Mass modes and secondary massive quark radiation II

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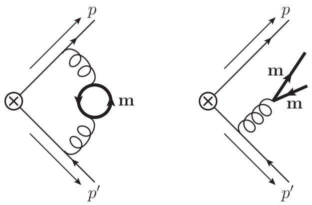
Outline

Outline

Aim

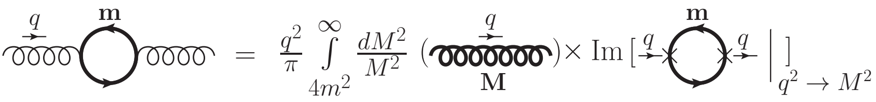
- **PART I**: technique for mass modes in SCET, massive gauge boson example
  arxiv:1302.4743
- **PART II** : two-loop extension $\Rightarrow$ *secondary* heavy quark radiation



- *primary* quarks massless
- to be specific $n_m = 1$
- factorization theorems at two-loop (same structure as massive gauge boson case, but some complications)
- quark bubble ($C_F T_F n_f$) contributions to $\mathcal{C}(Q, m, \mu)$, $J(s, m, \mu)$ and $S(\ell, m, \mu)$
  massless case $\longrightarrow$ Moch et al. (2005), Becher et al. (2007),
  Kelley et al. (2011), Hornig et al. (2011)

Dispersion relation

Observation: interpret fermion bubble insertion as "gluon" with massive propagator
Hoang (1995), PhD thesis



$$\Pi^{\text{eff}}_{\mu\nu}(q^2) \equiv \frac{(-i)^2 g_{\mu\rho}\Pi^{\rho\sigma}(q^2)g_{\sigma\nu}}{(q^2+i\epsilon)^2} = \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{dM^2}{M^2}\text{Im}\left[\Pi(M^2)\right]\frac{-i\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)}{q^2-M^2+i\epsilon}$$

- calculate one-loop massive gauge boson result
- convolute with Im $\left[\Pi(M^2)\right]$
- use same dispersive integral for full theory, collinear and soft sector
- convolution in $d = 4 - 2\epsilon$!!! (but for finite parts $d = 4$)

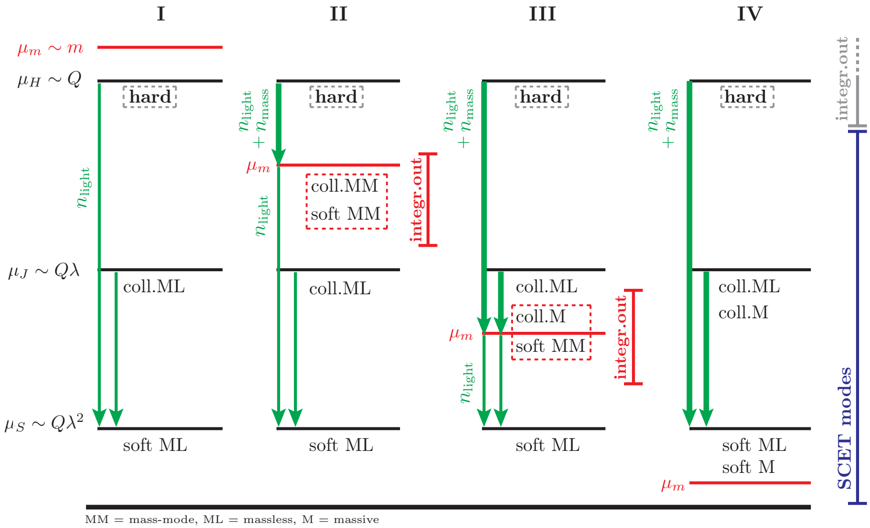
- Dispersive technique allows to address difficulties separately:
  1. separation of dynamical modes depending on m vs μ_H, μ_J, μ_S $\Rightarrow$ treated in massive “gluon” context
  2. secondary quark radiation and its influence on RG evolutions $\Rightarrow$ taken care of after final convolution
- **Part I**: Soft-bin subtraction can be carried out completely in the massive “gluon” context at $\mathcal{O}(\alpha_s)$.

