Theory of Flavor

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Definitions

- **Masses and mixings- two aspects of flavor**

Def. $$L_{mass} = \overline{Q}_L M_{q=u,d} Q_R + \bar{l}_L M_l l_R + \nu^T m_\nu \nu + h.c.$$

- **Mass basis:** $U_L M_{q,l} U_R^+ = M_{q,l}^{diag}$ **;** $U_\nu^* m_\nu U_\nu^\dagger = m_\nu^{diag}$

$$V_{CKM} = U_u U_d^+ \qquad U_{PMNS} = U_l U_\nu^+$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Flavor Puzzle

- Up quarks: $m_u/m_c \sim m_c/m_t \sim 1/200$
- Downs : $m_d : m_s : m_b = 0.002 : 0.03 : 1$
- Mixings: $V_{us} \approx 0.22; V_{cb} \approx 0.037; V_{ub} \approx 0.003$
- Leptons: $m_e : m_\mu : m_\tau = 0.0005 : 0.093 : 1.58$
- ν Masses: weak hierarchy $m_2 : m_3 \sim 0.2 : 1 \ (\theta_C : 1)$
- Mixings: $\sin^2\theta_{12} \cong .312;$ $\sin^2\theta_{23} \cong .466$ $\sin^2\theta_{13} \sim 0.02$
- Strong CP problem: why is $\theta_{QCD} \leq 10^{-10}$?

From data to theory via mass matrices

- Diagonal charged lepton mass matrix +

$$M_\nu \cong \begin{pmatrix} \varepsilon_1 & \varepsilon_3 & \varepsilon_3 \\ \varepsilon_3 & 1+\varepsilon_1 & -1+\varepsilon_3 \\ \varepsilon_3 & -1+\varepsilon_3 & 1+\varepsilon_1 \end{pmatrix}$$

$$\varepsilon_i \sim \lambda$$

Cabibbo angle

- Compare with

$$M_d \approx m_b \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Towards a theory of flavor

- Standard model: flavor resides in Yukawa couplings:

$$\mathcal{L}_Y = h_u \bar{Q}_L H u_R + h_d \bar{Q}_L \tilde{H} d_R + h_\ell \bar{L} \tilde{H} e_R + \mathcal{L}_\nu^{mass} + h.c.$$

- <H> leads to fermion masses and mixings.
- # of parameters: Majorana **ν** : 22

  Dirac **ν** : 20
- How do we understand these parameters ?

Symmetry approach

- Zero fermion masses (zero Yukawas)→ SM + RH nu → flavor symmetry group:

$$G_f = U(3)_Q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e \times U(3)_N$$

- Flavor: a consequence of breaking G_f
- Questions: G_f Gauge or global symmetry ?

  a) If global, should be discrete to avoid massless states: scale $M_f \geq M_{infl.}$ to avoid domain walls

  b) If local,no such restriction except phenomenology

This talk: Two classes of models-

(i) Gauged flavor approach

(ii) Discrete global flavor and GUTs

Gauged flavor approach with vector-like quarks, leptons

- Add vectorlike fermions→$U_{2/3}, D_{-1/3}, E_{-1}, N_0$
- Flavor is a high scale (multi-TeV) phenomenon but "trickles" down to low scales

$$\begin{array}{ccc} & q_R & Q_R \\ \begin{array}{c} q_L \\ Q_L \end{array} & \left(\begin{array}{c} 0 \\ \lambda_u v_R \end{array} \right. & \left. \begin{array}{c} \lambda_u v_L \\ < Y_u > \end{array} \right) \end{array}$$

unit matrix; flavor flat

$$h = \frac{\lambda v_R}{< Y >}$$

flavor structure –

- Gauged flavor guarantees this (SM:Greenstein,Redi,Villadoro'10; LR:Guadagnoli, Mohapatra, Sung'11) Cosmology OK.

Flavor from flavon vevs and solution to strong CP

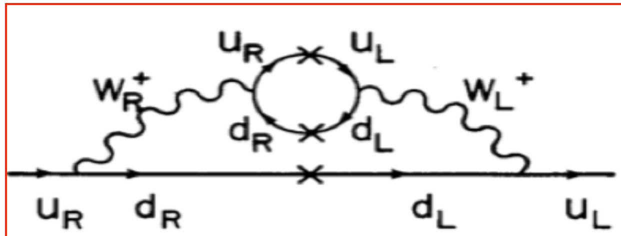
- Understanding flavor in this approach:
- Write potential $V(Y_u, Y_d)$ at M_f and minimize to get SM Yukawas:
- For MFV (SM) case problematic: (Alonso, Gavela, Merlo, Rigolin)

-Not for LR Gauged flavor case (preliminary)

