Running soft parameters in SUSY models with multiple U(1) gauge factors

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(R.M.F., Michal Malinský, Werner Porod and Florian Staub)

Outline

- U(1) mixing
- Renormalization group equations (RGEs) of SUSY models
- Inclusion of the U(1) mixing effect in the RGEs
- Some numerical results
- Conclusions

A new qualitative feature with $\mathrm{U}(1)^n$

- **Gauge theory**: the gauge group H is a direct product of abelian and simple factor groups - e.g: $\mathrm{H} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
- The most general Lagrangian invariant under H leads to the coupling of scalars and fermions to the gauge bosons
- There is one gauge coupling constant g_i for each factor group:

$$\underset{g_3}{\mathrm{SU}(3)} \times \underset{g_2}{\mathrm{SU}(2)} \times \underset{g_1}{\mathrm{U}(1)}$$

- By this reasoning $\mathrm{QED} \otimes \mathrm{QED}$ would have 2 gauge couplings. But this is wrong: **in reality there are 3 independent real gauge couplings**
- To see this, recall that gauge bosons transform under the adjoint representation, which is the trivial representation for U(1)'s, so **U(1) gauge bosons are singlets of the entire gauge group**

Multiple U(1)'s: use a gauge coupling matrix

- Assume that the gauge group is $\mathrm{U}(1)^n$ and there are some complex scalar fields ϕ_i (with $\#\phi \geq n$ for this to make sense)
- Introduce as usual n gauge bosons: A^a_μ, $a = 1, \cdots, n$
- The 'usual' gauge transformation is

$$\phi_i \to \exp\left(iQ^a_i \alpha^a\right)\phi_i = \exp\left(i\boldsymbol{Q}_{\boldsymbol{i}}^T \boldsymbol{\alpha}\right)$$

$$\boldsymbol{A}_{\boldsymbol{\mu}} \to \boldsymbol{A}_{\boldsymbol{\mu}} + \underline{\boldsymbol{G}}^{-1}\partial_\mu \boldsymbol{\alpha}$$

diagonal

therefore we can have the terms

$$-\tfrac{1}{4}\xi_{ab}F^a_{\mu\nu}F^{b\mu\nu} \equiv -\tfrac{1}{4}\boldsymbol{F}^T_{\boldsymbol{\mu\nu}}\boldsymbol{\xi}\boldsymbol{F}^{\boldsymbol{\mu\nu}}$$

$$-\tfrac{1}{2}M_{ab}\lambda_a\lambda_b + \mathrm{hc} \equiv -\tfrac{1}{2}\boldsymbol{\lambda}^T\boldsymbol{M}\boldsymbol{\lambda} + \mathrm{hc}$$

and

$$D_\mu = \partial_\mu - iQ^a_i g_a A^a_\mu \equiv \partial_\mu - i\boldsymbol{Q}^T_{\boldsymbol{i}}\boldsymbol{G}\boldsymbol{A}_{\boldsymbol{\mu}}$$

Put $A^a_\mu, \lambda^a, Q^a_i, \alpha^a$ in U(1)-space vectors

$$\boldsymbol{A}_{\boldsymbol{\mu}} = \begin{pmatrix} A^1_\mu \\ A^2_\mu \\ \vdots \\ A^n_\mu \end{pmatrix} \quad \boldsymbol{\lambda} = \begin{pmatrix} \lambda^1 \\ \lambda^2 \\ \vdots \\ \lambda^n \end{pmatrix}$$

$$\boldsymbol{Q}_{\boldsymbol{i}} = \begin{pmatrix} Q^1_i \\ Q^2_i \\ \vdots \\ Q^n_i \end{pmatrix} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha^1 \\ \alpha^2 \\ \vdots \\ \alpha^n \end{pmatrix}$$

