Neutrino Mass from d>5 Effective Operators in an SU(5) GUT with Discrete Symmetry

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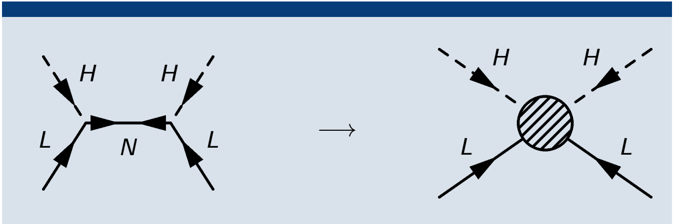
Motivation and Outline

Many new physics models come with an **extented Higgs sector** and additional **(discrete) symmetries**.

- Systematic study of neutrino mass generation by higher-dimensional effective operators
- New physics at the TeV scale and phenomenological implications at the LHC
- Embedding in SU(5) GUT and consequences for phenomenology

From new physics close to the GUT scale...

- The usual type I seesaw introduces new physics close to the GUT scale.

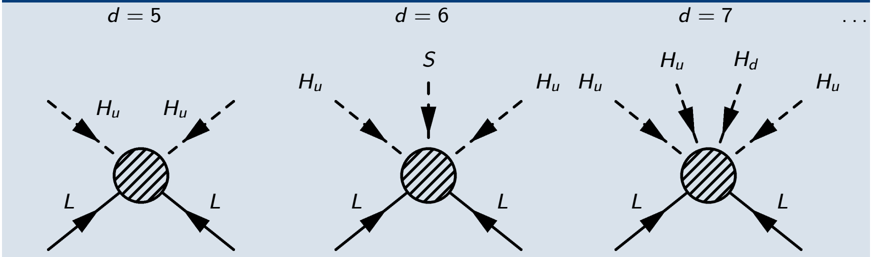


- At low energies the new physics effects can be described by the **Weinberg operator** $\mathcal{O}_W = (\overline{L^c} \mathrm{i}\tau^2 H)(H \mathrm{i}\tau^2 L)$ of $d = 5$.
- After EWSB $\frac{{Y_N}^2}{m_N} \langle H \rangle^2 \bar{\nu}^c \nu \rightarrow$ generates neutrino mass $m_\nu^{\text{eff}} \propto \frac{v^2}{\Lambda}$, with $\Lambda = m_N$

Not testable in experiments!

. . . down to the TeV scale

In theories with additional scalars (THDM, MSSM, NMSSM, . . .)
→ *Operators with* $d > 5$ *can have siginificant contribution to neutrino mass*



$\frac{1}{\Lambda}\langle H_u\rangle^2$ $\quad\frac{1}{\Lambda^2}\langle H_u\rangle^2\langle S\rangle$ $\quad\frac{1}{\Lambda^3}\langle H_u\rangle^2\langle H_u\rangle\langle H_d\rangle$

- Theories with discrete symmetries → operator can be forbidden at $d = 5$
- Operator with $d > 5$ as leading contribution to neutrino mass
- New physics scale can be at **lower energy**

Possible Effective Operators in the NMSSM

$$W_{\mathrm{NMSSM}} = y_u u^c Q H_u + y_d d^c Q H d + y_e e^c L H_d + \lambda S H_u H_d + \frac{1}{3}\kappa S^3$$

	Op.#	Effective interaction	Charge	Same as
$d=5$	1	LLH_uH_u	$2q_L + 2q_{H_u}$	
$d=6$	2	LLH_uH_uS	$2q_L + q_{H_u} - q_{H_d}$	
$d=7$	3	$LLH_uH_uH_dH_u$	$2q_L + 3q_{H_u} + q_{H_d}$	
	4	LLH_uH_uSS	$2q_L - 2q_{H_d}$	
$d=8$	5	$LLH_uH_uH_dH_uS$	$2q_L + 2q_{H_u}$	#1
	6	LLH_uH_uSSS	$2q_L + 2q_{H_u}$	#1
$d=9$	7	$LLH_uH_uH_dH_uH_dH_u$	$2q_L + 4q_{H_u} + 2q_{H_d}$	
	8	$LLH_uH_uH_dH_uSS$	$2q_L + q_{H_u} - q_{H_d}$	#2
	9	LLH_uH_uSSSS	$2q_L + q_{H_u} - q_{H_d}$	#2