-LR Advantage: $\theta_{QCD} = 0$

tree and 1-loop;→ 2-loop small

(Babu, RNM'90)



$$\bar{\theta} \sim \frac{g^4 m_b m_t}{(16\pi^2)^2 v_R^2} \sim 10^{-10}$$

New TeV scale Physics

- Sub-TeV vector-like fermions (top partner) ψ_t

  LHC: M_{ψ_t}> 760 GeV (ATLAS); > 475 GeV (CMS)
- 3rd gen mixing: $\theta_{t\psi_t} \leq 5\%$
- $t \rightarrow cg$ $\quad \kappa \approx 10^{-3}\left(\frac{TeV}{Y_{u,33}}\right)^{3}(TeV)^{-1}$;100 times SM
- Striking LHC signal 6b+2W

$$pp \rightarrow \psi_t \overline{\psi}_t + X$$

$$\psi_t \rightarrow t + H \rightarrow bWb\bar{b}$$

- 1-2% level CKM unitarity violation: (Branco, Botella, Nebot'12)

Strategy with discrete global flavor symmetry:

- Add SM singlet G_f non-singlet flavon fields ϕ_a
- For $\mu \ll M_f$, $\mathcal{L}_{eff} = \frac{1}{M_f^n}\bar{Q}_L \phi_a^n H u^c + ..$

- Minimize flavon potential→determines the Yukawa couplings i.e. $h_{u,d} = \frac{<\phi_a^n>}{M_f^n}$

Berezhiani, King, Ross;M-Varzielas; Altarelli, Feruglio, Hagedorn, Luhn, Merlo,Smirnov, Schmidt, ..

- Used extensively for neutrinos

→ Our example GUT model unifying quarks and leptons

GUT theory of flavor

- Many reasons to consider GUTs :
- GUTs unify couplings at very high scale:
- Interesting flavor relations at GUT scale: e.g.

$$m_b \approx m_\tau ; m_\mu \approx 3 m_s \qquad \frac{m_{sol}}{m_{atm}} \sim \theta_C ; \theta_{13} \sim \theta_C$$

- Seesaw for neutrinos points to GUT scale:

Type I: $m_\nu \cong -\frac{h_\nu^2 v_{wk}^2}{M_R}$ $M_R \approx 10^{14} GeV$; similar for Type II:

- Desirable to avoid domain wall problem $M_f \gg M_U$

Underlying structure in quark-lepton flavor for model building

- Suppose

$$M_u = M_0 + \delta_u$$

$$M_d = rM_0 + \delta_d$$

$$M_l = rM_0 + \delta_l$$

$$\delta_{u,d,l} << M_0$$

- Choose basis so m_ν diagonal.

→ **small quark mixings and large lepton mixings**

(Dutta, Mimura, RNM' PRD-09)

How to see that ?

- **Suppose:** $U_0 M_0 U_0^+ = M^{diag}$
- **Then** $VU_0(rM_0 + \delta_d)U_0^+ V^+ = M_d^{diag}$
- **Since** $\delta_{u,d,l} << M_0$ **off-diagonal elements of** V **are small.**

$$V_{CKM} = U_0 U_0^{+} V^{+} = V^{+}$$

- **On the other hand,** $U_{PMNS} = U_0$ **whose matrix elements are large for anarchic M$_0$**

- *Does not however explain mass hierarchies*

Rank mechanism and mass hierarchy

- **Assumption (II):** M0 **rank one** i.e.

$$M_0 = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \begin{pmatrix} a & b & c \end{pmatrix}$$

- mass for third gen fermions: t, b, τ with $m_b \cong m_\tau$

- Turn on $\delta_{u,d,l} << M_0$, $m_{c,s,\mu} << m_{t,b,\tau}$

- Relates mixings to masses;

(Rank idea: Balakrishna, Kagan, RNM; Babu, RNM'88-90; Berezhiani, Rattazi '95 Dobrescu, Fox,Ferreti, King, Romanino'2000+;...)

Illustration for 2-Gen. case

- Suppose $M_0 = \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} c & s \end{pmatrix}$ and $m_\nu = diag(\epsilon_1, \epsilon_2) \propto \delta_{u,d}$.
- c=cos θ; s=sin θ
- $\theta = Atm.$ angle; chosen large.
- **Predictions**: $m_\tau \cong m_b$, $\frac{m_s}{m_b} \approx -V_{cb} \tan\theta$ } consistent with observations
- Rest of talk :Realization in GUT theories

Rank idea in flavon approach

- Leading order charged fermion mass matrix:

$$M_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

$$\equiv < \phi_f >< \phi_f^T >$$

- Neutrinos:

$$M_\nu^0 \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} 0 & 1 & -1 \end{pmatrix}$$

SO(10) as the right group

- **Ansatz:** $$M_u = M_0 + \delta_u$$ $$M_d = rM_0 + \delta_d$$ **as** $\delta_{u,d} \to 0, M_u \propto M_d$

