Grating Optimisation for Generation of Smith-Purcell Radiation

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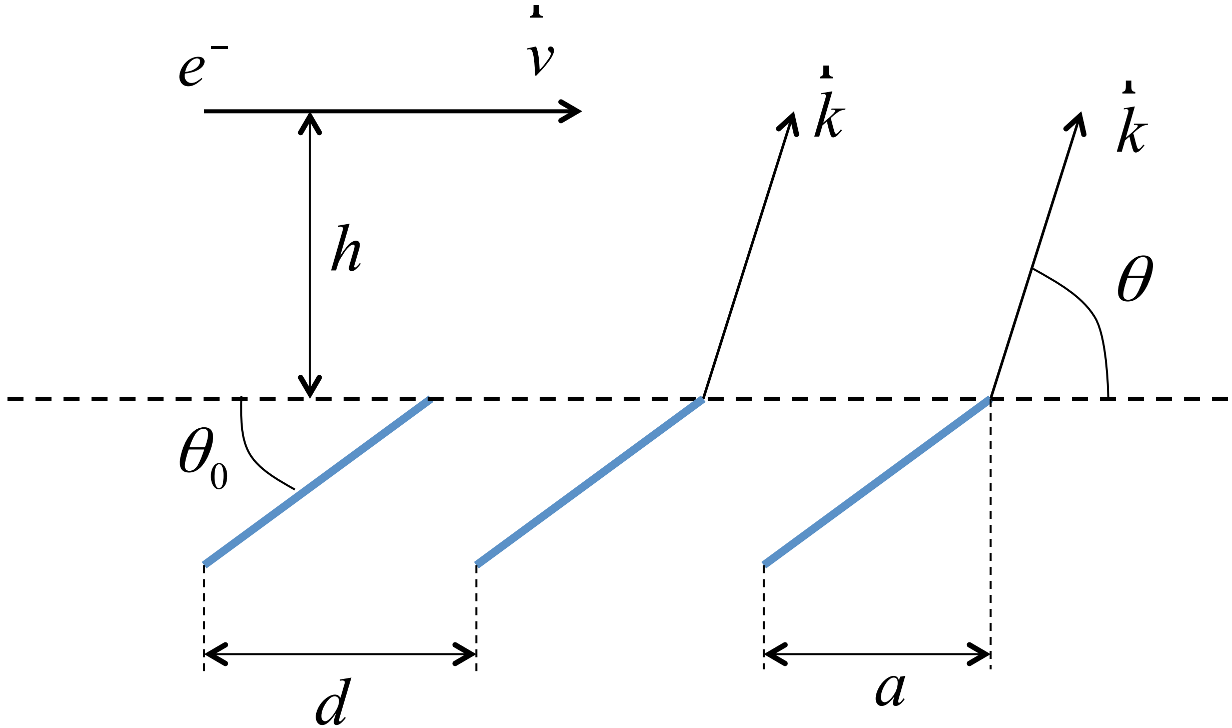
In order to achieve optimal performance of the radiation generation from a blind grating the following parameters have to be analysed:

- **Propagation of power**
  - **From each individual strip the power propagates in the mirror reflection direction**

- **Interference factor of the grating**
  - **defined by the dispersion relation;**
  - **responsible for the interference between multiple strips;**

- **Structural factor of the grating**
  - **Defined by the relation between the strip width;**
  - **Responsible for the diffraction effect, which is stronger at longer wavelength**



◆ **Propagation of power**

$$\theta = 2\theta_0$$

◆ **Interference factor of the grating**

$$\lambda_n = \frac{d}{n}\left(\frac{1}{\beta} - \cos\theta\right)$$

◆ **Structural factor of the grating**

$$ratio = d/a$$

Theoretical model

Smith – Purcell effect as resonant diffraction radiation, A.P. Potylitsyn, NIM B 145 (1998) 60 – 66.

Approximations used in the model:

- **Far-field approximation:**
  - **In mm-wavelength range and for some cases of SPR this approximation is not applicable;**
- **Infinitely thin strips:**
  - **Shadowing of the strips by each other is not taken into consideration**
- **Ideal conductor;**
- **Infinite strip length;**
- **Strip width must be much larger than the wavelength.**

$$\frac{d^2W_{RDR}}{d\omega d\Omega} = \frac{d^2W_{semiplane}}{d\omega d\Omega} F_{cell} F_N$$

Radiation distribution from a semiplane

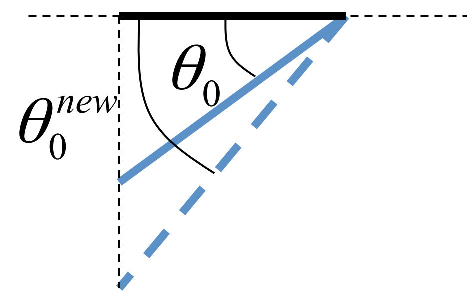
Strip (cell) geometry factor

Interference factor

$$F_{cell} = 4\exp[-2\chi]\left(\sinh^2\chi + \sin^2\frac{\Delta\varphi}{2}\right)$$

$$\chi = \frac{\pi(a/\cos\theta_0)\sin\theta_0}{\gamma\lambda}\sqrt{1+\gamma^2\theta_x^2}$$

$$\Delta\varphi = \frac{2\pi(a/\cos\theta_0)}{\lambda}\left[\cos(\theta_y - \theta_0) - \frac{\cos\theta_0}{\beta}\right]$$

