HADRON MASS EFFECTS IN POWER CORRECTIONS TO EVENT SHAPES

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IFIC - Valencia

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Collaboration with J. Thaler and I. Stewart Phys.Rev. D87 (2013) 014025

Builds on earlier work by
Salam and Wicke JHEP 0105 (2001) 061
Lee and Sterman Phys.Rev. D75 (2007) 014022

OUTLINE

- Introduction
- Power Corrections
- Universality
- Hadron Mass Effects
- Anomalous dimension of Power Correction
- Effects on the Cross Section and First Moment
- Conclusions

INTRODUCTION

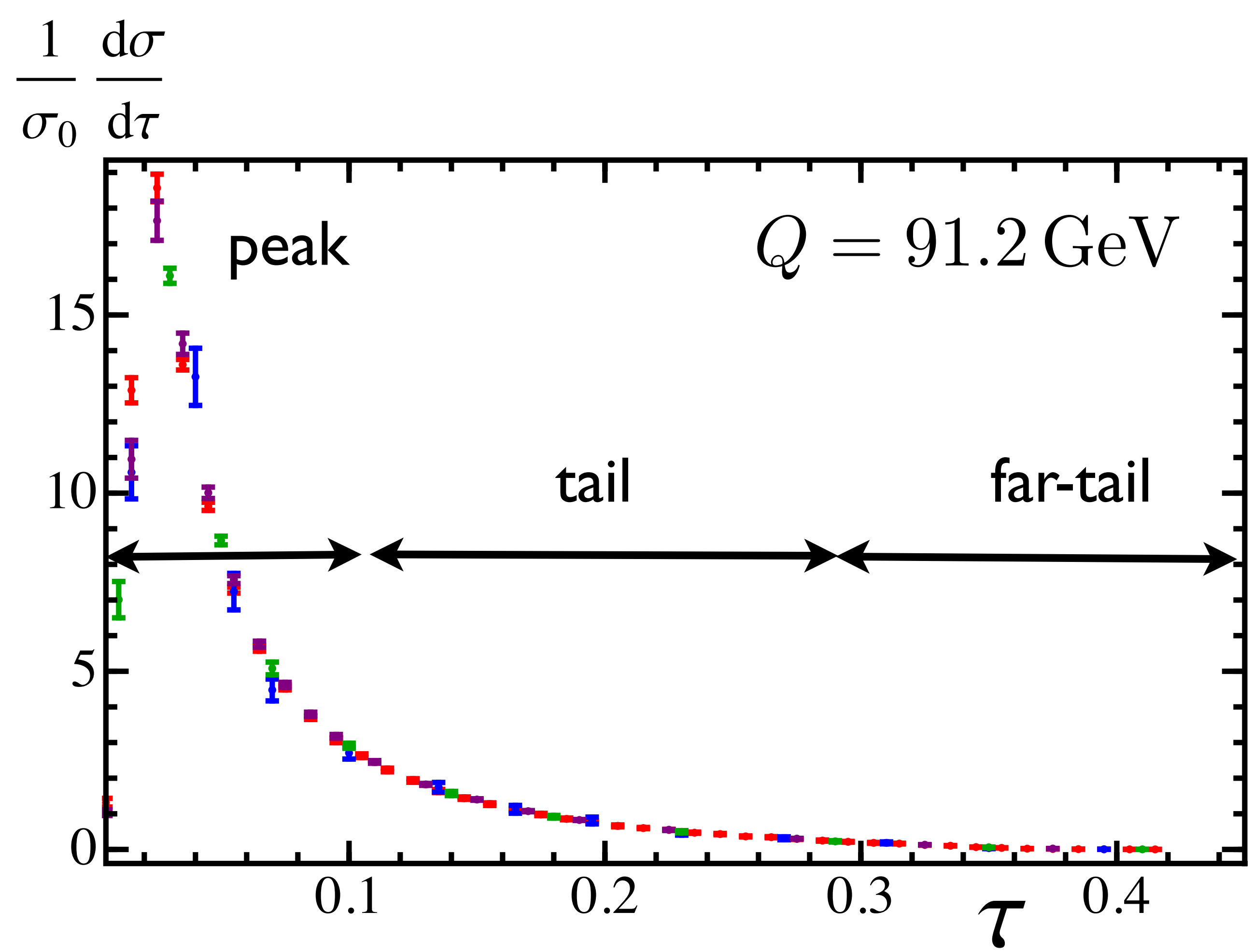
Event Shapes $e^+ e^- \to \text{jets}$

Event shapes characterize in a geometrical way the distribution of hadrons in the final state

Thrust is the most commonly studied event shape variable

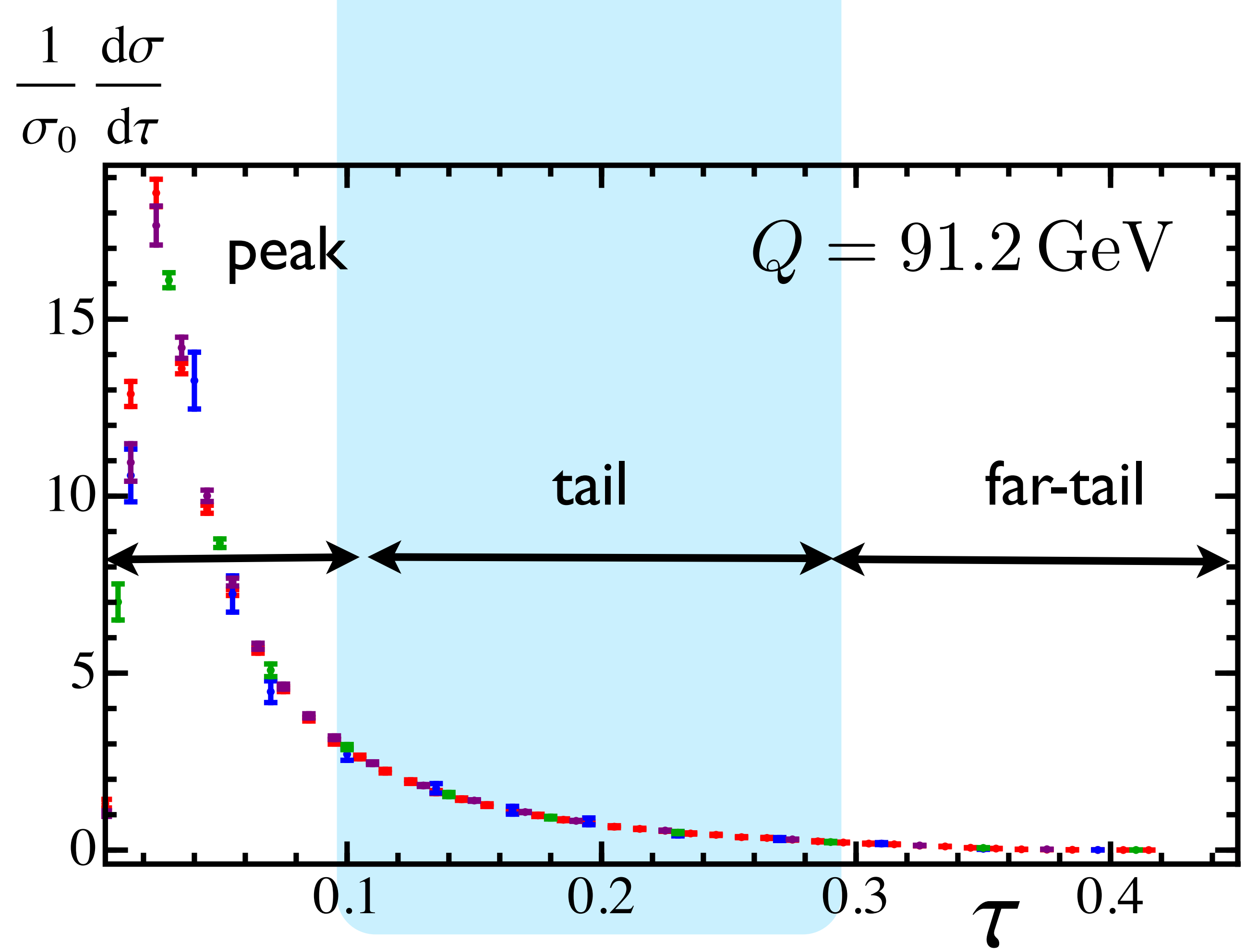
They are theoretically **more friendly than a Jet algorithm**

Continuous transition from 2-jet to 3-jet, ... multi-jet events



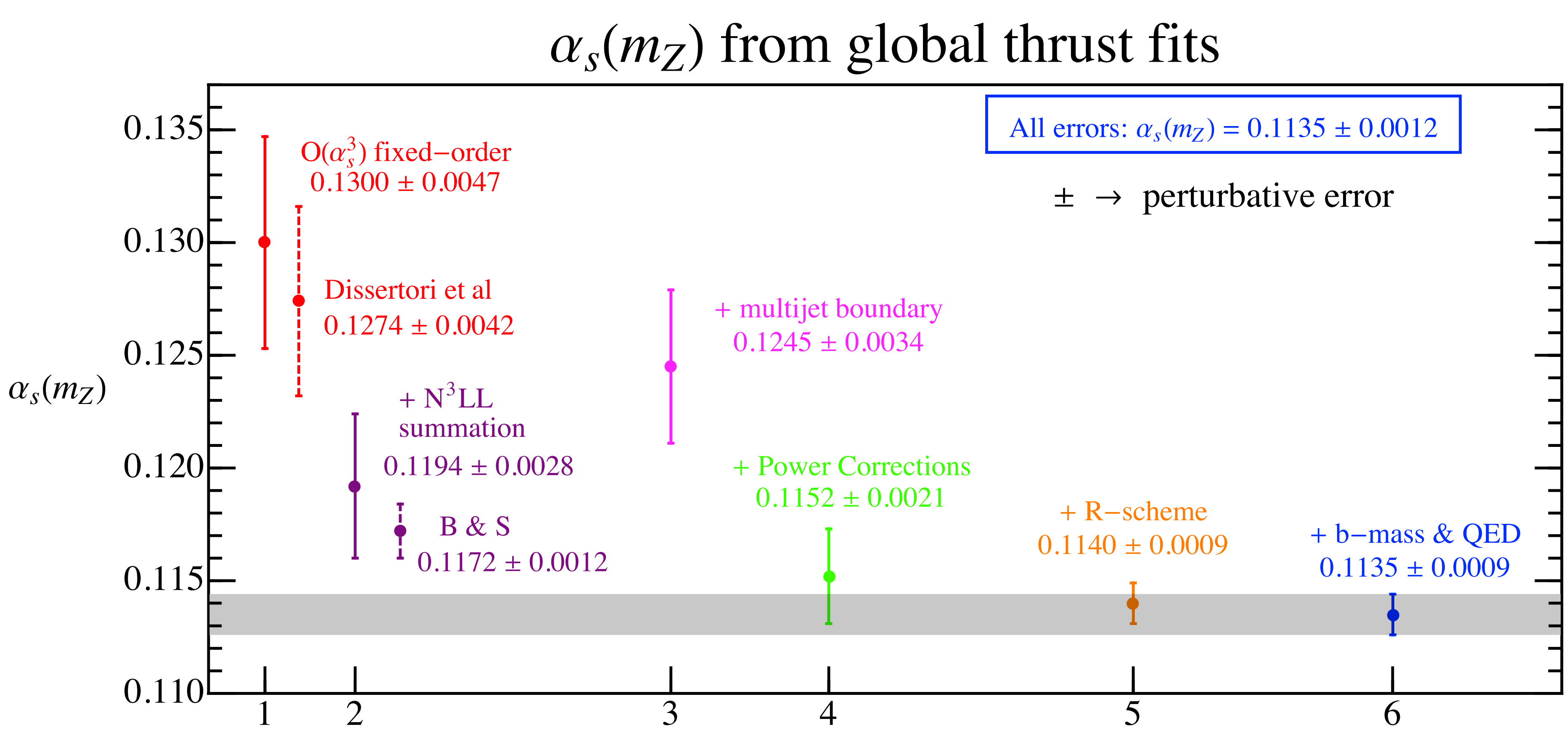
Event Shapes $e^+ e^- \to \text{jets}$

study power corrections and hadron mass effects in tail region, where an OPE is well defined