- In SM, u_R d_R are singlets- so M_u, M_d **unrelated.**
- We need a theory where, $\begin{pmatrix} u_R \\ d_R \end{pmatrix}$ are in a doublet.
- SU(5) cannot do: u_R , d_R in separate multiplets

- **SO(10) is precisely such a theory.**

Renormalizable SO(10)

- $16_m x 16_m = \{10\}_H + \{120\}_H + \{126\}_H$
- Fermion masses from Yukawa couplings as in SM

$$L_Y = h 16 \cdot 16 \cdot 10_H + f 16 \cdot 16 \cdot \overline{126}_H + h' 16 \cdot 16 \cdot [120,10]_H$$

- **Unifies quark and lepton flavors:** **(Babu, Mohapatra, 93)**
- (Fukuyama,Okada'02; Bajc, Senjanovic, Vissani'02; Goh, Mohapatra, Ng'03; Babu, Macesanu'05; Aulakh,Bajc,Melfo,Senjanovic, Vissani;Fukuyama,Ilakovic,Meljanac,Kikuchi,Okada; Dutta,Mimura,RNM;Bertolini,Frigerio,Malinsky; Joshipura,Patel'11;Altarelli,Blankenburg'11; Dev, RNM, Severson'11)

Fermion mass formulae in $SO(10)_{126}$

- Define $Y_f = M_f / v_{wk}$ (Babu, RNM'93)
- **SO(10) mass formulae:** **Compare with**

$$
\begin{aligned}
Y_u &= h + r_2 f + r_3 h' \\
Y_d &= r_1(h + f + h') \\
Y_e &= r_1(h - 3f + c_e h')
\end{aligned}
$$

$$
\begin{aligned}
M_u &= M_0 + \delta_u \\
M_d &= rM_0 + \delta_d \\
M_l &= rM_0 + \delta_l
\end{aligned}
$$

- Type II seesaw for neutrino mass→ $m_\nu \cong f v_\Delta$

Anstaz realized if $h \gg f, h'$-model fits data, predictive

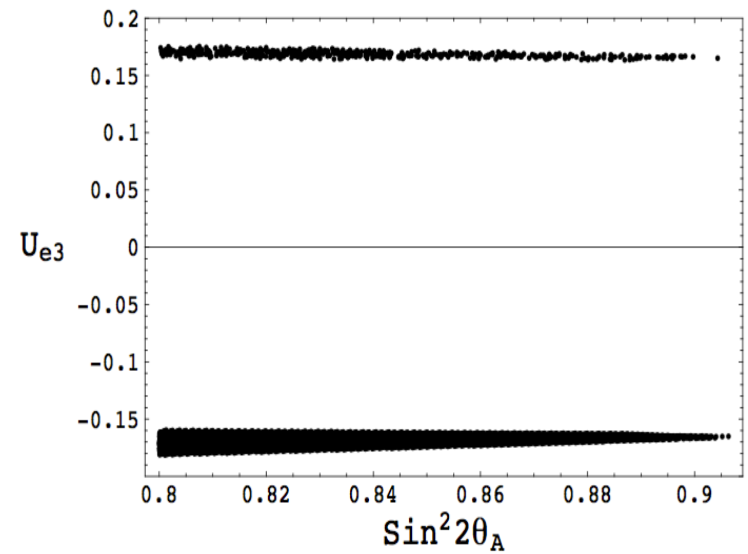
Theta_13 prediction of $SO(10)_{126}$ Models

- Predicts $\theta_{23},\ \theta_{12}$,solar mass in the right range:

- Type II seesaw case: →

Goh, RNM, Ng'03

$$\theta_{13} \simeq 0.17$$



- Babu and Macesanu'05:

$$\theta_{13} \simeq 0.156$$

Non-SUSY case (Type I) (Joshipura, Patel, '11; Severson'12)

$$\theta_{13} \simeq 0.156$$

(Daya Bay-RENO-DC expt ~0.15)

Some SO(10) details:

- Pre GUT superpotential with S_4 flavons, ϕ_1, ϕ_2, ϕ_3

$$W = (\phi_1\psi)(\phi_1\psi)H + (\phi_2\psi)(\phi_2\psi)\bar{\Delta} + (\phi_3\psi\psi)\bar{\Delta} + (\phi_2\psi\psi)H',$$

Δ (126); H, H' (10)

(Dutta, Mimura, RNM'09)

Yukawa pattern from dynamics

- Examples: S4 triplet flavon case: (Dutta, Mimura and RNM'09)

$$W = \frac{1}{2}m\phi^2 - \lambda\phi^3 = \frac{1}{2}m(x^2 + y^2 + z^2) - \lambda xyz.$$

$$\phi = \frac{m}{\lambda}\{(1,1,1) \text{ or } (1,-1,-1) \text{ or } (-1,1,-1) \text{ or } (-1,-1,1)\}.$$

