Mass modes & secondary heavy quark radiation I

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Mass effects in QCD

in QCD: e.g. DIS

- mass effects via different schemes (FFNS, VFNS, ...)
  $\longrightarrow$ correct limiting behavior, continuous description?
- ACOT scheme (VFNS): heavy quark production (3 scales: Q, m, Λ_{QCD})
  Aivazis, Collins, Olness, Tung (1994)
  factorization theorem interpolating between the regions
  $\longrightarrow m \gg Q$: full decoupling
  $\longrightarrow m \sim Q$: exact kinematics, mass effects in Wilson coefficients (FFNS)
  $\longrightarrow m \ll Q$: Log-resummation, massless kinematics, mass effects in pdf's

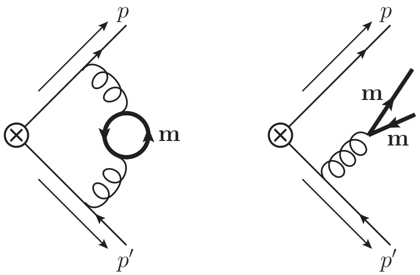
$\Rightarrow$ setup for additional scales? e.g. in endpoint region $x \to 1$: $Q^2(1-x)$

Mass effects in SCET

in SCET: event shapes for $e^+e^- \to jets$

- factorization and resummation for production of primary massive $t\bar{t}$-pairs
  `Fleming, Hoang, Mantry, Stewart (2008)`

$\Rightarrow$ still missing: systematic treatment of secondary massive quarks
$\rightarrow$ nontrivial setup with dynamical thresholds



Outline

1. Factorization theorem for thrust
2. Mass modes in SCET
3. Massive gauge boson radiation

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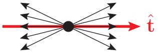
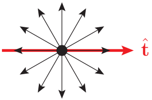
Thrust distribution

- thrust: $\tau \equiv 1 - \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i E_i} \in [0, \frac{1}{2}]$ (semantics: = 2-jettiness here!)

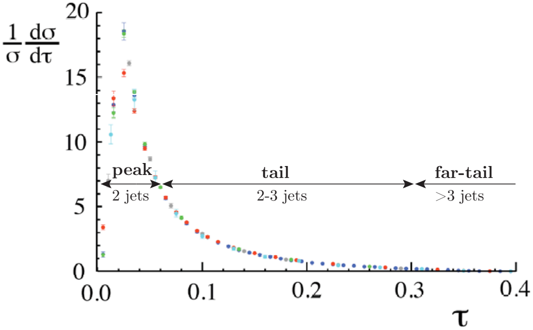
back-to-back: $\tau \to 0$



isotropic: $\tau \to \frac{1}{2}$



- thrust distribution from LEP data ($e^+e^- \to$ *jets*)

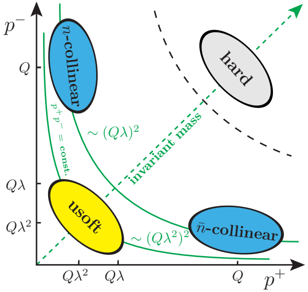


$\to$ peak region ($\tau \sim \Lambda_{QCD}/Q$): expansion parameter $\lambda = \sqrt{\Lambda_{QCD}/Q}$

$\to$ tail region ($\tau \gg \Lambda_{QCD}/Q$): expansion parameter $\lambda = \sqrt{\tau}$

Factorization theorem for thrust

Fleming, Hoang, Mantry, Stewart (2007)
Bauer, Fleming, Lee, Sterman (2008)



SCET result for $\tau \ll 1$:

$$\frac{d\sigma}{d\tau} = Q^2 \sigma_0 H_0(Q,\mu) \int d\ell\, J_0(Q\ell,\mu)\, S_0(Q\tau - \ell, \mu)$$

$Q \to$ CM energy, $\sigma_0 \to$ cross-section at LO

Ingredients:

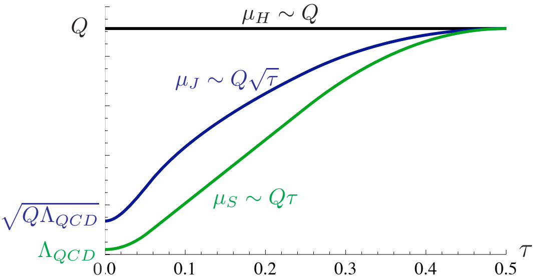
- hard function: $H_0(Q,\mu) = |C_V(Q,\mu)|^2$
- thrust jet function: $J_0(s,\mu) = \int ds'\; J_n(s',\mu) J_{\bar{n}}(s-s',\mu)$
- thrust soft function: $S_0(\ell,\mu) \equiv \int dk_R\, dk_L\, \delta(\ell - k_R - k_L)\, S_0^{\text{hemi}}(k_R,k_L,\mu)$
  $\to \mu_S \sim Q\lambda^2 \sim \Lambda_{QCD}$: $S_0 = S_0^{\text{model}}$: non-perturbative model
  $\to \mu_S \sim Q\lambda^2 \gg \Lambda_{QCD}$: $S_0 = S_0^{\text{part}} \otimes S_0^{\text{model}}$: S_0^{part} partonic piece (perturbative)

