An on-shell approach to soft-collinear factorization

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Work done with Matthew Schwartz

Outline

- A few comments on the traditional approach to SCET
- Approach soft/collinear factorization in a different way
  - Analyze tree-level amplitudes using spinor-helicity methods
  - Polarization-vector independence play a key role in tree-level factorization
  - Power count in physical observables
- Conclude

Traditional Approach

Effective field theory methods pushed to the limit

- Write $\mathcal{L}_{QCD}$ in terms of fields to which we can assign a definite power counting
- Expand in powers

The effective theory tells you the physics

e.g.

$$\xi_{n,\tilde{p}} = Y_n\, \xi^{(0)}_{n,\tilde{p}} \qquad\qquad A_{n,\tilde{p}} = Y_n\, A^{(0)}_{n,\tilde{p}}\, Y_n^\dagger$$

$$\mathcal{L}\big[\xi_{n,\tilde{p}}, A_{n,\tilde{q}}, A_{\text{soft}}\big] = \mathcal{L}\big[\xi^{(0)}_{n,\tilde{p}}, A^{(0)}_{n,\tilde{q}}\big]$$

This approach is complicated

$$\psi = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x}(\xi_{n,\tilde{p}} + \xi_{\bar{n},\tilde{p}})$$

$$A_{\rm coll} = \sum_{\tilde{q}} e^{-i\tilde{q}\cdot x} A_{n,\tilde{q}}$$

$$(\bar{n}\cdot p + in\cdot D)\xi_{\bar{n},\tilde{p}} = (\not{p}_\perp + i\not{D}_\perp)\xi_{n,\tilde{p}}$$

$$\mathcal{L} = \sum_{\tilde{p},\tilde{p}',\tilde{q}} e^{i(\tilde{p}'-\tilde{p})\cdot x}\bar{\xi}_{n,\tilde{p}'}\Big[in\cdot D_s + ge^{-i\tilde{q}\cdot x} n\cdot A_{n,\tilde{q}} + (\not{p}_\perp + i\not{D}_{s\perp} + ge^{-i\tilde{q}\cdot x}\not{A}^\perp_{n,\tilde{q}})$$
$$\times \frac{1}{\bar{n}\cdot p + i\bar{n}\cdot D_s + ge^{-i\tilde{q}\cdot x}\bar{n}\cdot A_{n,\tilde{q}}}(\not{p}_\perp + i\not{D}_{s\perp} + ge^{-i\tilde{q}\cdot x}\not{A}^\perp_{n,\tilde{q}})\Big]\frac{\not{\bar{n}}}{2}\xi_{n,\tilde{p}}$$

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Gauge transformations clash with power counting

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Leads to a complicated non-local Lagrangian

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$$\mathcal{L} = \sum_{\tilde{p},\tilde{p}',\tilde{q}} e^{i(\tilde{p}'-\tilde{p})\cdot x}\bar{\xi}_{n,\tilde{p}'}\Big[in\cdot D_s + ge^{-i\tilde{q}\cdot x}n\cdot A_{n,\tilde{q}} + (\not{p}_\perp + i\not{D}_{s\perp} + ge^{-i\tilde{q}\cdot x}\not{A}^\perp_{n,\tilde{q}})$$
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Field scaling is not physical

$$(\bar{n} \cdot A_{n,\tilde{q}}, n \cdot A_{n,\tilde{q}}, A^{\perp}_{n,\tilde{q}}) \sim (1, \lambda^2, \lambda) \sim q^{\mu}_{\mathrm{coll}}$$

$$\langle q | \, A^{\mu}(x) \, | 0 \rangle = e^{iq\cdot x} \epsilon^{\mu}(q)$$

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Especially not this scaling!

$$\begin{aligned} \epsilon^{\mu}_{\mathrm{SCET}} &\sim \frac{1}{\lambda}(1, \lambda^2, \lambda) + \mathcal{O}(\lambda) \\ &= \frac{1}{\lambda} n^{\mu} + \epsilon^{\mu}_{\mathrm{phys}} + \mathcal{O}(\lambda) \end{aligned}$$

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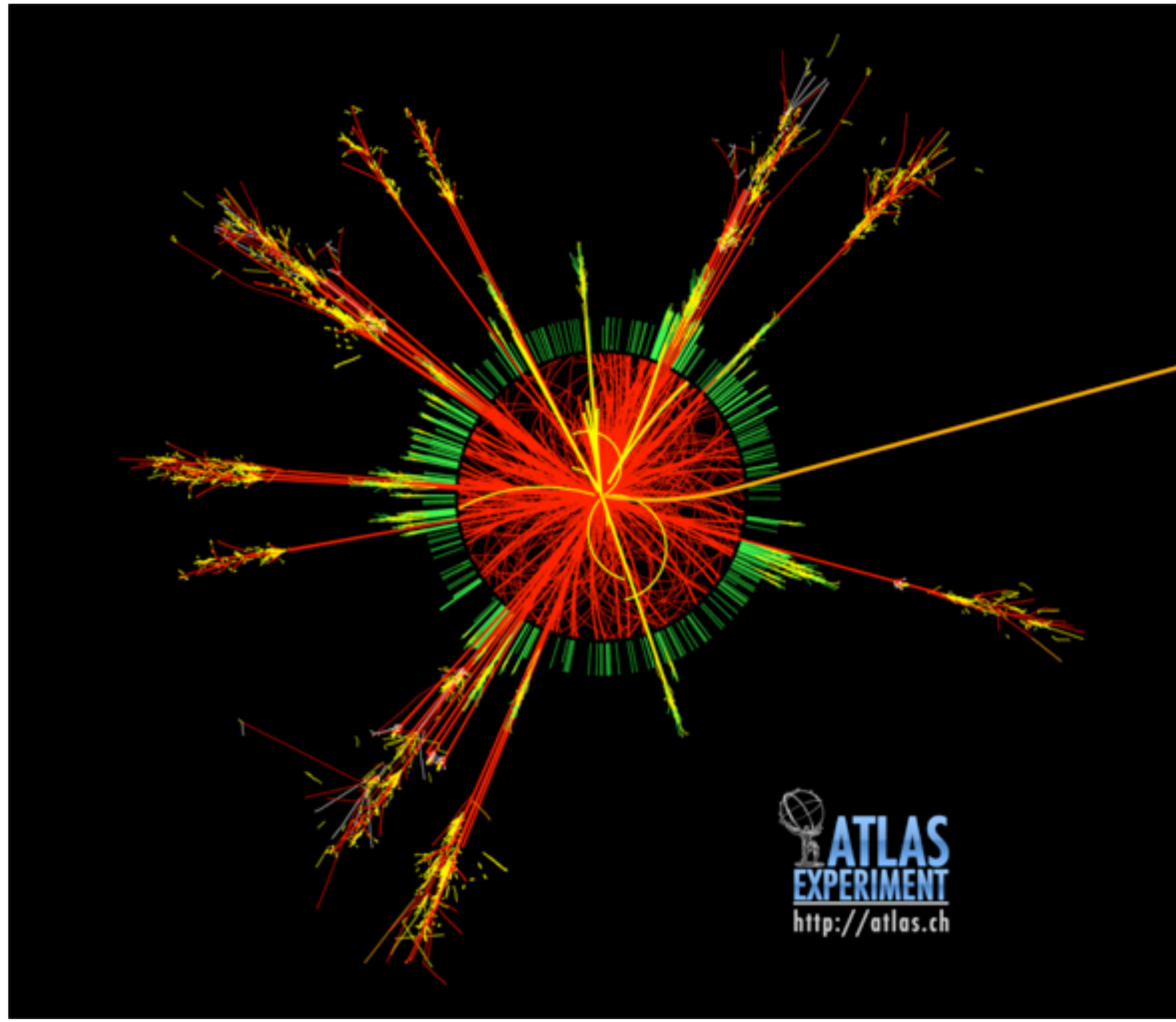

$$\mathrm{CET} \sim \frac{1}{\lambda}(1, \lambda^2, \lambda) + \mathcal{O}(\lambda)$$

$$= \frac{1}{\lambda} n^{\mu} + \epsilon^{\mu}_{\mathrm{phys}} + \mathcal{O}(\lambda)$$

But despite complicated intermediate steps, in the end physical results are simple:

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} = H \int d\Phi \, J(p_L^2)\, J(p_R^2)\, S(k)\, \delta(\tau - \tau[\Phi])$$

