

# Combining Higher-Order Resummation with Fully Exclusive NLO Calculations and Hadronization in GENEVA

GENEVA, 1211.7049

Jonathan Walsh, UC Berkeley

SCET X, Duke University  
March 16, 2013

The GENEVA Collaboration:

Simone Alioli, Christian Bauer, Calvin Berggren, Andrew Hornig,  
Frank Tackmann, Christopher Vermilion, JW, and Saba Zuberi



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Nicolas

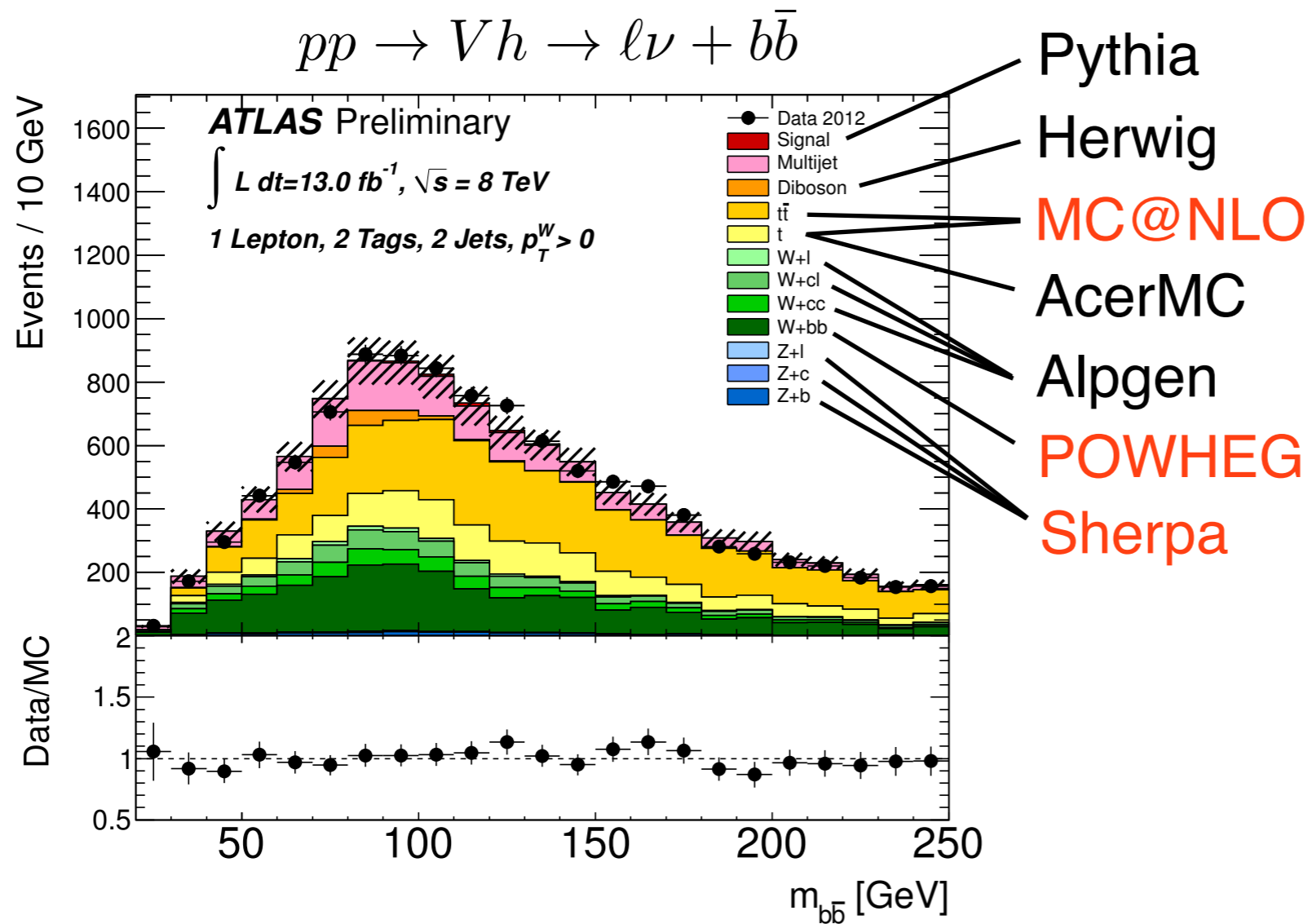


Zia



# Precision Calculations + Monte Carlo Generators

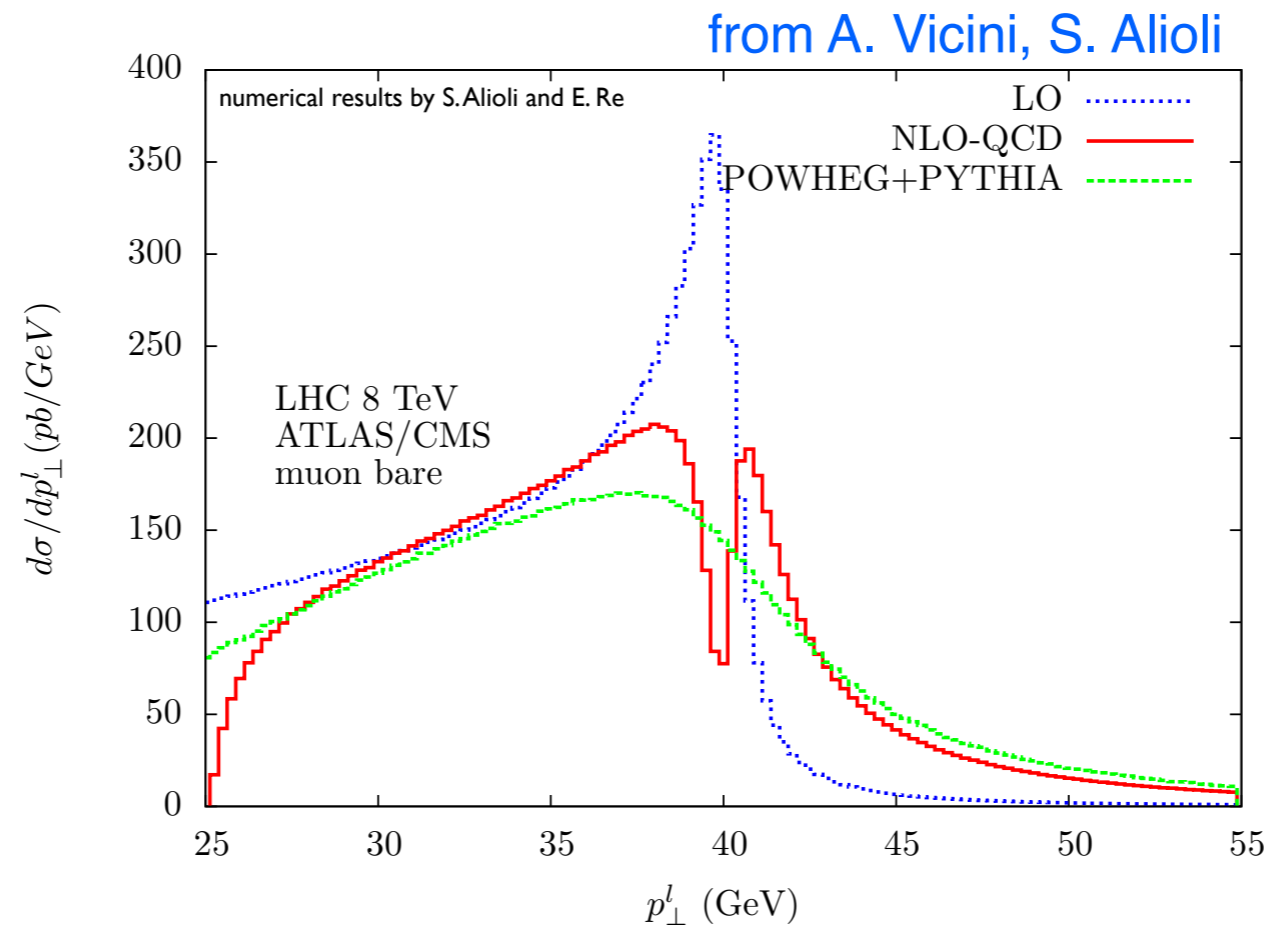
ATLAS-CONF-2012-161



Monte Carlo generators are often the only appropriate tool to understand data, ***even when precision is required***

# Precision Calculations + Monte Carlo Generators

But modern Monte Carlos are fundamentally  
***built from precision calculations***



NLO + resummation needed for an  
accurate description of a simple observable

# Outline

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The GENEVA  
Framework

Comparison to  
LEP Data

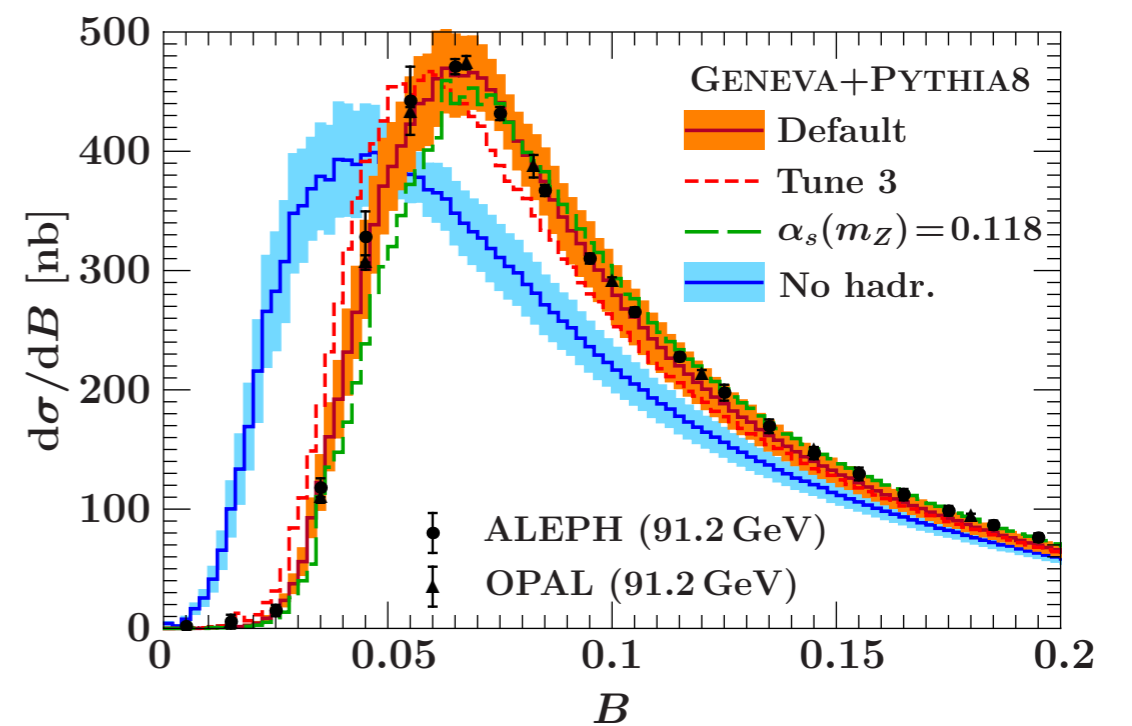
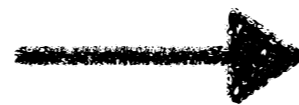
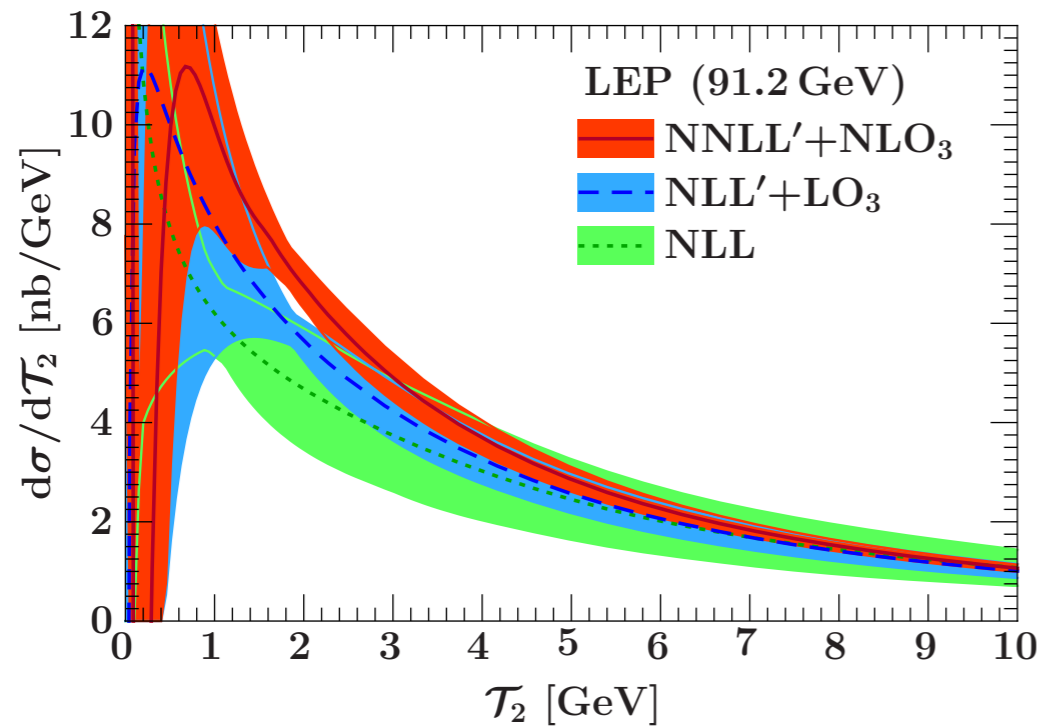
Applications for  
the LHC

