Analytical calculation of the spectral-angular characteristics of coherent Smith-Purcell radiation generated by the short bunches with THz repetition rate

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Contents

INTRODUCTION

Coherent Radiation from a single bunch

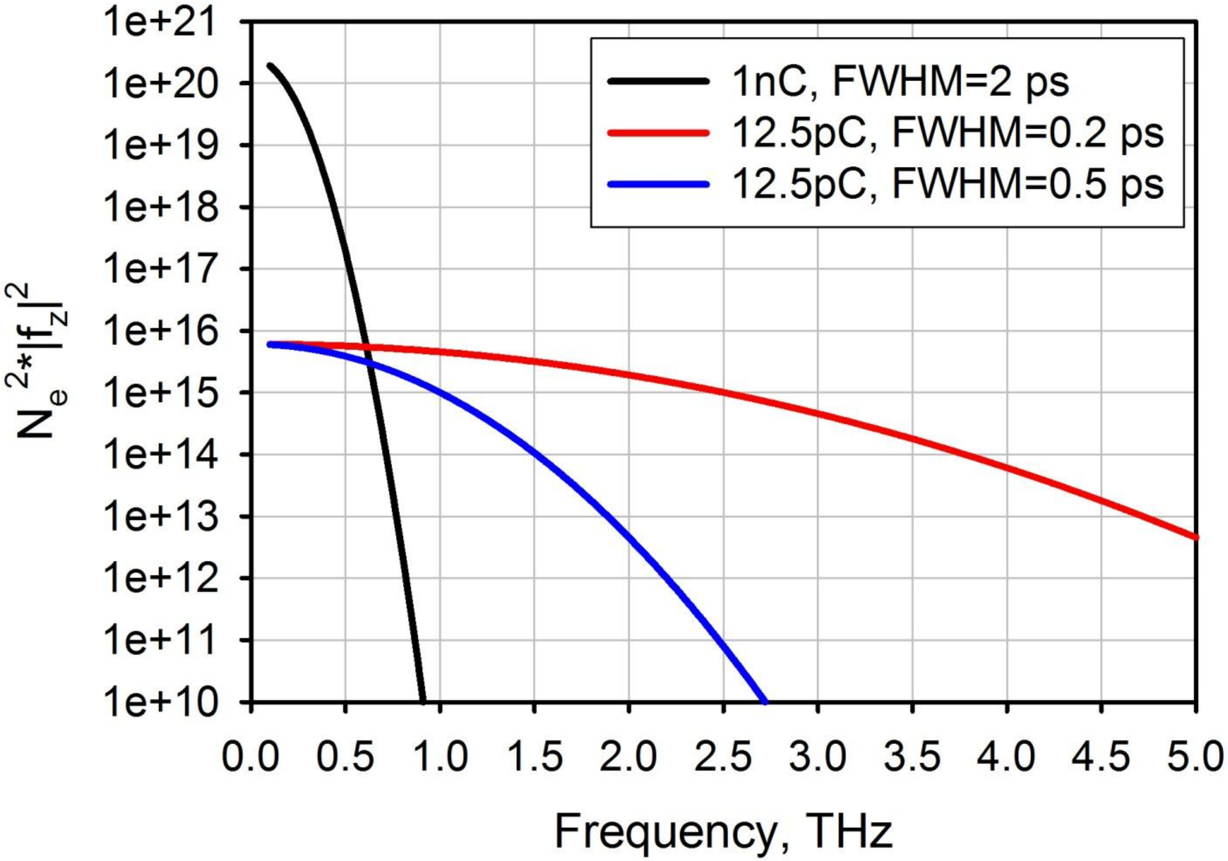
- Coherent radiation from a single bunch:

$$\frac{d^2W^s_{tot}}{d\omega d\Omega} = \frac{d^2W_{sing}}{d\omega d\Omega} N_e(1+(N_e-1)\,|\,f_l(\omega)\,|^2)$$

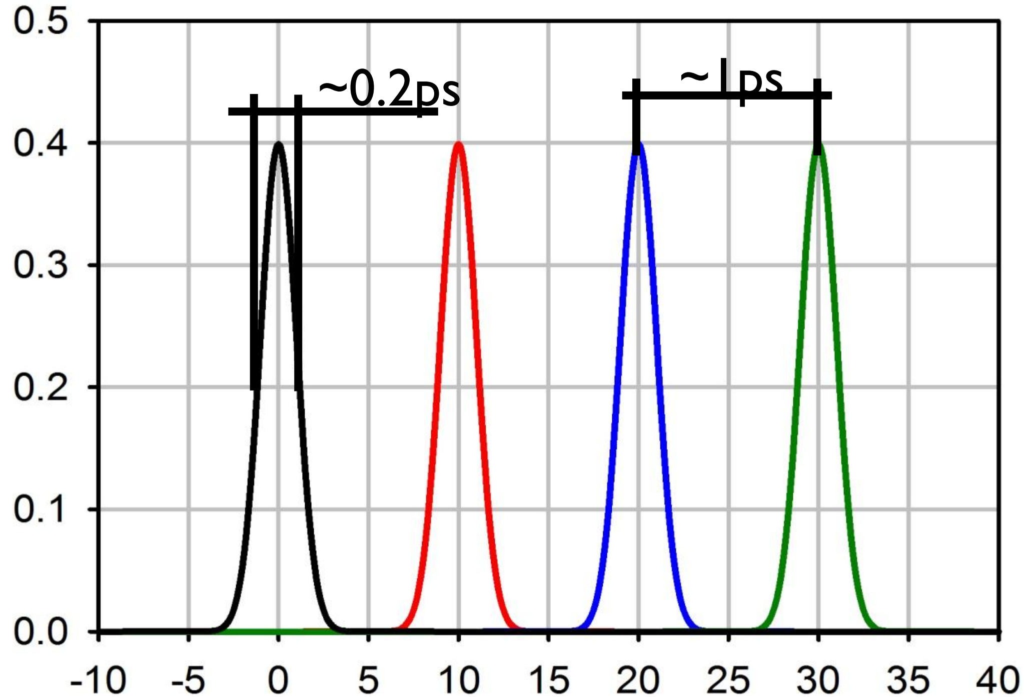
- Bunch form-factor:

$$f_l(\omega) = \int_{-\infty}^{\infty} dz \exp[-i\frac{\omega}{\beta c}z]\rho(z)$$

