# Final state interactions in factorizing and nonfactorizing cross sections

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Duke University

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Alexander Mitov, GS
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A. Final State Interactions (FSI) and Factorization: Comments and Motivation

B. Light Cone-Ordered Perturbation Theory and Detailed Unitarity

C. Cancellation from Detailed Unitarity for a LO FSI

D. From LO to AnyO: Hard-scattering Locality and Recoiling Radiation

E. A few Observations on Power Correction Estimates

The reasoning is simple, but requires a little detail.
The results are not surprising, especially since the work of Collins and Qiu (2007) and Mulders and Rogers (2010) on non-universality of TMDs for hadronic scattering – but the method may be interesting.

## A. Final State Interactions and Factorization: Comments and Motivation

- To be specific, consider

$$H_A(p_A) + H_B(p_B) \to t(p_t) + \bar{t}(p_{\bar{t}}) + X$$

- Collinear factorization for single-particle inclusive cross sections

$$\frac{d\sigma}{d^3p_t} = \sum_{ab} f_{a/A} \otimes f_{b/B} \otimes \hat{\sigma}^{\text{part}}_{ab\to t+X}(p_t) + \mathcal{O}(\Lambda/m_t)$$

  For the top with $p_T \sim m_t$, no fragmentation function is necessary.

- No role for soft FSI at leading power: universal pdfs and a short-distance cross section only.

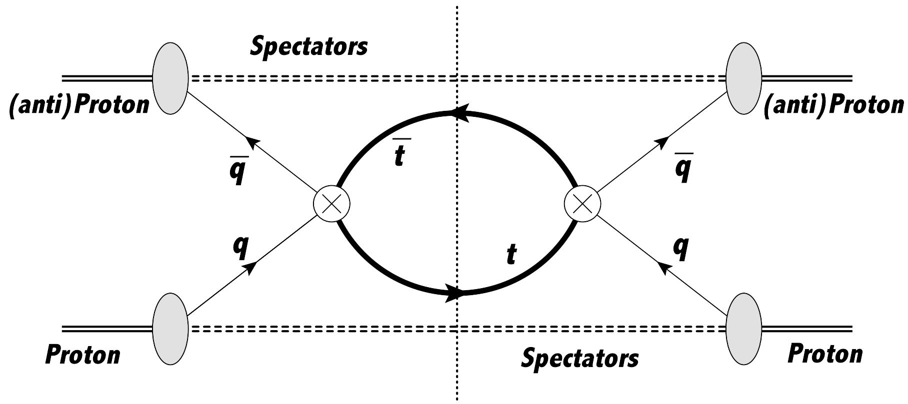
- This is why, in principle QCD prediction for the top AFB is robust.

- But what happened to the FSIs – how do they get into $\mathcal{O}(\Lambda/m_t)$? And how do they appear in a two-particle inclusive cross section – when is collinear factorization possible?

$$\frac{d\sigma}{d^3p_t d^3p_{\bar{t}}} = \sum_{ab} f_{a/A} \otimes f_{b/B} \otimes \hat{\sigma}^{\text{part}}_{ab\to t+\bar{t}+X}(p_t, p_{\bar{t}}) + C_{2PI}\,,$$

- Can we estimate the size of corrections, $C_{2PI}$ associated with the final state interactions of the top pair?

- **Heuristics: The production of a heavy pair is short distance & well localized. Low momentum-transfer FSIs are "too late" and "too diffuse" to interfere with the total cross section.**




- **Why do we claim factorization for a 1PI differential cross section?**

- **Well, don't worry – Jianwei Qiu already mentioned well reasoned arguments for factorization of 1PI cross sections.**

- **This talk: a variation of that reasoning to show how factorization final state interactions cancel or not depending on the observable.**

- **We'll concentrate on the classic final state interaction: scattering by "spectators": remnants of the beam.**

- **Heuristics of the result: for heavy particle production, the cancelation of FSIs requires that the production in the amplitude and the complex conjugate be localized at the same point. This requires a sum over hard "recoiling" radiation.**

- **And –illustrating with a point brought up in yesterday's talk by Huaxing Zhu – for top pair production at fixed $Q_{\mathrm{Tpair}}$, nonperturbative corrections due to FSIs are probably suppressed by the order $\Lambda_{\mathrm{QCD}}/\langle Q_{\mathrm{T\,pair}}\rangle$. Not like $1/m_t$, but not so bad either.**