# Outline

The GENEVA Framework

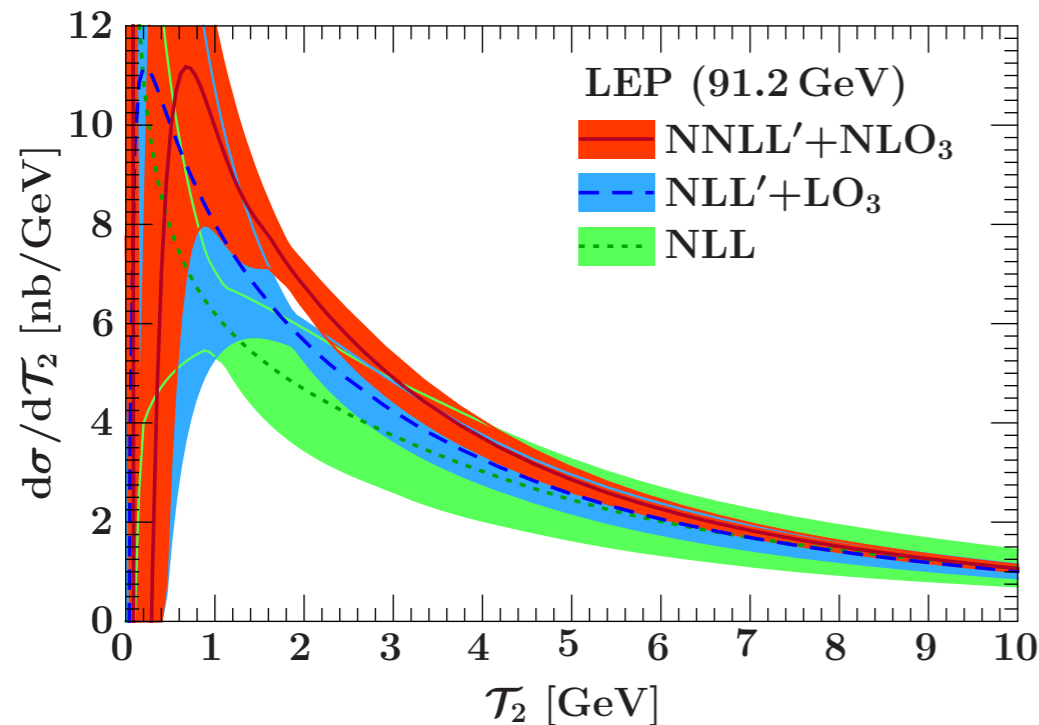
Comparison to LEP Data

Applications for the LHC



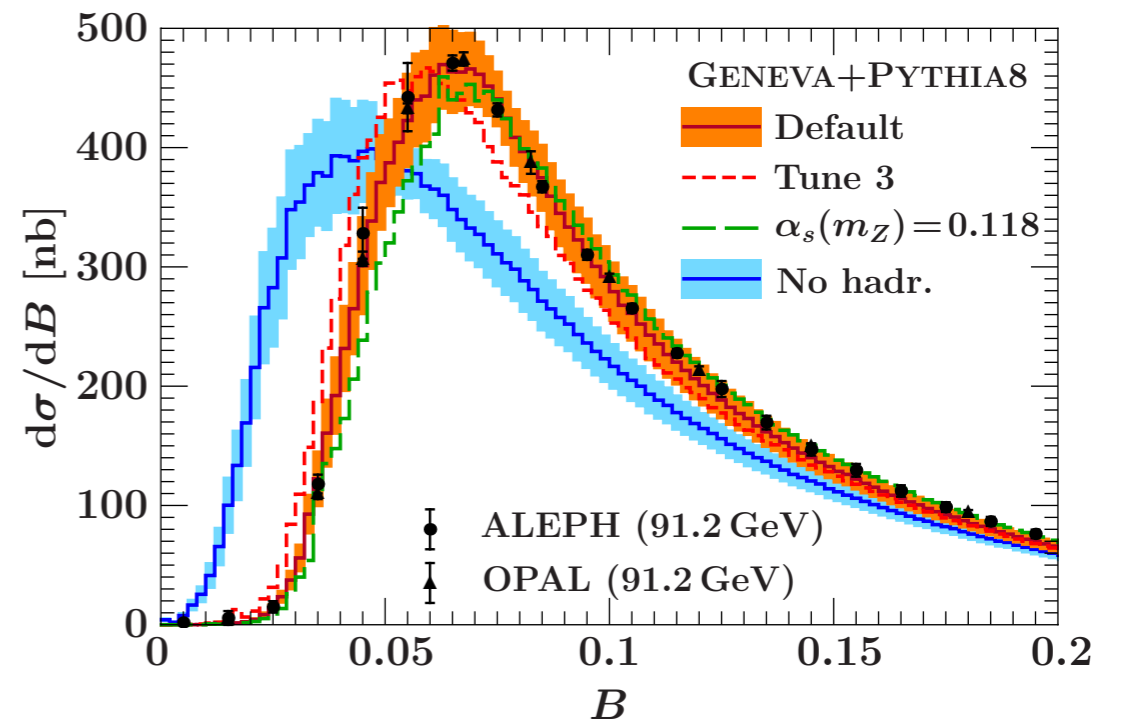
# The GENEVA Monte Carlo in a Nutshell

analytic thrust resummation



high accuracy prediction  
of a single observable

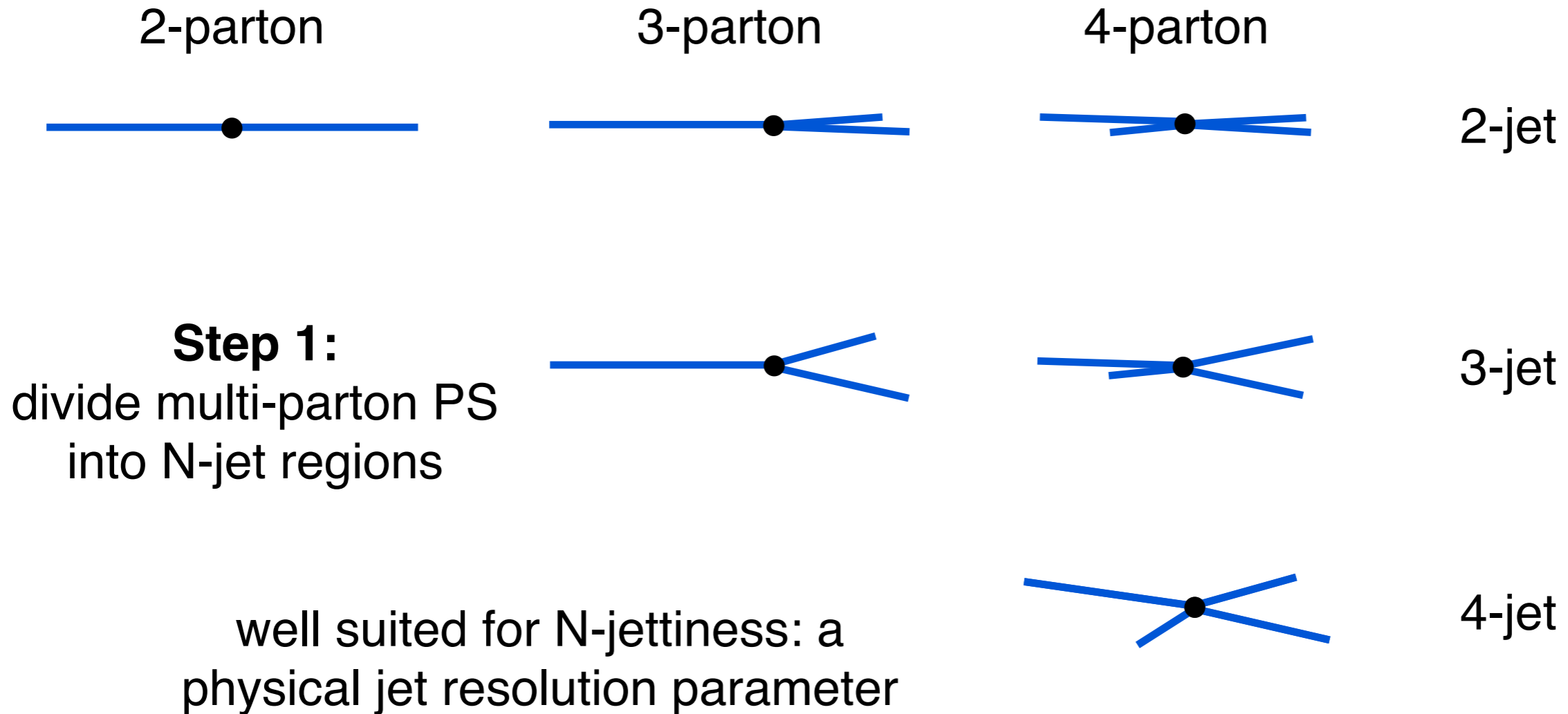
Monte Carlo hadronized events



reasonably accurate prediction  
of a broad palette of observables

- fully exclusive fixed order calculations
- high order resummation of jet resolution parameter
- need:
  - matching between jet multiplicities
  - parton shower to fill out jets
  - hadronization/MPI model for nonperturbative corrections

# The GENEVA Monte Carlo in a Nutshell



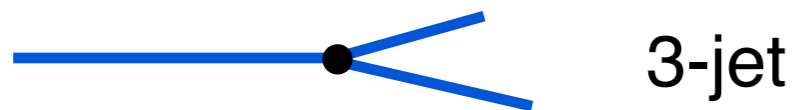
$$\begin{aligned} \mathcal{T}_2 < \mathcal{T}_2^{\text{cut}} & : 2\text{-jet exclusive} : \Phi_2 \\ \mathcal{T}_2 > \mathcal{T}_2^{\text{cut}} \text{ and } \mathcal{T}_3 < \mathcal{T}_3^{\text{cut}} & : 3\text{-jet exclusive} : \Phi_3 \\ \mathcal{T}_2 > \mathcal{T}_2^{\text{cut}} \text{ and } \mathcal{T}_3 > \mathcal{T}_3^{\text{cut}} & : 4\text{-jet inclusive} : \Phi_4 \end{aligned}$$

A N-jet PS point is  
represented by a  
N-parton event



# The GENEVA Monte Carlo in a Nutshell

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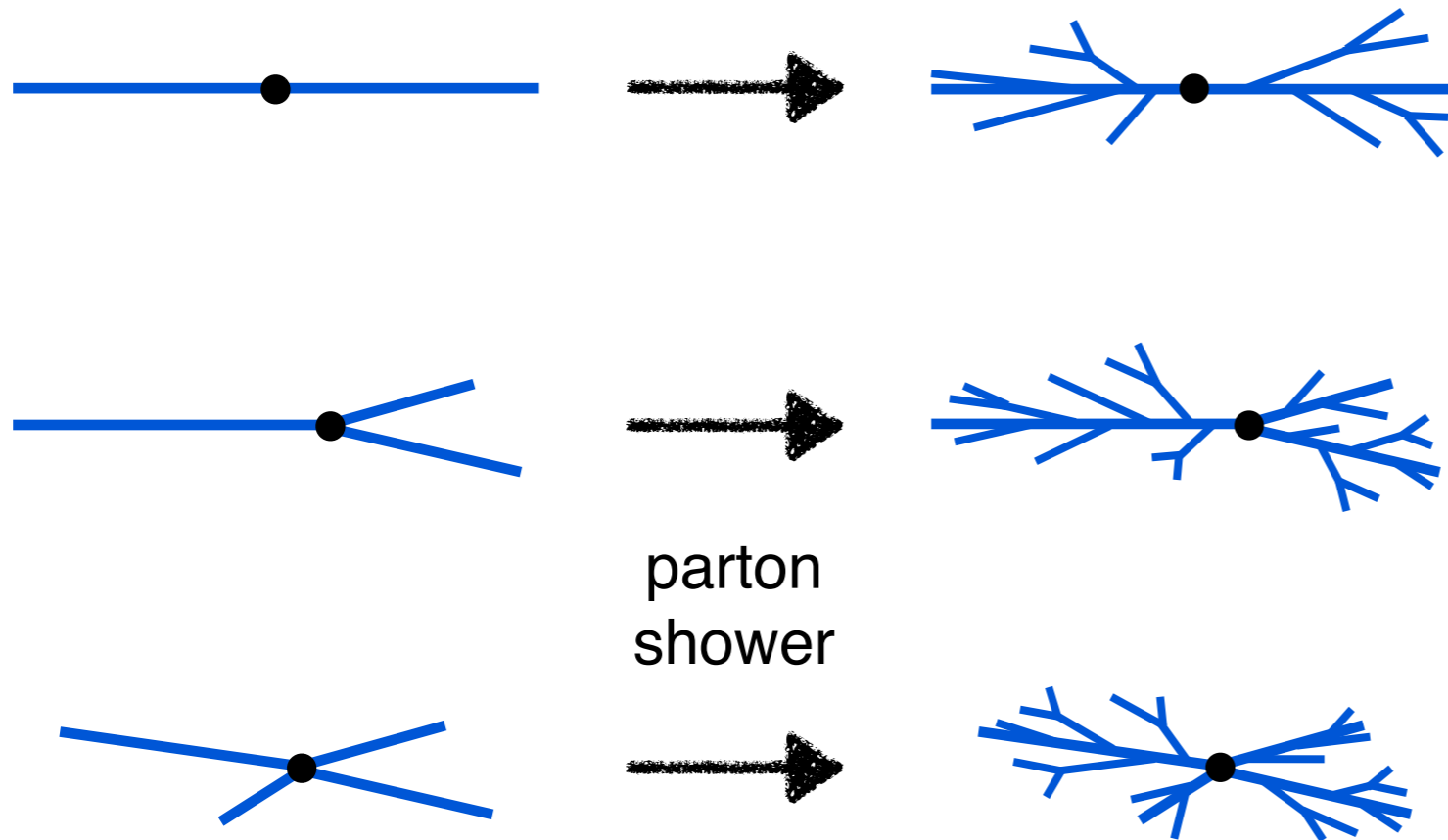
**Step 2:**  
assign each N-jet PS point a  
fully differential weight

$$\frac{d\sigma}{d\Phi_N}$$

We use an N-jettiness factorization  
theorem to compute the weight

# The GENEVA Monte Carlo in a Nutshell

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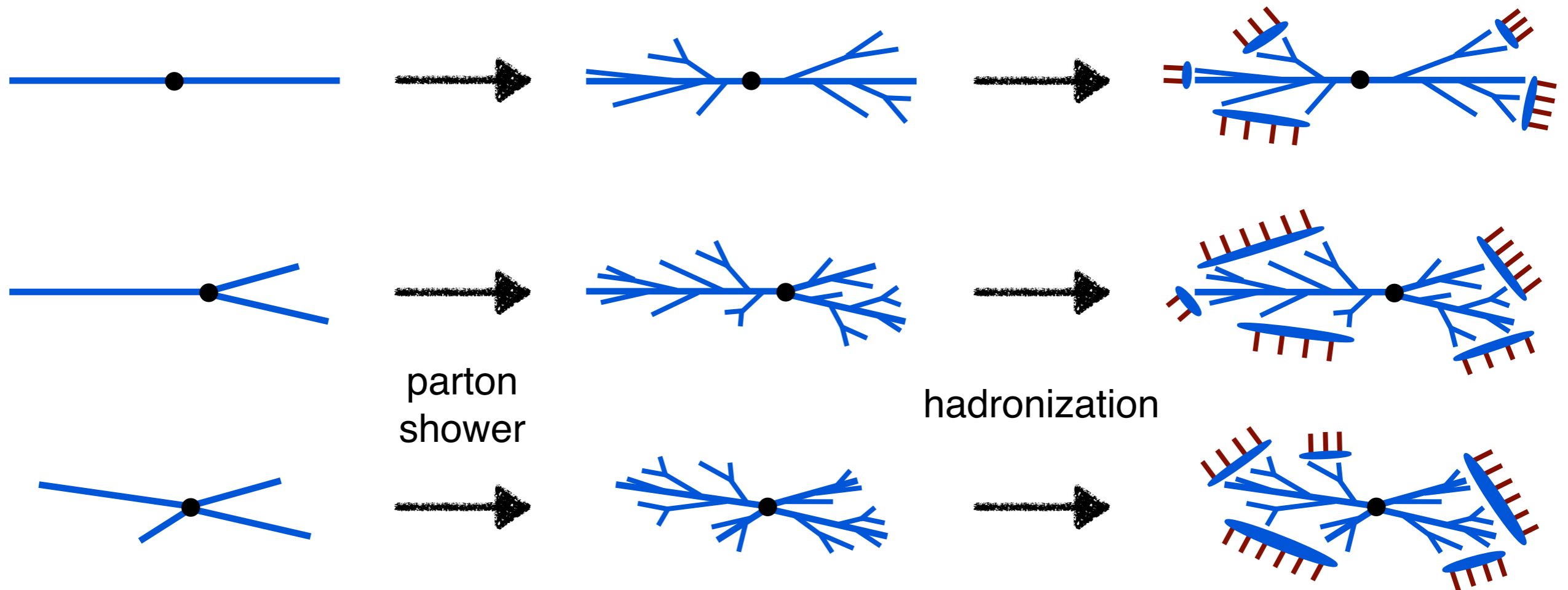
parton  
shower

**Step 3:**  
shower each event  
to “fill out” the jets

the shower is restricted  
based on N-jettiness

# The GENEVA Monte Carlo in a Nutshell

---



**Step 4:**  
hadronize the events

# Fully Exclusive Cross Sections in GENEVA

---

thrust  
spectrum

$$\frac{d\sigma}{d\Phi_2 d\mathcal{T}} = \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}} + \frac{d\sigma^{\text{FO}}}{d\Phi_2 d\mathcal{T}} - \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}} \Big|_{\text{exp}} \quad \Phi_2: \text{hard kinematics}$$

But we want to be fully differential in N-body PS

# Fully Exclusive Cross Sections in GENEVA

---

thrust  
spectrum

$$\frac{d\sigma}{d\Phi_2 d\mathcal{T}} = \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}} + \frac{d\sigma^{\text{FO}}}{d\Phi_2 d\mathcal{T}} - \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}} \Big|_{\text{exp}}$$

cancel in  
singular limit

cancel in  
nonsingular limit

multiplicative  
matching

$$\frac{d\sigma}{d\Phi_2 d\mathcal{T}} = \left[ \frac{d\sigma^{\text{FO}}}{d\Phi_2 d\mathcal{T}} / \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}} \Big|_{\text{FO}} \right] \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}}$$

alternative to additive matching

# Fully Exclusive Cross Sections in GENEVA

---

thrust  
spectrum

$$\frac{d\sigma}{d\Phi_2 d\mathcal{T}} = \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}} + \frac{d\sigma^{\text{FO}}}{d\Phi_2 d\mathcal{T}} - \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}} \Big|_{\text{exp}}$$

cancel in  
singular limit

cancel in  
nonsingular limit

multiplicative  
matching

$$\frac{d\sigma}{d\Phi_3} = \left[ \frac{d\sigma^{\text{FO}}}{d\Phi_3} / \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}} \Big|_{\text{FO}} \right] \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}}$$

fully differential  
3-body PS

$$\frac{d\sigma^{\text{FO}}}{d\Phi_3}$$

projects onto

$$\frac{d\sigma^{\text{FO}}}{d\Phi_2 d\mathcal{T}}$$

# GENEVA Master Formula

---

$$\frac{d\sigma^{\text{incl}}}{d\Phi_2} = \frac{d\sigma}{d\Phi_2}(\mathcal{T}_{\text{cut}}) + \int \frac{d\Phi_3}{d\Phi_2 d\Phi_3} \frac{d\sigma}{d\Phi_3}(\mathcal{T}) \theta(\mathcal{T} > \mathcal{T}_{\text{cut}})$$

*cumulant*  
exclusive 2-jet bin

*spectrum*  
inclusive 3-jet bin

# GENEVA Master Formula

---

$$\frac{d\sigma^{\text{incl}}}{d\Phi_2} = \frac{d\sigma}{d\Phi_2}(\mathcal{T}_{\text{cut}}) + \int \frac{d\Phi_3}{d\Phi_2} \frac{d\sigma}{d\Phi_3}(\mathcal{T}) \theta(\mathcal{T} > \mathcal{T}_{\text{cut}})$$