Profile functions

Profile functions: Parametrization of renormalization scales in terms of thrust
$\rightarrow$ continuous transition between peak, tail and far-tail region

Abbate, Fickinger, Hoang, Mateu, Stewart (2011)

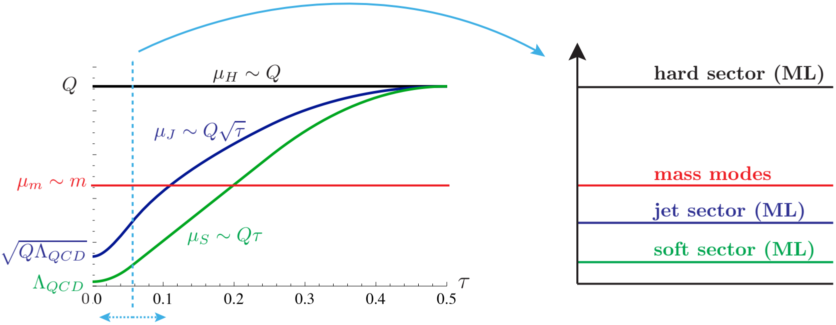
region		μ_H	μ_J	μ_S
peak	$\tau \sim \Lambda_{\rm QCD}/Q$	Q	$\sqrt{Q\Lambda_{\rm QCD}}$	$\Lambda_{\rm QCD}$
tail	$\Lambda_{\rm QCD}/Q \ll \tau \leq 1/3$	Q	$Q\sqrt{\tau}$	$Q\tau$
far-tail	$1/3 \leq \tau \leq 1/2$	Q	Q	Q



Scale hierarchies with a mass

introduce particle species with mass m
$\rightarrow$ several scale hierarchies in one single event shape spectrum (for $Q > m$)

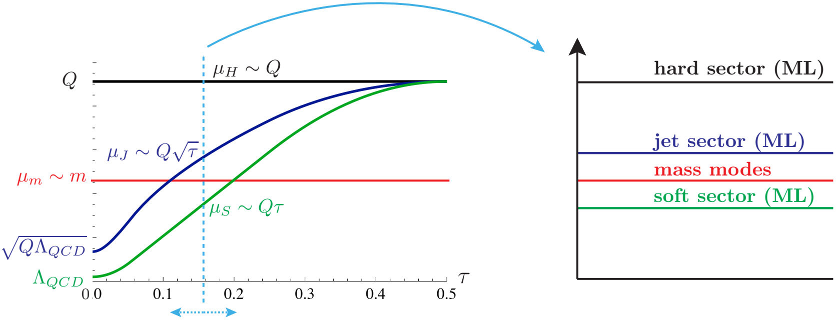
$$\mu_H > \mu_m > \mu_J > \mu_S$$



Scale hierarchies with a mass

introduce particle species with mass m
$\rightarrow$ several scale hierarchies in one single event shape spectrum (for $Q > m$)

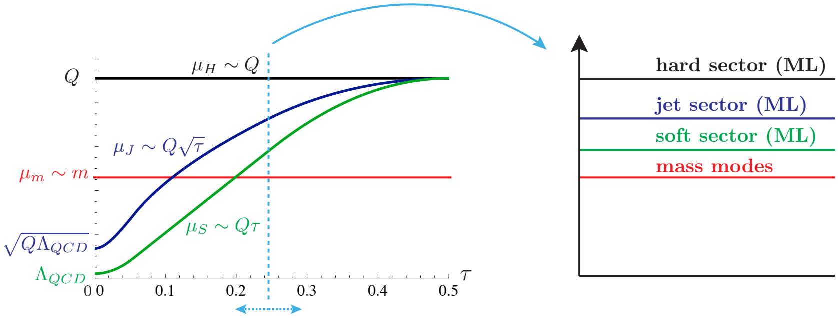
Scenario III: $\mu_H > \mu_J > \mu_m > \mu_S$



Scale hierarchies with a mass

introduce particle species with mass m
$\rightarrow$ several scale hierarchies in one single event shape spectrum (for $Q > m$)

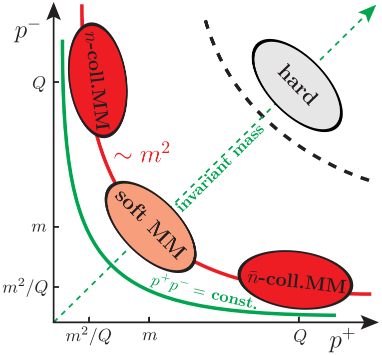
Scenario IV: $\mu_H > \mu_J > \mu_S > \mu_m$



Outline

Mass modes

- new degrees of freedom: "mass modes"
- additional scaling parameter: $\lambda_m = m/Q$



mode	$p^\mu = (+, -, \perp)$	p^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2

$\Rightarrow$ How to include mass modes into massless setup?