Individual copies of QCD

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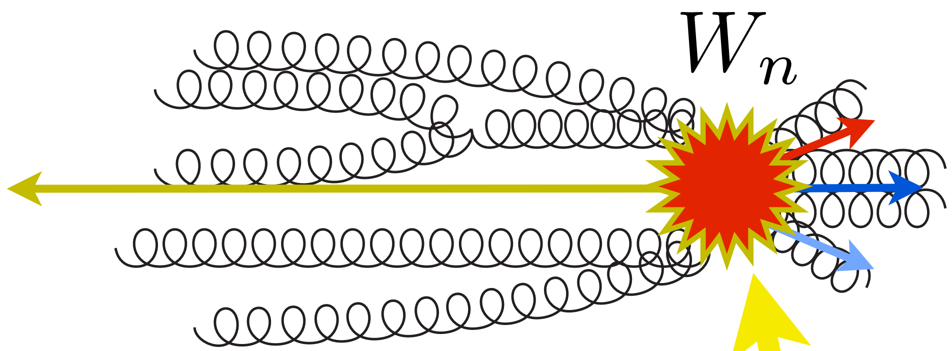
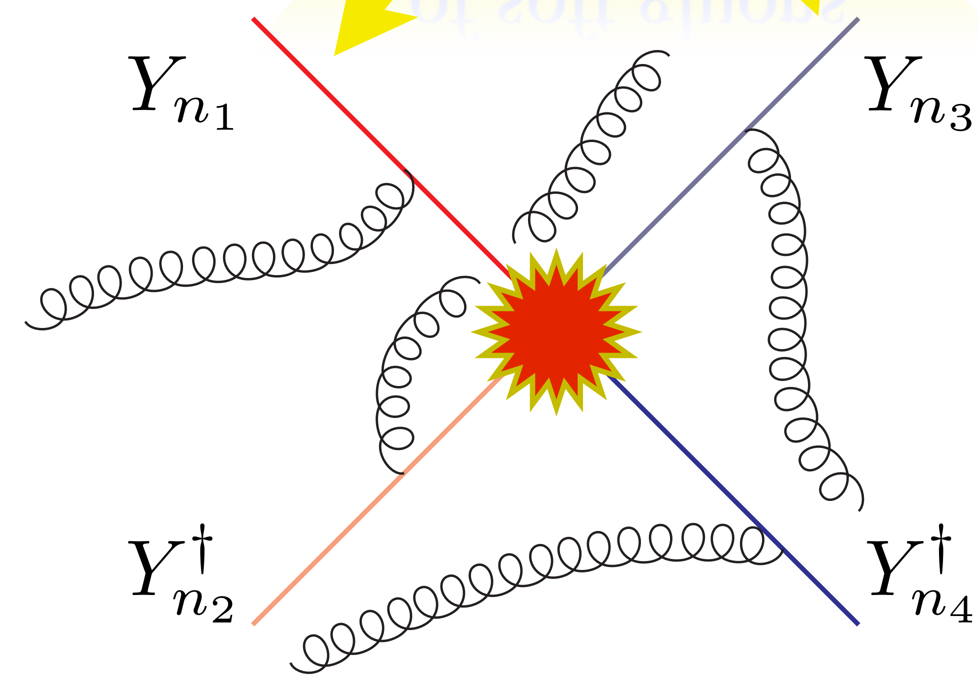
Individual copies of QCD

Collinear lines are classical sources of soft gluons

because the physics is simple



Collinear sector sees a source of collinear gluons



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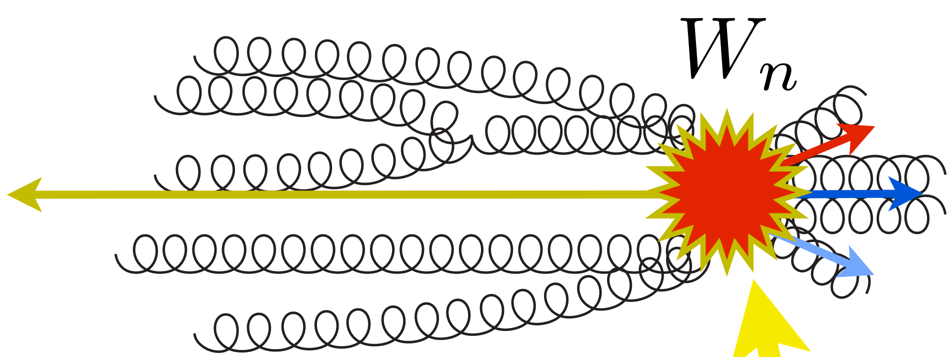

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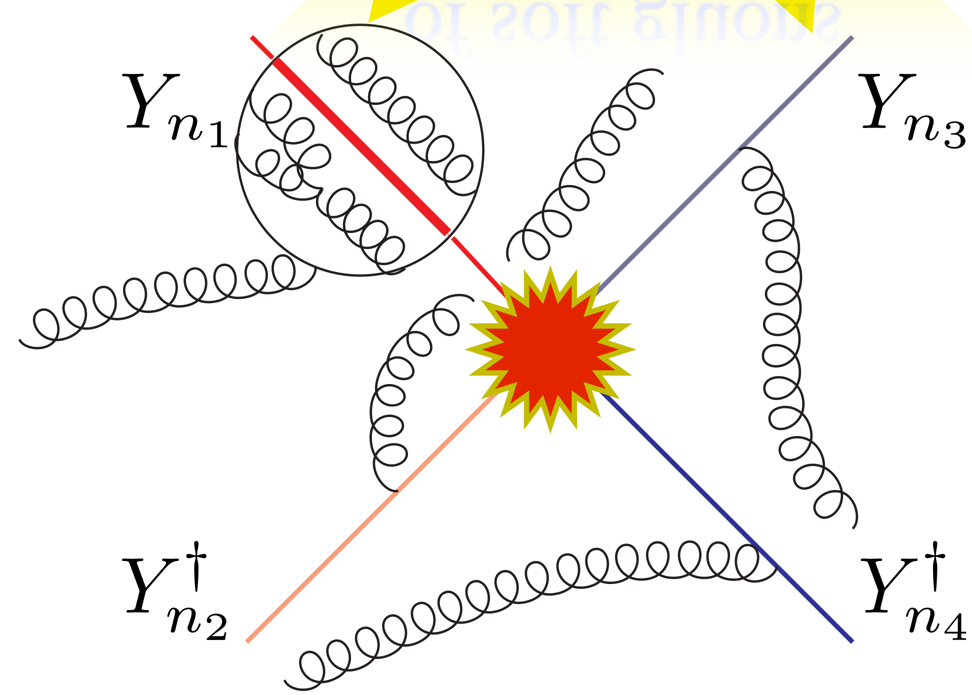

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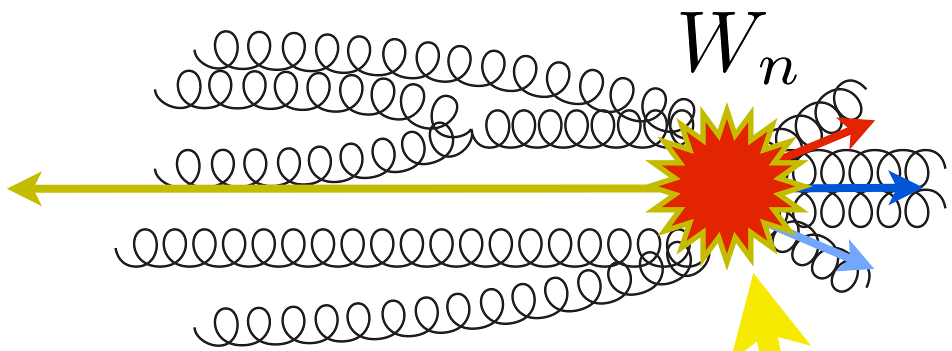

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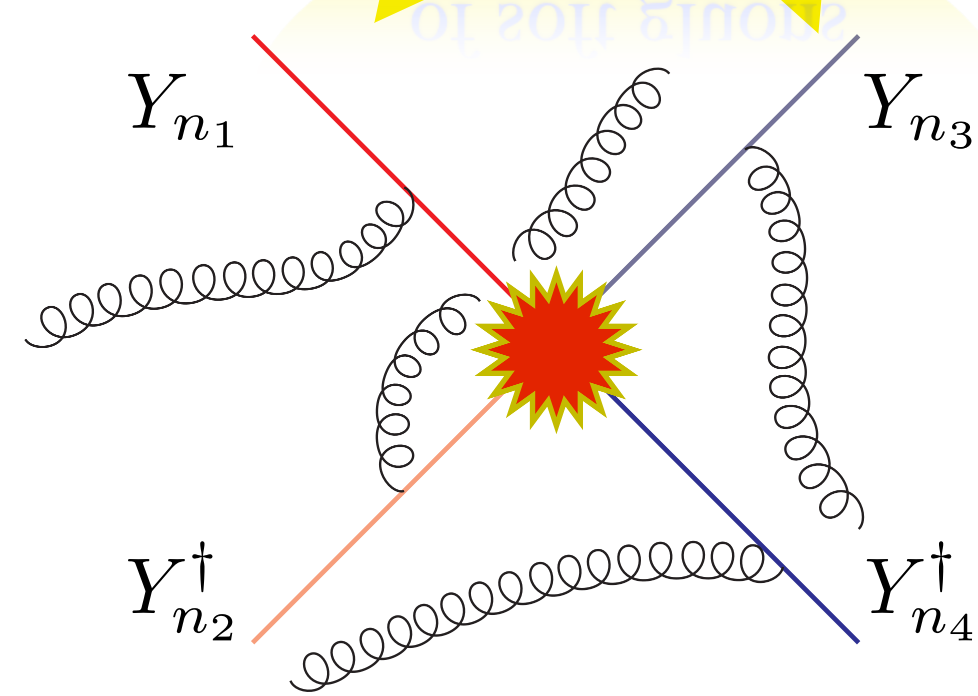


