

# Resummation for Higgs production

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# Introduction

## What are we looking at?

Higgs production cross section in gluon-gluon fusion

$$gg \rightarrow H + 0 \text{ jets}$$

with a jet veto on the jet transverse momentum:

$$p_T^{\text{Jet}} < p_T^{\text{Veto}} \sim 15 - 30 \text{ GeV}$$

## Why?

- Analysis is done in jet bins, precise prediction of the 0-jet bin, i.e. cross section with a jet veto needed.
- Veto on jet transverse momentum is experimentally motivated (essential to suppress background events)

# Resummation

Large double logarithms induced by the jet veto spoil convergence of the perturbative series

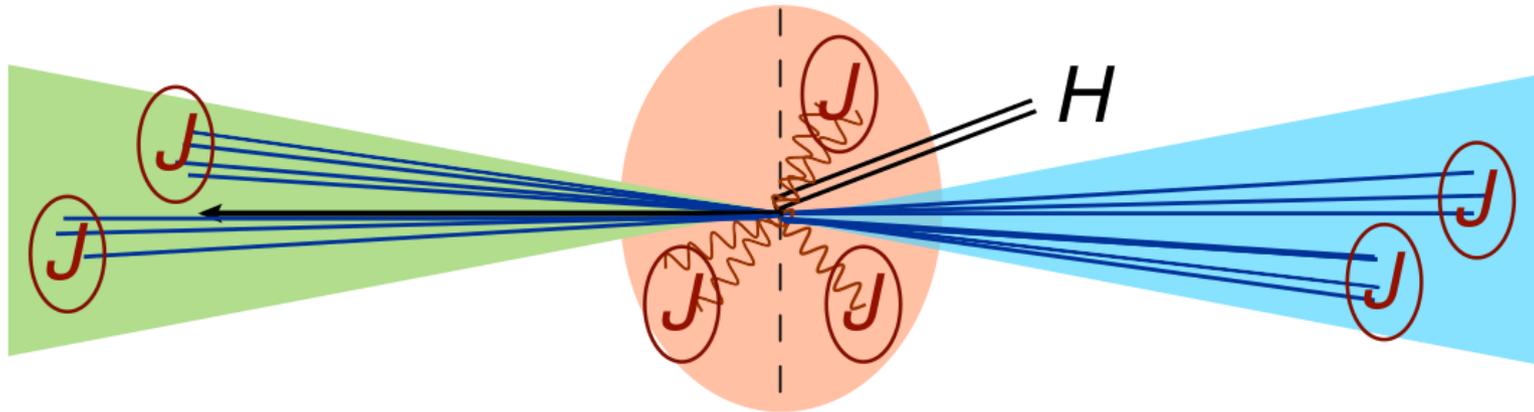
→ Need resummation!

Only very recently, resummed results for Higgs cross sections with a jet veto beyond LL became available:

- Resummation at NLL and NNLL using CAESAR:  
Banfi, Salam and Zanderighi 1203.5773 (BSZ)  
+ Monni 1206.4998 (BMSZ)
- Factorization and NNLL Resummation in SCET:  
Becher, Neubert 1205.3806 (BN)
- Resummation Properties of Jet Vetoes:  
Tackmann, Walsh, Zuberi 1206.4312 (TWZ)
- Higher jet bins:  
Liu and Petriello 1210.1906 (LP)

We will present our work and discuss how it is related to these.

# Jet clustering algorithm



- Jet clustering algorithm groups particles of different momentum scaling separately, if the jet radius

$$R < \ln(m_H/p_T) \approx 1.5$$

- Therefore the jet veto can be applied separately in each individual sector. → **Essential for soft-collinear factorization**

(TWZ argue that for  $R = O(1)$  soft-collinear emissions can cluster, giving contributions that are not reproduced by a soft-collinear factorization formula. Will be discussed later)

# All-order factorization theorem from SCET

BN proposed first all-order factorization formula for the cross section with a jet veto.

The naive factorization suffers from a collinear anomaly, leading to an **anomalous dependence** on the hard scale  $m_H$  (pure power in  $p_T$  space)

Hard function

$$\frac{d\sigma(p_T^{\text{veto}})}{dy} = \sigma_0(\mu) C_t^2(m_t^2, \mu) |C_S(-m_H^2, \mu)|^2 \left(\frac{m_H}{p_T^{\text{veto}}}\right)^{-2F_{gg}(p_T^{\text{veto}}, \mu)} \times \sum_{i,j} I_{g \leftarrow i}(p_T^{\text{veto}}, \mu) \otimes \phi_{i/P}(\mu) I_{g \leftarrow j}(p_T^{\text{veto}}, \mu) \otimes \phi_{j/P}(\mu)$$

Refactorized beam-jet functions

Beam functions: Stewart, Tackmann, Waalewijn '10

Sources of  $\text{Log}(m_H/p_T)$  are the anomaly and the RG evolution of the hard function.

# Ingredients for NNLL resummation

- $C_t$  and  $C_S$  at one-loop order
- One-loop collinear kernel functions:

$$I_{g \leftarrow i}(z, p_T^{\text{veto}}, \mu) = \delta(1-z) \delta_{gi} \left[ 1 + a_s \left( \Gamma_0^A \frac{L_\perp^2}{4} - \gamma_0^g L_\perp \right) \right] + a_s \left[ -\mathcal{P}_{g \leftarrow i}^{(1)}(z) \frac{L_\perp}{2} + \mathcal{R}_{g \leftarrow i}(z) \right]$$

where  $L_\perp = 2 \ln \left( \frac{\mu}{p_T^{\text{veto}}} \right)$

- Two-loop anomaly coefficient:

$$F_{gg}(p_T^{\text{veto}}, \mu) = a_s (\Gamma_0^A L_\perp + d_1^{\text{veto}}) + a_s^2 \left( \Gamma_0^A \beta_0 \frac{L_\perp^2}{2} + \Gamma_1^A L_\perp + d_2^{\text{veto}} \right)$$

vanishes
Anomaly coefficient: Introduces dependence on jet radius R at NNLL order

$$d_2^{\text{veto}}(R) = d_2^{\text{Higgs}} + \Delta d_2(R)$$

# Anomaly Coefficient at NNLL

- The part  $\Delta d_2(R)$  of the anomaly coefficient can be extracted from the anomalous part of diagrams with **two real emissions**. It is closely related to the function  $\mathcal{F}(R)$  computed by BSZ.
- BN extracted  $d_2^{\text{veto}}$  from  $\mathcal{F}(R)$ , taking the limit  $R \rightarrow \infty$ , under the assumption that the BSZ formula still holds in this limit. But later BMSZ showed that an extra constant term arises, which changes the value of  $d_2^{\text{veto}}$  significantly.