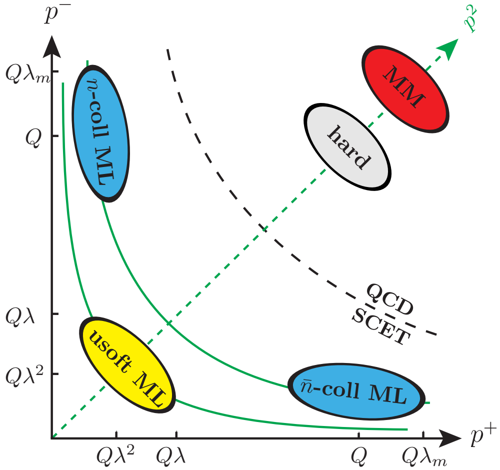
Mass mode setup

- construct a sequence of EFTs depending on $\lambda_m \leftrightarrow \lambda$

$$\text{I. } \lambda_m > 1 > \lambda > \lambda^2$$
$$\text{II. } 1 > \lambda_m > \lambda > \lambda^2$$
$$\text{III. } 1 > \lambda > \lambda_m > \lambda^2$$
$$\text{IV. } 1 > \lambda > \lambda^2 > \lambda_m$$

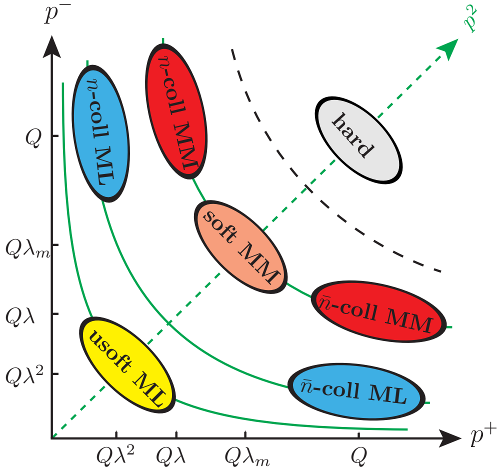
- aims:
  $\rightarrow$ continuity between scaling situations (“scenarios”)
  $\rightarrow$ mass-independent UV divergences
  $\rightarrow$ decoupling for $m \rightarrow \infty$
  $\rightarrow$ correct IR-finite massless limit for $m \rightarrow 0$

I. $\lambda_m > 1 > \lambda > \lambda^2$



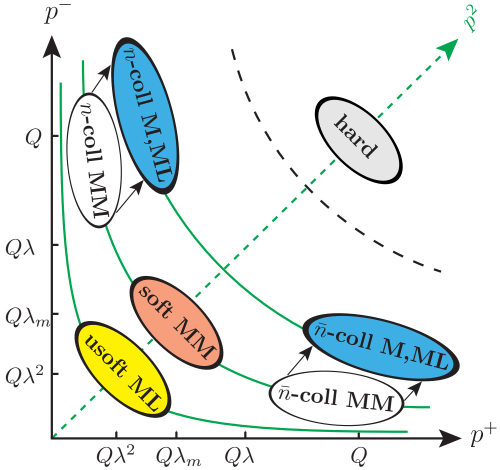
mode	$p^\mu = (+,-,\perp)$	p^2
hard	$Q(1,1,1)$	Q^2
n-coll ML	$Q(\lambda^2,1,\lambda)$	$Q^2\lambda^2$
usoft ML	$Q(\lambda^2,\lambda^2,\lambda^2)$	$Q^2\lambda^4$

II. $1 > \lambda_m > \lambda > \lambda^2$



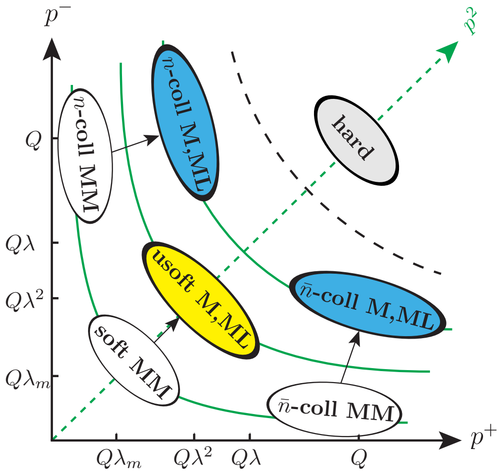
mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1, 1, 1)$	Q^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

III. $1 > \lambda > \lambda_m > \lambda^2$



mode	$p^\mu = (+, -, \perp)$	p^2
hard	$Q(1,1,1)$	Q^2
n-coll M	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
n-coll ML	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

IV. $1 > \lambda > \lambda^2 > \lambda_m$



mode	$p^\mu = (+, -, \perp)$	p^2
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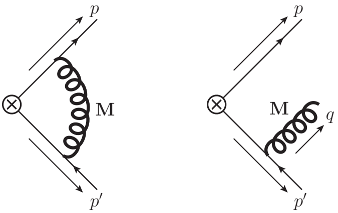
usoft ML	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$

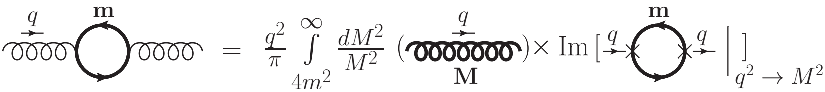
Outline

Motivation

Setup for massive gauge boson (mass *M*) with vector coupling (group factors denoted as in *SU*(3))

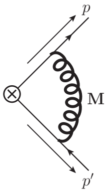


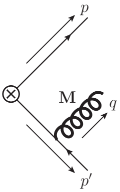
- dispersive technique:



- separate mass mode concept from technical issues at two-loop
- in principle applicable for EW effects, ...