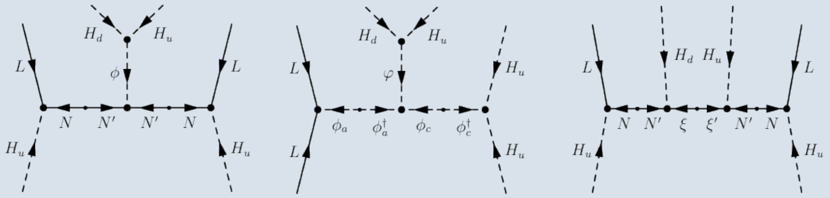
Characteristics

- Condition for discrete charges of fields from neutrality of superpotential
- Rules out some operators as leading contribution to neutrino mass
- Several possible fundamental theories can lead to the same effective operator

MBK, Ota, Porod, Winter (2011); *PRD 84, 115023*

(c.f. Bonnet, Hernandez, Ota, Winter (2009); *JHEP 0910, 076* for a study in the THDM)

Possible Decompositions for $d = 7$

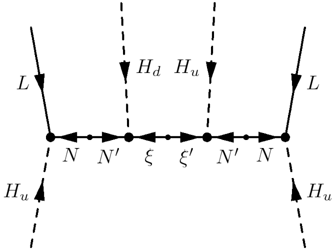


- Same external fields, different mediators
- Scalar mediators potentially problematic:
  VEV of scalar $\rightarrow$ induces $d = 6$ operator

A $d = 7$ Example

Superpotential

$$W = W_{\text{(N)MSSM}} + Y_N \hat{N}\hat{L} \cdot \hat{H}_u - \kappa_1 \hat{N}'\hat{\xi} \cdot \hat{H}_d + \kappa_2 \hat{N}'\hat{\xi}' \cdot \hat{H}_u + m_N \hat{N}\hat{N}' + m_\xi \hat{\xi} \cdot \hat{\xi}'$$



New fields:

- SM singlets N, N'
- $SU(2)_L$ doublets ξ, ξ'

Mass Matrix

- In the basis $f^0 = (\nu, N, N', \xi^0, \xi'^0)$ we obtain the mass matrix

$$M_f^0 = \begin{pmatrix} 0 & Y_N v_u & 0 & 0 & 0 \\ Y_N v_u & 0 & m_N & 0 & 0 \\ 0 & m_N & 0 & \kappa_1 v_d & \kappa_2 v_u \\ 0 & 0 & \kappa_1 v_d & 0 & m_\xi \\ 0 & 0 & \kappa_2 v_u & m_\xi & 0 \end{pmatrix}.$$

- By **integrating out** the heavy fields we obtain an **effective mass matrix** for the three SM neutrinos at **low energies**

$$m_\nu = v_u^3 v_d Y_N^2 \frac{\kappa_1 \kappa_2}{m_\xi m_N^2}$$

Masses at TeV scale for couplings $\mathcal{O}(10^{-3})$

LHC Phenomenology

Production of the new particles

- Rare production of $\hat{N}$ and $\hat{N}'$ due to small Yukawa couplings
- $SU(2)_L$ doublets can be produced in Drell-Yan processes ($\sigma \sim 10^2$ fb)

Characteristic Signals

- Displaced vertices due to small mixing between heavy and light neutrinos
- Lepton number violating processes
  - LNC cross-section for $pp \to W\ell\ell$ of $\mathcal{O}(10^2)$ fb
    LNV processes suppressed due to pseudo-Dirac pairs ($< \mathcal{O}(10^{-9})$ fb)
  - For $pp \to W\ell W\ell$ LNV processes larger than naively expected ($\mathcal{O}(10^{-2})$ fb)

MBK, Ota, Porod, Winter (2011); *PRD 84, 115023*

GUT Completion

- Additional particles modify running of the gauge couplings
- Spoils unification
- Add complete SU(5) multiplets to avoid this
  - Singlets: N, N', (S)
  - 5-plets:

$$\bar{5}_M = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L \qquad \bar{5}_{\xi'} = \begin{pmatrix} d_1'^c \\ d_2'^c \\ d_3'^c \\ \xi'^- \\ -\xi'^0 \end{pmatrix}_L \qquad 5_\xi = \begin{pmatrix} d_1'' \\ d_2'' \\ d_3'' \\ \xi^+ \\ -\xi^0 \end{pmatrix}_R$$

$$H_5 = \begin{pmatrix} H_1 \\ H_2 \\ H_3 \\ H_u^+ \\ H_u^0 \end{pmatrix} \qquad H_{\bar{5}} = \begin{pmatrix} H_1' \\ H_2' \\ H_3' \\ H_d^- \\ H_d^0 \end{pmatrix}$$