  $\Downarrow$

  No further soft-bin subtraction needed at $\mathcal{O}(\alpha_s^2)$. Convolution already regulated in dim. reg.
- **Beware:** setup is general, but dispersive treatment suitable for observables depending on invariant mass of secondary fermion pair

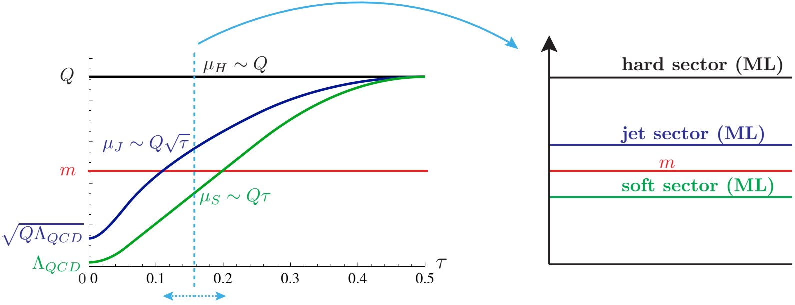
Setup

fermion mass ⇒ additional scale parameter, distinguishes four scenarios



Important features at one-loop

- several scenarios needed for one single thrust distribution



- patch four effective theories **continuously**

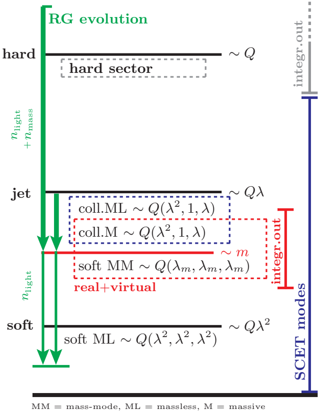
Important features at one-loop

- several scenarios needed for one single thrust distribution
- full explicit mass dependence (up to the order considered) of the singular contributions
- same anomalous dimension as in $M \to 0$ limit $\Rightarrow$ evolution factors affected only by number of massive gauge bosons
- reach all massless limits for hard, jet and soft functions smoothly
- interesting consistency condition $\to$ possible to rearrange mass-mode contributions into different functions depending on the choice of μ

$$2\,\mathrm{Re}\left[\delta F_{\mathrm{eff}}(Q,M,\mu)\right]\delta(\tau) - Q^2\delta J_m^{\mathrm{virt}}(Q^2\tau,M,\mu) - Q\delta S_m^{\mathrm{virt}}(Q\tau,M,\mu) = 0$$

Issues at two-loop

- Decoupling of α_s in evolution



- at scale μ_m integrate out massive flavours $\Rightarrow \alpha_s^{n_{\text{light}}+1} \longrightarrow \alpha_s^{n_{\text{light}}}$
- terms of the form

$$\frac{4}{3}\frac{T_F}{4\pi}\log\left(m^2/\mu_m^2\right) \times \text{one loop}$$

appear in factorization theorem

- Subtracted vs unsubtracted dispersive relation
- fixed order full QCD results not calculated yet

Issues at two-loop

- Decoupling of α_s in evolution
  $\Downarrow$
- Subtracted vs unsubtracted dispersive relation
  - unsubtracted and unrenormalized ($\overline{MS}$ scheme)

$$\Pi(q^2) \quad = \quad \frac{1}{\pi}\int_{4m^2}^{\infty} dM^2 \frac{1}{M^2 - q^2 - i\epsilon}\ \mathrm{Im}\left[\Pi(M^2)\right]$$

$$\text{use: } \Pi(q^2) - \frac{4}{3}\frac{1}{\epsilon}$$

  - heavy quark contributes to RGE
  - subtracted (on-shell scheme)

$$\Pi^{\mathrm{os}}(q^2) = \Pi(q^2) - \Pi(0) = \frac{q^2}{\pi}\int_{4m^2}^{\infty} \frac{dM^2}{M^2} \frac{1}{M^2 - q^2 - i\epsilon}\ \mathrm{Im}\left[\Pi(M^2)\right]$$

  - heavy quark does not contribute to any RGE

Use of unsubtracted relations crucial to cancel unwanted mass terms

- fixed order full QCD results not calculated yet
- Soft function with hemisphere (thrust) prescription is complicated

Issues at two-loop

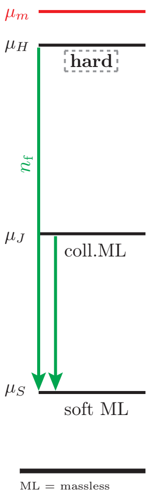
- Decoupling of α_s in evolution
- Subtracted vs unsubtracted dispersive relation
- fixed order full QCD results not calculated yet
  - virtual terms given by vertex function $F_{QCD}(Q^2, m^2)$
    Kniehl (1990), Hoang (1995)
  - and SCET gives all the singular terms
- Soft function with hemisphere (thrust) prescription is complicated

Issues at two-loop

- Decoupling of α_s in evolution
- Subtracted vs unsubtracted dispersive relation
- fixed order full QCD results not calculated yet
- Soft function with hemisphere (thrust) prescription is complicated
  - $S(k_L, k_R, m, \mu)$ not constrained by secondary quark invariant mass
  - $\Rightarrow$ dispersive approach only give correct UV and log structure