*cumulant*  
 exclusive 2-jet bin

*spectrum*  
 inclusive 3-jet bin

$$\frac{d\sigma}{d\Phi_2}(\mathcal{T}_{\text{cut}}) = \frac{d\sigma^{\text{resum}}}{d\Phi_2}(\mathcal{T}_{\text{cut}}) + \left[ \frac{d\sigma^{\text{FO}}}{d\Phi_2}(\mathcal{T}_{\text{cut}}) - \frac{d\sigma^{\text{resum}}}{d\Phi_2}(\mathcal{T}_{\text{cut}}) \Big|_{\text{FO}} \right]$$


$$\frac{d\sigma}{d\Phi_3}(\mathcal{T}) = \frac{d\sigma^{\text{FO}}}{d\Phi_3} \left[ \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}} \Big/ \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}} \Big|_{\text{FO}} \right]$$

can calculate the N-jet FO cross section to NLO:  $\text{NLO}_N$




# N-jet Weights from the Master Formula

---


**2-jet**

$$\frac{d\sigma}{d\Phi_2}(\mathcal{T}_{\text{cut}}) = \frac{d\sigma^{\text{resum}}}{d\Phi_2}(\mathcal{T}_{\text{cut}}) + \left[ \frac{d\sigma^{\text{FO}}}{d\Phi_2}(\mathcal{T}_{\text{cut}}) - \frac{d\sigma^{\text{resum}}}{d\Phi_2}(\mathcal{T}_{\text{cut}}) \Big|_{\text{FO}} \right]$$


**3-jet**

$$\frac{d\sigma}{d\Phi_3}(\mathcal{T}) = \frac{d\sigma^{\text{FO}}}{d\Phi_3} \left[ \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}} / \frac{d\sigma^{\text{resum}}}{d\Phi_2 d\mathcal{T}} \Big|_{\text{FO}} \right]$$


**4-jet**

$$\frac{d\sigma}{d\Phi_4}(\mathcal{T}) = \frac{d\sigma^{\text{LO}}}{d\Phi_4}$$

# Iterating To More Jet Multiplicities: Future Possibilities

---

$$\frac{d\sigma^{\text{incl}}}{d\Phi_2} = \frac{d\sigma}{d\Phi_2}(\mathcal{T}_2^{\text{cut}}) + \int \frac{d\Phi_3}{d\Phi_2} \frac{d\sigma}{d\Phi_3}(\mathcal{T}_2) \theta(\mathcal{T}_2 > \mathcal{T}_2^{\text{cut}})$$

$$\frac{d\sigma^{\text{incl}}}{d\Phi_3} = \frac{d\sigma}{d\Phi_3}(\mathcal{T}_3^{\text{cut}}) + \int \frac{d\Phi_4}{d\Phi_3} \frac{d\sigma}{d\Phi_3}(\mathcal{T}_3) \theta(\mathcal{T}_3 > \mathcal{T}_3^{\text{cut}})$$

⋮

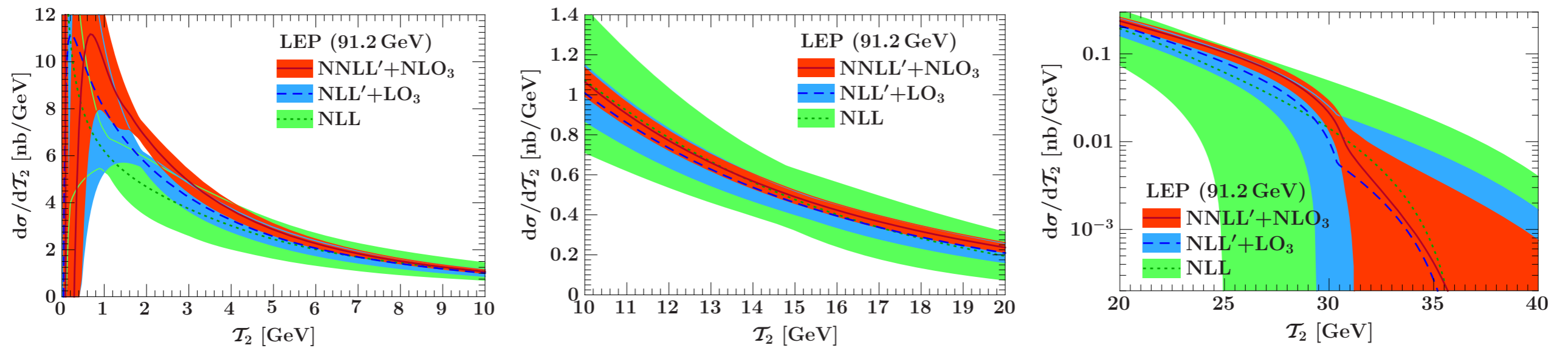
$$\frac{d\sigma^{\text{incl}}}{d\Phi_{N_{\text{max}}}} = \frac{d\sigma^{\text{LO}}}{d\Phi_{N_{\text{max}}}}$$

matching several jet multiplicities at NLO, with simultaneous resummation of N-jettiness for multiple N

where

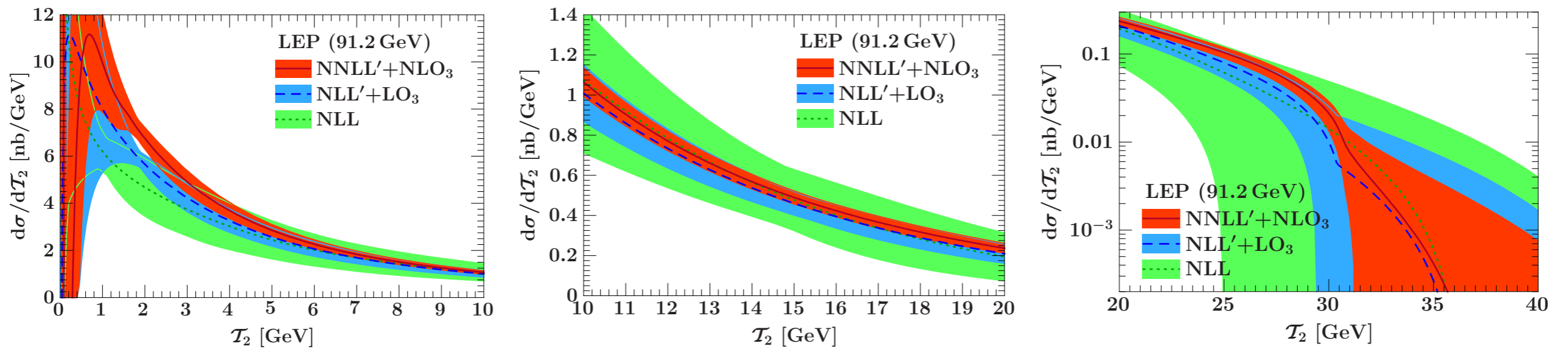
$$\frac{d\sigma}{d\Phi_{N+1}} = \frac{d\sigma^{\text{incl}}}{d\Phi_{N+1}} \left[ \frac{d\sigma^{\text{resum}}}{d\Phi_N d\mathcal{T}_N} \Big/ \frac{d\sigma^{\text{resum}}}{d\Phi_N d\mathcal{T}_N} \Big|_{\text{FO}} \right]$$

# GENEVA Reproduces Thrust Distribution

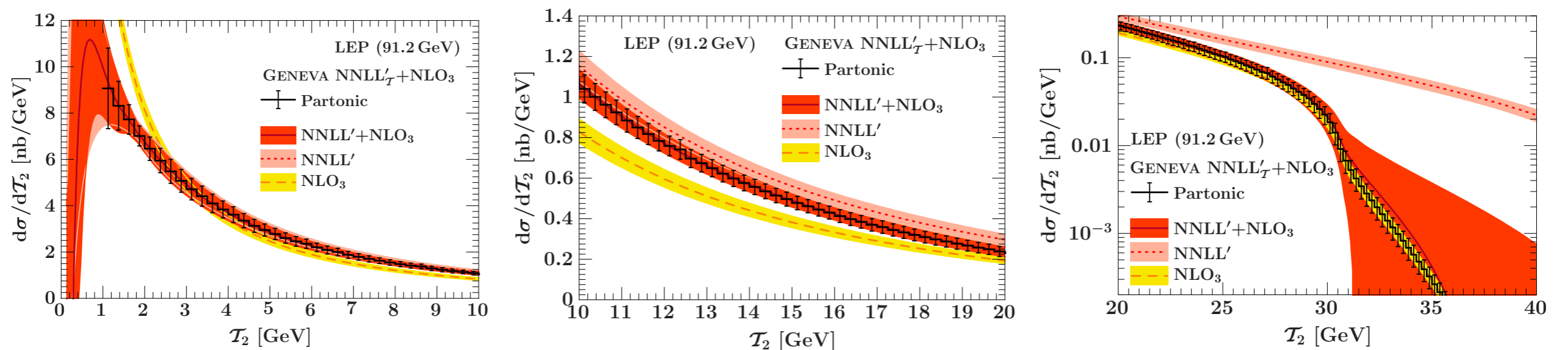


analytic calculation of thrust distribution using the usual nonsingular matching formula

# GENEVA Reproduces Thrust Distribution



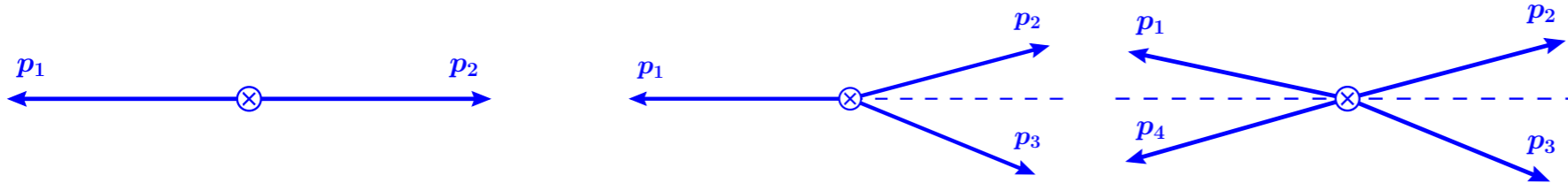
analytic calculation of thrust distribution using the usual nonsingular matching formula



**Step 2:** onto parton shower and hadronization

# Matching onto the Parton Shower

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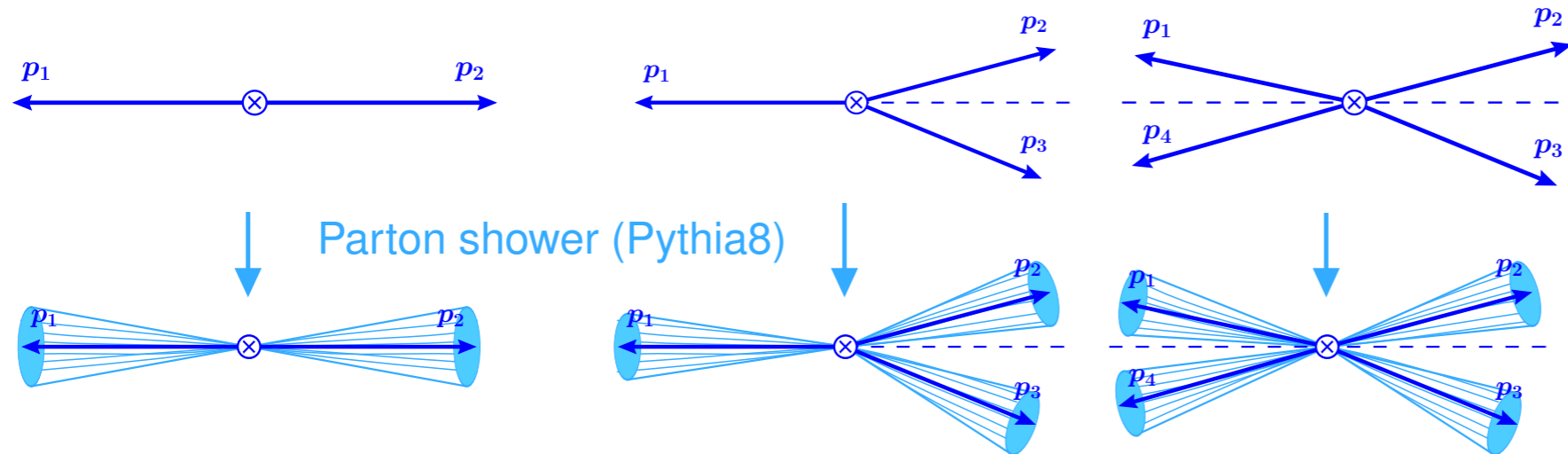
So far, we have assigned a weight to events that does two important things:

- Resums logs of the 2/3 jet resolution scale to NNLL'
- Has the 2 and 3 jet events calculated to NLO accuracy

But our events still only have 2, 3, or 4 partons  
→ *We need the parton shower to fill out our jets*

We will see the important effect of showering on other observables

# Matching onto the Parton Shower

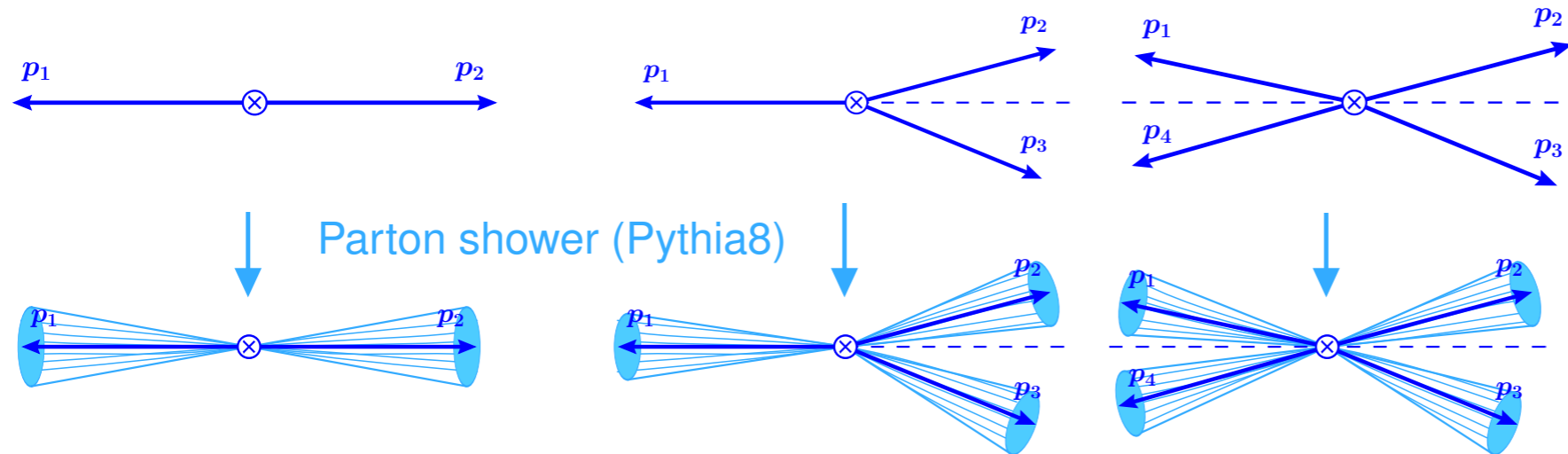


Use the Pythia8  
parton shower routine,  
*with modifications*

How do we want to limit Pythia's shower?

- We want to preserve the accuracy of our thrust distribution calculation
- We want 2/3 jet partonic events in GENEVA to shower into 2/3 jet events
- We want the 4 jet events to shower inclusively

# Matching onto the Parton Shower

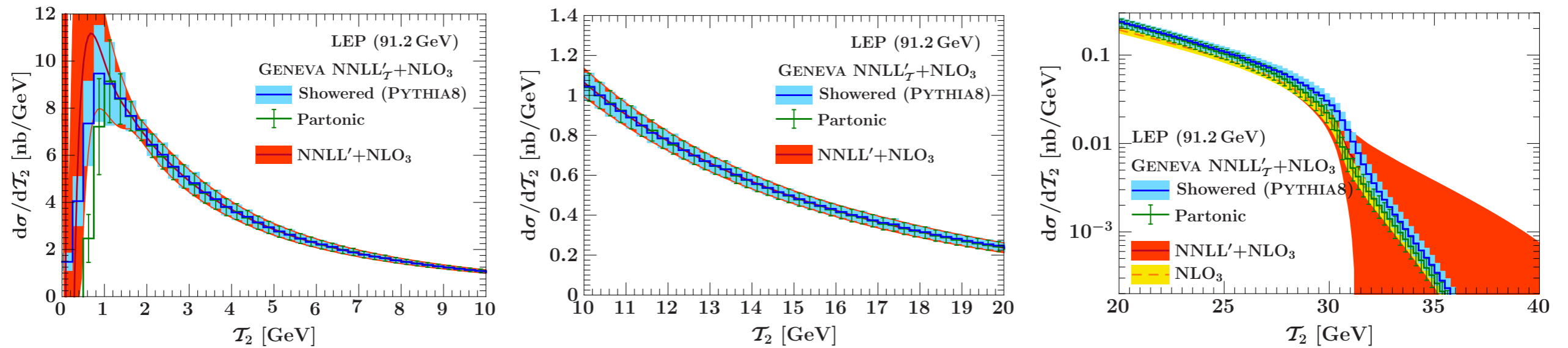


Use the Pythia8  
parton shower routine,  
*with modifications*

How do we want to limit Pythia's shower?