A Physical Approach

Focus on the physical quantities

- Analyze tree-level matrix elements
- Power count physical quantities
- Derive the formulation of Freedman & Luke

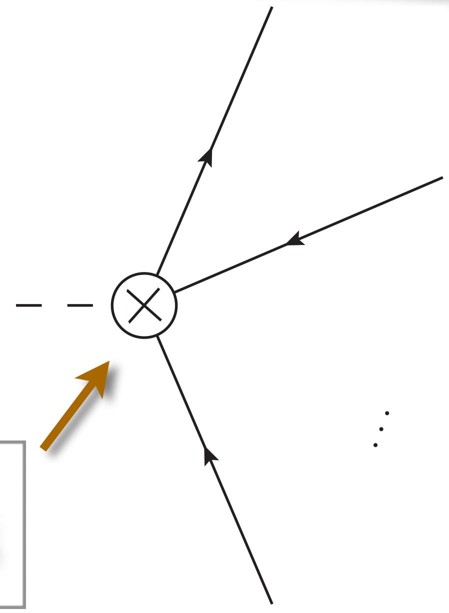
$$\mathcal{L}_{\text{SCET}} = \sum_j \mathcal{L}^j_{QCD} + \mathcal{L}^{\text{soft}}_{QCD}$$

This work could be thought of as a prequel to their paper

Soft Factorization

We want to separate :

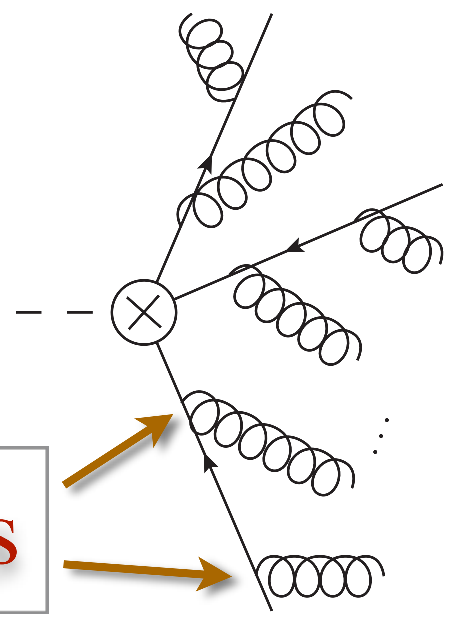
$$\langle p_1 \dots p_m | \, \bar{\psi}_1 \dots \psi_m \, |0\rangle =$$



Hard Process

From:

$$\langle p_1 \dots p_m ; k_1 \dots k_\ell | \, \bar{\psi}_1 \dots \psi_m \, |0\rangle =$$



Soft Processes

$k_\ell \sim \lambda^2 Q \ll p_i$

A result from QED

$$\langle p_1 \dots p_m ; k_1 \dots k_\ell | \, \bar{\psi}_1 \dots \psi_m \, | 0 \rangle$$

$$k_\ell \sim \lambda^2 Q \ll p_i$$

$$\prod_{j=1}^{\ell} \left(\sum_{i=1}^{m} e \eta_i \frac{n_i \cdot \epsilon_{k_j}}{n_i \cdot k_j} \right) \langle p_1 \dots p_m | \, \bar{\psi}_1 \dots \psi_m \, | 0 \rangle$$

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A result from SCET

$$Y_n^\dagger(x) = P \exp\left(-\, ie \int_0^\infty ds\, n \cdot A(x^\mu + s\, n^\mu)\, e^{-\epsilon s} \right)$$

$$\langle k_1 \ldots k_\ell |\, Y_n^\dagger(0)\, |0\rangle = \prod_{j=1}^{\ell} \left(e \frac{n \cdot \epsilon_{k_j}}{n \cdot k_j} \right)$$

A result from QED

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$$= \langle p_1 \ldots p_m; k_1 \ldots k_\ell | \, \left(\bar{\psi}_1 Y^\dagger_{n_1} \right) \ldots \left(Y_{n_m} \psi_m \right) \, |0\rangle_{\mathcal{L}_{\mathrm{SET}}}$$

Where

$$\mathcal{L}_{\mathrm{SET}} = \mathcal{L}^{QED}_{\text{not-soft}}$$

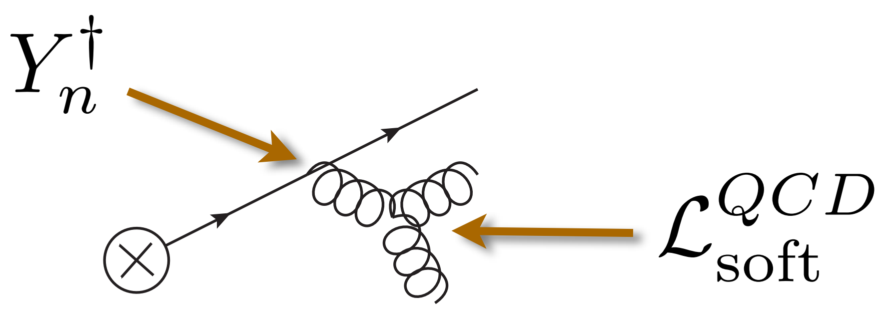
Soft Factorization

$$\langle p_1 \ldots p_m; k_1 \ldots k_\ell |\, \bar{\psi}_1 \ldots \psi_m \,|0\rangle$$

$k_\ell \sim \lambda^2 Q \ll p_i$

$$\langle k_1 \ldots k_\ell |\, Y_{n_1}^\dagger \ldots Y_{n_m} \,|0\rangle \, \langle p_1 \ldots p_m |\, \bar{\psi}_1 \ldots \psi_m \,|0\rangle$$

$$= \langle p_1 \ldots p_m; k_1 \ldots k_\ell |\, (\bar{\psi}_1 Y_{n_1}^\dagger) \ldots (Y_{n_m} \psi_m) \,|0\rangle_{\mathcal{L}_{\mathrm{SET}}}$$



$$\mathcal{L}_{\mathrm{SET}} = \mathcal{L}_{\text{not-soft}}^{QCD} + \mathcal{L}_{\mathrm{soft}}^{QCD}$$

Collinear Factorization

Now, we want to separate :

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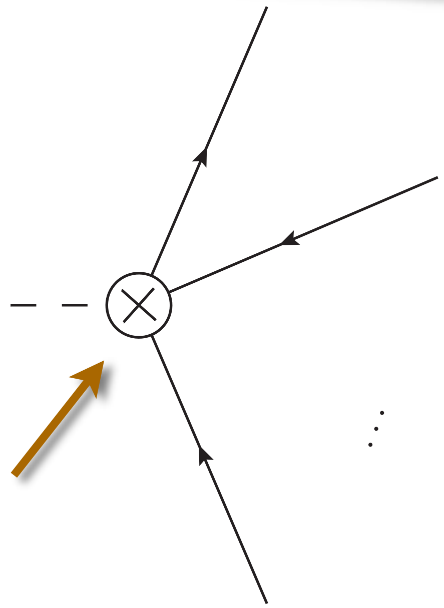

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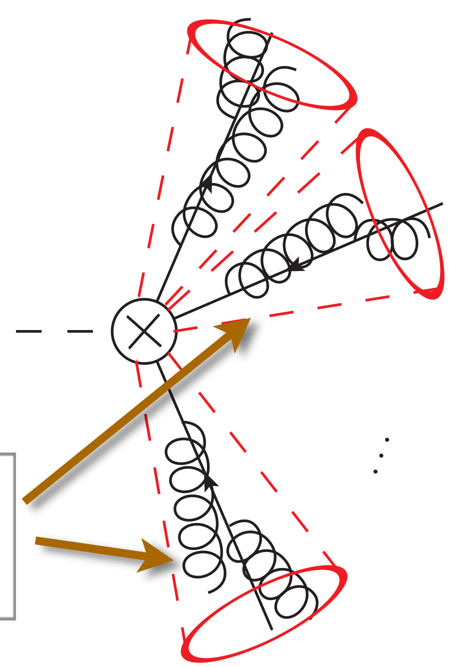

Collinear Processes

each $k_\ell \sim$ some p_i

Spinor-Helicity Formalism

Massless four-vectors: $p^\mu \;\to\; p\rangle[p\,, \quad 2p\cdot k = [p\,k]\langle k\,p\rangle$

$$\epsilon_k^-(r) \;\to\; \sqrt{2}\,\frac{k]\langle r}{\langle k\,r\rangle}\,, \qquad \epsilon_k^+(r) \;\to\; \sqrt{2}\,\frac{r]\langle k}{[r\,k]}$$