Motivations

- Event shapes have been extensively used to determine $\alpha_s(m_Z)$
- Power Corrections play an essential role in that determination
- Also important effects in Jet Substructure *[Boost 2012 proceedings]* *[Feige, Schwartz, Stewart, Thaler 2012]*
- Important in hadronization and underlying event at the LHC



[Abbate, Fickinger, Hoang, VM, Stewart]

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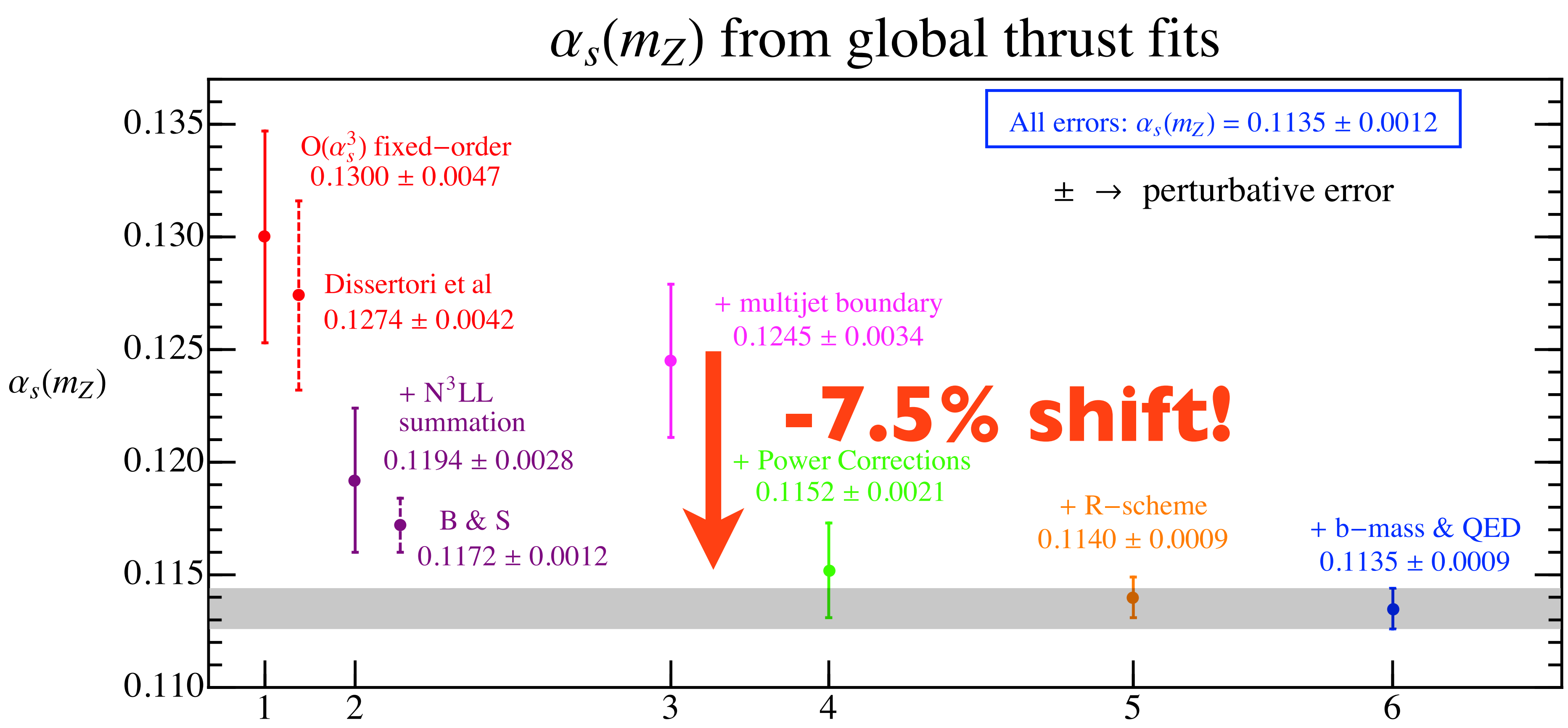

[Abbate, Fickinger, Hoang, VM, Stewart]

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Most common Event shapes

- Thrust $$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i\,|}$$ *[E. Farhi]*
- Angularities $$\tau_{(a)} = \frac{1}{Q} \sum_i E_i\,(\sin\theta_i)^a (1 - |\cos\theta_i|)^{1-a}$$ *[Berger, Kucs, Sterman]*
- Jet Masses $$\rho_\pm = \frac{1}{Q^2} \Big(\sum_{i\in\pm} p_i \Big)^2$$ *[Clavelli]* *[Chandramohan Clavelli]*
- Jet Broadening $$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$ *[Catani, Turnock, Webber]*
- C-parameter $$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2}$$ *[Parisi]* *[Donoghue, Low, Pi]*
- 2-Jettiness $$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$$ *[Stewart, Tackmann, Waalewijn]*

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2-jet event shapes

$e \to 0$

$\updownarrow$

dijet configuration

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Depend on a continuous parameter

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Our results do not apply in this case

Recoil sensitive

Most common Event shapes

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double sum

does not require minimization procedure

Factorization theorem for event shape distributions

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

[Korchemsky, Sterman]

[Berger, Kucs, Sterman]

[Fleming, Mantry, Hoang, Stewart]
thrust, jet masses

[Bauer, Fleming, Lee, Sterman]
general dijet case

Universal Wilson Coefficient

Jet function

Soft function

Nonsingular terms, power corrections

Calculable in perturbation theory $\mathcal{O}\left(\frac{1}{e}\right)$

Perturbative and nonperturbative components

In the dijet limit, event shape decomposes in collinear, soft and nonperturbative modes. This translates into a factorization theorem for differential distributions.

$$e = e_c + e_s + e_\Lambda$$

$e_c + e_s$: $\gg \frac{\Lambda_{\mathrm{QCD}}}{Q}$

e_Λ: $\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$

Factorization theorem for event shape distributions

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{\rm QCD}}{Q}\right)$$

$$S_e(\ell) = \langle\, 0\,|\,\overline{Y}^\dagger_{\bar n} Y^\dagger_n \delta(\ell - Q\hat{e}) Y_n \overline{Y}_{\bar n}\,|\,0\,\rangle$$

Leading power correction comes from soft function

Soft Wilson lines — event shape operator

$$S_e = \hat{S}_e \otimes F_e$$

perturbative — non-perturbative

[Korchemsky, Sterman]
[Korchemsky, Tafat]
[Ligeti, Tackmann, Stewart]
[Hoang, Stewart]

actually, it has **perturbative** too ! (more on this later)

[VM, Thaler, Stewart]

$$e = \underbrace{e_c + e_s}_{\gg \frac{\Lambda_{\rm QCD}}{Q}} + e_\Lambda$$

e_Λ: $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right)$

can drop hadron masses here by power counting

but not here!!

$m_H \sim \mathcal{O}(\Lambda_{QCD})$

$$\frac{d\sigma}{de} = \frac{d\hat{\sigma}}{de} \otimes F_e$$

POWER CORRECTIONS FOR EVENT SHAPES

Tree level OPE for nonperturbative corrections

[Lee & Sterman]

$$S_e(\ell) = \langle\, 0 \,|\, \overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \delta(\ell - Q\hat{e}) Y_n \overline{Y}_{\bar{n}} \,|\, 0 \,\rangle$$

For $e \gg \frac{\Lambda_{\rm QCD}}{Q}$

$$\delta(\ell - Q\hat{e}) \simeq \delta(\ell) - \delta'(\ell) Q\hat{e} + \ldots$$

Correct up to $\mathcal{O}(\alpha_s)$

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Shape function can be expanded in the tail

$$F_e(\ell) \simeq \delta(\ell) - \Omega_1 \delta'(\ell)$$

$$\Omega_1 = \langle\, 0 \,|\, \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger Q\hat{e} Y_n \overline{Y}_{\bar{n}} \,|\, 0 \,\rangle$$

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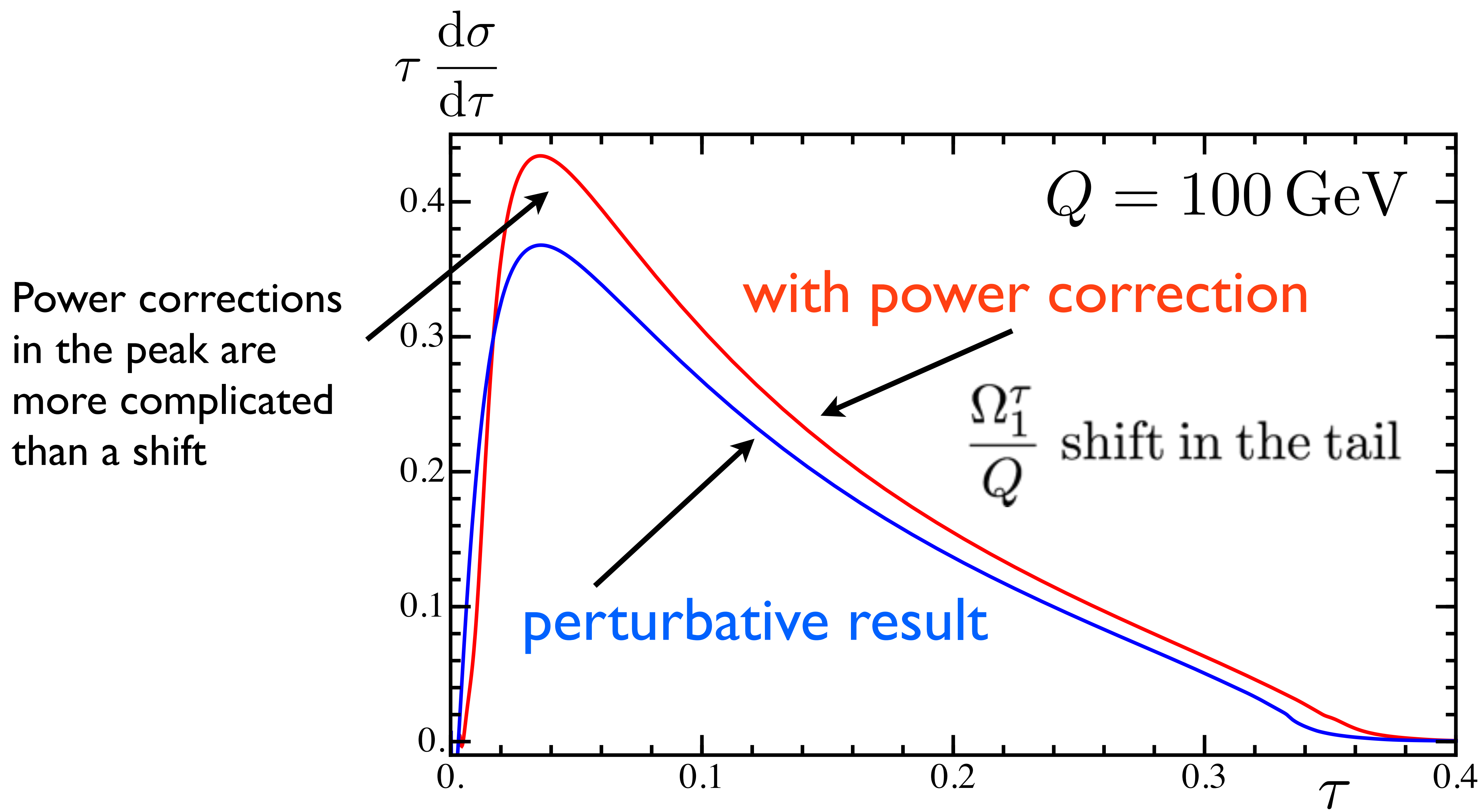
$$\frac{{\rm d}\sigma}{{\rm d}e} = \frac{{\rm d}\hat{\sigma}}{{\rm d}e} - \frac{\Omega_1}{Q}\frac{\rm d}{{\rm d}e}\frac{{\rm d}\hat{\sigma}}{{\rm d}e} \simeq \frac{{\rm d}\hat{\sigma}}{{\rm d}e}\Big(e - \frac{\Omega_1}{Q}\Big) + \mathcal{O}\left[\left(\frac{\Lambda_{\rm QCD}}{Q\,e}\right)^2\right]$$