Have now computed  $d_2^{\text{veto}}$  directly from a two-loop calculation in SCET.

The result is fully consistent with the BMSZ formula.

- What about soft-collinear mixing terms?

# Soft Collinear mixing terms

- Soft and collinear contributions are integrated over full phase space in SCET
- Double counting overlapping regions has to be avoided

Two possibilities:

multi-pole **expand the full integrand**  
(including all phase space constraints)

**“zero-bin” subtractions**

Manohar, Stewart '07

If the **integrand is fully expanded**, one ends up with a single scale integral such that all **zero-bins** are trivially zero (**scaleless**).

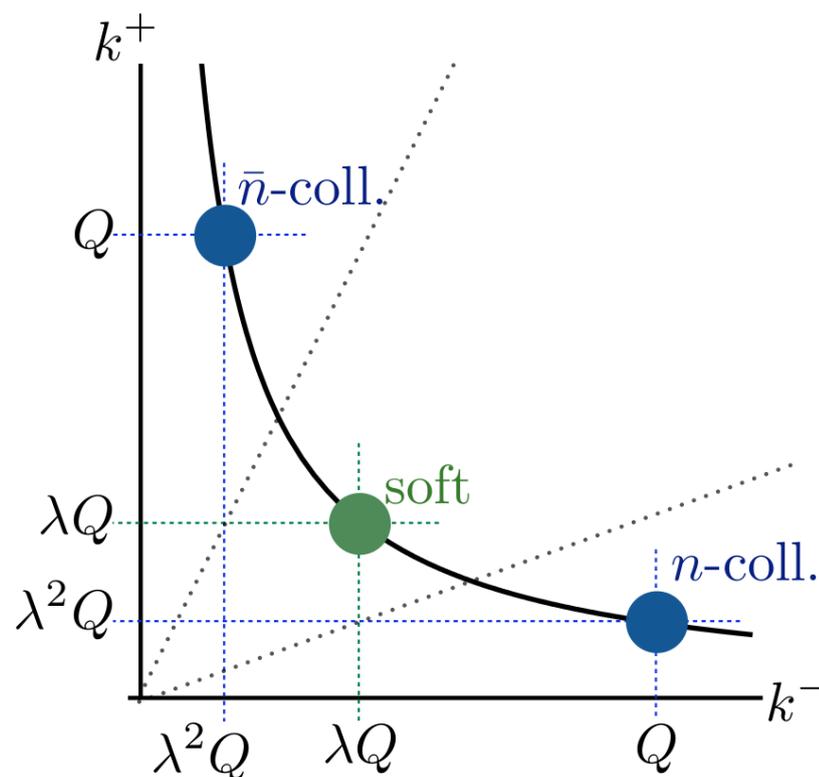


Figure from  
Chiu, Jain, Neill, Rothstein 1202.0814

# Soft Collinear mixing terms

On the Lagrangian level the factorization into soft, collinear and anti-collinear fields is manifest.

The measure (phase space constraints due to clustering and vetoes) can introduce factorization violating mixing contributions

$$\mathcal{M}_{ij}^{\text{Mix}} = \theta(\Delta R_{ij} < R) \theta(|p_T| < p_T^{\text{veto}}) \quad i, j = c, \bar{c}, s \quad \text{with} \quad i \neq j$$

↑  
veto on vector sum → factorization violated

We expand the full integrand (zero-bins scaleless) and get the regions

$$I_{\text{full}} = I_{cc} + I_{\bar{c}\bar{c}} + I_{cs} + I_{\bar{c}s} + I_{\bar{c}c} + I_{ss}$$

The dangerous mixing terms contained in  $I_{cs} + I_{\bar{c}s} + I_{\bar{c}c}$  are zero because the clustering of particles with different momentum scaling is power suppressed

$$\theta(\Delta R_{ij} < R) = 0 \quad i \neq j$$

↑  
if expanded (which we do)

# Soft Collinear mixing terms

$$\theta(R^2 - \Delta R_{ij}^2) \sim \theta \left( \overset{O(1)}{R^2} - \overset{\text{small (large)}}{y_i} - \overset{\text{large (small)}}{y_j} \right)^2 = 0 \quad i \neq j$$

No factorization violating mixing terms are present:

→ factorization at this order

For any number of particles, clustering of particles with different momentum scaling will be zero (power suppressed) → indicates factorization to all orders

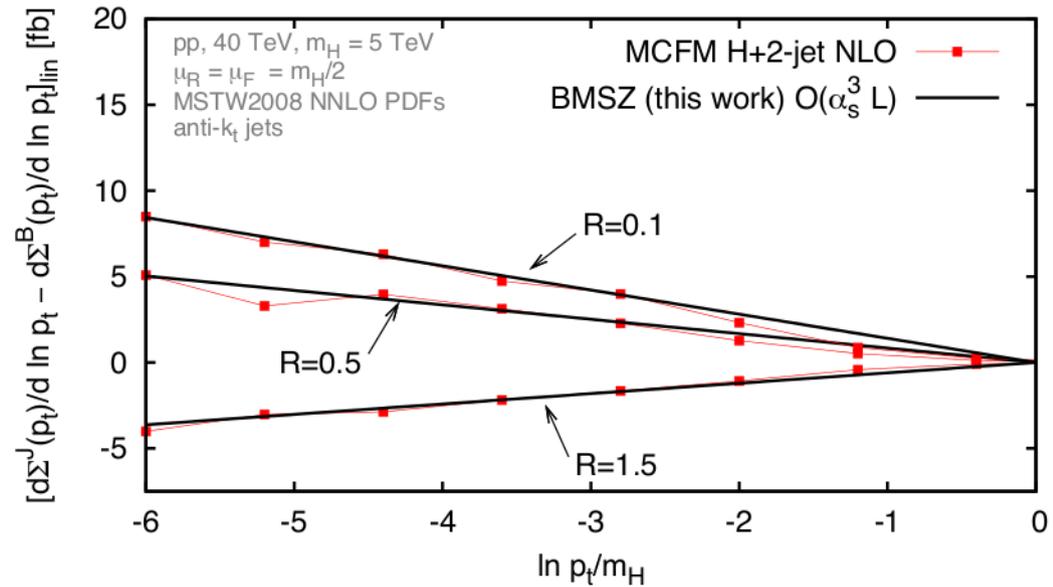
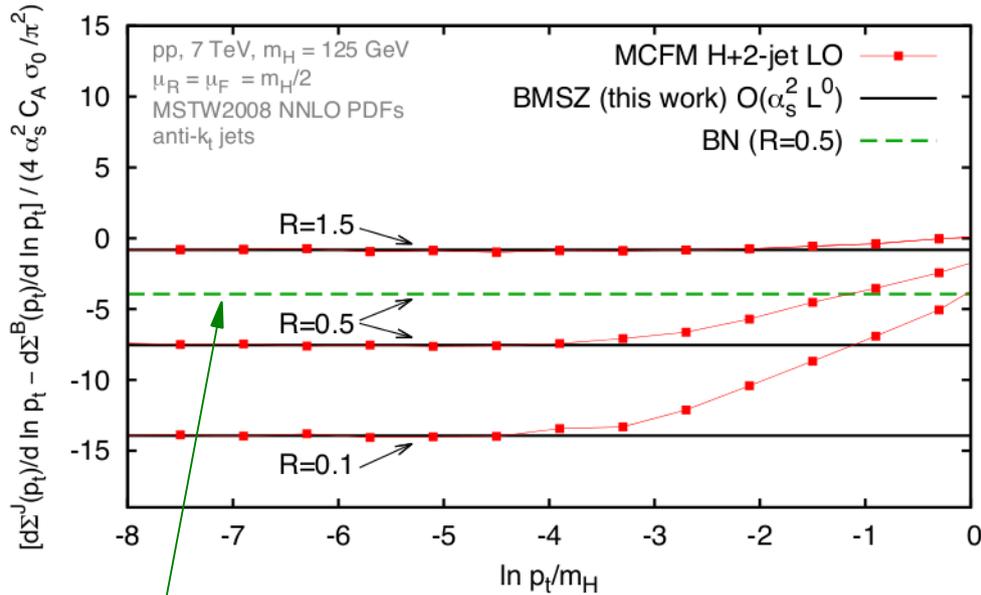
Note:  $\theta(R^2 - \ln \lambda) = \theta(e^{R^2} - \lambda)$

