Phenomenology of TMD's

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In collaboration with M. García Echevarría, A. Idilbi, **(EIS)** A. Schaefer, arXive:1208.1281, and work in progress

Some questions …and our answers

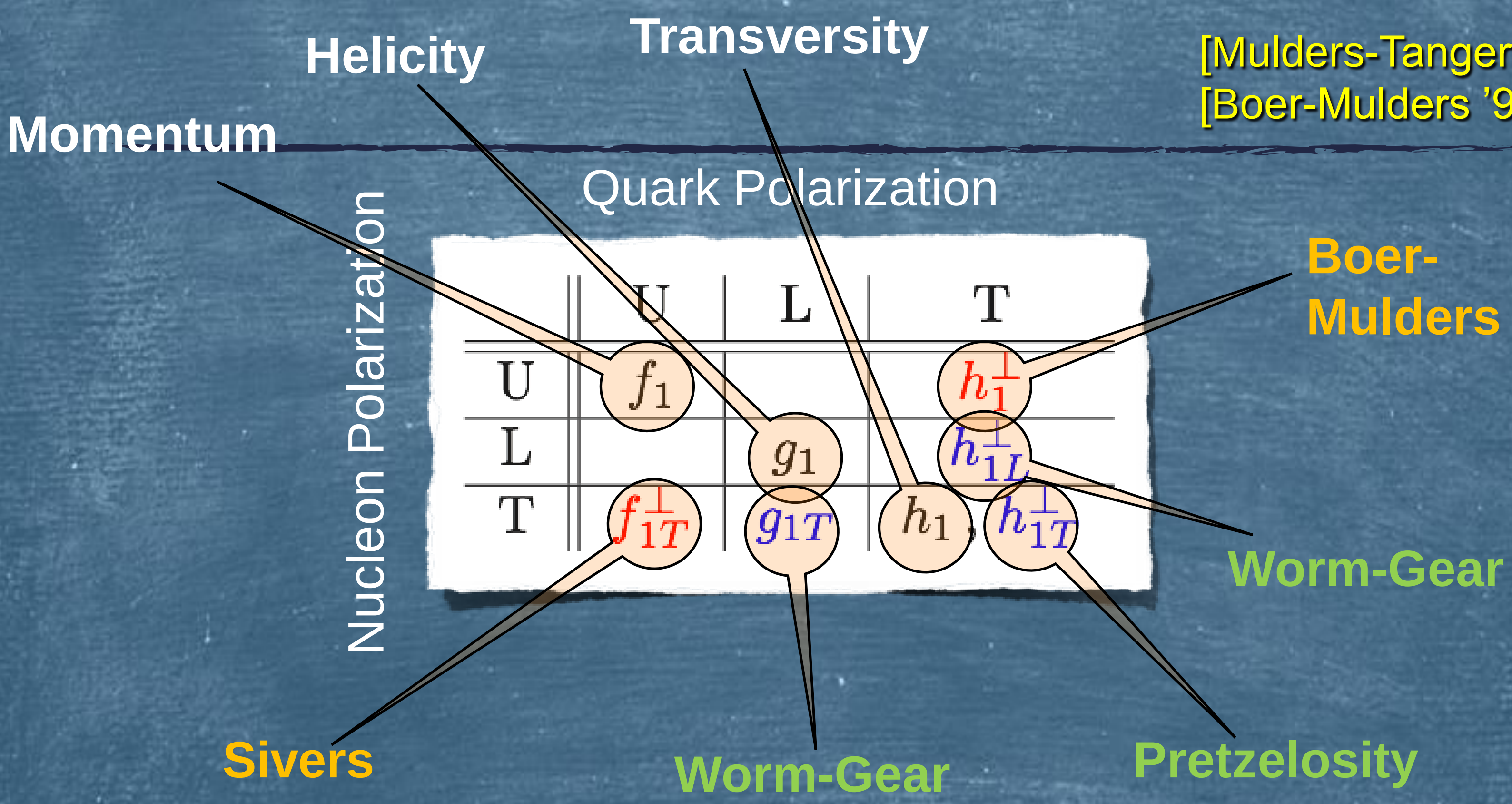
- Transverse Momentum distributions are fundamental in the factorization of DY at small qT and SIDIS and e+e- to 2j
- Can we formulate their definition independently of the IR/collinear regulators that we use? YES (Ahmad's talk)
- Are TMDs universal? See discussion
- How do we write the evolution of TMDs? Up to which order do we know their evolution?

We can up to NNLL..we could up NNNLL in some cases

- Is the evolution of all quark TMDs the same?YES
- Can we have a model independent evolution of the TMDs?YES, no effective strong coupling is necessary

TMDPDFs at Leading Twist

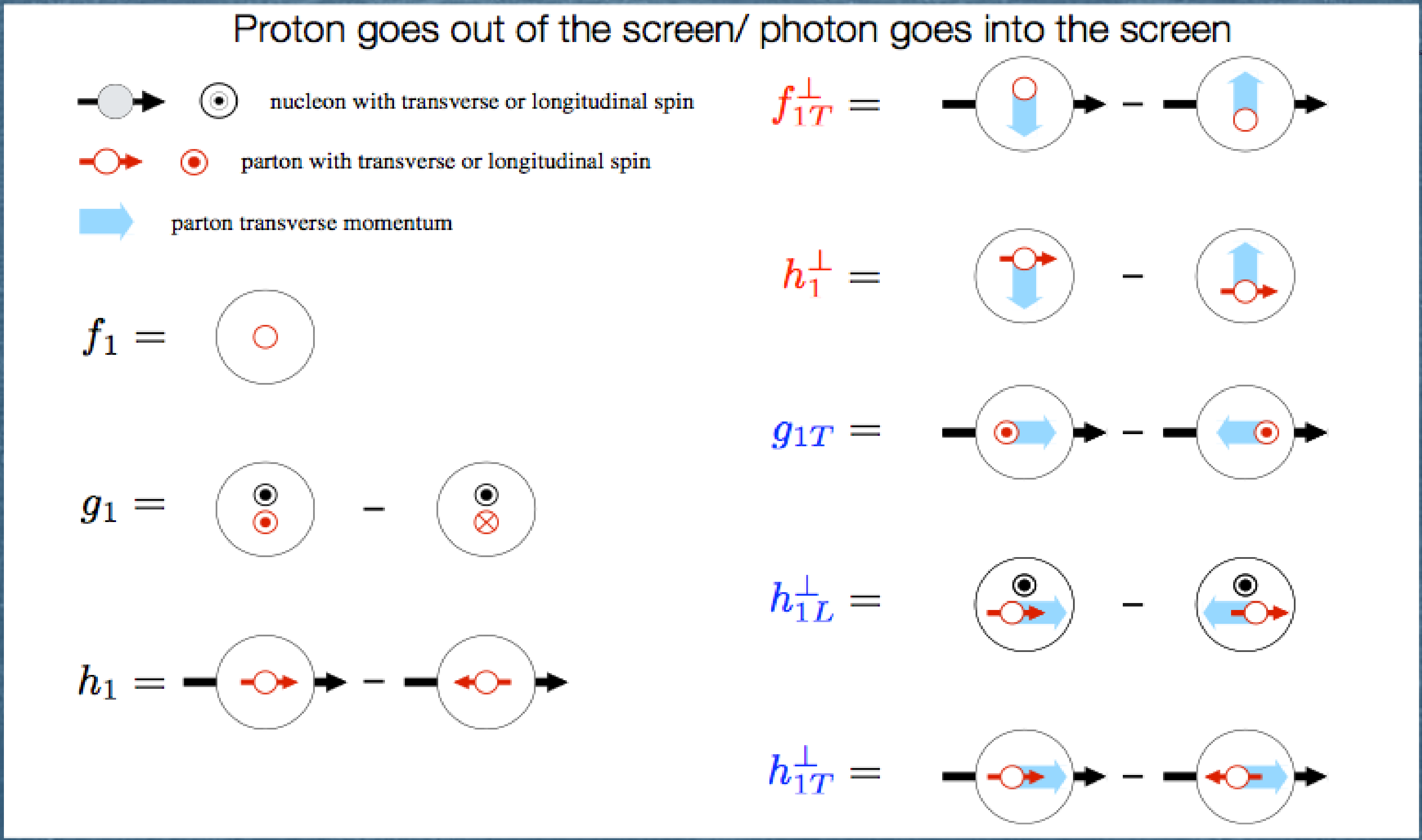
[Mulders-Tangerman'96]
[Boer-Mulders '98]