- We want to preserve the accuracy of our thrust distribution calculation
  - $\left| \mathcal{T}_{\text{GENEVA+PY}} / \mathcal{T}_{\text{GENEVA}} - 1 \right| < \lambda_N$
- We want 2/3 jet partonic events in GENEVA to shower into 2/3 jet events
  - We want to preserve the hard kinematics of the jets
- We want the 4 jet events to shower inclusively
  - Multijet tail of distributions only filled out by shower; need to match to higher order LO matrix elements

# Results on Thrust Distribution



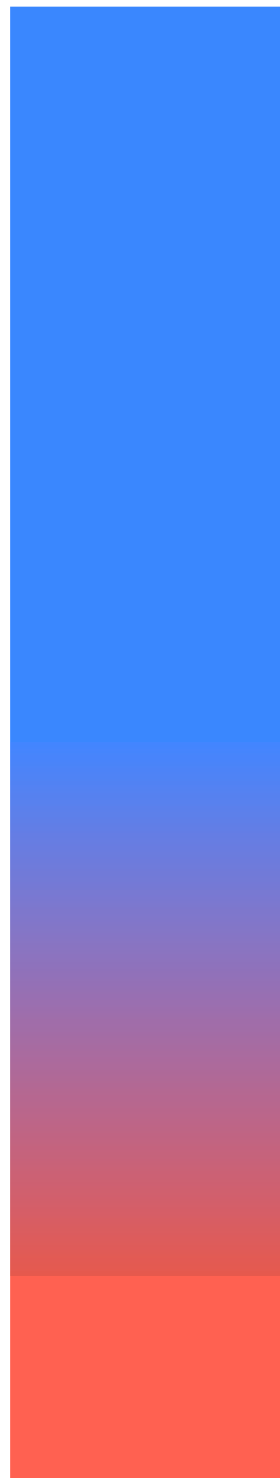
as advertised, showering does not change the thrust distribution

**Step 3:** next is to add hadronization



# Hadronization

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Pythia  
shower

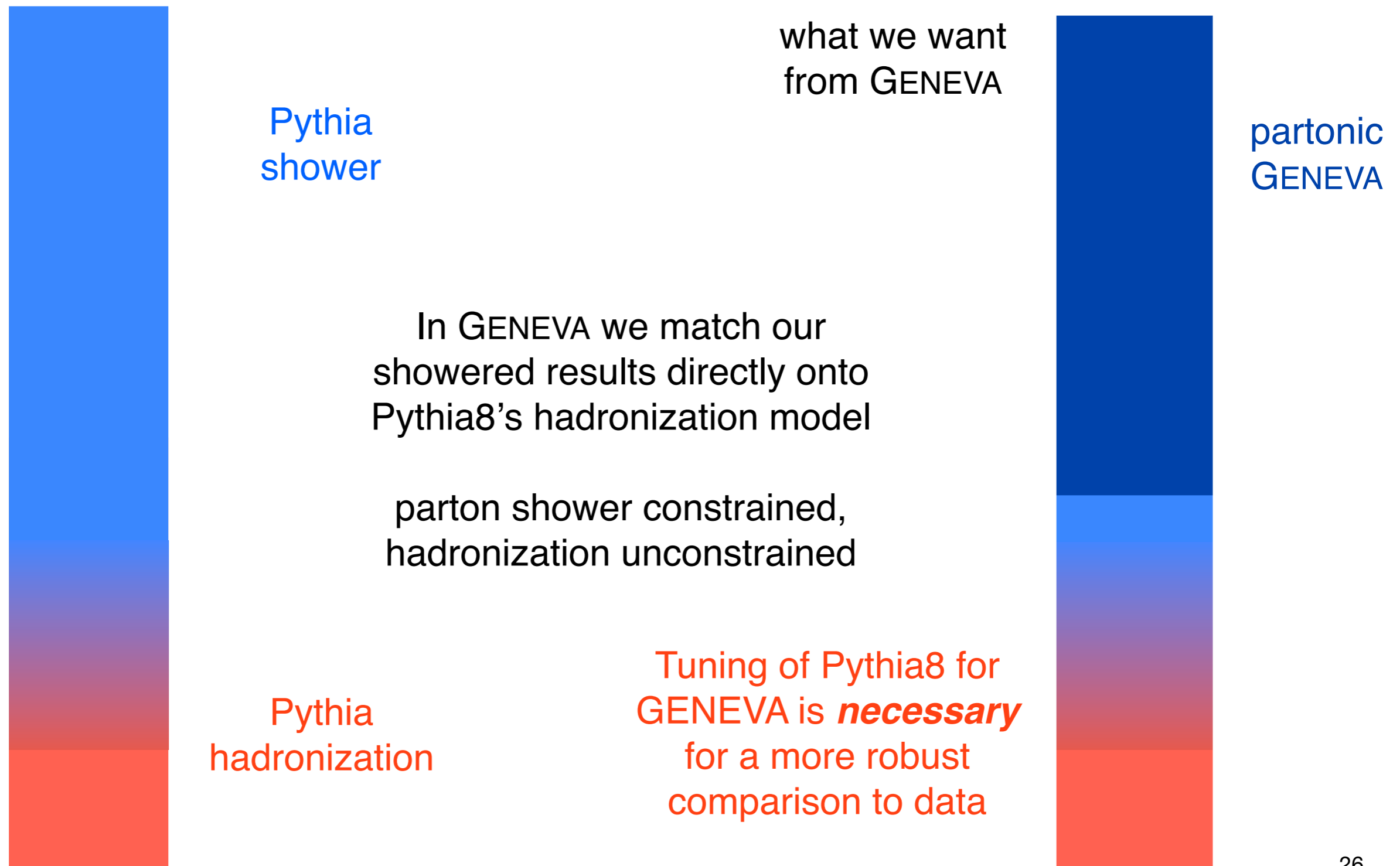
Pythia is tuned to match  
a palette of observables

This tuning effectively replaces  
higher order corrections

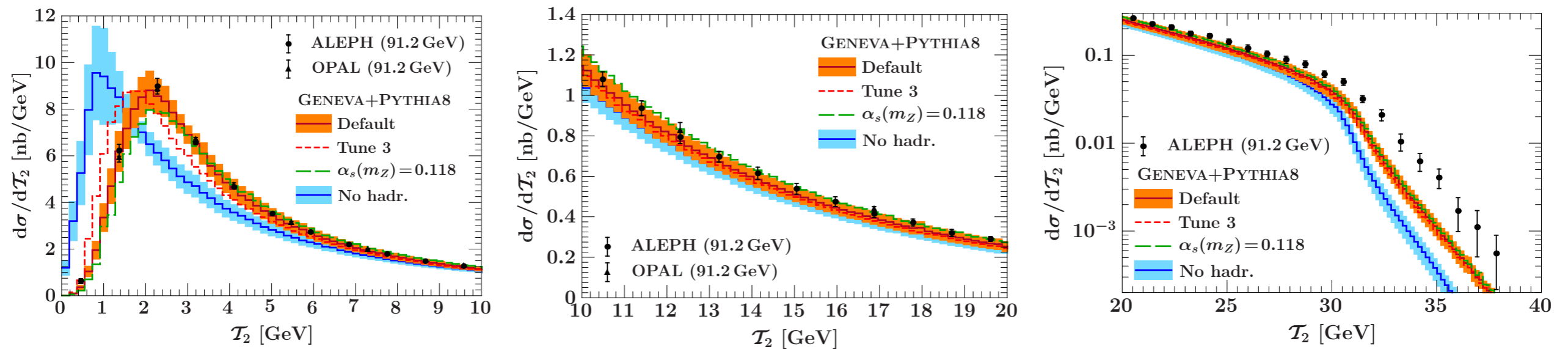
Pythia  
hadronization

# Hadronization

---



# Results on Thrust Distribution



✓ **Step 4:** excellent agreement with LEP data in peak and transition regions  
(tail region requires higher multiplicity matrix elements)

Compare GENEVA with two  $e^+e^-$  Pythia tunes (“tune 1”) and (“tune 3”) and two values of  $\alpha_s(M_Z)$  (0.1135, 0.118)

Find that for thrust, “tune 1” with  $\alpha_s = 0.1135$  works best (use as our default)

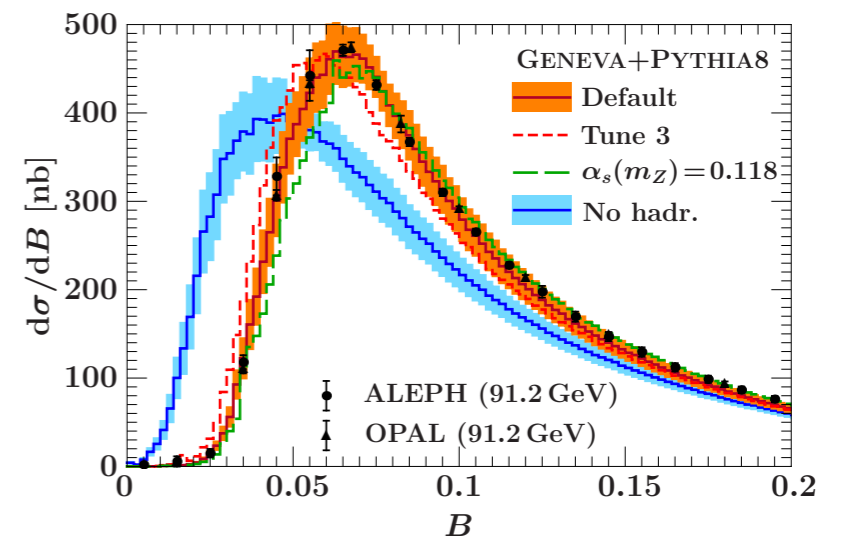
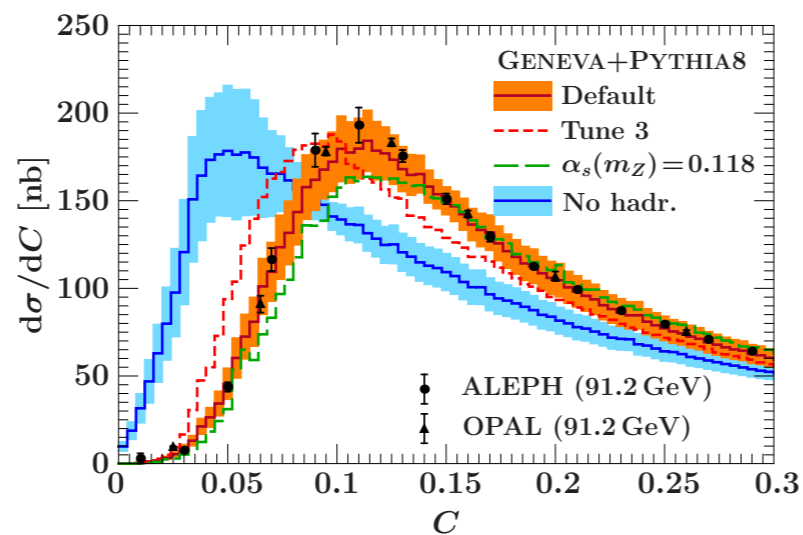
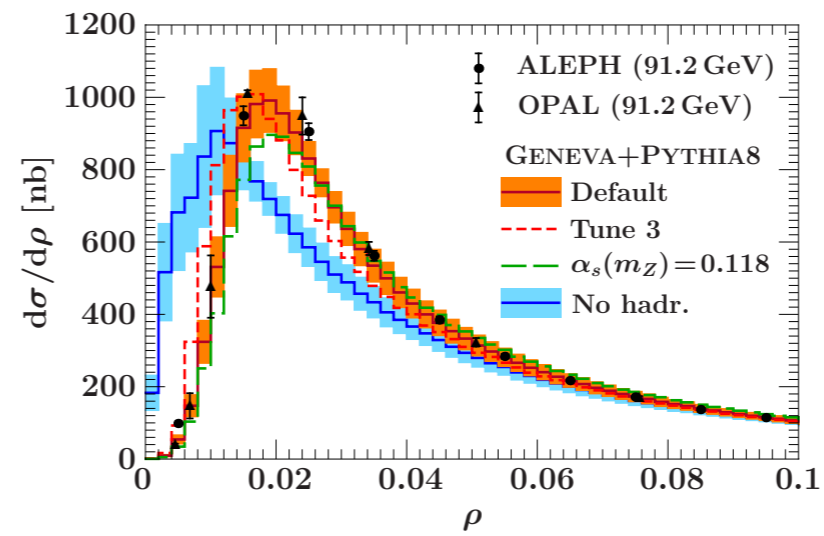
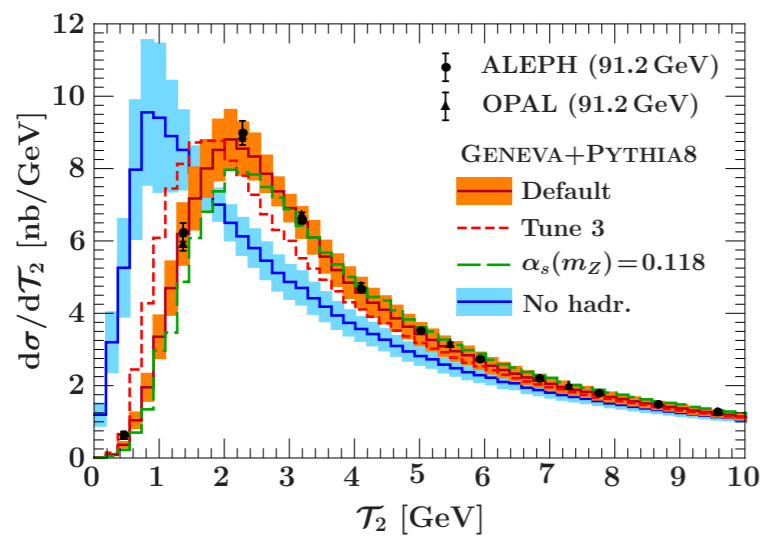
Also show “tune 3” with  $\alpha_s = 0.1135$  and “tune 1” with  $\alpha_s = 0.118$

# Outline

The GENEVA Framework

Comparison to LEP Data

Applications for the LHC

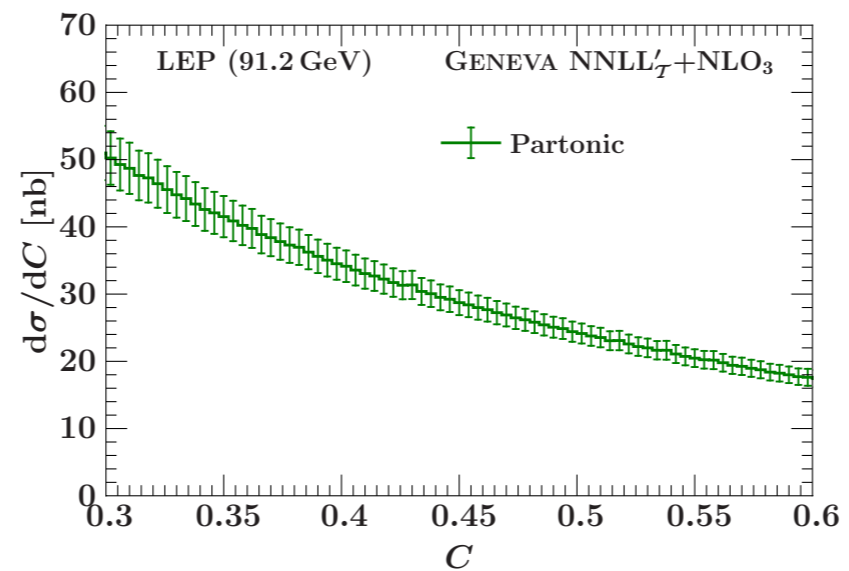
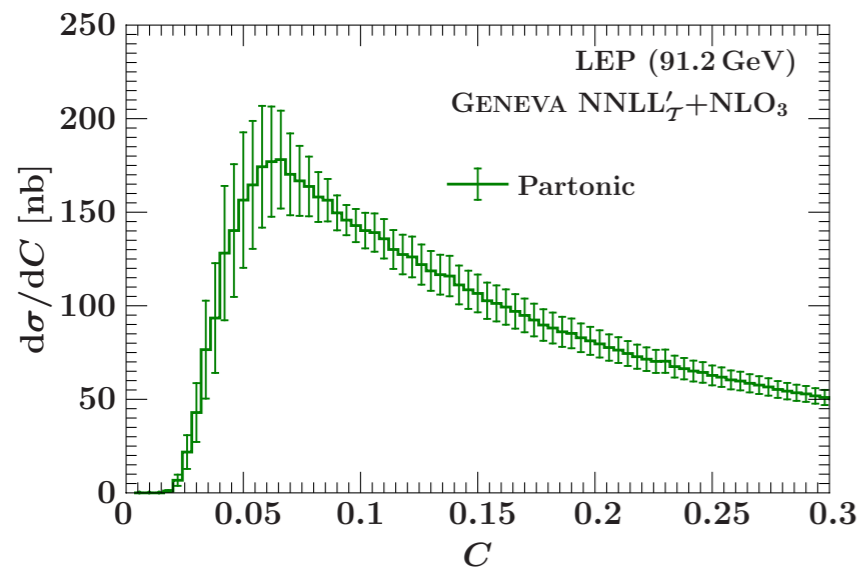