Reference-vector independence = Ward identity:

$$\epsilon_k^+(s) = \epsilon_k^+(r) + \sqrt{2}\frac{[sr]}{[kr][sk]}k\rangle[k$$

Power Counting: for $k^\mu \sim p^\mu = E\,n^\mu$

$$Q^2\lambda^2 \sim 2p\cdot k = [p\,k]\langle k\,p\rangle \;\Longrightarrow\; [p\,k] \sim \langle k\,p\rangle \sim Q\lambda$$

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for $r^\mu = \bar{n}^\mu$ $\quad \epsilon_k^+(r=\bar{n}) = \dfrac{1}{\sqrt{2}}\,(0,1,i,0)^\mu + \mathcal{O}(\lambda)$

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Physical polarizations

Spinor-Helicity Formalism

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Physical polarizations

for $r^\mu = n^\mu$ $\quad \epsilon_k^+(r=n) = \epsilon_k^+(r=\bar{n}) + \sqrt{2}\,\dfrac{[n\bar{n}]}{[k\bar{n}][nk]}\,k\rangle[k$

Spinor-Helicity Formalism

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Physical polarizations

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Physical polarizations

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Polarizations consistent with SCET power counting

Spinor-Helicity Formalism

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Physical polarizations

Not enhanced

for $r^\mu = n^\mu$ $\quad \epsilon_k^+(r=n) = \epsilon_k^+(r=\bar{n}) + \mathcal{O}\left(\frac{1}{\lambda}\right) k^\mu$

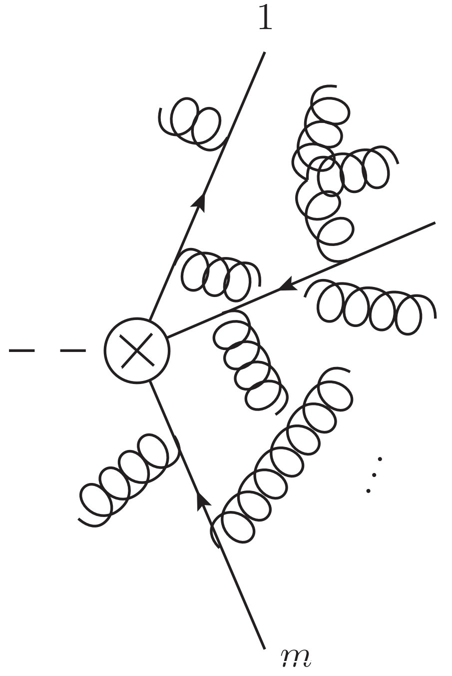
$$\sim \frac{1}{\lambda}(1,\lambda^2,\lambda) + \mathcal{O}(\lambda) \sim \frac{1}{\lambda}p^\mu + \mathcal{O}(\lambda)$$

Polarizations consistent with SCET power counting

Enhanced

Generically,

$$\langle p_1 \ldots p_m; k_1 \ldots k_\ell |\, \bar{\psi}_1 \ldots \psi_m \,|0\rangle = \sum$$



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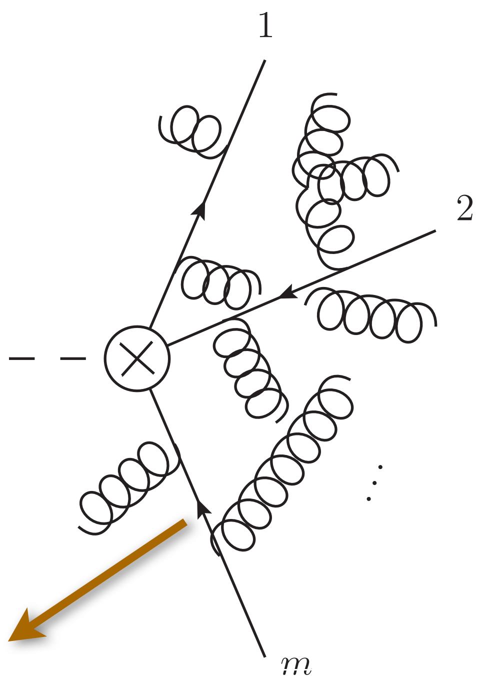

$$\ldots \frac{-i(\not{p} + \not{k})}{2p \cdot k} ig \not{\epsilon}_k v_p = \ldots g \left(\frac{p \cdot \epsilon_k}{p \cdot k} + \frac{\not{k}\not{\epsilon}_k}{2p \cdot k} \right) v_p$$

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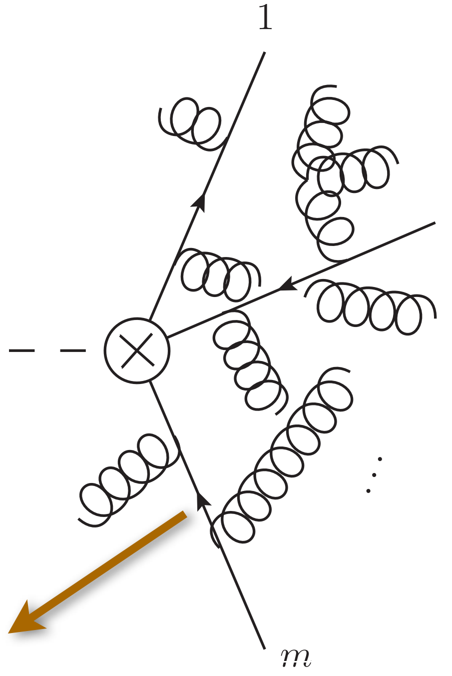

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$$\frac{\langle rp \rangle}{\langle kp \rangle \langle kr \rangle}$$

$$\sim \frac{1}{\langle kp \rangle} \quad \text{or} \quad 0$$

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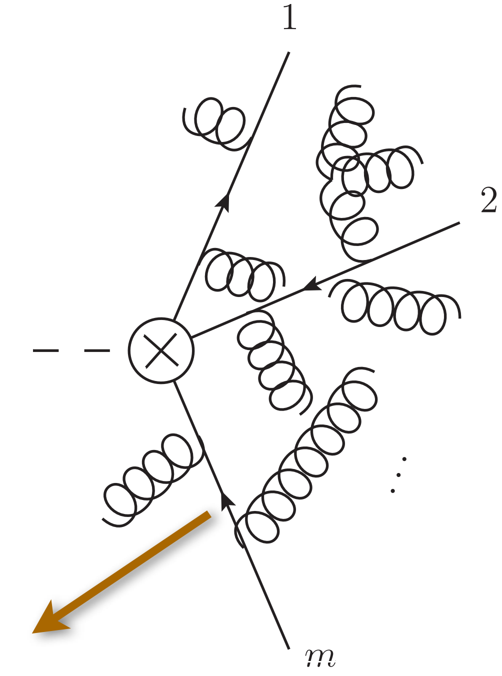

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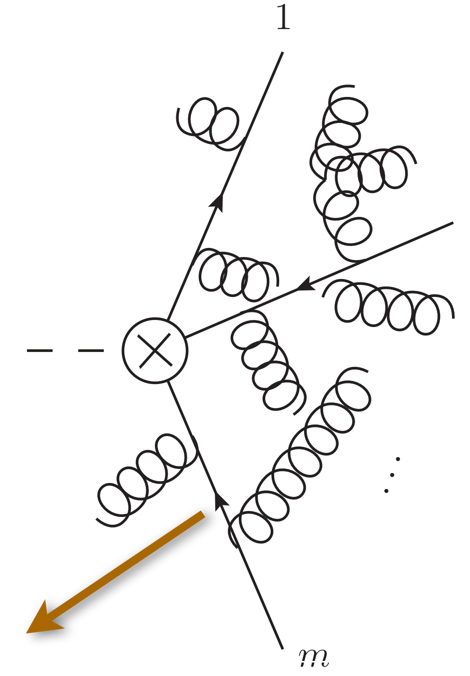
$$\sim \frac{1}{\langle kp \rangle} \quad \text{or} \quad 0$$

Independent of r and only enhanced when $k \sim p$

Generically,

$$\langle p_1 \dots p_m; k_1 \dots k_\ell |\, \bar\psi_1 \dots \psi_m \,|0\rangle = \sum$$



$$\dots \frac{-i(\not p + \not k)}{2p \cdot k} ig \not\epsilon_k v_p = \dots g\left(\frac{p \cdot \epsilon_k}{p \cdot k} + \frac{\not k \not\epsilon_k}{2p \cdot k}\right) v_p$$