Full theory result

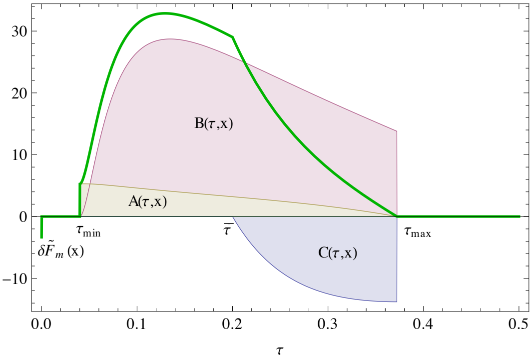




$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \frac{\alpha_s C_F}{4\pi}\Big\{\delta(\tau)\,\delta\tilde{F}_m^{\rm QCD}\left(\frac{M^2}{Q^2}\right) + \theta(\tau-\tau_{\rm min})\left[A\left(\tau,\frac{M^2}{Q^2}\right)+B\left(\tau,\frac{M^2}{Q^2}\right)\right] + \theta(\tau-\bar{\tau})\,C\left(\tau,\frac{M^2}{Q^2}\right)\Big\}$$

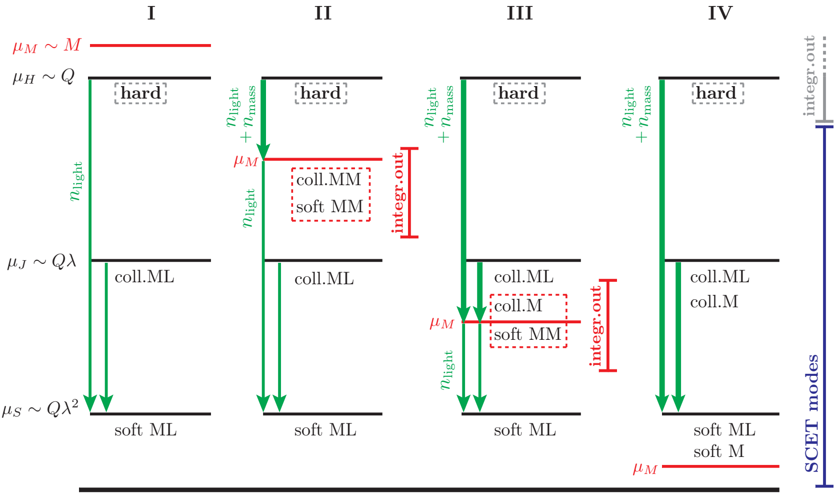
$\tau_{\rm min} = \frac{M^2}{Q^2} \longrightarrow$ threshold for real jet radiation

$\bar{\tau} = \frac{M}{Q} \longrightarrow$ threshold for real soft radiation



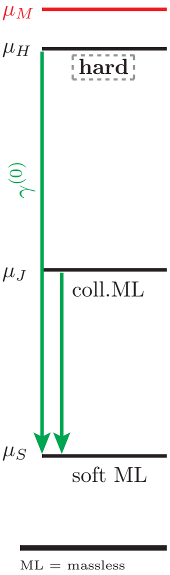
Mass mode setup: RG evolution

Top-down evolution: evolve to $\mu \sim \mu_S$



MM = mass-mode, ML = massless, M = massive

Scenario I: $\lambda_M > 1 > \lambda > \lambda^2$



integrate out mass modes at QCD level
$\rightarrow$ modification of hard matching coefficient,
otherwise like in massless SCET

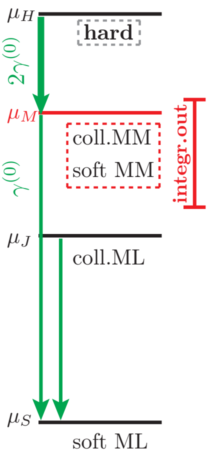
$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^I(\mu_H)\right|^2 U_H^{(0)}(\mu_H, \mu_S) \times \int d\ell \int ds\, J_0(s)\, U_J^{(0)}(Q\ell - s, \mu_S, \mu_J)\, S_0(Q\tau - \ell, \mu_S)$$

$U_H^{(0)}$, $U_J^{(0)}$: massless evolution factors

$$\mathcal{C}^I(\mu_H) = C_0(\mu_H) + \delta F_m^{\text{QCD}}$$

C_0: massless matching coefficient
δF_m^{QCD}: massive full theory contribution (OS)
$\rightarrow$ decoupling for $M/Q \rightarrow \infty$

Scenario II : $1 > \lambda_M > \lambda > \lambda^2$



ML = massless
MM = mass mode

mass modes enter SCET, but integrated out before the jet scale

$\rightarrow$ modification of the matching coefficient at μ_H

$\rightarrow$ additional matching contribution at μ_M

$\rightarrow$ massless jet & soft function

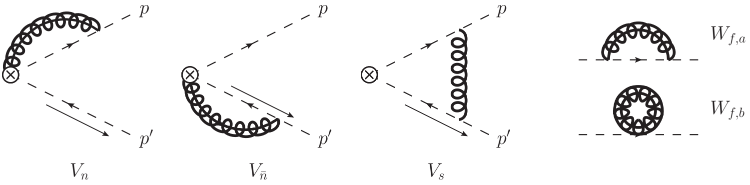
$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_M)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M,\mu_S) \times \int d\ell \int ds\, J_0(s,\mu_J)\, U_J^{(0)}(Q\ell - s,\mu_S,\mu_J)\, S_0(Q\tau-\ell,\mu_S)$$