  - matter 10-plet

MSSM scenario

Most general SU(5) invariant superpotential

$$
\begin{aligned}
W \quad = \quad & y_1\, N\, 5_\xi\, H_{\bar{5}} + y_2\, N\, \bar{5}_{\xi'}\, H_5 + y_3\, N\, \bar{5}_M\, H_5 + \\
& y_1' N'\, 5_\xi\, H_{\bar{5}} + y_2'\, N'\, \bar{5}_{\xi'}\, H_5 + y_3'\, N'\, \bar{5}_M\, H_5 + \\
& m_{\xi'}\, \bar{5}_M\, 5_\xi + m_\xi\, \bar{5}_{\xi'}\, 5_\xi + m_N N' N + m_{NN} NN + m_{N'N'} N' N' + \\
& y_d\, \bar{5}_M\, 10\, H_{\bar{5}} + y_d'\, \bar{5}_{\xi'}\, 10\, H_{\bar{5}} + y_u\, 10\, 10\, H_5 \,.
\end{aligned}
$$

If charged under a discrete symmetry,

Multiplet	$\bar{5}_M$	H_5	$H_{\bar{5}}$	N	N'	5_ξ	$\bar{5}_{\xi'}$	10
$\mathbb{Z}_3$ charge	1	1	1	1	2	0	0	1

the superpotential reduces to

$$
\begin{aligned}
W \quad = \quad & y_3\, N\, \bar{5}_M\, H_5 + y_1',\, N'\, 5_\xi\, H_{\bar{5}} + y_2'\, N'\, \bar{5}_{\xi'}\, H_5 + \\
& m_\xi\, \bar{5}_{\xi'}\, 5_\xi + m_N N' N \\
& y_d\, \bar{5}_M\, 10\, H_{\bar{5}} + y_u\, 10\, 10\, H_5 - \mu H_{\bar{5}} H_5 \,.
\end{aligned}
$$

Heavy d-Quarks

$$\begin{pmatrix} d_1'^c \\ d_2'^c \\ d_3'^c \\ \xi'^- \\ -\xi'^0 \end{pmatrix}_L$$

Interactions of d'

- Coloured components of mediator 5-plets
- Behave like heavy d-quarks
- RGE running leads to mass shift between quarks and lepton doublet
- Decay of d' protected by symmetry that forbids $d = 5$ operator

Cosmological constraints:

- From Big Bang Nucleosynthesis: Heavy nuclei
  $\rightarrow$ altering BBN processes $\rightarrow$ affecting observed abundancies of light elements
  e.g. Iocco *et. al.* (2009); *Phys.Rept. 472*
- Search for heavy hadrons in water excludes stable heavy d-like quarks
  Nardi, Roulet (1990); *Phys. Lett. B 245, 105*
- Effective operator $\epsilon^{ijklm}(5_\xi)_i(H_5)_j(H_5)_k(10)_{lm}$ leads to the decay

$$\overline{d}' \rightarrow H_u^+ \overline{u}$$

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Going to the NMSSM

- The term $\mu H_u H_d$ explicitly breaks the discrete symmetry (Otherwise every operator of the type $LLH_uH_u(H_uH_d)^n$ has same charge as Weinberg operator)
- μ-problem of the MSSM (μ has to be set to 100 GeV to few TeV by hand)
- Same issue with TeV mediator masses

Possible Alternative:
Use the NMSSM where μ and the mediator masses are generated by VEV of an additional scalar field S.