## B. Light Cone-Ordered Perturbation Theory and Detailed Unitarity

- Old-fashioned (here "light-cone ordered") perturbation theory. We can always do one integral per loop in every PT diagram. Here choose it as $l^-$ component, and get form:

$$\mathcal{G}_{\{p\}\to\{q\}} = \Sigma_{\text{vertex orderings}} \left\{ \Sigma_{\text{states}} \left[ \text{Product of } \frac{1}{(\text{“Energy Deficits”})} \times \text{ Numerators} \right] \right\}$$

$$= \sum_{\text{orderings}} \int \prod_{\text{loops }\{l\}} d^2 l_\perp dl^+ \prod_{\text{lines }\{k\}} \frac{\theta(k^+)}{2k^+} \prod_{\text{states }\{s\} \text{ in } T} \frac{1}{P^-_{\text{ext},s} - \sum_{k\in s}[k]^- + i\epsilon} N(k)$$

where

$$[k]^- = \frac{k_\perp^2}{2k^+}$$

Convenient kinematics for hadron-hadron: light-cone axis ($\hat{x}_3$) perpendicular to the beams:

$$\frac{1}{2}\sqrt{\frac{S}{2}} = p_A^\pm = p_B^\pm \,, \quad \frac{\sqrt{S}}{2} = p_A^0 = p_B^0 \,,$$

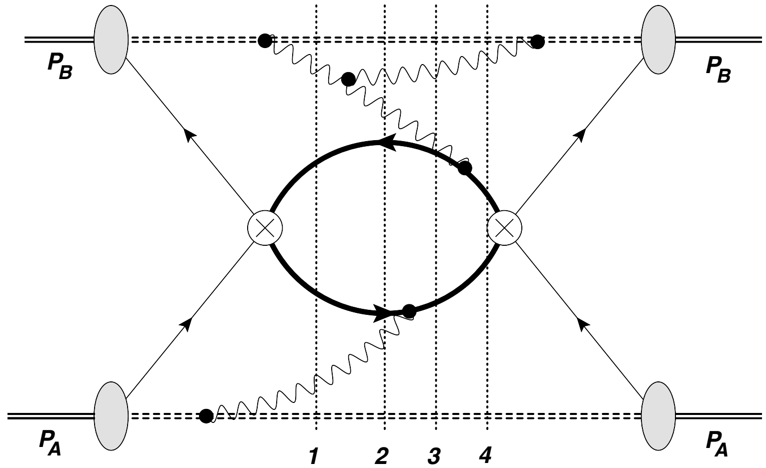
$$p_A^1 = p_A^0 \;\; p_B^1 = -p_B^0 \,.$$

"Perp" vs. "transverse" momenta:

$$k_\perp = (k^1, k^2) \,, \quad k_T = (k^2, k^3)$$

Beams have zero transverse, large perp.

- **LCO representation of a cross section: cut diagram.**




- **Initial and final states are manifest.**

$$2p_t^+ \frac{d\sigma^{(\Pi_{ab})}_{AB\to t\bar{t}+X}}{d^3p_t} = \sum_{\text{orderings T of } \Pi_{ab}} \int \prod_{\text{loops}\,\{l\}} d^2l_\perp dl^+ \prod_{\text{lines}\,\{k\}} \frac{\theta(k^+)}{2k^+}$$

$$\times \int_0^1 dx\ \delta\left(x - \frac{x_a p_A^- + x_b p_B^-}{P^-}\right) \mathcal{I}^{(T)*}_{ab/AB}\, \mathcal{F}^{(T)}_{ab}(p_t)\, \mathcal{I}^{(T)}_{ab/AB}$$

- **The final state factor for observed $\vec{p}_t$:**

$$\mathcal{F}^{(T)} = H^*_{ab} H_{ab} \sum_{j=1}^{S} \int d^3p_t^{(j)} \left(\prod_{i'=j+1}^{S} \frac{1}{P^-_{\text{ext i}'} - s_{i'} - i\epsilon}\right) 2\pi\delta\left(P^-_{\text{ext j}} - s_j\right) \delta^3(p_t - p_t^{(j)})$$

$$\times \left(\prod_{i=1}^{j-1} \frac{1}{P^-_{\text{ext i}} - s_i + i\epsilon}\right)$$

