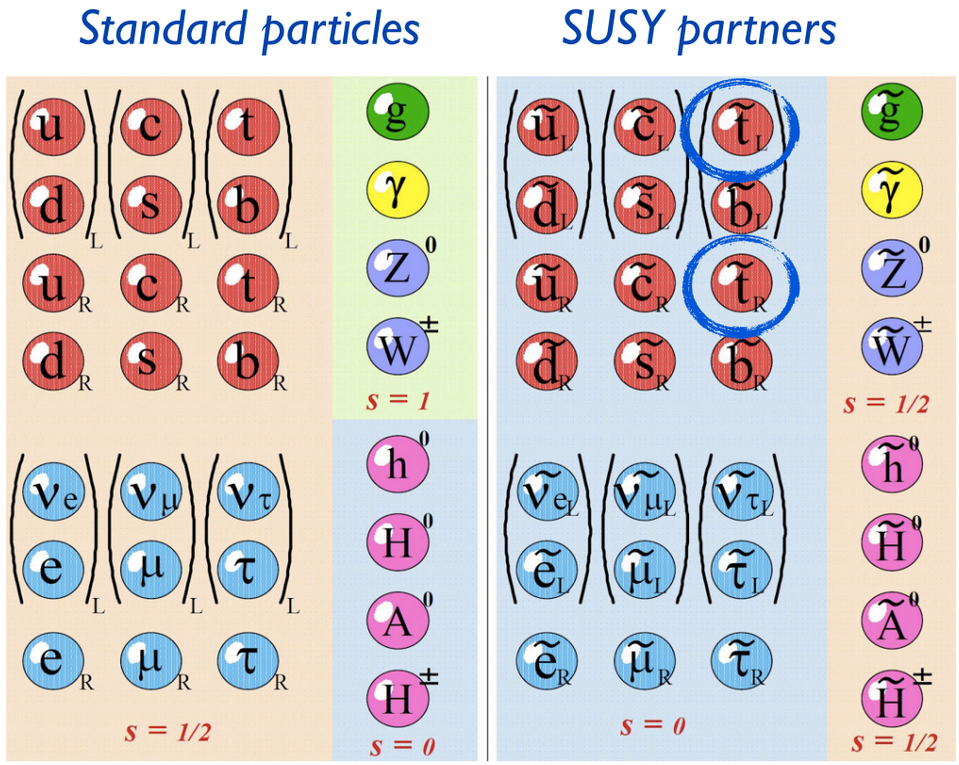
Outline

- Introduction: stop-pair production
- Pair Invariant Mass (PIM) and One Particle Inclusive (1PI) kinematics
- Approximate NNLO formulas
- Total cross section

Motivations for SUSY

- SUSY is a symmetry transformation which turns fermions into bosons and vice versa.
- It predicts the existence of "SUSY partners" for all of the SM particles (MSSM).



- It provides a solution to the Hierarchy Problem (systematic cancellation of dangerous contributions)
- It foresees the gauge coupling unification at the GUT scale (10^{16} GeV).
- If R-parity is conserved, it provides a dark matter candidate.

*One of the **main goal** of the **LHC** is to investigate the existence of **supersymmetric partners** of the known SM particles!*

Beyond NLO?

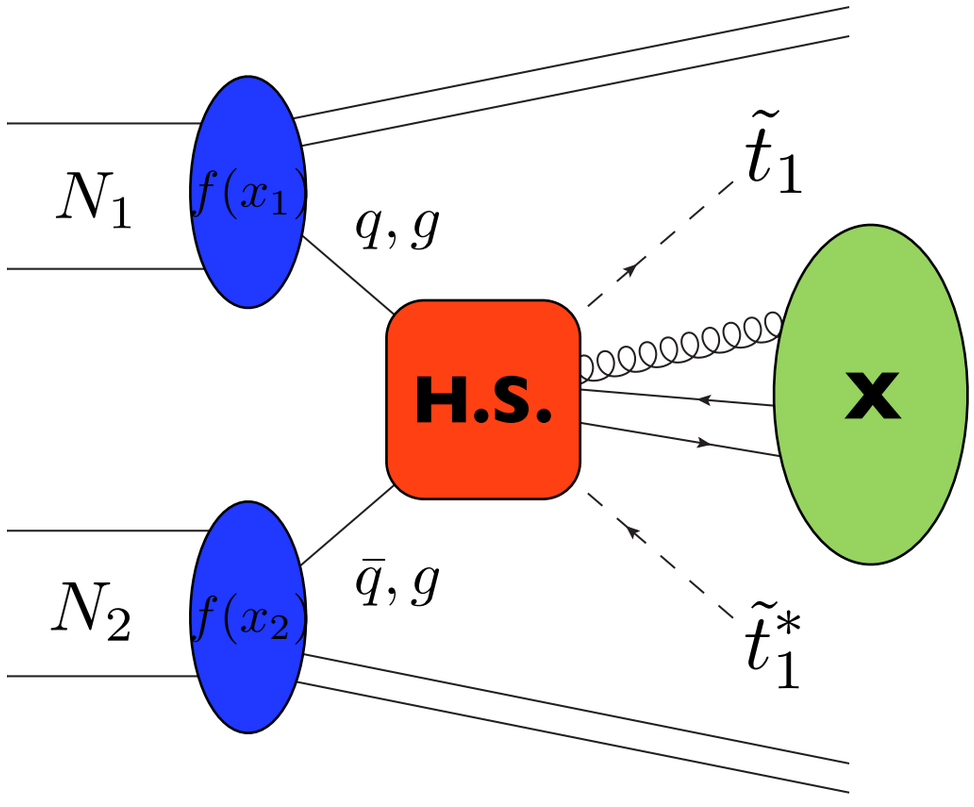
Leading contributions of the higher order terms
(Soft-gluon resummation, Approximate NNLO formulas)

Stop-pair production

Mass-eigenstates $\begin{pmatrix} \tilde{t}_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{t}} & -s_{\tilde{t}} \\ s_{\tilde{t}} & c_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$ Gauge-eigenstates

It is expected to be the lightest squark of the mass spectrum

- NLO SUSY-QCD corrections: [Beenakker, Kramer, Plehn, Spira, Zerwas 98']
- Soft-gluon emissions consists of a large portion of NLO SUSY-QCD corrections.
  Soft-gluon resummation at NLL $\beta \to 0$ by [Beenakker, Brensing, Kramer, Kulesza, Laenen, Niessen 10']
- NNLO (production-)threshold corrections $\beta \to 0$ by [Langenfeld, Moch 11']
- Soft and Coulomb resummation at NLL by [Beneke, Falgari, Schwinn 10'], [Falgari,, Schwinn, Wever 12']



$$N_1(P_1) + N_2(P_2) \to \tilde{t}_1(p_3) + \tilde{t}_1^*(p_4) + X(k)$$

At LO in P.T. two partonic channels contribute to the process

$$q(p_1) + \bar{q}(p_2) \to \tilde{t}_1(p_3) + \tilde{t}_1^*(p_4)$$

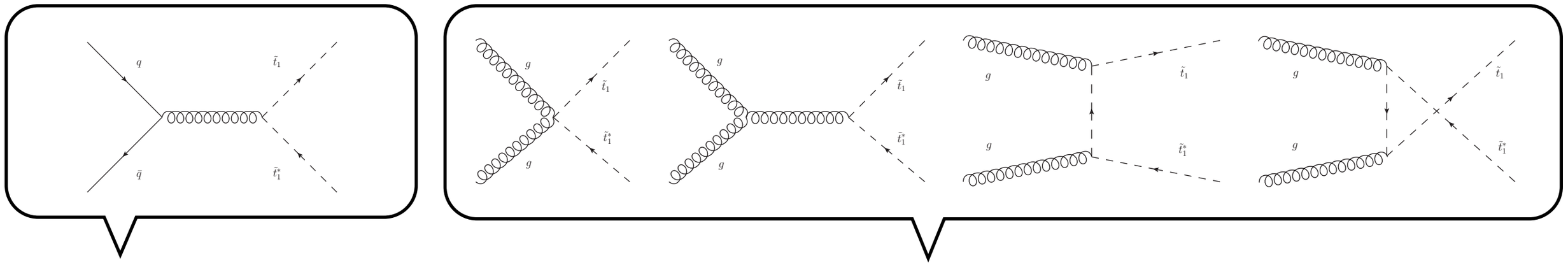
$$g(p_1) + g(p_2) \to \tilde{t}_1(p_3) + \tilde{t}_1^*(p_4)$$

where $p_i = x_i P_i \ (i = 1, 2).$

Stop-pair production

$$N_1(P_1) + N_2(P_2) \to \tilde{t}_1(p_3) + \tilde{t}_1^*(p_4) + X(k)$$

For LHC @ 14 TeV:



~ 12 %

~ 88 % *Dominant at the LHC!*

Hadronic invariants:

$$S = (P_1 + P_2)^2\,, \quad T_1 = (P_1 - p_3)^2 - m^2_{\tilde{t}_1}\,, \quad U_1 = (P_1 - p_4)^2 - m^2_{\tilde{t}_1}$$

Partonic invariants:

$$s = x_1 x_2 S = (p_1 + p_2)^2\,, \quad t_1 = x_1 T_1\,, \quad u_1 = x_2 U_1$$

$$M^2 = (p_3 + p_4)^2\,, \qquad s_4 = s + t_1 + u_1 = (p_4 + k)^2 - m^2_{\tilde{t}_1}$$

It is possible to define different threshold regions (soft limits) in different kinematics

Soft Limits

Ahrens, Ferroglia, Neubert, Pecjak, Yang

The Pair Invariant Mass kinematics (PIM)

$$(1-z) = 1 - \frac{M^2}{s} \to 0$$

to study the invariant mass distribution

$$\frac{d^2\sigma}{dM\, d\cos\theta}$$

Expansion parameter:

$$E_s/M = (1-z)/(2\sqrt{z})$$

One Particle Inclusive kinematics (1PI)

$$s_4 = (p_4 + k)^2 - m_{\tilde{t}_1}^2 \to 0$$

to study the transverse momentum and rapidity distributions

$$\frac{d^2\sigma}{dp_T dy}$$

Expansion parameter:

$$E_s/m_{\tilde{t}_1} = s_4/(2m_{\tilde{t}_1}\sqrt{m_{\tilde{t}_1}^2 + s_4})$$

- In PIM and 1PI threshold region the top squarks are not necessarily at rest ($\beta \neq 0$)
- Large logarithms appear in the partonic cross section when the "soft variable" is small
- These soft limits correspond to the region where the emitted gluons in the final state are soft
- The difference between PIM and 1PI comes from which combinations of momenta are counted as "small". In PIM $(p_3 + p_4)\cdot k \ll M^2$, in 1PI $p_4 \cdot k \ll m_{\tilde{t}_1}^2$

PIM kinematics

- The differential cross section in the **threshold region** reads

$$\frac{d^2\sigma}{dM\,d\cos\theta} = \frac{\pi\beta_{\tilde{t}_1}}{SM}\int_\tau^1 \frac{dz}{z}\Big[ff_{gg}\left(\frac{\tau}{z},\mu_f\right) C_{gg}\left(z,M,\cos\theta,\mu_f\right) + ff_{q\bar{q}}\left(\frac{\tau}{z},\mu_f\right) C_{q\bar{q}}\left(z,M,\cos\theta,\mu_f\right) + ff_{\bar{q}q}\left(\frac{\tau}{z},\mu_f\right) C_{\bar{q}q}\left(z,M,\cos\theta,\mu_f\right)\Big]$$

$$ff_{i,j}(y,\mu_f) = \int_y^1 \frac{dx}{x} f_{i/N_1}\left(x,\mu_f\right) f_{j/N_2}\left(\frac{y}{x},\mu_f\right)$$

- The hard scattering kernels $C_{ij}\left(z,M,\cos\theta,\mu_f\right)$ factorize into

$$C_{ij}\left(z,M,\cos\theta,\mu_f\right) = \mathrm{Tr}\left[\boldsymbol{H}_{ij}(M,\cos\theta,\mu_f)\boldsymbol{S}_{ij}(\sqrt{s}(1-z),\cos\theta,\mu_f)\right] + \mathcal{O}(1-z)$$

hard function: virtual corrections
contains ordinary functions of the kinematic variables

soft function: real emission of soft gluons
contains plus distributions

- where **H** and **S** are matrices in color space. Writing explicitly the dependence of **H** on the SUSY parameters: $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $m_{\tilde{q}}$, $m_{\tilde{g}}$, m_t, $\alpha_{\tilde{t}_1-\tilde{t}_2}$

- The soft functions **S** arise from the real emissions of soft gluons and contain large logarithms coming from distributions which are singular in the PIM threshold limit

$$\left[\frac{\ln^m(1-z)}{1-z}\right]_+, \quad (m=0,\cdots,2n-1)$$

← *Large Logs: need to be resummed*

1PI kinematics

- The differential cross section in the **threshold region** reads

$$\frac{d^2\sigma}{dp_T dy} = \frac{2\pi p_T}{S} \sum_{ij} \int_{x_1^{\min}}^1 \frac{dx_1}{x_1} \int_{x_2^{\min}}^1 \frac{dx_2}{x_2} f_{i/N_1}(x_1,\mu_f) f_{j/N_2}(x_2,\mu_f) C_{1\mathrm{PI},ij}\left(s_4, s, t_1, u_1, \mu_f\right)$$

- The hard scattering kernels $C_{ij}\left(s_4, s', t_1', u_1', \mu\right)$ factorize into

$$C_{ij}\left(s_4, s', t_1', u_1', \mu\right) = \mathrm{Tr}\left[\boldsymbol{H}_{ij}\left(s', t_1', u_1', \mu\right) \boldsymbol{S}_{ij}\left(s_4, s', t_1', u_1', \mu\right)\right] + \mathcal{O}(s_4)$$

- where **H** and **S** are matrices in color space. **H** is the same for PIM and 1PI, **S** is different in PIM and 1PI. Writing explicitly the dependence of **H** on the SUSY parameters: $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $m_{\tilde{q}}$, $m_{\tilde{g}}$, m_t, $\alpha_{\tilde{t}_1 - \tilde{t}_2}$

- The soft functions **S** arise from the real emissions of soft gluons and contain large logarithms coming from distributions which are singular in the 1PI threshold limit

$$\left[\frac{\ln^m(s_4/m_{\tilde{t}_1}^2)}{s_4}\right]_+, \qquad (m = 1, \cdots, 2n-1)$$

Large Logs: need to be resummed

Approximate NNLO formulas

AB, Ferroglia, Neubert, Vernazza, Yang

- **H** and **S** satisfy RG equations of the form:

$$\frac{d}{d\ln\mu}\boldsymbol{H} = \boldsymbol{\Gamma}_H \boldsymbol{H} + \boldsymbol{H}\boldsymbol{\Gamma}_H^\dagger$$

$$\frac{d}{d\ln\mu}\tilde{\mathbf{s}}_{\{\mathrm{PIM,1PI}\}} = \boldsymbol{\Gamma}^\dagger{}_{s\,\{\mathrm{PIM,1PI}\}}\tilde{\mathbf{s}}_{\{\mathrm{PIM,1PI}\}} + \tilde{\mathbf{s}}_{\{\mathrm{PIM,1PI}\}}\boldsymbol{\Gamma}_{s\,\{\mathrm{PIM,1PI}\}}$$