Gaussian form-factors



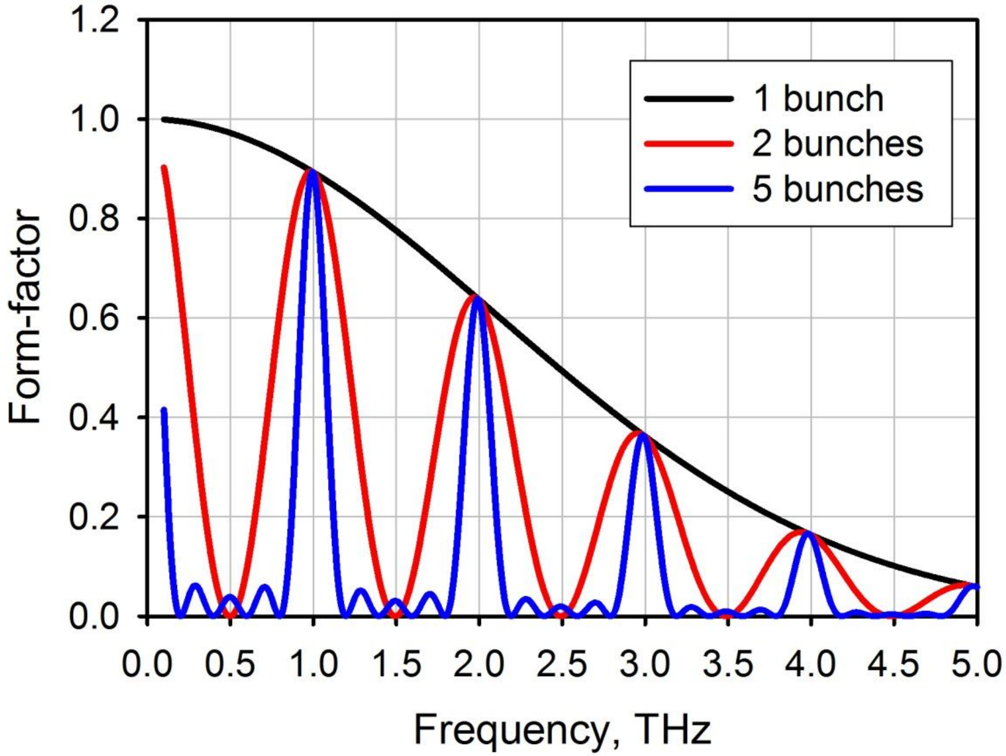
Coherent Radiation from a train of bunches



$$\frac{d^2W_{tot}^{s}}{d\omega d\Omega} = \frac{d^2W_{sing}}{d\omega d\Omega} N_e(1+(N_e-1)|f_l(\omega)|^2)$$

$$\frac{d^2W_{tot}^{s}}{d\omega d\Omega} = \frac{d^2W_{sing}}{d\omega d\Omega} N_e\left(1+(N_e-1)\frac{\sin^2[\frac{N_b\omega\lambda_{RF}}{2\beta c}]}{\sin^2[\frac{\omega\lambda_{RF}}{2\beta c}]}|f_l(\omega)|^2\right)$$

CR spectrum



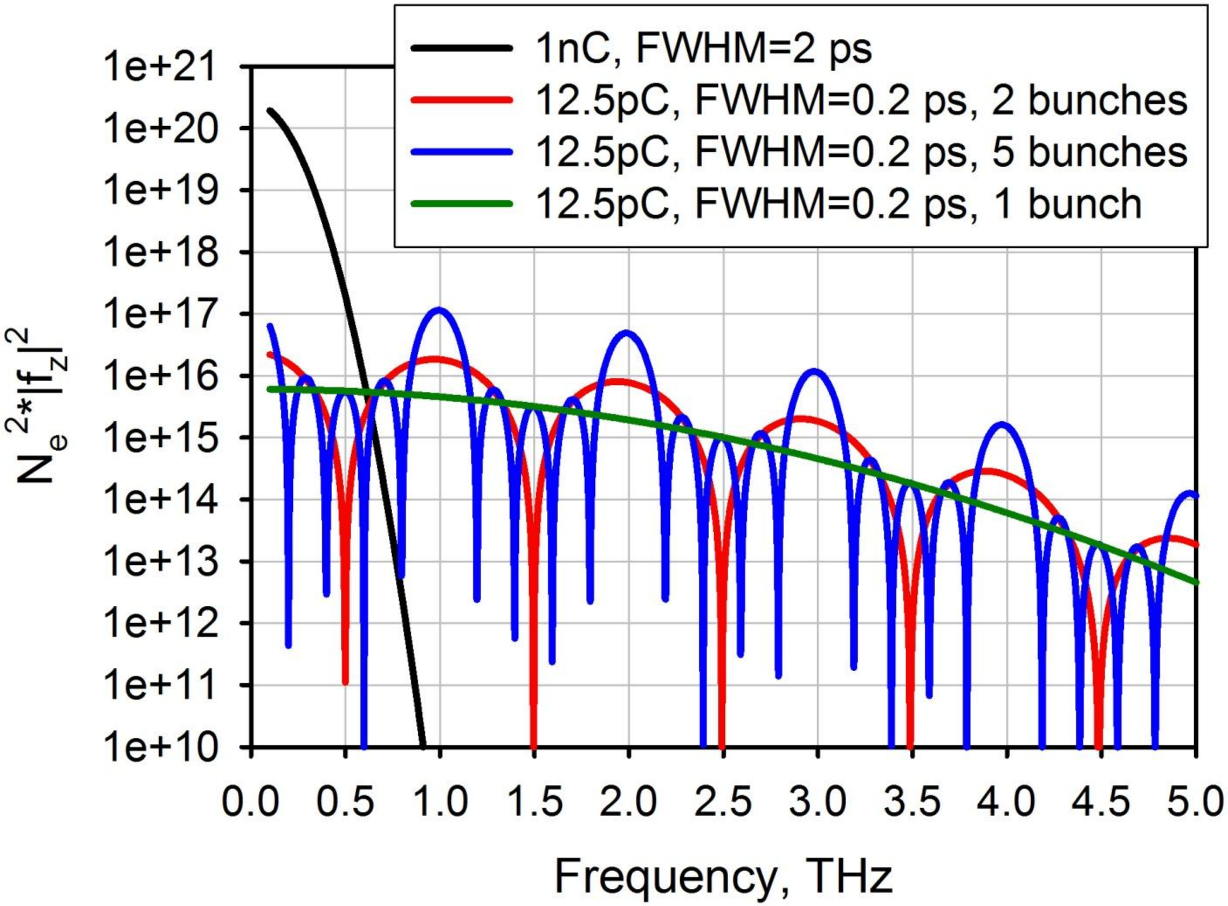
$$\frac{1}{N_b^2}\frac{\sin^2[\frac{N_b\omega\lambda_{RF}}{2\beta c}]}{\sin^2[\frac{\omega\lambda_{RF}}{2\beta c}]}|f_l(\omega)|^2$$