$= 0$ for $r = p$

$$\frac{\langle rp\rangle}{\langle kp\rangle\langle kr\rangle}$$

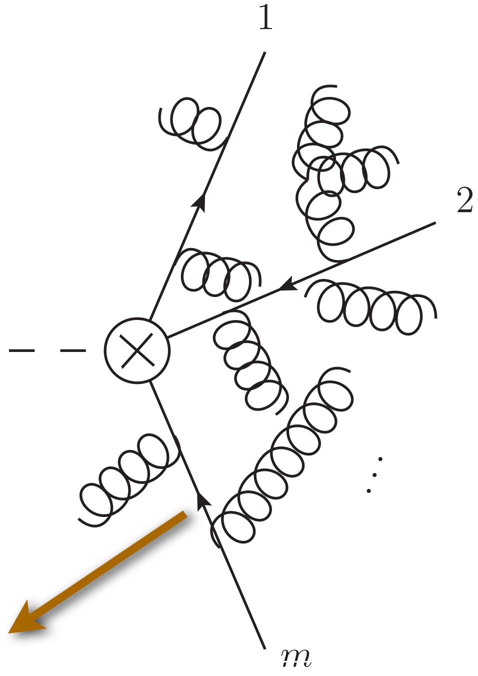
$\sim \dfrac{1}{\langle kp\rangle}$ or 0

for $r \not\propto p_j \ \forall\, j$:
only enhanced
when $k \sim p$

Independent of r
and only enhanced
when $k \sim p$

Generically,

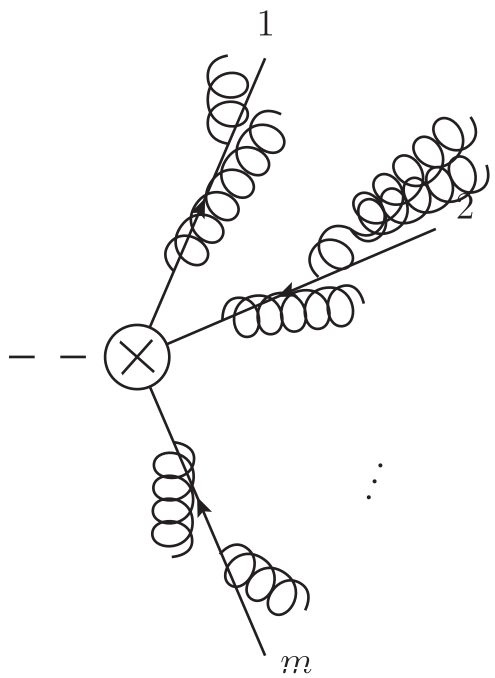
$$\langle p_1 \ldots p_m; k_1 \ldots k_\ell | \, \bar{\psi}_1 \ldots \psi_m \, |0\rangle = \sum$$



$$\ldots \frac{-i(\not{p} + \not{k})}{2p \cdot k} ig \not{\epsilon}_k v_p = \ldots g \left(\frac{p \cdot \epsilon_k}{p \cdot k} + \frac{\not{k} \not{\epsilon}_k}{2p \cdot k} \right) v_p$$

Therefore, for $r \not\sim p_j \;\; \forall \, j$:

$$\langle p_1 \ldots p_m; k_1 \ldots k_\ell | \, \bar{\psi}_1 \ldots \psi_m \, |0\rangle = \sum$$



$$\sim \left(\frac{g}{\langle kp \rangle} \right)^\ell = \left(\frac{g}{\lambda} \right)^\ell \quad \text{and} \quad \epsilon_k^\mu \sim \mathcal{O}(1)$$

$$= \langle p_1; k_1 \ldots k_{\ell_1} | \, \bar{\psi}_1 \, |0\rangle_{\mathcal{L}^1_{QCD}} \ldots \langle p_m; k_{\ell_{m-1}} \ldots k_\ell | \, \psi_m \, |0\rangle_{\mathcal{L}^m_{QCD}}$$

For $r \not\sim p_j \;\; \forall\, j$:

$$\langle p_1 \ldots p_m; k_1 \ldots k_\ell |\, \bar{\psi}_1 \ldots \psi_m \,|0\rangle$$

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But now, each $\mathcal{L}^j_{QCD}$ has a different quantum number, j

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Gauge Invariance $\Rightarrow$

$$\langle p_1 \ldots p_m; k_1 \ldots k_\ell |\, \bar{\psi}_1 \ldots \psi_m \,|0\rangle$$

$$= \langle p_1; k_1 \ldots k_{\ell_1} |\, \bar{\psi}_1 W_1 \,|0\rangle_{\mathcal{L}^1_{QCD}}$$

$$\times \cdots \times \langle p_m; k_{\ell_{m-1}} \ldots k_\ell |\, W^\dagger_m \psi_m \,|0\rangle_{\mathcal{L}^m_{QCD}}$$

where

$$W_j(x) = P \exp\left(ig \int_{-\infty}^{x} dx_\mu A^\mu \right)$$

$\langle k \ldots | \, W_j \, |0\rangle \sim \mathcal{O}(1)$

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Gauge Invariance

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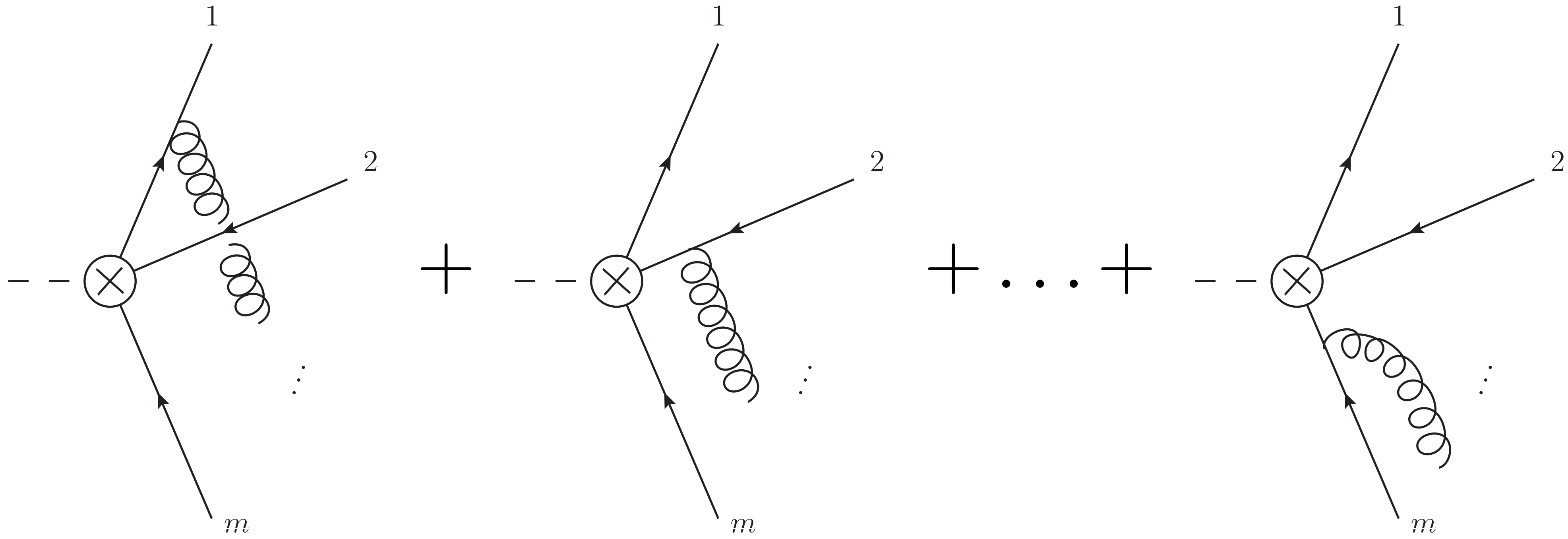
$$\times \cdots \times \langle p_m; k_{\ell_{m-1}} \ldots k_\ell |\, W_m^\dagger \psi_m \,|0\rangle_{\mathcal{L}^m_{QCD}}$$