- **For all final states, $P_{\text{ext}} = P^- = p_A^- + p_B^-$.**

- General feature: infrared cancelations are a consequence of unitarity $\leftrightarrow$ hermitian Hamiltonian.
- Unitarity is manifest in LCOPT through a generalization of:

$$2\pi\delta(y) = i\left(\frac{1}{y+i\epsilon} - \frac{1}{y-i\epsilon}\right)$$

- Which generalizes to a completely differential version of the optical theorem ("detailed unitarity") at the level of fixed loop momenta in individual diagrams (after the minus integrals)

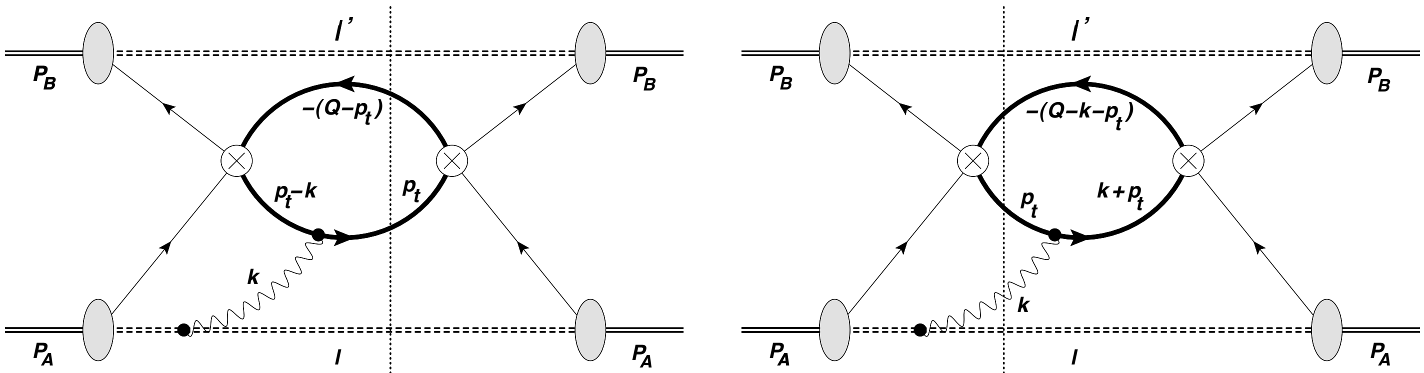
$$\sum_{j=1}^{n} \prod_{i'=j+1}^{n} \frac{1}{y'_i - i\epsilon}\, \delta(y_j) \prod_{i=1}^{j-1} \frac{1}{y_i + i\epsilon} \quad = \quad 2\mathrm{Im} \prod_{i=1}^{n} \frac{1}{y_i + i\epsilon}$$

Proof: apply the LO identity and cancel terms pairwise.

- Implication for our case: the sum of terms in a cut diagram with $\pm i\epsilon$ denominators with gives the difference of an integral with all $+i\epsilon$ and its complex conjugate. This can mean momentum integrals are not pinched between singularities $\rightarrow$ if so can avoid infinities of the integrand $\rightarrow$ finite integrals and PT is OK!
- This little identity has many uses – in particular showing that jet cross sections and event shapes are infrared safe.
- In the following – apply to spectator-pair quark FSIs.

## C. Cancellation from Detailed Unitarity for a LO FSI

- **The simplest example in a little detail:** [$Q \sim x_a p_A + x_b p_B$, $x = Q^-/P^-$ (as above).]




$$\mathcal{F} \;=\; H^*_{ab}\, H_{ab} \quad \left[ 2\pi\delta\left(D_2^{(2)}\right) \frac{1}{D_1^{(2)} + i\epsilon} + \frac{1}{D_2^{(1)} - i\epsilon} 2\pi\delta\left(D_1^{(1)}\right)\right]$$