- The two loop anomalous dimensions including massive partons were computed by [Ferroglia, Neubert, Pecjak, Yang 09']

- The large logarithms could be resummed to all-orders by solving the RG equations for **H** and **S**, but here we follow a different possibility. The resummed formulas can be re-expanded to obtain fixed-oder formulas:

$$C_{\mathrm{PIM}}\left(z, M, \cos\theta, \mu\right) = \tilde{c}_{\mathrm{PIM}}\left(\partial_\eta, M, \cos\theta, \mu\right)\left(\frac{M}{\mu}\right)^{2\eta}\frac{e^{-2\gamma_E\eta}}{\Gamma\left(2\eta\right)}\frac{z^{-\eta}}{\left(1-z\right)^{1-2\eta}}\Bigg|_{\eta=0}$$

$$C_{\mathrm{1PI}}\left(s_4, s', t_1', u_1', \mu\right) = \tilde{c}_{\mathrm{1PI}}\left(\partial_\eta, s', t_1', u_1', \mu\right)\frac{e^{-2\gamma_E\eta}}{\Gamma\left(2\eta\right)}\frac{1}{s_4}\left(\frac{s_4}{\sqrt{m_{\tilde{t}_1}^2 + s_4\mu}}\right)^{2\eta}\Bigg|_{\eta=0}$$

where

$$\tilde{c}_{\mathrm{PIM}}\left(\partial_\eta, M, \cos\theta, \mu\right) = \mathrm{Tr}[\boldsymbol{H}\left(M, \cos\theta, \mu\right)\tilde{\boldsymbol{s}}_{\mathrm{PIM}}\left(\partial_\eta, M, \cos\theta, \mu\right)]$$

$$\tilde{c}_{\mathrm{1PI}}\left(\partial_\eta, s', t_1', u_1', \mu\right) = \mathrm{Tr}[\boldsymbol{H}\left(s', t_1', u_1', \mu\right)\tilde{\boldsymbol{s}}_{\mathrm{1PI}}\left(\partial_\eta, s', t_1', u_1', \mu\right)]\,,$$

Laplace-transformed hard scattering kernels

Approximate NNLO formulas

AB, Ferroglia, Neubert, Vernazza, Yang

- The perturbative expansion of the hard-scattering kernels reads

$$\tilde{c} = \alpha_s^2 \left[\tilde{c}^{(0)} + \frac{\alpha_s}{4\pi} \tilde{c}^{(1)} + \left(\frac{\alpha_s}{4\pi} \right)^2 \tilde{c}^{(2)} + \mathcal{O}(\alpha_s^3) \right]$$

hard-scattering kernels

$$\tilde{c}^{(0)} = \mathrm{Tr}\left[\boldsymbol{H}^{(0)} \tilde{\boldsymbol{s}}^{(0)} \right] , \qquad \tilde{c}^{(1)} = \mathrm{Tr}\left[\boldsymbol{H}^{(0)} \tilde{\boldsymbol{s}}^{(1)} \right] + \mathrm{Tr}\left[\boldsymbol{H}^{(1)} \tilde{\boldsymbol{s}}^{(0)} \right] = \sum_{j=0}^{2} c_j^{(1)} L^j$$

$$\tilde{c}^{(2)} = \mathrm{Tr}\left[\boldsymbol{H}^{(0)} \tilde{\boldsymbol{s}}^{(2)} \right] + \mathrm{Tr}\left[\boldsymbol{H}^{(1)} \tilde{\boldsymbol{s}}^{(1)} \right] + \mathrm{Tr}\left[\boldsymbol{H}^{(2)} \tilde{\boldsymbol{s}}^{(0)} \right] = \sum_{j=0}^{4} c_j^{(2)} L^j$$

constant term unknown — *scale dep. part is known*

- By knowing the analytical expressions for **H** and **S** at NLO in both kinematics, we can solve explicitly the RG equations for **H** and **S** at NNLO
- In this way it is possible to reconstruct the coefficients $c_i^{(2)} \ (i = 1, \cdots, 4)$ and the scale dependent part of $c_0^{(2)}$
- In order not to artificially reduce the scale dependence of the cross section, we drop the scale dependent part of $\mathbf{H}^{(2)}$ from our numerical results

Hard & Soft functions

AB, Ferroglia, Neubert, Vernazza, Yang

- The hard function matrices are fundamental ingredients of this calculation. They have a perturbative expansion in powers of α_s

$$\boldsymbol{H} = \alpha_s^2 \frac{1}{d_R}\left(\boldsymbol{H}^{(0)} + \frac{\alpha_s}{4\pi}\boldsymbol{H}^{(1)} + \dots\right)$$

tree level

$$H_{IJ}^{(0)} = \frac{1}{4}\frac{1}{\langle c_I|c_I\rangle\langle c_J|c_J\rangle}\langle c_I|\mathcal{M}_{\rm ren}^{(0)}\rangle\langle\mathcal{M}_{\rm ren}^{(0)}|c_J\rangle$$

color basis

$$\boldsymbol{H}_{q\bar{q}}^{(0)} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}\frac{1}{t_1+u_1}\left[\frac{t_1u_1}{t_1+u_1} + m_{\tilde{t}_1}^2\right]$$

$$\boldsymbol{H}_{gg}^{(0)} = \begin{pmatrix} \frac{1}{N^2} & \frac{1}{N}\frac{t_1-u_1}{M^2} & \frac{1}{N} \\ \frac{1}{N}\frac{t_1-u_1}{M^2} & \frac{(t_1-u_1)^2}{M^4} & \frac{t_1-u_1}{M^2} \\ \frac{1}{N} & \frac{t_1-u_1}{M^2} & 1 \end{pmatrix}\frac{m_{\tilde{t}_1}^4}{2t_1u_1}\left[-\frac{2M^2}{m_{\tilde{t}_1}^2} + \frac{t_1u_1}{m_{\tilde{t}_1}^4} + 4 + 2\frac{t_1^2+u_1^2}{t_1u_1}\right]$$

one loop

$$H_{IJ}^{(1)} = \frac{1}{4}\frac{1}{\langle c_I|c_I\rangle\langle c_J|c_J\rangle}\left[\langle c_I\,|\,\mathcal{M}_{\rm ren}^{(0)}\rangle\langle\mathcal{M}_{\rm ren}^{(1)}\,|\,c_J\rangle + \langle c_I\,|\,\mathcal{M}_{\rm ren}^{(1)}\rangle\langle\mathcal{M}_{\rm ren}^{(0)}\,|\,c_J\rangle\right]$$

- The calculation involves **40** diagrams in the **qq channel** and **152** diagrams in the **gg channel.** (As a consistency check 30 diagrams in the ghost channel were computed). I used **Form** for the algebraic manipulations and **Reduze** [Studerus, Von Manteuffel] for the reductions to scalar integrals.

- NLO Soft functions for PIM and 1PI already known from top-pair production [Ahrens, Ferroglia, Neubert, Pecjak, Yang 10', 11']. Soft interactions do not depend on the spin.