# Other 2-jet Event Shapes

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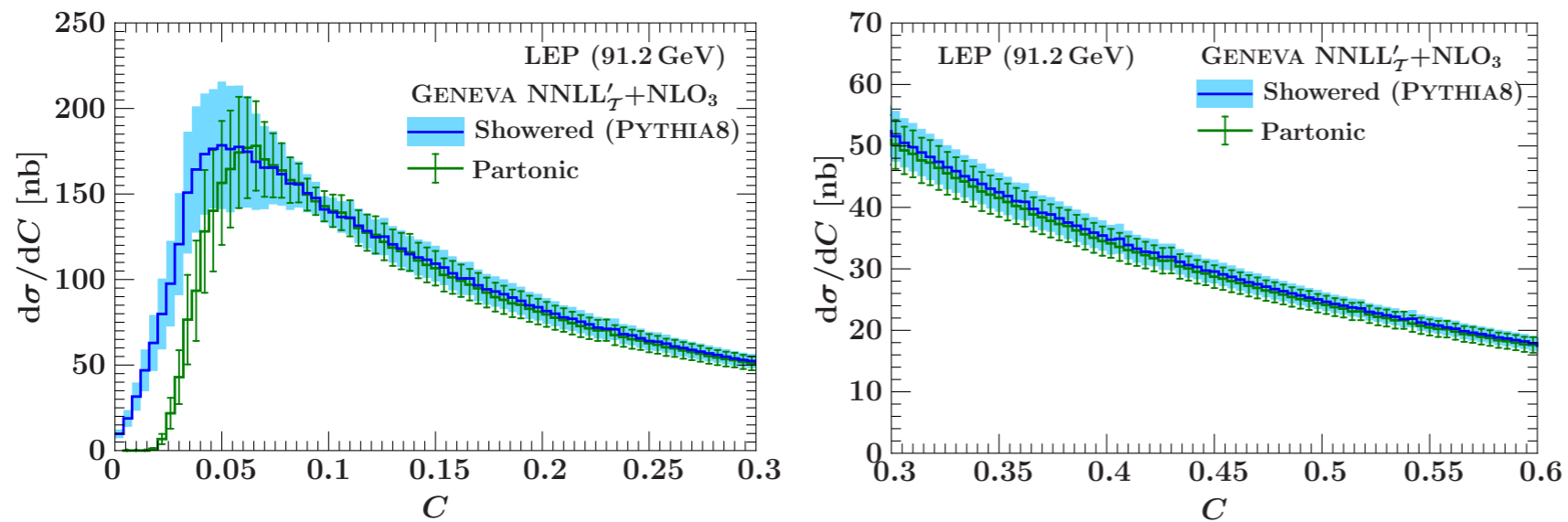
- C-parameter
  - Log structure essentially the same as thrust, with different nonsingular terms, power corrections, and nonperturbative effects
- Heavy Jet Mass
  - Log structure a different projection of dijet mass distribution
- Jet Broadening
  - SCET<sub>II</sub> observable, tests the ability of GENEVA to describe other observables with different log series
- GENEVA is making a *prediction* for these observables: important to validate against their known resummation

# Parton Shower: C-parameter



partonic GENEVA

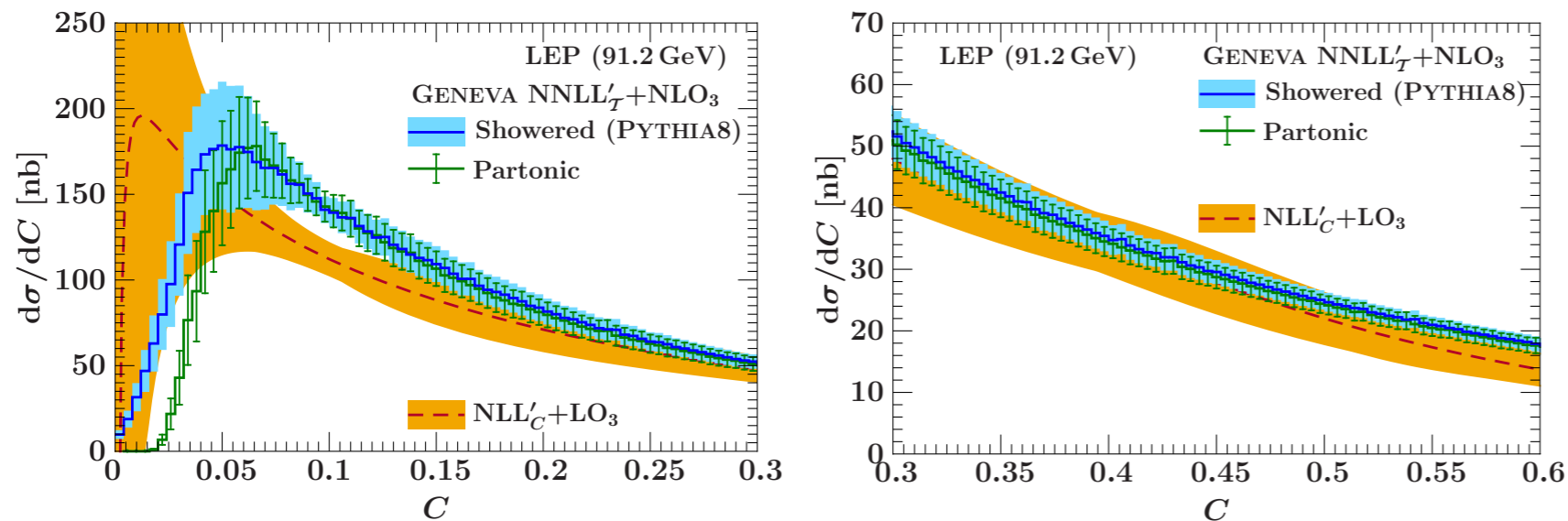
# Parton Shower: C-parameter



showered GENEVA

The parton shower does not significantly change the  $C$  spectrum and fills out the 2-jet region

# Parton Shower: C-parameter

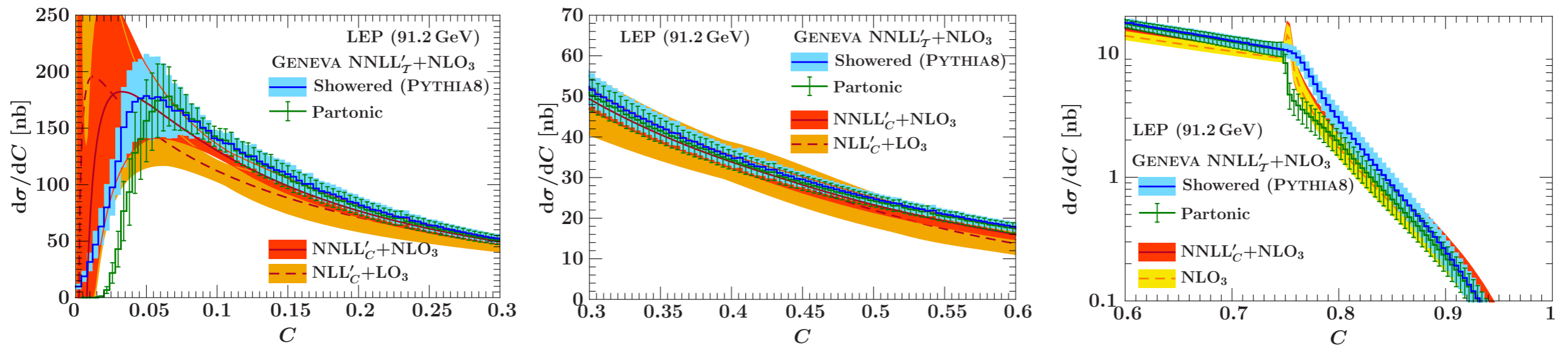


showered GENEVA  
vs. NLL' + LO3 analytic  
resummation

The parton shower does not significantly change the  $C$  spectrum  
and fills out the 2-jet region



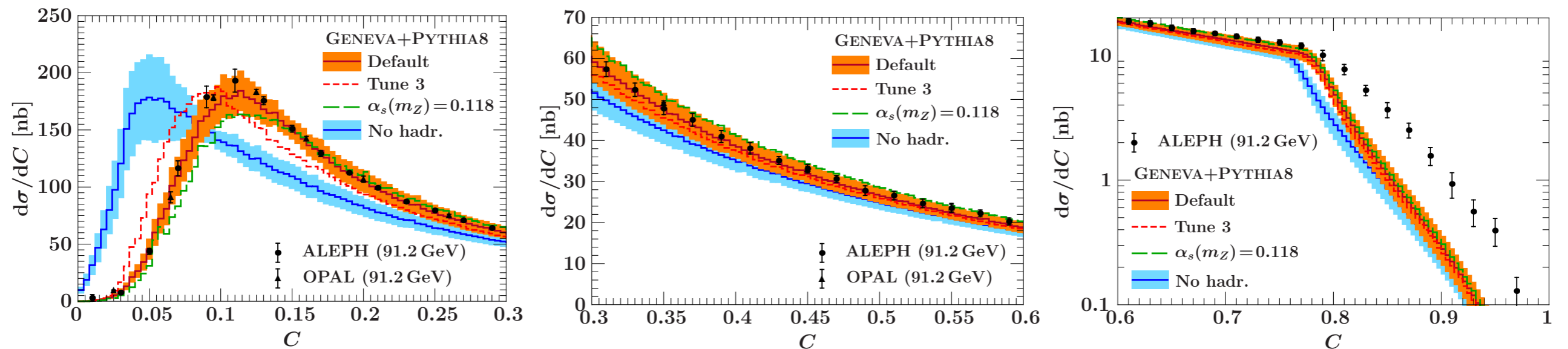
# Parton Shower: C-parameter



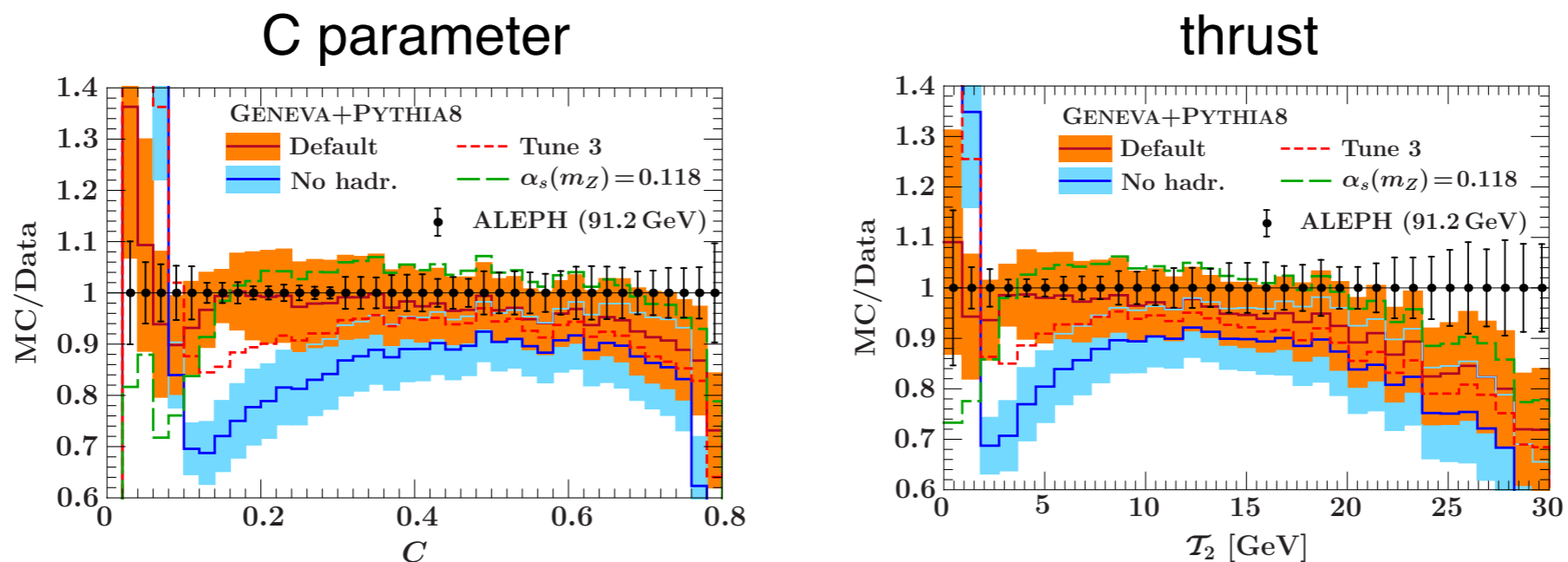
The parton shower does not significantly change the  $C$  spectrum and fills out the 2-jet region

Solid agreement between analytic NNLL'<sub>T</sub> resummation and the GENEVA prediction

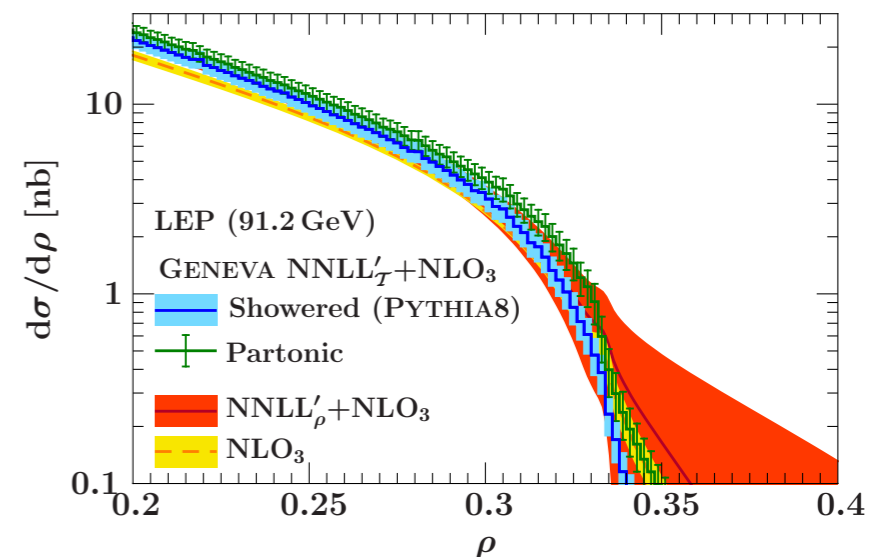
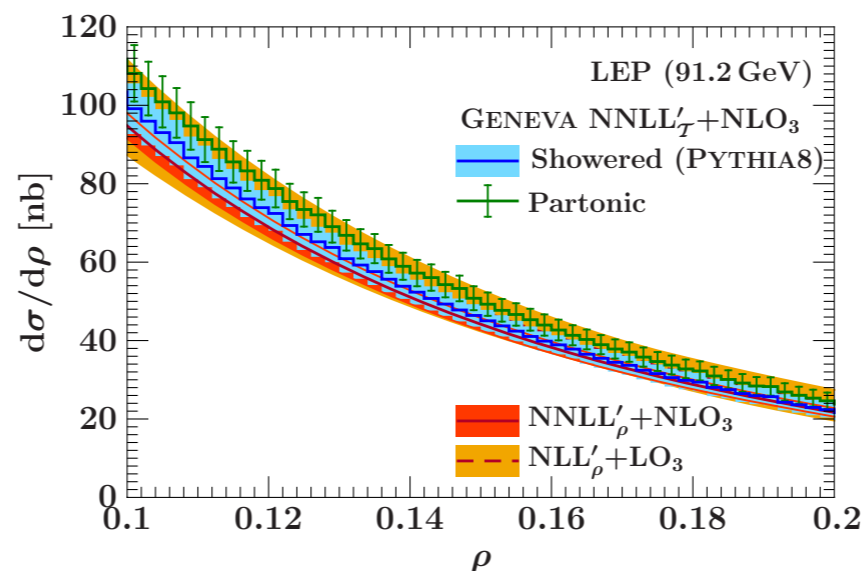
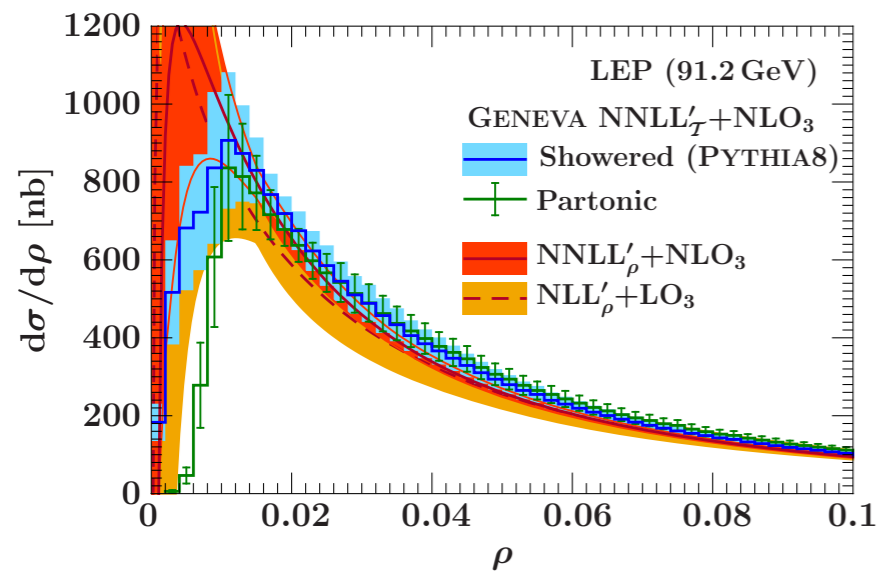
# LEP Comparison: C-parameter