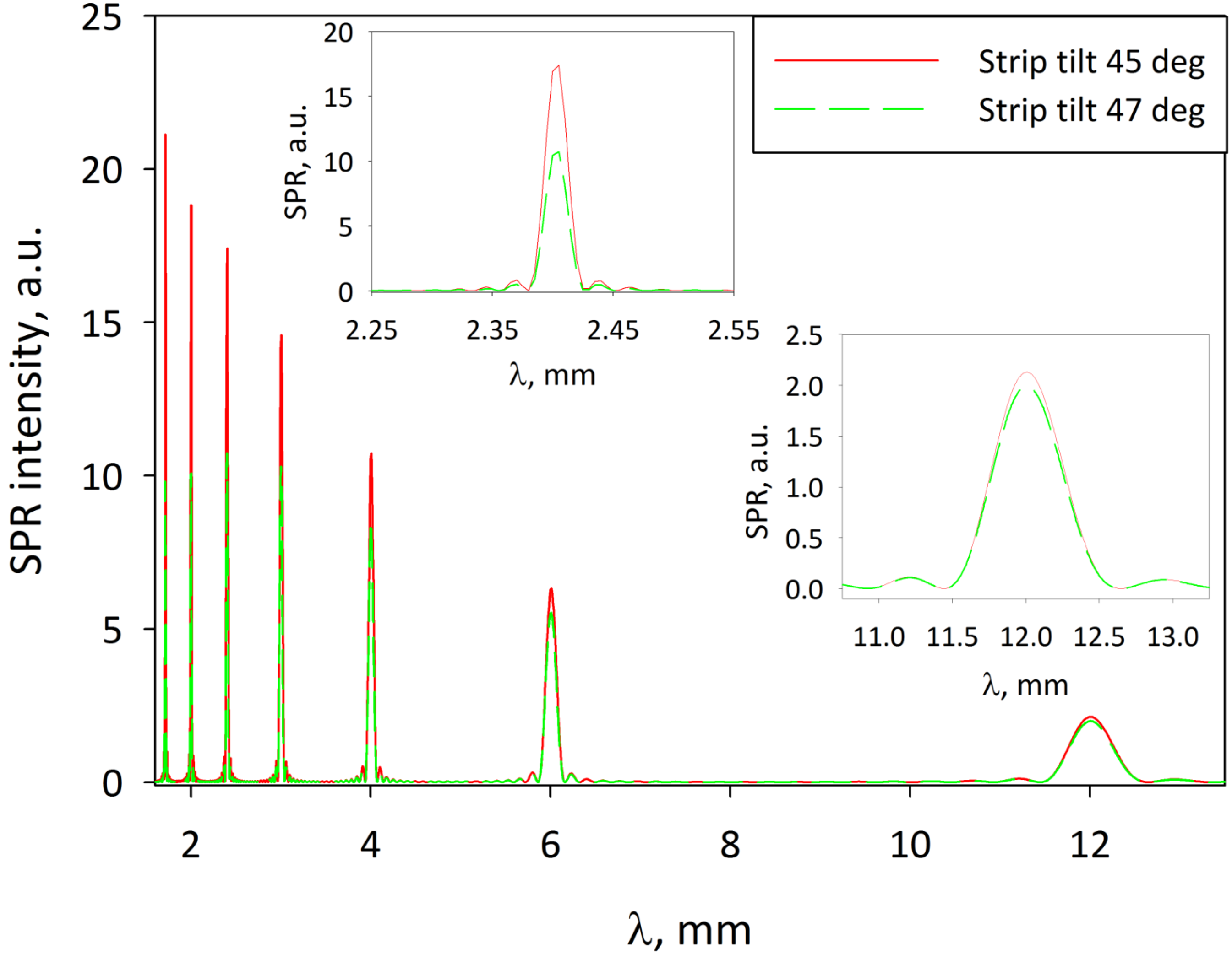


$$\theta_x = 0; \theta_y = 2\theta_0$$

Direction of the mirror reflection from a strip.