Approximate NNLO formulas

AB, Ferroglia, Neubert, Vernazza, Yang

$$C_{ij} = \alpha_s^2 \left[C_{ij}^{(0)} + \frac{\alpha_s}{4\pi} C_{ij}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_{ij}^{(2)} + \mathcal{O}(\alpha_s^3) \right],$$

where for PIM

$$C_{\rm PIM}^{(1)} = D_1^{(1,\rm PIM)} \left[\frac{\ln(1-z)}{1-z}\right]_+ + D_0^{(1,\rm PIM)} \left[\frac{1}{1-z}\right]_+ + C_0^{(1,\rm PIM)} \delta(1-z) + R^{(1,\rm PIM)}(z)\,,$$

$$C_{\rm PIM}^{(2)} = D_3^{(2,\rm PIM)} \left[\frac{\ln^3(1-z)}{1-z}\right]_+ + D_2^{(2,\rm PIM)} \left[\frac{\ln^2(1-z)}{1-z}\right]_+ + D_1^{(2,\rm PIM)} \left[\frac{\ln(1-z)}{1-z}\right]_+ + D_0^{(2,\rm PIM)} \left[\frac{1}{1-z}\right]_+ + C_0^{(2,\rm PIM)} \delta(1-z) + R^{(2,\rm PIM)}(z)\,.$$

This would require a complete 2 loop calculation

and for 1PI

$$C_{\rm 1PI}^{(1)} = D_1^{(1,\rm 1PI)} \left[\frac{\ln(s_4/m_{\tilde{t}_1}^2)}{s_4}\right]_+ + D_0^{(1,\rm 1PI)} \left[\frac{1}{s_4}\right]_+ + C_0^{(1,\rm 1PI)} \delta(s_4) + R^{(1,\rm 1PI)}(s_4)\,,$$

$$C_{\rm 1PI}^{(2)} = D_3^{(2,\rm 1PI)} \left[\frac{\ln^3(s_4/m_{\tilde{t}_1}^2)}{s_4}\right]_+ + D_2^{(2,\rm 1PI)} \left[\frac{\ln^2(s_4/m_{\tilde{t}_1}^2)}{s_4}\right]_+ + D_1^{(2,\rm 1PI)} \left[\frac{\ln(s_4/m_{\tilde{t}_1}^2)}{s_4}\right]_+ + D_0^{(2,\rm 1PI)} \left[\frac{1}{s_4}\right]_+ + C_0^{(2,\rm 1PI)} \delta(s_4) + R^{(2,\rm 1PI)}(s_4)\,.$$

Regular terms are formally subleading ,but they improve the numerical agreement between PIM and 1PI

Approximate NLO vs NLO

AB, Ferroglia, Neubert, Vernazza, Yang

The Total Cross Section is obtained by integrating over the differential distributions

Scale Choice

$$\mu_R = \mu_f = \mu$$

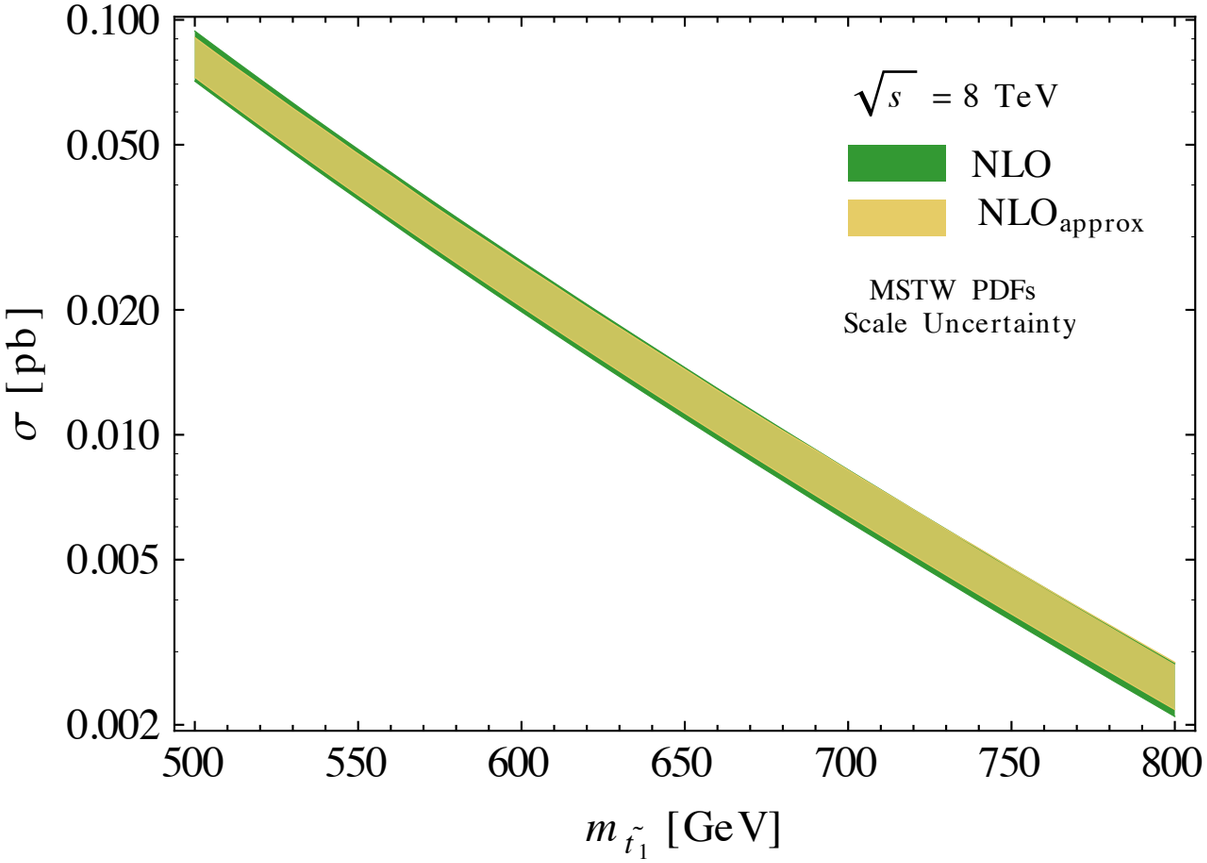
$$m_{\tilde{t}_1}/2 < \mu < 2m_{\tilde{t}_1}$$

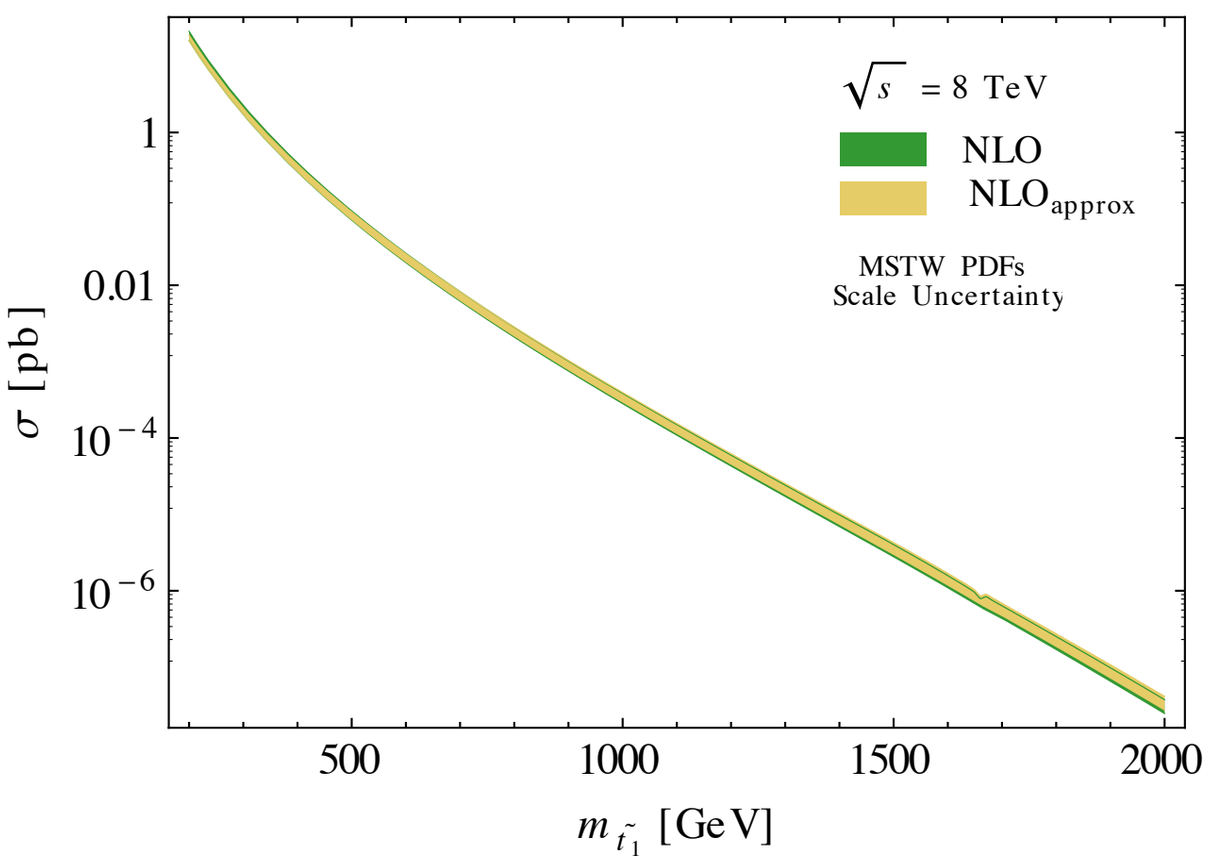
$$\sigma = \frac{1}{2}\left(\sigma_{\rm PIM} + \sigma_{\rm 1PI}\right) ,$$

