Transverse Momentum Distributions: a Regularization-Scheme free definition

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M.G. Echevarría, Ahmad Idilbi, Ignazio Scimemi. [arXiv: 1211.1947]
MGE, AI, Andreas Schäfer, IS. [arXiv: 1208.1281]
MGE, AI, IS. JHEP 07 (2012) 002. [arXiv: 1111.4996]

SCET Workshop 2013
Duke University
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Outline

★ **Hadronic ME with TMD**

★ **Definition of TMDPDF**

★ **Collins' ("JCC") Definition and Comparison with EIS**

★ **Conclusions and Outlook**

★ ***Q^2-Resummation, Evolution, Phenomenology... - See Ignazio's Talk***

Prelude: Integrated PDF (1/2)

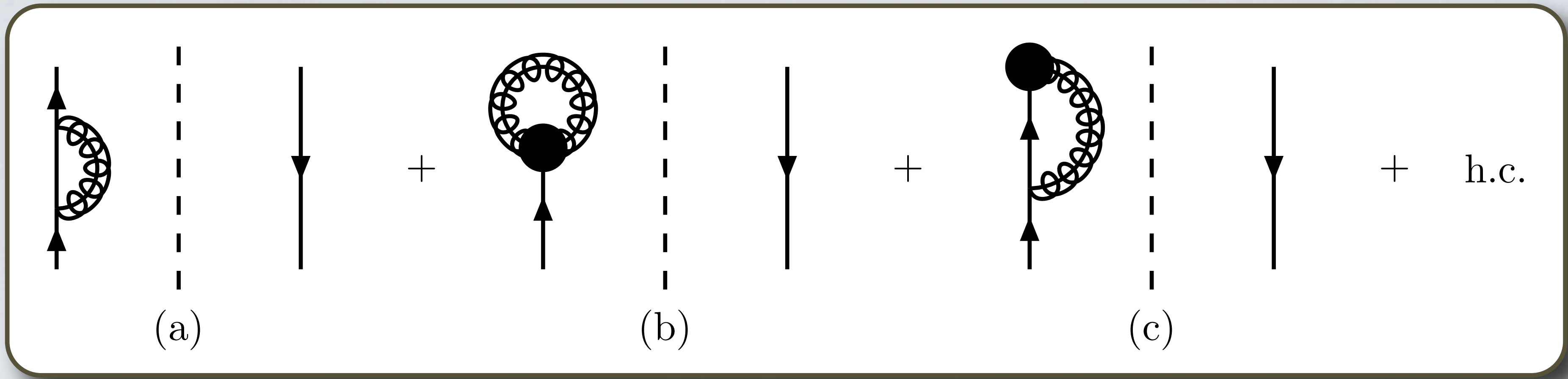
$$\boxed{f_n(0^+, y^-, \vec{0}_\perp) = \frac{1}{2}\sum_\sigma \langle P,\sigma|\; \left[\bar{\xi}_n W_n\right](0^+, y^-, \vec{0}_\perp)\,\frac{\bar{\not{n}}}{2}\;\left[W_n^\dagger \xi_n\right](0)\,|P,\sigma\rangle}$$

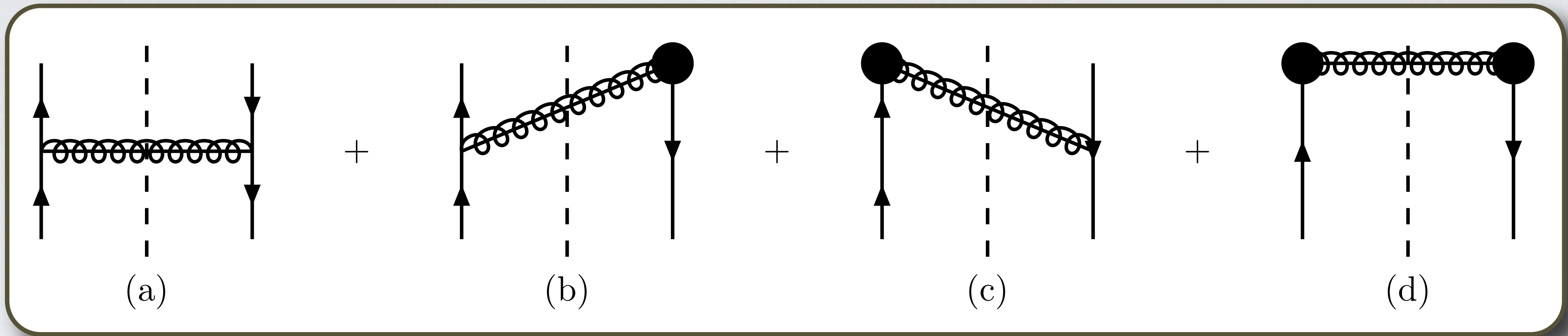
$$\boxed{W_n(x) = \bar{P}\exp\left[ig\int_{-\infty}^{0} ds\,\bar{n}\cdot A_n(x+s\bar{n})\right]}$$

- Regulator:

$$\boxed{\begin{array}{l} \dfrac{i(\not{p}+\not{k})}{(p+k)^2+i\Delta^-} \longrightarrow \dfrac{1}{k^-+i\delta^-}\,,\ \delta^- = \dfrac{\Delta^-}{p^+} \\ \dfrac{i(\bar{\not{p}}-\not{k})}{(\bar{p}-k)^2+i\Delta^+} \longrightarrow \dfrac{1}{-k^++i\delta^+}\,,\ \delta^+ = \dfrac{\Delta^+}{\bar{p}^-} \end{array}}$$

Integrated PDF (2/2)





$$f_n = \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\left\{\mathcal{P}_{q/q}\left(\frac{1}{\varepsilon_{\mathrm{UV}}} - \ln\frac{\Delta^-}{\mu^2}\right) - \frac{1}{4}\delta(1-x) - (1-x)[1+\ln(1-x)]\right\}$$

- The UV pole is cancelled by renormalization
- The IR pole (logarithm) is washed out by confinement

Transverse Dependence

- The next-to-simple hadronic matrix element:

$$F_n^{naive}(0^+, y^-, \vec{y}_\perp) = \frac{1}{2}\sum_\sigma \langle P,\sigma| \, [\bar{\xi}_n W_n]\,(0^+, y^-, \vec{y}_\perp)\, \frac{\bar{\not{n}}}{2}\, [W_n^\dagger \xi_n]\,(0)\,|P,\sigma\rangle$$

We would also need ***transverse*** *gauge links to maintain gauge invariance*

[Collins-Soper '82]

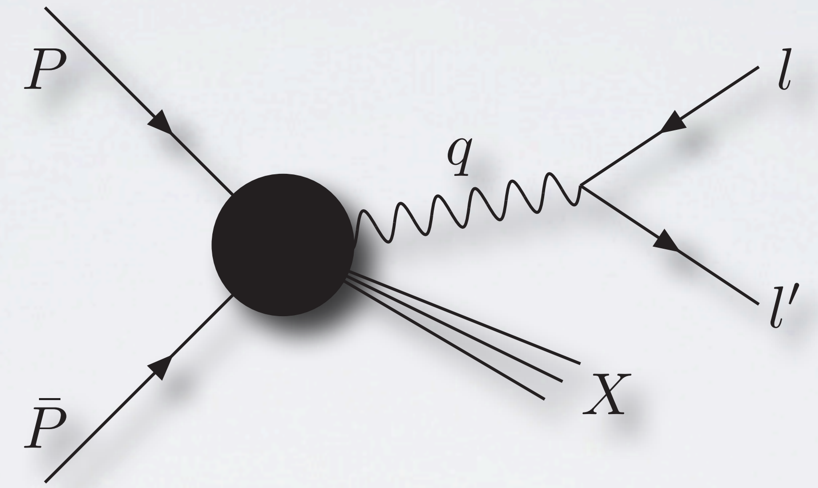
$$\begin{aligned}\tilde{F}_n^{naive} &= \delta(1-x) + \frac{\alpha_s C_F}{2\pi}\Bigg\{\delta(1-x)\Bigg[\frac{2}{\varepsilon_{\rm UV}}\ln\frac{\Delta^+}{Q^2} + \frac{3}{2\varepsilon_{\rm UV}} \\ &\quad -\frac{1}{4} + \frac{3}{2}L_T + 2L_T\ln\frac{\Delta^+}{Q^2}\Bigg] \\ &\quad -(1-x)\ln(1-x) - \mathcal{P}_{q/q}\ln\frac{\Delta^-}{\mu^2} - L_T\mathcal{P}_{q/q}\Bigg\}\end{aligned}$$