Leading nonperturbative correction in the tail is a shift of the distribution

Power correction for Thrust



The main effect of the power correction is shifting the distribution to the right. The shift is proportional to $\frac{1}{Q}$

Dispersive approach

[Dokshitzer & Webber]

Assume that α_s is replaced by an effective coupling below certain cutoff μ_I

Subtract from perturbation theory contributions at scales below μ_I

It is believed that this procedure removes all renormalons

Initial approach relied on one gluon exchange

The Milan factor accounts for two-gluon exchange *[Dokshitzer, Webber Salam]*

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The Milan factor accounts for two-gluon exchange *[Dokshitzer, Webber Salam]*

It predicts that leading power correction is universal up to a calculable coefficient

Effect on first moment

$$\langle e \rangle = \langle e \rangle_{\mathrm{PT}} + c_e \frac{\mathcal{P}}{Q}$$

Effect on distributions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e}\left(e - c_e \frac{\mathcal{P}}{Q}\right)$$

c_e = universality constant

(more on this later)

$$\mathcal{P} = \frac{4C_F}{\pi^2}\mathcal{M}\frac{\mu_I}{Q}\left\{\alpha_0(\mu_I) - \alpha_s(\mu_R) - \beta_0\frac{\alpha_s^2}{2\pi}\left(\ln\frac{\mu_R}{\mu_I} + \frac{K}{\beta_0} + 1\right)\right\}$$

Milan Factor $\simeq 1.49$

Shape function approach

Soft function is convolution of perturbative soft function and shape function

$$S_e(\ell) = \int \mathrm{d}p\, \hat{S}_e(\ell - p) F_e(p) \quad \Longrightarrow \quad \frac{\mathrm{d}\sigma}{\mathrm{d}e} = \int \mathrm{d}\ell\, \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e}\left(\mathrm{e} - \frac{\ell}{\mathrm{Q}}\right) \mathrm{F_e}(\ell)$$

perturbative nonperturbative
(and perturbative...)

Non pert. distribution is convolution of pert. distribution with shape function
This is valid on the peak of the distribution as well

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perturbative   nonperturbative (and perturbative...)

Non pert. distribution is convolution of pert. distribution with shape function
This is valid on the peak of the distribution as well

Effect on moments $$\langle e^n \rangle = \langle e^n \rangle_{\mathrm{PT}} + n \langle e^{n-1} \rangle_{\mathrm{PT}} \frac{\Omega_1^e}{Q}$$

Effect on distributions $$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e} - \frac{\Omega_1^e}{Q} \frac{\mathrm{d}}{\mathrm{d}e} \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e}$$

Massless universality $$\Omega_1^e = c_e\, \Omega_1^\rho$$ *[Lee & Sterman]*

UNIVERSALITY

Studies of Universality

- Dispersive approach *[Dokshitzer & Webber 1995]*
  - Predicts universality for a bunch of event shapes, including recoil sensitive ones.
  - They are based on a model and on the one-gluon approximation. Modification of (effective coupling) below a cutoff scale.
  - Milan factor takes into account two-gluon effects. *[Dokshitzer, Webber, Salam]*
- SCET-CSS approach *[Lee & Sterman 2006]*
  - Predicts universality for non-recoil-sensitive event shapes.
  - They are model-independent, formulated in terms of QCD matrix elements.
  - Do not rely on one-gluon approximation.

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Both approaches assume particles are massless!!

Massless predictions for universality

Thrust	$\tau = 1 - \max_{\vec{n}} \frac{\sum_i \lvert\vec{p}_i \cdot \vec{n}\rvert}{\sum \lvert\vec{p}_i\rvert}$	$c_\tau = 2$
Two-Jetiness	$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i \lvert\vec{p}_i \cdot \vec{n}\rvert}{Q}$	$c_{\tau_2} = 2$
C-parameter	$C = \frac{3}{2} \frac{\sum_{i,j} \lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert \sin^2(\theta_{ij})}{(\sum_i \lvert\vec{p}_i\rvert)^2}$	$c_C = 3\pi$
Angularities	$\tau_{(a)} = \frac{1}{Q} \sum_i E_i \, (\sin\theta_i)^a (1 - \lvert\cos\theta_i\rvert)^{1-a}$	$c_{\tau_{(a)}} = \frac{2}{1-a}$
Jet Masses	$\rho_\pm = \frac{1}{Q^2} \Big(\sum_{i \in \pm} p_i \Big)^2$	$c_\rho = 1$

Massless Universality in SCET-CSS

In the massless limit one has

$$e(N) = \frac{1}{Q}\sum_{i\in N} p_i^{\perp}\, f_e(1, y_i)$$

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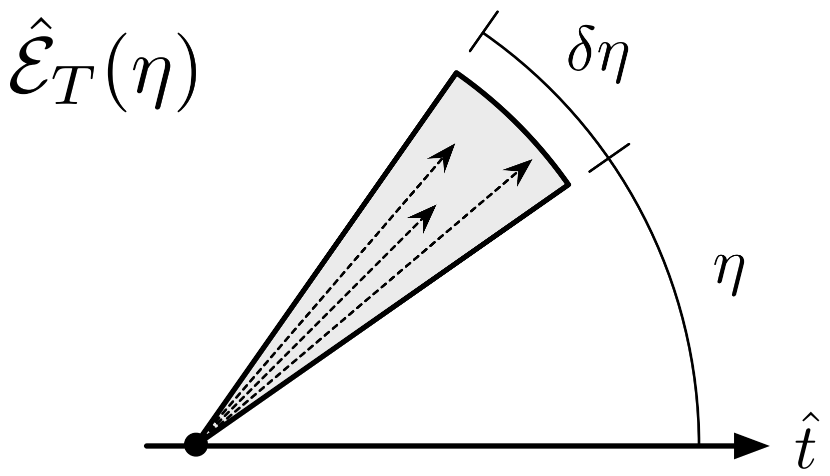
Transverse energy-flow operator

$$\mathcal{E}_T(y)|\,N\,\rangle = \sum_{i\in N} p_i^{\perp}\delta(y-y_i)|\,N\,\rangle$$

[Lee Sterman, Korchemsky Oderda Sterman, Sveshnikov and F.V.Tkachov Ore Sterman]



[Bauer, Fleming, Lee, Sterman]

Measures all momenta flowing in a given rapidity

$$\mathcal{E}_T(y) = \frac{1}{\cosh^3 y}\int_0^{2\pi} \mathrm{d}\phi \lim_{R\to\infty}\int_0^{\infty} \mathrm{d}t\,\hat{n}_i T_{0i}(t, R\hat{n})$$

[unfortunately there is no physical limit in which this is the correct operator to use for power correction...]

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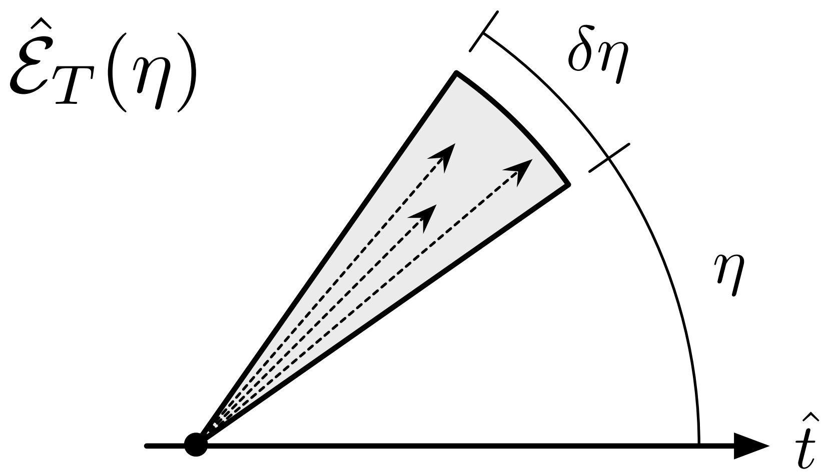

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Measures all momenta flowing in a given rapidity

$$Q\,\hat{e} = \int \mathrm{d}y\, f_e(1, y)\mathcal{E}_T(y) \longrightarrow \hat{e}\,|\,N\,\rangle = e(N)\,|\,N\,\rangle$$

Event shape operator

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$$\Omega_1^e = \langle\, 0\,|\, \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger Q\hat{e} Y_n \overline{Y}_{\bar{n}}\,|\,0\,\rangle$$

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Boost invariance requires this term is y-independent

Universal power correction

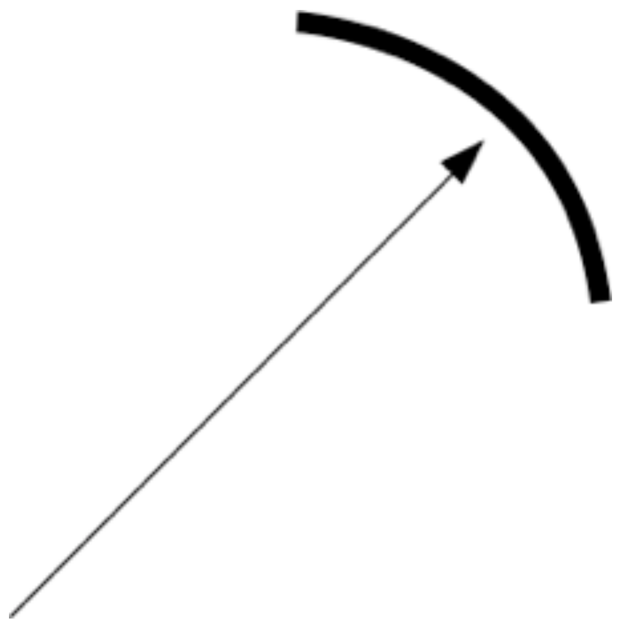
Calculable coefficient, depends on the event shape

$$\Omega_1^E = \langle\, 0\,|\,\overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(0) Y_n \overline{Y}_{\bar{n}}\,|\,0\,\rangle$$

Operator definition of power correction

HADRON MASS EFFECTS ON POWER CORRECTIONS

Hadron masses and Schemes



What can be measured when a particle hits the detector?