The default tune +  $\alpha_s = 0.1135$  gives as good of an agreement as thrust



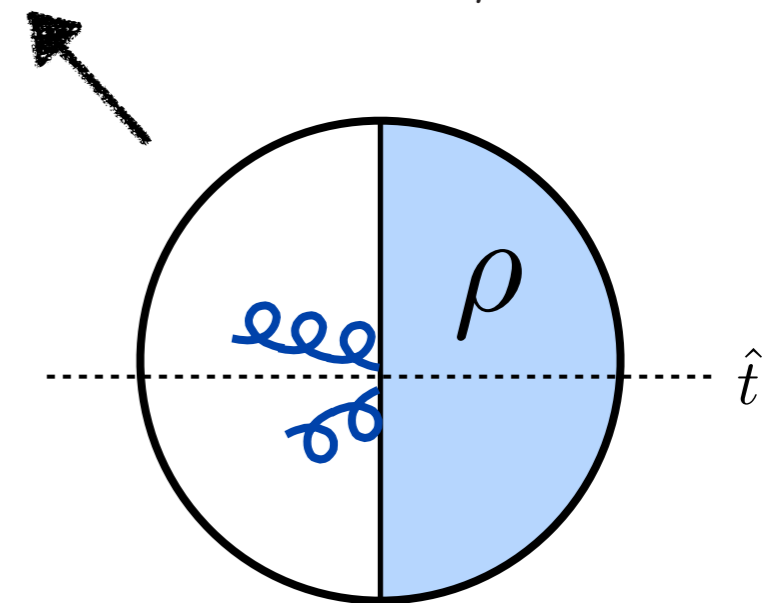
# Parton Shower: Heavy Jet Mass



$$\frac{d\sigma}{d\rho} = \int dm_1^2 dm_2^2 \frac{d\sigma}{dm_1^2 dm_2^2} \delta(\rho - \max(m_1^2, m_2^2)/Q^2)$$

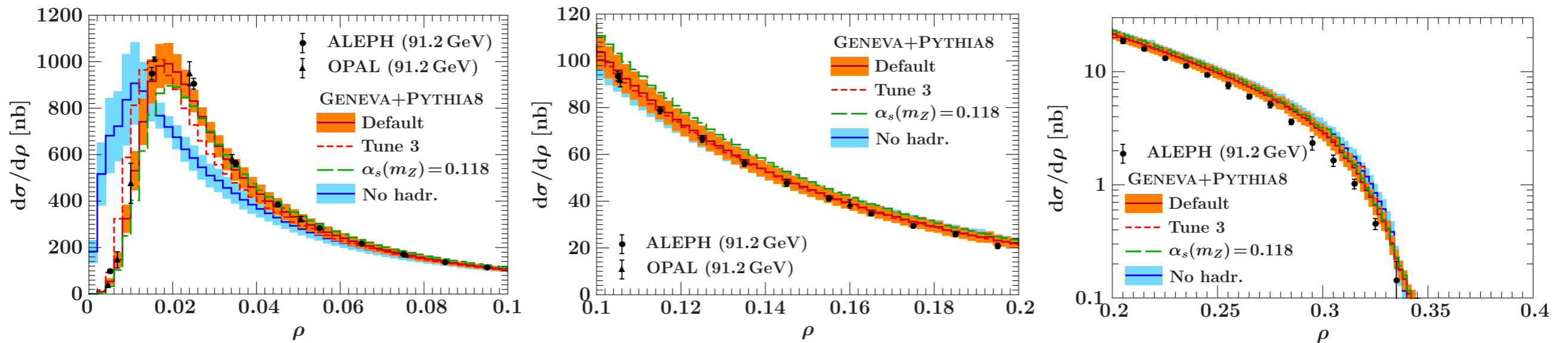
$$\frac{d\sigma}{d\tau} = \int dm_1^2 dm_2^2 \frac{d\sigma}{dm_1^2 dm_2^2} \delta(\tau - (m_1^2 + m_2^2)/Q^2)$$

The GENEVA + Pythia prediction agrees better than partonic GENEVA with the analytic NNLL' result (as it should)



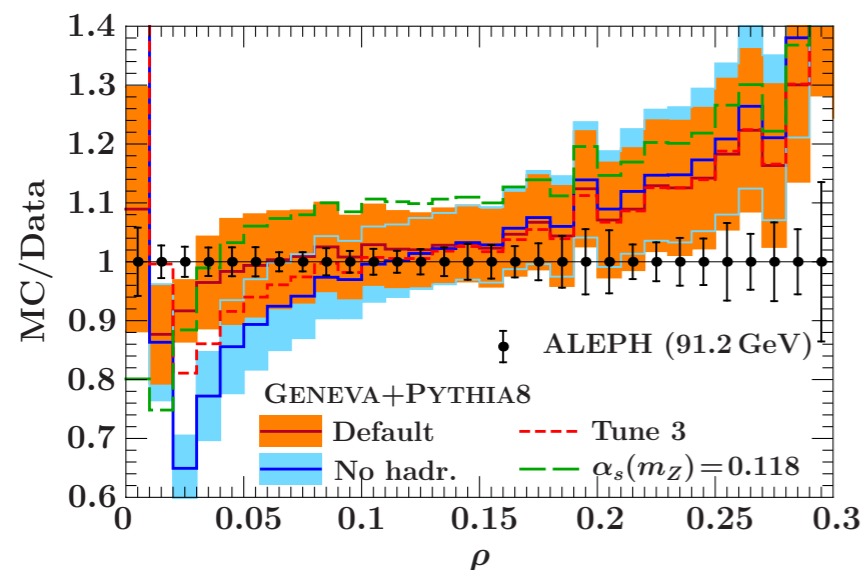
“backshowering” into light hemisphere

# LEP Comparison: Heavy Jet Mass



fast transition into the tail region

again the default tune performs better than larger  $\alpha_s$  or tune 3

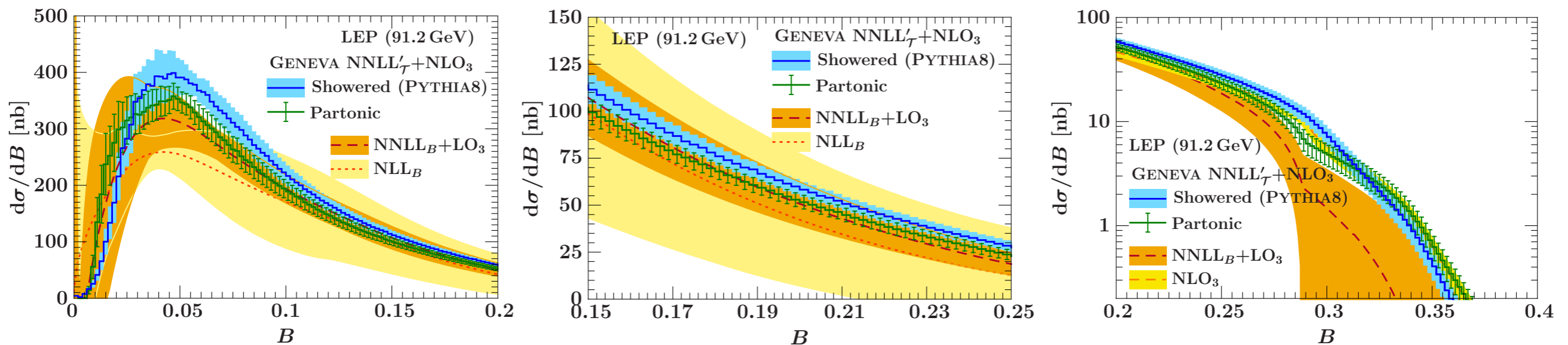


the need for matching out to high jet multiplicity is apparent

even matching to 5 jets at LO will improve the tail region

# Parton Shower: Jet Broadening

Becher, Bell  
1210.0580



Pythia changes the spectrum from partonic GENEVA  
due to the lack of correlation with thrust

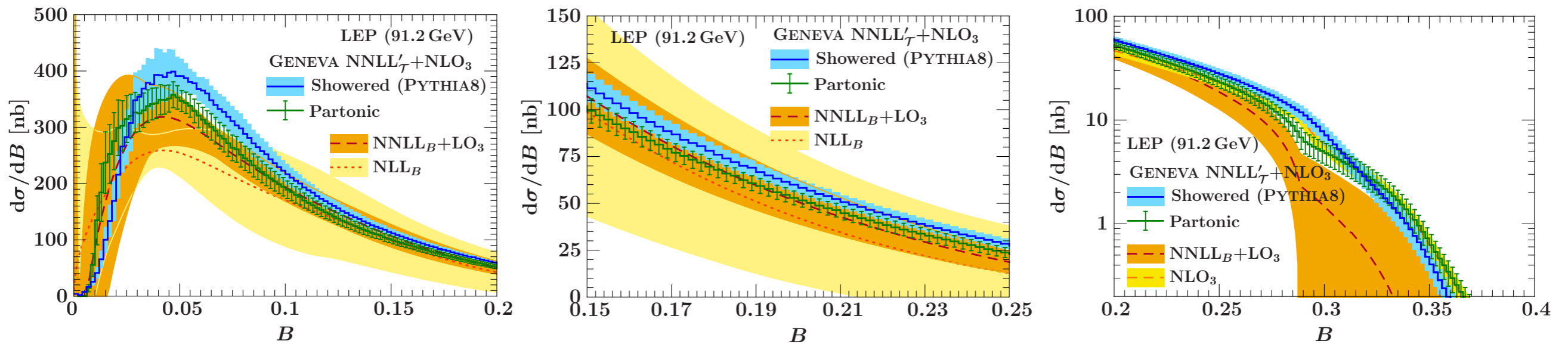
GENEVA showered result closer to the NNLL than  
NLL resummation for jet broadening

$$\text{SCET}_{\parallel} : \frac{d\sigma}{dB} = \frac{\alpha_s C_F}{2\pi} \left( \frac{-8 \ln B - 6}{B} \right) + \dots$$

$$\text{SCET}_{\perp} : \frac{d\sigma}{d\tau} = \frac{\alpha_s C_F}{2\pi} \left( \frac{-4 \ln \tau - 3}{\tau} \right) + \dots$$

interesting to ask how accurately  
GENEVA is describing  
uncorrelated observables

# Interlude on Uncertainties



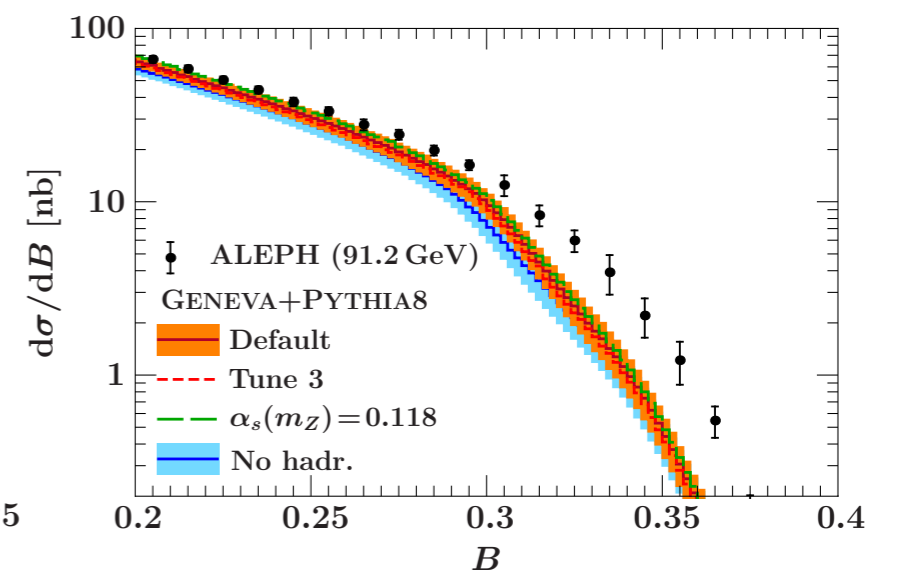
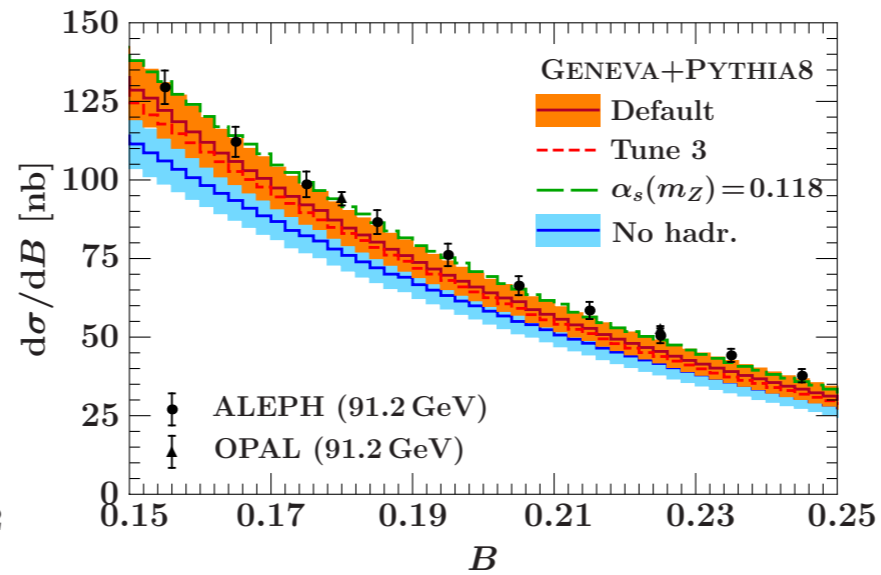
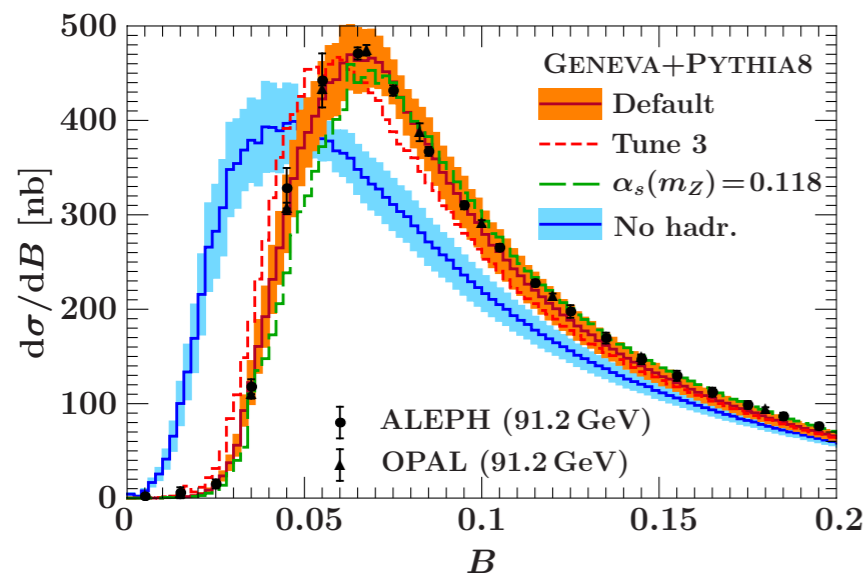
GENEVA provides FO and scale variation uncertainties event-by-event

We do not (yet) give uncertainties associated with Pythia

A reasonable proxy for parton shower uncertainties could be the spread between partonic and showered GENEVA