# NNLL calculation and factorization

Complete agreement (analytically) of our computation (SC-mixing terms are absent) with the QCD computation of BMSZ

BMSZ have explicitly verified (MCFM) that they correctly reproduce the logarithmic terms at  $O(\alpha_s^2)$  and  $O(\alpha_s^3)$

→ established factorization at this order



BN with missing constant in  $d_2^{\text{veto}}$ . Now agreement!

# Zero-bin subtractions

Mixing part of measure:

$$\mathcal{M}_{ij}^{\text{Mix}} = \theta(\Delta R_{ij} < R) \theta(|p_T| < p_T^{\text{veto}}) \quad i, j = c, \bar{c}, s \quad \text{with} \quad i \neq j$$

If the measure is not expanded (TWZ), this term is non-zero. Factorization violating mixing terms

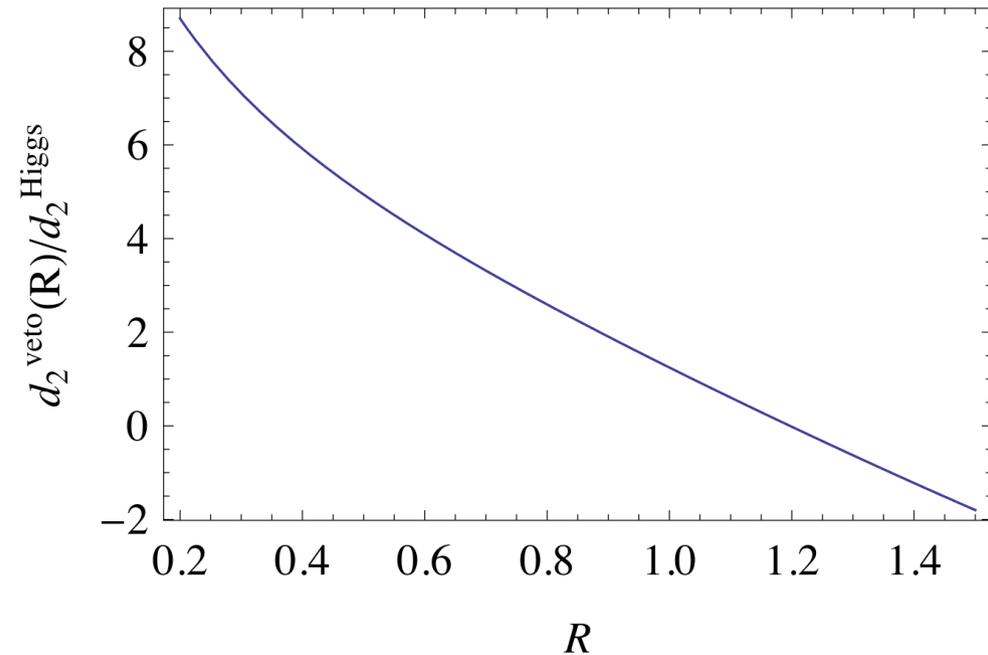
$$\delta\sigma_{CS} \quad \delta\sigma_{\bar{C}S} \quad \delta\sigma_{C\bar{C}}$$

are present ( $\propto R^2$ ). But also zero-bin subtractions are needed.

TWZ have calculated  $\delta\sigma_{SC}$ , but not explicitly  $\sigma_{CC}$ ,  $\sigma_{\bar{C}\bar{C}}$ ,  $\delta\sigma_{C\bar{C}}$  and its zero bins. Then the mixing terms (which should cancel after adding all zero-bins) seem to be present.

# NNLL+NNLO results

- $d_2^{\text{veto}}$  very large for small  $R$  (leads to large scale dependence)
- Depending on matching scheme scale dependence reduces
  - Indicates **cancellations** (depending on the scheme)



- Can we trust this? Best would be to **separate** uncertainties of prefactor, leading power terms and power corrections and then combine. Not possible for NNLL+ NNLO
- At NNLL the large constant  $d_2^{\text{veto}}$  appeared, which is underestimated by a scale variation at NLL ( $R$  independent).