$$\lambda_{RF} = 1 \text{ ps}$$

$$\text{FWHM} = 0.2 \text{ fs}$$

Radiation line width is proportional to N_b^{-1}

Gaussian form-factors for several bunches



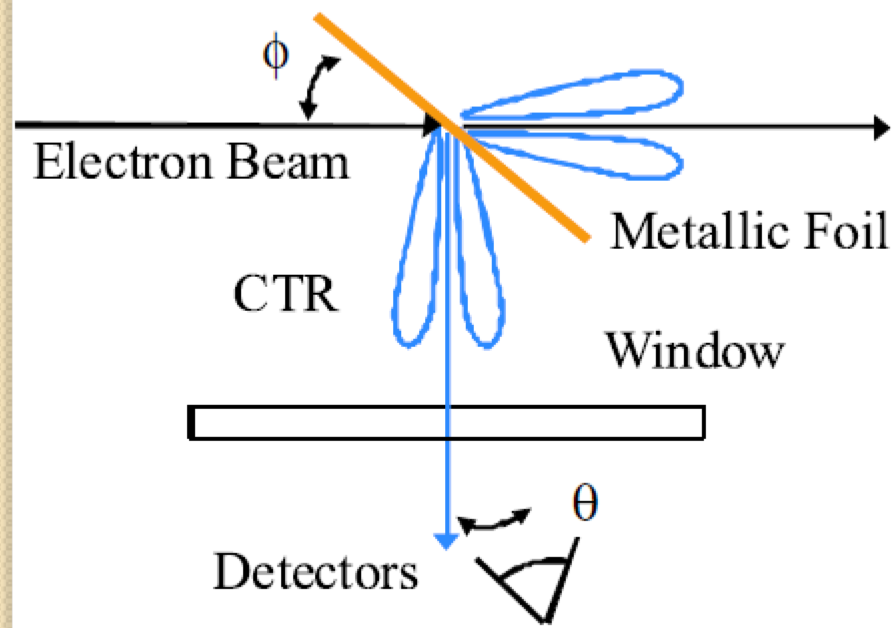
$\lambda_{RF} = 1$ ps

Frequency-locked CTR

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **10**, 082801 (2007)

Absolute scale power measurements of frequency-locked coherent transition radiation

Roark A. Marsh, Amit S. Kesar, and Richard J. Temkin
Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
(Received 15 May 2007; published 7 August 2007)



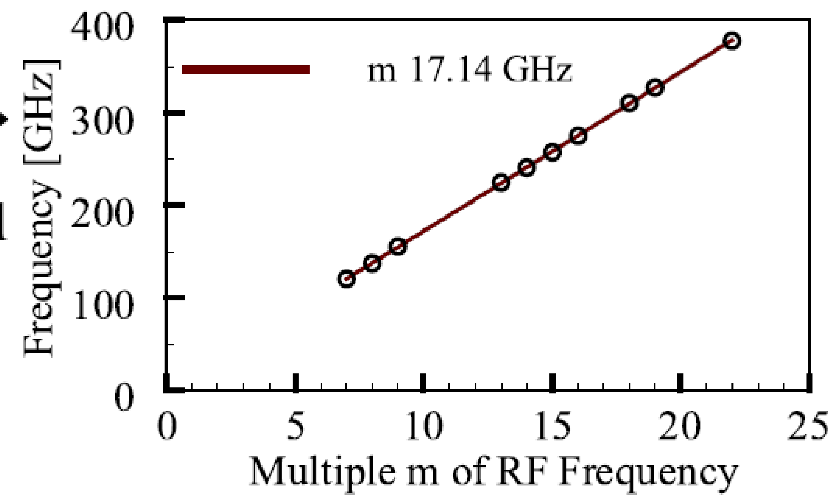
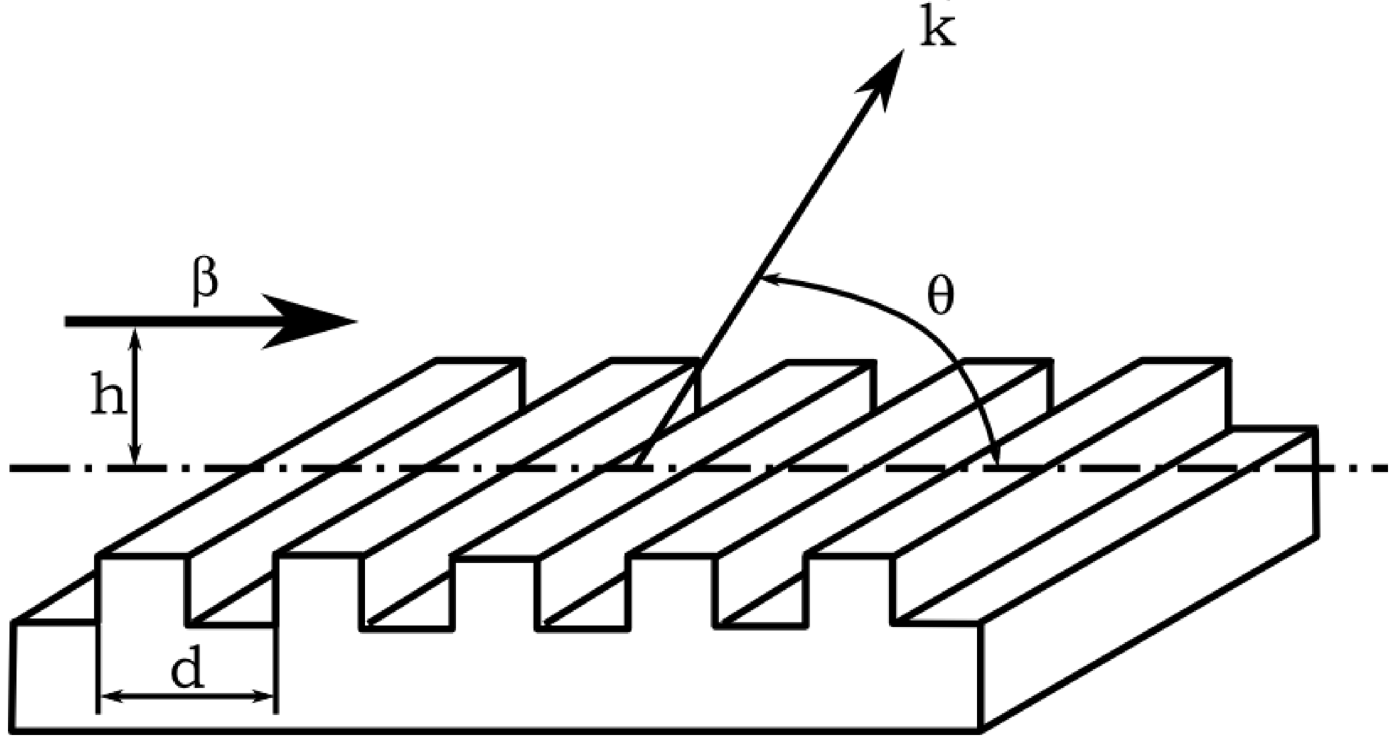


FIG. 6. (Color) CTR frequencies measured with the double heterodyne receiver, open circles. Radiation is only observed at integer harmonics of the rf frequency. The straight line is a fit to the data and has a slope consistent with the accelerator frequency of 17.14 GHz.