Counting the degrees of freedom in $\boldsymbol{G}$

- **Normalizing canonically** the gauge bosons, $\boldsymbol{\xi} = \mathbf{1}$, **leads to a non-diagonal coupling matrix $\boldsymbol{G}$**

$$\phi_i \to \exp\left(iQ_i^a \alpha^a\right)\phi_i = \exp\left(i\boldsymbol{Q_i^T}\boldsymbol{\alpha}\right)$$
$$\boldsymbol{A_\mu} \to \boldsymbol{A_\mu} + \boldsymbol{G}^{-1}\partial_\mu \boldsymbol{\alpha}$$

$$-\frac{1}{4}\boldsymbol{F_{\mu\nu}^T}\boldsymbol{F^{\mu\nu}}$$
$$-\frac{1}{2}\boldsymbol{\lambda}^T\boldsymbol{M}\boldsymbol{\lambda} + \text{hc}$$
$$D_\mu = \partial_\mu - i\boldsymbol{Q_i^T}\boldsymbol{G}\boldsymbol{A_\mu}$$

- So we are swapping a diagonal $\boldsymbol{G}$ and a symmetric $\boldsymbol{\xi}$ by an arbitrary $\boldsymbol{G}$ - let's **count the number of independent parameters**

diagonal $\boldsymbol{G}$: **0 parameters** (we can normalize the A_μ^a's so that $\boldsymbol{G}$=1)

symmetric $\boldsymbol{\xi}$: $\boldsymbol{\frac{n(n+1)}{2}}$ **parameters**

arbitrary real $\boldsymbol{G}$: $\boldsymbol{\frac{n(n+1)}{2}}$ **parameters**

QR decomposition

Polar decomposition

n diagonal; $\frac{n(n-1)}{2}$ off-diagonal

Symmetries of $\boldsymbol{G}$, $\boldsymbol{Q_i}$ and $\boldsymbol{A_\mu}$

- Since the interaction part of the covariant derivative is $-i\boldsymbol{Q_i^T}\boldsymbol{G}\boldsymbol{A_\mu}$, there is the following **symmetry** in the Lagrangian:

$$\boldsymbol{Q_i} \to \boldsymbol{Q_i'} = O_1\boldsymbol{Q_i} \qquad \boldsymbol{A_\mu} \to \boldsymbol{A_\mu'} = O_2\boldsymbol{A_\mu}$$

$$\boldsymbol{G} \to \boldsymbol{G'} = O_1\boldsymbol{G}O_2^T \qquad \boldsymbol{\lambda} \to \boldsymbol{\lambda'} = O_2\boldsymbol{\lambda}$$

$$\boldsymbol{M} \to \boldsymbol{M'} = O_2\boldsymbol{M}O_2^T$$

where O_1 and O_2 are orthogonal matrices

- In fact, we can get rid of O_1 by working with $\boldsymbol{V_i} \equiv \boldsymbol{G}^T\boldsymbol{Q_i}$; $\boldsymbol{G}$ and $\boldsymbol{Q_i}$ always appear in the Lagrangian through this combination

$$\boldsymbol{V_i} \to \boldsymbol{V_i'} = O_2\boldsymbol{V_i}$$

- This is a powerful tool to generalize single-U(1) expressions:

Example

$$\frac{d}{dt}g = \frac{1}{16\pi^2}g^3S(R) = \frac{1}{16\pi^2}g^3\sum_i q_i^2 \longrightarrow \frac{d}{dt}\boldsymbol{G} = \frac{1}{16\pi^2}\boldsymbol{G}\sum_i \boldsymbol{V_i}\boldsymbol{V_i^T}$$

There is no other way!

RGEs: some one loop formulas

- The one-loop RGEs for gauge couplings, gaugino masses and soft sparticle masses are:

$$\frac{d}{dt}\boldsymbol{G} = \frac{1}{16\pi^2}\boldsymbol{G}\boldsymbol{G}^{\boldsymbol{T}}\gamma\boldsymbol{G}$$

$\gamma \equiv \sum_i \boldsymbol{Q_i}\boldsymbol{Q_i^T}$

$$\frac{d}{dt}\boldsymbol{M} = \frac{1}{16\pi^2}\left(\boldsymbol{M}\boldsymbol{G}^{\boldsymbol{T}}\gamma\boldsymbol{G} + \boldsymbol{G}^{\boldsymbol{T}}\gamma\boldsymbol{G}\boldsymbol{M}\right)$$

$$\frac{d}{dt}m_i^2 = -\frac{1}{2\pi^2}\boldsymbol{Q_i^T}\boldsymbol{G}\boldsymbol{M}\boldsymbol{M}^\dagger\boldsymbol{G}^{\boldsymbol{T}}\boldsymbol{Q_i}$$