NNLL scale variation does not estimate  $\ln^2(R)$  terms **and** two loop ( $l \times \Phi$ ) becomes  $R$  dependent (similar effect?) → Strong motivation for N<sup>3</sup>LL

# Ingredients for N<sup>3</sup>LL

- C<sub>t</sub> and C<sub>s</sub> to two loop order
- Two-loop collinear kernel functions (will be  $R$  dependent)  $I_{g \leftarrow i}(R)$
- Three-loop anomaly function

$$F_{gg}(p_T^{\text{veto}}, \mu) = a_s(\Gamma_0^A L_\perp + d_1^{\text{veto}}) + a_s^2 \left[ \Gamma_0^A \beta_0 \frac{L_\perp^2}{2} + \Gamma_1^A L_\perp + d_2^{\text{veto}}(R) \right] \\ + a_s^3 \left[ \Gamma_0^A \beta_0^2 \frac{L_\perp^3}{3} + (\Gamma_0^A \beta_1 + 2\Gamma_1^A \beta_0) \frac{L_\perp^2}{2} + (\Gamma_2^A + 2\beta_0 d_2^{\text{veto}}(R)) L_\perp + d_3^{\text{veto}}(R) \right]$$

Everything known except anomaly coefficient  $d_3^{\text{veto}}$

# NNLL and N<sup>3</sup>LL resummed results

Goal to disentangle different sources of scale dependence (anomaly, beam functions, power corrections) →

- We factor out

$$P = \sigma_0 C_t^2(m_t^2, \mu) |C_S(-m_H^2, \mu)|^2 \left( \frac{m_H}{p_T^{\text{veto}}} \right)^{-2F_{gg}(L_\perp, a_s)} e^{2h_A(L_\perp, a_s)}$$

↑  
scale independent!

Extra factor removed from beam function  
↓

Full cross section is then given by **two separately scale independent parts**:

$$\sigma(p_T^{\text{veto}}) = P \cdot \underbrace{(\bar{I} \otimes \phi)^2}_{\text{independent of } m_H \text{ (for fixed } m_H^2/s)}$$

“bar” indicates that factor  $h_A$  has been removed

Also enables us to **extract two loop ( $I \times \Phi$ )** using **HNNLO fixed-order code by Grazzini**

- Run HNNLO at high  $m_H$  (suppress power corrections) and factor out  $P$

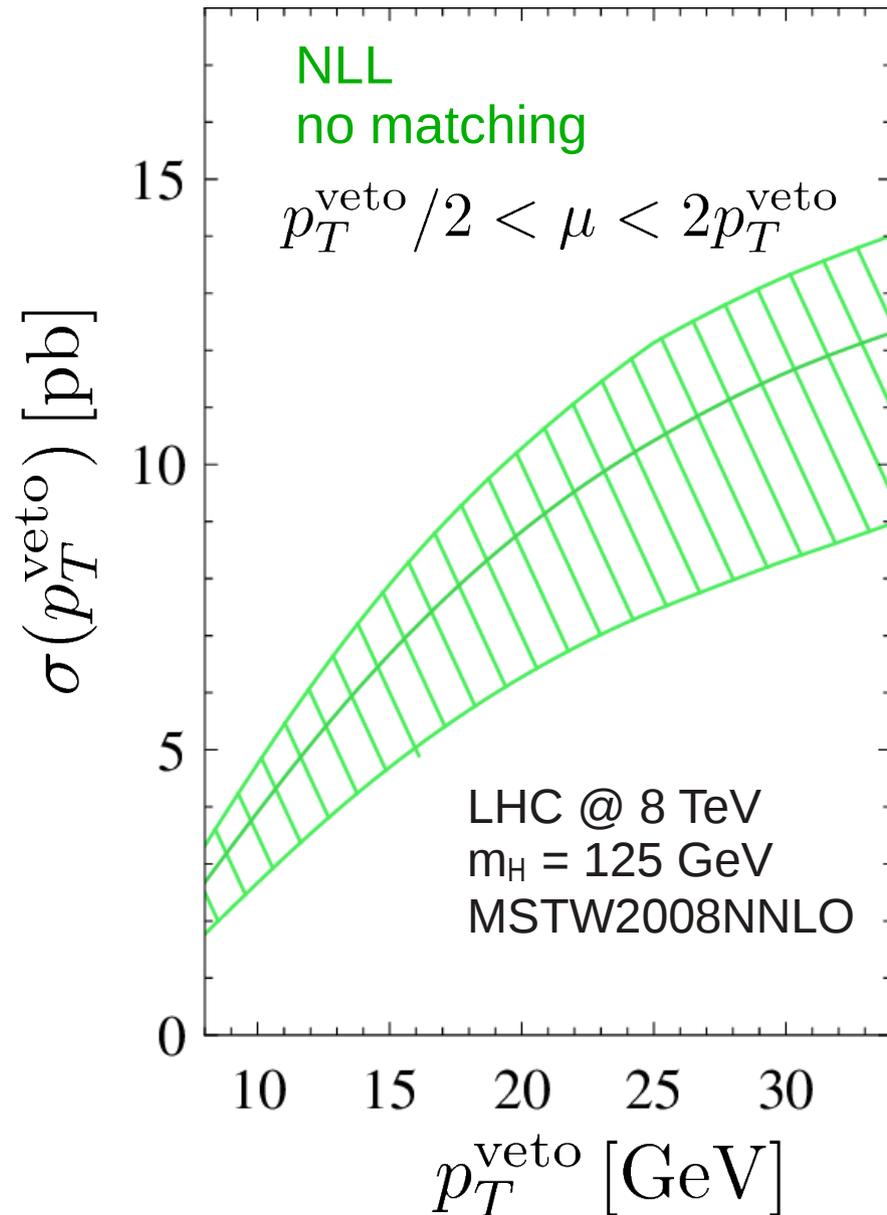
# Numerical results

I will show various plots, of the cross section.

The plots have the same format as the one on the right.