where

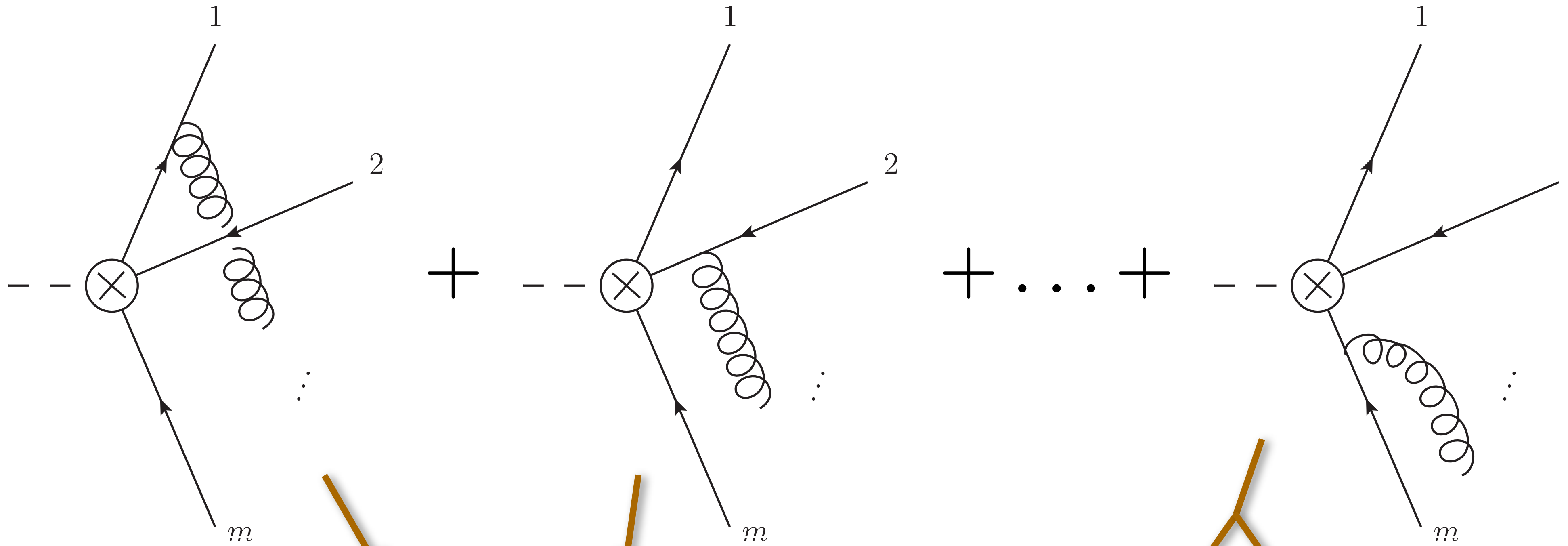
$$W_j(x) = P \exp\left(ig \int_{-\infty}^{x} dx_\mu A^\mu \right)$$

We cannot deduce the path this way

Example:



Example:

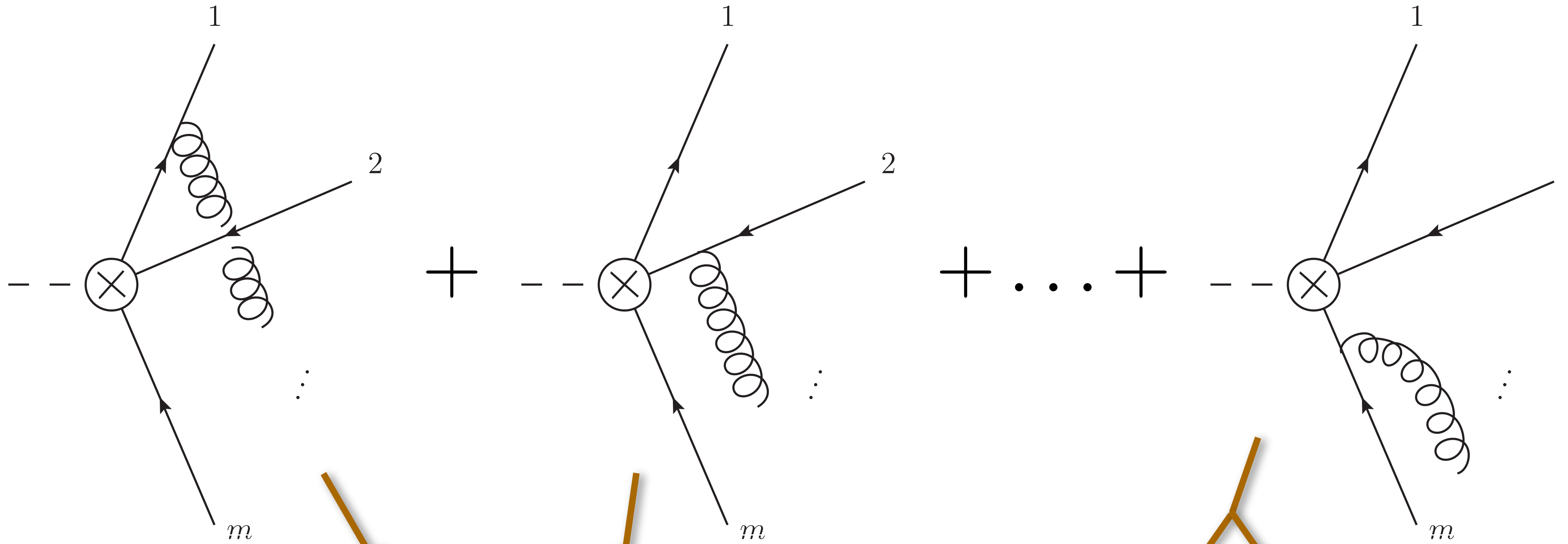


$\not{k}\not{\epsilon}_k \sim 1$

for $j \neq m$

$$\left(\sum_{j<m} \eta_j \frac{p_j \cdot \epsilon_k}{p_j \cdot k}\right) + \frac{p_m \cdot \epsilon_k}{p_m \cdot k} + \frac{\not{k}\not{\epsilon}_k}{2p_m \cdot k}$$

Example:



$$\not{k}\not{\epsilon}_k \sim 1$$

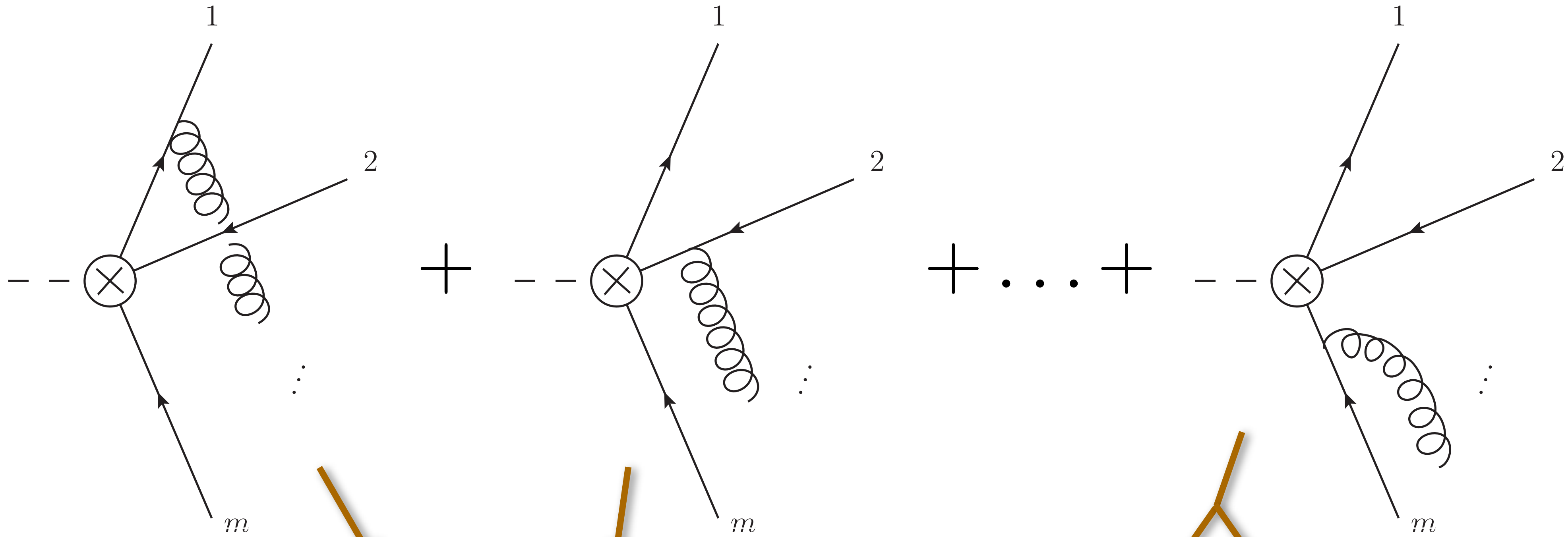
for $j \neq m$

$$\frac{p_j \cdot \epsilon_k}{p_j \cdot k} \approx \frac{\bar{p}_m \cdot \epsilon_k}{\bar{p}_m \cdot k}$$

$$\left(\sum_{j<m} \eta_j \frac{p_j \cdot \epsilon_k}{p_j \cdot k} \right) + \frac{p_m \cdot \epsilon_k}{p_m \cdot k} + \frac{\not{k}\not{\epsilon}_k}{2p_m \cdot k}$$

$$\frac{\bar{p}_m \cdot \epsilon_k}{\bar{p}_m \cdot k} \left(\sum_{j<m} \eta_j \right) + \frac{p_m \cdot \epsilon_k}{p_m \cdot k} + \frac{\not{k}\not{\epsilon}_k}{2p_m \cdot k}$$

Example:



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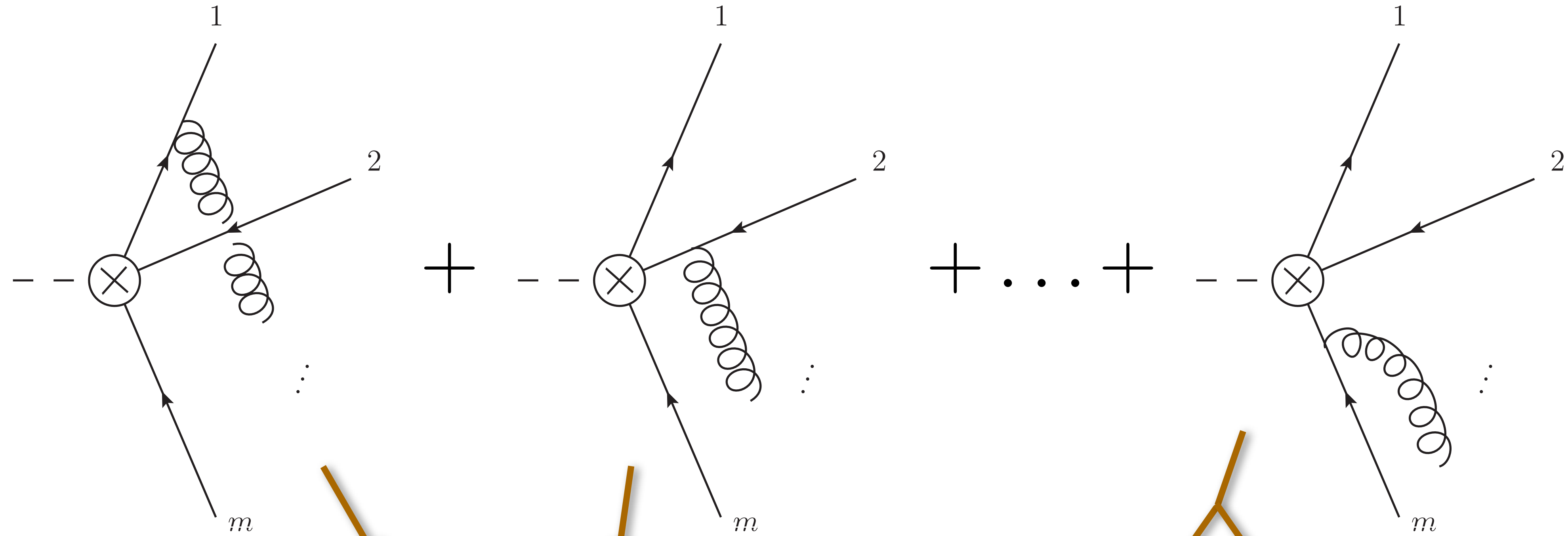
$$\frac{\bar{p}_m \cdot \epsilon_k}{\bar{p}_m \cdot k} \left(\sum_{j<m} \eta_j \right) + \frac{p_m \cdot \epsilon_k}{p_m \cdot k} + \frac{\not{k}\not{\epsilon}_k}{2p_m \cdot k}$$

Example:



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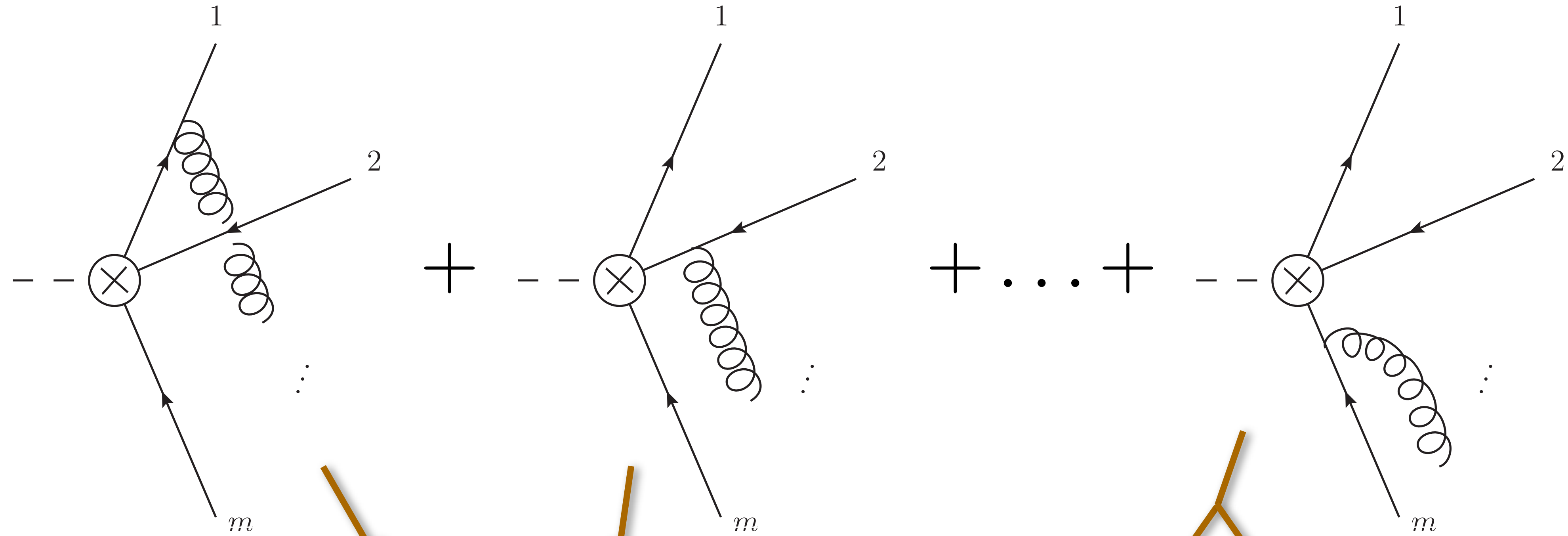
$$\frac{\bar{p}_m \cdot \epsilon_k}{\bar{p}_m \cdot k} \left(-\eta_m \right) + \frac{p_m \cdot \epsilon_k}{p_m \cdot k} + \frac{\not{k}\not{\epsilon}_k}{2p_m \cdot k}$$

Example:



$$\not{k}\not{\epsilon}_k \sim 1$$

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$$-\frac{\bar{p}_m \cdot \epsilon_k}{\bar{p}_m \cdot k} + \frac{p_m \cdot \epsilon_k}{p_m \cdot k} + \frac{\not{k}\not{\epsilon}_k}{2p_m \cdot k}$$

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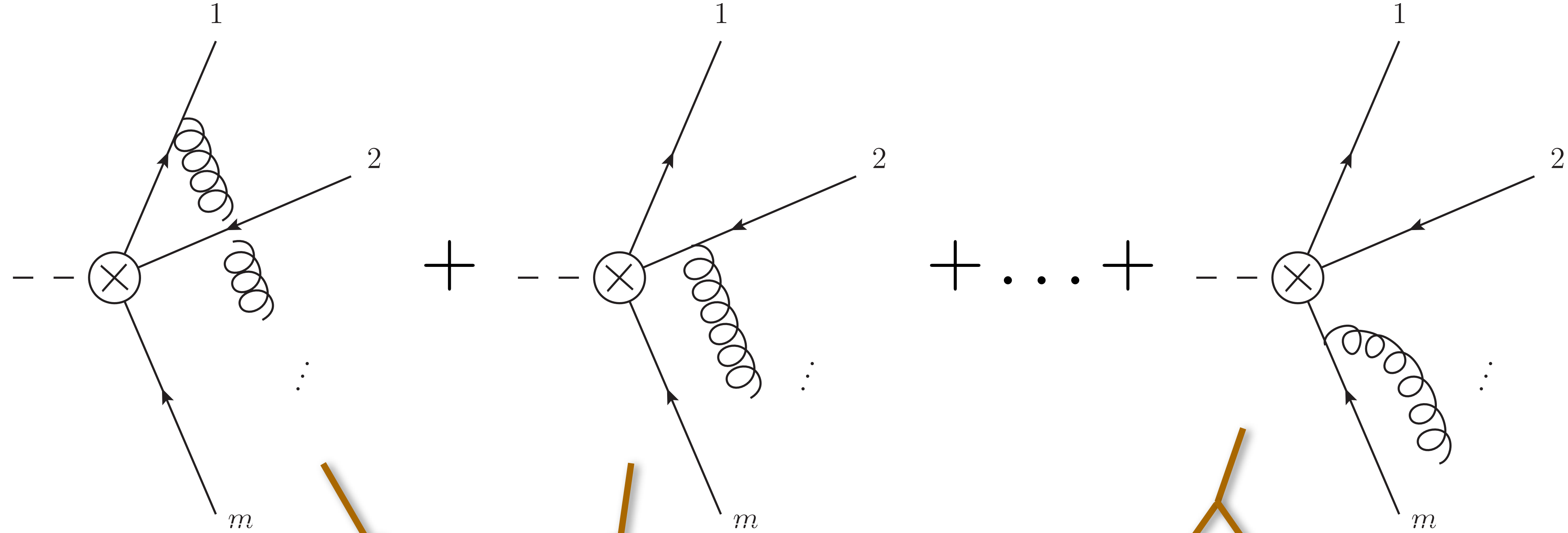