Ideally we would like **energy and momentum separately measured**, but that is not always possible.

If a **particle** is **not identified**, mass is not known, no information on magnitude of momentum.

One can assume all particles are pions [default scheme]

Alternatively one can use only energy and directions [E scheme] $|\vec{p}| \to E$

Finally one can use only momenta and directions [P scheme] $E \to |\vec{p}|$

These considerations are irrelevant in perturbation theory, but have important consequences for power corrections!

Kinematics of Event Shapes

We will concentrate on event shapes that are **not recoil sensitive**

and can be written in the dijet limit as

$$e(N) = \frac{1}{Q} \sum_{i \in N} m_i^{\perp} f_e(r_i, y_i)$$

$$y = \frac{1}{2} \log \left(\frac{E + p_z}{E - p_z} \right)$$ rapidity

$$r \equiv \frac{p^{\perp}}{m^{\perp}}$$ transverse velocity

All event shapes can be expressed in terms of these two variables

$$m^{\perp} = \sqrt{p_T^2 + m^2}$$ transverse mass

$$\eta = \ln \left(\frac{\sqrt{r^2 + \sinh^2 y} + \sinh y}{r} \right)$$ pseudo-rapidity

$$v = \frac{\sqrt{r^2 + \sinh^2 y}}{\cosh y}$$ velocity

massless limit

$$v = r = 1$$

$$y = \eta$$

$$m^{\perp} = p^{\perp}$$

Mass Effects on Power Corrections

Salam & Wicke 2001
have studied mass effects on power corrections

- Use the flux tube model (later refined with QCD effects)
- Predict that hadron masses break universality
- Find a privileged scheme (E-scheme) which preserves universality
- Predict that hadron multiplicity translates into log(Q) effects on power corrections

$$\Omega_1 \to \Omega_1 + K\left(\log\frac{Q}{\Lambda}\right)^{\frac{4C_A}{\beta_0}}$$

Mass Effects in SCET

[VM, I.W. Stewart, J.Thaler]
arXiv: 1209.3781

$$e(N) = \frac{1}{Q}\sum_{i\in N} m_i^{\perp}\, f_e(r_i, y_i)$$

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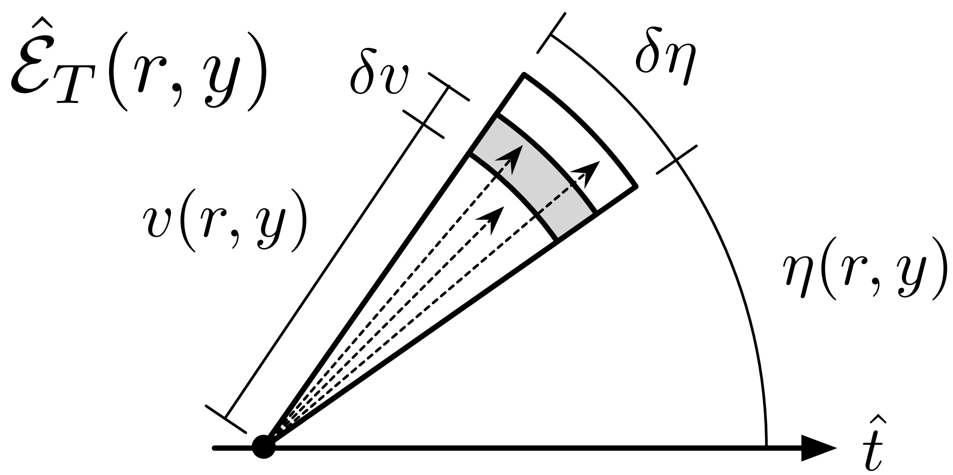
One has to generalize the transverse energy flow operator

Transverse velocity operator

$$\mathcal{E}_T(r,y)|\,N\,\rangle = \sum_{i\in N} m_i^\perp \delta(r-r_i)\delta(y-y_i)|\,N\,\rangle$$



$$v = v(r,y)$$
$$\eta = \eta(r,y)$$

measures momenta of particles with given transverse velocity flowing at a given rapidity

$$\hat{e} = \frac{1}{Q}\int \mathrm{d}y\,\mathrm{d}r\,\mathcal{E}_T(r,y) f_e(r,y)$$

two integrals

$$\mathcal{E}_T(v,\eta) = -\frac{v(1-v^2\tanh^2 y)^{\frac{3}{2}}}{\cosh\eta}\lim_{R\to\infty} R^3 \int_0^{2\pi} \mathrm{d}\phi\,\hat{n}_i\, T_{0i}(R, v\,R\,\hat{n})$$

Mass Effects in SCET

[VM, I.W. Stewart, J.Thaler]
arXiv: 1209.3781

$$e(N) = \frac{1}{Q}\sum_{i\in N} m_i^{\perp}\, f_e(r_i, y_i)$$

One has to generalize the transverse energy flow operator

$$\Omega_1^e = \int \mathrm{d}r\, \mathrm{d}y\, f_e(r,y) \langle\, 0 \,|\, \overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(r,y) Y_n \overline{Y}_{\bar{n}} \,|\, 0 \,\rangle$$

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Boost invariance requires this term is y-independent

Operator definition of power correction $\Omega_1(r) = \langle\, 0\, |\, \overline{Y}_{\bar n}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(r,0) Y_n \overline{Y}_{\bar n}\, |\, 0\, \rangle$

Same as for massless computation $c_e = \int_{-\infty}^{\infty} \mathrm{d}y\, f_e(1,y)$

$$g_e(r) = \frac{1}{c_e}\int \mathrm{d}y\, f_e(r,y)$$

encodes all mass effects

each $g_e(r)$ defines a universality class of events with same power correction

Event shapes considered

Thrust

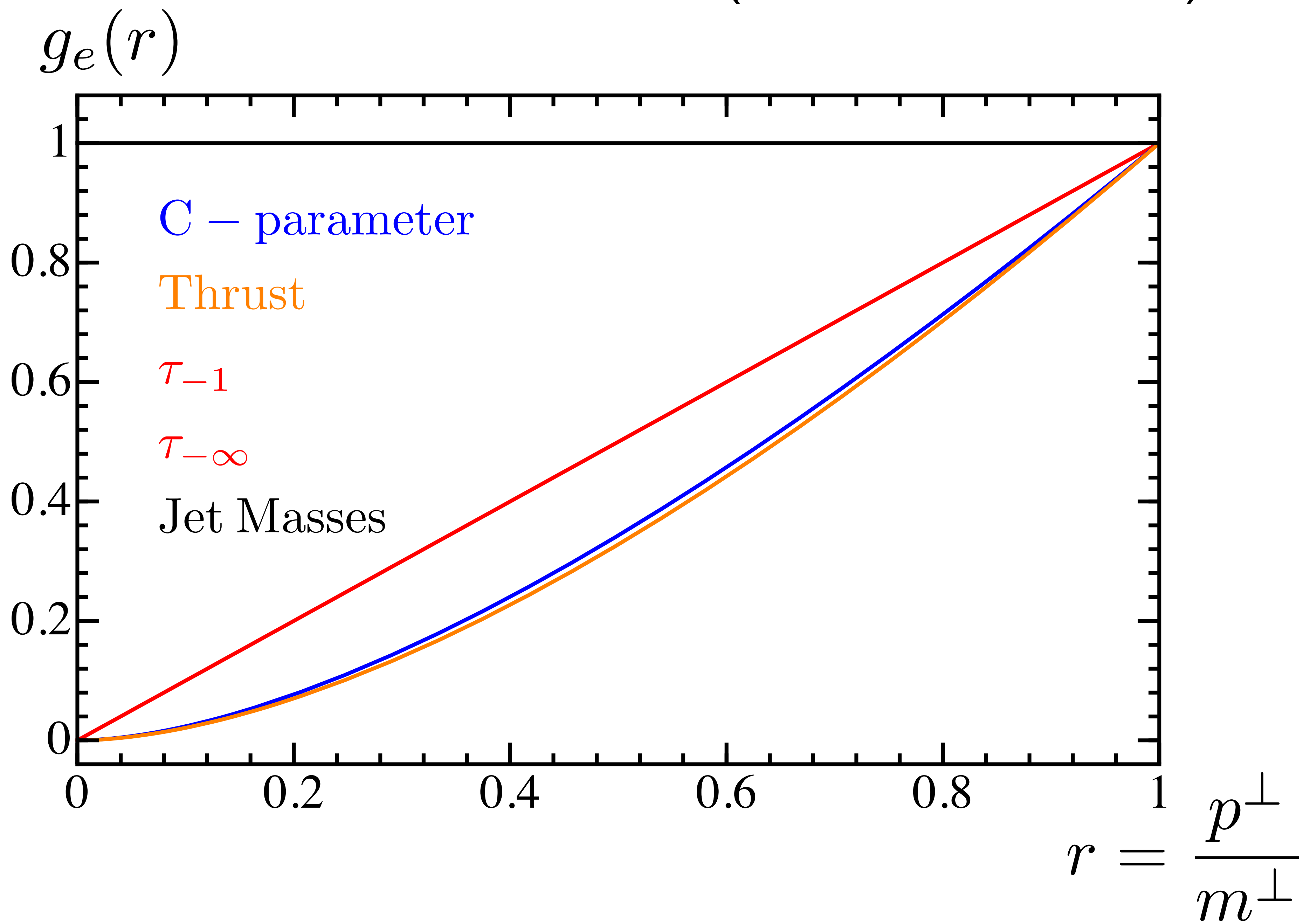
Jet Masses

C-parameter

Angularities

2-Jettiness

mass scheme (default definition)