Smith-Purcell Radiation



$$\lambda_n = \frac{d}{n}(\beta^{-1} - \cos[\theta])$$

Frequency-locked CSPR

PRL **94,** 054803 (2005) PHYSICAL REVIEW LETTERS week ending 11 FEBRUARY 2005

Observation of Frequency-Locked Coherent Terahertz Smith-Purcell Radiation

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(Received 22 September 2004; published 11 February 2005)

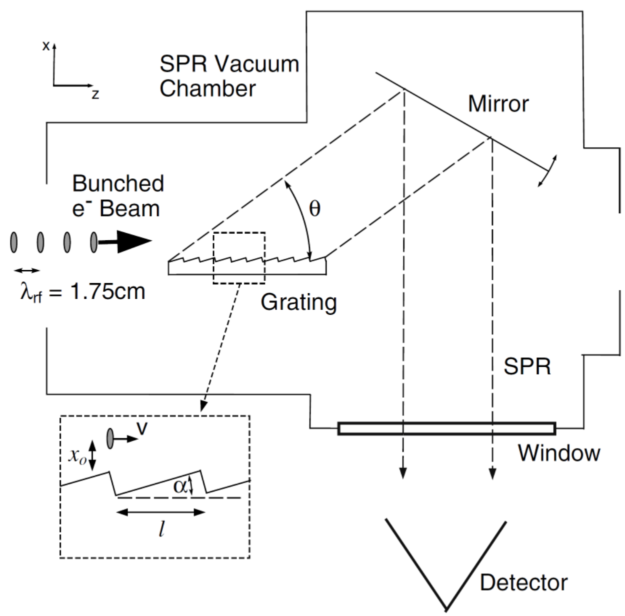

FIG. 1. SPR experimental setup. The train of bunched electrons is traveling along the z direction above a metallic echelle grating. The diffracted radiation is measured by a detector located outside of the vacuum chamber.

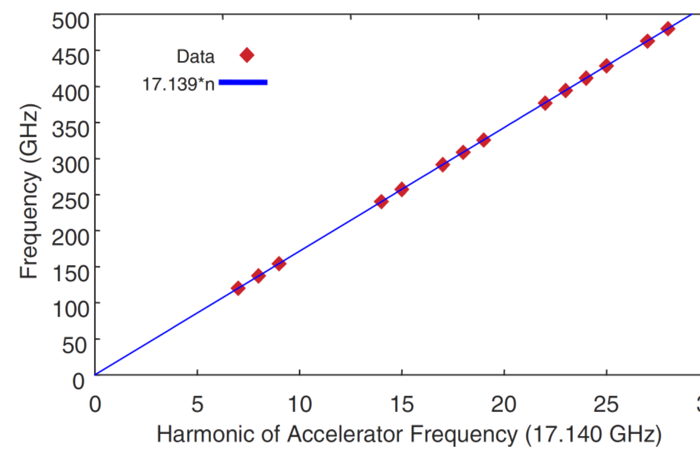

FIG. 4 (color online). Plot of the frequency spectrum of SPR as measured by the double heterodyne receiver. The accelerator rf frequency was 17.140 GHz.

Bunched CSPR power measurement

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **9**, 022801 (2006)

Power measurement of frequency-locked Smith-Purcell radiation

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(Received 19 October 2005; published 7 February 2006)

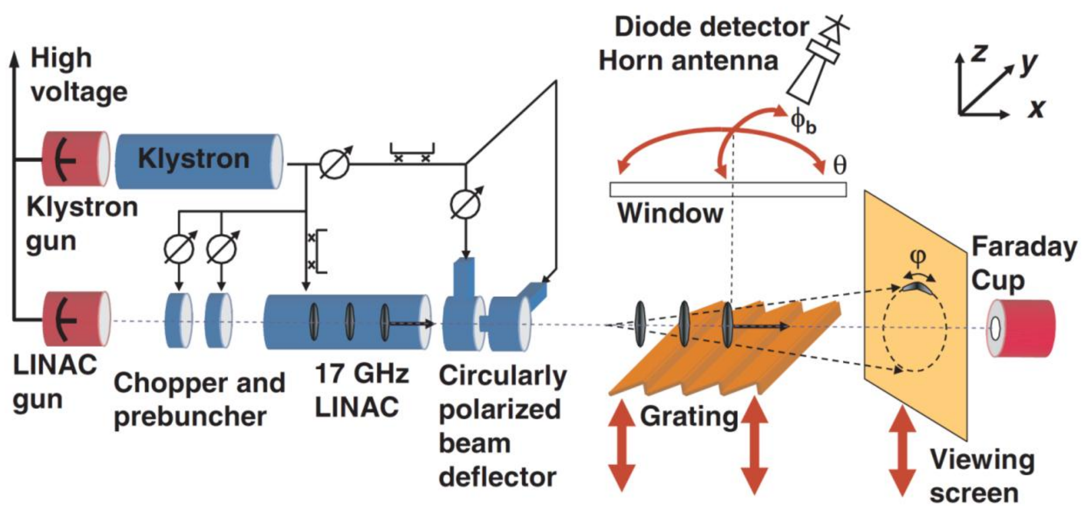


FIG. 1. (Color) SPR experimental setup (not to scale) including the klystron, linac, deflecting cavities and screen, and the grating.

Bunched CSPR power measurement

TABLE I. Smith-Purcell experiment parameters

Average current I_b	80 mA
Train relativistic factor γ	30
Train frequency f_{RF}	17.140 GHz
Height above the grating, $b_{\min}$	2 mm
Bunch length σ_x	170 μm
Grating period D_g	2.54 mm
Blaze angle α	30°
Number of periods, N_g	20
Grating width, W	100 mm

Bunch charge 4.67 pC

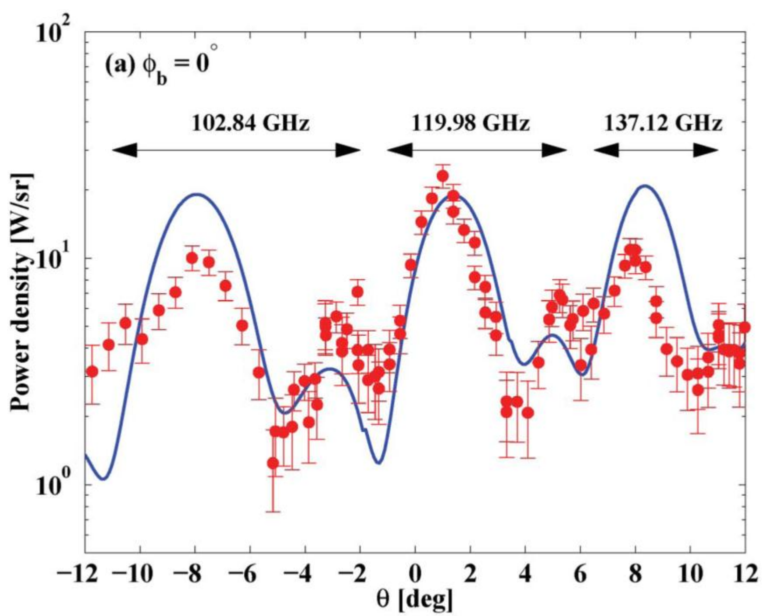


FIG. 3. (Color) Measured power density in W/sr (dots with error bars). The measurement is compared to the first-order radiated power density by the EFIE model (solid line). The power is plotted versus θ when $\phi_b = 0°$ (a) and $\phi_b = 7.6°$ (b). In these figures, each arrow spans over a range of angles in which the power is dominated by one discrete frequency (see Fig. 4).