Valid approximation for 1st and 2nd generations and assuming unification of soft masses

- **Interesting comparisons**:

$\left(\boldsymbol{G}^{\boldsymbol{T}}\right)^{-1}\boldsymbol{M}\boldsymbol{G}^{-1}$ is an RGE invariant just like $\frac{M}{g^2}$ with no mixing

$\frac{d}{dt}\left(\frac{\boldsymbol{G}\boldsymbol{G}^{\mathsf{T}}}{4\pi}\right)^{-1} = -\frac{1}{2\pi}\gamma$ looks like $\frac{d}{dt}\alpha^{-1} = -\frac{1}{2\pi}b$ so $\frac{\boldsymbol{G}\boldsymbol{G}^{\mathsf{T}}}{4\pi} \equiv \boldsymbol{A}$

$$m_i^2(t_1) - m_i^2(t_0) = 2M_{1/2}^2\alpha_G^{-2}\boldsymbol{Q_i^T}\boldsymbol{A}_0\left[\gamma^{-1} - \boldsymbol{A}_1\boldsymbol{A}_0^{-1}\gamma^{-1}\boldsymbol{A}_0^{-1}\boldsymbol{A}_1\right]\boldsymbol{A}_0\boldsymbol{Q_i}$$

$$\left(\boldsymbol{G}^{\boldsymbol{T}}\right)^{-1}\boldsymbol{M}\boldsymbol{G}^{-1} = 4\pi M_{1/2}^2\alpha_G^{-1}\mathbf{1}$$

The rotated basis

- If we are interested only on the gauge couplings and if at some scale $\boldsymbol{G} = g\mathbf{1}$ then with a **rotation** of the charges and the gauge bosons

For example if all U(1)'s come from a common gauge factor group

$$\boldsymbol{Q_i} \to \boldsymbol{Q_i'} = O\boldsymbol{Q_i}$$
$$\boldsymbol{A_\mu} \to \boldsymbol{A_\mu'} = O\boldsymbol{A_\mu}$$
$$\boldsymbol{G} \to \boldsymbol{G'} = O\boldsymbol{G}O^T = \boldsymbol{G} = g\mathbf{1}$$

we can bring $\gamma \equiv \sum_i \boldsymbol{Q_i}\boldsymbol{Q_i^T}$ to a diagonal form as well

- So **in this rotated basis there is no mixing**:

$$\frac{d}{dt}\boldsymbol{G'} = \frac{1}{16\pi^2}\boldsymbol{G'}\boldsymbol{G'^T}\gamma'\boldsymbol{G'}$$

- If the gaugino mass matrix $\boldsymbol{M}$ happens to be $\propto \mathbf{1}$ at the same scale we can still use this technique (otherwise we cannot)

- Two-loop RGEs of $\boldsymbol{G}$ and $\boldsymbol{M}$ involve Yukawa and soft SUSY breaking trilinear couplings – the rotated basis method fails

U(1) mixing in the literature

- All this (and more) has been discussed in the literature

Holdom 1986
del Aguila, Coughlan, Quirós 1988
del Aguila, Gonzalez, Quirós 1988
Babu, Kolda, March-Russell 1998
Jack, Jones 2001
Luo, Xiao 2003
Malinský, Romão, Valle 2005
Bertolini, Luzio, Malinský 2009
Braam, Reuter 2011
Romari, Hirsch, Malinský 2011

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- But **for SUSY models, there were no complete two-loop RGEs** which included U(1) mixing effects
- Our main objective was to fill this gap