NMSSM scenario

Most general SU(5) invariant superpotential

$$\begin{aligned} W \quad = \quad & y_1\, N\, 5_\xi\, H_{\bar{5}} + y_2\, N\, \bar{5}_{\xi'}\, H_5 + y_3\, N\, \bar{5}_M\, H_5 + \\ & y'_1,\, N'\, 5_\xi\, H_{\bar{5}} + y'_2\, N'\, \bar{5}_{\xi'}\, H_5 + y'_3\, N'\, \bar{5}_M\, H_5 + \\ & \lambda_{\xi'}\, S\, \bar{5}_M\, 5_\xi + \lambda_\xi\, S\, \bar{5}_{\xi'}\, 5_\xi + \lambda_N S\, N' N + \lambda_{NN} S\, NN + \lambda_{N'N'} S\, N' N' + \\ & y_d\, \bar{5}_M\, 10\, H_{\bar{5}} + y'_d\, \bar{5}_{\xi'}\, 10\, H_{\bar{5}} + y_u\, 10\, 10\, H_5\,. \end{aligned}$$

BUT if all masses generated by $\langle S \rangle$ the effective operators become

$$\frac{1}{\langle S \rangle} LLH_uH_u\,, \qquad \frac{1}{\langle S \rangle^3}(LLH_uH_d)(H_uH_d)\,, \quad \ldots$$

$\Rightarrow \langle S \rangle$ breaks discrete symmetry

Superpotential constrains charges in a way that we always will have a $d = 5$ *contribution.*

Extended NMSSM scenario

We introduce an additional scalar S' and obtain the superpotential

$$W = \quad y_3\, N\, \overline{5}_M\, H_5 + y'_1,\, N'\, 5_\xi\, H_{\overline{5}} + y'_2\, N'\, \overline{5}_{\xi'}\, H_5 + \lambda_\xi\, S'\, \overline{5}_{\xi'}\, 5_\xi + \lambda_N S'\, N' N$$
$$+ y_d\, \overline{5}_M\, 10\, H_{\overline{5}} + y_u\, 10\, 10\, H_5 + \lambda_S S H_{\overline{5}} H_5 + \kappa S^3 + \lambda'_S S' H_{\overline{5}} H_5 + \kappa' {S'}^3\, .$$

we can choose the charges

Multiplet	$\bar{5}_M$	H_5	$H_{\bar{5}}$	N	N'	5_ξ	$\bar{5}_{\xi'}$	10	S	S'
$\mathbb{Z}_3$ charge	1	1	1	1	2	0	0	1	1	0

- The term λ'_S breaks the symmetry softly.

Soft breaking of the discrete symmetry

- Couplings breaking the symmetry must be small!
  (Supression of the Weinberg operator)
- Soft breaking term $y_3' \, N' \, \overline{5}_M \, H_5 \to d = 5$ contribution to m_ν

$$m_\nu^{d=5} = \frac{y_3 y_3' v_u^2}{\langle S' \rangle} \, .$$

- We require

$$m_\nu^{d=5} < m_\nu^{d=7} = \frac{y_1 y_2 y_3^2 v_u^3 v_d}{\langle S' \rangle^3}$$
$$\Rightarrow y_3' < \frac{y_1 y_2 y_3 v_u v_d}{\langle S' \rangle^2}$$

- d' decay via the soft breaking operator $\overline{5}_\xi H_{\overline{5}} 10$

Summary

- Possible to use effective operators with $d > 5$ to generate neutrino masses
- New physics at TeV scale, phenomenological implications at LHC
- Full SU(5) multiplets necessary to not spoil unification
- Additional d-quarks $\rightarrow$ consider cosmological constraints, decay via effective operator or by soft symmetry breaking
- NMSSM realization with softly broken symmetry