$U_H^{(1)}$: evolution factor ($\gamma_H^{(1)} = 2\gamma_H^{(0)}$)

$$\mathcal{C}^{II}(\mu_H) = \mathcal{C}^{I}(\mu_H) - \delta F_m^{\text{eff}}(\mu_H)$$

δF_m^{eff}: massive SCET contribution

Hard function contribution δF_m^{eff}



$$\delta F_m^{\text{eff}}(Q, M, \mu) = \frac{\alpha_s C_F}{4\pi} \left\{ \ln\left(\frac{M^2}{\mu^2}\right) \left[2 \ln\left(\frac{-Q^2}{\mu^2}\right) - \ln\left(\frac{M^2}{\mu^2}\right) - 3 \right] - \frac{5\pi^2}{6} + \frac{9}{2} \right\}$$

Chiu, Golf, Kelley, Manohar (2008)

Chiu, Fuhrer, Hoang, Kelley, Manohar (2009)

double counting of mass mode effects!

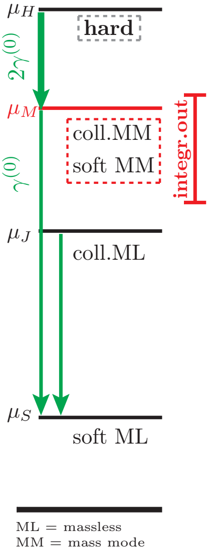
$\rightarrow$ subtraction of collinear diagrams with soft scaling necessary = soft-bin subtraction

$\rightarrow$ allows to obtain regulator-independent, gauge-invariant result

correct massless limit for matching coefficient:

$$\mathcal{C}^{II}(Q, M, \mu_H) = \mathcal{C}^{I}(\mu_H) - \delta F_m^{\text{eff}}(\mu_H) \stackrel{M \to 0}{\longrightarrow} 2C_0(Q, \mu_H)$$

Scenario II : $1 > \lambda_M > \lambda > \lambda^2$



$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_M)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M,\mu_S) \times \int d\ell \int ds\, J_0(s,\mu_J)\, U_J^{(0)}(Q\ell - s,\mu_S,\mu_J)\, S_0(Q\tau-\ell,\mu_S)$$

$U_H^{(1)}$: evolution factor ($\gamma_H^{(1)} = 2\gamma_H^{(0)}$)

$$\mathcal{C}^{II}(\mu_H) = \mathcal{C}^{I}(\mu_H) - \delta F_m^{\text{eff}}(\mu_H)$$

$$\mathcal{M}_H(\mu_M) = 1 + \delta F_m^{\text{eff}}(\mu_M)$$

δF_m^{eff}: massive SCET contribution

continuity to scenario I for $\mu_M = \mu_H$:

$$\left|\mathcal{C}^{II}(\mu_H)\right|^2 \left|\mathcal{M}_H(\mu_H)\right|^2 = \left|\mathcal{C}^{I}(\mu_H)\right|^2$$

Scenario II: Matching to full theory

- expansion of the most singular terms, i.e. for $\tau \sim M^2/Q^2 \ll 1$ and $\tau \sim M/Q \ll 1$

$$\frac{1}{\sigma_0}\,\frac{d\sigma^{\text{full th.}}}{d\tau}\bigg|_{\rm FO} = \delta(\tau) + 2\,\mathrm{Re}\left[\delta F_m^{\rm QCD}\left(\frac{M^2}{Q^2}\right)\right]\,\delta(\tau)$$
$$+\,Q^2\theta\left(\tau-\frac{M^2}{Q^2}\right)\delta J_m^{\rm real}(Q^2\tau) + Q\,\theta\left(\tau-\frac{M}{Q}\right)\delta S_m^{\rm real}(Q\tau)$$

Scenario II: Matching to full theory

- expansion of the most singular terms, i.e. for $\tau \sim M^2/Q^2 \ll 1$ and $\tau \sim M/Q \ll 1$

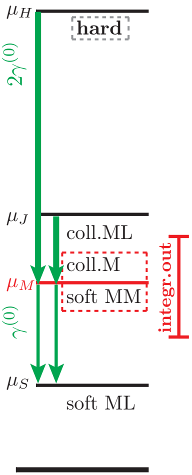
$$\frac{1}{\sigma_0}\frac{d\sigma^{\text{full th.}}}{d\tau}\bigg|_{\text{FO}} = \delta(\tau) + 2\,\text{Re}\left[\delta F_m^{\text{QCD}}\left(\frac{M^2}{Q^2}\right)\right]\delta(\tau) + Q^2\theta\left(\tau - \frac{M^2}{Q^2}\right)\delta J_m^{\text{real}}(Q^2\tau) + Q\,\theta\left(\tau - \frac{M}{Q}\right)\delta S_m^{\text{real}}(Q\tau)$$