- **It is ill-defined!! Mixed UV/Rapidity divergences!!**

$$L_T = \ln\frac{\mu^2 b^2}{4e^{-2\gamma_E}}$$

DY q_T-Spectrum

$$M = H(Q^2/\mu^2) \int d^4y\, e^{-iq\cdot y} J_n^{(0)}(0^+, y^-, \vec{y}_\perp)\, S(0^+, 0^-, \vec{y}_\perp)\, J_{\bar{n}}^{(0)}(y^+, 0^-, \vec{y}_\perp)$$



$$J_n^{(0)}(0^+, y^-, \vec{y}_\perp) = \frac{1}{2} \sum_{\sigma_1} \langle N_1(P, \sigma_1)|\, \bar{\chi}_n(0^+, y^-, \vec{y}_\perp)\, \frac{\bar{n}\!\!\!/}{2}\, \chi_n(0)\, |N_1(P, \sigma_1)\rangle\Big|_{\text{zb subtracted}}$$

$$J_{\bar{n}}^{(0)}(y^+, 0^-, \vec{y}_\perp) = \frac{1}{2} \sum_{\sigma_2} \langle N_2(\bar{P}, \sigma_2)|\, \bar{\chi}_{\bar{n}}(0)\, \frac{n\!\!\!/}{2}\, \chi_{\bar{n}}(y^+, 0^-, \vec{y}_\perp)\, |N_2(\bar{P}, \sigma_2)\rangle\Big|_{\text{zb subtracted}}$$

$$S(0^+, 0^-, \vec{y}_\perp) = \langle 0|\, \mathrm{Tr}\, \bar{\mathbf{T}}\big[S_n^{T\dagger} S_{\bar{n}}^T\big](0^+, 0^-, \vec{y}_\perp)\mathbf{T}\big[S_{\bar{n}}^{T\dagger} S_n^T\big](0)\, |0\rangle$$

$$\chi_n = W_n^{T\dagger} \xi_n$$

Definition of TMDPDF
(1st attempt...)

- The hadronic tensor is:

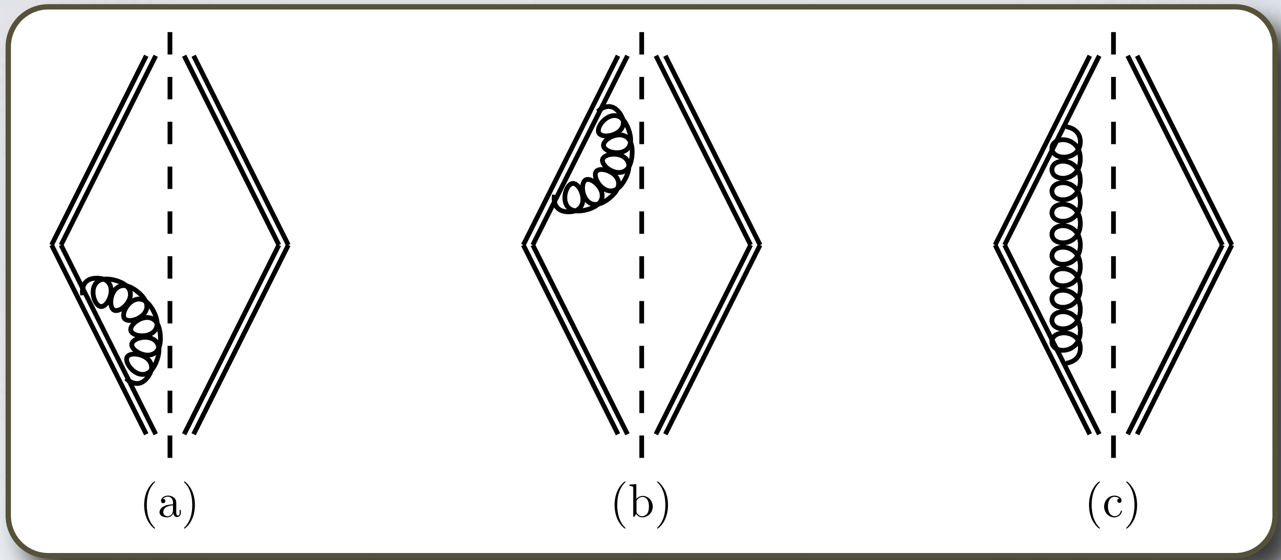
$$M = H(Q^2/\mu^2) \int d^4y\, e^{-iq\cdot y} J_n^{(0)}(0^+, y^-, \vec{y}_\perp)\, S(0^+, 0^-, \vec{y}_\perp)\, J_{\bar{n}}^{(0)}(y^+, 0^-, \vec{y}_\perp)$$

- In QCD there are no mixed divergences, so they are cancelled in the combination of collinear, anti-collinear and soft.

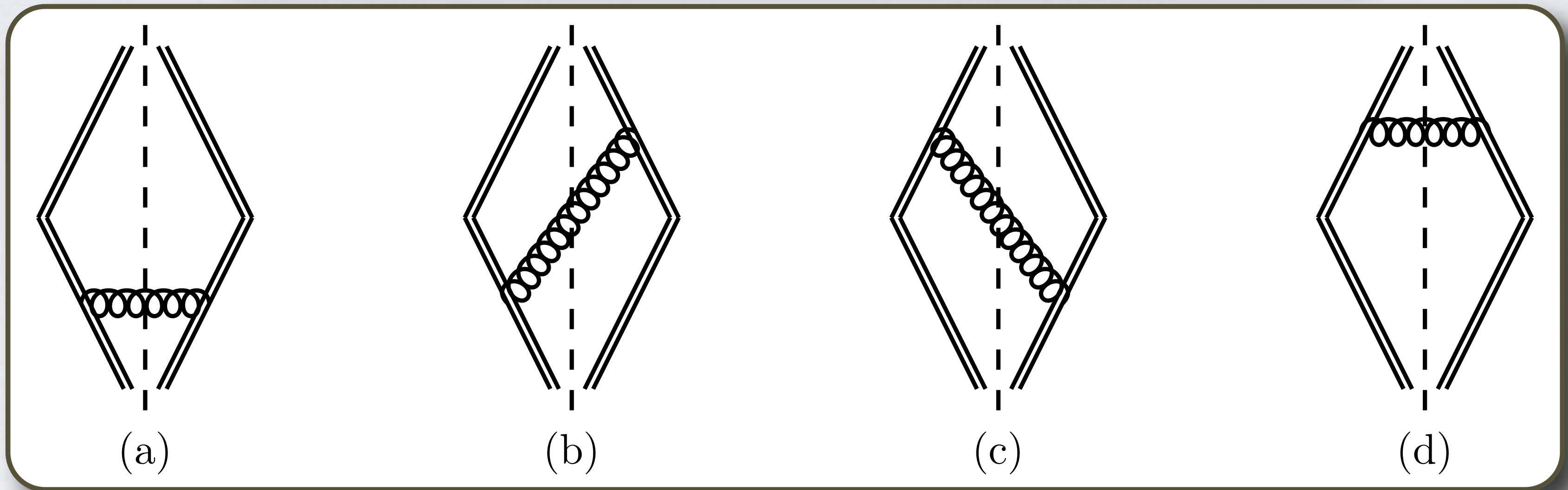
- Thus we can try to define the TMDPDF "by symmetry" as:

$$F_n^{trial}(x, \vec{k}_\perp) = \frac{1}{2} \int \frac{dr^- d^2\vec{r}_\perp}{(2\pi)^3} e^{-i(\frac{1}{2}r^- xP^+ - \vec{r}_\perp \cdot \vec{k}_\perp)} \times J_n^{(0)}(0^+, r^-, \vec{r}_\perp) \sqrt{S(0^+, 0^-, \vec{r}_\perp)}$$

Results at One Loop: Soft Function



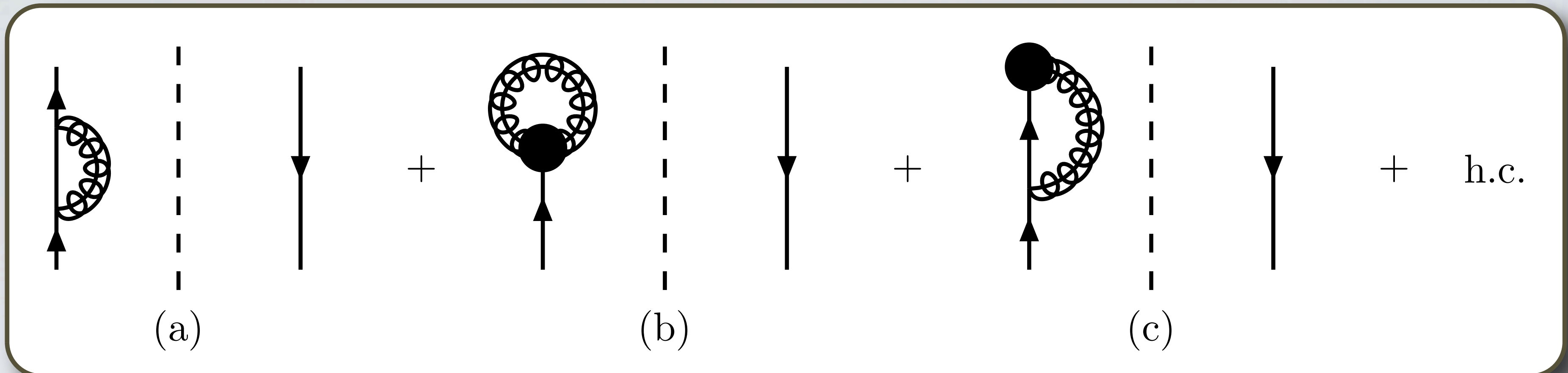
+ h.c.

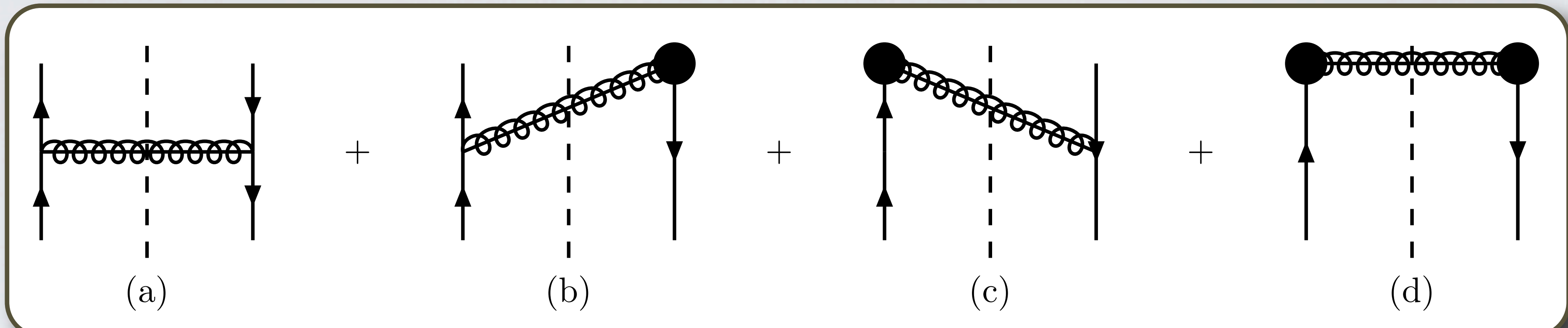


$$\tilde{S}_1(\Delta,\Delta) = \frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\varepsilon_{\rm UV}^2} + \frac{2}{\varepsilon_{\rm UV}}\ln\frac{\Delta^2}{\mu^2 Q^2} + L_T^2 + 2L_T\ln\frac{\Delta^2}{\mu^2 Q^2} + \frac{\pi^2}{6}\right]$$

$$L_T = \ln\frac{\mu^2 b^2}{4e^{-2\gamma_E}}$$

Results at One Loop: Pure Collinear





$$\tilde{J}^{(0)}_{n1}(\Delta) = \frac{\alpha_s C_F}{2\pi}\left\{\delta(1-x)\left[\frac{2}{\varepsilon^2_{\mathrm{UV}}} - \frac{2}{\varepsilon_{\mathrm{UV}}}\ln\frac{\Delta}{\mu^2} + \frac{3}{2\varepsilon_{\mathrm{UV}}} - \frac{1}{4} - \frac{2\pi^2}{12} - L_T^2 + \frac{3}{2}L_T - 2L_T\ln\frac{\Delta}{\mu^2}\right] - (1-x)\ln(1-x) - \mathcal{P}_{q/q}\ln\frac{\Delta}{\mu^2} - L_T\mathcal{P}_{q/q}\right\}$$

- We subtract the zero-bin for each diagram!!

$$L_T = \ln\frac{\mu^2 b^2}{4e^{-2\gamma_E}}$$

Results at One Loop: TMDPDF

$$\tilde{F}_n^{trial} = \tilde{J}_n^{(0)} \sqrt{\tilde{S}}$$

$$\tilde{F}_{n1}^{trial} = \frac{\alpha_s C_F}{2\pi} \left\{ \delta(1-x) \left[\frac{1}{\varepsilon_{\rm UV}^2} - \frac{1}{\varepsilon_{\rm UV}} \ln\frac{Q^2}{\mu^2} + \frac{3}{2\varepsilon_{\rm UV}} - \frac{1}{2}L_T^2 + \frac{3}{2}L_T - L_T \ln\frac{Q^2}{\mu^2} - \frac{\pi^2}{12} \right] + (1-x) - L_T \mathcal{P}_{q/q} - \mathcal{P}_{q/q} \ln\frac{\Delta}{\mu^2} - \frac{1}{4}\delta(1-x) - (1-x)[1+\ln(1-x)] \right\}$$

[EIS, JHEP 2012]

$$\tilde{F}_{n1}^{trial} = \frac{\alpha_s C_F}{2\pi} \left\{ \delta(1-x_n) \left[\frac{1}{\varepsilon_{\rm UV}^2} - \frac{1}{\varepsilon_{\rm UV}} \ln\frac{Q^2 \Delta^-}{\mu^2 \Delta^+} + \frac{3}{2\varepsilon_{\rm UV}} - \frac{1}{2}L_T^2 + \frac{3}{2}L_T - L_T \ln\frac{Q^2 \Delta^-}{\mu^2 \Delta^+} - \frac{\pi^2}{12} \right] + (1-x_n) - L_T \mathcal{P}_{q/q} - \mathcal{P}_{q/q} \ln\frac{\Delta^-}{\mu^2} - \frac{1}{4}\delta(1-x_n) - (1-x_n)[1+\ln(1-x_n)] \right\}$$

- **There are mixed UV/nUV divergences!!**
- **What is going on?? The definition I provided has a catch...**

Factorization of Modes (1/2)

- The factorization of the relevant modes in tricky...