- **The only ones that survive in the collinear limit (when we integrate over qT)** • **They are T-odd**

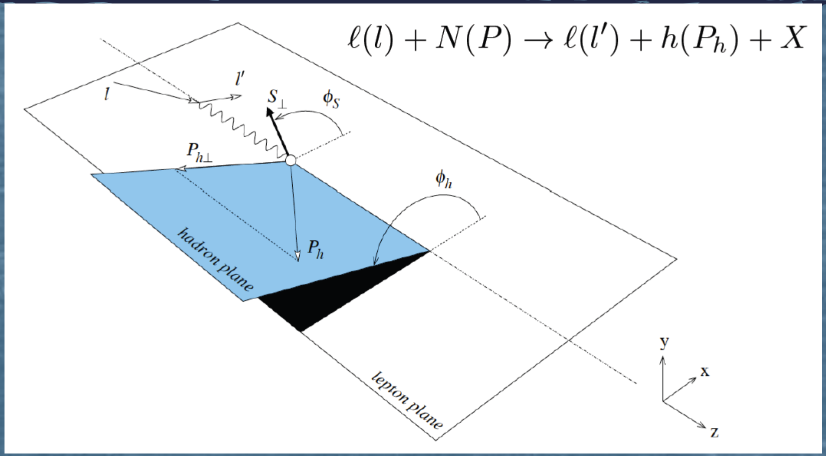
- *There are similar families for gluon-TMDPDFs and quark/gluon-TMDFFs*
- *They are **distributions** that give us information about the inner structure of the nucleons*

Probabilistic Interpretation

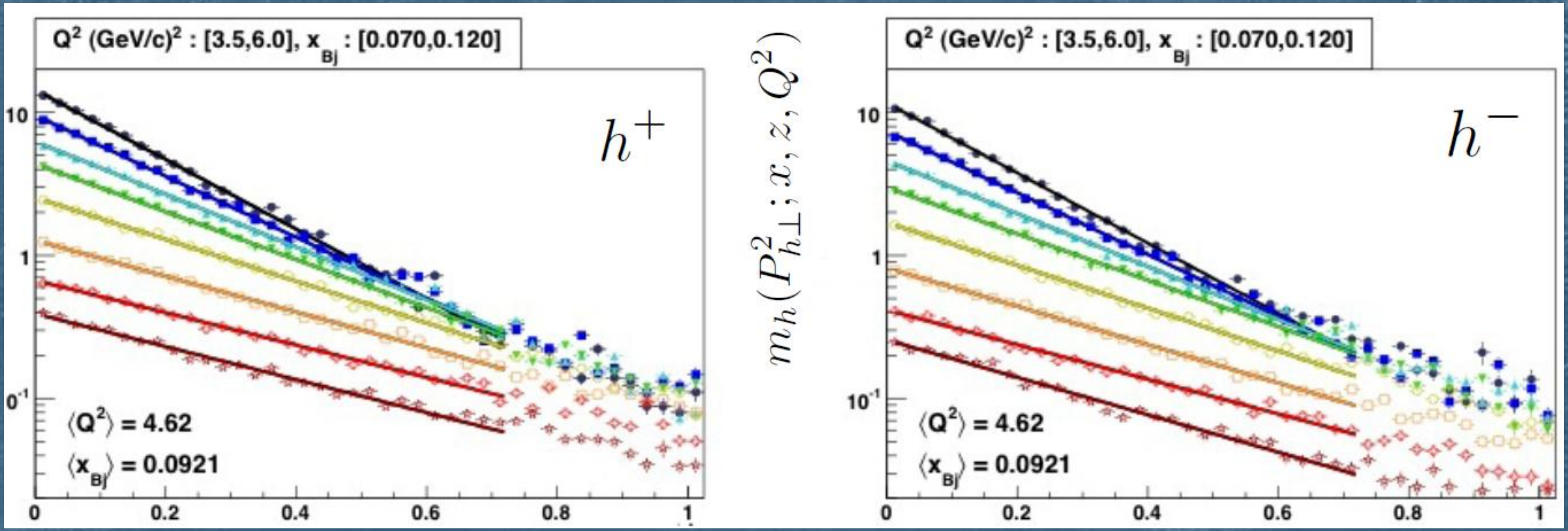


From a talk of A. Bacchetta

Preliminaries…

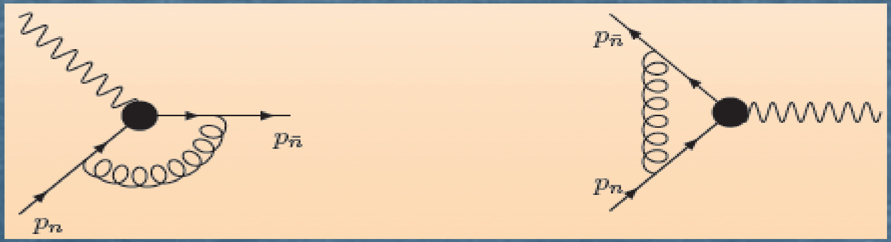


Diploma Thesis of A. Signori, 2012
Compass Coll. Cern
Preliminary data



Universality of the TMD's

- The extraction of all TMD's requires a contemporary analysis of DY, SIDIS, e+e- to 2j. Different experiments (Hermes, Jlab, EIC?, Compass, Tevatron, LHC, LEP, Belle, Babar,…), different energies
- The collinear and soft matrix element are the same in DY and SIDIS



- The definition of Wilson lines in DY and SIDIS is different

DY

$$W_n(x) = \bar{P}\exp\left[ig\int_{-\infty}^{0} ds\bar{n}\cdot A_n(x+s\bar{n})\right]$$

$$S_n(x) = P\exp\left[ig\int_{-\infty}^{0} ds\bar{n}\cdot A_s(x+s\bar{n})\right]$$

SIDIS

$$W_n(x) = \bar{P}\exp\left[-ig\int_{-\infty}^{0} ds\bar{n}\cdot A_n(x+s\bar{n})\right]$$

$$S_n(x) = P\exp\left[-ig\int_{-\infty}^{0} ds\bar{n}\cdot A_s(x+s\bar{n})\right]$$

Universality of the unpolarized TMDPDF at one loop

EIS, JHEP (2012)

- Universality of the Soft Function

$$S_1^{r,DIS} = S_1^{r,DY} + \frac{\alpha_s C_F}{2\pi}\delta^{(2)}(\vec{k}_T)\pi^2; \qquad S_1^{v,DIS} = S_1^{v,DY} - \frac{\alpha_s C_F}{2\pi}\delta^{(2)}(\vec{k}_T)\pi^2$$

- Universality of the Collinear Function

$$j_{n1}^{r,DIS} = j_{n1}^{r,DY} - \frac{\alpha_s C_F}{2\pi}\delta^{(2)}(\vec{k}_T)\pi^2; \qquad j_{n1}^{vDIS} = j_{n1}^{v,DY} + \frac{\alpha_s C_F}{2\pi}\delta^{(2)}(\vec{k}_T)\pi^2$$

Both Naive Collinear And Soft ME Are Universal!

