The Adjoint SU(5) constrained by a Z_4 Flavour Symmetry

Catarina Simões

In colaboration with

D. Emmanuel-Costa and M. Tortóla (in progress)

Suported by FCT through the fellowship SFRH/BD/61623/2009

Outline

1) Motivation
2) The Model
3) Gauge Couplings Unification
4) Neutrino Sector
5) Results
6) Conclusions

Motivation

D. Emmanuel-Costa , C. S., Phys.Rev.D.85.016003

- Minimal SU(5) model plus 3 right-handed neutrinos and 2 Higgs quintets; where M_u and M_d had NNI through a Z_4 flavour symmetry

$$NNI = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

- Solve some problems:
  - To correct the mass relation $M_e = M_d^T$, that is not compatible with down type quark and charged lepton masses @ low energy
    → non renormalisable higher dimension operators
    → 5_H substituted by 45_H
  - Unification scale Λ obtained from the numerics $\Lambda = 1.9 \times 10^{14}$ GeV was lower that the one obtained from the proton decay via the exchange of heavy X and Y gauge bosons, $\Lambda_{X,Y} = (4.0 - 5.1) \times 10^{15}$ GeV
  - Unnaturally large splitting between the mass of Σ_8 and Σ_3

↓

Adjoint SU(5)

P. Fileviez Perez, Phys.Lett. B 654 (2007) 189

Fermion sector

3 x
$$\bar{5}=(d^c, L)=(\bar{3},1,\frac{1}{3})+(1,2,-\frac{1}{2})$$
$$10=(Q, u^c, e^c)=(3,2,\frac{1}{6})+(\bar{3},1,-\frac{2}{3})+(1,1,1)$$
$$24=(\rho_8, \rho_3, \rho_{(3,2)}, \rho_{(\bar{3},2)}, \rho_0)=(8,1,0)+(1,3,0)+(3,2,-\frac{5}{6})+(\bar{3},2,\frac{5}{6})+(1,1,0)$$

Higgs sector

$$5_H=(T_1, H_1)=(3,1,\frac{-1}{3})+(1,2,\frac{1}{2})$$

$$24_H=(\Sigma_8, \Sigma_3, \Sigma_{(3,2)}, \Sigma_{(\bar{3},2)}, \Sigma_0)$$
$$=(8,1,0)+(1,3,0)+(3,2,-\frac{5}{6})+(\bar{3},2,\frac{5}{6})+(1,1,0)$$

$$45_H=(S_8, S_{(\bar{6},1)}, S_{(3,3)}, S_{(\bar{3},2)}, S_{(\bar{3},1)}, T_2, H_2)$$
$$(8,2,\frac{1}{2})+(\bar{6},1,-\frac{1}{3})+(3,3,-\frac{1}{3})+(\bar{3},2,-\frac{7}{6})+(\bar{3},1,\frac{4}{3})+(3,1,-\frac{1}{3})+(1,2,\frac{1}{2})$$

The model: the breaking

The 24 adjoint field, Σ breaks spontaneously the SU(5) gauge group down to the SM group, $SU(3)_c \times SU(2)_L \times U(1)_Y$ through the VEV,

$$\sum_{j=1}^{5} 24_i^j = 0 \qquad \langle \Sigma \rangle = \frac{v}{\sqrt{2}\sqrt{30}} diag(2,2,2,-3,-3)$$

At the GUT scale, the 45_H field acquires a vacuum expectation value VEV

$$\langle 45_\alpha^{\beta 5} \rangle = v_{45}(\delta_\alpha^\beta - 4\delta_4^\alpha \delta_\beta^4)$$

$$45_k^{ij} = -45_k^{ji} \qquad \sum_{j=1}^{5} 45_k^{ij} = 0 \qquad \sum_{j=1}^{3} 45_j^{ji} = 3\mathrm{H}_2^i \qquad \sum_{j=4}^{5} 45_j^{ji} = -3\mathrm{H}_2^i$$

$$\langle 45_1^{15} \rangle = \langle 45_2^{25} \rangle = \langle 45_3^{35} \rangle = v_{45} \qquad \langle 45_4^{45} \rangle = -3v_{45} \qquad \langle 45_j^{ik} \rangle = 0$$