$$\Delta\sigma^+ = \max\left\{\sigma_{\rm PIM} + \Delta\sigma^+_{\rm PIM}, \sigma_{\rm 1PI} + \Delta\sigma^+_{\rm 1PI}\right\} - \sigma$$

$$\Delta\sigma^- = \min\left\{\sigma_{\rm PIM} + \Delta\sigma^-_{\rm PIM}, \sigma_{\rm 1PI} + \Delta\sigma^-_{\rm 1PI}\right\} - \sigma$$

The scale uncertainty also include a kinematical uncertainty



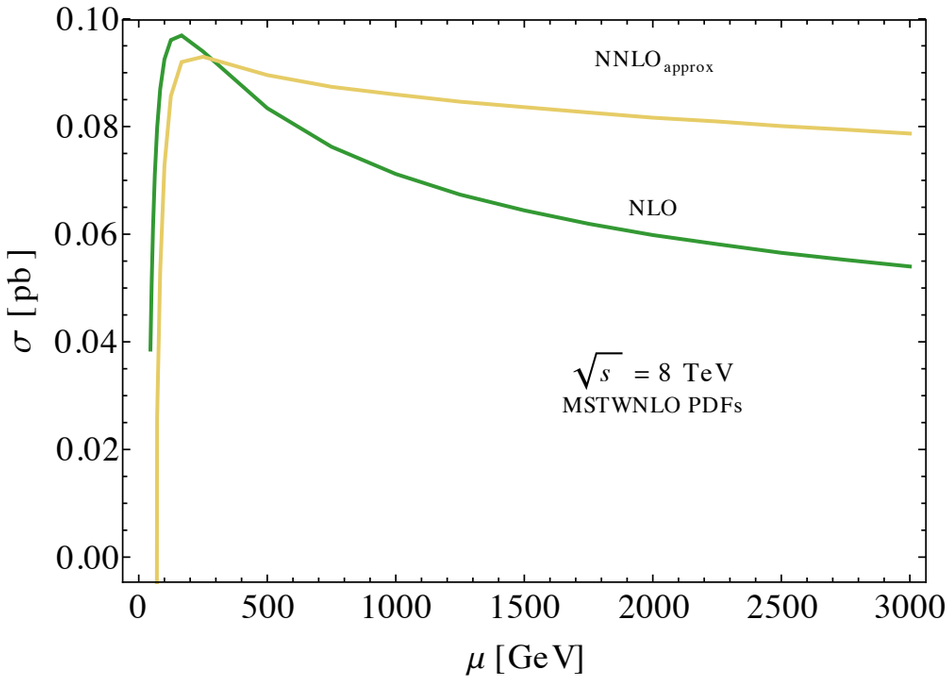


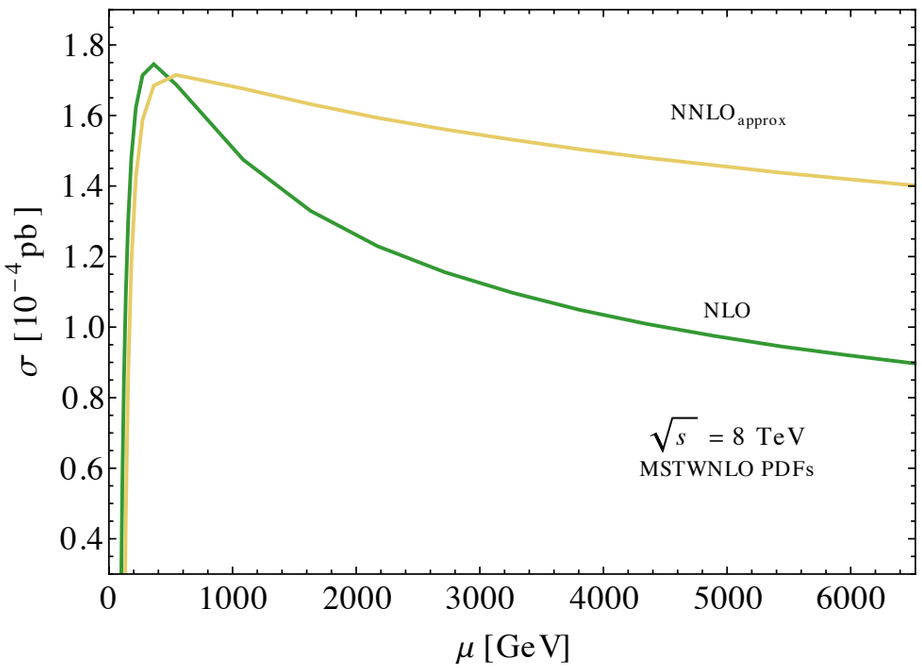
Scale Dependence

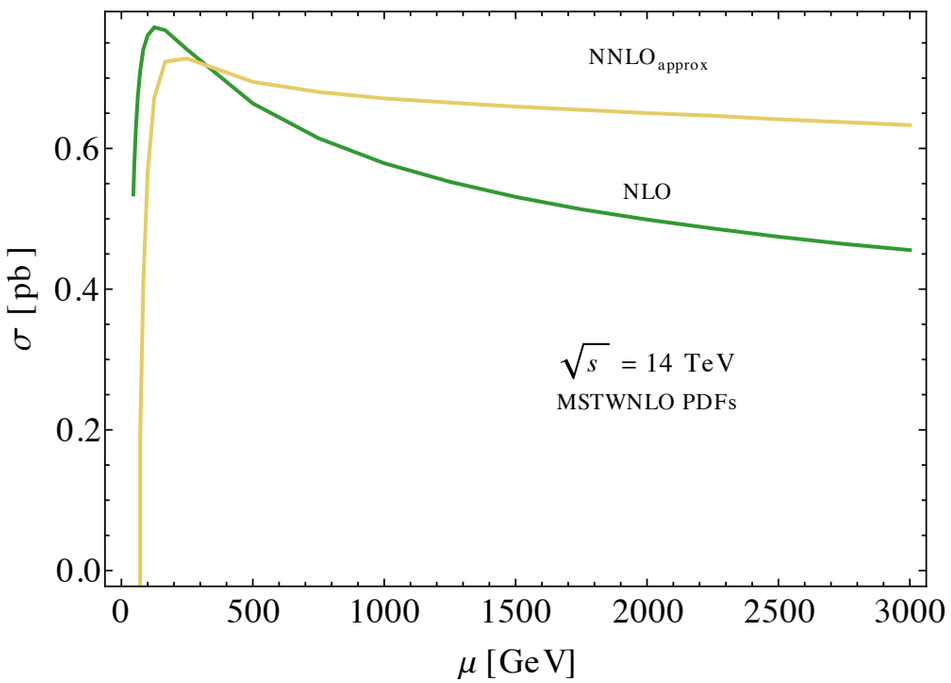
AB, Ferroglia, Neubert, Vernazza, Yang

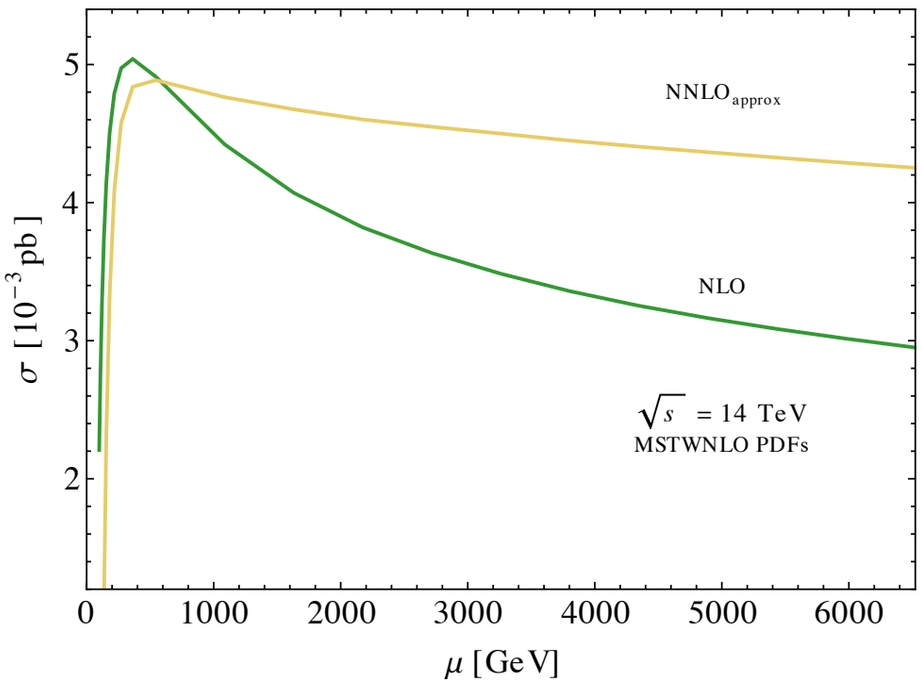
The approximate NNLO results are matched with the exact NLO calculations

$$\sigma_k^{\mathrm{NNLO,\ approx}} = \sigma^{\mathrm{NLO}} + \sigma_k^{(2),\,\mathrm{approx}}$$