- matching with SCET result at fixed order gives mass matching functions in scenario II, $\tau < M^2/Q^2$:

$$\frac{1}{\sigma_0}\frac{d\sigma^{\text{SCET}}}{d\tau}\bigg|^{\text{II}} = \delta(\tau) + 2\,\text{Re}\left[\delta F_m^{\text{QCD}}\left(\frac{M}{Q}\right)\right]\delta(\tau) + 2\,\text{Re}\left[\delta F_m^{\text{eff}}(Q,M,\mu) + \mathcal{M}_H^{(1)}(Q,M,\mu)\right]\delta(\tau)$$

$$\rightarrow \text{Re}\left[\mathcal{M}_H^{(1)}(Q,M,\mu)\right] = -\text{Re}\left[\delta F_m^{\text{eff}}(Q,M,\mu)\right]$$

Scenario III: $1 > \lambda > \lambda_M > \lambda^2$



massive and massless collinear modes fluctuate over comparable scales ($\lambda_M \leq \lambda$)
→ assign collinear massless scaling (keep $M \neq 0$)
→ modification of the jet function at μ_J
→ additional jet matching contribution at μ_M
→ massless soft function

$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}''(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_m)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M,\mu_S) \times \int d\ell \int ds \int ds' \int ds'' J_{0+m}(s,\mu_J)\, U_J^{(1)}(s'-s,\mu_M,\mu_J) \times \mathcal{M}_J(s''-s',\mu_M)\, U_J^{(0)}(s''-Q\ell,\mu_S,\mu_M)\, S_0(Q\tau-\ell,\mu_S)$$

$$J_{0+m}(s,\mu_J) = J_0(s,\mu_J) + \delta J_m^{\rm virt}(s,\mu_J) + \theta(s-M^2)\,\delta J_m^{\rm real}(s)$$

$\delta J_m^{\rm virt}$: virtual piece of jet function (distributive structure)
$\delta J_m^{\rm real}$: real radiation piece of jet function (function)

Jet function

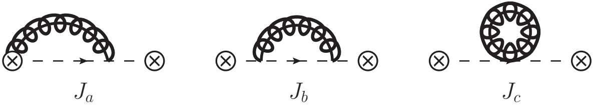


diagram J_a individually not well-defined $\to$ soft-bin subtractions are crucial!

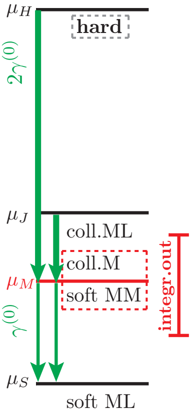
$$J_{0+m}(s,M,\mu) = J_0(s,\mu) + \delta J_m^{\rm virt}(s,M,\mu) + \theta(s-M^2)\,\delta J_m^{\rm real}(s,M)$$

$$\mu^2 \delta J_m^{\rm virt}(s,M,\mu) = \frac{\alpha_s C_F}{4\pi}\left\{\delta(\bar{s})\left[-4\ln^2\left(\frac{M^2}{\mu^2}\right) - 6\ln\left(\frac{M^2}{\mu^2}\right) + 9 - 2\pi^2\right] + 8\ln\left(\frac{M^2}{\mu^2}\right)\left[\frac{\theta(\bar{s})}{\bar{s}}\right]_+\right\}$$

$$\delta J_m^{\rm real}(s,M) = \frac{\alpha_s C_F}{4\pi}\left\{\frac{2(M^2-s)(3s+M^2)}{s^3} + \frac{8}{s}\ln\left(\frac{s}{M^2}\right)\right\}$$

$\to$ $\delta J_m^{\rm virt}$ = virtual radiation ($\bar{s} \equiv s/\mu^2$)
$\to$ $\delta J_m^{\rm real}$ = real radiation for $s > M^2$, continuous: $\delta J_m^{\rm real}(s = M^2, M) = 0$
$\to$ correct massless limit: $J_{0+m}(s,M,\mu_J) \xrightarrow{M\to 0} 2J_0(s,\mu_J)$

Scenario III: $1 > \lambda > \lambda_M > \lambda^2$



$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_m)\left|\mathcal{M}_H(\mu_M)\right|^2 U_H^{(0)}(\mu_M,\mu_S) \times \int d\ell \int ds \int ds' \int ds'' J_{0+m}(s,\mu_J)\, U_J^{(1)}(s'-s,\mu_M,\mu_J) \times \mathcal{M}_J(s''-s',\mu_M)\, U_J^{(0)}(s''-Q\ell,\mu_S,\mu_M)\, S_0(Q\tau-\ell,\mu_S)$$

$$J_{0+m}(s,\mu_J) = J_0(s,\mu_J) + \delta J_m^{\rm virt}(s,\mu_J) + \theta(s-M^2)\,\delta J_m^{\rm real}(s)$$

$$\mathcal{M}_J(s,\mu_M) = \delta(s) - \delta J_m^{\rm virt}(s,\mu_M)$$

$\delta J_m^{\rm virt}$: virtual piece of jet function (distributive structure)
$\delta J_m^{\rm real}$: real radiation piece of jet function (function)

continuity to scenario II for $\mu_M = \mu_J$ ($\mu_M \le M$):

$$J_{0+m}(s,\mu_J)\,\mathcal{M}_J(s,\mu_J) = J_0(s,\mu_J)$$

ML = massless
MM = mass mode
M = massive

Scenario III: Matching to full theory

- expansion of the most singular terms, i.e. for $\tau \sim M^2/Q^2 \ll 1$ and $\tau \sim M/Q \ll 1$