Scenario I : $\lambda_m > 1 > \lambda > \lambda^2$

heavy quark integrated out at QCD level $\Rightarrow \mathcal{C}^I(Q, m, \mu_H)$



$$\frac{d\sigma}{d\tau} = Q\sigma_0 \left|\mathcal{C}^I(Q, m, \mu_H)\right|^2 U_H^{(n_f)}(Q, \mu_H, \mu_S) \int ds \int ds' \times U_J^{(n_f)}(s - s', \mu_S, \mu_J) J_0(s', \mu_J) S_0\left(Q\tau - \frac{s}{Q}, \mu_S\right)$$

$$U_i^{(n_f)}(Q, \mu_H, \mu_S) = \text{massless RG factors, } n_f \text{ light flavours}$$

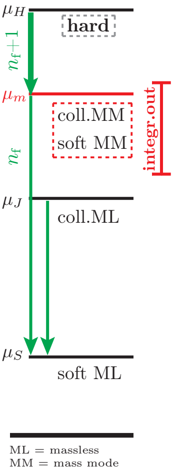
$$\mathcal{C}^I(Q, m, \mu) = 1 + \alpha_s^{(n_f)} C_0^{(1)} + (\alpha_s^{(n_f)})^2 \left(C_0^{(2)} + F_{\mathrm{QCD}}^{(2)}(Q, m)\Big|^{\mathrm{OS}}\right)$$

Hoang (1995)

- $F_{\mathrm{QCD}}^{(2)}(Q, m \to \infty)\Big|^{\mathrm{OS}} \to 0$ (decoupling)
- $F_{\mathrm{QCD}}^{(2)}(Q, m \to 0)\Big|^{\mathrm{OS}} \to a_1 \ln^3(-x) + a_2 \ln^2(-x) + a_3 \ln(-x) + \ldots$

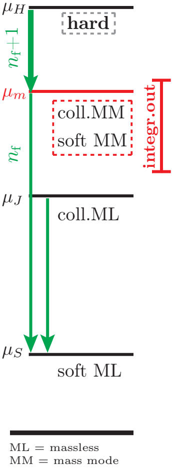
$$x = m^2/Q^2$$

Scenario II : $1 > \lambda_m > \lambda > \lambda^2$



$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(Q,m,\mu_H)\right|^2 U_{H_Q}^{(n_f+1)}(Q,\mu_H,\mu_m)\left|\mathcal{M}_{H_Q}(Q,m,\mu_m)\right|^2$$
$$\times U_{H_Q}^{(n_f)}(Q,\mu_m,\mu_S)\int ds\int ds' U_J^{(n_f)}(s-s',\mu_S,\mu_J)$$
$$\times J_{0,\tau}(s',\mu_J)S_{0,\tau}\left(Q\tau-\frac{s}{Q},\mu_S\right)$$

Scenario II : $1 > \lambda_m > \lambda > \lambda^2$

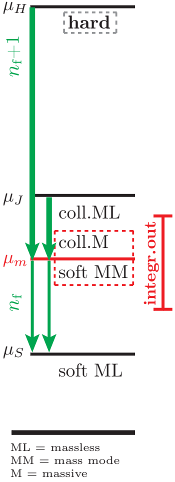


$$\begin{aligned}
\mathcal{C}^{II}(Q,m,\mu) \;&=\; 1 + C_0^{(1)}\alpha_s^{(n_f+1)} + (\alpha_s^{(n_f+1)})^2 \\
&\quad \times \left(C_0^{(2)} + C_m^{(2)}(Q,m,\mu) \right)
\end{aligned}$$

$$\mathcal{M}_{H_Q}(Q,m,\mu_m) \;=\; 1 + (\alpha_s^{(n_f+1)})^2 \left[F_{\mathrm{SCET}}^{(2)}(Q,m,\mu_m)\Big|^{\overline{\mathrm{MS}}} + \alpha_s \text{ dec.} \right]$$

- $C_m^{(2)}(Q,m,\mu) \rightarrow$ mass modes contributions
- in $C_m^{(2)}(Q,m,\mu)$ cancellation of divergences takes place
- sum and subtract same terms at different scales (μ_H vs μ_m)
- continuous transition scenario I $\leftrightarrow$ II