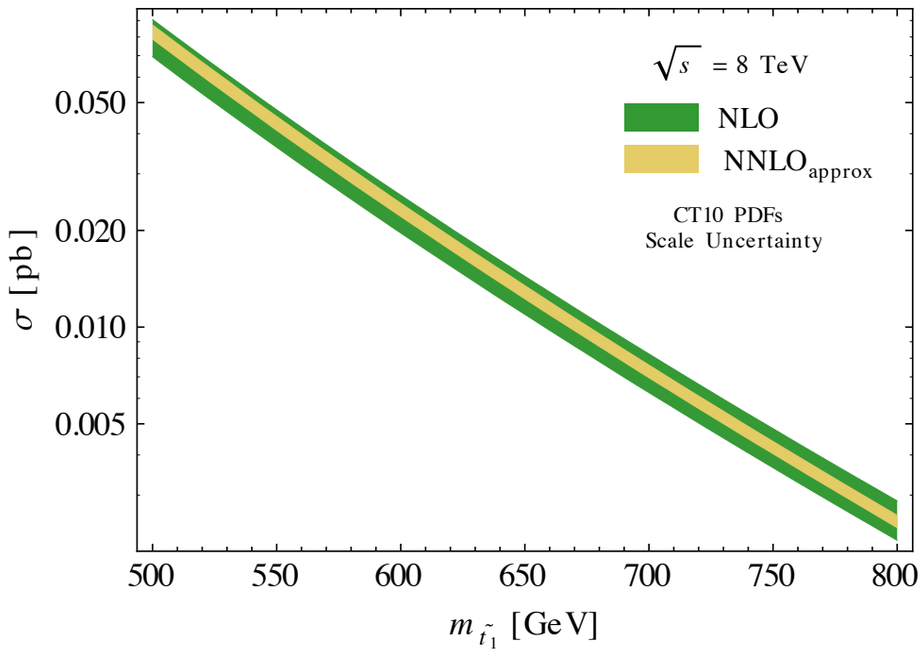


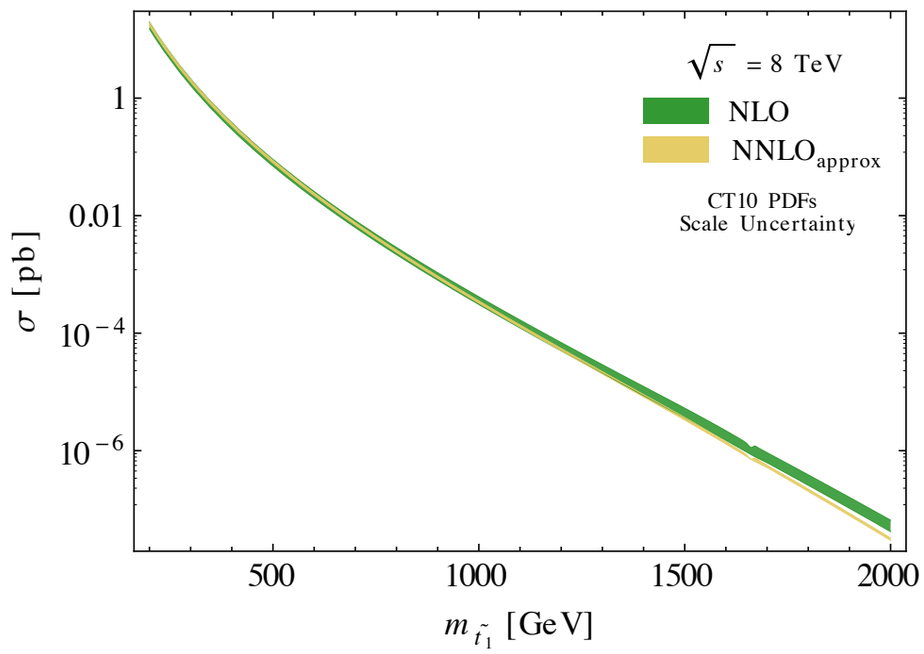


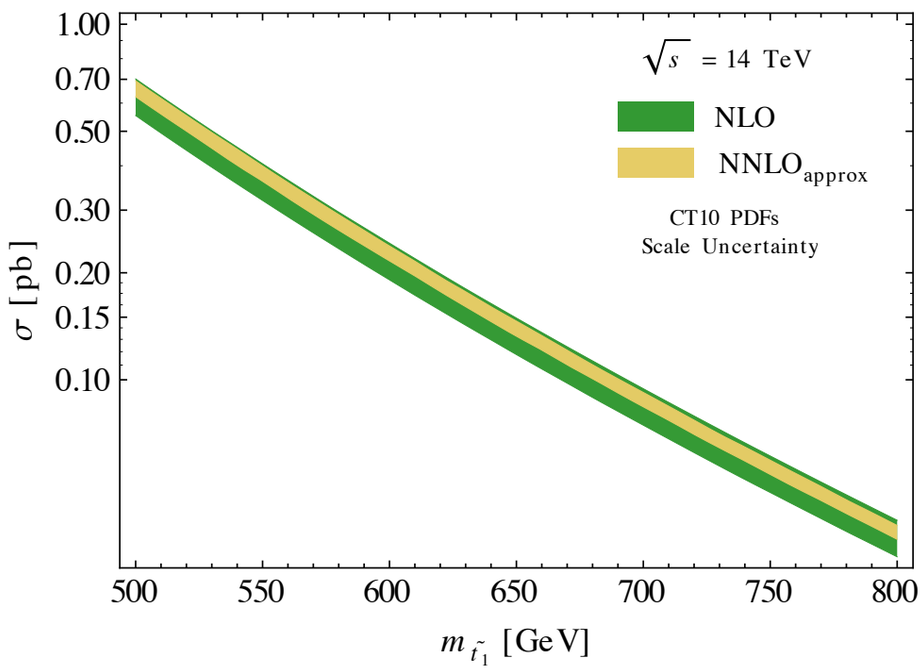
Total Cross Section as a function of the stop mass

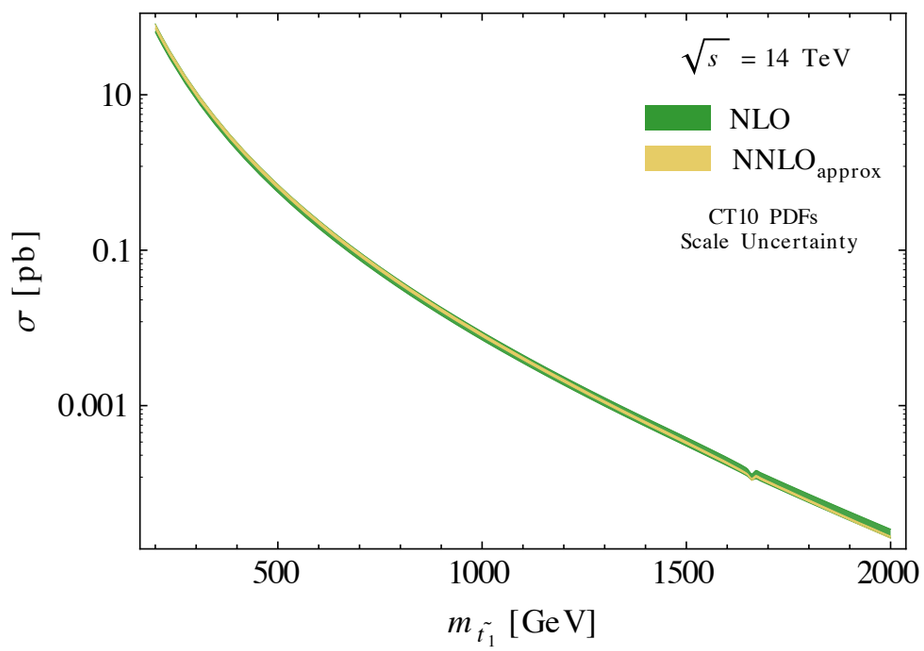
AB, Ferroglia, Neubert, Vernazza, Yang

$$m_{\tilde{t}_1} = 1087.17\,\text{GeV}\,, m_{\tilde{t}_2} = 1319.87\,\text{GeV}\,, m_{\tilde{q}} = 1460.3\,\text{GeV}\,,$$
$$m_{\tilde{g}} = 1489.98\,\text{GeV}\,, m_t = 173.3\,\text{GeV}\,, \alpha_{\tilde{t}} = 68.4^\circ\,, \mu = m_{\tilde{t}_1}$$









Small dependence on SUSY parameters other than the stop mass!

Total Cross Sections

AB, Ferroglia, Neubert, Vernazza, Yang

Scale uncertainties and PDFs + α_s uncertainties

LHC 8 TeV	CT10	
$m_{\tilde{t}_1}$ [GeV]	500	1087.17
$(\sigma \pm \Delta\sigma_\mu \pm \Delta_{\rm pdf+\alpha_s})_{\rm LO}$ [pb]	$54.0^{+21.2+11.0}_{-14.2-8.3} \times 10^{-3}$	$10.6^{+4.8+6.6}_{-3.1-3.2} \times 10^{-5}$
$(\sigma \pm \Delta\sigma_\mu \pm \Delta_{\rm pdf+\alpha_s})_{\rm NLO}$ [pb]	$80.9^{+9.8+16.6}_{-11.4-13.1} \times 10^{-3}$	$16.5^{+2.3+10.4}_{-2.7-5.3} \times 10^{-5}$