5_H breaks the SM gauge group down to $SU(3)_c \times U(1)_{em}$ and together with the 45_H generate the fermion masses via the Yukawa interactions @ electroweak scale. $M_Z = 91.1876 \pm 0.0021\, GeV$

$$\langle 5_H \rangle = diag(0,0,0,0,v_5)$$

CFTP

TÉCNICO LISBOA

The model: Higgs potential (i)

The potential is given by

$$V = V(5_H) + V(24_H) + V(45_H) + V(5_H, 24_H) + V(5_H, 45_H) + V(24_H, 45_H) + V(5_H, 24_H, 45_H)$$

where

$$V(5_H) = -\frac{1}{2}\mu_5^2 5^\alpha \overline{5}_\alpha + \frac{1}{4} a_4 (5^\alpha \overline{5}_\alpha)^2$$

$$V(24_H) = -\frac{\mu_{24}^2}{2} 24^\alpha_\beta 24^\beta_\alpha + \frac{a_1}{2}(24^\alpha_\beta 24^\beta_\alpha)^2 + \frac{a_2}{3} 24^\alpha_\beta 24^\beta_\gamma 24^\gamma_\alpha + \frac{a_3}{2} 24^\alpha_\beta 24^\beta_\gamma 24^\gamma_\delta 24^\delta_\alpha$$

$$\begin{aligned} V(45_H) = & -\frac{1}{2}\mu_{45}^2 (45^{\alpha\beta}_\gamma \overline{45}^\gamma_{\alpha\beta}) + \lambda_1 (45^{\alpha\beta}_\gamma \overline{45}^\gamma_{\alpha\beta})^2 + \lambda_2 45^{\alpha\beta}_\gamma \overline{45}^\delta_{\alpha\beta} 45^{k\lambda}_\delta \overline{45}^\gamma_{k\lambda} \\ & + \lambda_3 45^{\alpha\beta}_\gamma \overline{45}^\delta_{\alpha\beta} 45^{k\gamma}_\lambda \overline{45}^\lambda_{k\delta} + \lambda_4 45^{\alpha\delta}_\beta \overline{45}^\beta_{\alpha\gamma} 45^{k\gamma}_\lambda \overline{45}^\lambda_{k\delta} \\ & + \lambda_5 45^{\alpha\gamma}_\delta \overline{45}^\beta_{\gamma\epsilon} 45^{k\delta}_\alpha \overline{45}^\epsilon_{k\beta} + \lambda_6 45^{\alpha\gamma}_\delta \overline{45}^\beta_{\gamma\epsilon} 45^{k\epsilon}_\alpha \overline{45}^\delta_{k\beta} \\ & + \lambda_7 45^{\alpha\gamma}_\delta \overline{45}^\beta_{\gamma\epsilon} 45^{k\delta}_\beta \overline{45}^\epsilon_{k\alpha} + \lambda_8 45^{\alpha\gamma}_\delta \overline{45}^\beta_{\gamma\epsilon} 45^{k\epsilon}_\beta \overline{45}^\delta_{k\alpha} \end{aligned}$$