Ergo, <u>the unpolarized TMDPDF is Universal</u>

Universality of the Sivers functions

- For the Sivers functions the universality is peculiar.

$$f_{1T}^{\perp,SIDIS} = -f_{1T}^{\perp,DY}$$

Evolution of the TMDPDF

- The hadronic tensor is RG scale independent

$$\tilde{M} = H(Q^2/\mu^2) F_n(x;\vec{b}_\perp, Q, \mu) F_{\bar{n}}(z;\vec{b}_\perp, Q, \mu)$$

$$\frac{d \ln \tilde{M}}{d \ln \mu} = 0 = \gamma_H + \gamma_n + \gamma_{\bar{n}} = \gamma_H + 2\gamma_{\bar{n}} = \gamma_H + 2\gamma_n$$

$$\gamma_H = A(\alpha_s) \ln \frac{Q^2}{\mu^2} + B(\alpha_s); \quad F_n(x;\vec{b}_\perp, Q, \mu) = \exp\left[\int_{\mu_I}^{\mu} \frac{d\mu'}{\mu'} \gamma_n\right] F_n(x;\vec{b}_\perp, Q, \mu_I)$$

$$H(Q^2/\mu^2) = |C(Q^2/\mu^2)|^2$$

Comes from the matching of currents: It is spin independent

The hard coefficient is the same as for inclusive DY! *Ergo,*

WE KNOW THE AD of the 8 TMDPDF up to 3-LOOPS

OPE of the TMDPDF on to the PDF

- When qT is in the perturbative region the TMDPDF can be factorized in a Wilson coefficient and a PDF like in OPE

$$F_f(x;\vec{b}_\perp, Q, \mu) = \sum_{j=q,g} \int_x^1 \frac{dx'}{x'} \tilde{C}_{f/j}\left(\frac{x}{x'}; b, Q, \mu\right) f_{j/P}(x';\mu)$$

The coefficient C works as any other Wilson coefficient
IT IS INDEPENDENT OF IR-SCALES

BUT THERE IS STILL A Q^2 DEPENDENCE

$$\tilde{C}_n(x;b,Q,\mu) = \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\left[-P_{q/q} L_T + (1-x) - \delta(1-x)\left(\frac{1}{2}L_T^2 - \frac{3}{2}L_T + \ln\frac{Q^2}{\mu^2}L_T + \frac{\pi^2}{12}\right)\right]$$

THESE TERMS HAVE TO BE RESUMMED!!

$$L_T = \ln\frac{\mu^2 b^2}{4e^{-2\gamma_E}}$$

Q^2-Resummation

- Using Lorentz invariance and dimensional analysis

$$\ln F_n = \ln j_n - \frac{1}{2}\ln S$$

$$\ln j_n = R_n\left(x;\alpha_s, L_T, \ln\frac{\Delta}{Q^2}\right), \qquad \ln S = R_\phi\left(\alpha_s, L_T, \ln\frac{\Delta^2}{Q^2\mu^2}\right)$$

Since the TMDPDF (Wilson coefficients and PDFs) is free from rapidity divergences to all orders in perturbation theory:

$$\frac{d}{d\ln\Delta}\ln F_n = 0$$

Q^2-Resummation

- From the fact that the TMDPDF is free from rapidity divergencies we can extract and exponentiate the Q^2-dependence.

- But we can also extract it just applying the RGE to the hadronic tensor:

$$\frac{d\ln\tilde{F}_n}{d\ln\mu} = -\frac{1}{2}\gamma_H = -\frac{1}{2}A(\alpha_s)\ln\frac{Q^2}{\mu^2} - \frac{1}{2}B(\alpha_s)$$

$$\ln\tilde{F}_n = \ln\tilde{F}_n^{\not Q} - D(\alpha_s, L_T)\ln\frac{Q^2}{\mu^2}$$

Independent of Q^2!!

$$\tilde{C}_{f/j}(x,b;Q^2,\mu) = \left(\frac{Q^2}{\mu^2}\right)^{-D(b;\mu)} \tilde{C}_{f/j}^{\not Q}(x,b;\mu)$$

$$\frac{dD(b;\mu)}{d\ln\mu} = \Gamma_{cusp}(\alpha_s)$$

$$A(\alpha_s) = 2\Gamma_{cusp}$$

- The Q^2-factor is extracted for each TMDPDF individually.
- We do not need Collins-Soper evolution equation to resum the logs of Q^2.
- We know cusp AD at 3-loops, **so we know D at 2-loops!!**

Q^2-Resummation

- The final form of the TMD in IPS is

$$\ln F_n = \ln F_n^{sub} - D(\alpha_s, L_T)\left(\ln\frac{Q^2}{\mu^2} + L_T\right)$$

$$F_n(x;\vec{b}_\perp, Q, \mu) = \left(\frac{Q^2 b^2 e^{2\gamma_E}}{4}\right)^{-D(\alpha_s, L_T)} C_n(x;\vec{b}_\perp, \mu) \otimes f_n(x;\mu)$$

$$\frac{dD(\alpha_s, L_T)}{d\ln\mu} = \Gamma_{\text{cusp}}(\alpha_s)$$

$$D(\alpha_s, L_T) = \sum_{n=1}^{\infty} d_n(L_T)\left(\frac{\alpha_s}{4\pi}\right)^n$$

$$d_n'(L_\perp) = \frac{1}{2}\Gamma_{n-1} + \sum_{m=1}^{n-1} m\beta_{n-1-m} d_m(L_\perp)$$

The cusp AD is known at 3-loops!!

→ The function D is known up to order α^2

Resumming!

$$F_{f/P}(x;\vec{b}_\perp,Q^2,\mu=Q)=\sum_{j=q,g}\exp\left[\int_{\mu_I}^{\mu}\frac{d\mu'}{\mu'}\gamma_n\right]\left(\frac{Q^2}{\mu^2}\right)^{-D(b,\mu_I)} C_{f/j}\left(x;\vec{b}_\perp,\mu_I\right)\otimes f_{j/P}(x;\mu_I)$$

Order	γ	Γcusp	C	D
LL	-	α	tree	-
NLL	α	α^2	tree	α
NNLL	α^2	α^3	α	α^2
NNNLL	α^3	α^4	α^2	α^3

Green: Aybat, Collins , Qiu, Rogers; Aybat, Rogers; Anselmino, Boglione,Melis

Yellow: Our Group

Blue: Known pieces for unpolarized TMDs from Catani et al. ' 12 And Gehrmann et al. '12

See Thomas Lübbert talk

The Evolution of all quark TMDs

- The hard matching coefficient H does not depend on spin! And its AD governs all evolution of the TMDs and also the evolution of the D-function! (EIS+S, ´12) even when the TMDs do not match on PDFs

$$F_{\alpha\beta}(x,\vec{k}_\perp)=\frac{1}{2}\int\frac{dr^- d^2\vec{r}_\perp}{(2\pi)^3}e^{-i(\frac{1}{2}r^- xP^+ - \vec{r}_\perp\cdot\vec{k}_\perp)}\Phi^q_{\alpha\beta}(0^+,r^-,\vec{r}_\perp)\sqrt{S(0^+,0^-,\vec{r}_\perp)}$$

$$\Phi^q_{\alpha\beta}(0^+,r^-,\vec{r}_\perp)=\langle P\vec{S}\,\|[\bar{\xi}_{n\alpha}W_n^T](0^+,y^-,\vec{y}_\perp)[W_n^{T\dagger}\xi_{n\beta}](0)|\,P\vec{S}\rangle$$

$$S=\langle 0|\,\mathrm{Tr}\left[S_n^{T\dagger}S_{\bar{n}}^T\right](0^+,0^-,\vec{y}_\perp)[S_{\bar{n}}^{T\dagger}S_n^T](0)\,|\,0\rangle,\quad \alpha,\beta=\text{Dirac indeces}$$