U(1) mixing in two-loop RGEs

Martin, Vaughn 1994, 2008

“Finally, we must note that there is an exceptional case when the gauge group contains (unlike the MSSM) a direct product of more than one $U(1)$. One should then choose the basis for the $U(1)$ subgroups so that the matrix $\mathrm{Tr}[q_a q_b]$ is diagonal, where the trace is over all chiral superfields and q_a denotes the $U(1)_a$ charge. This is always possible, since $\mathrm{Tr}[q_a q_b]$ is always a real symmetric matrix. Then the only non-trivial rule is that the term $g^4 C(i)\mathrm{Tr}[S(r)m^2]$ in (2.20) becomes a sum over non-$U(1)$ subgroups as before, plus a contribution

$$g^4 C(i)\mathrm{Tr}[S(r)m^2] \to \sum_a \sum_b g_a^2 g_b^2 (q_a)_i (q_b)_i \mathrm{Tr}[q_a q_b m^2] \qquad (3.18)$$

where $\sum_a$ and $\sum_b$ are sums over $U(1)$ subgroups, and $(q_a)_i$ denotes the $U(1)_a$ charge of the chiral superfield carrying the index i.”

Rotated basis scheme – as discussed, this only works on special one-loop scenarios

Presentation of the RGEs of a generic model

- Gather the model's information (superpotential and soft SUSY breaking terms) in tensors:

$$V_{\text{soft}} = \left(S^i \phi_i + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \text{c.c.} \right)$$

$$W(\Phi) = L_i \Phi_i + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j + \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k \qquad + (m^2)^i{}_j \phi_i \phi_j^* + \frac{1}{2} M \lambda_a \lambda_a$$

- The β functions of a simple gauge group model are written as a function of these tensor:

Example

$$\left[\beta_b^{(1)}\right]^{ij} = \frac{1}{2} b^{il} Y_{lmn} Y^{mnj} + \frac{1}{2} Y^{ijl} Y_{lmn} b^{mn} + \mu^{il} Y_{lmn} h^{mnj} - 2 \left(b^{ij} - 2M \mu^{ij} \right) g^2 C(i) + (i \leftrightarrow j)$$

- To generalize these formulas to product groups, a list of substitutions is given:

Example

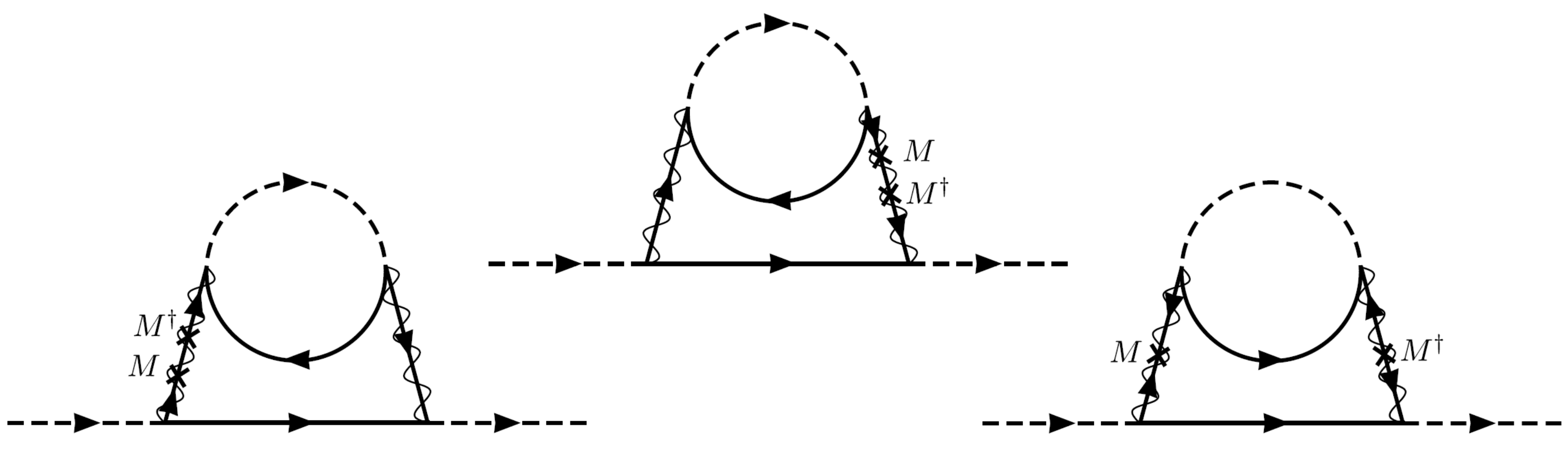
$$48 g^4 M M^\dagger C(i)^2 \to \sum_a \sum_b g_a^2 g_b^2 C_a(i) C_b(i) \times \times \left(32 M_a M_a^\dagger + 8 M_a M_b^\dagger + 8 M_b M_a^\dagger \right)$$