- While for

$$W = \frac{1}{2}m\phi^2 - \frac{\kappa_1}{M}(\phi^4)_1 - \frac{\kappa_2}{M}(\phi^4)_2$$

$$\vec{a} = (0,0,\pm1),\ (0,\pm1,0),\ (\pm1,0,0),\ \vec{b} = (\pm1,\pm1,\pm1),\ \text{and}\ \vec{c} = (0,\pm1,\pm1),$$

- Slightly perturbed around this vacuum W $+\delta\ \phi_2\phi_3$

Fits and Predictions

- Fernion masses:

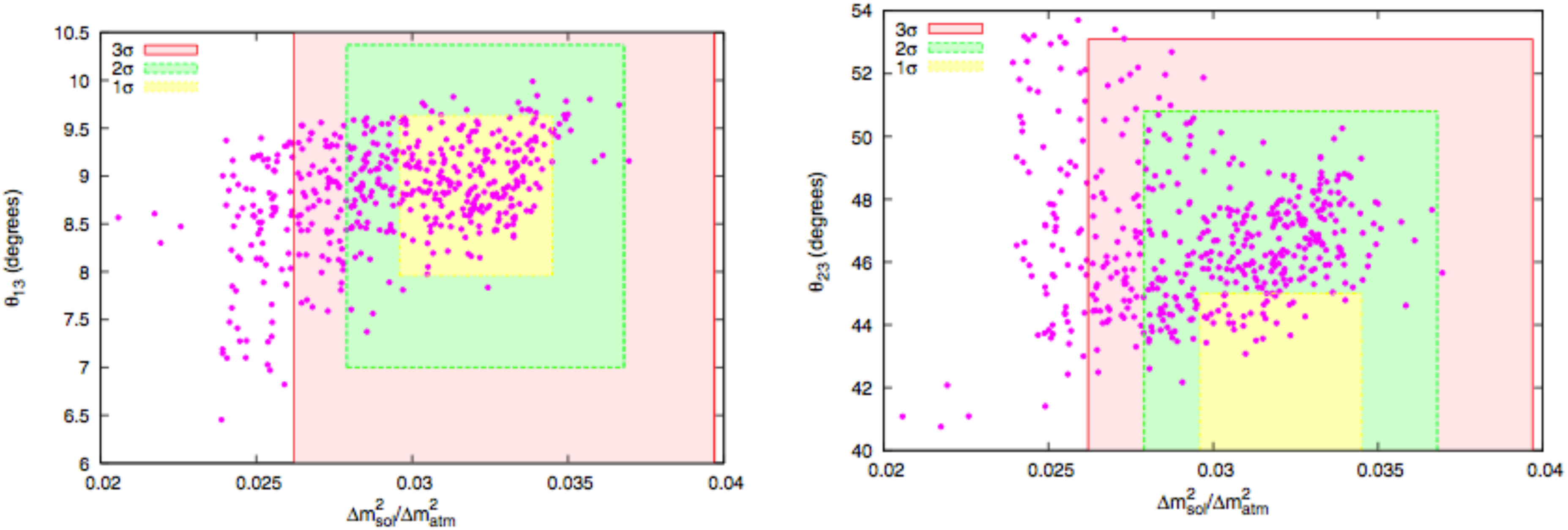
(17 parameters)→5 predictions

	Predicted value	3σ exp range
θ_{12}	33.77°	(30.6–36.8)°
θ_{23}	44.82°	(35.7–53.1)°
θ_{13}	9.02°	(1.8–12.1)° [(5.9–11.6)°]
δ_D	−165.28°	

(Severson, Dutta, Dev, RNM'12)

	Best fit	Exp value
m_e (MeV)	0.3585	$0.3585^{+0.0003}_{-0.003}$
m_μ (MeV)	75.6719	$75.6715^{+0.0578}_{-0.0501}$
m_τ (GeV)	1.2922	$1.2922^{+0.0013}_{-0.0012}$
m_d (MeV)	0.8960	$1.5036^{+0.4235}_{-0.2304}$
m_s (MeV)	21.9535	$29.9454^{+4.3001}_{-4.5444}$
m_b (GeV)	1.0627	$1.0636^{+0.1414}_{-0.0865}$
m_u (MeV)	0.7284	$0.7238^{+0.1365}_{-0.1467}$
m_c (MeV)	209.8979	$210.3273^{+19.0036}_{-21.2264}$
m_t (GeV)	84.1739	$82.4333^{+30.2676}_{-14.7686}$
V_{us}	0.2243	0.2243 ± 0.0016
V_{ub}	0.0033	0.0032 ± 0.0005
V_{cb}	0.0351	0.0351 ± 0.0013
J	2.19×10^{-5}	$(2.2 \pm 0.6) \times 10^{-5}$
$\Delta m^2_\odot / \Delta m^2_{\text{atm}}$	0.0321	0.0320 ± 0.0025
χ^2	4.05	