THIS IS SPIN INDEPENDENT:
Same evolution for all 8 TMD's
Up to NNLL!

$$\gamma_F=\frac{-1}{2}\gamma_H$$

Evolution Kernel

- If we want to connect two TMDPDFs at two different scales:

$$\tilde{F}_n(x, b; Q_f^2) = \tilde{F}_n(x, b; Q_i^2)\, \tilde{R}(b; Q_i, Q_f)$$

$$\tilde{R}(b; Q_i, Q_f) = \left(\frac{Q_f^2}{Q_i^2}\right)^{-D(\alpha_s(Q_i), L_T(Q_i))} \exp\left[\int_{Q_i}^{Q_f} \frac{d\mu'}{\mu'} \gamma_F\left(\alpha_s(\mu'), \ln\frac{Q_f^2}{\mu'^2}\right)\right]$$

- The evolution is given in terms of the function D and the AD

- When we Fourier transform back, we **need** to resum large logs in the D...

$$L_T = \ln\frac{Q^2 b^2}{4e^{-2\gamma_E}}$$

- I will show you **TWO** methods: the “traditional” CSS and the one we propose.

Resummation of R: CSS

$$\frac{dD(b;\mu)}{d\ln\mu} = \Gamma_{cusp}(\alpha_s)$$

$$D\left(b^*;Q_i\right) = D(b^*;\mu_b) + \int_{\mu_b}^{Q_i} \frac{d\bar{\mu}}{\bar{\mu}}\Gamma_{\text{cusp}}$$

$$L_T = \ln\frac{\mu^2 b^2}{4e^{-2\gamma_E}}$$

$$\mu_{b^*} = \frac{2e^{-\gamma_E}}{b^*}$$

$$b_*(b) = \frac{b}{\sqrt{1+b^2/b_{max}^2}}$$

Non-perturbative model (BLNY)

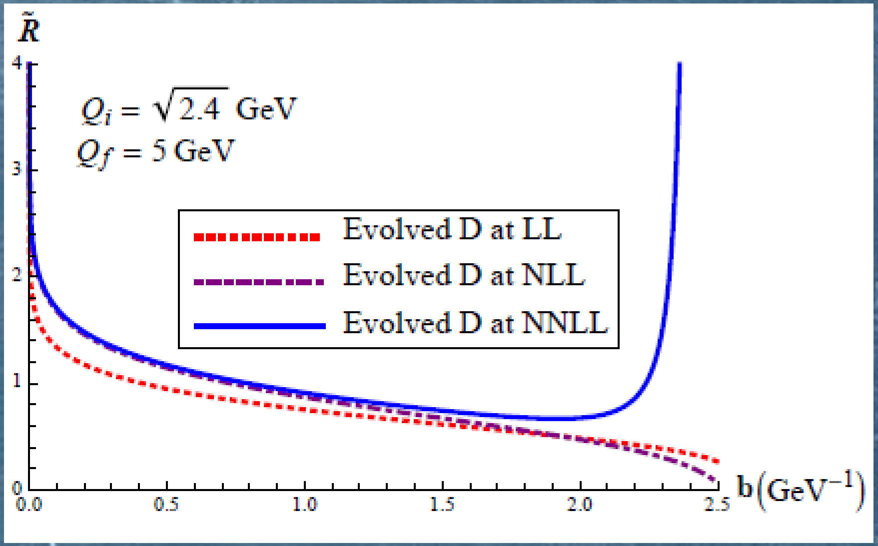
$$\tilde{R}^{CSS}(b,Q_i,Q_f) = \exp\left\{-\frac{1}{2}g_2 b^2 \ln\frac{Q_f}{Q_i}\right\}$$

$$\times\left(\frac{Q_f^2}{Q_i^2}\right)^{-\left[D(b^*,\mu_b)-\int_{\mu_b}^{Q_i}\frac{d\bar{\mu}}{\bar{\mu}}\Gamma_{cusp}\right]} \exp\left[\int_{Q_i}^{Q_f}\frac{d\mu'}{\mu'}\gamma_F\left(\alpha_s(\mu'),\ln\frac{Q_f^2}{\mu'^2}\right)\right]$$

Perturbative pieces

Resummation of R: CSS

$$\mu_b = \frac{2e^{-\gamma_E}}{b}$$



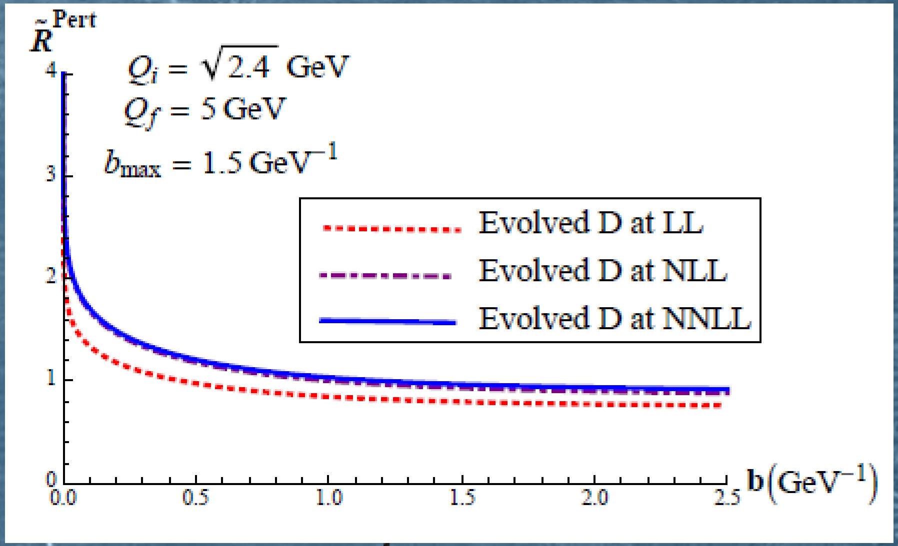
$$\tilde{R}^{Pert}(b, Q_i, Q_f) = \left(\frac{Q_f^2}{Q_i^2}\right)^{-\left[D(b,\mu_b) - \int_{\mu_b}^{Q_i} \frac{d\bar{\mu}}{\mu}\Gamma_{cusp}\right]} \exp\left[\int_{Q_i}^{Q_f} \frac{d\mu'}{\mu'}\gamma_F\left(\alpha_s(\mu'), \ln\frac{Q_f^2}{\mu'^2}\right)\right]$$