What we did: generalize the substitutions

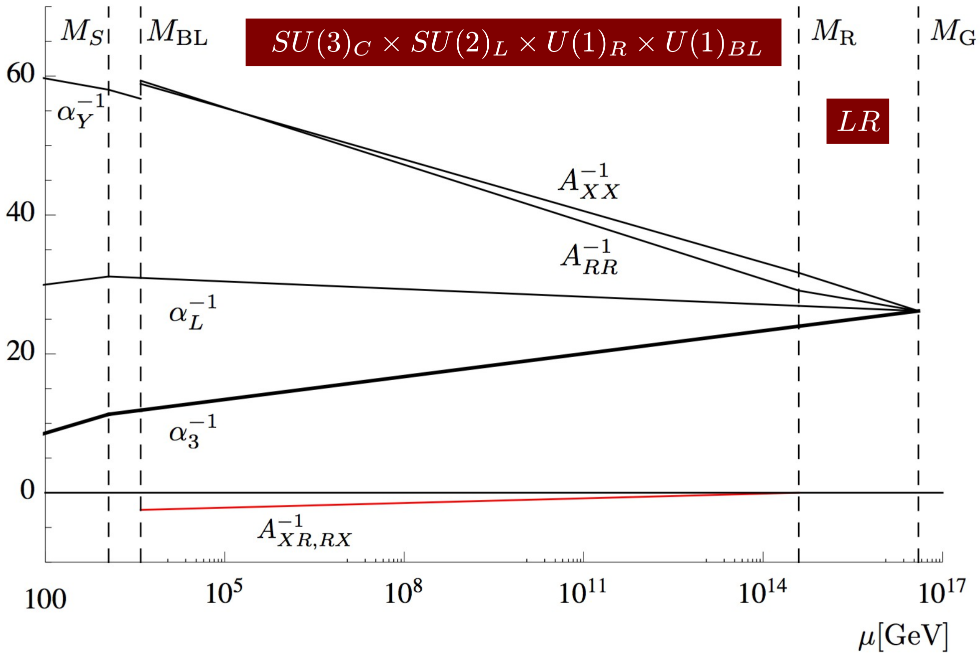
- By carefully analysing the structure of each term and, in doubt, looking at the underlying Feynman diagram, we generalized the substitutions rules:

Example

$$
\begin{aligned}
48g^4 M M^* C(r)^2 \rightarrow & \sum_{A,B} g_A^2 g_B^2 C_A(r)\, C_B(r) \left[32 M_A M_A^* + 8 M_A M_B^* + 8 M_B M_A^*\right] \\
& + \sum_{A} g_A^2 C_A(r) \left[32 M_A M_A^* \left(V_r^T V_r\right) + 16 M_A \left(V_r^T M^\dagger V_r\right) + 16 M_A^* \left(V_r^T M V_r\right) + 32 \left(V_r^T M M^\dagger V_r\right)\right] \\
& + \left[32 \left(V_r^T M M^\dagger V_r\right)\left(V_r^T V_r\right) + 16 \left(V_r^T M V_r\right)\left(V_r^T M^\dagger V_r\right)\right]
\end{aligned}
$$