Pavel et al, Phys.Rev. D78 (2008) 015013

The model: Higgs potential (ii)

$$V(24_H, 45_H) = a_5 45^{\alpha\beta}_{\gamma} 24^{\gamma}_{\delta} \overline{45}^{\delta}_{\alpha\beta} + a_6 (45^{\alpha\beta}_{\gamma} \overline{45}^{\gamma}_{\alpha\beta}) 24^{\delta}_{\epsilon} 24^{\epsilon}_{\delta}$$
$$+ \beta_1 45^{\alpha\beta}_{\gamma} 24^{\delta}_{\alpha} 24^{\epsilon}_{\beta} \overline{45}^{\gamma}_{\delta\epsilon} + \beta_2 45^{\alpha\beta}_{\gamma} 24^{\gamma}_{\beta} 24^{\delta}_{\epsilon} \overline{45}^{\epsilon}_{\alpha\delta}$$
$$+ \beta_3 45^{\alpha\beta}_{\gamma} 24^{\gamma}_{\epsilon} 24^{\delta}_{\beta} \overline{45}^{\epsilon}_{\alpha\delta} + \beta_4 45^{\alpha\beta}_{\gamma} 24^{k}_{\alpha} 24^{\lambda}_{k} \overline{45}^{\gamma}_{\lambda\beta}$$
$$+ \beta_5 45^{\alpha\beta}_{\gamma} 24^{\gamma}_{k} 24^{k}_{\lambda} \overline{45}^{\lambda}_{\alpha\beta}$$

$$V(24_H, 5_H) = \beta_6 \overline{5}_{\alpha} 24^{\alpha}_{\beta} 5^{\beta} + \beta_7 \overline{5}_{\alpha} 5^{\alpha} 24^{\beta}_{\gamma} 24^{\gamma}_{\beta}$$
$$+ \beta_8 \overline{5}_{\alpha} 24^{\alpha}_{\beta} 24^{\beta}_{\gamma} 5^{\gamma}$$

$$V(5_H, 45_H) = c_1 (45^{\alpha\beta}_{\gamma} \overline{45}^{\gamma}_{\alpha\beta}) \overline{5}_{\delta} 5^{\delta} + c_2 45^{\alpha\beta}_{\delta} \overline{5}_{\gamma} \overline{45}^{\gamma}_{\alpha\beta} 5^{\delta}$$
$$+ c_3 45^{\alpha\beta}_{\gamma} \overline{45}^{\gamma}_{\alpha\delta} \overline{5}_{\beta} 5^{\delta}$$

$$V(24_H, 45_H, 5_H) = c_4 \overline{5}_{\alpha} 24^{\gamma}_{\beta} 45^{\alpha\beta}_{\gamma} + c_5 \overline{5}_{\alpha} 24^{\gamma}_{\delta} 24^{\delta}_{\beta} 45^{\alpha\beta}_{\gamma}$$
$$+ c_6 \overline{5}_{\alpha} 24^{\alpha}_{\beta} 24^{\gamma}_{\delta} 45^{\beta\delta}_{\gamma} + H.c.$$

Pavel et al, Phys.Rev. D78 (2008) 015013

The model: Z_4 flavour symmetry

$$NNI = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$$

Z_4 Flavour Symmetry

A field R transforms as $R \to R' = e^{\left(2i\frac{\pi}{4} Q(R)\right)} R, \quad Q(R) \in Z_4$

- $Q(5_H) = \phi_1$
- $Q(45_H) = \phi_2$
- $Q(24_H) = 0$ to ensure the Z_4 symmetry below the GUT scale

In order to have the NNI form for the up- and down-quark and charged lepton mass matrices, the Z_4 charges of the 10 and $\bar{5}$ fermionic fields must be of the follow form

$$\phi_2 = 2\phi_1$$

$$Q(\bar{5}_i) = (q_3 + 2\phi_1, -3q_3, -q_3 + \phi_1)$$

$$Q(10_i) = (3q_3 + \phi_1, -q_3 - \phi_1, q_3)$$

$$Q(24_i) = (n_1, n_2, n_3)$$

The model: Yukawa and mass terms (i)

The **Yukawa interactions** of the up- and down-quark and charged lepton sectors can be written as,

$$-L_Y = (\Gamma_u^1)_{ij} 10_i 10_j 5_H + (\Gamma_u^2)_{ij} 10_i 10_j 45_H + (\Gamma_d^1)_{ij} 10_i \overline{5}_j \overline{5}_H + (\Gamma_d^2)_{ij} 10_i \overline{5}_j \overline{45}_H + H.c.$$