$(\sigma \pm \Delta\sigma_\mu \pm \Delta_{\rm pdf+\alpha_s})_{\rm approx.NNLO}$ [pb]	$83.6^{+3.6+19.0}_{-4.8-12.3} \times 10^{-3}$	$15.2^{+0.3+8.1}_{-1.0-4.7} \times 10^{-5}$
$K_{\rm NLO}$	1.50	1.56
$K_{\rm approx.NNLO}$	1.55	1.44

LHC 14 TeV	CT10	
$m_{\tilde{t}_1}$ [GeV]	500	1087.17
$(\sigma \pm \Delta\sigma_\mu \pm \Delta_{\rm pdf+\alpha_s})_{\rm LO}$ [pb]	$42.6^{+14.4+5.0}_{-10.1-4.3} \times 10^{-2}$	$30.1^{+11.3+7.8}_{-7.7-5.2} \times 10^{-4}$
$(\sigma \pm \Delta\sigma_\mu \pm \Delta_{\rm pdf+\alpha_s})_{\rm NLO}$ [pb]	$63.2^{+7.0-7.6}_{-7.8-6.6} \times 10^{-2}$	$44.1^{+4.8+11.7}_{-5.8-8.1} \times 10^{-4}$
$(\sigma \pm \Delta\sigma_\mu \pm \Delta_{\rm pdf+\alpha_s})_{\rm approx.NNLO}$ [pb]	$65.9^{+3.4-8.2}_{-3.4-6.6} \times 10^{-2}$	$44.6^{+1.3+12.1}_{-2.1-7.8} \times 10^{-4}$
$K_{\rm NLO}$	1.48	1.47
$K_{\rm approx.NNLO}$	1.55	1.48

Scale dependence:

- *NLO +13% -15%*
- *Approx-NNLO +4% -6%*

Impact on the central value

- *K(Approx-NNLO)~[0.88-1.06]*

*Approx NNLO central values in **very good** agreement with NLO+NLL [1006.4771]*

The main theoretical uncertainties are coming from PDFs uncertainties!

A Fortran program will be soon available!