[Manohar-Stewart '06]

- Soft and Collinear modes have the **same invariant mass**.
- Only can be distinguished by their **relative rapidities**:

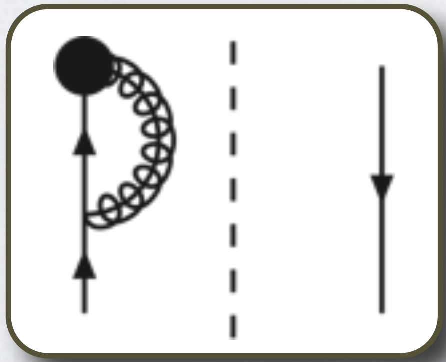
$$\begin{array}{lcl} k_n \sim Q(1, \lambda^2, \lambda) & \to & y \gg 0 \\ k_{\bar{n}} \sim Q(\lambda^2, 1, \lambda) & \to & y \ll 0 \\ k_s \sim Q(\lambda, \lambda, \lambda) & \to & y \approx 0 \end{array}$$

$$k_n^2 \sim k_{\bar{n}}^2 \sim k_s^2 \sim q_T^2$$

$$y = \frac{1}{2} \ln \left| \frac{k^+}{k^-} \right|$$

$$\lambda \sim \frac{q_T}{Q}$$

- **Modes can be mixed under boosts,** so we need rapidity cuts.



- Rapidity divergence when k^+ goes to 0
- We need a lower rapidity cutoff

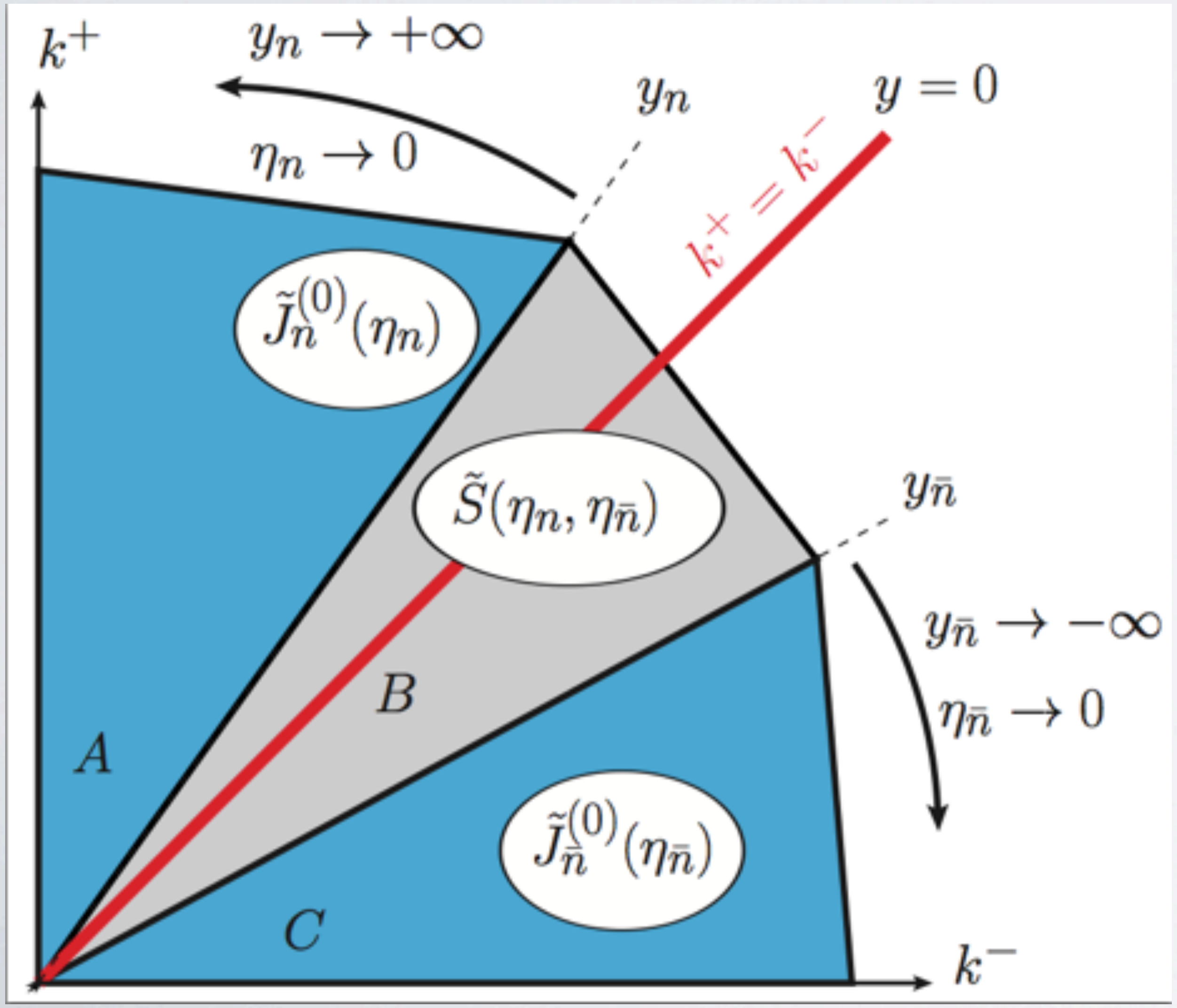
$$\mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d} \frac{p^+ + k^+}{[k^+ - i\varepsilon][(p+k)^2 + i\varepsilon][k^2 + i\varepsilon]}$$

Factorization of Modes (2/2)

- We need to impose **rapidity cutoffs** to separate the modes:

$$\boxed{H(Q^2)\,\tilde{J}_n^{(0)}(\eta_n)\,\tilde{S}(\eta_n,\eta_{\bar{n}})\,\tilde{J}_{\bar{n}}^{(0)}(\eta_{\bar{n}})}$$

$$y = \frac{1}{2}\ln\left|\frac{k^+}{k^-}\right|$$



- A is collinear
- B is soft
- C is anti-collinear
- Soft function is NOT symmetric w.r.t. the "separating line" $k^+=k^-$ when $y^+ \neq y^-$.

- We proved that the soft function can be split in two "hemispheres"

- And we will identify positive & negative rapidity quanta with each TMDPDF!!

Splitting of the Soft Function (1/3)

- The soft function can be split in two "pieces".
- I will use the Δ-regulator, but the arguments are regulator-INdependent!!