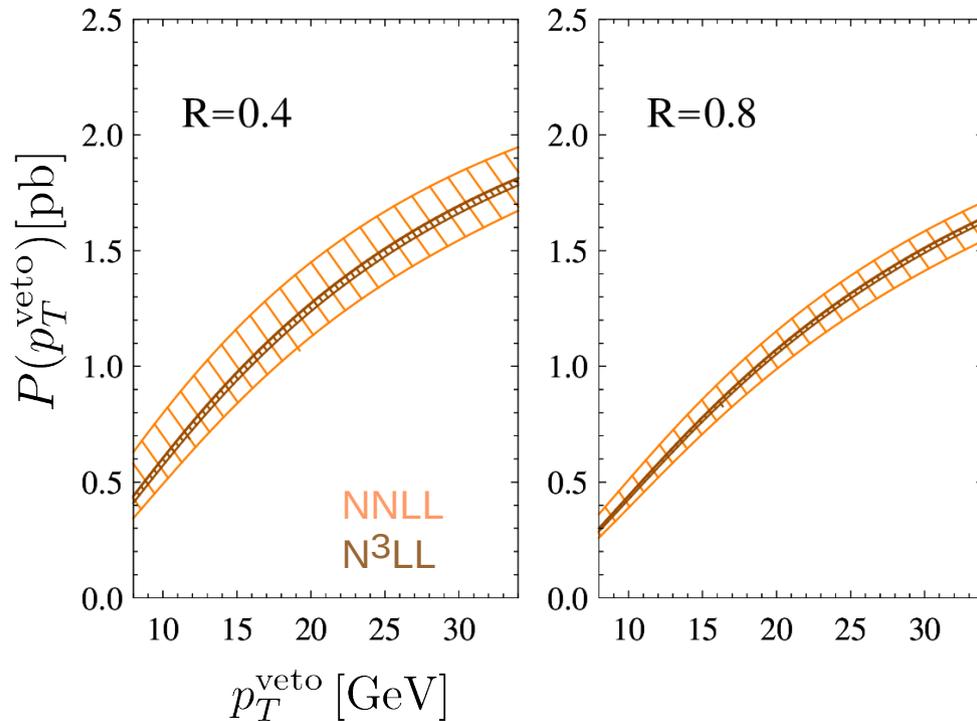
The results are

**PRELIMINARY**



# NNLL and N<sup>3</sup>LL resummed (different variations)

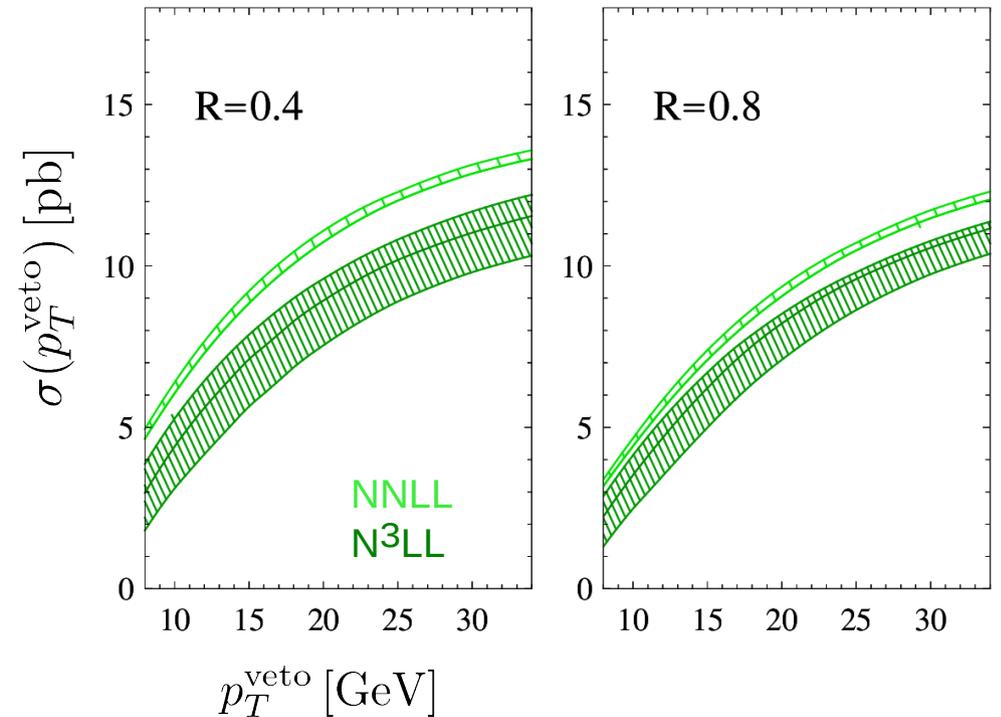
Scale variation of prefactor only



Scale variation reduces significantly.

(however  $d_3^{\text{veto}}$  is not included → estimate its effect)

Scale variation of  $(I \times \Phi)^2$  only



Uncertainty increases! At NNLO first  $R$  dependence arises in  $(I \times \Phi)^2$

→ small scale variation of  $(I \times \Phi)^2$  at NLO not reliable!

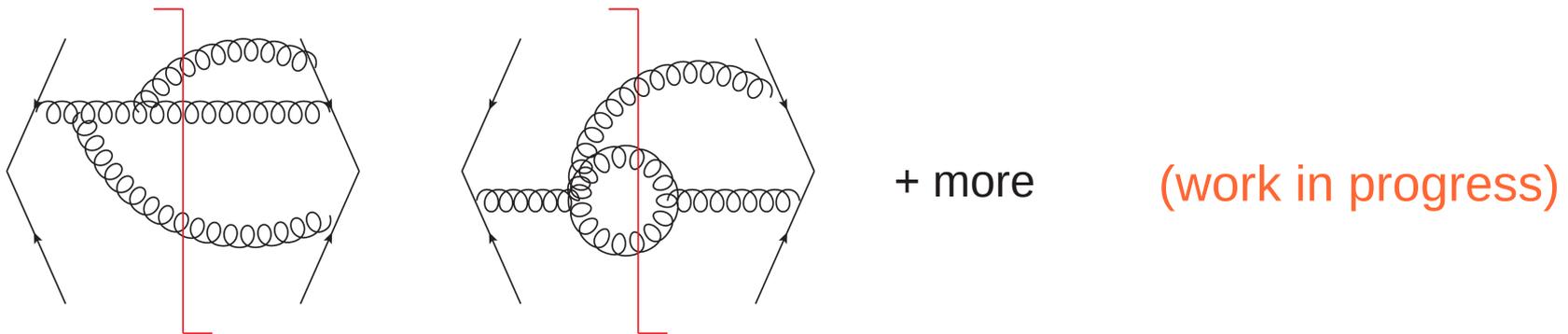
# Three loop anomaly coefficient $d_3^{\text{veto}}$

Uncertainty from the unknown coefficient  $d_3^{\text{veto}}$

- Leading color part of  $d_2^{\text{veto}}$  approximately  $d_2^{\text{veto}} \sim 2C_A^2 4^2 \ln \frac{2}{R}$
- Motivates estimate for the **leading logarithmic  $R$  dependence** of  $d_3^{\text{veto}}$