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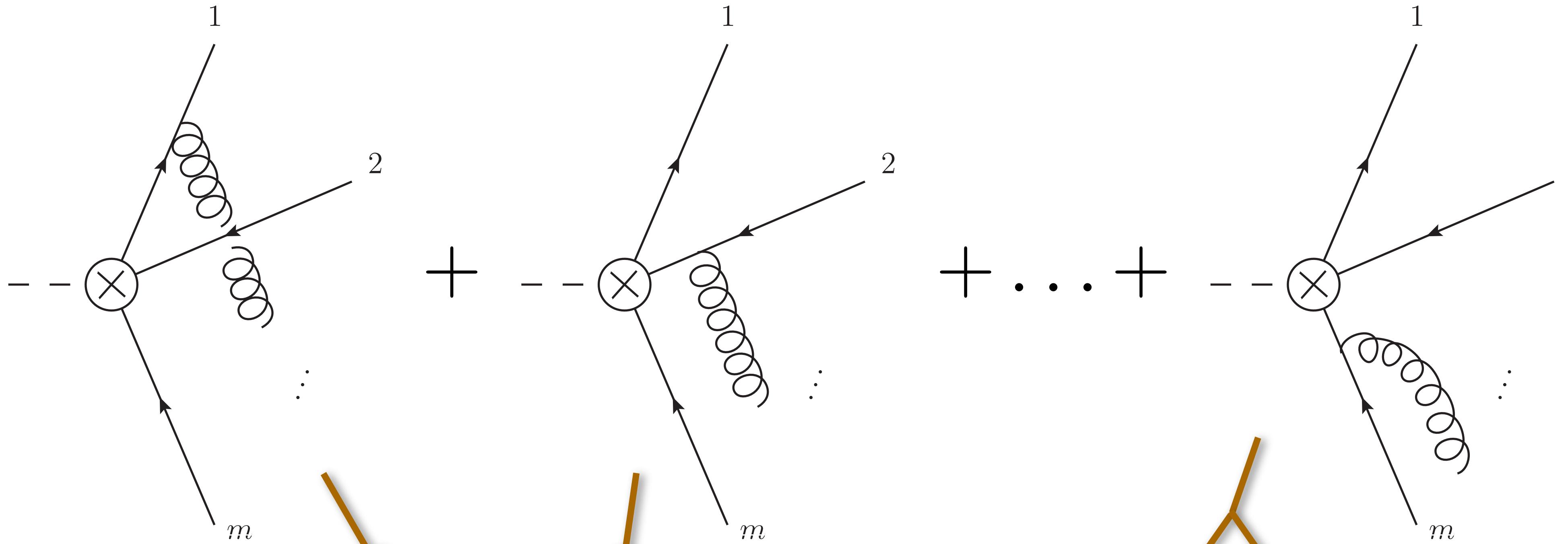
$$= \langle p_1 \ldots p_m ; k_m | \, \bar{\psi}_1 \ldots (W_m^\dagger \psi_m) \, |0\rangle$$

where

$$W_m^\dagger(x) = P \exp\left(ig \int_0^\infty ds \, \bar{p}_m \cdot A(x + \bar{p}_m \, s) \right)$$

Example:



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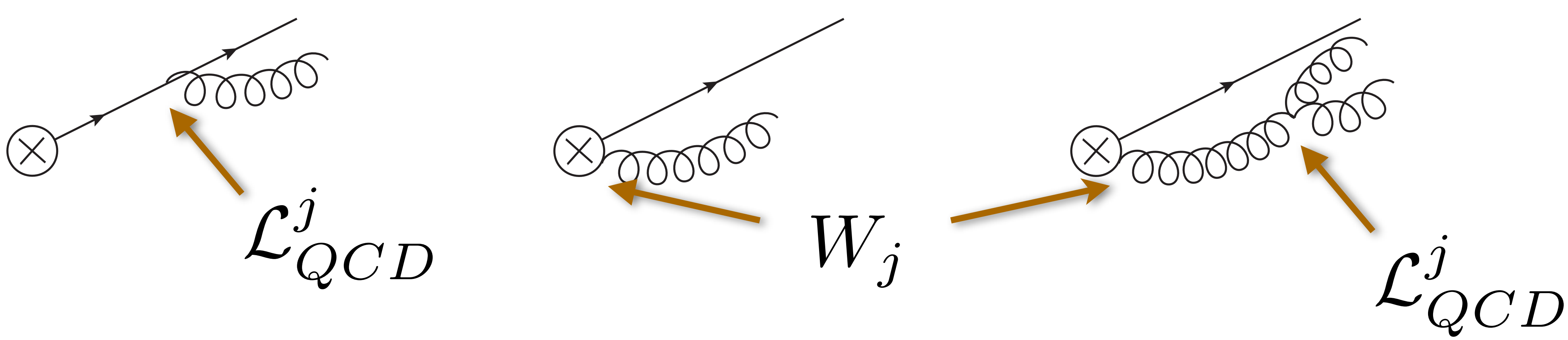
Collinear Factorization

$$\langle p_1 \dots p_m; k_1 \dots k_\ell | \, \bar{\psi}_1 \dots \psi_m \, |0\rangle$$

each $k_\ell \sim$ some p_i

$$\langle p_1; k_1 \dots k_{\ell_1} | \, \bar{\psi}_1 W_1 \, |0\rangle_{\mathcal{L}^1_{QCD}} \dots \langle p_m; k_{\ell_{m-1}} \dots k_\ell | \, W^\dagger_m \psi_m \, |0\rangle_{\mathcal{L}^m_{QCD}}$$

$$= \langle p_1 \dots p_m; k_1 \dots k_\ell | \, (\bar{\psi}_1 W_1) \dots (W^\dagger_m \psi_m) \, |0\rangle_{\mathcal{L}_{\rm CET}}$$



$$\mathcal{L}_{\rm CET} = \sum_{j=1}^{m} \mathcal{L}^j_{QCD}$$

A Physical Approach

For tree-level amplitudes

$$\langle X_{n_1}, \ldots, X_{n_m}; X_s | \, \bar{\psi}_1 \ldots \psi_m \, |0\rangle$$

at leading power

$$\langle X_{n_1}, \ldots, X_{n_m}; X_s | \, (\bar{\psi}_1 W_1 Y_1^\dagger) \ldots (Y_m W_m^\dagger \psi_m) \, |0\rangle_{\mathcal{L}_{\rm SCET}}$$

$$\mathcal{L}_{\rm SCET} = \sum_j \mathcal{L}_{QCD}^j + \mathcal{L}_{QCD}^{\rm soft}$$

Conclusions

Analyzed tree-level amplitudes with soft/collinear enhancements in QCD

- Spinor-helicity methods offered insight into diagrammatic power counting
- Reference-vector independence played a key role in tree-level factorization
- Derived the formulation of Freedman & Luke

$$\mathcal{L}_{\rm SCET} = \sum_j \mathcal{L}^j_{QCD} + \mathcal{L}^{\rm soft}_{QCD}$$

Thank you!

Spinor-Helicity Formalism:

$$p_{\dot\alpha\alpha} = \bar\sigma^{\mu}_{\dot\alpha\alpha}\, p_\mu \quad \text{and} \quad p^\mu = \frac{1}{2}\sigma^{\mu\,\alpha\dot\alpha}\, p_{\dot\alpha\alpha}$$

$$2p\cdot q = \sigma^{\mu\,\alpha\dot\alpha}\, p_{\dot\alpha\alpha}\, \sigma_\mu^{\beta\dot\beta}\, q_{\dot\beta\beta} = \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\, p_{\dot\alpha\alpha}\, q_{\dot\beta\beta} = p_{\dot\alpha\alpha}\, q^{\alpha\dot\alpha}$$

$$m^2 = 0 = \det p \implies p^{\alpha\dot\alpha} = \lambda^\alpha\tilde\lambda^{\dot\alpha} = p]\langle p \;\;\&\;\; p_{\dot\alpha\alpha} = \tilde\lambda_{\dot\alpha}\lambda_\alpha = p\rangle[p$$

$$2p\cdot q = p_{\dot\alpha\alpha}\, q^{\alpha\dot\alpha} = \tilde\lambda_{\dot\alpha}\lambda_\alpha\chi^\alpha\tilde\chi^{\dot\alpha} = [p\, q]\langle q\, p\rangle$$

$$\big(\epsilon_k^-(r)\big)^{\alpha\dot\alpha} \equiv \sqrt{2}\,\frac{k]\langle r}{\langle k\, r\rangle} \quad \text{and} \quad \big(\epsilon_k^+(r)\big)^{\alpha\dot\alpha} \equiv \sqrt{2}\,\frac{r]\langle k}{[r\, k]}$$