Backup-Slides

Decompositions

#	Operator	Mediators	SU(5) multiplets
1	$(H_u \mathrm{i}\tau^2 \overline{L^c})(H_u \mathrm{i}\tau^2 L)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{1}^R_0,\ \mathbf{1}^L_0,\ \mathbf{1}^s_0$	$\mathbf{1}, \mathbf{1}, \mathbf{1}$
2	$(H_u \mathrm{i}\tau^2 \vec{\tau}\overline{L^c})(H_u \mathrm{i}\tau^2 L)(H_d \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{3}^R_0,\ \mathbf{3}^L_0\ \mathbf{1}^R_0,\ \mathbf{1}^L_0,\ \mathbf{3}^s_0$	$\mathbf{24}, \mathbf{24}, (\mathbf{1}), (\mathbf{1}), \mathbf{24}$
3	$(H_u \mathrm{i}\tau^2 \vec{\tau}\overline{L^c})(H_u \mathrm{i}\tau^2 \vec{\tau} L)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{3}^R_0,\ \mathbf{3}^L_0,\ \mathbf{1}^s_0$	$\mathbf{24}, \mathbf{24}, \mathbf{1}$
4	$(-\mathrm{i}\epsilon^{abc})(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(H_u \mathrm{i}\tau^2 \tau^b L)(H_d \mathrm{i}\tau^2 \tau^c H_u)$	$\mathbf{3}^R_0,\ \mathbf{3}^L_0,\ \mathbf{3}^s_0$	$\mathbf{24}, \mathbf{24}, \mathbf{24}$
5	$(\overline{L^c} \mathrm{i}\tau^2 \vec{\tau} L)(H_d \mathrm{i}\tau^2 H_u)(H_u \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{3}^s_{+1},\ \mathbf{3}^s_{+1},\ \mathbf{1}^s_0$	$\mathbf{15}, \mathbf{15}, \mathbf{1}$
6	$(-\mathrm{i}\epsilon_{abc})(\overline{L^c} \mathrm{i}\tau^2 \tau_a L)(H_d \mathrm{i}\tau^2 \tau_b H_u)(H_u \mathrm{i}\tau^2 \tau_c H_u)$	$\mathbf{3}^s_{+1},\ \mathbf{3}^s_{+1},\ \mathbf{3}^s_0$	$\mathbf{15}, \mathbf{15}, \mathbf{24}$
7	$(H_u \mathrm{i}\tau^2 \overline{L^c})(L \mathrm{i}\tau^2 \vec{\tau} H_d)(H_u \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{1}^R_0,\ \mathbf{1}^L_0,\ \mathbf{3}^R_{-1},\ \mathbf{3}^L_{-1},\ \mathbf{3}^s_{+1}$	$\mathbf{1}, \mathbf{1}, \mathbf{15}, \overline{\mathbf{15}}, \mathbf{15}$
8	$(-\mathrm{i}\epsilon^{abc})(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(L \mathrm{i}\tau^2 \tau^b H_d)(H_u \mathrm{i}\tau^2 \tau^c H_u)$	$\mathbf{3}^R_0,\ \mathbf{3}^L_0,\ \mathbf{3}^R_{-1},\ \mathbf{3}^L_{-1},\ \mathbf{3}^s_{+1}$	$\mathbf{24}, \mathbf{24}, \mathbf{15}, \overline{\mathbf{15}}, \mathbf{15}$
9	$(H_u \mathrm{i}\tau^2 \overline{L^c})(\mathrm{i}\tau^2 H_u)(L)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{1}^R_0,\ \mathbf{1}^L_0,\ \mathbf{2}^R_{-1/2},\ \mathbf{2}^L_{-1/2},\ \mathbf{1}^s_0$	$\mathbf{1}, \mathbf{1}, \mathbf{5}, \overline{\mathbf{5}}, \mathbf{1}$
10	$(H_u \mathrm{i}\tau^2 \vec{\tau}\overline{L^c})(\mathrm{i}\tau^2 \vec{\tau} H_u)(L)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{3}^R_0,\ \mathbf{3}^L_0,\ \mathbf{2}^R_{-1/2},\ \mathbf{2}^L_{-1/2},\ \mathbf{1}^s_0$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \overline{\mathbf{5}}, \mathbf{1}$
11	$(H_u \mathrm{i}\tau^2 \overline{L^c})(\mathrm{i}\tau^2 H_u)(\vec{\tau} L)(H_d \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{1}^R_0,\ \mathbf{1}^L_0,\ \mathbf{2}^R_{-1/2},\ \mathbf{2}^L_{-1/2},\ \mathbf{3}^s_0$	$\mathbf{1}, \mathbf{1}, \mathbf{5}, \overline{\mathbf{5}}, \mathbf{24}$