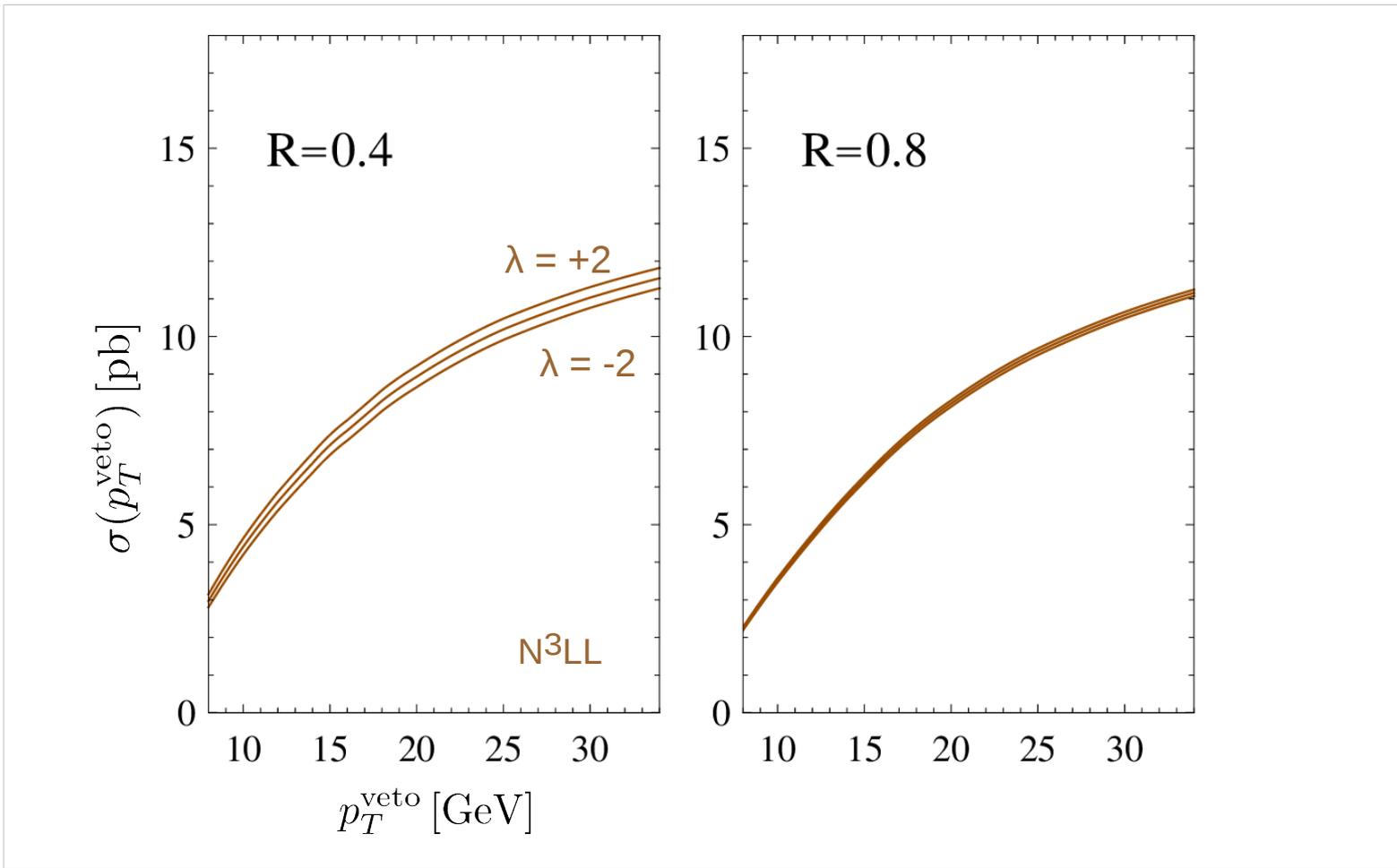
$$d_3^{\text{veto}} \sim \lambda 2C_A^3 4^3 \ln^2 \frac{2}{R} \quad \lambda = -2, \dots, +2$$

- Possibility to extract  $\lambda$  from three-emission soft function in SCET:



# N<sup>3</sup>LL with d<sub>3</sub><sup>estimate</sup>

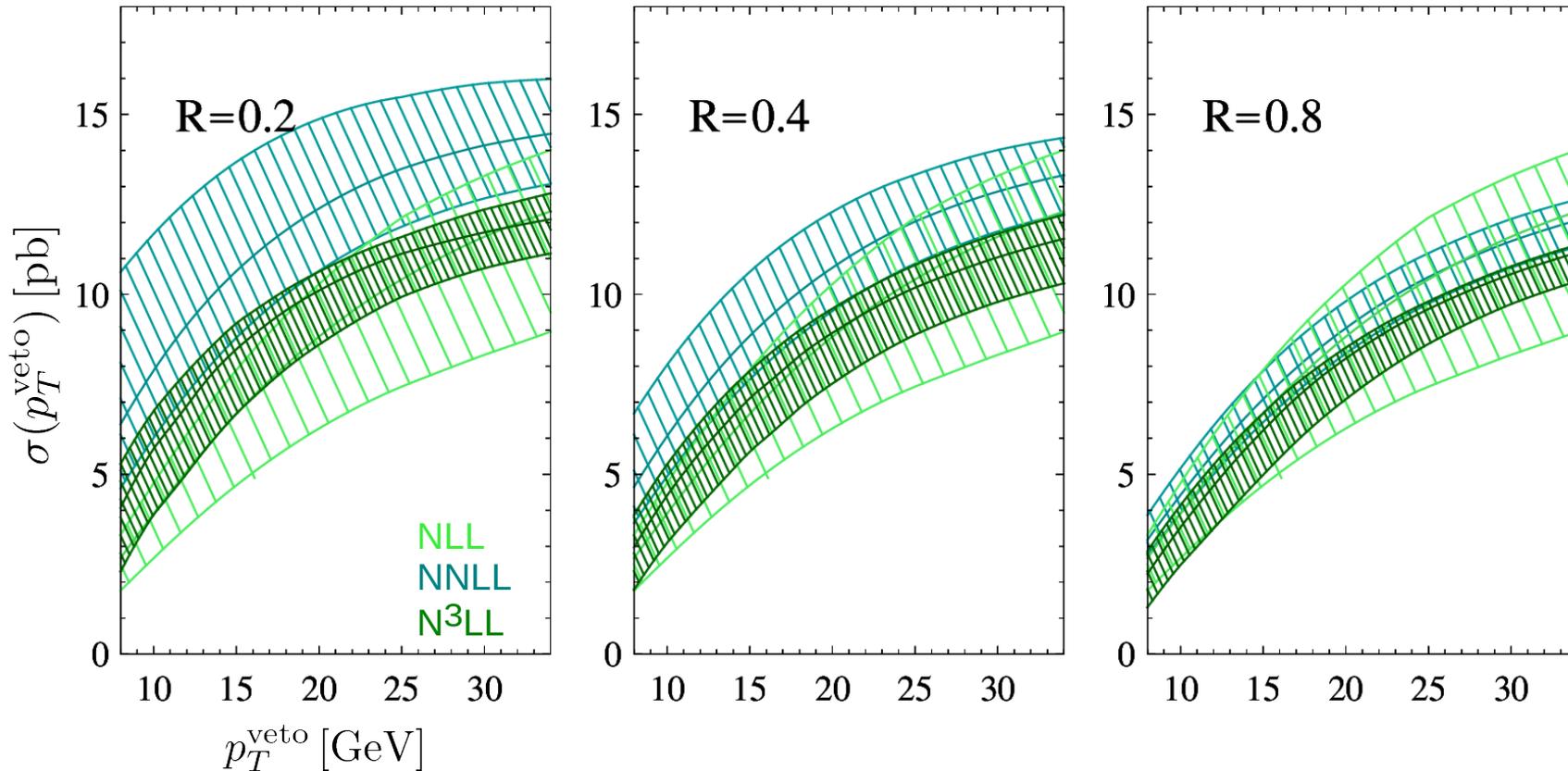
$$d_3^{\text{veto}} \sim \lambda 2C_A^3 4^3 \ln^2 \frac{2}{R}$$