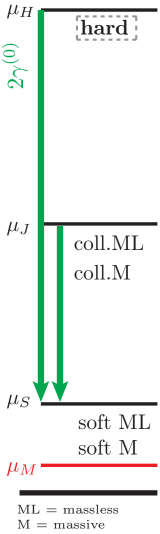
$$\frac{1}{\sigma_0}\frac{d\sigma^{\text{full th.}}}{d\tau}\bigg|_{\rm FO} = \delta(\tau) + 2\,\text{Re}\left[\delta F_m^{\rm QCD}\left(\frac{M^2}{Q^2}\right)\right]\delta(\tau) + Q^2\theta\left(\tau - \frac{M^2}{Q^2}\right)\delta J_m^{\rm real}(Q^2\tau) + Q\,\theta\left(\tau - \frac{M}{Q}\right)\delta S_m^{\rm real}(Q\tau)$$

- matching with SCET result at fixed order gives mass matching functions e.g. in scenario III, $M/Q > \tau > M^2/Q^2$:

$$\begin{aligned}\frac{1}{\sigma_0}\frac{d\sigma^{\rm SCET}}{d\tau}\bigg|^{\rm III} = {} & \delta(\tau) + 2\,\text{Re}\left[\delta F_m^{\rm QCD}\left(\frac{M}{Q}\right)\right]\delta(\tau) \\ & + 2\,\text{Re}\left[\delta F_m^{\rm eff}(Q,M,\mu) + \mathcal{M}_H^{(1)}(Q,M,\mu)\right]\delta(\tau) \\ & + Q^2\theta\left(\tau - \frac{M^2}{Q^2}\right)\delta J_m^{\rm real}(Q^2\tau, M) \\ & + Q^2\left[\delta J_m^{\rm virt}(Q^2\tau, M, \mu) + \mathcal{M}_J^{(1)}(Q^2\tau, M, \mu)\right]\end{aligned}$$

$\rightarrow \mathcal{M}_J^{(1)}(Q^2\tau, M, \mu) = -\delta J_m^{\rm virt}(Q^2\tau, M, \mu)$ (integrate out virtual contributions)
$\rightarrow$ no real radiation appearing in mass matching functions

Scenario IV: $1 > \lambda > \lambda^2 > \lambda_M$



massive soft and massless usoft modes fluctuate over comparable scales ($\lambda_M \leq \lambda^2$)

$\rightarrow$ assign usoft massless scaling (keep $M \neq 0$)!

$\rightarrow$ all structures get massive contributions

$\rightarrow$ massive modes stay in the game to the end

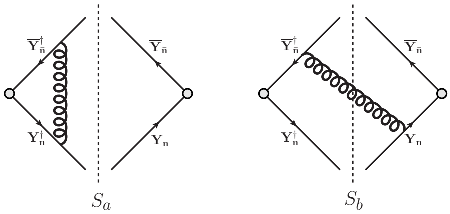
$$\frac{d\sigma}{d\tau} \sim \left|C^{ll}(\mu_H)\right|^2 U_H^{(1)}(\mu_H, \mu_S) \times \int d\ell \int ds\, J_{0+m}(s, \mu_J)\, U_J^{(1)}(Q\ell - s, \mu_S, \mu_J)\, S_{0+m}(Q\tau - \ell, \mu_S)$$

$$S_{0+m}(\ell, \mu_S) = S_0(\ell, \mu_S) + \delta S_m^{\text{virt}}(\ell, \mu_S) + \theta(\ell - M)\, \delta S_m^{\text{real}}(\ell)$$

δS_m^{virt}: virtual piece of massive soft function (distributive structure)

δS_m^{real}: real radiation piece of massive soft function (function)

Soft function



$$S_{0+m}(\ell, M, \mu) = S_0(\ell,\mu) + \delta S_m^{\rm virt}(\ell, M, \mu) + \theta(\ell - M)\,\delta S_m^{\rm real}(\ell, M)$$

$$\mu\,\delta S_m^{\rm virt}(\ell, M, \mu) = \frac{\alpha_s C_F}{4\pi}\left\{\delta(\bar{\ell})\left[2\ln^2\left(\frac{M^2}{\mu^2}\right) + \frac{\pi^2}{3}\right] - 8\ln\left(\frac{M^2}{\mu^2}\right)\left[\frac{\theta(\bar{\ell})}{\bar{\ell}}\right]_+\right\}$$

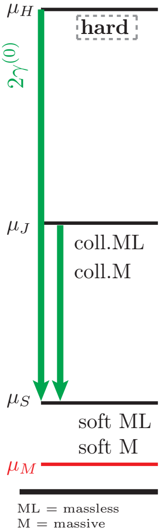
$$\delta S_m^{\rm real}(\ell, M) = \frac{\alpha_s C_F}{4\pi}\left\{-\frac{8}{\ell}\ln\left(\frac{\ell^2}{M^2}\right)\right\}$$

$\longrightarrow$ $\delta S_m^{\rm virt}$ = virtual radiation ($\bar{\ell} \equiv \ell/\mu$)