$$\begin{aligned}
D_1^{(1)} &= xP^- - [Q - p_t - k]^- - [p_t]^- + d_1 = xP^- - [Q - p_t]^- - [p_t]^- + \beta_{Q-p_t}\cdot k + d_1 + \mathcal{O}(k^2)\,,\\
D_1^{(2)} &= xP^- - [Q - p_t]^- - [p_t - k]^- + d_1 = xP^- - [Q - p_t]^- - [p_t]^- + \beta_{p_t}\cdot k + d_1 + \mathcal{O}(k^2)\,,\\
D_2^{(1)} &= xP^- - [Q - p_t - k]^- - [p_t + k]^- + d_2 \; = xP^- - [Q - p_t]^- - [p_t]^-\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad + (\beta_{Q-p_t} - \beta_{p_t})\cdot k + d_2 + \mathcal{O}(k^2)\,,\\
D_2^{(2)} &= xP^- - [Q - p_t]^- - [p_t]^- + d_2
\end{aligned}$$

- $\beta_p \equiv \frac{1}{p^+}\left(0^+, [p]^-, p_\perp\right)$, & $d_1(l, l')$ and $d_2(l, l')$ are $\sim \mathcal{O}(k)$ and depend only on choice of out state.

- **Apply the unitarity identity to the two diagrams ...**

$$
\begin{aligned}
-i\mathcal{F} \;=\; H^*_{ab}\,H_{ab}\,\Bigg[ & \frac{1}{D_2^{(2)}+i\epsilon}\times\frac{1}{D_1^{(2)}+i\epsilon} \;-\; \frac{1}{D_2^{(2)}-i\epsilon}\times\frac{1}{D_1^{(2)}+i\epsilon} \\
& +\frac{1}{D_2^{(1)}-i\epsilon}\times\frac{1}{D_1^{(1)}+i\epsilon} \;-\; \frac{1}{D_2^{(1)}-i\epsilon}\times\frac{1}{D_1^{(1)}-i\epsilon}\Bigg]
\end{aligned}
$$

- **Terms with two $+i\epsilon$s or two $-i\epsilon$s are ok – no pinch near $x = ([Q-p_t]^- + [p_t]^-)/P^-$ [because forward jets are smooth functions of the total minus momenta that flow into the hard scattering.]**

- **Drop nonsingular terms and trivially reorganize the other two:**

$$
\begin{aligned}
-i\mathcal{F} &= H^*_{ab}\,H_{ab}\,\left[\frac{1}{D_2^{(1)}-i\epsilon}\times\frac{1}{D_1^{(1)}+i\epsilon} - \frac{1}{D_2^{(2)}-i\epsilon}\times\frac{1}{D_1^{(2)}+i\epsilon}\right] \\
&= H^*_{ab}\,H_{ab}\,\left[\frac{1}{D_2^{(2)}+\left(D_2^{(1)}-D_2^{(2)}\right)-i\epsilon}\times\frac{1}{D_1^{(2)}+\left(D_1^{(1)}-D_1^{(2)}\right)+i\epsilon} - \frac{1}{D_2^{(2)}-i\epsilon}\times\frac{1}{D_1^{(2)}+i\epsilon}\right]
\end{aligned}
$$

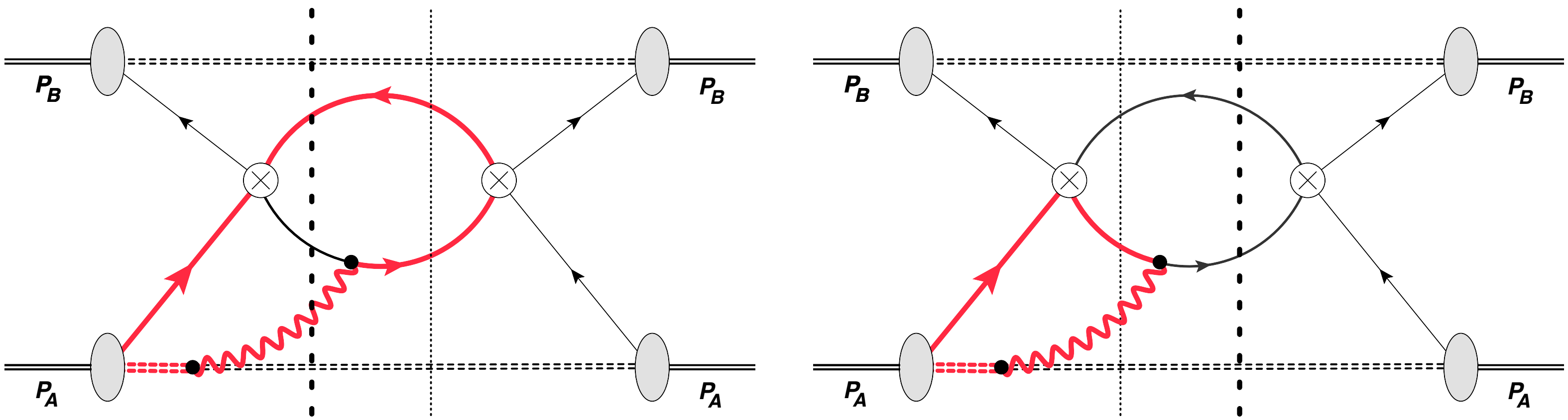
$$
\begin{aligned}
D_1^{(1)} - D_1^{(2)} &= -[p_t]^- - [Q-p_t-k]^- + [p_t-k]^- + [Q-p_t]^- \\
&= (\beta_{Q-p_t} - \beta_{p_t})\cdot k + \mathcal{O}(k^2)
\end{aligned}
$$