The Evolution Kernel with the effective coupling hits the **Landau Pole!!** :-(

Resummation of R à la CSS

$$\mu_{b^*} = \frac{2e^{-\gamma_E}}{b^*}$$

$$b_*(b) = \frac{b}{\sqrt{1+b^2/b^2_{max}}}$$



b

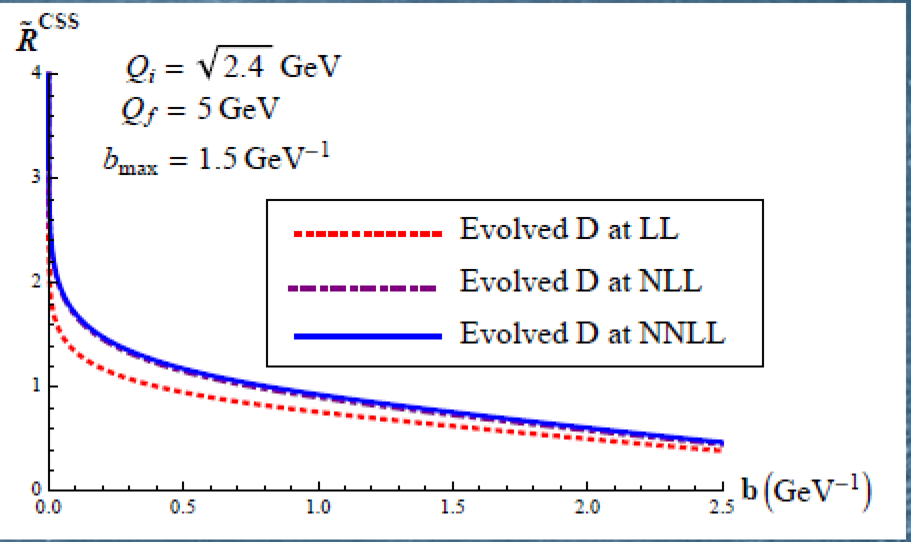
$$\tilde{R}^{Pert}(b,Q_i,Q_f) = \left(\frac{Q_f^2}{Q_i^2}\right)^{-\left[D(b^*,\mu_{b^*}) - \int_{\mu_{b^*}}^{Q_i} \frac{d\bar{\mu}}{\mu}\Gamma_{cusp}\right]} \exp\left[\int_{Q_i}^{Q_f} \frac{d\mu'}{\mu'}\gamma_F\left(\alpha_s(\mu'), \ln\frac{Q_f^2}{\mu'^2}\right)\right]$$

We impose a cutoff over b writing b*(b) instead of b.

But we loose information at large b!!

Resummation of R: CSS

$$\tilde{R}^{\rm NP} = \exp\left\{-\frac{1}{2} g_2 b^2 \ln\frac{Q_f}{Q_i}\right\}$$



The value of g_2 and b_{max} are extracted from fits to experimental data

$$\tilde{R}(b; Q_i, Q_f) = \tilde{R}\left(b^*(b); Q_i, Q_f\right)\ \tilde{R}^{\rm NP}(b; Q_i, Q_f)$$

We need to add a **non-perturbative model in the evolution** extracted from **data**...

- ***But there is a complete different way to resum the logs...***

D-Resummation

- We are going to write D as a **series** and resum it directly:

$$\frac{dD(b;\mu)}{d\ln\mu} = \Gamma_{cusp}(\alpha_s)$$

$$D(b;\mu) = \sum_{n=1}^{\infty} d_n(L_\perp)\left(\frac{\alpha_s}{4\pi}\right)^n$$

$$\Gamma_{\text{cusp}}(\alpha_s) = \sum_{n=1}^{\infty} \Gamma_{n-1}\left(\frac{\alpha_s}{4\pi}\right)^n$$

$$\beta(\alpha_s) = -2\alpha_s \sum_{n=1}^{\infty} \beta_{n-1}\left(\frac{\alpha_s}{4\pi}\right)^n$$

$$\frac{d}{dL_\perp} d_n(L_\perp) = \frac{1}{2}\Gamma_{n-1} + \sum_{m=1}^{n-1} m\beta_{n-1-m} d_m(L_\perp)$$

Recurrence relation

D-Resummation

$$D^R = \sum_{n=1}^{\infty} d_n(L_\perp) a^n =$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\left\{X^n\left(\frac{\Gamma_0}{\beta_0}\frac{1}{n}\right) + aX^{n-1}\left(\frac{\Gamma_0\beta_1}{\beta_0^2}\left(-1+H_{n-1}^{(1)}\right)|_{n\geq 3} + \frac{\Gamma_1}{\beta_0}|_{n\geq 2}\right)\right.$$

$$+a^2X^{n-2}\left((n-1)2d_2(0)|_{n\geq 2} + (n-1)\frac{\Gamma_2}{2\beta_0}|_{n\geq 3} + \frac{\beta_1\Gamma_1}{\beta_0^2}s_n|_{n\geq 4} + \frac{\beta_1^2\Gamma_0}{\beta_0^3}t_n|_{n\geq 5}\right.$$

$$\left.\left.+\frac{\beta_2\Gamma_0}{2\beta_0^2}(n-3)|_{n\geq 4}\right) + \ldots\right\},$$

$$X = a\beta_0 L_\perp$$

$$D^R = -\frac{\Gamma_0}{2\beta_0}\ln(1-X) + \frac{1}{2}\left(\frac{a}{1-X}\right)\left[-\frac{\beta_1\Gamma_0}{\beta_0^2}(X+\ln(1-X)) + \frac{\Gamma_1}{\beta_0}X\right]$$

$$+\frac{1}{2}\left(\frac{a}{1-X}\right)^2\left[2d_2(0) + \frac{\Gamma_2}{2\beta_0}(X(2-X)) + \frac{\beta_1\Gamma_1}{2\beta_0^2}\left(X(X-2) - 2\ln(1-X)\right) + \frac{\beta_2\Gamma_0}{2\beta_0^2}X^2\right.$$

$$+\frac{\beta_1^2\Gamma_0}{\beta_0^3}\frac{+121X^6 - 188X^5 + 13X^4 + 30X^3 + 12X^2(1-\mathrm{Li}_2(X)) + 12X(X+1)\ln(1-X)}{24X^2}$$

$$\left.+\frac{\beta_1^2\Gamma_0}{2\beta_0^3}(1-X)^2\sum_{n=5}^{\infty}X^{n-2}(n-1)\left[H_{n-1}^{(1)}\right]^2\right] + \ldots,$$

D-Resummation

$$X = a\beta_0 L_\perp$$

$$X = 1 \rightarrow b_X = \frac{2e^{-\gamma_E}}{Q_i} \exp \frac{2\pi}{\beta_0 \alpha_s(Q_i)}$$

New expansion!

$$\begin{aligned}
D^R = & -\frac{\Gamma_0}{2\beta_0}\ln(1-X) + \frac{1}{2}\left(\frac{a}{1-X}\right)\left[-\frac{\beta_1\Gamma_0}{\beta_0^2}(X+\ln(1-X)) + \frac{\Gamma_1}{\beta_0}X\right] \\
& + \frac{1}{2}\left(\frac{a}{1-X}\right)^2\Bigg[2d_2(0) + \frac{\Gamma_2}{2\beta_0}(X(2-X)) + \frac{\beta_1\Gamma_1}{2\beta_0^2}(X(X-2) - 2\ln(1-X)) + \frac{\beta_2\Gamma_0}{2\beta_0^2}X^2 \\
& + \frac{\beta_1^2\Gamma_0}{\beta_0^3}\frac{+121X^6 - 188X^5 + 13X^4 + 30X^3 + 12X^2(1-\mathrm{Li}_2(X)) + 12X(X+1)\ln(1-X)}{24X^2} \\
& + \frac{\beta_1^2\Gamma_0}{2\beta_0^3}(1-X)^2\sum_{n=5}^{\infty}X^{n-2}(n-1)\left[H_{n-1}^{(1)}\right]^2\Bigg] + \ldots ,
\end{aligned}$$

D-Resummation

Properties of DR:

- The resummation works for all X<1
- The sign of DR is the same at all orders (that we checked)
- Asymptotically, when X→1

$$D^R|_{X\to 1^-} = -\frac{\Gamma_0}{2\beta_0}\ln(1-X)\left[1+\left(\frac{a}{1-X}\right)\frac{\beta_1}{\beta_0}+\left(\frac{a}{1-X}\right)^2\frac{\beta_1\Gamma_1}{\beta_0\Gamma_0}+\ldots\right]$$