Neutrino predictions:

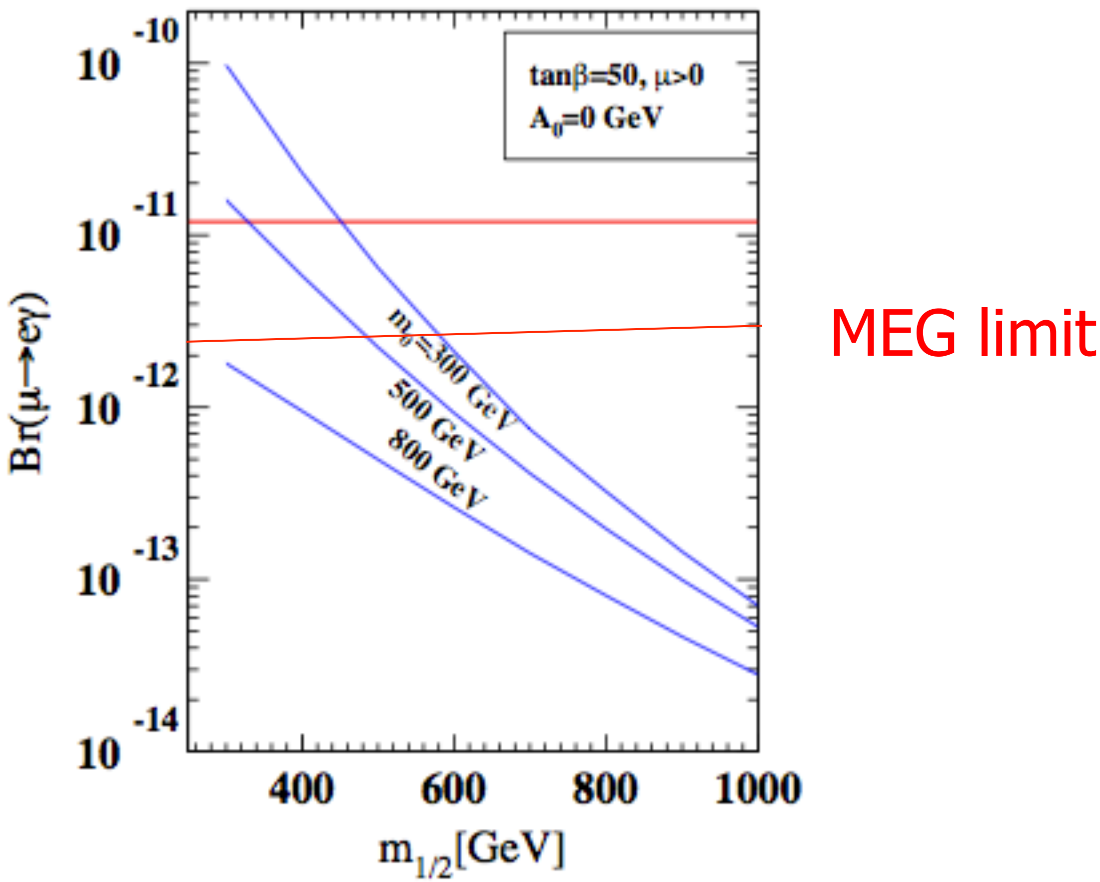


- Requires threshold corrections which make predictions for susy parameters.

(Severson, Dutta, Dev, RNM'12)

Prediction for $\mu \to e + \gamma$

- .



Predictions for stop and gluion masses

- Large threshold corrections require large A terms and gluino masses which are required to fit Higgs mass 125 GeV.

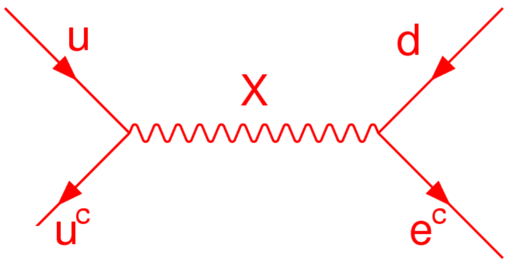
$$M_{\tilde{G}} \geq 1.5TeV; M_{\tilde{t}} \geq 700GeV$$