Scenario III : $1 > \lambda > \lambda_m > \lambda^2$

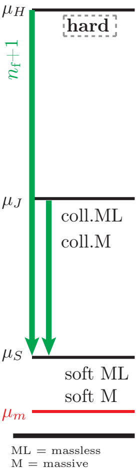


$$
\begin{aligned}
\frac{d\sigma}{d\tau} \quad \sim \quad & \left|\mathcal{C}^{II}(Q,m,\mu_H)\right|^2 U_{H_Q}^{(n_f+1)}(Q,\mu_H,\mu_m)\left|\mathcal{M}_{H_Q}(Q,m,\mu_m)\right|^2 \\
& \times U_{H_Q}^{(n_f)}(Q,\mu_m,\mu_S)\int ds\, ds'\, ds''\, dt\, J_{0+m}(s'',m,\mu_J) \\
& \times U_J^{(n_f+1)}(s'-s'',\mu_m,\mu_J)\mathcal{M}_J(t,m,\mu_m)U_J^{(n_f)}(s-s',\mu_S,\mu_m) \\
& \times S_0\left(Q\tau - s/Q - t/Q,\mu_S\right)
\end{aligned}
$$

$$
\begin{aligned}
J_{0+m}(s,m,\mu) \; &= \; 1+\alpha_s^{(n_f+1)}J_0^{(1)} + (\alpha_s^{(n_f+1)})^2\Big(J_0^{(2)} \\
& \qquad + J_m^{\mathrm{virt},(2)}(s,m,\mu)\Big|^{\overline{\mathrm{MS}}} + \theta(s-4m^2)J_m^{\mathrm{real},(2)}(s,m^2)\Big) \\
\mathcal{M}_J(s,m,\mu_m) \; &= \; \delta(s) + (\alpha_s^{(n_f+1)})^2\left[-J_m^{\mathrm{virt},(2)}(s,m,\mu_m)\Big|^{\overline{\mathrm{MS}}} + \alpha_s \text{ dec.}\right]
\end{aligned}
$$

- match before real radiation sets in (i.e. $\tau_m < 4m^2/Q^2$)
- $J_m^{\mathrm{real},(2)}(s,m^2) = 0$ at threshold

Scenario IV: $1 > \lambda > \lambda^2 > \lambda_m$



$$
\begin{aligned}
\frac{d\sigma}{d\tau} \quad \sim \quad & \left|\mathcal{C}^{II}(Q,m,\mu_H)\right|^2 U_{H_Q}^{(n_f+1)}(Q,\mu_H,\mu_S) \int ds\, ds' \\
& U_J^{(n_f+1)}(s-s',\mu_S,\mu_J) J_{0+m}(s',m,\mu_J) \\
& \times S_{0+m}\left(Q\tau - \frac{s}{Q}, m, \mu_S\right)
\end{aligned}
$$

$$
S_m^{(2)}(\ell,m,\mu) = S_{\mathrm{m}}^{\mathrm{virt},(2)}(\ell,m,\mu) + \theta(\ell-2m) S^{\mathrm{real},(2)}(\ell,m)
$$

- consistency relation:

$$
\begin{aligned}
0 \quad = \quad & 2\mathrm{Re}\left[F_{\mathrm{SCET}}\Big|^{\overline{\mathrm{MS}}}\right]\delta(\tau) - Q^2 J_m^{\mathrm{virt},(2)}\Big|^{\overline{\mathrm{MS}}} - Q S_m^{\mathrm{virt},(2)}\Big|^{\overline{\mathrm{MS}}} \\
& + \frac{4}{3}\ln\left(\frac{m^2}{\mu_m^2}\right)\left\{2\mathrm{Re}\left[C_0^{(1)}\right]\delta(\tau) + Q^2 J_0^{(1)} + Q S_0^{(1)}\right\}
\end{aligned}
$$

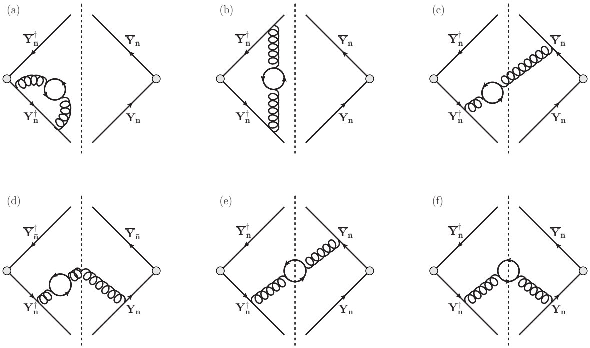
- $\Rightarrow$ continuous transition III $\leftrightarrow$ IV

Outline

1. From one-loop to two-loop
2. Soft function with mass modes
3. Effects on thrust
4. Summary & Outlook

Calculation for the soft function

- $m = 0$ calculation Kelley et al. (2011), Hornig et al. (2011)