Numerical results: one-loop



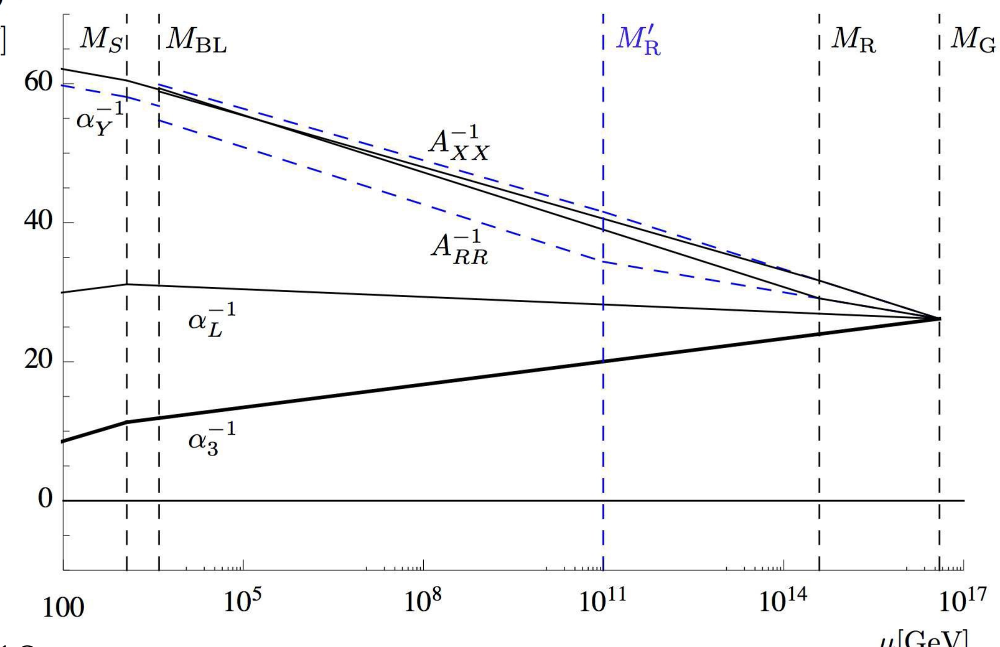
With U(1) mixing

(recall that $\frac{GG^\intercal}{4\pi} \equiv A$)

Without U(1) mixing

$\sim 4\%$ difference in α_Y^{-1} at low energies

To compensate for this M_R would have to be shifted 4 order of magnitude



Numerical results: two-loops

The hypercharge gauge coupling does not change because the additional states not present in the MSSM are hypercharge neutral

Even so, these numbers suggest that in general U(1) mixing can have a percent level effect on the hypercharge gauge coupling

	One-loop results			Two-loop results			
	No kinetic mixing	Rotated basis method	Complete RGEs	No kinetic mixing	Complete RGEs	No kinetic mixing	Complete RGEs
g_{YY}	0.4511	0.4700	0.4700	0.4487	0.4677	0.4487	0.4686
g_{BLBL}	0.4083	0.4243	0.4243	0.4070	0.4231	0.4131	0.4298
g_{BLY}, g_{YBL}	0.	-0.0723	-0.0723	0.0	-0.0725	0.0	-0.0725
g_Y	0.4511	0.4511	0.4511	0.4487	0.4487	0.4487	0.4500
M_{YY} [GeV]	196.34	218.13	218.13	185.82	207.96	185.80	208.71
M_{BLBL} [GeV]	160.83	178.67	178.67	154.88	173.19	144.26	161.97
M_{BLY}, M_{YBL} [GeV]	0.0	- 62.39	- 62.39	0.0	-63.10	0.0	-62.15
M_Y [GeV]	196.34	196.34	196.34	185.82	185.96	185.80	187.04
	Exact unification					$g_{BL}^{\rm GUT} = 1.05\, g_Y^{\rm GUT}$	

Two-loop mixing effects in the Bino mass can be of the percent level

simulate the inclusion of threshold effects

Conclusions

- When there are multiple U(1)'s a **new qualitative feature** emerges: U(1) mixing
- These models can appear naturally in **Grand Unified Theories** which have a group with rank 5 or higher: SO(10), E6, ...
- In practice, instead of n gauge couplings and n gaugino masses we must consider $n \times n$ matrices $\boldsymbol{G}$ and $\boldsymbol{M}$
- Failure to include this effect can result in errors of the order of a few percent
- **There were no two loop renormalization group equations for SUSY models with U(1) mixing** – we derived them by extending the the previously known formulas which were valid only in the absence of mixing