Same **color** means same **power correction**

Event shapes considered

Thrust

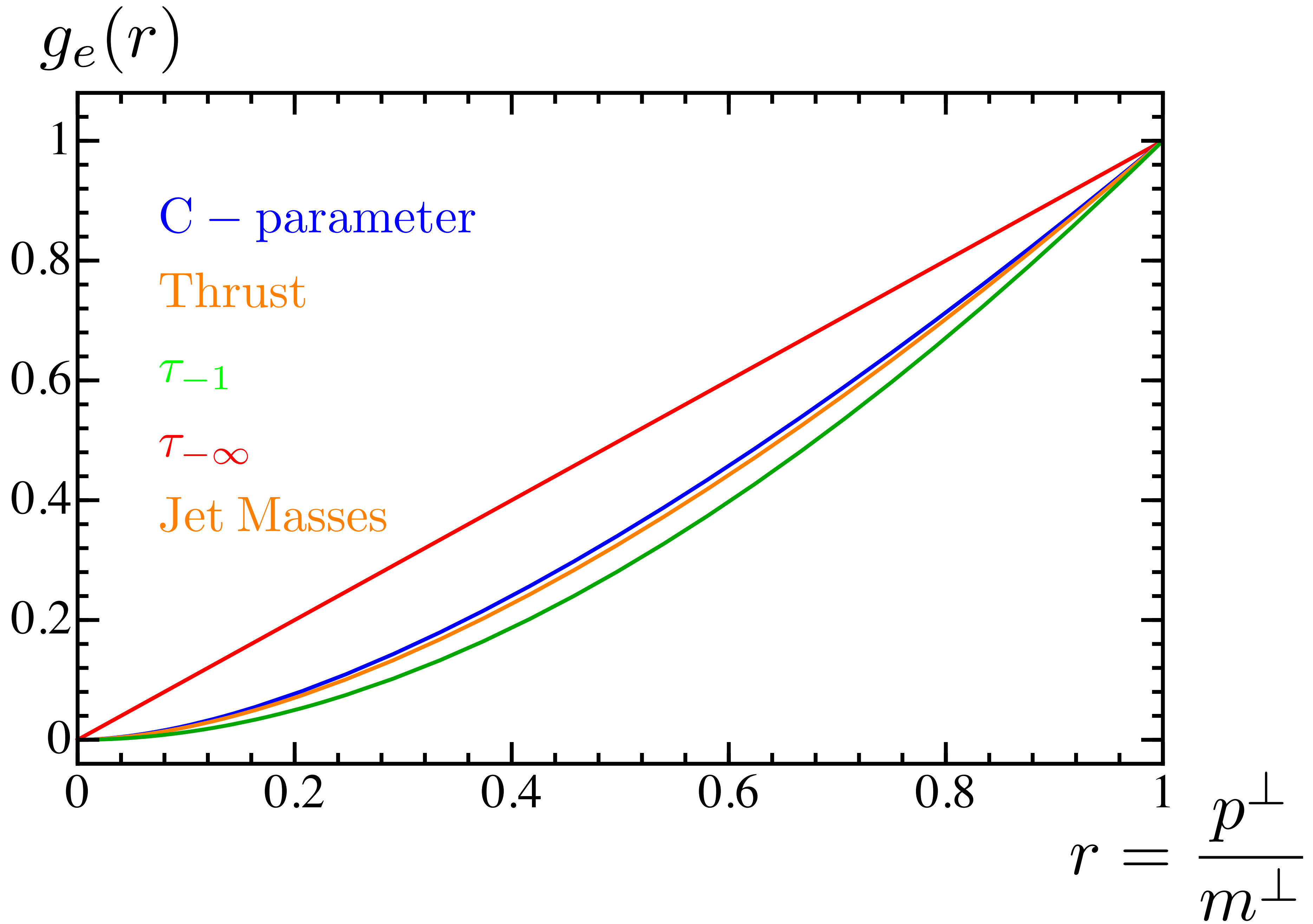
Jet Masses

C-parameter

Angularities

2-Jettiness

P-scheme



→ Scheme changes event shape definition

Event shapes considered

Thrust

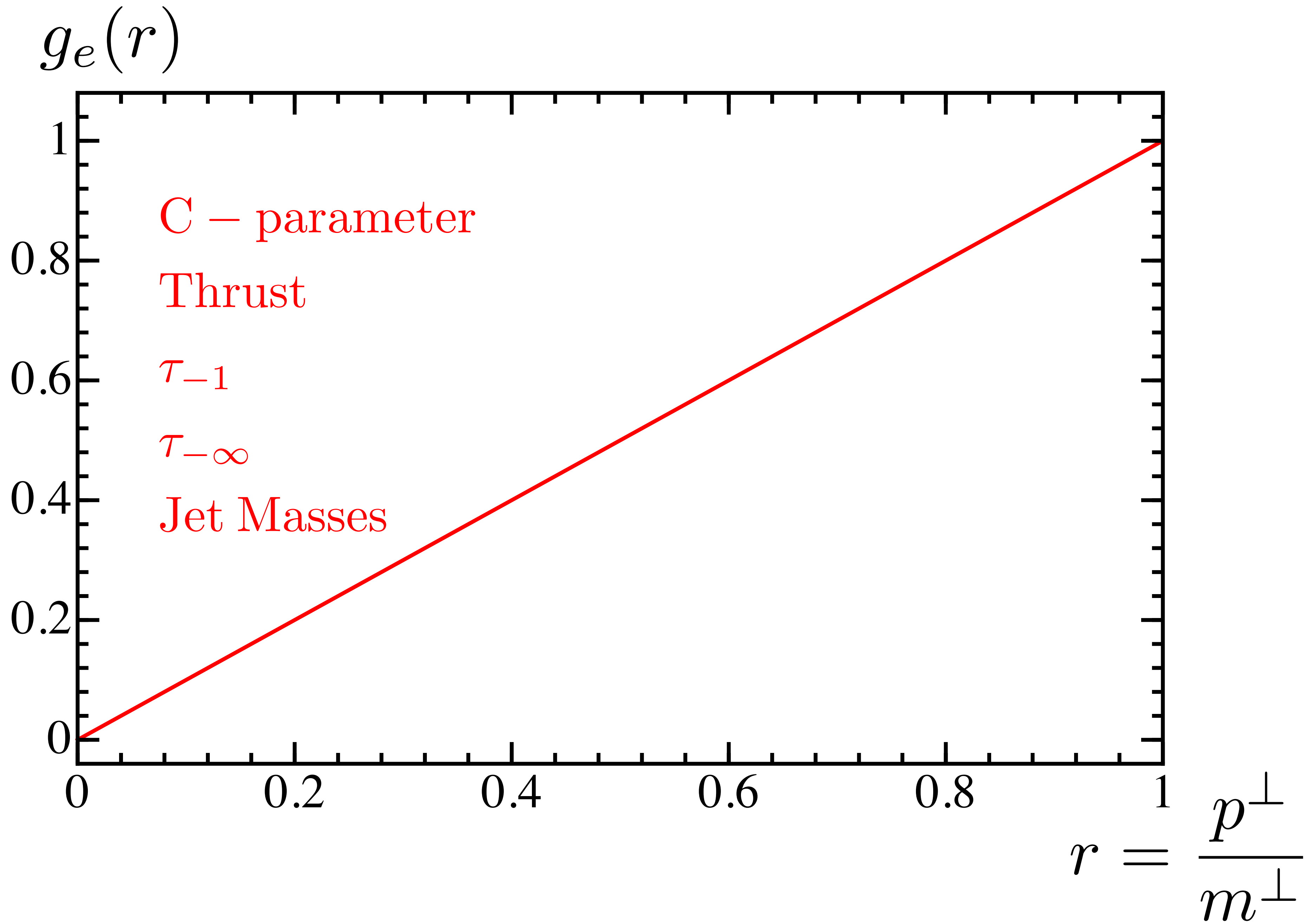
Jet Masses

C-parameter

Angularities

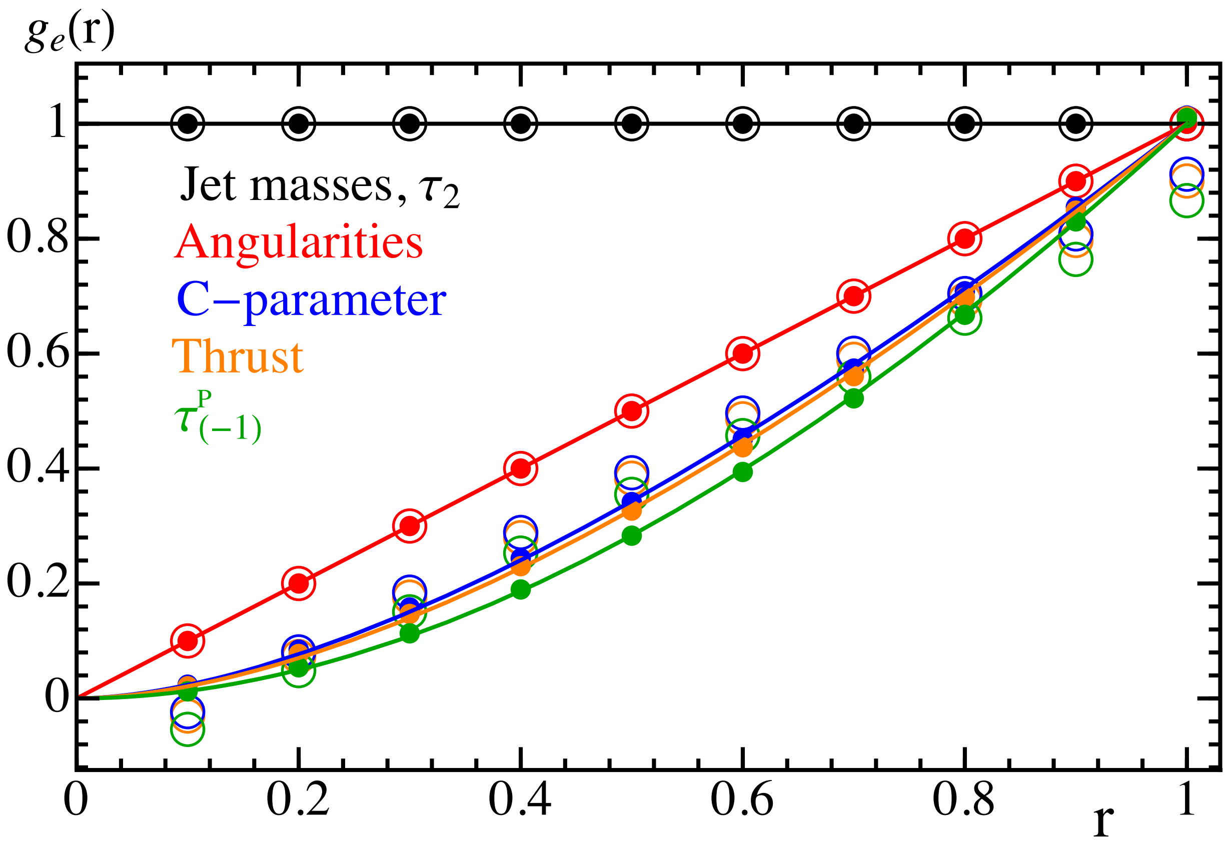
2-Jettiness

E-scheme



Scheme changes event shape definition

Effective parametrization



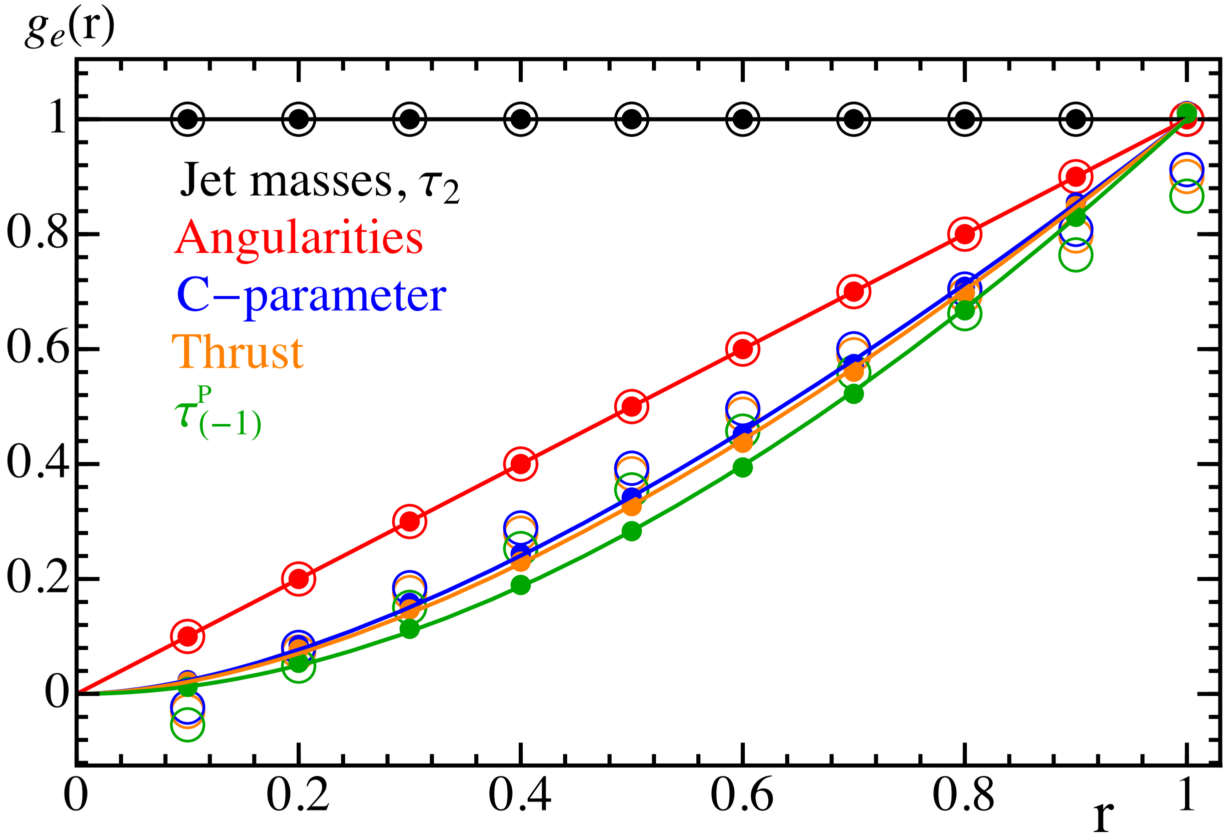
$g_e(r)$ functions are different, but it seems they could be approximated well by some suitable set of orthogonal polynomial