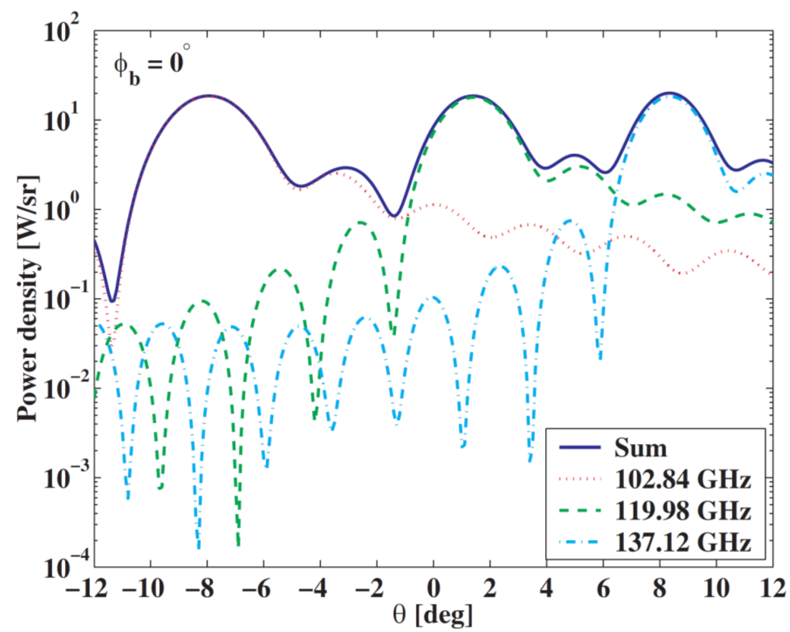
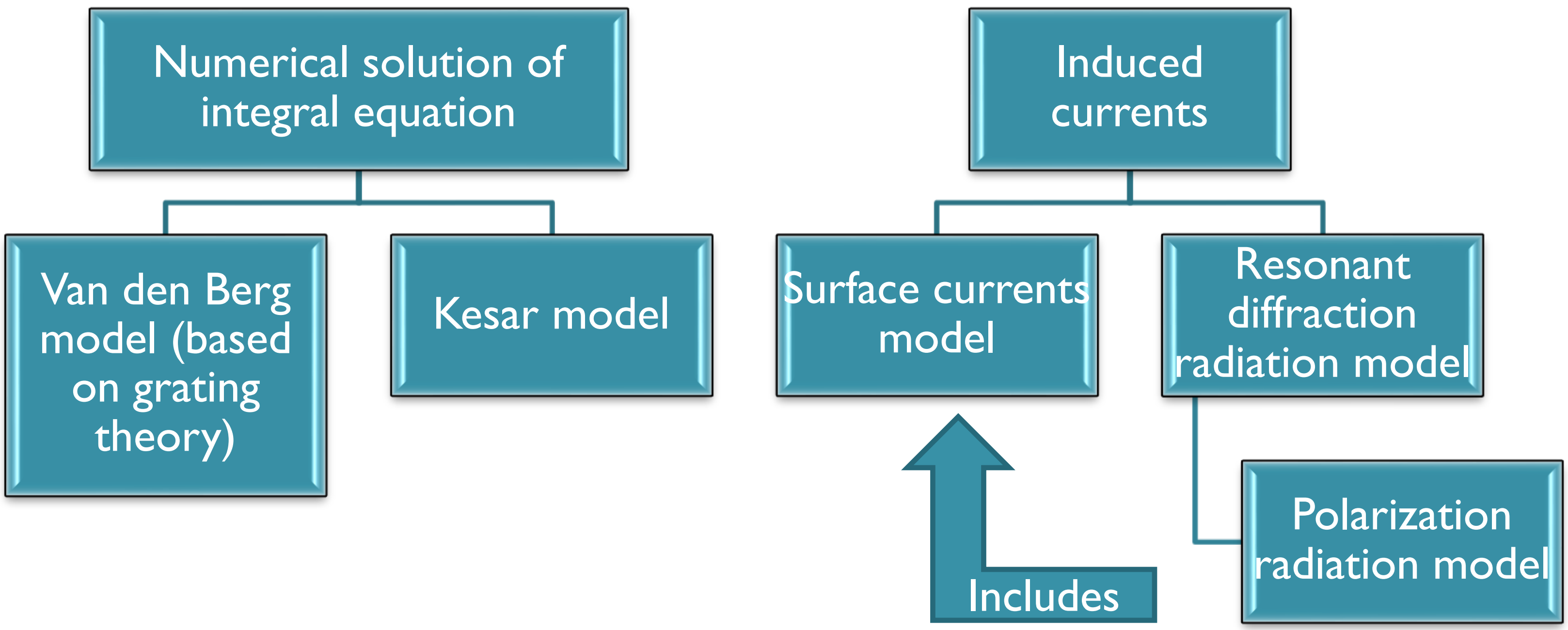


FIG. 4. (Color) First-order radiated power density calculated for $\phi_b = 0°$ by the EFIE model and Eq. (3) (solid line). This calculation was composed from the 6th (dotted line), 7th (dashed line), and 8th (dash-dotted line) harmonics of the accelerator frequency.