- On one hand:

$$\tilde{M} = H(Q^2/\mu^2)\, \tilde{J}_n^{(0)}(\Delta^-)\, \tilde{J}_{\bar{n}}^{(0)}(\Delta^+)\, \tilde{S}(\Delta^-,\Delta^+)$$

$$\ln\tilde{M} = \ln H + \ln\tilde{J}_n^{(0)} + \ln\tilde{J}_{\bar{n}}^{(0)} + \ln\tilde{S}$$

$$\begin{aligned}
\ln\tilde{J}_n^{(0)} &= \mathcal{R}_n\left(x_n, \alpha_s, L_\perp, \ln\frac{\Delta^-}{\mu^2}\right) \\
\ln\tilde{J}_{\bar{n}}^{(0)} &= \mathcal{R}_{\bar{n}}\left(x_{\bar{n}}, \alpha_s, L_\perp, \ln\frac{\Delta^+}{\mu^2}\right) \\
\ln\tilde{S} &= \mathcal{R}_s\left(\alpha_s, L_\perp, \ln\frac{\Delta^-\Delta^+}{Q^2\mu^2}\right)
\end{aligned}$$

Splitting of the Soft Function (2/3)

- On the other:

$$\boxed{\begin{aligned}\tilde{M} &= H(Q^2/\mu^2)\,\tilde{C}_n(x_n; L_T, Q^2/\mu^2)\,\tilde{C}_{\bar{n}}(x_{\bar{n}}; L_T, Q^2/\mu^2)\\ &\quad\times f_n(x_n; \Delta^-/\mu^2)\,f_{\bar{n}}(x_{\bar{n}}; \Delta^+/\mu^2)\end{aligned}}$$

$$\boxed{\ln\tilde{M} = \ln H + \ln\tilde{C}_n + \ln\tilde{C}_{\bar{n}} + \ln f_n + \ln f_{\bar{n}}}$$

$$\boxed{\begin{aligned}\ln f_n &= \mathcal{R}_{f1}(x_n, \alpha_s) + \mathcal{R}_{f2}(x_n, \alpha_s)\ln\frac{\Delta^-}{\mu^2}\\ \ln f_{\bar{n}} &= \mathcal{R}_{f1}(x_{\bar{n}}, \alpha_s) + \mathcal{R}_{f2}(x_{\bar{n}}, \alpha_s)\ln\frac{\Delta^+}{\mu^2}\end{aligned}}$$

- We have used the fact that the IR collinear divergence in the PDFs is encoded in a single log (or single pole if we use dimensional regularization)

Splitting of the Soft Function (3/3)

- Then we have:

$$\ln \tilde{J}_n^{(0)} = \mathcal{R}_{n1}(x_n, \alpha_s, L_\perp) + \mathcal{R}_{n2}(x_n, \alpha_s, L_\perp) \ln \frac{\Delta^-}{\mu^2}$$

$$\ln \tilde{J}_{\bar{n}}^{(0)} = \mathcal{R}_{\bar{n}1}(x_{\bar{n}}, \alpha_s, L_\perp) + \mathcal{R}_{\bar{n}2}(x_{\bar{n}}, \alpha_s, L_\perp) \ln \frac{\Delta^+}{\mu^2}$$

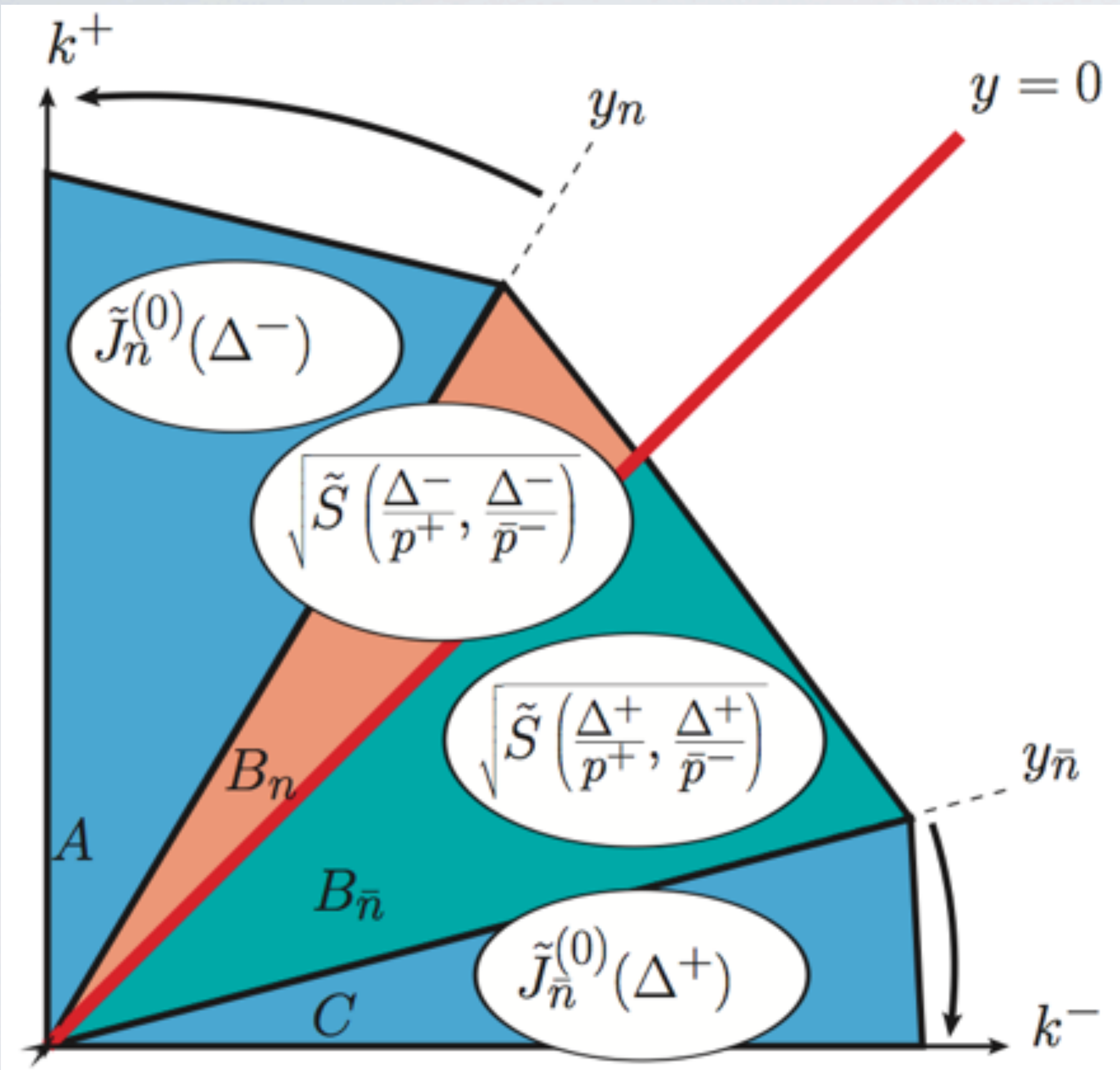
$$\ln \tilde{S} = \mathcal{R}_{s1}(\alpha_s, L_\perp) + \mathcal{R}_{s2}(\alpha_s, L_\perp) \ln \frac{\Delta^- \Delta^+}{Q^2 \mu^2}$$

$$Q^2 = p^+ \bar{p}^-$$

$$\tilde{S}\left(\frac{\Delta^-}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right) = \sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+}, \frac{\Delta^-}{\bar{p}^-}\right)} \sqrt{\tilde{S}\left(\frac{\Delta^+}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right)}$$

Definition of TMDPDF

- Collecting all the **positive** & **negative** rapidity quanta we (re)define the TMDPDFs as:



$$\tilde{F}_n(x_n, b; Q, \mu) = \tilde{J}_n^{(0)}(\Delta^-)\,\sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+},\frac{\Delta^-}{\bar{p}^-}\right)}$$

$$\tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q, \mu) = \tilde{J}_n^{(0)}(\Delta^+)\,\sqrt{\tilde{S}\left(\frac{\Delta^+}{p^+},\frac{\Delta^+}{\bar{p}^-}\right)}$$

$$\tilde{M} = H(Q^2/\mu^2)\,\tilde{F}_n(x_n, b; Q^2, \mu^2)\,\tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

No soft function in the factorization theorem!!