$$h_n(r) = \sqrt{2n+1}\,P_n(2x+1)$$

$$g_e(r) = \sum_{n=0}^{\infty} b_n^e\,h_n(r)$$

$\Omega_1(r)$ can be expanded as well $\Omega_1(r) = \Omega_1^\rho\,h_0(r) + \sqrt{3}(2\Omega_1^E - \Omega_1^\rho)h_1(r) + \Omega_1^\delta h_2(r) + \ldots$

Effective parametrization



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$$\Omega_1^\tau = 1.034\,\Omega_1^E - 0.135\,\Omega_1^\rho + 0.050\,\Omega_1^\delta$$

$$\Omega_1^C = 1.039\,\Omega_1^E - 0.127\,\Omega_1^\rho + 0.046\,\Omega_1^\delta$$

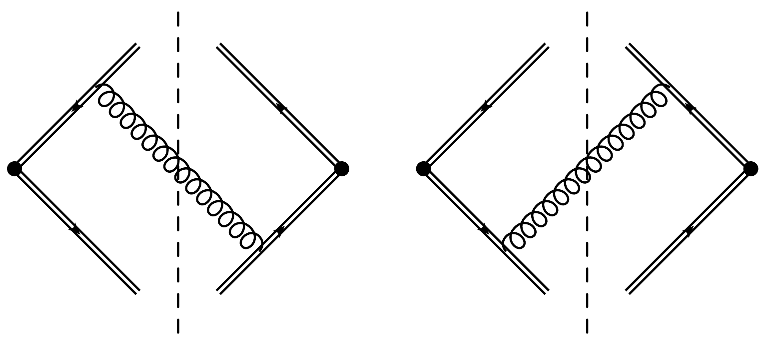
$$\Omega_1^{\tau^P_{-1}} = 1.022\,\Omega_1^E - 0.156\,\Omega_1^\rho + 0.064\,\Omega_1^\delta$$

small correction

ANOMALOUS DIMENSION OF POWER CORRECTION

[VM, I.W. Stewart, J.Thaler]
arXiv: 1209.3781

Anomalous dimension computation



$$\Omega_1(r) = \langle\, 0\, |\overline{Y}^{\dagger}_{\bar n} Y^{\dagger}_{n} \mathcal{E}_T(r,0) Y_n \overline{Y}_{\bar n} |\, 0\, \rangle$$

One needs to compute diagrams that probe the operator

The measured probe gluon corresponds to a source

$$A^{\mu A}(x) \to A^{\mu A}(x) + J^{\mu A}(x)$$

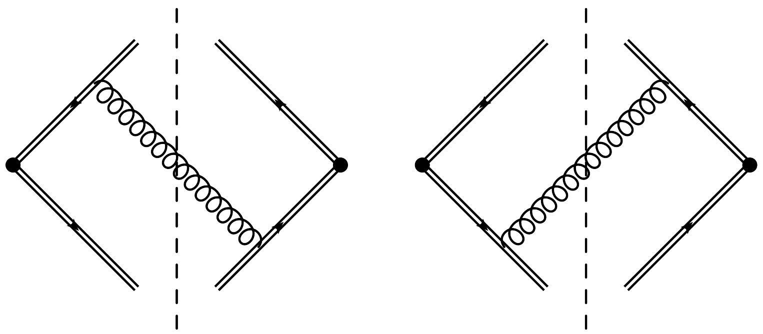
massless quantum field

off-shell background gauge field

$$r \neq 1$$

$$M_1^{\rm tree}(r) = \frac{2\alpha_s C_F}{\pi}\,\frac{m\, r}{(1-r^2)^{\frac{3}{2}}}$$

Anomalous dimension computation



$$\Omega_1(r) = \langle\, 0\,|\overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(r,0) Y_n \overline{Y}_{\bar{n}} |\, 0\,\rangle$$

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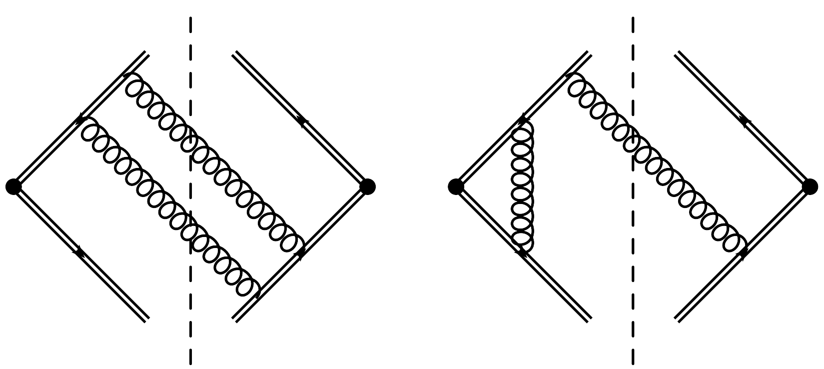
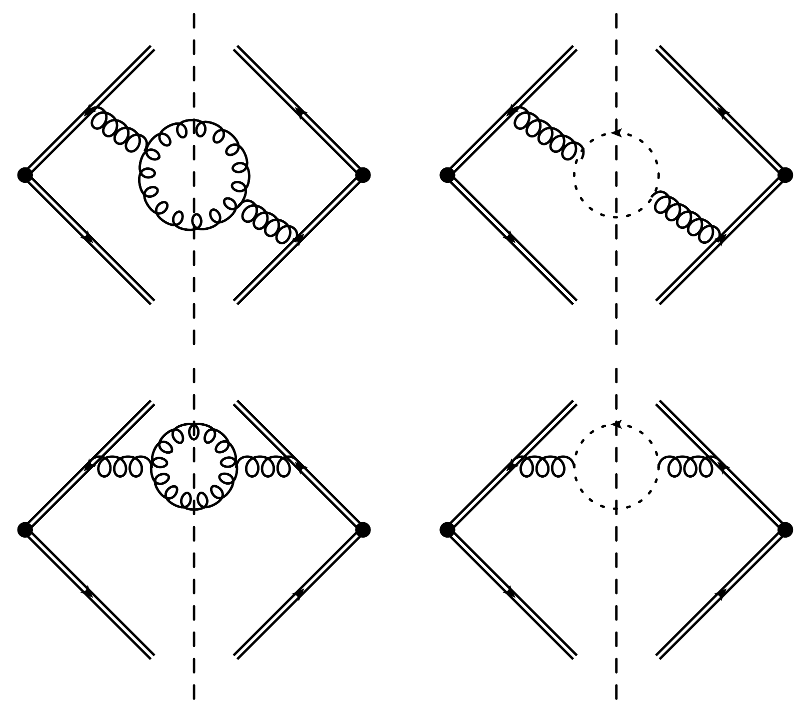
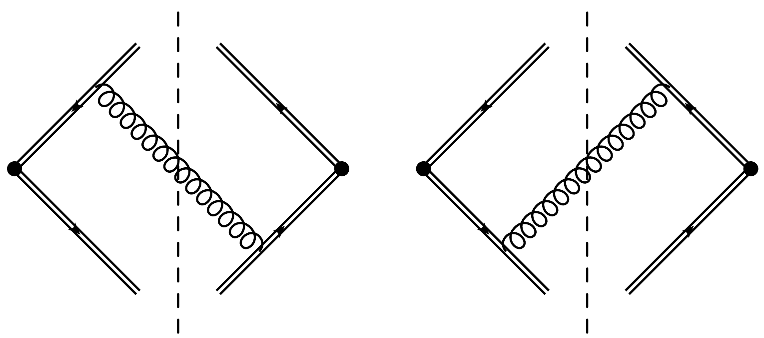




Abelian contribution exactly vanish when adding real and virtual radiation

Self-energy diagrams are IR and UV finite

Anomalous dimension computation



$$\Omega_1(r) = \langle\, 0 \,|\overline{Y}^{\dagger}_{\bar{n}} Y^{\dagger}_{n} \mathcal{E}_T(r,0) Y_n \overline{Y}_{\bar{n}} |\, 0 \,\rangle$$

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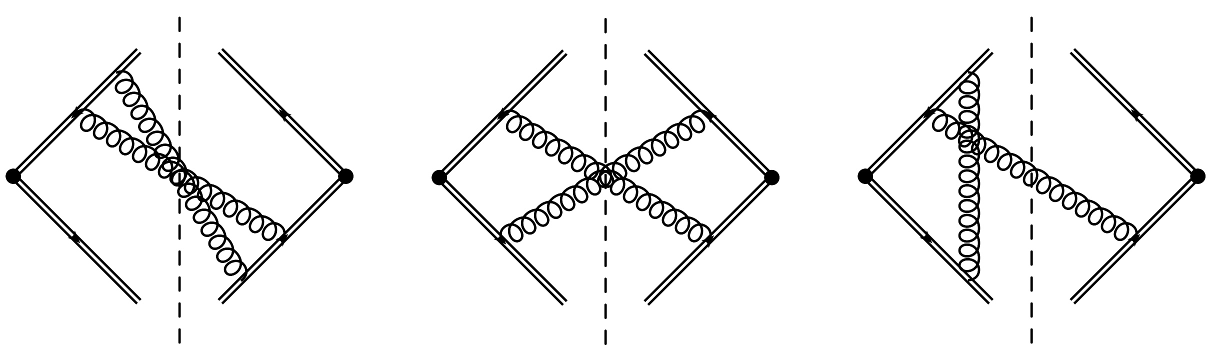