$$
\begin{aligned}
D_2^{(1)} - D_2^{(2)} &= -[p_t+k]^- - [Q-p_t-k]^- + [p_t]^- + [Q-p_t]^- \\
&= (\beta_{Q-p_t} - \beta_{p_t})\cdot k + \mathcal{O}(k^2)
\end{aligned}
$$

- **These two terms have pinches, but they cancel at leading power if we shift $x$ in the first term by $\delta x = (\beta_{Q-p_t} - \beta_{p_t})\cdot k$.**

D. From LO to AnyO: Hard-scattering Locality and Recoiling Radiation

- What have we done? The shift in $x$ simply re-routes momentum $k$ through the $\bar{t}$ line:




- This mechanism is completely general, but requires that we integrate over the "unobserved" $p_{\bar{t}}$.

- It will not work for the 2PI cross section, or even the cross section at measured pair transverse momentum. Such cross sections are not independent of FSIs, and do not factorize in general (either collinear or TMD), although they can factor in the presence of unobserved high-$p_t$ radiation.

- Can't "re-route" radiation of energy $\geq \Lambda_{\mathrm{QCD}}$ through spectators.

- Lesson: we need recoiling radiation hard enough to "absorb" the effects of spectator FSIs. The presence of such radiation produces locality – the elimination of interference between pair production at separated space-time points. (This is the origin of evolution for fragmentation – viz. "cut vertices")

- Special case to note: the generic event in top pair production will have many GeV of radiation. Suggests the cancelation is still good at the nonperturbative level.

- **The general form** ($D_i^{(j)}$ **is the LC denominator for final state** $i$ **when state** $j$ **is the out state.)**

$$
\begin{aligned}
-i\mathcal{F} &= -i \sum_{j=1}^{S} \left( \prod_{i'=j+1}^{S} \frac{1}{D_{i'}^{(j)} - i\epsilon} \right) 2\pi\delta\left(D_j^{(j)}\right) \left( \prod_{i=1}^{j-1} \frac{1}{D_i^{(j)} + i\epsilon} \right) \\
&= \sum_{j=1}^{S} \left( \prod_{i'=j+1}^{S} \frac{1}{D_{i'}^{(j)} - i\epsilon} \right) \left( \prod_{i=1}^{j} \frac{1}{D_i^{(j)} + i\epsilon} \right) - \sum_{j=1}^{S} \left( \prod_{i'=j}^{S} \frac{1}{D_{i'}^{(j)} - i\epsilon} \right) \left( \prod_{i=1}^{j-1} \frac{1}{D_i^{(j)} + i\epsilon} \right) \\
&= \sum_{j=1}^{S-1} \mathcal{F}^{(j)} + \text{no pinch}\,,
\end{aligned}
$$

- **Again, we drop terms in which the** $x$ **integral can be deformed. The other terms are going to cancel at leading power:**