→ if  $d_3^{\text{veto}}$  is indeed the size of our estimate, it will have a small effect (for phenomenologically relevant values of  $R$ )

# NLL, NNLL and N<sup>3</sup>LL resummed results

Full resummed results (errors of P and  $(l \times \Phi)^2$  quadratically added)

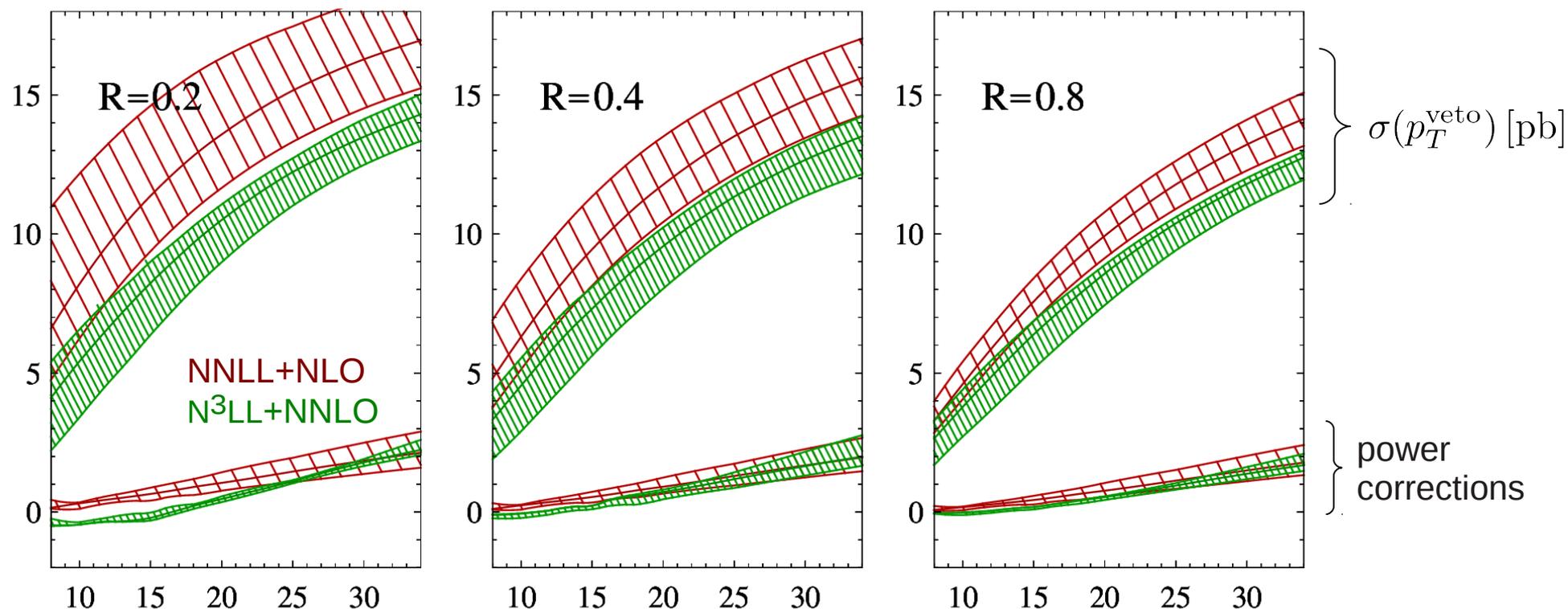


Low R: Size of the scale bands reduces only slowly with growing order:

- **NNLL**: leading R-dependent term appears in  $d_2^{\text{veto}}$
- **N<sup>3</sup>LL**: leading R-dependent term appears in two loop  $(l \times \Phi)^2$ .