Thank you for your time

Example: $LR \to SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{BL}$

- The matching condition is

from $SU(2)_R$

$$\boldsymbol{G}(t_{LR}) = \begin{pmatrix} g_R & 0 \\ 0 & g_{BL} \end{pmatrix}$$

from $U(1)_{BL}$

and $= \sqrt{\frac{3}{8}}(B-L)$

$$\gamma = \sum_i \begin{pmatrix} q_{R\,i} \\ q_{BL\,i} \end{pmatrix} \begin{pmatrix} q_{R\,i} & q_{BL\,i} \end{pmatrix}$$

These T's are matrices in chiral-superfield space

$$= \sum_i \begin{pmatrix} q_{R\,i}^2 & q_{R\,i}q_{BL\,i} \\ q_{R\,i}q_{BL\,i} & q_{BL\,i}^2 \end{pmatrix} = \begin{pmatrix} \mathrm{Tr}\left(T_R T_R\right) & \mathrm{Tr}\left(T_R T_{BL}\right) \\ \mathrm{Tr}\left(T_{BL} T_{BL}\right) & \mathrm{Tr}\left(T_R T_{BL}\right) \end{pmatrix}$$

- The off-diagonal entries are small if both T's come from the same group and few fields have been integrated out – remember that $\mathrm{Tr} T_a T_b \propto \delta_{ab}$ for the generators of a simple gauge group

e.g.: $SO(10)$

Example: $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{BL} \to MSSM$

- Only one combination of the two U(1)'s is preserved:

$$q_Y = \sqrt{\tfrac{3}{5}} q_R + \sqrt{\tfrac{2}{5}} q_{BL}$$

sometimes called α_X

- Does this mean that $\alpha_Y^{-1} = \frac{3}{5}\alpha_R^{-1} + \frac{2}{5}\alpha_{BL}^{-1}$? **No**
- To see why, rotate the charges to the basis of q_Y and $q_\perp$:

orthogonal charge to q_Y

$$\begin{pmatrix} q_Y \\ q_\perp \end{pmatrix} = \underbrace{\begin{pmatrix} \sqrt{\frac{3}{5}} & \sqrt{\frac{2}{5}} \\ -\sqrt{\frac{2}{5}} & \sqrt{\frac{3}{5}} \end{pmatrix}}_{\equiv \boldsymbol{P}} \begin{pmatrix} q_R \\ q_{BL} \end{pmatrix}$$

Example: $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{BL} \to MSSM$

- The coupling matrix

$$\boldsymbol{G} = \begin{pmatrix} g_{RR} & g_{RBL} \\ g_{BLR} & g_{BLBL} \end{pmatrix}$$

is transformed to $\boldsymbol{G}' = P\boldsymbol{G}O_2^T$ where O_2 is a rotation of the gauge bosons which we do not care about - $\boldsymbol{G}'\boldsymbol{G}'^T = P\boldsymbol{G}\boldsymbol{G}^T P^T$

- After these two rotation the gauge bosons' mass matrix

$$M^2_{\boldsymbol{A}'} = \boldsymbol{G}'^T \begin{pmatrix} 0 & 0 \\ 0 & \cdot \end{pmatrix} \boldsymbol{G}'$$

is diagonal. This means that $\boldsymbol{G}' = \begin{pmatrix} g_Y & \cdot \\ 0 & \cdot \end{pmatrix}$ (upper triangular)

- Using the relation between $\boldsymbol{G}$ and $\boldsymbol{G}'$ **we conclude that**

$$g_Y^{-2} = \frac{3\left(g^2_{BLBL}+g^2_{BLR}\right)+2\left(g^2_{RR}+g^2_{RBL}\right)-2\sqrt{6}(g_{RR}g_{BLR}+g_{RBL}g_{BLBL})}{5(g_{RR}g_{BLBL}-g_{RBL}g_{BLR})^2}$$