Optimization of the strip tilt angle

θ_x =0 deg;
θ_y =90 deg;
γ=20;
h=0.15 mm;
a=d=12 mm;
N=20



Optimization of the strip tilt angle

θ_x =0 deg.;
θ_y =90 deg.;
γ=20;
h=0.15 mm;
a=d=12 mm;
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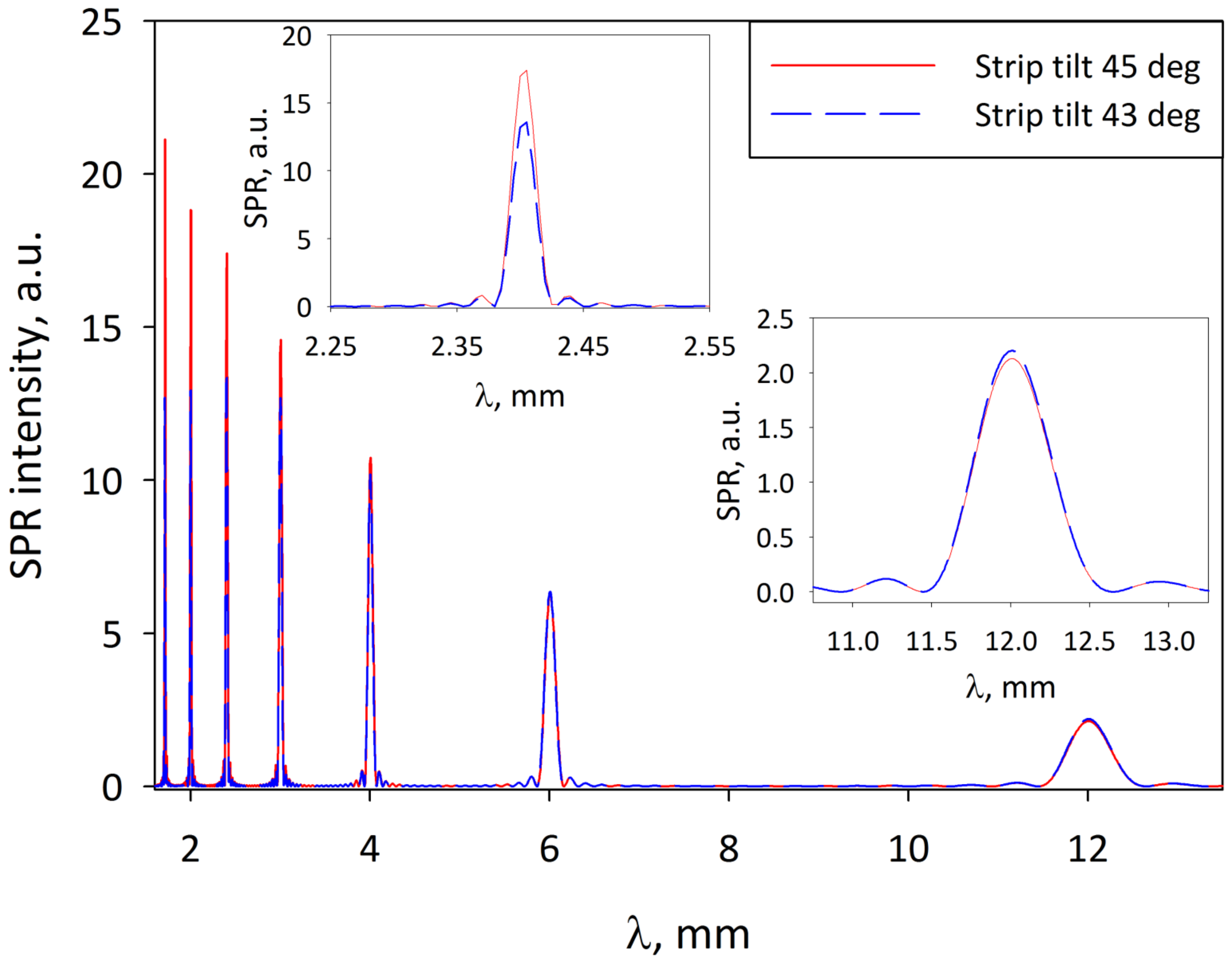
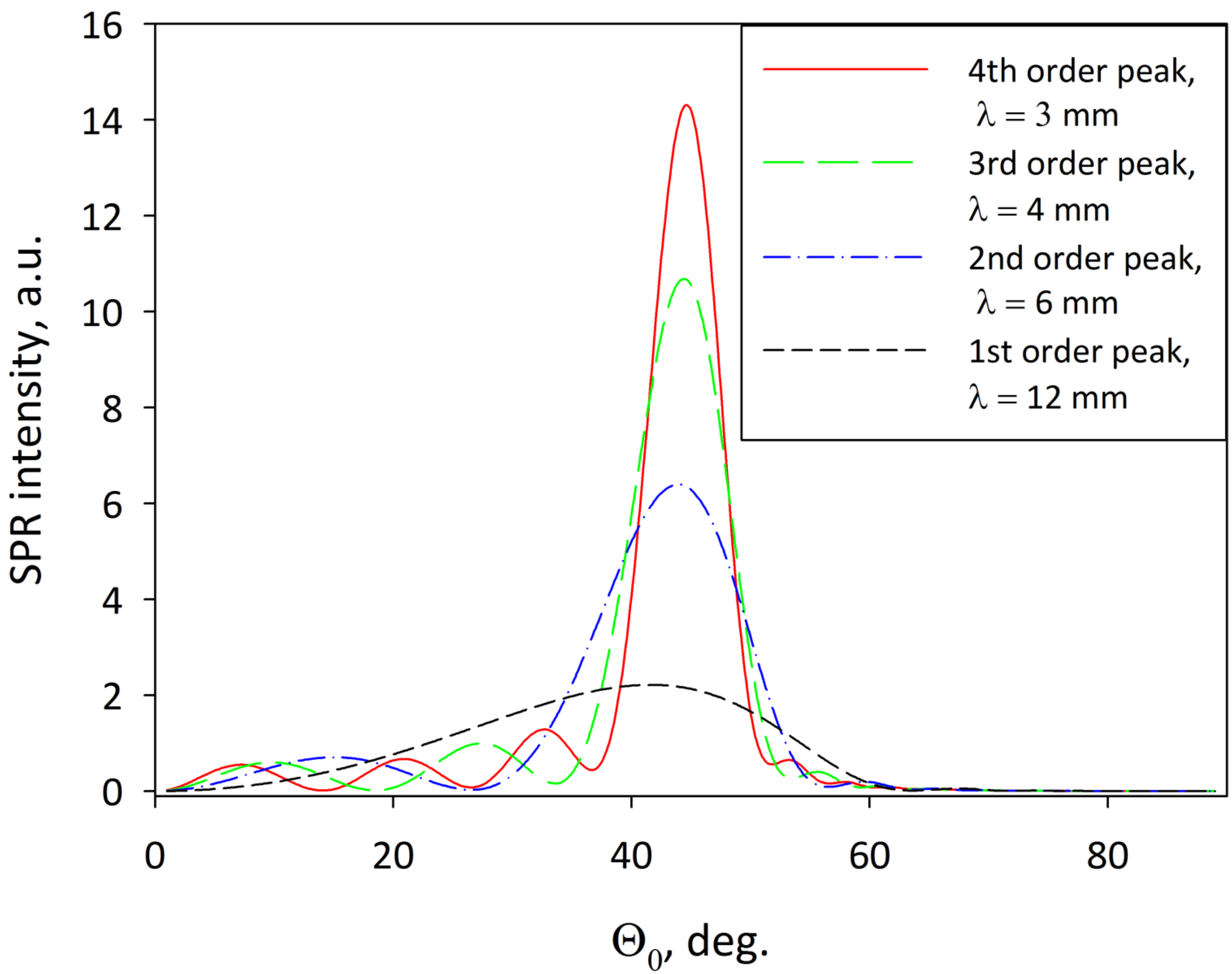