Conclusions

- We computed approximate NNLO formulas for **stop-pair production** in PIM and 1PI kinematics
- This was possible after the calculation of the Hard Functions at NLO
- We obtained precise predictions for stop-pair production total cross section
- The main theoretical uncertainties on the total cross sections are due to PDFs uncertainties
- Future Goals: NNLL resummation, improved predictions for squark-pair production

Anomalous dimensions for stop production

$$\mathbf{\Gamma}_H^{\mathrm{PIM}}\left(M,\cos\theta,\alpha_s\right)=\Gamma_{\mathrm{cusp}}(\alpha_s)\left(\ln\frac{M^2}{\mu^2}-i\pi\right)+\boldsymbol{\gamma}^h\left(M,\cos\theta,\alpha_s\right)$$

$$\mathbf{\Gamma}_H^{\mathrm{1PI}}\left(s',t_1',u_1',\alpha_s\right)=\Gamma_{\mathrm{cusp}}(\alpha_s)\left(\ln\frac{s'}{\mu^2}-i\pi\right)+\boldsymbol{\gamma}^h\left(s',t_1',u_1',\alpha_s\right)\ ,$$

$$\tilde{\mathbf{\Gamma}}_{s\mathrm{PIM}}=-\left[\Gamma_{\mathrm{cusp}}(\alpha_s)\ln\frac{M^2}{\mu^2}+2\gamma^\phi\left(\alpha_s\right)\right]\mathbf{1}-\boldsymbol{\gamma}^h\left(M,\cos\theta,\alpha_s\right)$$

$$\tilde{\mathbf{\Gamma}}_{s\mathrm{1PI}}=-\left[\Gamma_{\mathrm{cusp}}(\alpha_s)\ln\frac{s'}{\mu^2}+2\gamma^\phi\left(\alpha_s\right)+\Gamma_{\mathrm{cusp}}(\alpha_s)\log\frac{s'm_{\tilde{t}_1}^2}{t_1'u_1'}\right]\mathbf{1}-\boldsymbol{\gamma}^h\left(s',t_1',u_1',m_{\tilde{t}_1},\alpha_s\right)$$

NNLO solution of the RGE for the Soft function

[arXiv:1103.0550]

$$\tilde{s}(L,\alpha_s(\mu)) = 1 + \frac{\alpha_s(\mu)}{4\pi}\sum_{n=0}^{2} s^{(1,n)} L^n + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 \sum_{n=0}^{4} s^{(2,n)} L^n + \dots$$

$$\tilde{s}(L,\alpha_s(\mu)) = 1 + \frac{\alpha_s(\mu)}{4\pi}\left[\frac{\Gamma_0}{2}L^2 + L\gamma_0^s + s^{(1,0)}\right]$$

$$+ \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 \left[\frac{\Gamma_0^2}{8}L^4 + \left(-\frac{\beta_0\Gamma_0}{6} + \frac{\Gamma_0\gamma_0^s}{2}\right)L^3 + \frac{1}{2}\left(\Gamma_1 - \beta_0\gamma_0^s + (\gamma_0^s)^2 + \Gamma_0 s^{(1,0)}\right)L^2\right.$$

$$\left. + (\gamma_1^s - \beta_0 s^{(1,0)} + \gamma_0^s s^{(1,0)})L + s^{(2,0)}\right].$$

Approximation schemes

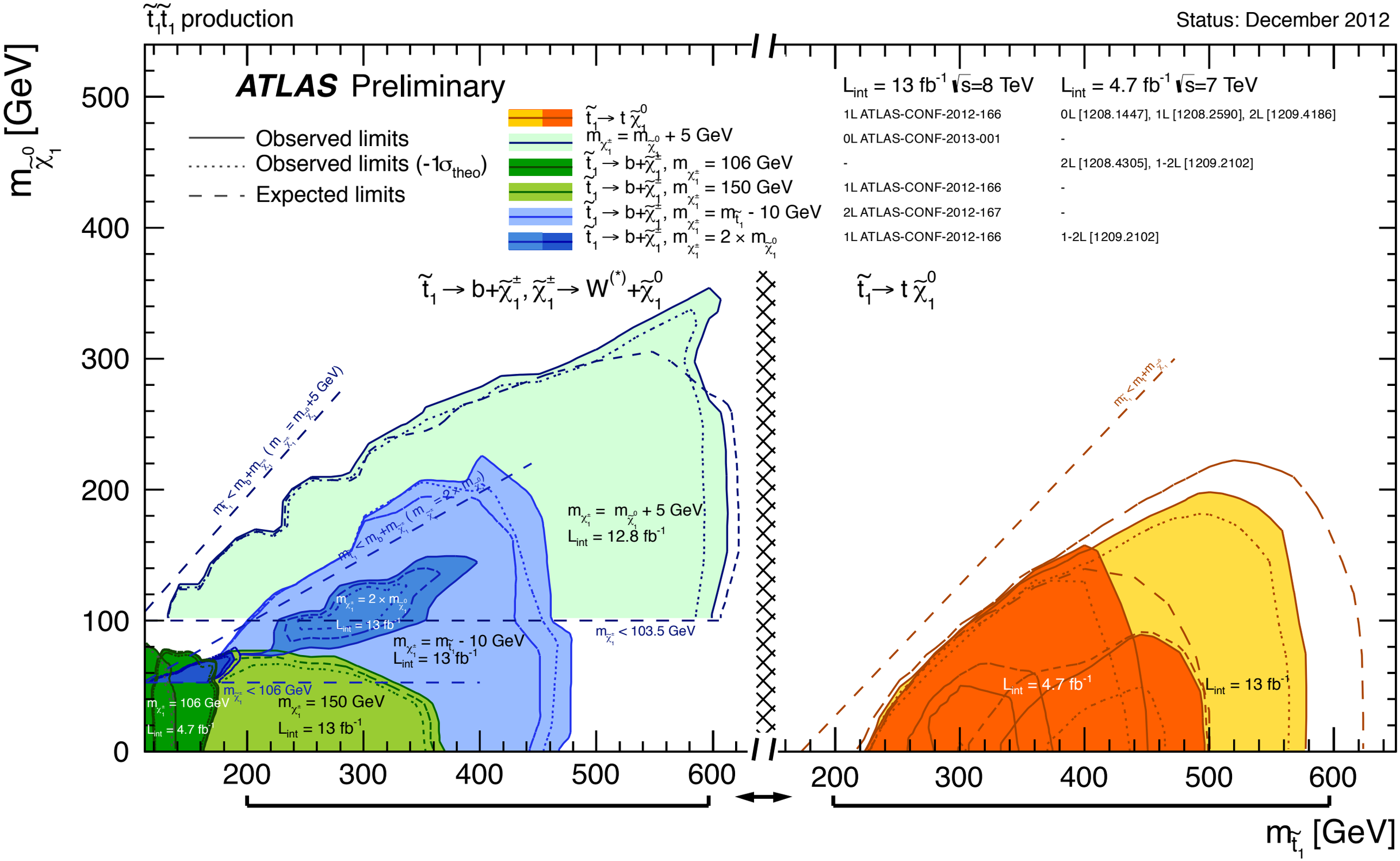
[Becher, Neubert, Xu, 07']

RG-impr. PT	Log. approx.	Accuracy $\sim \alpha_s^n L^k$	$\Gamma_{\rm cusp}$	γ^V, γ^ϕ	C_V, $\widetilde{s}_{\rm DY}$
—	LL	$k = 2n$	1-loop	tree-level	tree-level
LO	NLL	$2n-1 \leq k \leq 2n$	2-loop	1-loop	tree-level
NLO	NNLL	$2n-3 \leq k \leq 2n$	3-loop	2-loop	1-loop
NNLO	NNNLL	$2n-5 \leq k \leq 2n$	4-loop	3-loop	2-loop

For Sudakov problems the counting of the logarithms is done in the exponent!

The large logarithms count as $1/\alpha_s$, it is always possible to rewrite a log of a ratio of two scales as

$$\ln \frac{\nu}{\mu'} = \int_{\alpha_s(\mu')}^{\alpha_s(\nu)} \frac{d\alpha}{\beta(\alpha)} \qquad \beta(\alpha_s) = -2\alpha_s \left[\beta_0 \left(\frac{\alpha_s}{4\pi}\right) + \beta_1 \left(\frac{\alpha_s}{4\pi}\right)^2 + \mathcal{O}(\alpha_s^3)\right]$$

$$\beta(\alpha_s) \sim \alpha_s^2$$

Stop exclusion limits