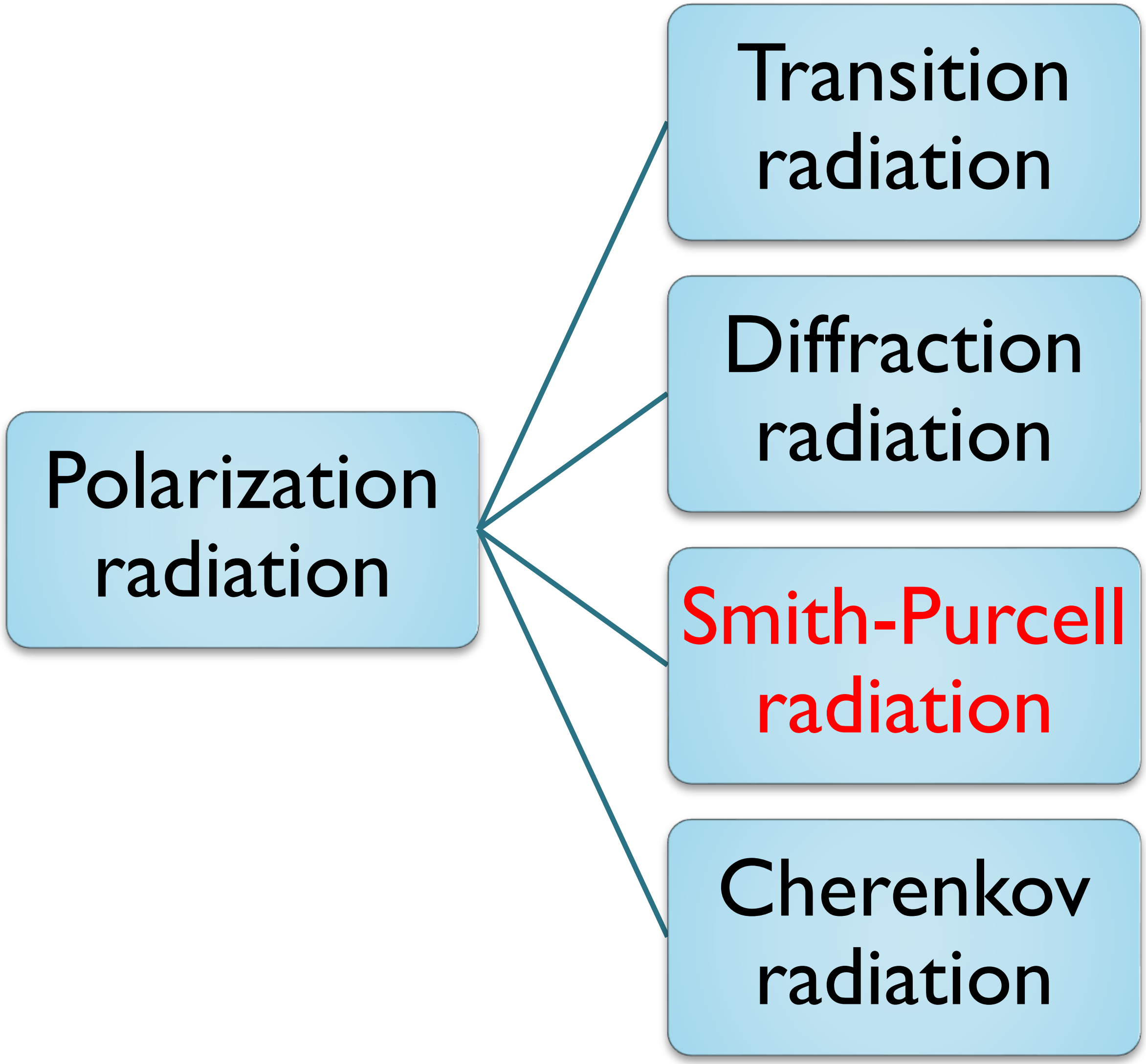
Smith-Purcell radiation models



It is a big challenge to compare all models with known experimental data and to find/create the best one

POLARIZATION RADIATION THEORY

Polarization radiation



Theory 1

- Polarization radiation theory was developed by D. V. Karlovets and A.P. Potylitsyn:

1. D.V. Karlovets and A.P. Potylitsyn, JETP Lett. 2009, Volume 90, Number 5, Pages 326-331
2. D.V. Karlovets, JETP, 2011, Volume 113, Number 1, Pages 27-45

Theory 2

For non magnetic media a polarization current is a linear function of the full field

$$\mathbf{j}(\mathbf{r},\omega)_{\rm pol} = \sigma(\mathbf{r},\omega)(\mathbf{E}^0 + \mathbf{E}^{\rm pol}(\mathbf{j}_{\rm pol}))$$

where:

$$\sigma(\mathbf{r},\omega) = (\varepsilon(\mathbf{r},\omega) - 1)\omega/4\pi i.$$

Maxwell equations may be written as:

$$\left(\Delta + \varepsilon(\mathbf{r},\omega)\frac{\omega^2}{c^2}\right)\mathbf{H}^{\rm pol}(\mathbf{r},\omega) = -\frac{4\pi}{c}\Big(\sigma(\mathbf{r},\omega){\rm rot}\,\mathbf{E}^0 - $$
$$- (\mathbf{E}^0 + \mathbf{E}^{\rm pol}) \times \nabla\sigma(\mathbf{r},\omega)\Big).$$

Unwanted term

Theory 3

For the simplest case of the flat vacuum-medium boundary:

$$\sigma(\mathbf{r},\omega) = \Theta(z)\sigma(\omega)$$

And

$$(\mathbf{E}^0 + \mathbf{E}^{\mathrm{pol}}) \times \nabla\sigma(\mathbf{r},\omega) = \sigma(\omega)\delta(z)(\mathbf{E}^0 + \mathbf{E}^{\mathrm{pol}}) \times \mathbf{n}$$

where $\mathbf{n} = \{0,0,1\}$ - Surface normal

Due to boundary conditions:

$$(\mathbf{E}^0 + \mathbf{E}^{\mathrm{pol}}) \times \mathbf{n}|_{z=0} = \mathbf{E}^0 \times \mathbf{n}$$

The unwanted term disappears.

Theory 3

The exact solution of the Maxwell equations may be written as following:

$$\mathbf{H}^{\mathrm{pol}}(\mathbf{r},\omega) = \mathrm{rot}\frac{1}{c}\int\limits_{V_T} \mathbf{j}_{\mathrm{pol}}^{(0)}(\mathbf{r}',\omega)\frac{e^{i\sqrt{\varepsilon(\omega)}\omega|\mathbf{r}-\mathbf{r}'|/c}}{|\mathbf{r}-\mathbf{r}'|}d^3r'$$

where

$$\mathbf{j}_{\mathrm{pol}}^{(0)} = \sigma(\mathbf{r},\omega)\mathbf{E}^0(\mathbf{r},\omega)$$