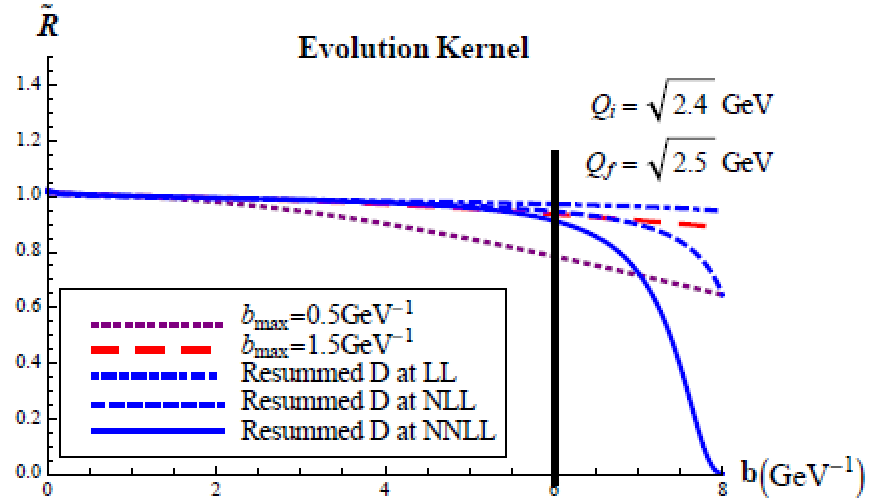
$$\stackrel{n_f=5}{=} -\frac{\Gamma_0}{2\beta_0}\ln(1-X)\left[1+\left(\frac{a}{1-X}\right)5.04+\left(\frac{a}{1-X}\right)^2 34.84+\ldots\right]$$

Truncation of DR:

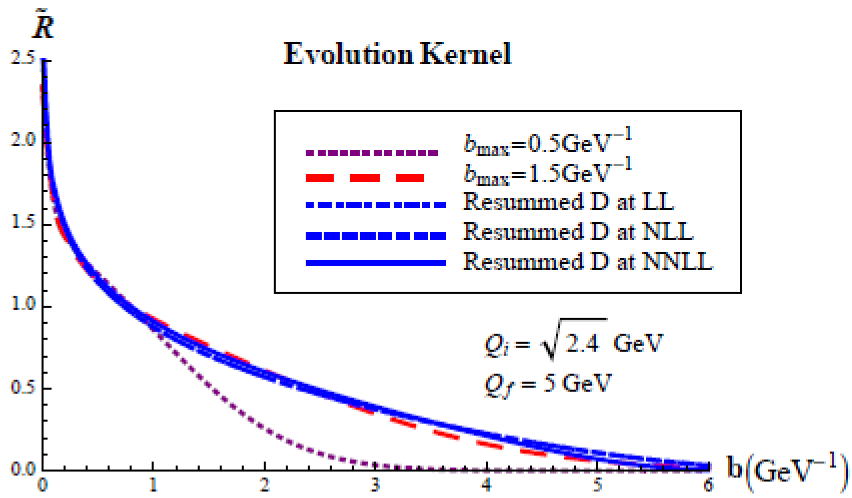
- We can think to truncate DR when *a/(1-X)~1*
- We have tried the truncation at bc such that

$$X(b_X)=1; \quad a(Q)/(1-X(b_{c1}))=1; \quad a(Q)/(1-X(b_{c2}))=0.2$$

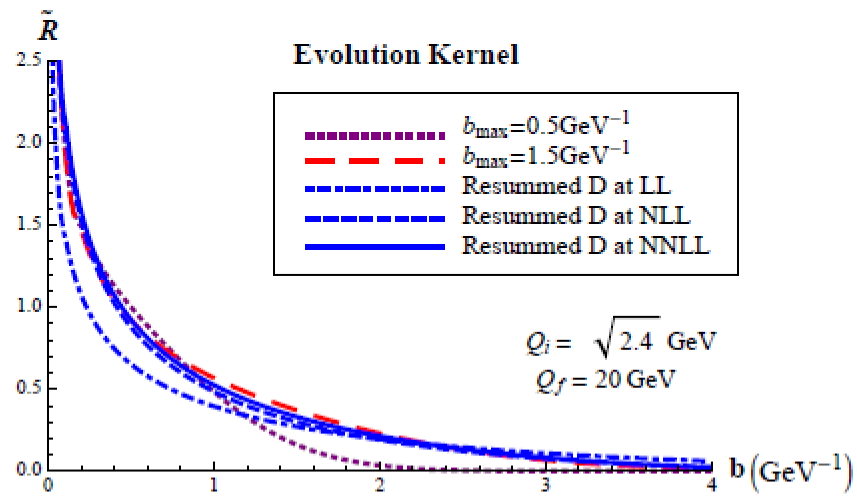
Results



$\tilde{R}$
Evolution Kernel
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
$Q_i = \sqrt{2.4}$ GeV
$Q_f = \sqrt{2.5}$ GeV
b_{max}=0.5GeV^{-1}
b_{max}=1.5GeV^{-1}
Resummed D at LL
Resummed D at NLL
Resummed D at NNLL
b(GeV^{-1})