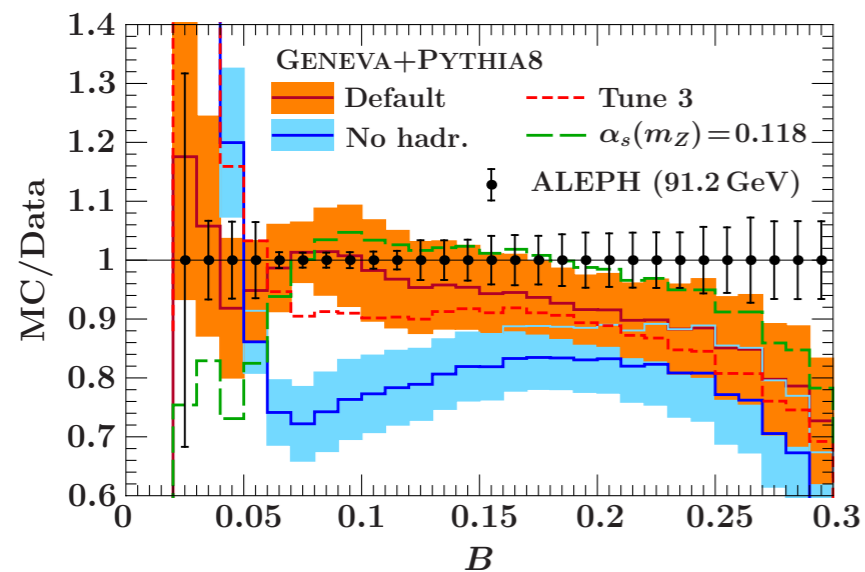
Hadronization uncertainties are more delicate, and would require proper tuning of hadronization parameters

# LEP Comparison: Jet Broadening

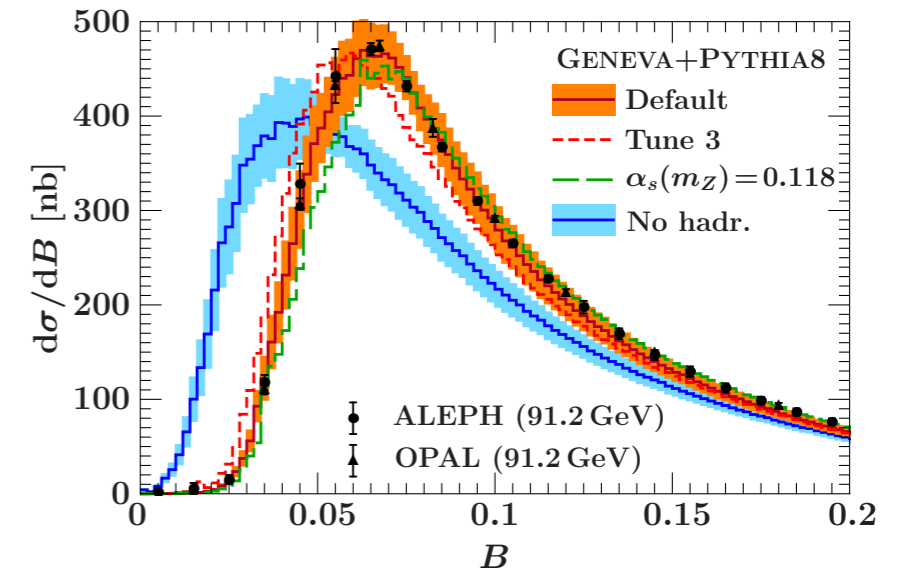
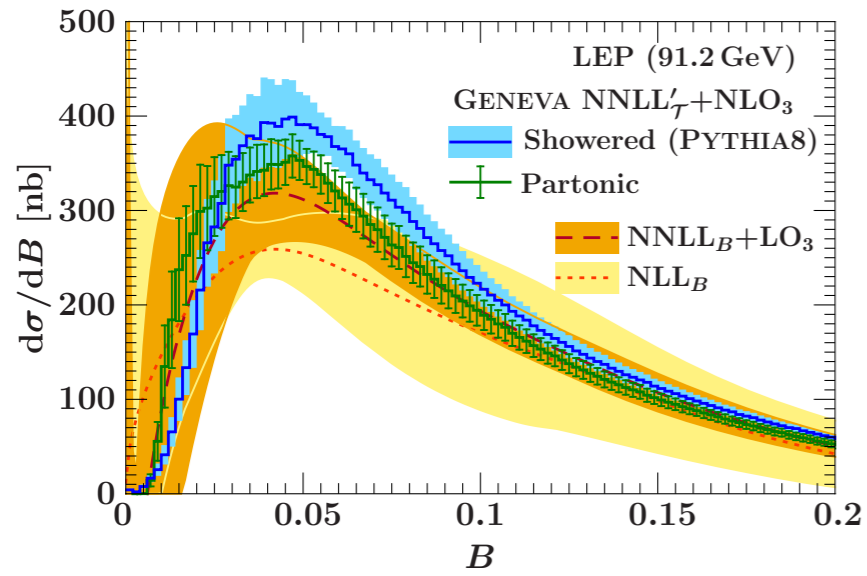


good agreement with data  
through peak/early transition

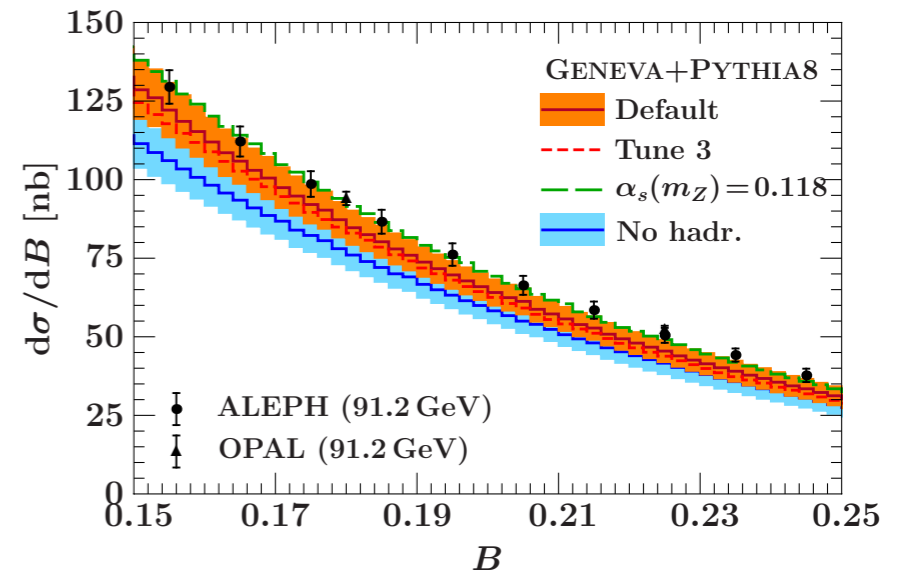
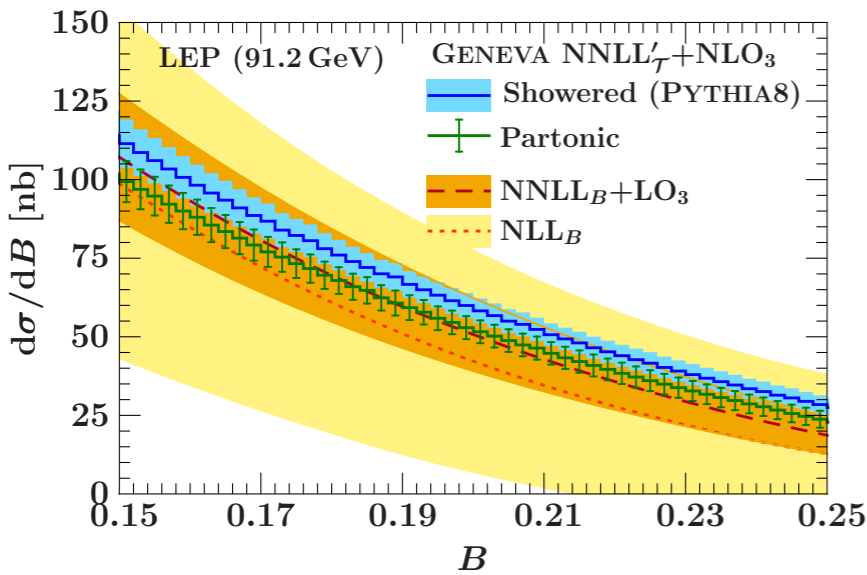
multijet corrections  
in the tail region



# Jet Broadening: GENEVA Prediction



Showered agrees with analytic resummation, hadronized agrees with data



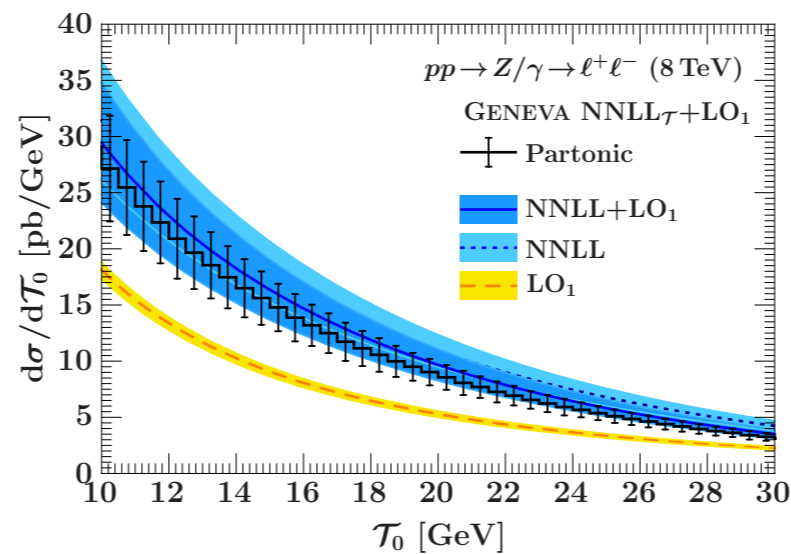
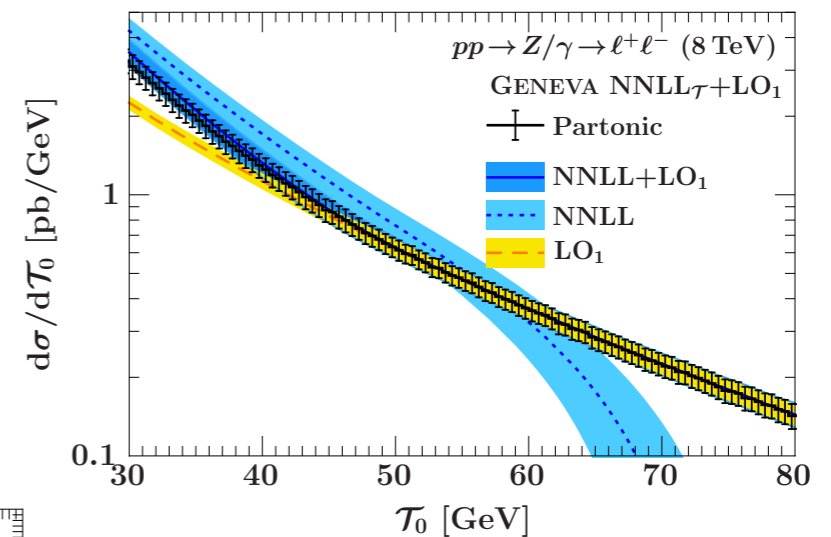
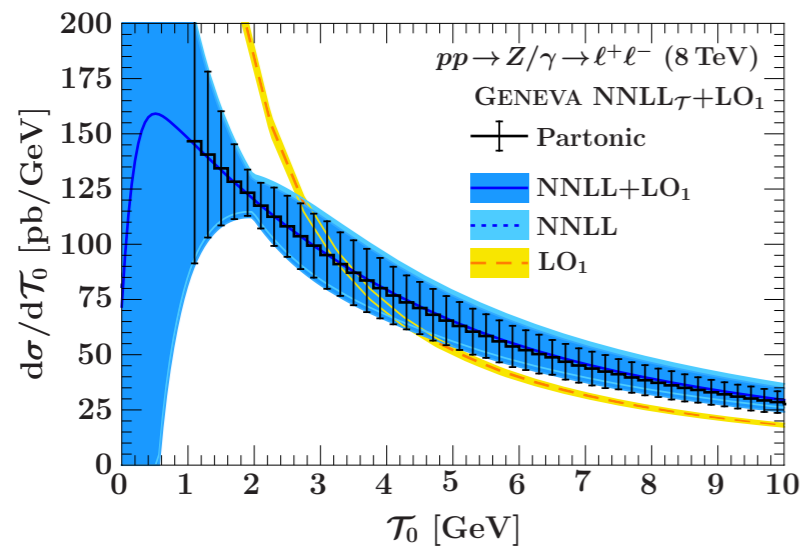


# Outline

The GENEVA Framework

Comparison to LEP Data

Applications for the LHC



# From $e^+e^-$ to pp Collisions

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Can use N-jettiness as in  $e^+e^-$  to distinguish jet multiplicities

master formula: 
$$\frac{d\sigma^{\text{incl}}}{d\Phi_0} = \frac{d\sigma}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) + \int \frac{d\Phi_1}{d\Phi_0} \frac{d\sigma}{d\Phi_1}(\mathcal{T}_0) \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})$$

The essential perturbative physics translates to pp collisions

$$\frac{d\sigma}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) = \frac{d\sigma^{\text{resum}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) + \left[ \frac{d\sigma^{\text{FO}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) - \frac{d\sigma^{\text{resum}}}{d\Phi_0}(\mathcal{T}_0^{\text{cut}}) \Big|_{\text{FO}} \right]$$

$$\frac{d\sigma}{d\Phi_1}(\mathcal{T}_0) = \frac{d\sigma^{\text{FO}}}{d\Phi_1} \left[ \frac{d\sigma^{\text{resum}}}{d\Phi_0 d\mathcal{T}_0} / \frac{d\sigma^{\text{resum}}}{d\Phi_0 d\mathcal{T}_0} \Big|_{\text{FO}} \right]$$

# From $e^+e^-$ to pp Collisions

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Can use N-jettiness as in  $e^+e^-$  to distinguish jet multiplicities

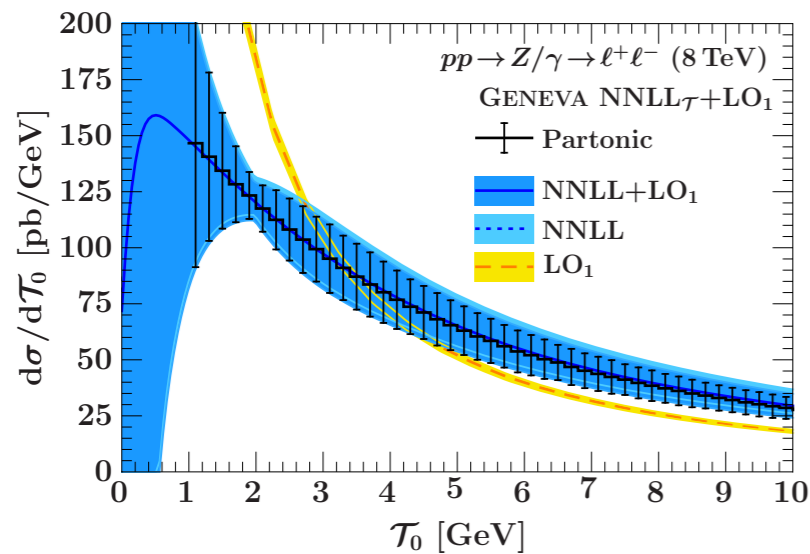
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Initial state radiation provides conceptual, technical challenges

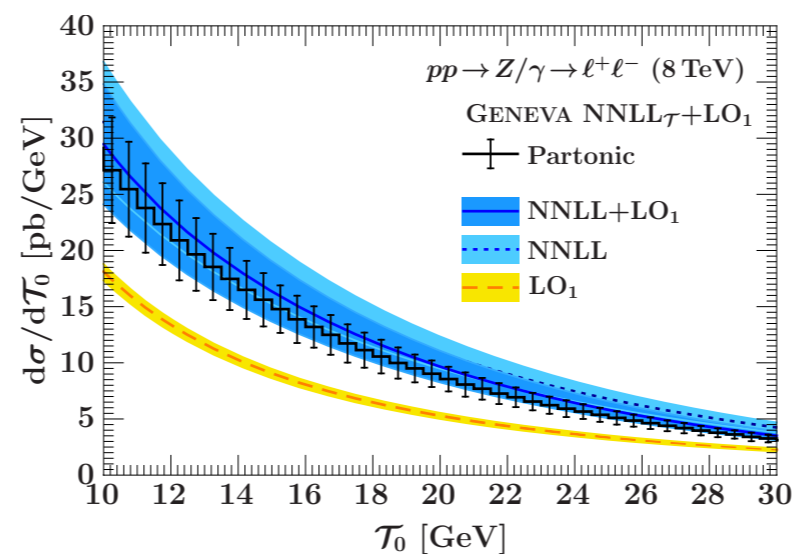
- Resummation involves beam functions, sum over partonic channels
- FO calculations more challenging
- Requires matching GENEVA to an initial state parton shower
- pp collisions require multiple parton interaction (MPI) model