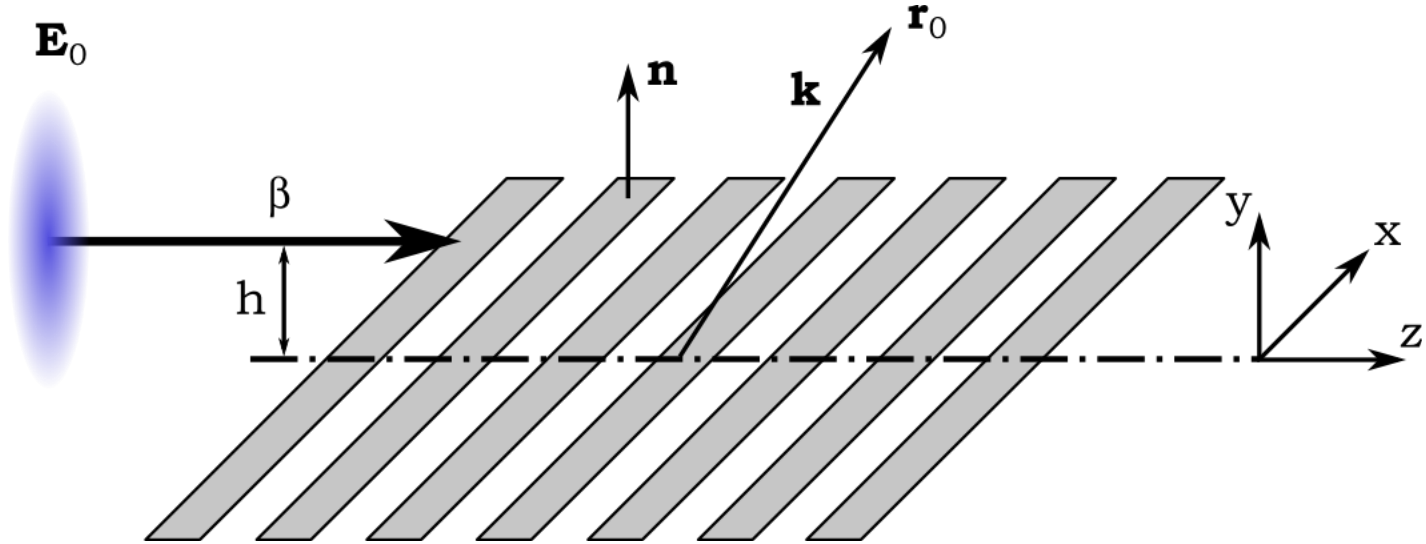
This is exact field of polarization radiation inside the target with arbitrary permittivity. Additional manipulations are required to find the field outside the target. One should use the reciprocity theorem and Fresnel coefficients.

SUPER-RADIANT SMITH-PURCELL RADIATION MODEL

Smith-Purcell radiation model

In the case of ideal-conducting thin grating the radiation field in far field assumption may be written as:

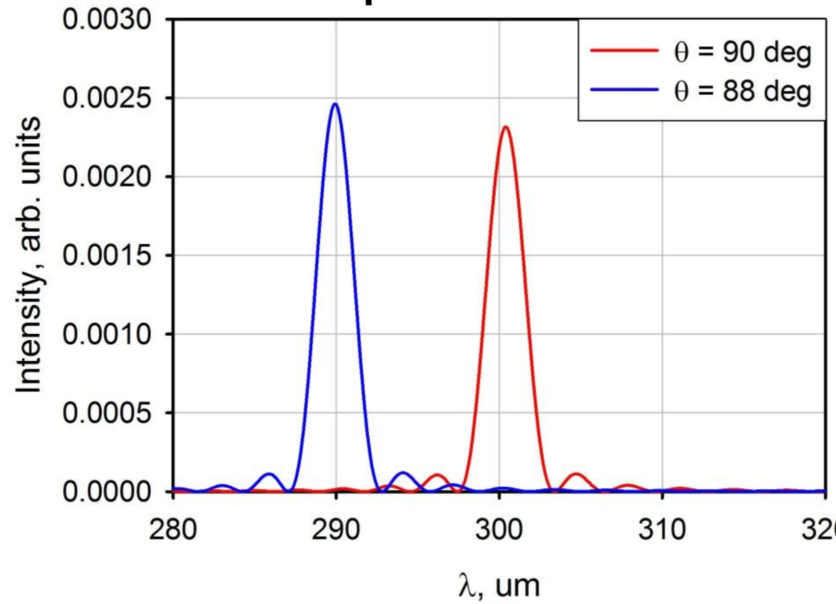
$$\mathbf{E}^{\mathbf{R}}(\mathbf{r_0},\omega) = -i\frac{e^{ikr_0}}{r_0}\mathbf{k}\times\int_S dz\,[\mathbf{n},\mathbf{E_0}(k_x,y=0,z,\omega)]\,e^{-ik_z z}$$



Smith-Purcell radiation spectrum

Parameter	Value
Electron energy, E_e	10 MeV
Grating period, d	300 um
Number of strips, N	101
Impact-parameter, h	1 mm
Observation angle, θ	On the figure
Microbunch length, σ	0
# of microbunches, N_b	1
Distance between microbunches, λ_{rf}	-

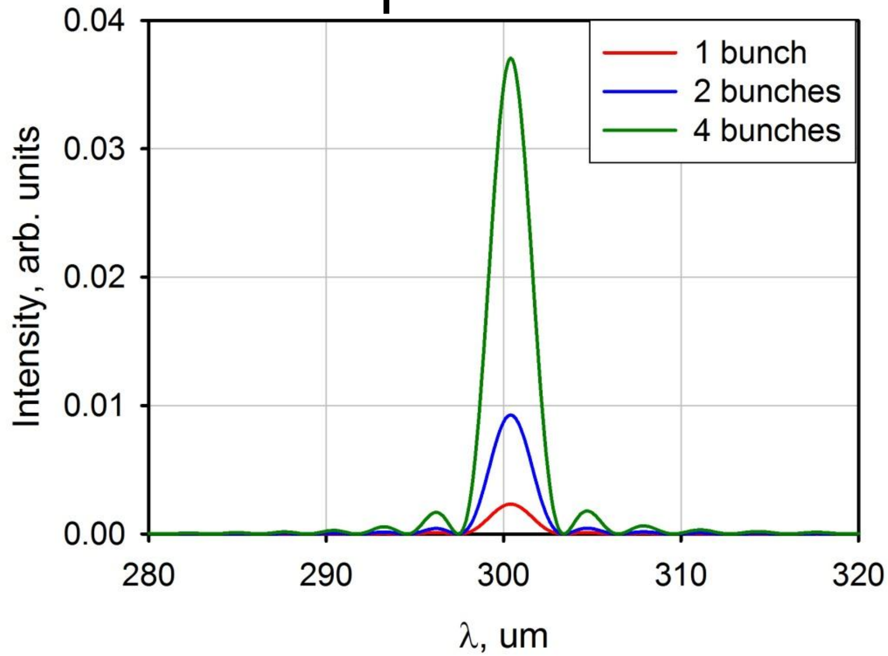
Smith-Purcell radiation spectrum



Smith-Purcell radiation gain due to several microbunches

Parameter	Value
Electron energy, E_e	10 MeV
Grating period, d	300 um
Number of strips, N	101
Impact-parameter, h	1 mm
Observation angle, θ	90 degree
Microbunch length, σ	0
# of microbunches, N_b	On the figure
Distance between microbunches, λ_{rf}	300 um