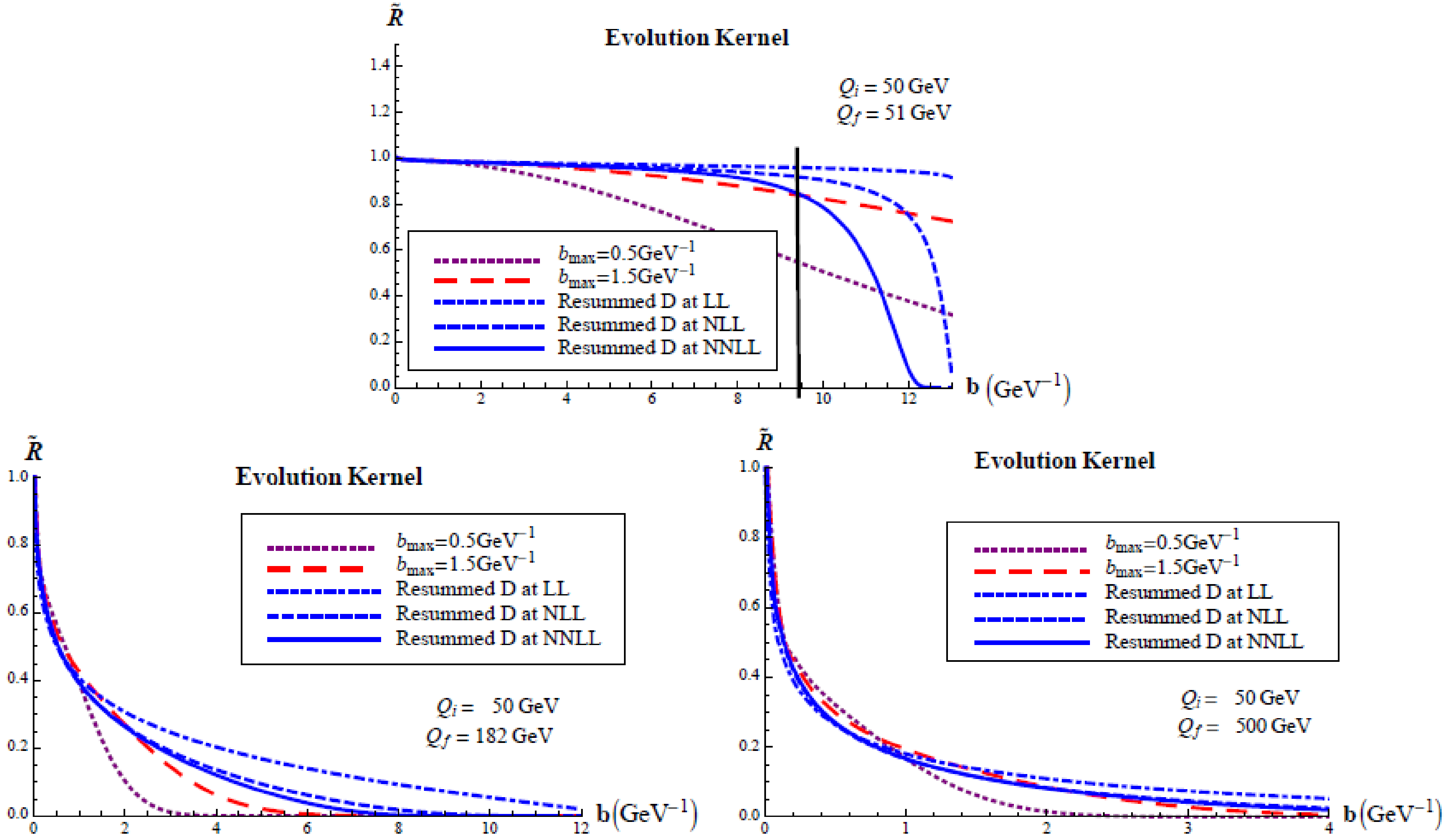


$\tilde{R}$
Evolution Kernel
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
6
b_{max}=0.5GeV^{-1}
b_{max}=1.5GeV^{-1}
Resummed D at LL
Resummed D at NLL
Resummed D at NNLL
$Q_i = \sqrt{2.4}$ GeV
Q_f = 5 GeV
b(GeV^{-1})



$\tilde{R}$
Evolution Kernel
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
b_{max}=0.5GeV^{-1}
b_{max}=1.5GeV^{-1}
Resummed D at LL
Resummed D at NLL
Resummed D at NNLL
$Q_i = \sqrt{2.4}$ GeV
Q_f = 20 GeV
b(GeV^{-1})

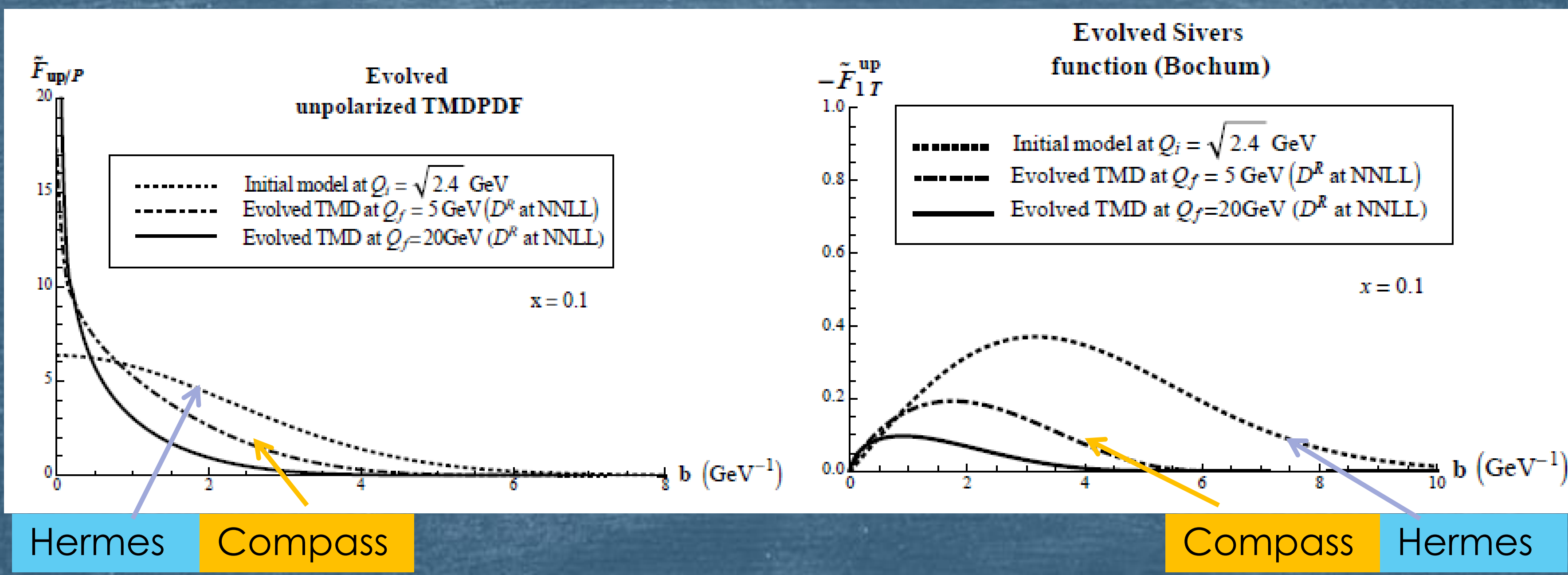
Results



Evolution Kernel
$\tilde{R}$
Q_i = 50 GeV
Q_f = 51 GeV
b_{max}=0.5GeV^{-1}
b_{max}=1.5GeV^{-1}
Resummed D at LL
Resummed D at NLL
Resummed D at NNLL
b (GeV^{-1})
Evolution Kernel
Q_i = 50 GeV
Q_f = 182 GeV
Evolution Kernel
Q_i = 50 GeV
Q_f = 500 GeV

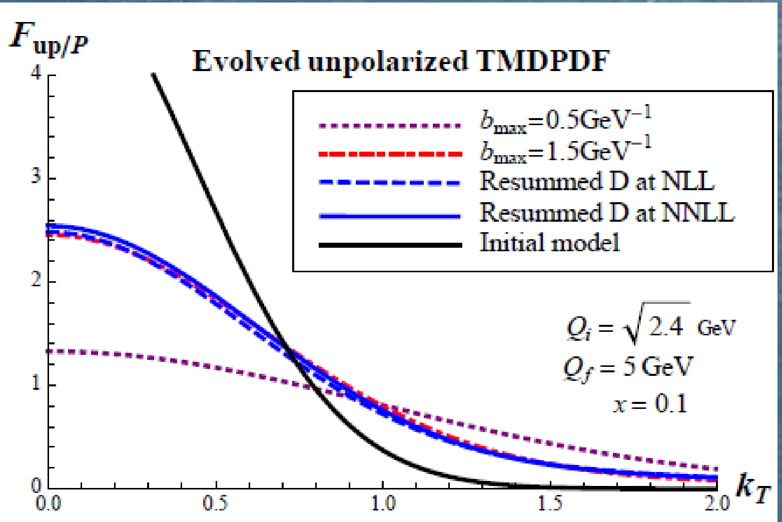
Results

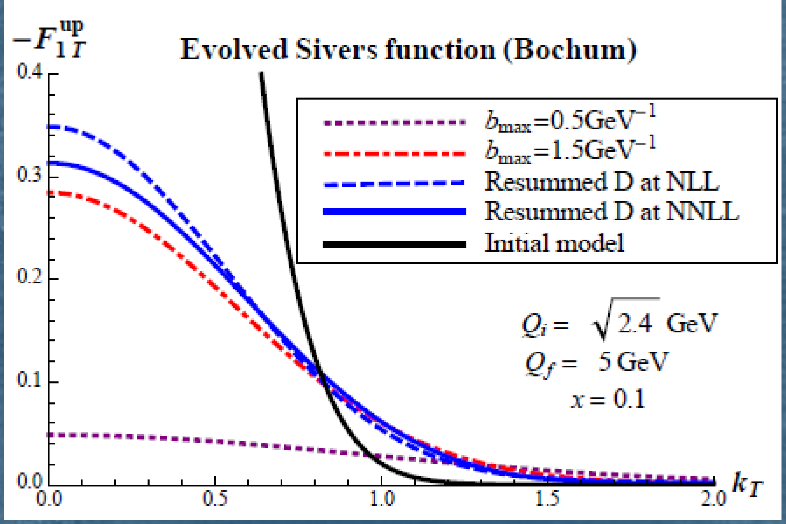
In practice the TMD are concentrated on a region of IPS shorter than the range of validity of the evolutor