# Initial LHC Plots: Drell-Yan Production

partonic GENEVA, NNLL + LO<sub>1</sub>



agrees with resummation  
in the peak



agrees with FO  
in the tail

Drell-Yan is a good test process - lots of physics, lots of data

W + jets, Higgs production, top production are next steps

# Road to Improvements for pp Collisions

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- Improve resummation accuracy to NNLL'
- Improve FO accuracy to NLO<sub>1</sub>
- Add parton shower, hadronization, MPI
- Combine and test against DY studies at the LHC, Tevatron
- Add other processes

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We are in the process of validating the combination  
NNLL' + NLO<sub>1</sub> + Pythia

# Conclusions

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- Precision physics is going to play a large role in Higgs measurements and BSM physics at the LHC
  - Higgs property measurements improved through more accurate calculations
  - More challenging new physics searches require higher precision background estimation
- GENEVA combines the high accuracy from the calculation of single observables with the flexibility of a Monte Carlo
  - Excellent agreement with LEP data for a variety of observables
  - Look for more pp results in the next several months
  - Initial code release planned this summer

# Extra Slides

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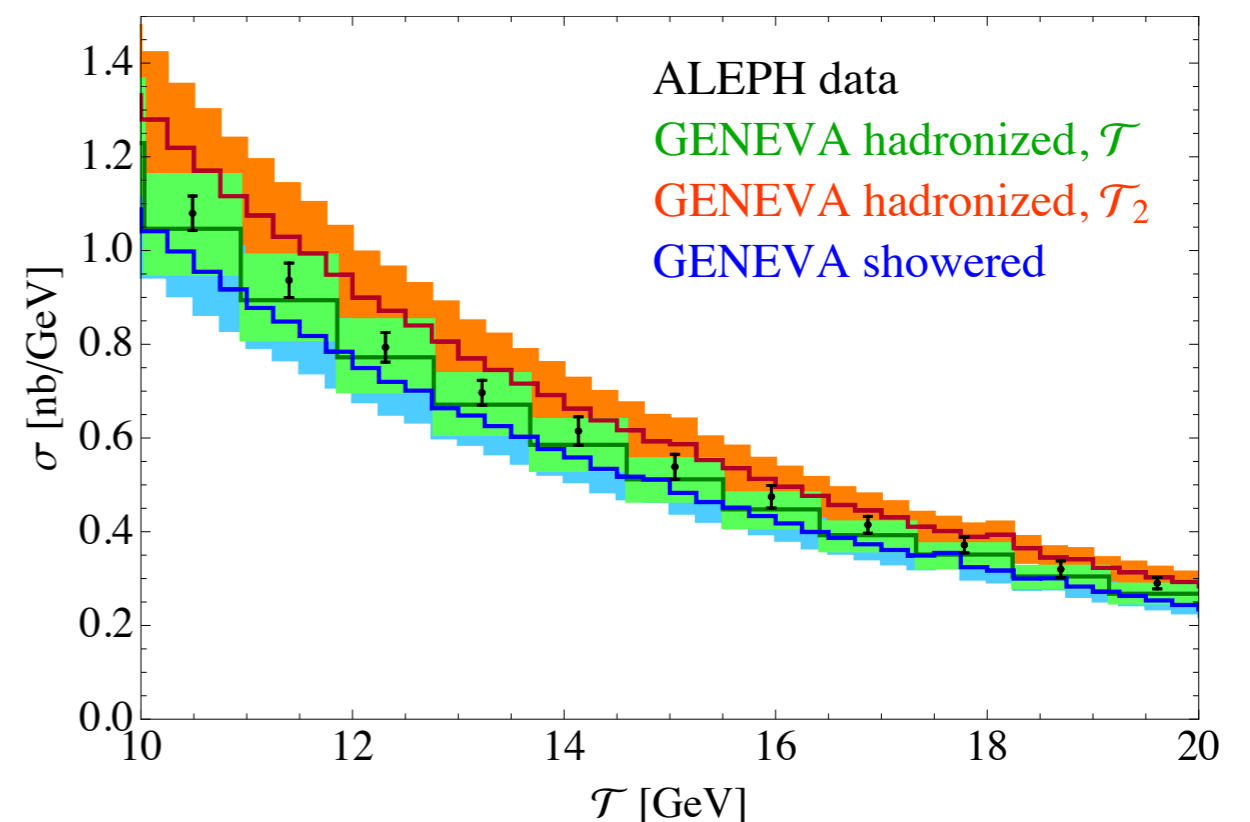
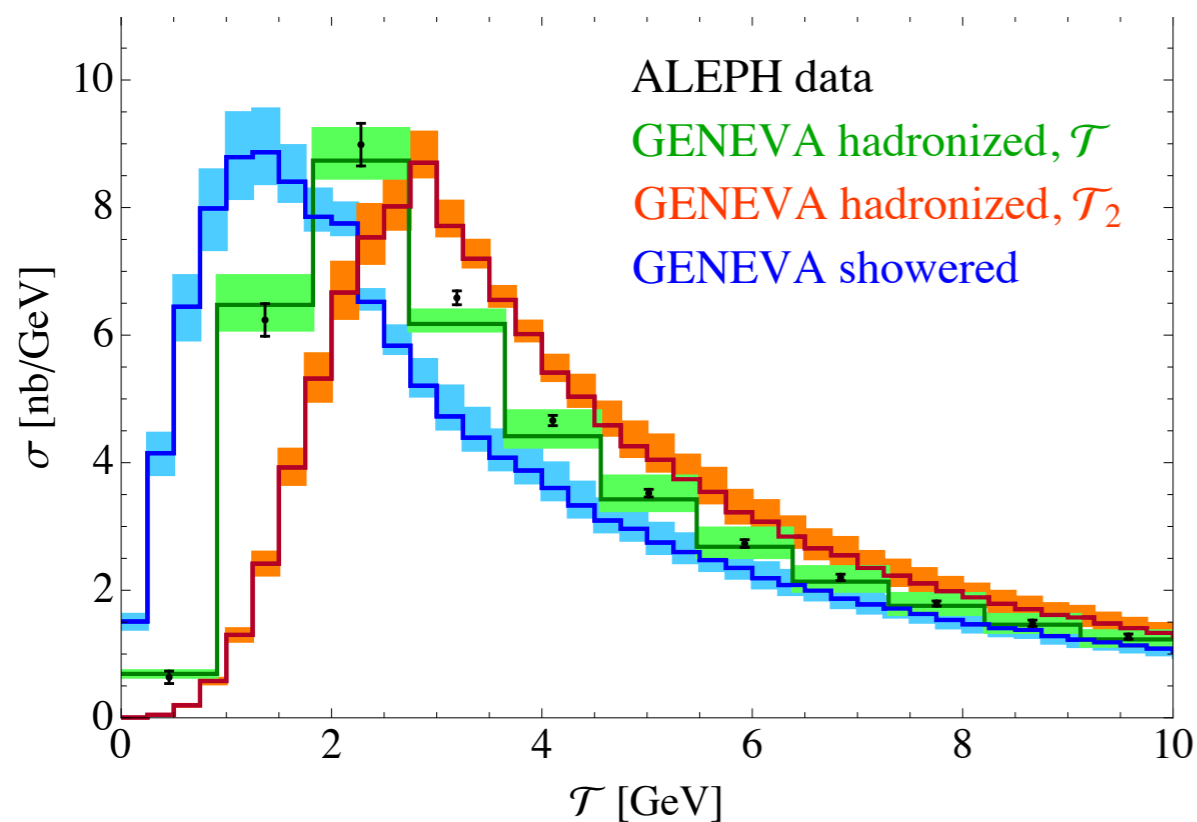


# Power Corrections from Hadron Masses

power corrections for thrust, 2-jettiness differ:  $\Omega_1^{\mathcal{T}_2} = 2\Omega_1^{\mathcal{T}}$

Mateu, Stewart, Thaler 1209.3781

Observe an 1 scaling with thrust and 2-jettiness that agrees with **MST**



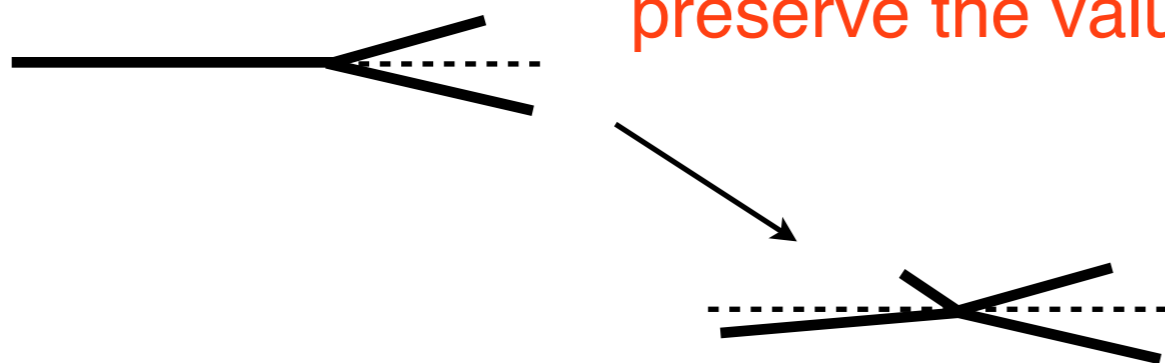
# Fixed Order Calculation

$$\frac{d\sigma^{\text{NLO}}}{d\Phi_3}(\mathcal{T}) = \overset{\text{Born}}{B_3(\Phi_3, \mathcal{T})} + \overset{\text{virtual}}{V_3(\Phi_3, \mathcal{T})} + \int \frac{d\Phi_4}{d\Phi_3} B_4(\Phi_4) \delta[\mathcal{T}(\Phi_4(\Phi_3)) - \mathcal{T}(\Phi_3)]$$

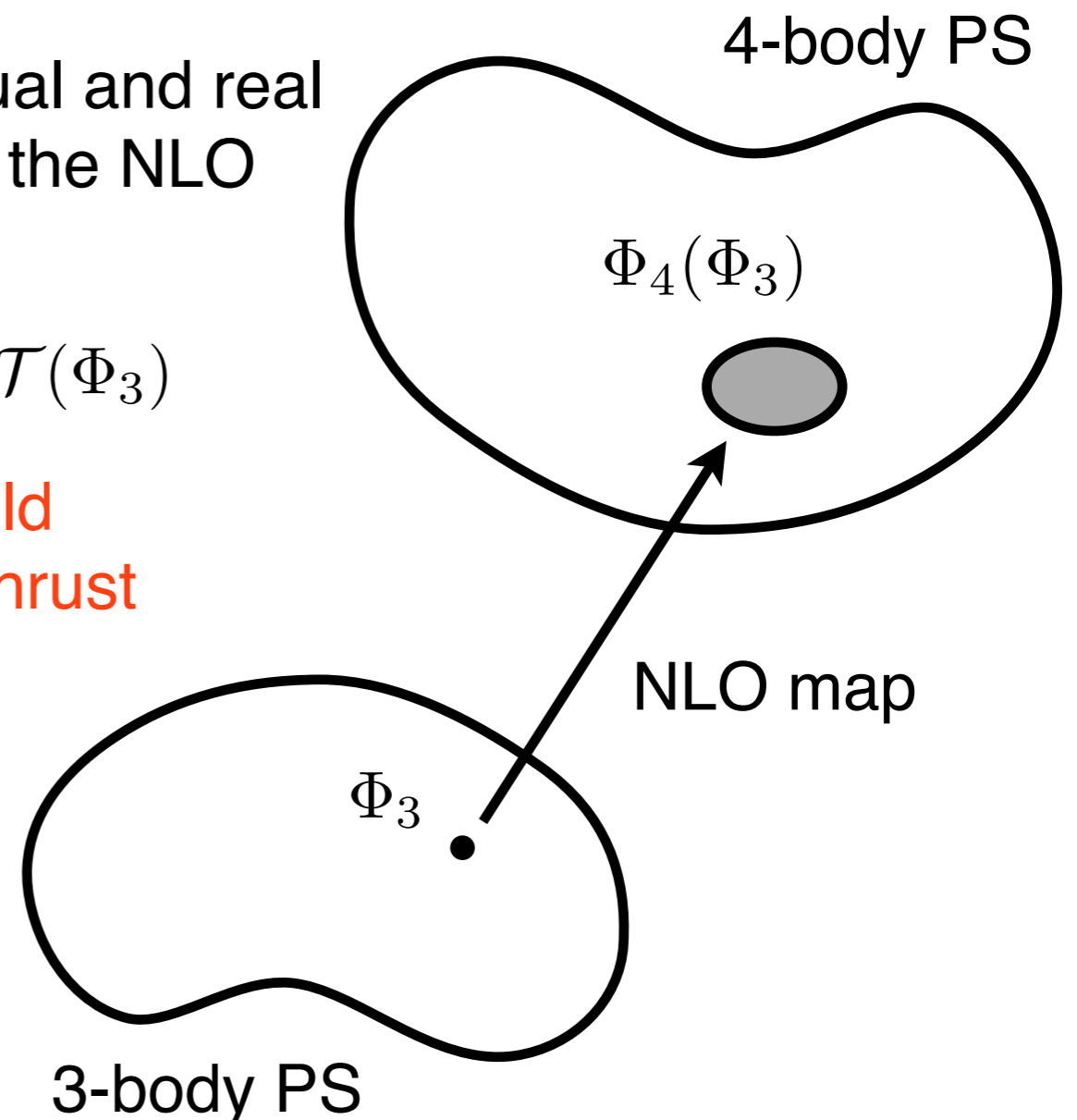
We use FKS subtractions to render the virtual and real emission calculations finite and calculate the NLO

need  $\mathcal{T}(\Phi_4(\Phi_3)) = \mathcal{T}(\Phi_3)$

real emissions should preserve the value of thrust



We have designed a PS map that does this



# Resummation in a Monte Carlo vs. Direct Calculation

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matching formula for the spectrum:

$$\frac{d\sigma}{d\Phi_1}(\mathcal{T}_0) = \frac{d\sigma^{\text{FO}}}{d\Phi_1} \left[ \frac{d\sigma^{\text{resum}}}{d\Phi_0 d\mathcal{T}_0} \Big/ \frac{d\sigma^{\text{resum}}}{d\Phi_0 d\mathcal{T}_0} \Big|_{\text{FO}} \right]$$

our usual formula for the resummation:

$$\frac{d\sigma^{\text{resum}}}{d\Phi_0 d\mathcal{T}_0} = \sum_{\kappa} H_0^{\kappa}(\Phi_0) \int dY B_a^{\kappa}(Y, \mathcal{T}_0) \otimes B_b^{\kappa}(Y, \mathcal{T}_0) \otimes S(\mathcal{T}_0)$$

each part of the matching formula is a sum over contributions from separate parton channels

$$\{u\bar{u}, d\bar{d}, c\bar{c}, s\bar{s}, b\bar{b}\} \rightarrow \ell^+ \ell^-$$

events are generated in individual flavor channels

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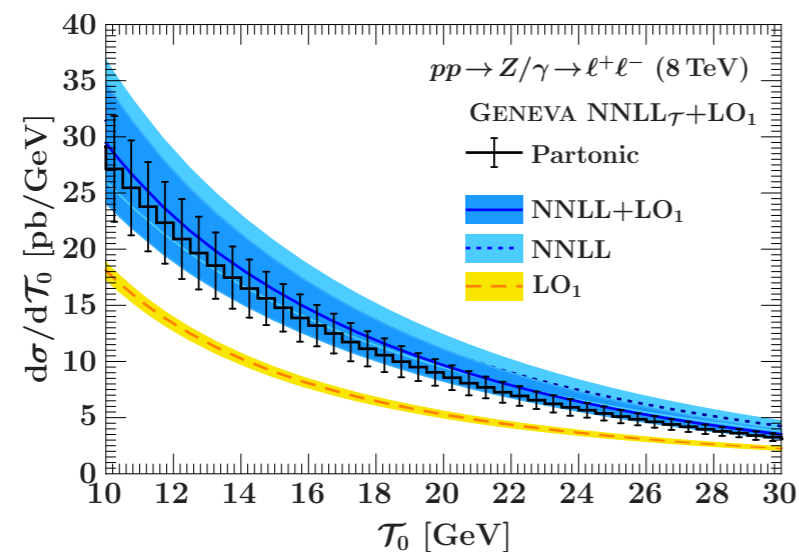
events are generated in individual flavor channels

# Resummation in a Monte Carlo vs. Direct Calculation

In GENEVA we pull the sum over parton channels out:

$$\frac{d\sigma}{d\Phi_1}(\mathcal{T}_0) = \sum_{\kappa} \frac{d\sigma_{\kappa}^{\text{FO}}}{d\Phi_1} \left[ \frac{d\sigma_{\kappa}^{\text{resum}}}{d\Phi_0 d\mathcal{T}_0} / \frac{d\sigma_{\kappa}^{\text{resum}}}{d\Phi_0 d\mathcal{T}_0} \Bigg|_{\text{FO}} \right]$$

The difference is small but noticeable:



Both matching formulas are equally valid, as they correctly interpolate in the singular/nonsingular limits