Results at One Loop

- The pure collinear is the same as before.

$$\tilde{J}^{(0)}_{n1}(\Delta^-) = \frac{\alpha_s C_F}{2\pi}\left\{\delta(1-x)\left[\frac{2}{\varepsilon_{\rm UV}^2} - \frac{2}{\varepsilon_{\rm UV}}\ln\frac{\Delta^-}{\mu^2} + \frac{3}{2\varepsilon_{\rm UV}} - \frac{1}{4} - \frac{2\pi^2}{12} - L_T^2 + \frac{3}{2}L_T - 2L_T\ln\frac{\Delta^-}{\mu^2}\right] - (1-x)\ln(1-x) - \mathcal{P}_{q/q}\ln\frac{\Delta^-}{\mu^2} - L_T\mathcal{P}_{q/q}\right\}$$

- The soft function is split in two pieces:

$$\tilde{S}_1\left(\frac{\Delta^-}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right) = \frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\varepsilon_{\rm UV}^2} + \frac{2}{\varepsilon_{\rm UV}}\ln\frac{\Delta^+\Delta^-}{\mu^2 Q^2} + L_T^2 + 2L_T\ln\frac{\Delta^+\Delta^-}{\mu^2 Q^2} + \frac{\pi^2}{6}\right]$$
$$= \frac{1}{2}\left[\tilde{S}_1\left(\frac{\Delta^-}{p^+}, \frac{\Delta^-}{\bar{p}^-}\right) + \tilde{S}_1\left(\frac{\Delta^+}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right)\right]$$

$$\tilde{S}_1\left(\frac{\Delta^-}{p^+}, \frac{\Delta^-}{\bar{p}^+}\right) = \frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\varepsilon_{\rm UV}^2} + \frac{2}{\varepsilon_{\rm UV}}\ln\frac{\Delta^-\Delta^-}{\mu^2 Q^2} + L_T^2 + 2L_T\ln\frac{\Delta^-\Delta^-}{\mu^2 Q^2} + \frac{\pi^2}{6}\right]$$

TMDPDF

$$\tilde{F}_n(x_n, b; Q, \mu) = \tilde{J}_n^{(0)}(\Delta^-)\sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+}, \frac{\Delta^-}{\bar{p}^-}\right)}$$

- Positive rapidity modes

$$\tilde{F}_{n1} = \frac{\alpha_s C_F}{2\pi}\left\{\delta(1-x)\left[\frac{1}{\varepsilon_{\rm UV}^2} - \frac{1}{\varepsilon_{\rm UV}}\ln\frac{Q^2}{\mu^2} + \frac{3}{2\varepsilon_{\rm UV}}\right.\right.$$
$$\left. -\frac{1}{2}L_T^2 + \frac{3}{2}L_T - L_T\ln\frac{Q^2}{\mu^2} - \frac{\pi^2}{12}\right] + (1-x) - L_T\mathcal{P}_{q/q}$$
$$\left. -\mathcal{P}_{q/q}\ln\frac{\Delta^-}{\mu^2} - \frac{1}{4}\delta(1-x) - (1-x)[1+\ln(1-x)]\right\}$$

- *First line:* UV contribution **<u>without</u>** any mixed UV/nUV divergences.
- *Second line:* Matching coefficient of the TMDPDF onto integrated PDF. Does not depend on any nUV regulator!!
- *Third line:* integrated PDF

End of story for TMDPDF!! [EIS, arXiv 1211.1947]

Evolution: Anomalous Dimension

- Applying the RGE to the hadronic tensor we get the AD of the TMDPDF:

$$\tilde{M} = H(Q^2/\mu^2)\, \tilde{F}_{f/P}(x_1, b; Q^2, \mu)\, \tilde{F}_{\bar{f}/\bar{P}}(x_2, b; Q^2, \mu)$$

$$\frac{d\ln\tilde{M}}{d\ln\mu} = 0 = \gamma_H + \gamma_n + \gamma_{\bar{n}}$$

$$\gamma_n = \gamma_{\bar{n}} = -\frac{1}{2}\gamma_H$$

$$\tilde{F}_n(x, b; Q^2, \mu_f^2) = \tilde{F}_n(x, b; Q^2, \mu_i^2) \times \exp\left\{\int_{\mu_i}^{\mu_f} \frac{d\mu'}{\mu'}\, \gamma_n\left(\alpha_s(\mu), \ln\frac{Q^2}{\mu^2}\right)\right\}$$

$$\gamma_H = A(\alpha_s)\ln\frac{Q^2}{\mu^2} + B(\alpha_s)$$

$$\gamma_{n1} = -\frac{1}{2}\gamma_{H1} = \frac{\alpha_s C_F}{2\pi}\left(3 + 2\ln\frac{\mu^2}{Q^2}\right)$$

[Manohar '03]

- A and B are known up to three loops!!
- **We have by free the AD of the TMDPDF at 3-loops!!**

[Moch-Vermaseren-Vogt '04-'05] [Idilbi-Ji-Yuan '05]

Properties...

- We showed explicitly at one-loop that:

$$S_1^{v,DIS} = S_1^{v,DY} + \frac{\alpha_s C_F}{2\pi} \delta^{(2)}(\vec{k}_T)\, \pi^2$$

$$J_{n1}^{v,DIS} = J_{n1}^{v,DY} + \frac{\alpha_s C_F}{2\pi} \delta(1-x) \delta^{(2)}(\vec{k}_{nT}) \pi^2$$

$$S_1^{r,DIS} = S_1^{r,DY} - \frac{\alpha_s C_F}{2\pi} \delta^{(2)}(\vec{k}_T)\, \pi^2$$

$$J_{n1}^{r,DIS} = J_{n1}^{r,DY} - \frac{\alpha_s C_F}{2\pi} \delta(1-x) \delta^{(2)}(\vec{k}_{nT}) \pi^2$$