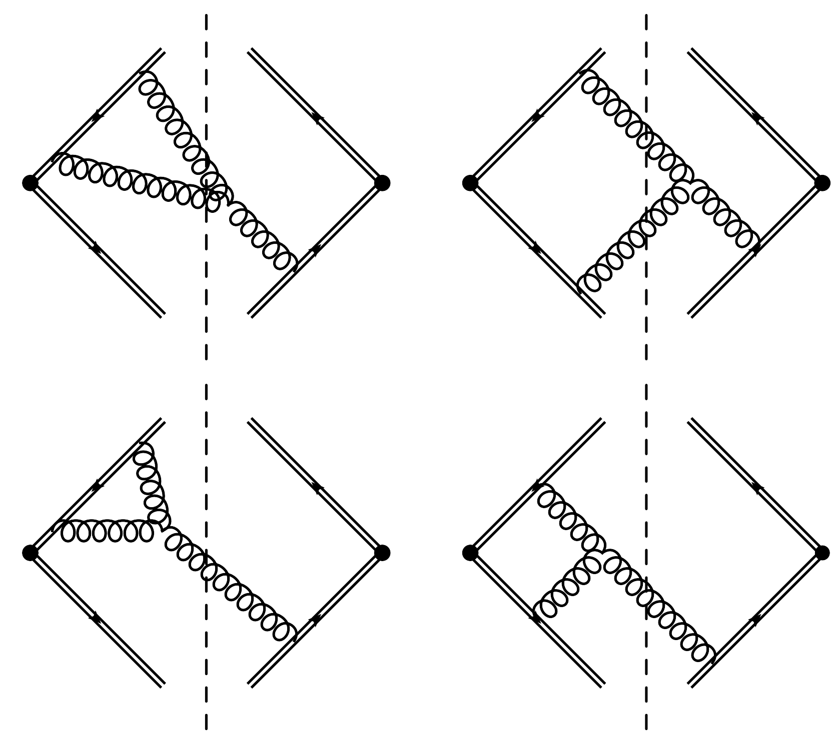

Only purely non-abelian diagrams contribute

We obtain an IR finite anomalous dimension

Results and consequences

$$\gamma^{\Omega_1} = -\frac{\alpha_s C_A}{\pi} \log(1 - r^2)$$

r-dependent anomalous dimension
no mixing between various r values

RGE solution at NLL

$$\Omega_1(r,\mu) = \Omega_1(r,\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{2\,C_A}{\beta_0}\log(1-r^2)}$$

$$\sim \Omega_1(r,\mu_0)\left[1 - \frac{\alpha_s(\mu_0)C_A}{\pi}\log\left(\frac{\mu}{\mu_0}\right)\log(1-r^2)\right]$$

Expanded out result

Results and consequences

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Expanded out result

Not a resummation formula for Ω_1^e

$$\Omega_1^e(\mu) = \int \mathrm{dr}\, \mathrm{g_e(r)}\, \Omega_1(\mathrm{r},\mu_0)\left(\frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(\mu_0)}\right)^{\frac{2\,\mathrm{C_A}}{\beta_0}\log(1-\mathrm{r}^2)}$$

Unknown function !

Using expanded out result

$$\Omega_1^e(\mu) = \Omega_1^e(\mu_0) - \frac{\alpha_s(\mu_0) C_A}{\pi}\log\left(\frac{\mu}{\mu_0}\right)\Omega_{\log}^e(\mu_0)$$

$$\Omega_{\log}^e(\mu_0) = \int \mathrm{dr}\, \log(1-\mathrm{r}^2)\, \mathrm{g_e(r)}\, \Omega_1(\mathrm{r},\mu_0)$$

New nonperturbative parameter

Matching computation

At one loop one has:

$$\delta(\ell - Qe_{\rm pert} - Qe_{\rm np})$$

$$\simeq \delta(\ell - Qe_{\rm pert}) - Qe_{\rm np}\delta'(\ell - Qe_{\rm pert})$$

$$\simeq \delta(\ell) - Q(e_{\rm np} + e_{\rm pert})\delta'(\ell)$$

This corrects the tree level OPE result

Full theory computation

Effective theory computation (anomalous dimension)

Matching computation

At one loop one has:

$$\delta(\ell - Qe_{\rm pert} - Qe_{\rm np})$$

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Full theory computation

Effective theory computation (anomalous dimension)

Same diagrams as for anomalous dimension computation, but different measurement

EFT diagrams have to be subtracted from full theory result

Matching coefficient compensates μ dependence of Ω_1

Matching computation

At one loop one has:

$$\delta(\ell - Qe_{\rm pert} - Qe_{\rm np})$$
$$\simeq \delta(\ell - Qe_{\rm pert}) - Qe_{\rm np}\delta'(\ell - Qe_{\rm pert})$$
$$\simeq \delta(\ell) - Q(e_{\rm np} + e_{\rm pert})\delta'(\ell)$$

This corrects the tree level OPE result

Full theory computation

Effective theory computation (anomalous dimension)

$$F_e(\ell) = \delta(\ell) + \int {\rm d}r\, C_1^e(\ell, r, \mu)\, c_e\, g_e(r)\, \Omega_1(r,\mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{\ell^3}\right)$$

$$C_1^e(\ell, r, \mu) = -\,\delta'(\ell) + \frac{C_A \alpha_s(\mu)}{\pi} \ln(1-r^2) \frac{\rm d}{{\rm d}\ell}\left(\frac{1}{\mu}\left[\frac{\mu}{\ell}\right]_+\right) + \frac{\alpha_s(\mu)}{\pi}\delta'(\ell)\, d_1^e(r) + \mathcal{O}(\alpha_s^2)$$

explicitly checked (the $\ln(1-r^2)$ term)

needs a full matching computation (the $d_1^e(r)$ term)

EFFECTS ON OBSERVABLES

Effect of hadron masses

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e} - \frac{1}{Q}\left(\Omega_1^e(\mu) + \frac{\alpha_s(\mu)}{\pi}\,\Omega_1^{e,\,d}(\mu)\right)\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}e^2}(e) + \frac{\Omega_1^{e,\,\ln}(\mu)}{Q}\,\frac{\alpha_s(\mu)C_A}{\pi}\left\{\ln\left(\frac{\mu}{eQ}\right)\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}e^2}(e) - \int_0^{eQ}\frac{\mathrm{d}\ell}{\ell}\left[\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}e^2}\left(e-\frac{\ell}{Q}\right) - \frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}e^2}(e)\right]\right\},$$

Distribution

$$\langle e\rangle = \langle e\rangle_{\mathrm{pert}} + \frac{\Omega_1^e(\mu)}{Q} + \frac{\alpha_s(\mu)}{\pi}\,\frac{\Omega_1^{e,\,d}}{Q} + \frac{\Omega_1^{e,\,\ln}(\mu)}{Q}\,\frac{C_A\alpha_s(\mu)}{\pi} \times\int_0^{e_{\max}}\mathrm{d}e\,\frac{1}{\hat{\sigma}}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e}(e)\left[\ln\left(\frac{\mu}{Q(e_{\max}-e)}\right) - \frac{e^2}{e_{\max}(e_{\max}-e)}\right]$$

First moment

Effect of hadron masses

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e} - \frac{1}{Q}\left(\Omega_1^e(\mu) + \frac{\alpha_s(\mu)}{\pi}\,\Omega_1^{e,\,d}(\mu)\right)\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}e^2}(e)$$

usual shift

$$+\frac{\Omega_1^{e,\,\ln}(\mu)}{Q}\,\frac{\alpha_s(\mu)C_A}{\pi}\left\{\ln\left(\frac{\mu}{eQ}\right)\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}e^2}(e) - \int_0^{eQ}\frac{\mathrm{d}\ell}{\ell}\left[\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}e^2}\left(e-\frac{\ell}{Q}\right) - \frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}e^2}(e)\right]\right\},$$

additional term (not a shift)

$$\langle e\rangle = \langle e\rangle_{\mathrm{pert}} + \frac{\Omega_1^e(\mu)}{Q} + \frac{\alpha_s(\mu)}{\pi}\,\frac{\Omega_1^{e,\,d}}{Q} + \frac{\Omega_1^{e,\,\ln}(\mu)}{Q}\,\frac{C_A\alpha_s(\mu)}{\pi}$$

usual shift

$$\times\int_0^{e_{\max}}\mathrm{d}e\,\frac{1}{\hat{\sigma}}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e}(e)\left[\ln\left(\frac{\mu}{Q(e_{\max}-e)}\right) - \frac{e^2}{e_{\max}(e_{\max}-e)}\right]$$

additional term

Effect of hadron masses

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e} - \frac{1}{Q}\left(\Omega_1^e(\mu) + \frac{\alpha_s(\mu)}{\pi}\,\Omega_1^{e,\,d}(\mu)\right)\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}e^2}(e) + \frac{\Omega_1^{e,\,\ln}(\mu)}{Q}\,\frac{\alpha_s(\mu)C_A}{\pi}\left\{\ln\left(\frac{\mu}{eQ}\right)\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}e^2}(e) - \int_0^{eQ}\frac{\mathrm{d}\ell}{\ell}\left[\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}e^2}\left(e-\frac{\ell}{Q}\right) - \frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}e^2}(e)\right]\right\},$$

$$\Omega_1^{e,\,d_1}(\mu) = \int \mathrm{d}r\, d_1^e(r)\, c_e\, g_e(r)\, \Omega_1(r,\mu)$$

perturbatively suppressed
another power correction

$$\langle e\rangle = \langle e\rangle_{\rm pert} + \frac{\Omega_1^e(\mu)}{Q} + \frac{\alpha_s(\mu)}{\pi}\frac{\Omega_1^{e,\,d}}{Q} + \frac{\Omega_1^{e,\,\ln}(\mu)}{Q}\frac{C_A\alpha_s(\mu)}{\pi} \times \int_0^{e_{\max}}\mathrm{d}e\,\frac{1}{\hat{\sigma}}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e}(e)\left[\ln\left(\frac{\mu}{Q(e_{\max}-e)}\right) - \frac{e^2}{e_{\max}(e_{\max}-e)}\right]$$

CONCLUSIONS

CONCLUSIONS

- Operator description of hadron mass effects.
- These effects break universality. Not a simple a correction.
- Set of privileged classes in which there is universality. Approximate universality among classes.

$$\Omega_1^C \simeq \frac{3\pi}{2}\Omega_1^\tau$$

$$\Omega_1^{\mathrm{HJM}} \neq 2\,\Omega_1^\tau$$

- Computation of anomalous dimension predicts log(Q) dependence. Complete matching computation is w.i.p.
- Small effect on fits to α_s: additional 0.0005 error.