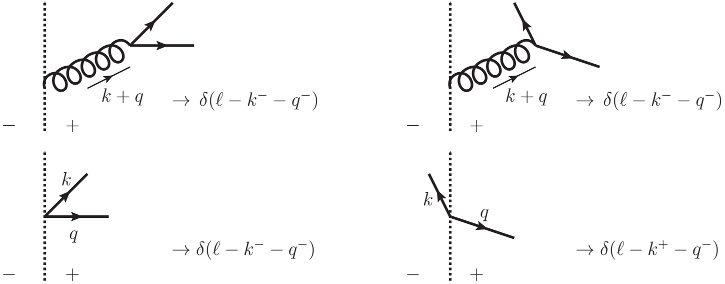


Remember: dispersive technique suitable for observables depending on fermion pair invariant mass $(k+q)^2$

"*quark hemisphere*" prescription $\Rightarrow$ k and q treated separately if both are in different hemispheres: $S^{(QQ)}(k_L, k_R, m, \mu)$

Prescriptions

First approximation: "*gluon hemisphere*" prescription $\Rightarrow k+q$: $\quad S^{(g)}(k_L, k_R, m, \mu)$



same hemisphere: $k^+ > k^-$, $q^+ > q^-$

opposite hemisphere: $k^- > k^+$, $q^+ > q^-$
$k^+ + q^+ > k^- + q^-$

- quarks in same hemisphere $\Rightarrow S^{(g)}(k_L, k_R, m, \mu)\big|_{\text{same}} = S^{(QQ)}(k_L, k_R, m, \mu)\big|_{\text{same}}$
- quarks in opposite hemispheres

$$\Rightarrow S^{(g)}(k_L, k_R, m, \mu)\big|_{\text{opp}} \neq S^{(QQ)}(k_L, k_R, m, \mu)\big|_{\text{opp}}$$

Method

- $S^{(g)}(k_L, k_R, m, \mu)$: same UV divergences of $S^{(QQ)}(k_L, k_R, m, \mu)$ (RG consistency)
-

$$S^{(QQ)}(k_L, k_R, m, \mu) = S^{(g)}(k_L, k_R, m, \mu) + \underbrace{\left(S^{(QQ)}(k_L, k_R, m, \mu)\Big|_{\text{opp}} - S^{(g)}(k_L, k_R, m, \mu)\Big|_{\text{opp}} \right)}_{\text{finite}=\Delta S(k_L, k_R, m)}$$

- $\Delta S(k_L, k_R, m)$ not easy analytically
- $\Delta S(k_L, k_R, m)$ finite! so numerically doable in 4 dimensions

Thrust soft function at two-loop

$$S_\tau(\ell, m, \mu) = \int dk_L \, dk_R \; S(k_L, k_R, m, \mu) \, \delta(\ell - k_L - k_R)$$

$$\begin{aligned} S_\tau^{(QQ),(2)}(\ell, m, \mu) \sim \alpha_s^2 \Bigg\{ & \delta(\ell) \left[\frac{1}{18} L_m^3 - \frac{5}{18} L_m^2 + \left(\frac{28}{27} - \frac{\pi^2}{18} \right) L_m + \ldots \right] \\ & + \frac{1}{\mu} \left[\frac{\mu \, \theta(\ell)}{\ell} \right]_+ \left(\frac{1}{3} L_m^2 - \frac{10}{9} L_m + \frac{28}{27} \right) \\ & + \frac{1}{\mu} \left[\frac{\mu \, \theta(\ell) \ln(\ell/\mu)}{\ell} \right]_+ \frac{4}{3} L_m \Bigg\} \\ & + \theta(\ell - 2m) S^{\mathrm{real},(2)}(\ell, m) + \Delta S_\tau(\ell, m) \end{aligned}$$

- $L_m \equiv \ln(m/\mu)$

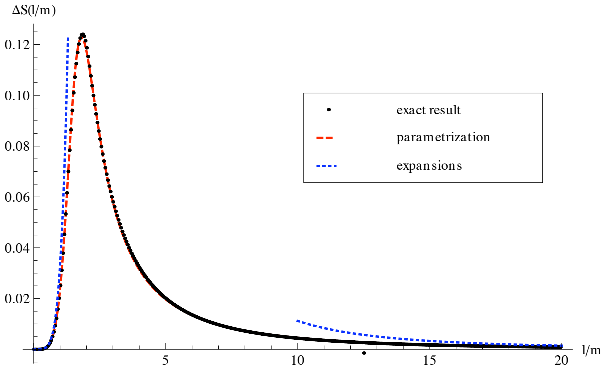
- $S_\tau^{(QQ),(2)}(\ell, m, \mu)$ reaches the massless limit
- same anomalous dimension as massless function
- massive quark radiation described by $\theta(\ell - 2m) S^{\mathrm{real},(2)}(\ell, m)$ (as in jet function)
- also $\Delta S_\tau(\ell, m)$ corresponds to real radiation