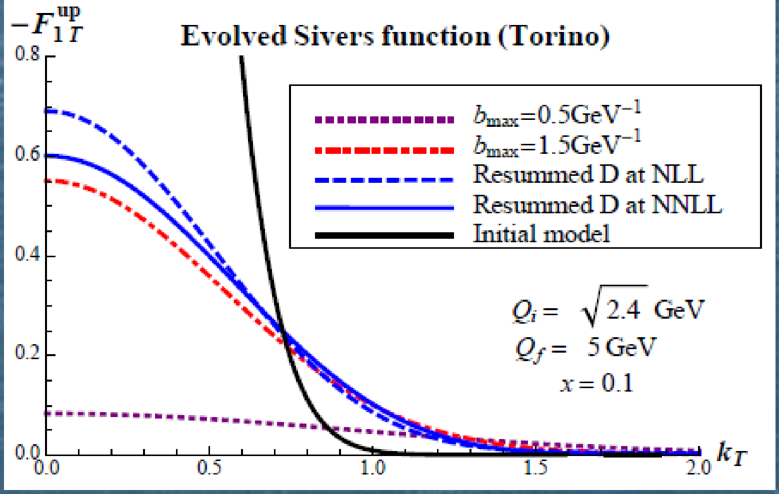
Results

We compare with CSS and
bmax=0.5, Collins ideal
bmax=1.5, fitted from
Phenomenology
(Konychev, Nadolsky'06)





All graphs show an agreement
With the bmax=1.5 choice



CONCLUSIONS

- We have a formulation of factorization on-the-light-cone (no parameters on any matching coefficient!)
- We can relate the AD of the hard matching coefficient to the AD of the TMDPD's → WE KNOW THE EVOLUTION OF ALL TMDPDF UP TO NNLL
- We can build an evolutor for TMDPDF removing the problem of the Landau pole in a model independent way (agreement with fits that use bmax=1.5)
- We need experiments to get a mapping of TMDs as precise as for PDFs

BACKUP SLIDES

Formulas for Λ_{QCD}

$$\Lambda_{\mathrm{QCD}} = Q\exp[G(t_Q)]$$

$$t_Q \equiv -2\pi/(\beta_0\alpha_s(Q))$$

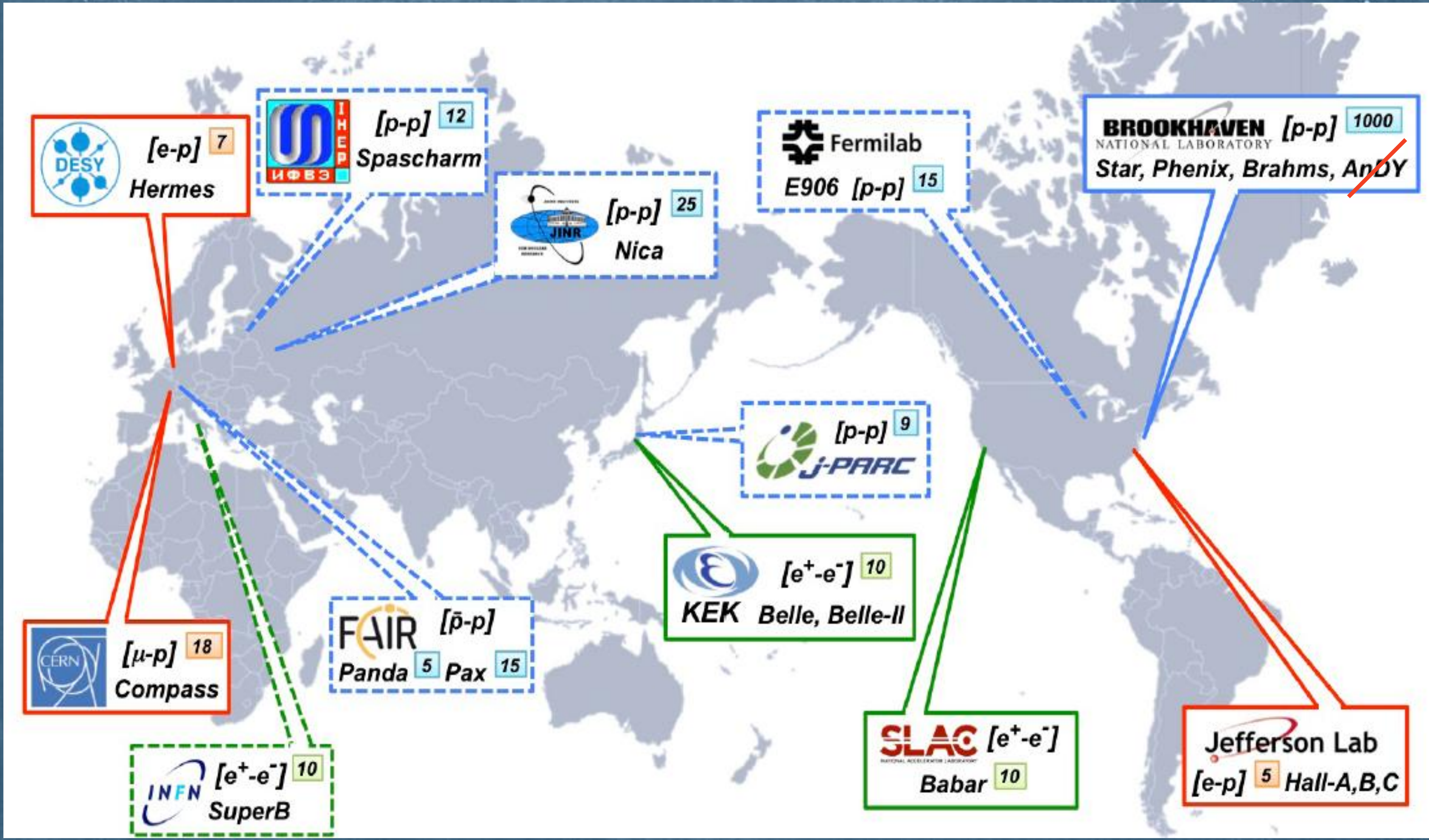
$$G(t) = t + \frac{\beta_1}{2\beta_0^2}\ln(-t) - \frac{\beta_1^2-\beta_0\beta_2}{4\beta_0^4}\frac{1}{t} - \frac{\beta_1^3-2\beta_0\beta_1\beta_2+\beta_0^2\beta_3}{8\beta_0^6}\frac{1}{2t^2} + \ldots$$

$$\alpha_s(M_Z) = 0.117$$

$$\mathrm{n}_f = 5$$

$$\Lambda_{\mathrm{QCD}} \approx 157\,\mathrm{MeV}$$

$$b_\Lambda = \frac{2e^{-\gamma_E}}{\Lambda_{\mathrm{QCD}}} \approx 7.15\,\mathrm{GeV}^{-1}$$



DESY
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Hermes
IHEP
ИФВЭ
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Spascharm
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