BACKUP SLIDES

COMPARISONS TO PYTHIA AND HERWIG

Comparisons to MC generators

Define generalized angularities, useful to compare to MC

$$\tau_{(n,a)} \equiv \sum_i m_i^{\perp} r_i^n e^{-|y_i|(1-a)} \quad \begin{cases} g_{(n,a)} = r^n \\ c_{(n,a)} = \dfrac{2}{1-a} \end{cases}$$

Comparisons to MC generators

Define generalized angularities, useful to compare to MC

$$\tau_{(n,a)} \equiv \sum_i m_i^{\perp} r_i^n e^{-|y_i|(1-a)} \quad \begin{cases} g_{(n,a)} = r^n \\ c_{(n,a)} = \dfrac{2}{1-a} \end{cases}$$

We study the first moment of the distributions
Taking differences of classes we obtain:

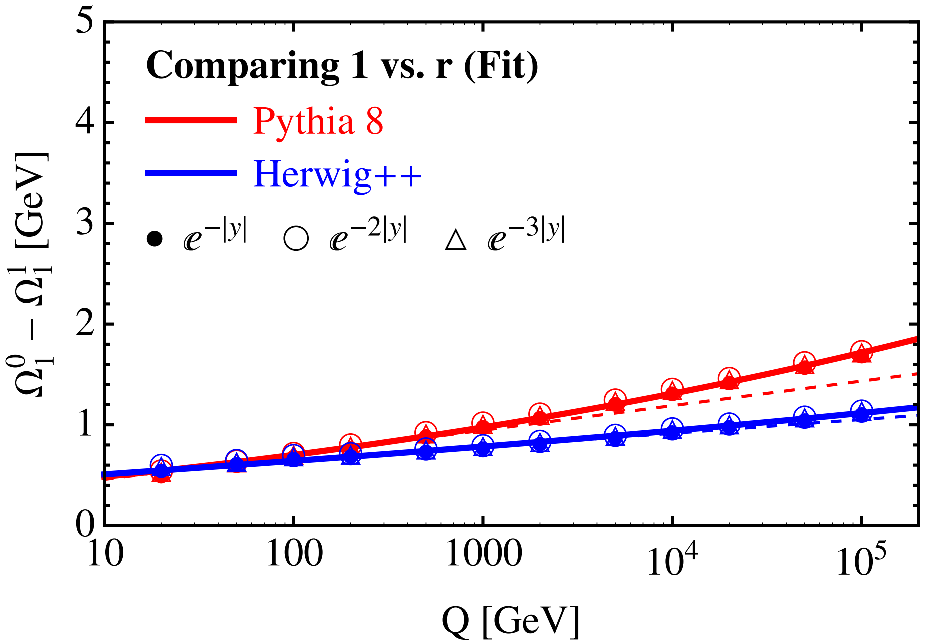
$$\Omega_1^0(\mu_Q) - \Omega_1^n(\mu_Q) = \frac{Q}{c_a}\left(\langle\tau_{(0,a)}\rangle - \langle\tau_{(n,a)}\rangle\right)$$

Perturbative moment is class-independent and vanishes in the difference

Comparisons to MC generators

Define generalized angularities, useful to compare to MC

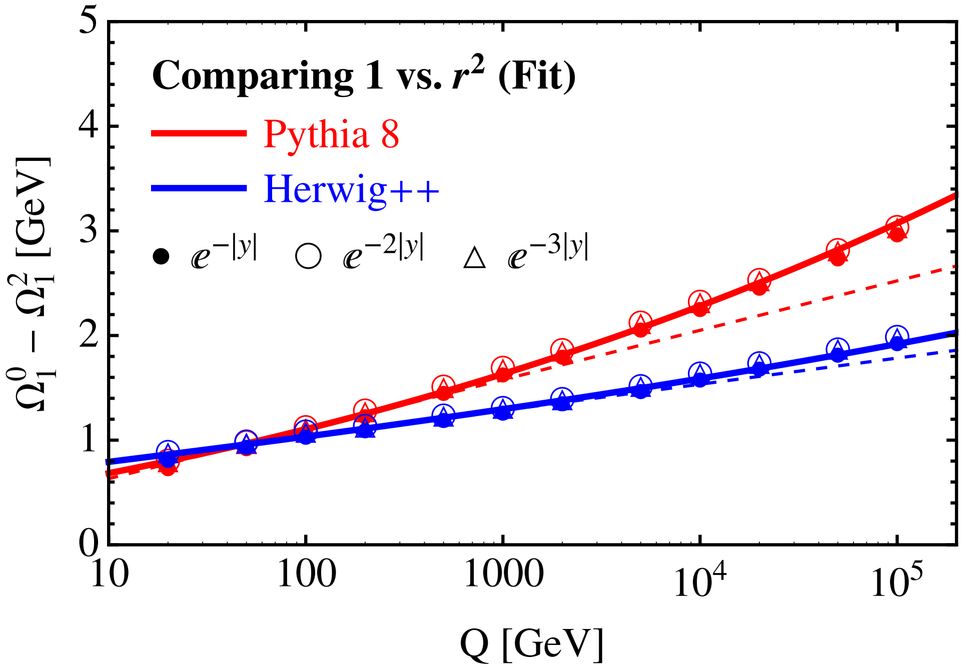
$$\tau_{(n,a)} \equiv \sum_i m_i^\perp r_i^n e^{-|y_i|(1-a)} \quad \begin{cases} g_{(n,a)} = r^n \\ c_{(n,a)} = \dfrac{2}{1-a} \end{cases}$$

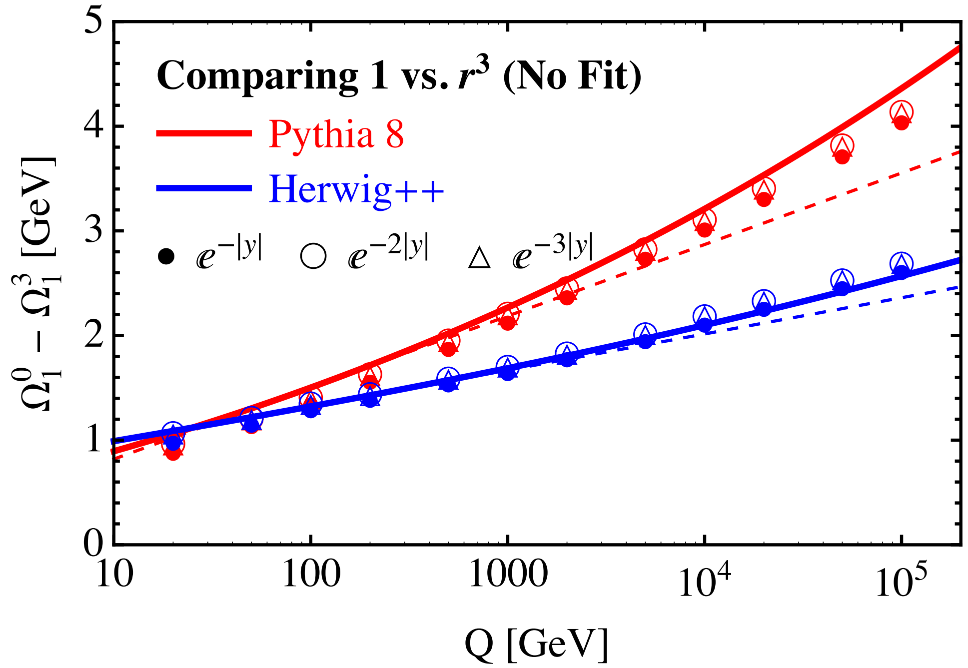


resummed expression

expanded out expression

fit function basis $\{1, r, (1-r)^{-\frac{1}{4}}\}$



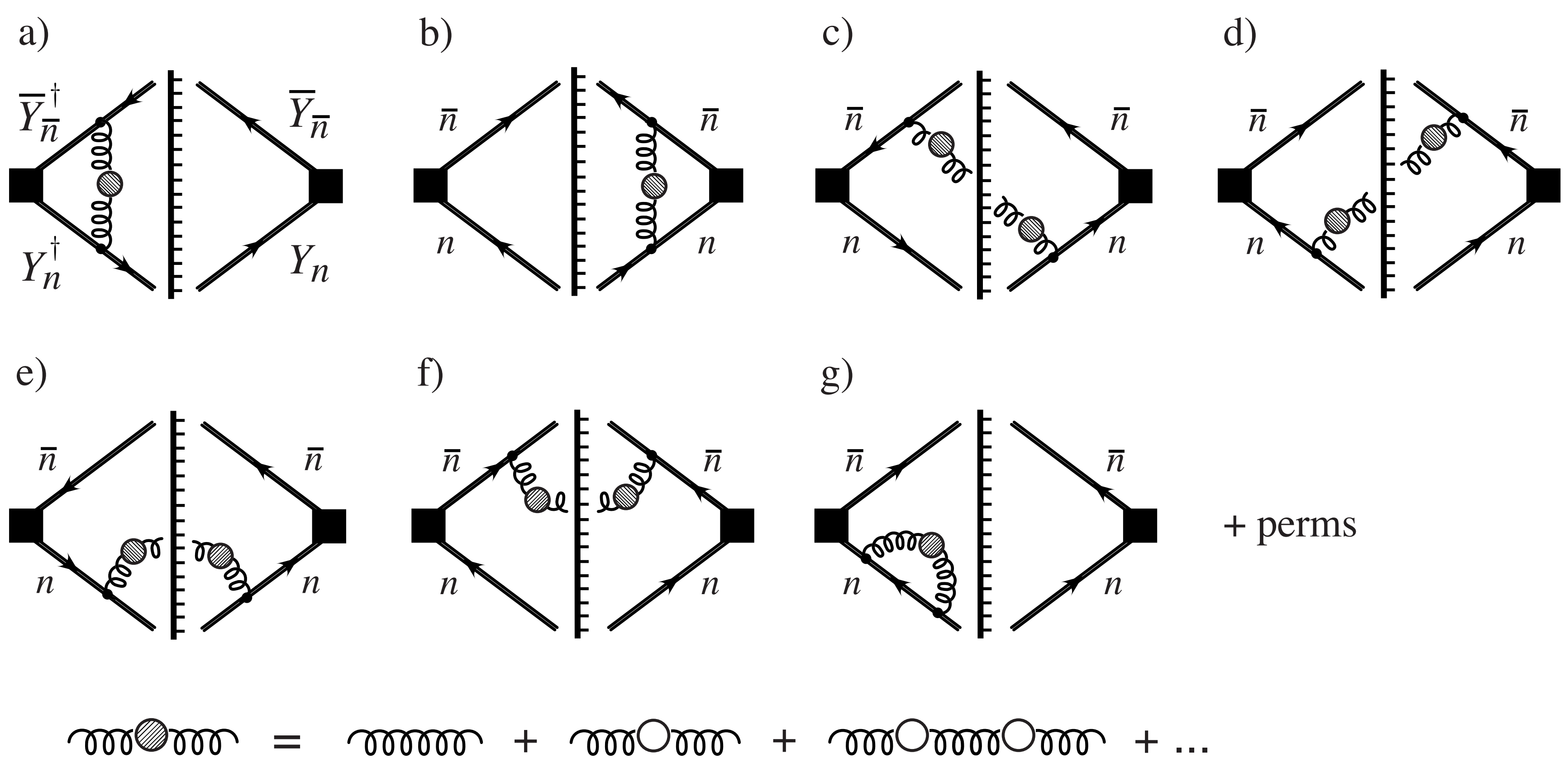


Renormalon subtractions

There is a $u = \frac{1}{2}$ renormalon in Ω_1

It can be removed by appropriate subtractins

$$B\left[\ln S_e^{\mathrm{bubbles}}(x,\mu)\right](u) = c_e \frac{8C_F e^{5/6}}{\pi\beta_0(u-\frac{1}{2})}\ (ix\mu)$$



Renormalon subtractions

There is a $u = \frac{1}{2}$ renormalon in Ω_1
It can be removed by appropriate subtractins

$$\hat{\sigma}_e(x) \to \tilde{\sigma}_e(x) = \hat{\sigma}_e(x)\, e^{-ix\,\delta_e(R,\mu)/Q}$$

[Hoang & Kluth]

$$\delta_e(R,\mu) = \frac{c_e}{c_{e'}} R e^{\gamma_E} \frac{\mathrm{d}}{\mathrm{d}\ln(ix)} \ln S_{e'}^{\mathrm{pert}}(x,\mu)\Big|_{x=(iRe^{\gamma_E})^{-1}}$$

[VM, Thaler, Stewart]

$$\frac{\mathrm{d}^3\vec{p}}{(2\pi)^3 2E_p} = \frac{\mathrm{d}y}{4\pi}\frac{\mathrm{d}^2\vec{p}_\perp}{(2\pi)^2}$$

allows to compute the renormalon for all event shapes simultaneously

Renormalon inside

$$\Omega_1^e(R,\mu) \equiv \Omega_1^e(\mu) - \delta_e(R,\mu)$$

Renormalon free