Maximum of the radiation intensity is observed at $\theta_0 = 45$ deg

θ_x =0 deg.;
θ_y =90 deg.;
γ=20;
h=0.15 mm;
a=d=12 mm;
N=20



Comparison between two types of grating

$\theta_x = 0$ deg.;

$\theta_y = 90$ deg.;

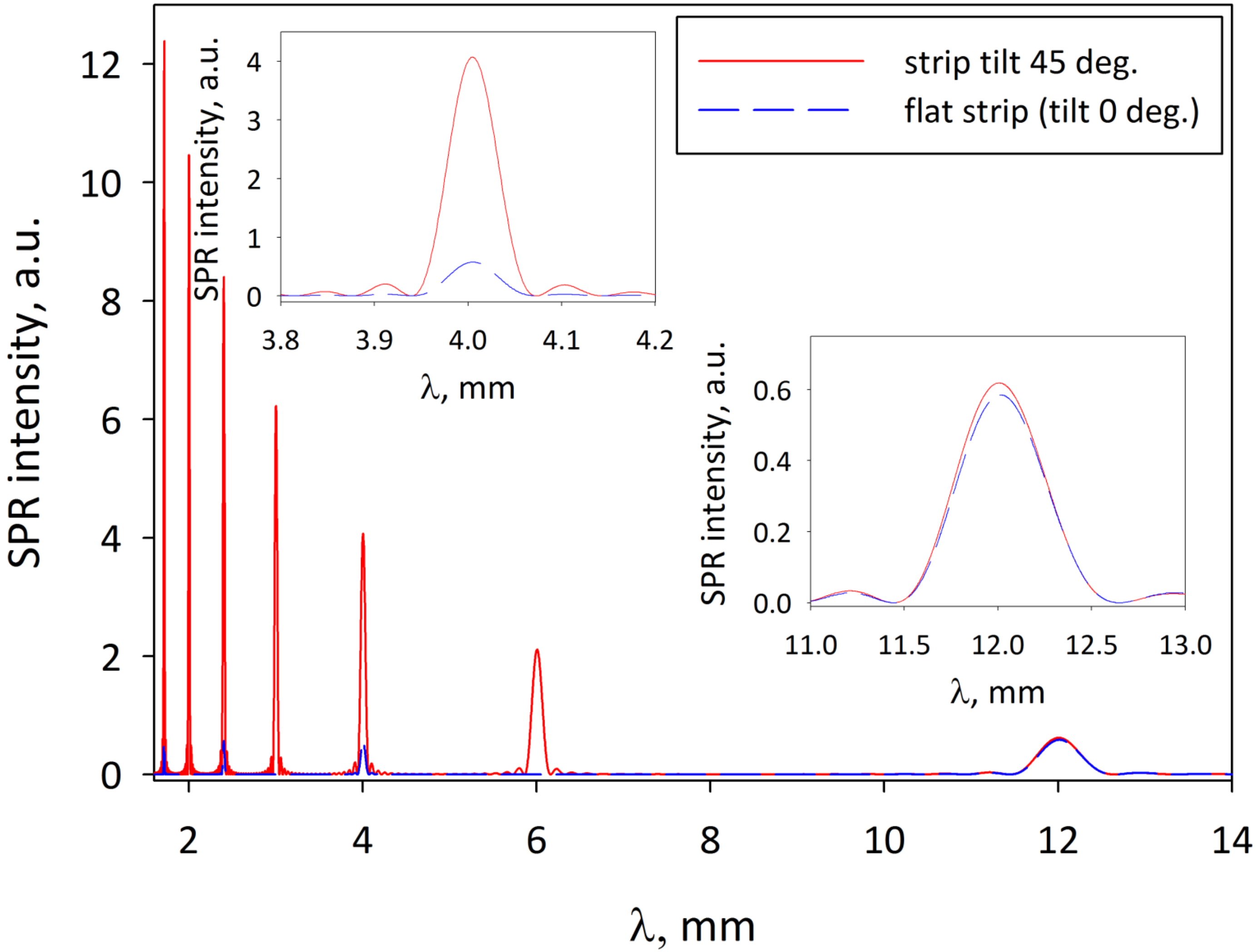
$\gamma = 20$;