The correction $\Delta S_\tau(\ell, m)$

- crucial to find correct massless limit

$$\Delta S_\tau(\ell, m) \overset{m\to 0}{\longrightarrow} \delta(\ell) \times \left\{ -\frac{4}{9} + \frac{13\pi^2}{54} - \frac{4\zeta(3)}{3} \right\}$$

  checked numerically and analytically!
- contains no thresholds even though describes real radiation
- numerically small $< 5\%$ of $S_\tau(\ell, m, \mu)$
- analytical asymptotic expansions



- fit to a Breit Wigner type function (4 free parameters)
- 7 parameters constrained with asymptotic expansions and normalization

$$\Delta S_\tau(x) \sim \frac{1}{m}\frac{(ax)^\alpha}{(1+(ax)^\beta)^{\gamma/\beta}}\left[b\,\log^2\left(1+Ax+Bx^2\right)+c\,\log\left(1+Cx+Dx^2\right)+d\right]$$

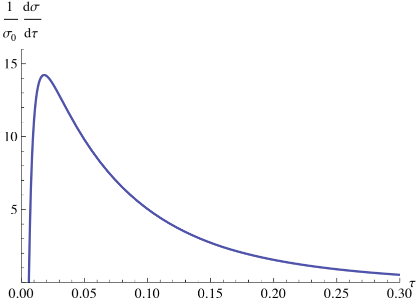
$$x \equiv \ell/m$$

Outline

Plots for $Q = 14, 35$ GeV

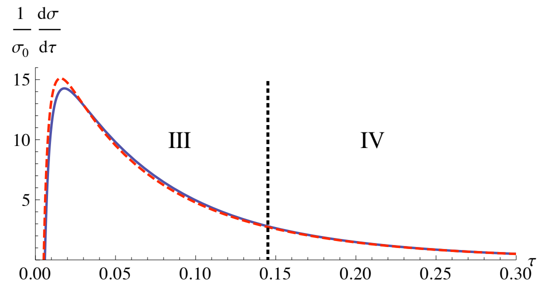
- numerical code for thrust distribution at $\mathrm{N}^3\mathrm{LL}$
- **preliminary**, so far no non-perturbative physics!
- $Q = 14, 35$ GeV $\leftrightarrow$ data from PETRA
- determination of $\alpha_s(M_z)$: $Q = 35...207$ GeV Abbate et al. (2011, 2012)
- massless: 5 light flavours vs. massive: 4 light + 1 massive b ($m_b = 4.2$ GeV)

Thrust distribution: massive ($Q = 14$ GeV, $\alpha_s(M_z) = 0.118$)

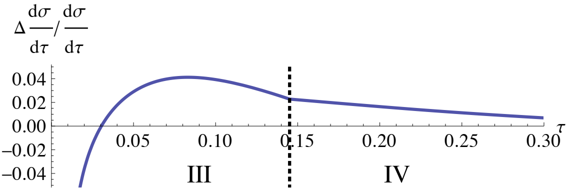


$Q = 14$ GeV

Thrust distribution: massive vs. massless

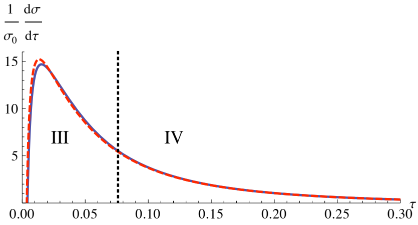


relative mass effects

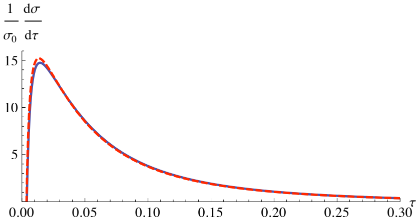


$Q = 35$ GeV: possible effect on α_s

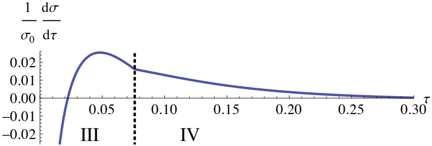
massive vs. massless



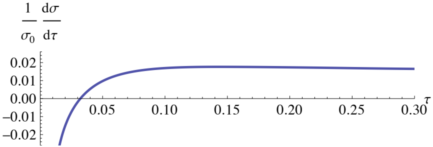
$\alpha_s(M_z) = 0.119$ vs. $\alpha_s(M_z) = 0.118$