- Testable in colliders

GUTs and Proton decay

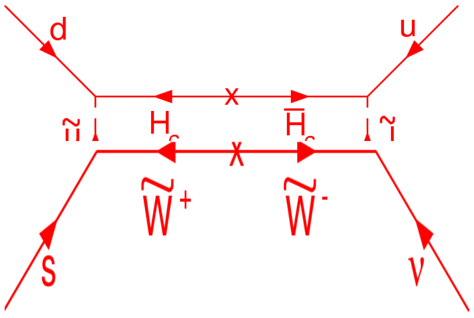
- **Beware of Proton decay problem !!**
- SUSY GUTs - two generic sources:

(i) Gauge exchange:



$$p \to e^+\pi^0,\ \tau_p^{-1} \approx \left[\frac{g^2}{M_X^2}\right]^2 m_p^5 \approx [10^{36\pm1} yr]^{-1}$$

(ii) Higgsino exchange:(dangerous one)



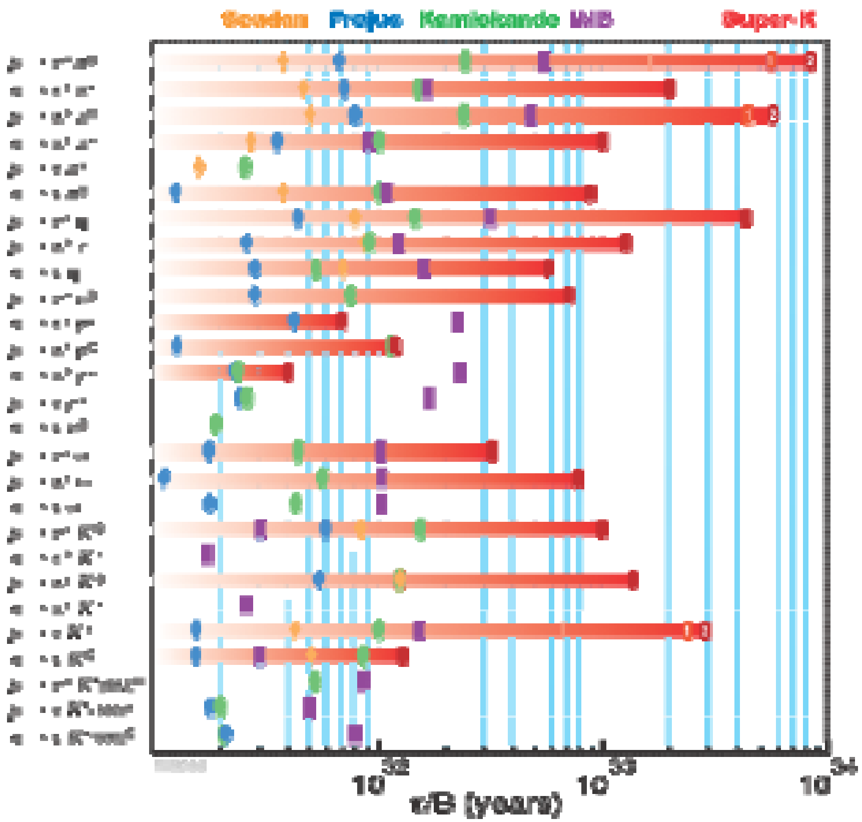
$$p \to \bar{\nu}K^+$$

$$\tau_p^{-1} \approx [\frac{f^2}{M_{H_c}M_{SUSY}}]^2(\frac{\alpha}{4\pi})^2 m_p^5 \approx [10^{28} - 10^{32} yr]^{-1}$$

- **Present limit:** $\tau_{\bar{\nu}K^+} > 2.3\times10^{33}\, yrs$

Present experimental limits

- Super-K, Soudan, IMB, Frejus

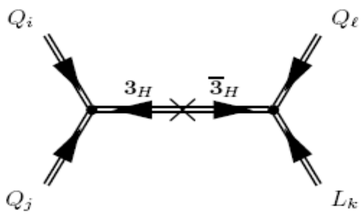


Ed Kearns @ LBNV-09

; $\tau_{n\bar{n}} \geq 2 \times 10^{8}\ sec.$

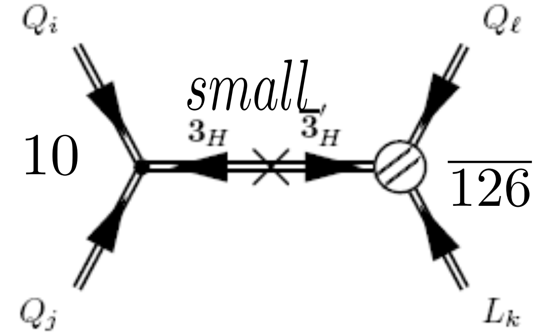
Rank one keeps susy proton decay mode small

- Proton decay problem in SU(5): one Higgs pair s



$$\rightarrow A_p \propto Y_u Y_d$$

- In SO(10), there are more Higgs fields and if flavor structure is such that triplet Higgs do not connect, no p-decay problem:



- Choice flavor structure that does it (Dutta, Mimura, RNM' 05)

$$h_{10} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad h_{126} = \begin{pmatrix} 0 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix};$$

$$h_{120} = \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix};$$

10

$$\tau_{p\rightarrow K\nu} \leq 10^{34} yrs$$

Predicts seesaw linked B-L violating proton decays

- Seesaw breaks B-L by two units:
- Leads to B-L =2 proton decay modes via dim 6 operators e.g. $u^c d^c d^c L H_u$ $d^c d^c d^c L H_d$

(Babu, Mohapatra, Phys. Rev. Lett. 109 (2012) 091803)

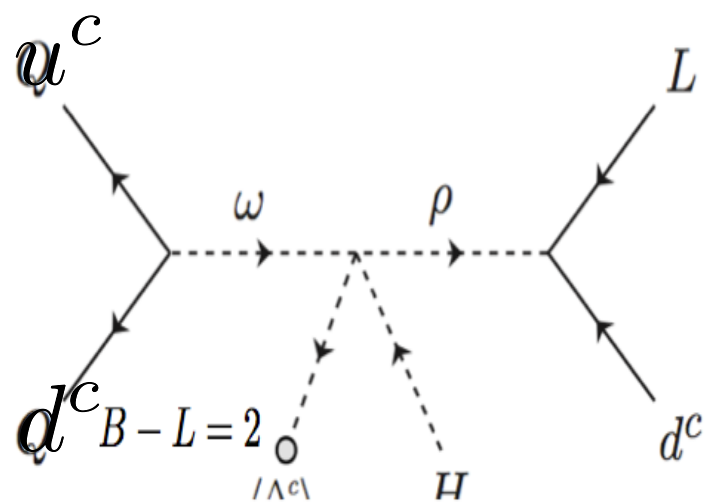
- Observable Modes

$$n \rightarrow e^- K^+, e^- \pi^+$$

- Diagram from exchange of

126 fields

$$\tau_p \approx 4 \times 10^{33} \ yrs$$



Conclusion:

(i) pre-GUT scale Flavon approach promising to unify diverse profiles of quark and lepton flavor patterns using LR symmetric GUTs

(ii) SO(10) GUT with type II seesaw :S_4 example that realizes ansatz.

(iii) Predicts θ_{13} in agreement with observations and solves proton decay problem, predicts new modes

Extra Slides

- They follow:

Possible solution to strong CP problem in SO(10)

- Slightly different version: 10+126+120 with CP
- Under CP, 10→10*, 126→ 126*; 120→-120*
- Makes 10, 126 Yukawas real, 120 Yukawa imaginary: <120> breaks CP → hermitean mass matrices

  ---Non-trivial CKM but

  ---Arg Det M_u M_d =0→ θ_{QCD} =0 tree

From SO(10) down to the Std Model

SO(10) **Nu mass**

$$\downarrow$$

- **LR Sym.** $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ $\begin{pmatrix} 0 & 0 \\ 0 & M \end{pmatrix}$

$$\Downarrow \quad \Delta(B-L) \neq 0$$

- **Standard Model-** $SU(2)_L \times U(1)_Y \times SU(3)_c$

-> seesaw $\begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$

$$\downarrow$$

$$SU(3)_c \times U(1)_{em}$$

A specific realization with predictive textures:

- Group: SO(10)x**S4** $\supset 3_1 + 3_2 + 2 + 1_1 + 1_2$
- **Consider flavons** $\phi_{1,2,3} \subset 3_{1,2}$ **; matter {16}**$\subset 3_2$
- **Inv effective superpotential at GUT scale:**

$$W = (\phi_1\psi)(\phi_1\psi)H + (\phi_2\psi)(\phi_2\psi)\bar{\Delta} + \phi_3\psi\psi\bar{\Delta} + \phi_2\psi\psi H'$$

- **The flavon vevs align as:** $\phi_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$

- **Leading to f=** $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ **and h' =** $\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

- **Gives realistic model for fermion masses and mixings**

Naturally large neutrino mixings

- Diagonalize M_0;

$$\hat{M}_e \equiv U_0 M_e U_0^T = \mathrm{diag}(0,0,m_3) + U_0(\delta M_e)U_0^T$$