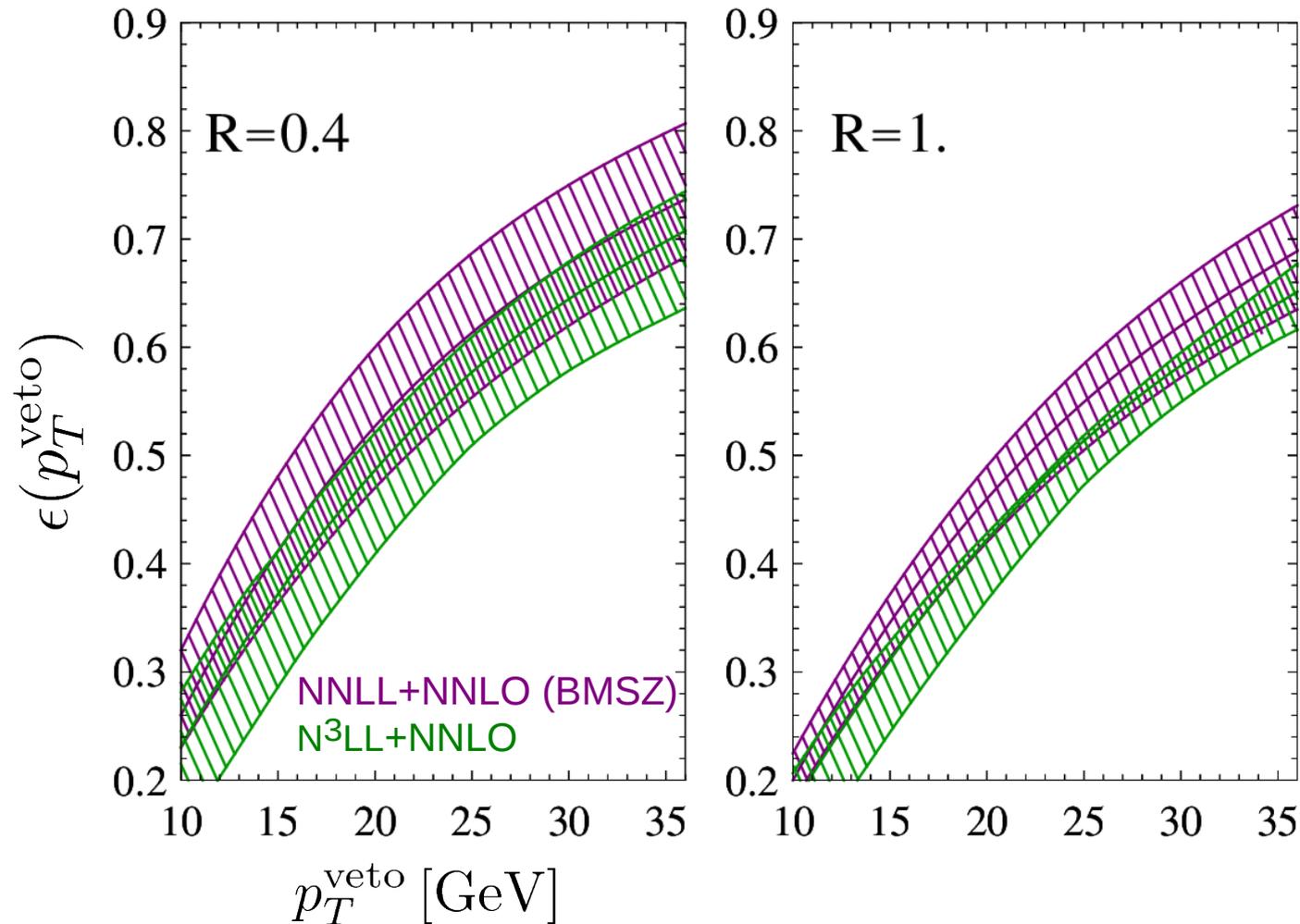
Large R: good perturbative behavior

# Matching to fixed-order numerical results



- The scale variations of  $P$ ,  $(I \times \Phi)^2$  and power corr. are done separately and then quadratically added (avoid accidental cancellation of large corrections)
- All two loop ingredients in place  $\rightarrow$  power corrections are small/suppressed ( $p_T/m_H$ ) (numerics needs improvement for small  $R$ )
- At N<sup>3</sup>LL+NNLO leading  $R$ -dependent effects are included in both  $P$  and  $(I \times \Phi)^2$

# Comparison to BMSZ result



We directly predict the cross section, not the efficiency. The above efficiencies are obtained using  $\sigma_{\text{tot}}=19.65$  pb from RGHiggs.

# Summary

Have presented new theoretical predictions for the Higgs production rate with a jet veto at **N<sup>3</sup>LL+NNLO accuracy**:

- Analytic computation of  $d_2^{\text{veto}}$  → result fully compatible with BMSZ (no SC mixing)
- Estimated  $d_3^{\text{veto}}$  effect small. This uncertainty could be reduced by a numerical computation of the  $\log^2(R)$  part → Nontrivial but feasible
- Reduce scheme dependence by choosing a scheme, which disentangles different sources of uncertainty (avoids accidental cancellations)
- Extract two loop  $(I \times \Phi)$  (**HNNLO code**)
- At this order **leading R dependence** (which leads to large uncertainty bands) included **both** in  $P$  and  $(I \times \Phi)^2$

# Extra Slides

# Simple example (zero-bins vs. multi-pole expansion)

Let us illustrate how double counting of overlapping regions can be avoided using either of the two methods.

Consider the following simple rapidity integral:

$$I = \int_{-\infty}^{\infty} dy \theta(R^2 - (y - y_0)^2)$$

Rapidity of a fixed collinear particle

It can easily be evaluated

$$I = \int_{y_0 - R}^{y_0 + R} dy = 2R$$

Now turn to a SCET framework and consider different regions (method of regions)

$y$  can either be **collinear** or **soft**

# Simple example: multi-pole expansion

The Integrand is just 1 and has no expansion into different regions.

The phase-space constraint however does:

collinear region:  $I_{cc} = I$

soft region:

$$\theta \left( \underset{\substack{\uparrow \\ \text{O}(1)}}}{R^2} - \left( \underset{\substack{\uparrow \\ \text{O}(1)}}}{y} - \underset{\substack{\uparrow \\ \text{Large}}}{y_0} \right)^2 \right) = 0 \quad \Rightarrow \quad I_{cs} = 0$$

zero-bin subtraction gives a vanishing integral  $I_{c(c \rightarrow s)}$

For the full result we then have to add all the regions

$$(I_{cc} - I_{c(c \rightarrow s)}) + I_{cs} = I$$

Note:  $\theta(R^2 - \ln \lambda) = \theta(e^{R^2} - \lambda)$

# Simple example: Zero-bin subtraction

If the phase-space constraint is not expanded, all regions give the same result.

$$I_{cc} = I \quad I_{cs} = I \quad I_{c(c \rightarrow s)} = I$$

And the final result  $(I_{cc} - I_{c(c \rightarrow s)}) + I_{cs} = I$

Notice that here the **zero-bin subtraction** was **essential** in order to get the correct result. The zero-bin canceled the soft-collinear mixing term.

# Backup: expanded measure

$$\begin{aligned}
 & \theta(R^2 - (y_s - y_c)^2) = \theta(R - |(y_s - y_c)|) \\
 & = \theta(R - (y_s - y_c)) \theta(y_s - y_c) + \theta(R - (y_c - y_s)) \theta(y_c - y_s) \\
 & = \theta\left(e^R - \frac{k_+^s k_T^c}{k_T^s k_+^c}\right) \theta(k_+^s k_T^c - k_T^s k_+^c) + \theta\left(e^R - \frac{k_T^s k_+^c}{k_+^s k_T^c}\right) \theta(k_T^s k_+^c - k_+^s k_T^c) \\
 & = \theta(\underbrace{e^R k_T^s k_+^c}_{\lambda^3} - \underbrace{k_+^s k_T^c}_{\lambda^2}) \theta(\underbrace{k_+^s k_T^c}_{\lambda^2} - \underbrace{k_T^s k_+^c}_{\lambda^3}) + \theta(\underbrace{e^R k_+^s k_T^c}_{\lambda^2} - \underbrace{k_T^s k_+^c}_{\lambda^3}) \theta(\underbrace{k_T^s k_+^c}_{\lambda^3} - \underbrace{k_+^s k_T^c}_{\lambda^2}) \\
 & = \theta(-k_+^s k_T^c) \theta(k_+^s k_T^c) + \theta(e^R k_+^s k_T^c) \theta(-k_+^s k_T^c) = 0
 \end{aligned}$$

# Backup

$h_A$  in prefactor  $P$  explicitly:

$$h_A(L_{\perp}, a_s) = a_s \left[ \Gamma_0^A \frac{L_{\perp}^2}{4} - \gamma_0^g L_{\perp} \right] + a_s^2 \left[ \Gamma_0^A \beta_0 \frac{L_{\perp}^3}{12} + (\Gamma_1^A - 2\gamma_0^g \beta_0) \frac{L_{\perp}^2}{4} - \gamma_1^g L_{\perp} \right]$$