12	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\mathrm{i}\tau^2 \tau^a H_u)(\tau^b L)(H_d \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}^R_0,\ \mathbf{3}^L_0,\ \mathbf{2}^R_{-1/2},\ \mathbf{2}^L_{-1/2},\ \mathbf{3}^s_0$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \overline{\mathbf{5}}, \mathbf{24}$
13	$(H_u \mathrm{i}\tau^2 \overline{L^c})(L)(\mathrm{i}\tau^2 H_u)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{1}^R_0,\ \mathbf{1}^L_0,\ \mathbf{2}^s_{+1/2},\ \mathbf{1}^s_0$	$\mathbf{1}, \mathbf{1}, \mathbf{5}, \mathbf{1}$
14	$(H_u \mathrm{i}\tau^2 \vec{\tau}\overline{L^c})(\vec{\tau} L)(\mathrm{i}\tau^2 H_u)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{3}^R_0,\ \mathbf{3}^L_0,\ \mathbf{2}^s_{+1/2},\ \mathbf{1}^s_0$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \mathbf{1}$
15	$(H_u \mathrm{i}\tau^2 \overline{L^c})(L)(\mathrm{i}\tau^2 \vec{\tau} H_u)(H_d \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{1}^R_0,\ \mathbf{1}^L_0,\ \mathbf{2}^s_{+1/2},\ \mathbf{3}^s_0$	$\mathbf{1}, \mathbf{1}, \mathbf{5}, \mathbf{24}$
16	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\tau^a L)(\mathrm{i}\tau^2 \tau^b H_u)(H_d \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}^R_0,\ \mathbf{3}^L_0,\ \mathbf{2}^s_{+1/2},\ \mathbf{3}^s_0$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \mathbf{24}$
17	$(H_u \mathrm{i}\tau^2 \overline{L^c})(H_d)(\mathrm{i}\tau^2 H_u)(H_u \mathrm{i}\tau^2 L)$	$\mathbf{1}^R_0,\ \mathbf{1}^L_0,\ \mathbf{2}^R_{-1/2},\ \mathbf{2}^L_{-1/2}$	$\mathbf{1}, \mathbf{1}, \mathbf{5}, \overline{\mathbf{5}}$
18	$(H_u \mathrm{i}\tau^2 \vec{\tau}\overline{L^c})(\vec{\tau} H_d)(\mathrm{i}\tau^2 H_u)(H_u \mathrm{i}\tau^2 L)$	$\mathbf{3}^R_0,\ \mathbf{3}^L_0,\ \mathbf{2}^R_{-1/2},\ \mathbf{2}^L_{-1/2},\ \mathbf{1}^R_0,\ \mathbf{1}^L_0$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \overline{\mathbf{5}}, (\mathbf{1}), (\mathbf{1})$
19	$(H_u \mathrm{i}\tau^2 \overline{L^c})(H_d)(\mathrm{i}\tau^2 \vec{\tau} H_u)(H_u \mathrm{i}\tau^2 \vec{\tau} L)$	$\mathbf{1}^R_0,\ \mathbf{1}^L_0,\ \mathbf{2}^R_{-1/2},\ \mathbf{2}^L_{-1/2},\ \mathbf{3}^R_0,\ \mathbf{3}^L_0$	$(\mathbf{1}), (\mathbf{1}), \mathbf{5}, \overline{\mathbf{5}}, \mathbf{24}, \mathbf{24}$
20	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\tau^a H_d)(\mathrm{i}\tau^2 \tau^b H_u)(H_u \mathrm{i}\tau^2 \tau^b L)$	$\mathbf{3}^R_0,\ \mathbf{3}^L_0,\ \mathbf{2}^R_{-1/2},\ \mathbf{2}^L_{-1/2},$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \overline{\mathbf{5}}$
21	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a L)(H_u \mathrm{i}\tau^2 \tau^a)(\tau^b H_d)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}^s_{+1},\ \mathbf{2}^s_{+1/2}\ ,\ \mathbf{3}^s_{+1}$	$\mathbf{15}, \mathbf{5}, \mathbf{15}$
22	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a L)(H_d \mathrm{i}\tau^2 \tau^a)(\tau^b H_u)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}^s_{+1},\ \mathbf{2}^s_{+3/2},\ \mathbf{3}^s_{+1}$	$\mathbf{15}, \mathbf{40}, \mathbf{15}$
23	$(\overline{L^c} \mathrm{i}\tau^2 \vec{\tau} L)(H_u \mathrm{i}\tau^2 \vec{\tau})(H_u)(H_u \mathrm{i}\tau^2 H_u)$	$\mathbf{3}^s,\ \mathbf{2}^s,\ \mathbf{1}^s$	$\mathbf{15}, \mathbf{5}, \mathbf{1}$
24			