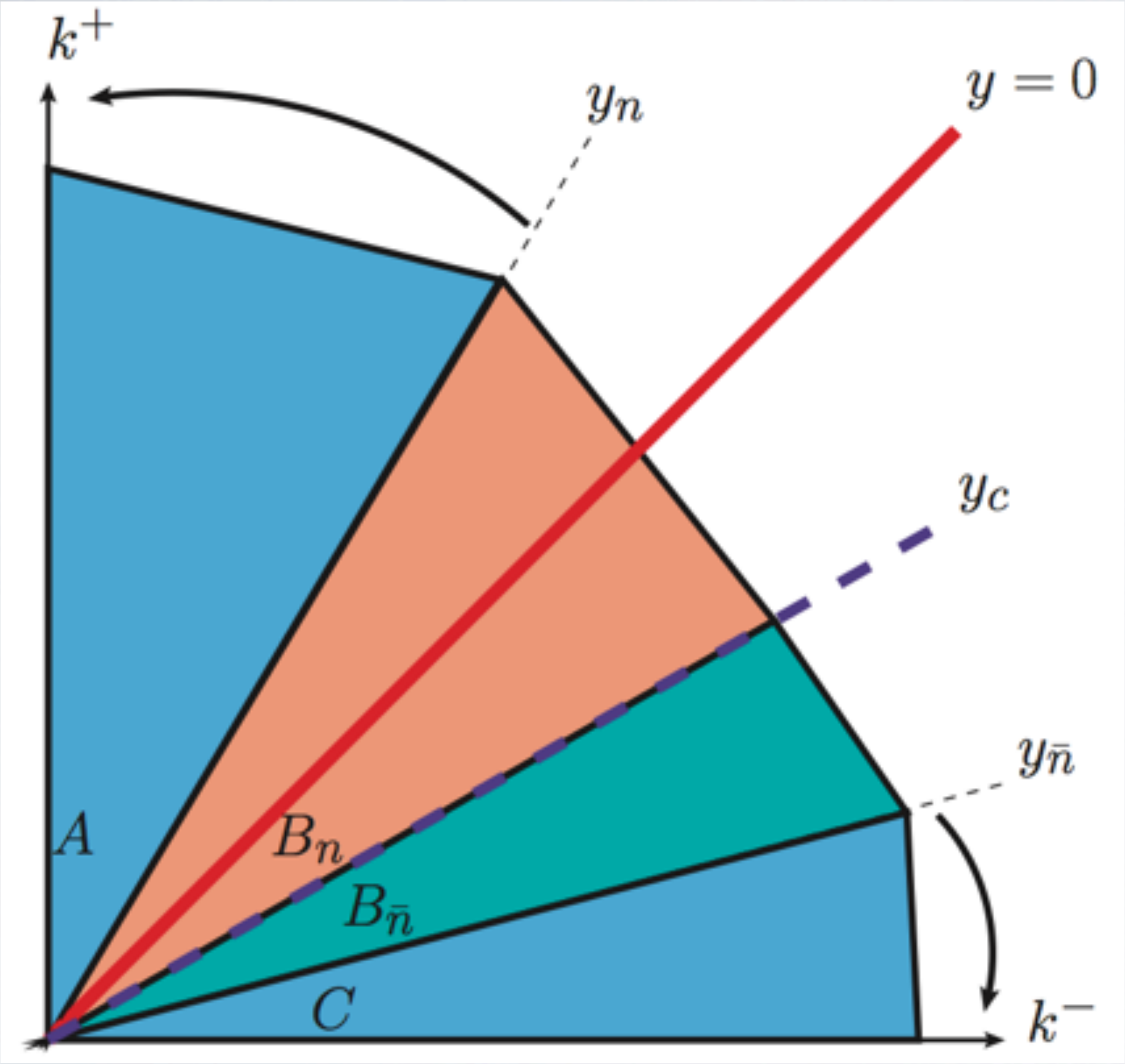
So the TMDPDF is UNIVERSAL!!

(UN-polarized TMDPDF)

- Also that when integrated over the transverse momentum we recover the PDF:

$$\int d^2\vec{k}_{nT}\, F_n(x, \vec{k}_{nT}; Q^2, \mu^2) = f_n(x; \mu)$$

JCC Definition of TMDPDF



$$\tilde{F}_n^{\mathrm{JCC(naive)}}(x_n, b; \sqrt{\zeta_n}, \mu) = \lim_{y_{\bar{n}}\to-\infty} \frac{\tilde{J}_n(y_{\bar{n}})}{\tilde{S}(y_c, y_{\bar{n}})}$$

$$\bar{n} = (-e^{2y_{\bar{n}}}, 1, \vec{0}_\perp)$$
$$n_c = (1, -e^{-2y_c}, \vec{0}_\perp)$$

$$\tilde{J}_n(y_{\bar{n}}) = A + B_n + B_{\bar{n}}$$
$$\tilde{S}(y_c, y_{\bar{n}}) = B_{\bar{n}}$$
$$\tilde{F}_n = A + B_n$$

- If we take into account the Wilson lines' self-energy for finite rapidity y_c, then:

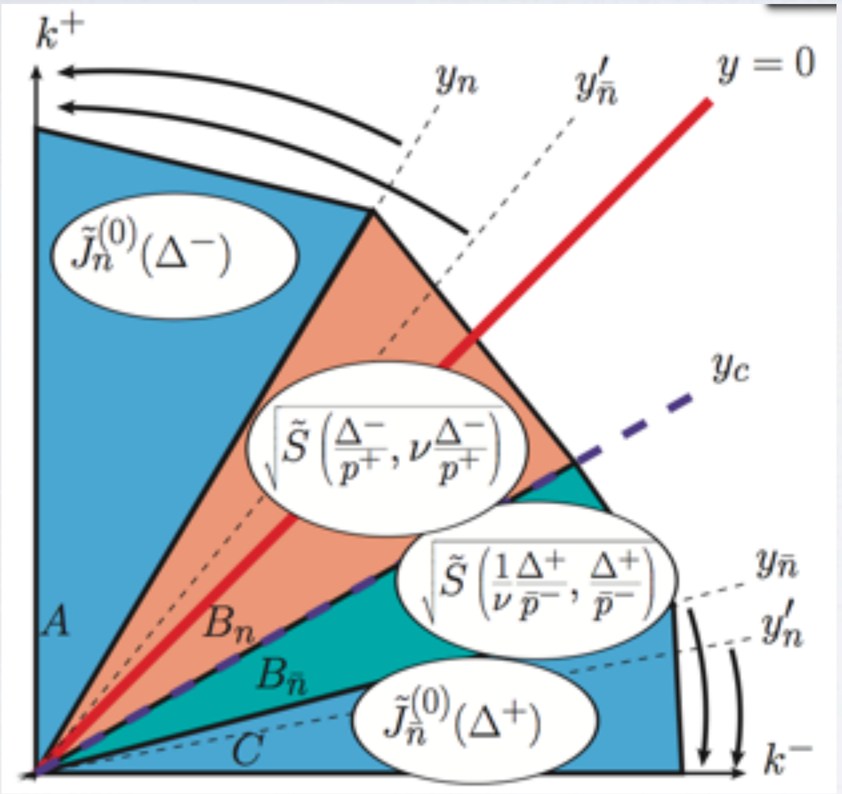
$$\tilde{F}_n^{\mathrm{JCC}}(x_n, b; \sqrt{\zeta_n}, \mu) = \lim_{\substack{y_n\to+\infty \\ y_{\bar{n}}\to-\infty}} \tilde{J}_n(y_{\bar{n}}) \sqrt{\frac{\tilde{S}(y_n, y_c)}{\tilde{S}(y_c, y_{\bar{n}})\,\tilde{S}(y_n, y_{\bar{n}})}}$$