relative mass effects



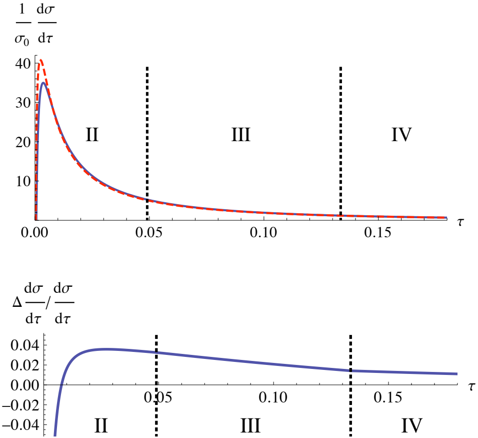
relative correction



$Q = 500$ GeV

- $Q = 500$ GeV $\leftrightarrow$ ILC
- massless: 6 light flavours vs. massive: 5 light + 1 massive t ($m_t = 175$ GeV)

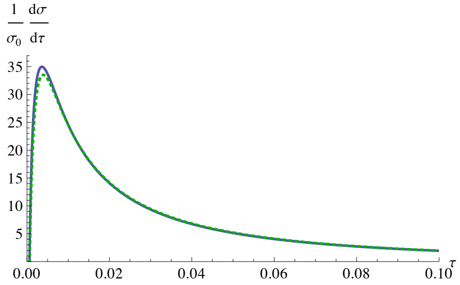
Thrust distribution: massive vs. massless

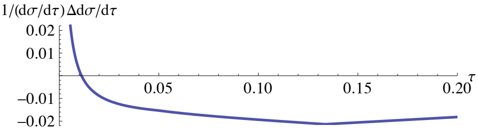


$Q = 500$ GeV (decoupling)

- $Q = 500$ GeV $\leftrightarrow$ ILC
- massless: 5 light flavours vs. massive: 5 light + 1 massive t ($m_t = 175$ GeV)

Thrust distribution: massive vs. massless



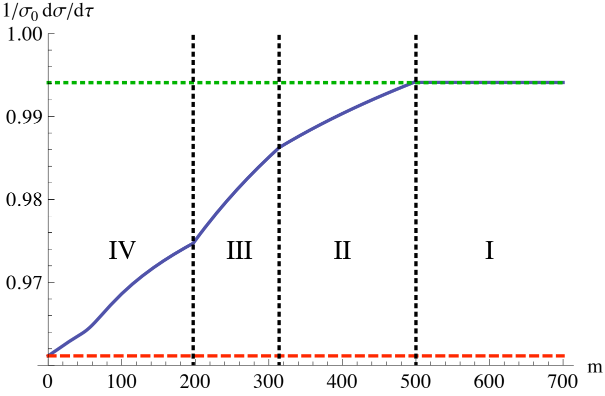


$Q = 500$ GeV: Consistency check

Thrust distribution: $\tau = 0.15$ fixed, vary mass m

massless limit (6 flavours): dashed

decoupling limit (5 flavours): dotted



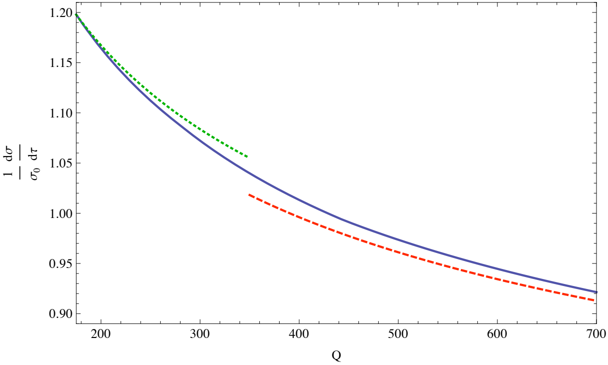
continuous transitions and correct limiting behaviour!

Extreme approach

Thrust distribution: $\tau = 0.15$, $m_t = 175$ GeV fixed, vary Q

massless limit (6 flavours): dashed

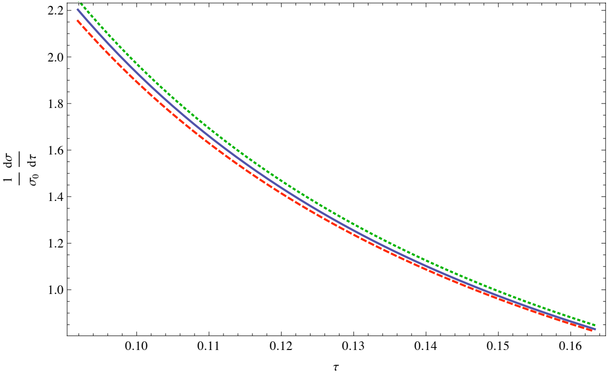
decoupling limit (5 flavours): dotted



Extreme approach vs ours

- $Q = 500$ GeV
- $\tau \approx m_t^2/Q^2$

Thrust distribution: $m_t = 175$ GeV fixed, vary Q
massless limit (6 flavours): dashed
decoupling limit (5 flavours): dotted