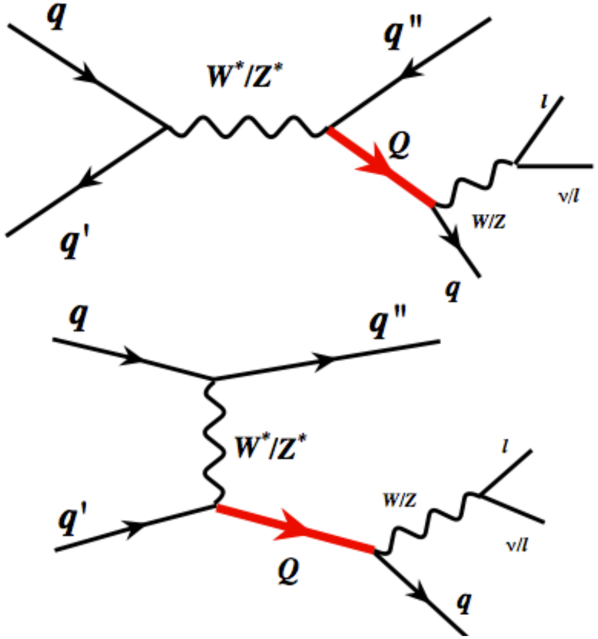
$$U_0 = \begin{pmatrix} \cos\theta_s & \sin\theta_s & 0 \\ -\cos\theta_a \sin\theta_a & \cos\theta_a \cos\theta_s & -\sin\theta_a \\ -\sin\theta_a \sin\theta_s & \sin\theta_a \cos\theta_s & \cos\theta_a \end{pmatrix}$$

$$\tan\theta_s = \frac{c}{b} \qquad \tan\theta_a = \frac{\sqrt{b^2+c^2}}{a}$$

- In the mass basis θ_a =atmos θ_s =solar angles and are generically large.

LHC searches for vectorlike quarks

- Production: ATLAS 1.04fb^{-1} : 3rd gen. partner



$M_Q > 760$ GeV.

For TeV mass $\sigma \sim 10\, fb$

- CMS: pp->QQ-bar→t+Z+t-bar+Z→ $M_Q > 475$ GeV

$$Z \downarrow \ell\bar{\ell}$$

Other consequences

- Reduction of top width
- $t \to cg$ probe

Parameterize: $L_{eff} = \kappa \bar{t}_R \sigma^{\mu\nu} c_L G_{\mu\nu}$

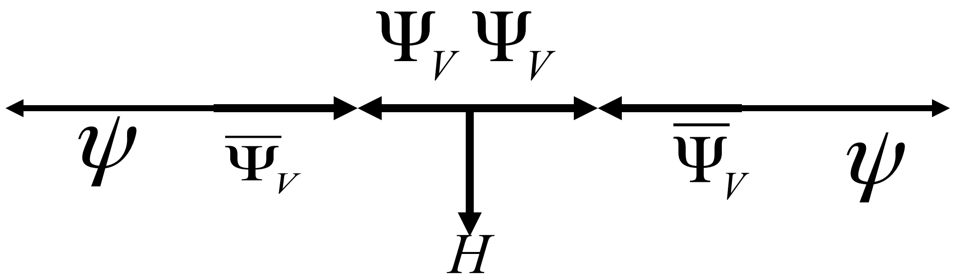
SM prediction $\kappa \sim 10^{-5} (TeV)^{-1}$ D0: <.018 $(TeV)^{-1}$

Our model: intermediate top partner mediated graph

→ $$\kappa \approx 10^{-3} \left(\frac{TeV}{Y_{u,33}} \right)^3 (TeV)^{-1}$$

Detailed flavon approach in SUSY SO(10): example

- Rank one in an effective theory using vector like matter $\Psi_V\{16\} + \overline{\Psi}_V\{\overline{16}\}$



- Replace Yukawa couplings by flavon fields ϕ_i whose value is determined by the minimum of the theory via family discrete sym constraints.

Perturbed flavon vacuum and Phenomenology

- Perturb: vev alignment: (Dev, Dutta, RNM, Severson'12)

$$\phi_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \qquad \phi_2 = \begin{pmatrix} \epsilon \\ a \\ b \end{pmatrix}, \qquad \phi_3 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

$$(\epsilon, a, b) = (-0.080, -0.752, 0.692), \text{ and } (x, y, z) = (0.937, 0.928, 0.936)$$

$$W = \frac{1}{2} m\phi^2 - \frac{\kappa_1}{M}(\phi^4)_1 - \frac{\kappa_2}{M}(\phi^4)_2 + \delta\ \phi_2\phi_3$$

Predicts Yukawa matrices:

$$\bar{f} = m_0 \begin{pmatrix} \epsilon^2 & \epsilon a & \epsilon b \\ a\epsilon & a^2 & ab \\ b\epsilon & ba & b^2 \end{pmatrix} + m_1 \begin{pmatrix} 0 & z & y \\ z & 0 & x \\ y & x & 0 \end{pmatrix},$$

$$h = \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1.946 \end{pmatrix}$$

$$\bar{h}' = \delta \begin{pmatrix} 0 & b & a \\ b & 0 & \epsilon \\ a & \epsilon & 0 \end{pmatrix},$$