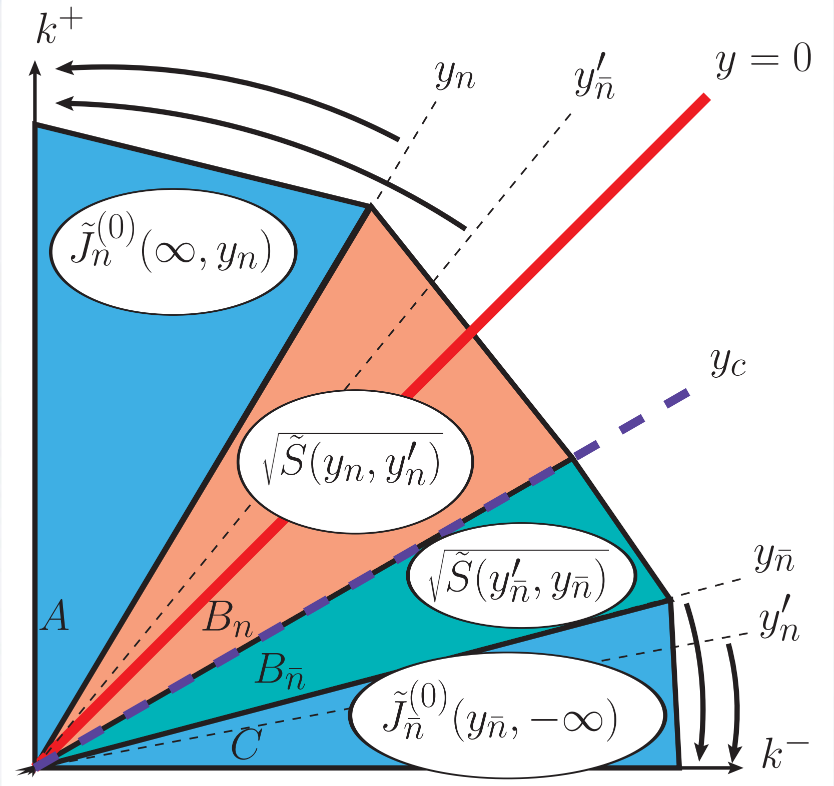
$$\zeta_n = (p^+)^2 e^{-2y_c}$$

JCC and EIS Definitions

$$\lim_{\Delta^-\to 0}\sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+},\nu\frac{\Delta^-}{p^+}\right)}\Bigg|_{\text{EIS}} = \lim_{\substack{y_n\to+\infty\\ y'_n\to-\infty}}\sqrt{\tilde{S}\left(y_n,y'_n\right)}\Bigg|_{\text{EIS}} = \lim_{\substack{y_n\to+\infty\\ y_{\bar{n}}\to-\infty}}\sqrt{\frac{\tilde{S}(y_n,y_c)\,\tilde{S}(y_n,y_{\bar{n}})}{\tilde{S}(y_c,y_{\bar{n}})}}\Bigg|_{\text{JCC}} = B_n$$

$$\lim_{\Delta^+\to 0}\sqrt{\tilde{S}\left(\frac{1}{\nu}\frac{\Delta^+}{\bar{p}^-},\frac{\Delta^+}{\bar{p}^-}\right)}\Bigg|_{\text{EIS}} = \lim_{\substack{y'_{\bar{n}}\to+\infty\\ y_{\bar{n}}\to-\infty}}\sqrt{\tilde{S}\left(y'_{\bar{n}},y_{\bar{n}}\right)}\Bigg|_{\text{EIS}} = \lim_{\substack{y_n\to+\infty\\ y_{\bar{n}}\to-\infty}}\sqrt{\frac{\tilde{S}(y_c,y_{\bar{n}})\,\tilde{S}(y_n,y_{\bar{n}})}{\tilde{S}(y_n,y_c)}}\Bigg|_{\text{JCC}} = B_{\bar{n}}$$





$$y_c = \lim_{\substack{y_n\to+\infty\\ y'_n\to-\infty}}\frac{1}{2}(y_n+y'_n) = \lim_{\substack{y'_{\bar{n}}\to+\infty\\ y_{\bar{n}}\to-\infty}}\frac{1}{2}(y'_{\bar{n}}+y_{\bar{n}}) = \frac{1}{2}\ln\nu$$

JCC and EIS Definitions

- By symmetry in the center of mass frame, and since the TMDPDF is boost-invariant:

$$\boxed{\tilde{F}_n(x_n, b; Q, \mu) = \tilde{J}_n^{(0)}(\Delta^-) \sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+}, \frac{\Delta^-}{\bar{p}^-}\right)}}$$

$$\nu = \frac{p^+}{\bar{p}^-}$$

$$\boxed{\tilde{F}_n^{\rm JCC}(x_n, b; \sqrt{\zeta_n} = Q, \mu) = \lim_{\substack{y_n \to +\infty \\ y_{\bar{n}} \to -\infty}} \tilde{J}_n(y_{\bar{n}}) \sqrt{\frac{\tilde{S}(y_n, y_c = 0)}{\tilde{S}(y_c = 0, y_{\bar{n}})\, \tilde{S}(y_n, y_{\bar{n}})}}}$$

- JCC definition reduces to ours with y_c=0.
- It has no rapidity-regulator! Instead, one combines integraNDS... Hard to apply beyond one-loop!
- Ours can be implemented with other regulators: offshellnesses, nu-regulator,...

Conclusions & Outlook

- We have defined a renormalizable and regularization-scheme free TMDPDF.
- Properties: free from mixed UV/nUV divergences, we can recover the integrated PDF, universal (unpolarized), gauge invariant, boost invariant.
- We know its AD at 3-loops based on the factorization theorem.
- Although equivalent to, simpler to implement than JCC definition

★ The definition of quark-TMDPDFs can be extended to gluon TMDPDFs and quark/gluon TMDFFs.

★ It can also be applied to polarized TMDs (Sivers functions, Collins function, Boer-Mulders function, etc...)

Back up slides

EIS Definition Revisited

- We already showed that:

$$\boxed{\ln \tilde{S} = \mathcal{R}_{s1}(\alpha_s, L_\perp) + \mathcal{R}_{s2}(\alpha_s, L_\perp) \ln \frac{\Delta^- \Delta^+}{Q^2 \mu^2}}$$

$$\boxed{\ln \tilde{S}\left(\frac{\Delta^-}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right) = \frac{1}{2}\ln \tilde{S}\left(\frac{\Delta^-}{p^+}, \frac{\Delta^-}{\bar{p}^-}\right) + \frac{1}{2}\ln \tilde{S}\left(\frac{\Delta^+}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right)}$$

- It can be generalized:

$$\boxed{\ln \tilde{S}\left(\frac{\Delta^-}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right) = \frac{1}{2}\ln \tilde{S}\left(\frac{\Delta^-}{p^+}, \nu\frac{\Delta^-}{p^+}\right) + \frac{1}{2}\ln \tilde{S}\left(\frac{1}{\nu}\frac{\Delta^+}{\bar{p}^-}, \frac{\Delta^+}{\bar{p}^-}\right)}$$

- nu transforms like $(p^+)^2$ under boosts

Splitting of the Soft Revisited

$$\ln\tilde{S}\left(\frac{\Delta^-}{p^+},\frac{\Delta^+}{\bar{p}^-}\right)=\frac{1}{2}\ln\tilde{S}\left(\frac{\Delta^-}{p^+},\nu\frac{\Delta^-}{p^+}\right)+\frac{1}{2}\ln\tilde{S}\left(\frac{1}{\nu}\frac{\Delta^+}{\bar{p}^-},\frac{\Delta^+}{\bar{p}^-}\right)$$

- nu transforms like $(p^+)^2$ under boosts

$$y_n=\ln\frac{\mu p^+}{\Delta^-}\qquad y_{\bar{n}}=\ln\frac{\Delta^+}{\mu\bar{p}^-}$$

$$y'_n=\ln\frac{\nu\Delta^-}{\mu p^+}\qquad y'_{\bar{n}}=\ln\frac{\nu\mu\bar{p}^-}{\Delta^+}$$

$$\lim_{\substack{y_n\to+\infty\\ y_{\bar{n}}\to-\infty}}\tilde{S}(y_n,y_{\bar{n}})=\lim_{\substack{y_n\to+\infty\\ y_{\bar{n}}\to-\infty\\ y'_n\to-\infty\\ y'_{\bar{n}}\to+\infty}}\sqrt{\tilde{S}(y_n,y'_n)}\sqrt{\tilde{S}(y'_{\bar{n}},y_{\bar{n}})}$$

$$y_c=\lim_{\substack{y_n\to+\infty\\ y'_n\to-\infty}}\frac{1}{2}(y_n+y'_n)=\lim_{\substack{y'_{\bar{n}}\to+\infty\\ y_{\bar{n}}\to-\infty}}\frac{1}{2}(y'_{\bar{n}}+y_{\bar{n}})=\frac{1}{2}\ln\nu$$