deviation from extreme approach can be up to $\approx 4\%$

Outline

Summary & Outlook

- inclusion of heavy quark masses important for high precision collider physics
- setup for secondary massive quarks in terms of mass modes
- calculation of all ingredients for thrust distribution at N^3LL
- **NEXT:**
  - double hemisphere soft function $S^{(QQ)}(k_L, k_R, m, \mu)$
  - renormalon analysis
  - possible applications
    1. bottom mass effects in α_s determination at N^3LL
    2. analysis of LEP data (at $Q = 14$ GeV) and data from B factories
    3. gluino bounds Kaplan & Schwartz (2008)
    4. massive effects for parton distribution functions in heavy quark production
    5. hard photoproduction with a heavy quark jet
    6. ...

A note on QCD form factor

Use of two different dispersive relations $\Rightarrow F_{\text{QCD}}(Q, m, \mu)\Big|^{\overline{\text{MS}}}$ vs $F_{\text{QCD}}(Q, m, \mu)\Big|^{\text{OS}}$

$$F^{(2)}_{\text{QCD}}(Q, m, \mu)\Big|^{\overline{\text{MS}}} = F^{(2)}_{\text{QCD}}(Q, m)\Big|^{\text{OS}} - \underbrace{\left(\Pi(0) - \frac{4}{3}\frac{1}{\epsilon}\right) \times (\text{1-loop QCD (in d-dim!)})}_{(\text{OS}\leftrightarrow\overline{\text{MS}})}$$

- $(\text{OS} \leftrightarrow \overline{\text{MS}})$ contains IR divergences which exactly cancel those from SCET diagrams

- "quark prescription" :

$$
\begin{aligned}
F(k_R, k_L) = (-2\pi i)^2\, \delta(k^2)\, \delta(q^2)\, \Theta(k^+ + k^-)\, \Theta(q^+ + q^-) \\
\times \big[\Theta(k^+ - k^-)\, \Theta(q^- - q^+)\, \delta(q^+ - k_R)\, \delta(k^- - k_L) \\
+ \Theta(k^- - k^+)\, \Theta(q^+ - q^-)\, \delta(k^+ - k_R)\, \delta(q^- - k_L) \\
+ \Theta(k^- - k^+)\, \Theta(q^- - q^+)\, \delta(k^+ + q^+ - k_R)\, \delta(k_L) \\
+ \Theta(k^+ - k^-)\, \Theta(q^+ - q^-)\, \delta(k^- + q^- - k_L)\, \delta(k_R) \big]
\end{aligned}
$$

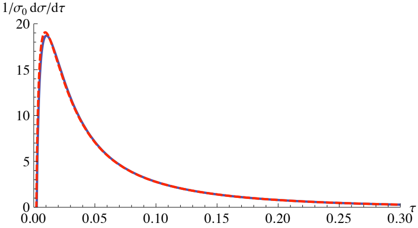
- "gluon prescription"

$$
\begin{aligned}
F^{(g)}(k_R, k_L) = (-2\pi i)^2\, \delta(k^2)\, \delta(q^2)\, \Theta(k^+ + k^-)\, \Theta(q^+ + q^-) \\
\times \big[\Theta(k^+ + q^+ - k^- - q^-)\, \delta(k_R)\, \delta(k^- + q^- - k_L) \\
+ \Theta(k^- + q^- - k^+ - q^+)\, \delta(k_L)\, \delta(k^+ + q^+ - k_R) \big]
\end{aligned}
$$

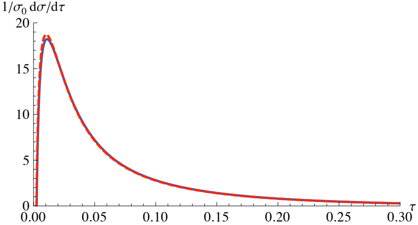
- $k_L, k_R =$ light cone components of left, right hemispheres momenta

$Q = 91$ GeV: possible effect on α_s

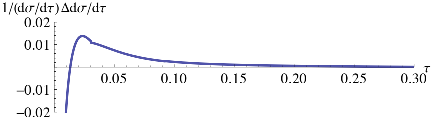
massive vs. massless



$\alpha_s(M_z) = 0.119$ vs. $\alpha_s(M_z) = 0.118$



relative mass effects



relative correction