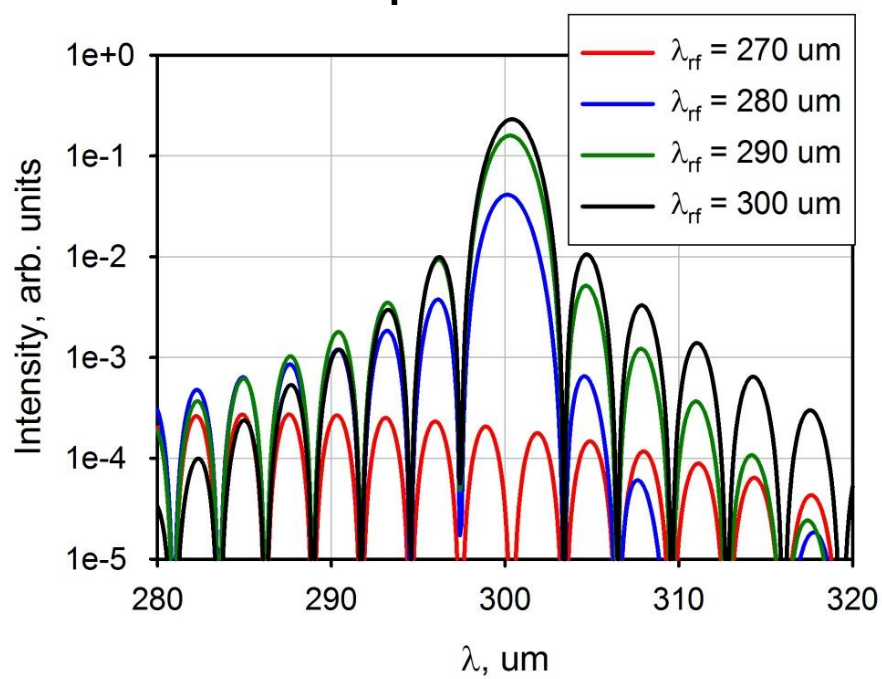
Smith-Purcell radiation spectrum



Radiation gain strongly depends on bunching frequency…

Parameter	Value
Electron energy, E_e	10 MeV
Grating period, d	300 um
Number of strips, N	101
Impact-parameter, h	1 mm
Observation angle, θ	90 deg
Microbunch length, σ	0
# of microbunches, N_b	10
Distance between microbunches, λ_{rf}	On the figure

Smith-Purcell radiation spectrum



Tilted grating

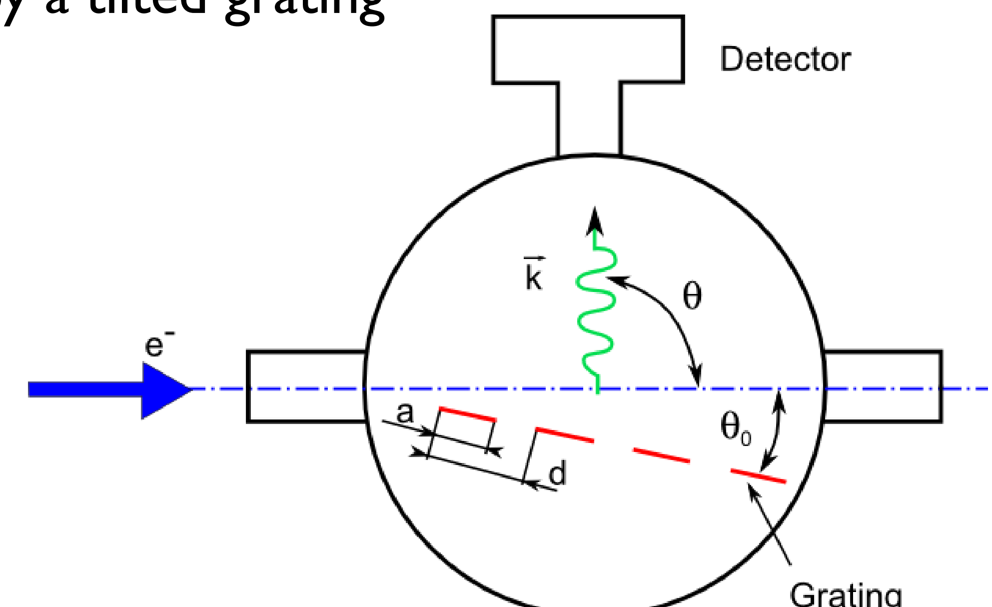
- For the first time was calculated by P. Karataev et al.

PHYSICAL REVIEW E VOLUME 61, NUMBER 6 JUNE 2000

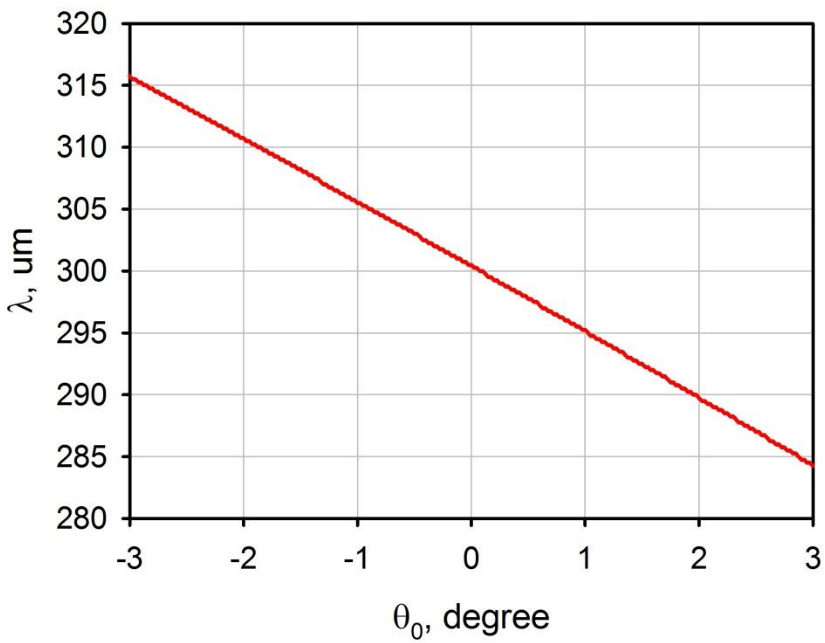
Resonant diffraction radiation from an ultrarelativistic particle moving close to a tilted grating

A. P. Potylitsyn, P. V. Karataev,* and G. A. Naumenko

- The developed model allows calculation of spectral-angular parameters of super-radiant Smith-Purcell radiation generated by a tilted grating



Line shift



Parameter	Value
Electron energy, E_e	10 MeV
Grating period, d	300 um
Number of strips, N	101
Impact-parameter, h	1 mm
Observation angle, θ	90 deg
Microbunch length, σ	0
# of microbunches, N_b	1
Distance between microbunches, λ_{rf}	-

CONCLUSION

Conclusion

- Smith-Purcell radiation intensity significantly increases due to beam microbunching at the frequencies that correspond to the microbunching frequency.
- Microbunching frequency control and diagnostics is really important.
- One may try to use the grating tilt for such control.

THANK YOU FOR YOUR ATTENTION