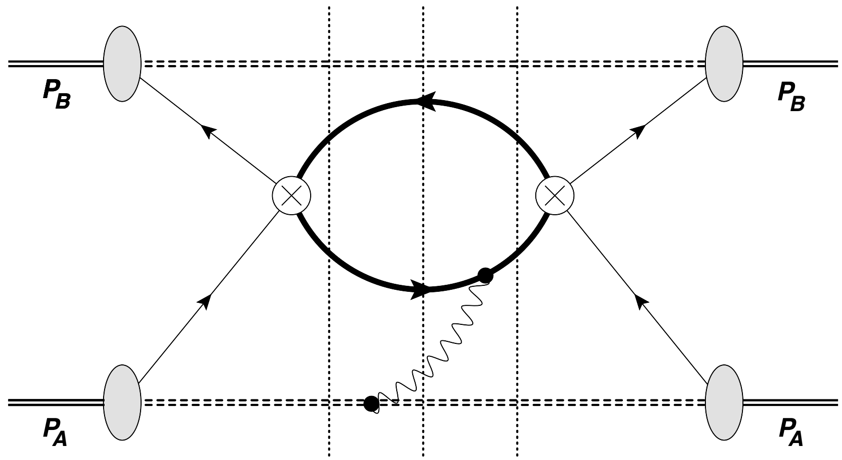
$$
-i\mathcal{F}^{(j)} = \left( \prod_{i'=j+1}^{S} \frac{1}{D_{i'}^{(j)} - i\epsilon} \right) \left( \prod_{i=1}^{j} \frac{1}{D_i^{(j)} + i\epsilon} \right) - \left( \prod_{i'=j+1}^{S} \frac{1}{D_{i'}^{(j+1)} - i\epsilon} \right) \left( \prod_{i=1}^{j} \frac{1}{D_i^{(j+1)} + i\epsilon} \right)
$$

- **As above . . .** $\quad D_i^{(j)} - D_i^{(j+1)} = P^-\,\delta x^{(j)} + \mathcal{O}(k^2)\,,$

- **The RHS is independent of** $i$ **at order** $k$**, and the same shift matches all LCO denominators.**

## E. A few Observations on Power Correction Estimates

- The most interesting FSI are scattering completely in the final state,




- Interesting feature for spectator with $l_T \sim \Lambda$, $l^- \sim P^-$:
- In this case, the final state that includes gluon $k$ is leading power only for $k_T < \Lambda^2/P^-$, but the other two final states are leading power all the way to $k_T \sim \Lambda$.
- The latter describe 'true' final state rescatterings, related to "Glauber" or "Coulomb" regions, and produce the non-universality demonstrated by Collins and Qiu (2007) and Mulders and Rogers (2010) in the context of TMDs.

- The case of 'one unobserved jet' with recoiling momentum, $p_{\rm jet}$ much softer than the pair mass.

- E.g.: an extra final state gluon, of momentum $p'$. In this case, the leading behavior is (dropping color factors)

$$H^*_{ab\to t\bar t\ +g(p')}\,H_{ij\to t\bar t\ +g(p')} \ = \ \sum_{c,d=a,b,t,\bar t}\frac{p_c\cdot p_d}{p_c\cdot p'\,p_d\cdot p'}H^*_{ij\to t\bar t}\,H_{ij\to t\bar t}$$

$$\frac{p_a\cdot p_b}{p_a\cdot p'\,p_b\cdot p'}=\frac{1}{2}\,\frac{1}{{p'_T}^{\,2}}$$

- Expand: the gluon either carries the momentum of a soft rescattering of the pair or not.

$$\frac{1}{(p'\pm k)^2_T}-\frac{1}{{p'_T}^{\,2}}$$

- A power series in $1/p_T$.

- Because all divergences in inclusive cross sections are logarithmic, $\mathcal{O}(k)$ suppression $\to$ pQCD is OK at an arbitrary pinch surface involving final state interactions. But, NP soft gluons can still give an infrared Landau pole. • OPE (Mueller (1984)) • DIS (Dasgupta & Webber) • event shapes in electron-positron annihilation (Webber, Dokshitzer, Marchesini, Salam . . . Korchemsky, GS, Zhakarov, Akhoury, 1995 et seq. up to talks yesterday by Mateu Barreda and Daniel Kolodrubetz).

- Estimate tof power corrections ias in (Qiu, GS 1990): additive nonperturbative corrections to leading-power factorization $\Rightarrow$ generic $\langle k\rangle/Q$ relative to leading-power behavior.

- Clearly, much more to be learned . . .

- It is plausible to anticipate an effective theory operator classification of the expansion in final-state momentum transfers $k$.