$h = 0.15$ mm;

$a = \frac{d}{2} = 6$ mm;

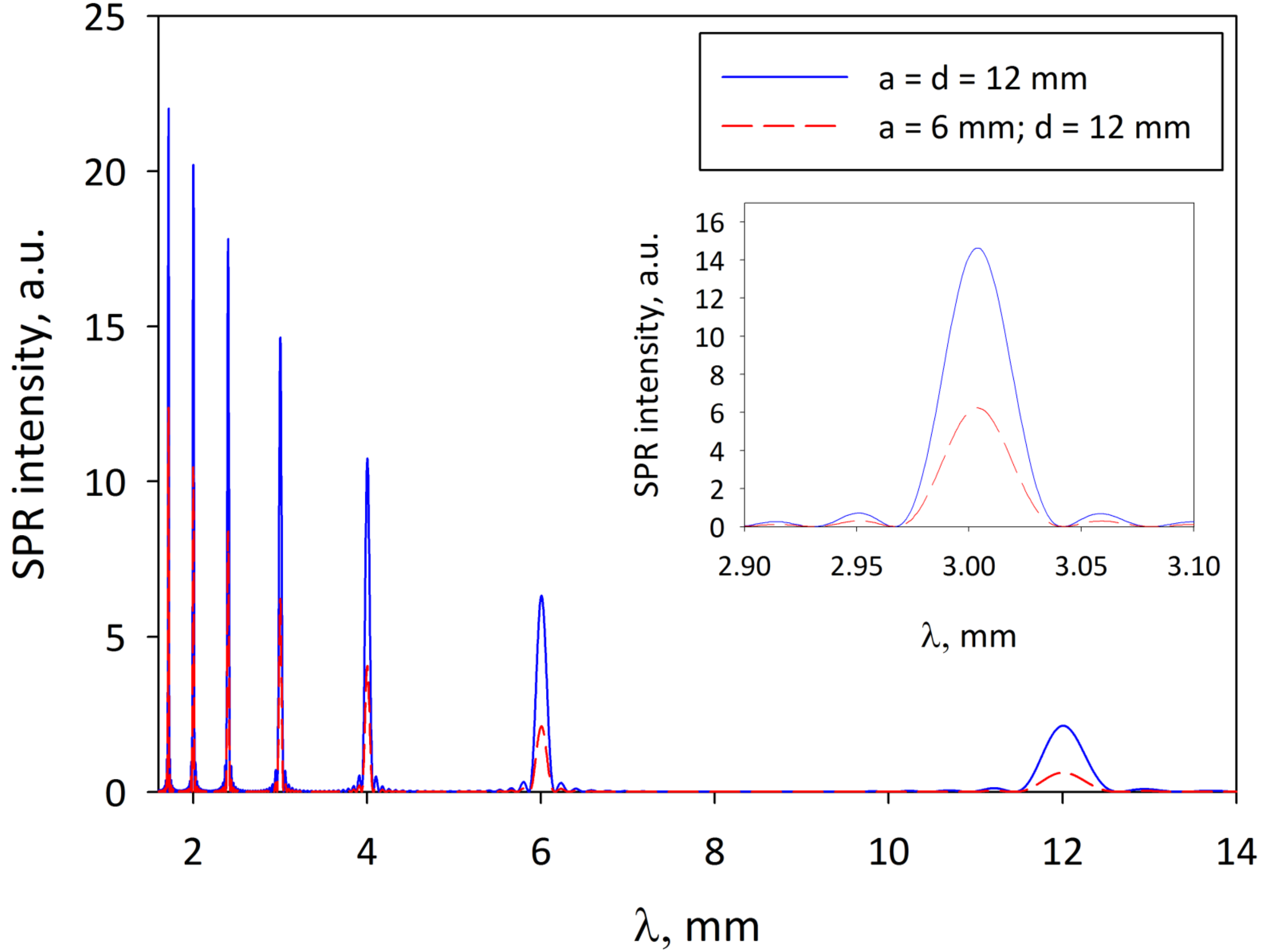
$d = 12$ mm

$N = 20$



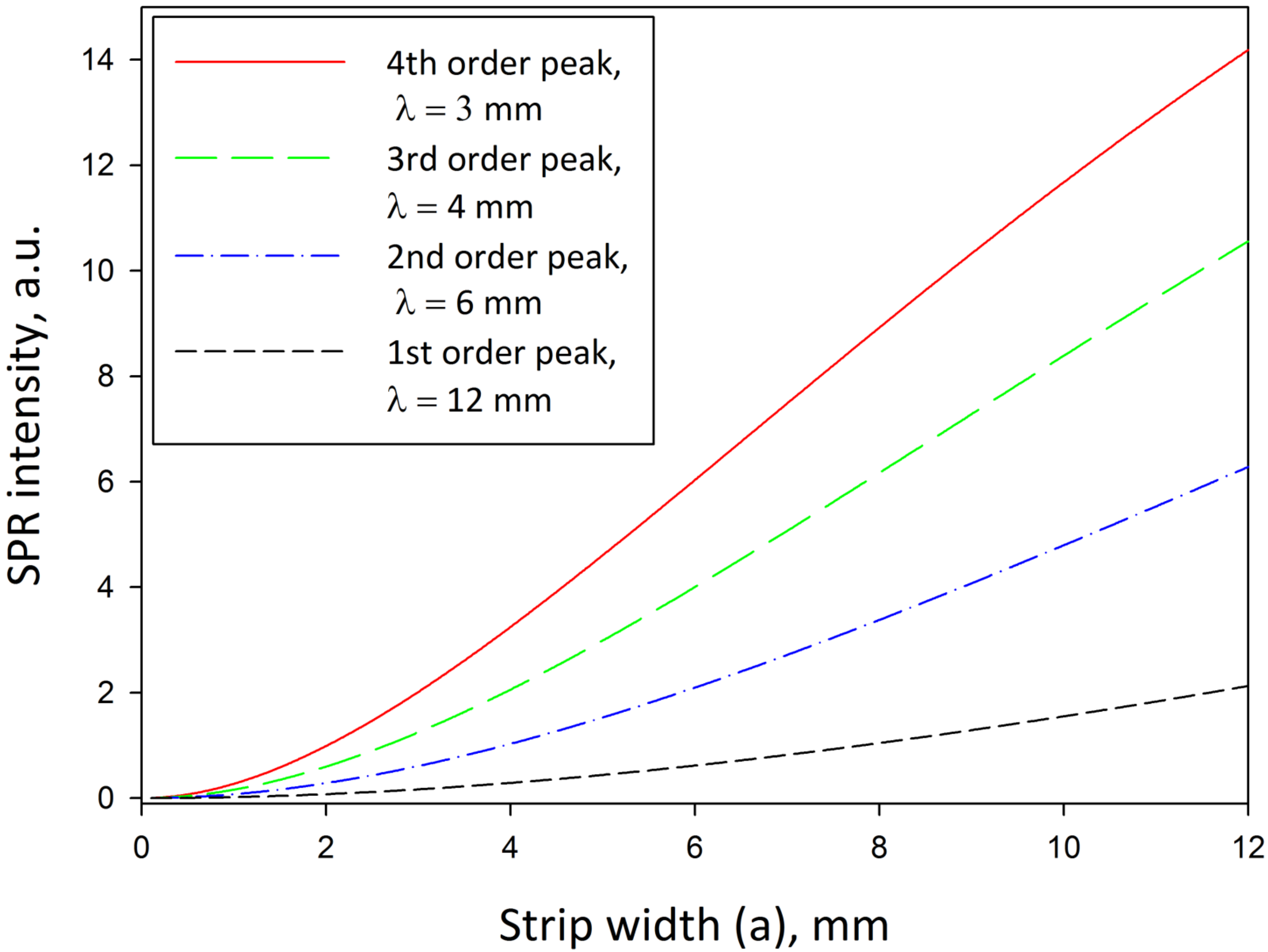
Optimization of the grating structural factor

$\theta_x = 0$ deg.;
$\theta_y = 90$ deg.;
$\theta_0 = 45$ deg.;
$\gamma = 20$;
$h = 0.15$ mm;
$N = 20$



SPR intensity at the maxima of spectral peaks as a function of a strip width.