$\longrightarrow$ $\delta S_m^{\rm real}$ = real radiation for $\ell > M$, continuous: $\delta S_m^{\rm real}(\ell = M, M) = 0$

$\longrightarrow$ correct massless limit: $S_{0+m}(\ell, M, \mu_S) \xrightarrow{M\to 0} 2 S_0(\ell, \mu_S)$

Scenario IV: $1 > \lambda > \lambda^2 > \lambda_M$



$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_H^{(1)}(\mu_H,\mu_S) \times \int d\ell \int ds\, J_{0+m}(s,\mu_J)\, U_J^{(1)}(Q\ell - s,\mu_S,\mu_J)\, S_{0+m}(Q\tau - \ell,\mu_S)$$

$$S_{0+m}(\ell,\mu_S) = S_0(\ell,\mu_S) + \delta S_m^{\rm virt}(\ell,\mu_S) + \theta(\ell - M)\,\delta S_m^{\rm real}(\ell)$$

$\delta S_m^{\rm virt}$: virtual piece of massive soft function (distributive structure)
$\delta S_m^{\rm real}$: real radiation piece of massive soft function (function)

agreement with expanded full theory result at fixed order

continuity to scenario III for $\mu_M = \mu_J$:
$\rightarrow$ consistency relation between virtual modes :

$$2\,\mathrm{Re}\left[\delta F_m^{\rm eff}(Q,\mu)\right]\delta(\tau) - Q^2\delta J_m^{\rm virt}(Q^2\tau,\mu) - Q\,\delta S_m^{\rm virt}(Q\tau,\mu) = 0$$

(Intermediate) Summary

Achieved: EFT setup for massive gauge boson radiation in thrust distribution for different scale hierarchies incorporating

- $\surd$ continuous transitions (up to $\mathcal{O}(\alpha_s^2)$), no large power corrections at the transition!
- $\surd$ anomalous dimensions mass-independent (same result for M and ML)
- $\surd$ decoupling limit reached

$$\mathcal{C}^{I}(Q, M, \mu_H) \xrightarrow{M\to\infty} C_0(Q, \mu_H)$$

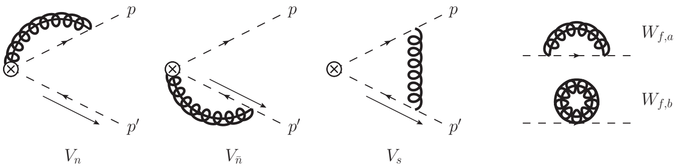
- $\surd$ massless limit reached

$$\mathcal{C}^{II}(Q, M, \mu_H) \xrightarrow{M\to 0} 2C_0(Q, \mu_H)$$
$$J_{0+m}(s, M, \mu_J) \xrightarrow{M\to 0} 2J_0(s, \mu_J)$$
$$S_{0+m}(\ell, M, \mu_S) \xrightarrow{M\to 0} 2S_0(\ell, \mu_S)$$

Outline

Soft-bin subtractions

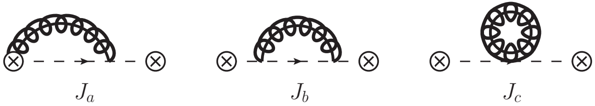
- hard function



$$\delta F_m^{\text{eff}} \sim V_n - V_{n,0M} + V_{\bar{n}} - V_{\bar{n},0M} + V_s - W_f$$
$$\sim V_n + V_{\bar{n}} - V_s - W_f$$

Idilbi, Mehen (2007)
no regulator required if suitable combinations of integrals taken

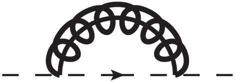
- jet function



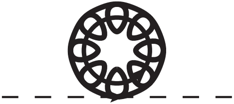
$$\delta J_m \sim J_a - J_{a,0M} + J_b + J_c$$

→ remark: no soft diagrams appearing here

Soft-bin subtractions to self-energy diagrams



I_a

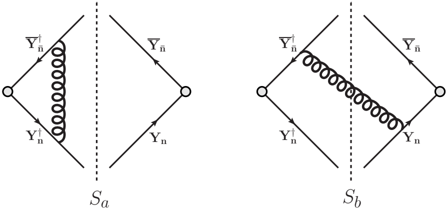


I_b

- SCET reproduces QCD wave-function renormalization
- in scenario II: $Q < M < Q\lambda$
  $\rightarrow$ off-shellness for interactions with collinear mass mode gauge bosons:
  $s = (p+k)^2 \sim M^2$
  $\rightarrow$ off-shellness for interactions with soft mass mode gauge bosons:
  $s = (p+k)^2 \sim QM$

  $\Rightarrow$ soft-bin subtractions to (collinear) self-energy power-suppressed by M/Q

Soft function calculation



additional divergences in the lightcone components (not regularized by DIMREG)
$\rightarrow$ S_a and S_b individually ill-defined
$\rightarrow$ we use analytic α-regulator (DIMREG in lightcone components)
`Becher, Bell (2011)`

$$\int dk^- \int dk^+ \rightarrow \int dk^- \left(\frac{\nu_1}{k^-}\right)^{\alpha_1} \int dk^+ \left(\frac{\nu_2}{k^+}\right)^{\alpha_2}$$

$\rightarrow S_a = 0,\ \delta S_m = S_b$