θ_x =0 deg.;
θ_y =90 deg.;
θ_0 =45 deg.;
γ=20;
h=0.15 mm;
d=12 mm;
N=20



θ_x =0 deg.;

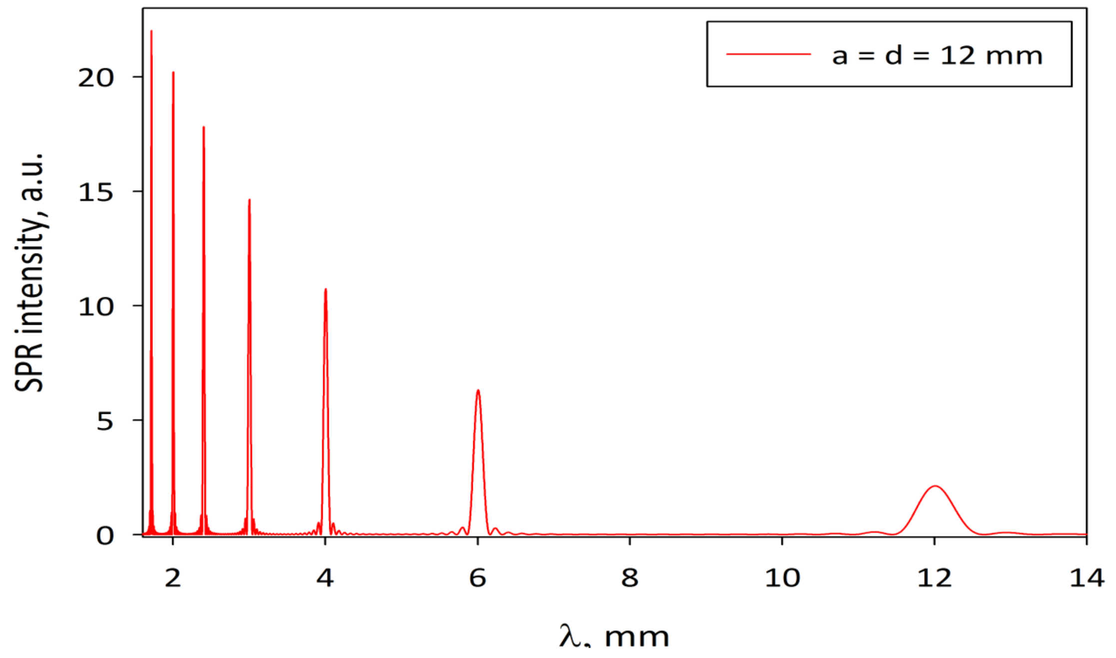
θ_y =90 deg.;

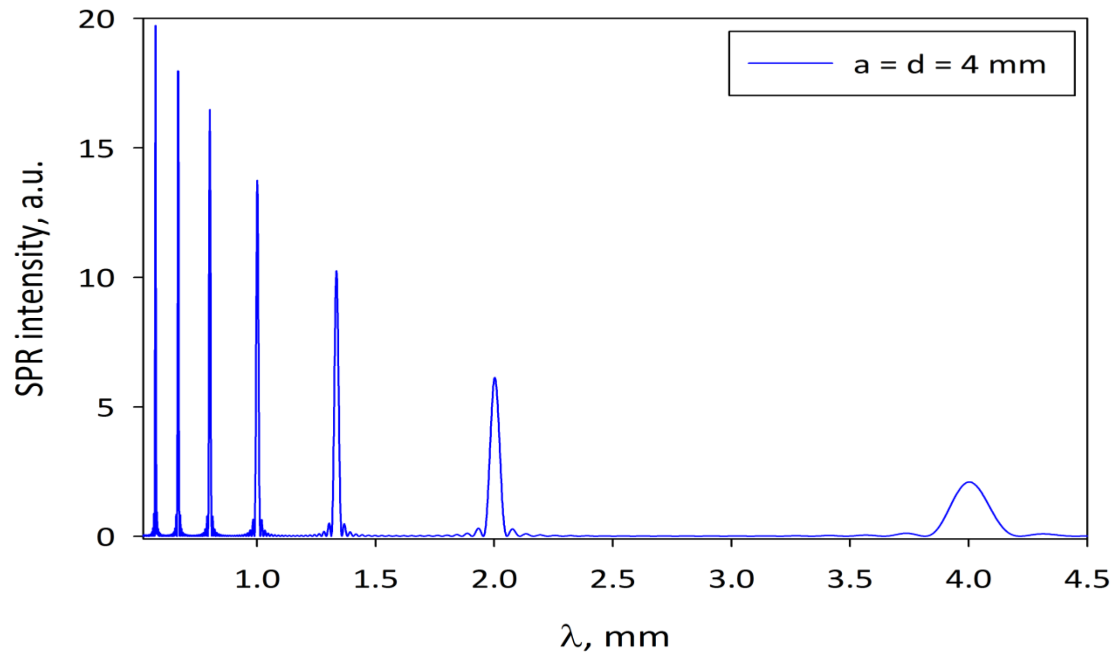
θ_0=45 deg.;

γ=20;

h=0.15 mm;

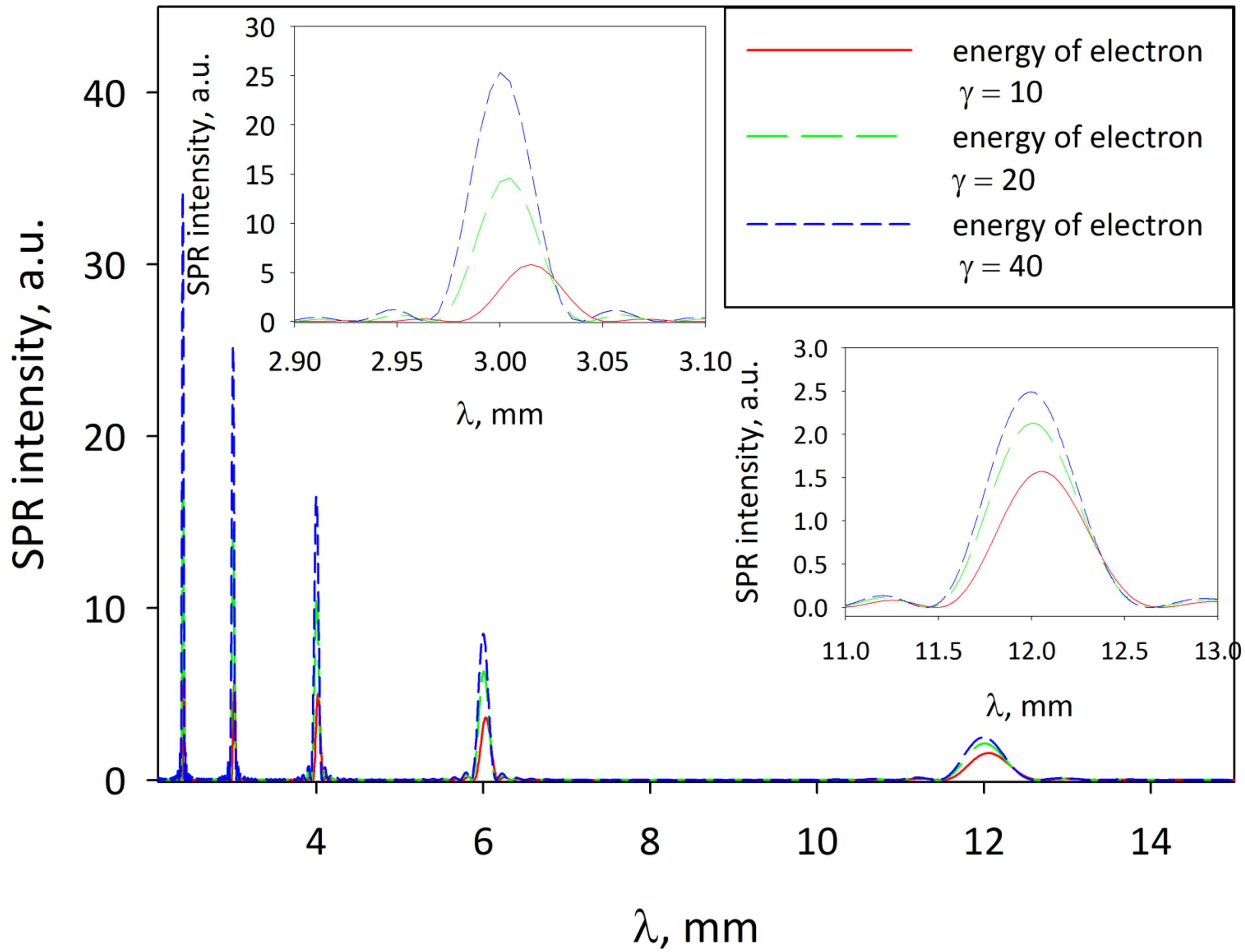
N=20





For the increased energy of an electron high order peaks become more intense compared to the first order peak.

θ_x =0 deg.;
θ_y =90 deg.;
θ_0 =45 deg.;
h=0.15 mm;
a=d=12 mm;
N=20



Summary

- **Grating Optimization:**
  - An optimal observation angle coincides with mirror reflection direction;
  - An optimal period should assure the direction of resonance order in the direction of specular reflection;
    - The order of interest is still to be decided;
  - The structural factor of unity gives the highest intensity.
- **For a given set of parameters:**
  - While the tilt angle of the strips is changed a relative decrease of the radiation intensity for the high order peaks is larger than for the first order peak;
  - While the strip width changes a relative decrease of radiation intensity for lower order peaks is higher;
  - Larger radiation yields of high order peaks compared to the first order peak are observed due to diffraction and becomes even larger for higher beam energies.