The up- and down-quark masses as well as the charged lepton masses are given by

$$M_u = 4(\Gamma_u^1 + \Gamma_u^{1T}) v_5 - 8(\Gamma_u^2 - \Gamma_u^{2T}) v_{45}$$
$$M_d = \Gamma_d^1 v_5^* + 2\Gamma_d^2 v_{45}^*$$
$$M_e = \Gamma_d^{1T} v_5^* - 6\Gamma_d^{2T} v_{45}^*$$

The mass relation is given by

$$M_d - M_e^T = 8\Gamma_d^2 v_{45}^*$$

$$\Gamma_u^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_u \\ 0 & b_u & 0 \end{pmatrix} \quad \Gamma_u^2 = \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & 0 \\ 0 & 0 & c_u \end{pmatrix}$$

The Yukawa coupling matrices :

$$\Gamma_d^1 = \begin{pmatrix} 0 & a_d & 0 \\ a_d' & 0 & 0 \\ 0 & 0 & c_d \end{pmatrix} \quad \Gamma_d^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_d \\ 0 & b_d' & 0 \end{pmatrix}$$

Gauge Couplings Unification (i)

Unification of the gauge couplings at $\Lambda \approx M_V$

$$M_V > (4.0 - 5.1) \times 10^{15} \text{ GeV} \quad \text{for} \quad \alpha_U^{-1} \approx 25 - 40$$

PDG'12

Partial proton lifetime

$$\tau(p \rightarrow \pi^0 e^+) > 8.2 \times 10^{33} \, years$$

Proton decay width

$$\Gamma \approx \alpha_U^2 \frac{m_p^5}{M_V^4}$$

$$\frac{d}{dt}\alpha_i^{-1} = -\frac{b_i}{2\pi k_i}$$

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi k_i} \ln\left(\frac{\mu}{M_Z}\right)$$

$$\alpha_U = \alpha_1(\Lambda) = \alpha_2(\Lambda) = \alpha_3(\Lambda)$$

CFTP

TÉCNICO LISBOA

Gauge Couplings Unification (ii)

The B-Test

$$B_{ij} \equiv B_i - B_j \qquad B_i = \frac{1}{k_i}\left(b_i^{SM} + \sum_I b_i^I r_I\right) \qquad r_i = \frac{\ln\left(\frac{\Lambda}{M_I}\right)}{\ln\left(\frac{\Lambda}{M_Z}\right)}$$

$$B_{12} \ln\left(\frac{M_Z}{\mu}\right) = \frac{2\pi}{\alpha}\left(\frac{1}{k_1} - \left(\frac{1}{k_1} + \frac{1}{k_2}\right)\sin^2\theta_w\right)$$

$$B_{23} \ln\left(\frac{M_Z}{\mu}\right) = \frac{2\pi}{\alpha k_2}\left(\sin^2\theta_w - \frac{\alpha k_2}{\alpha_s k_3}\right)$$

$$B \equiv \frac{B_{23}}{B_{12}} = \frac{\sin^2\theta_w - \frac{\alpha k_2}{\alpha_s k_3}}{\frac{k_2}{k_1} - \left(1 + \frac{k_2}{k_1}\right)\sin^2\theta_w}$$

$$(b_{1,}^{SM}\, b_{2,}^{SM}\, b_3^{SM}) = \left(\frac{41}{6}, -\frac{19}{6}, -7\right)$$

for SU(5)

$$(k_{1,} k_{2,} k_3) = (5/3, 1, 1)$$

CFTP

TÉCNICO LISBOA

Gauge Couplings Unification (iii)

PDG'12

$$\alpha_s(M_Z)=0.1184\pm0.0007$$

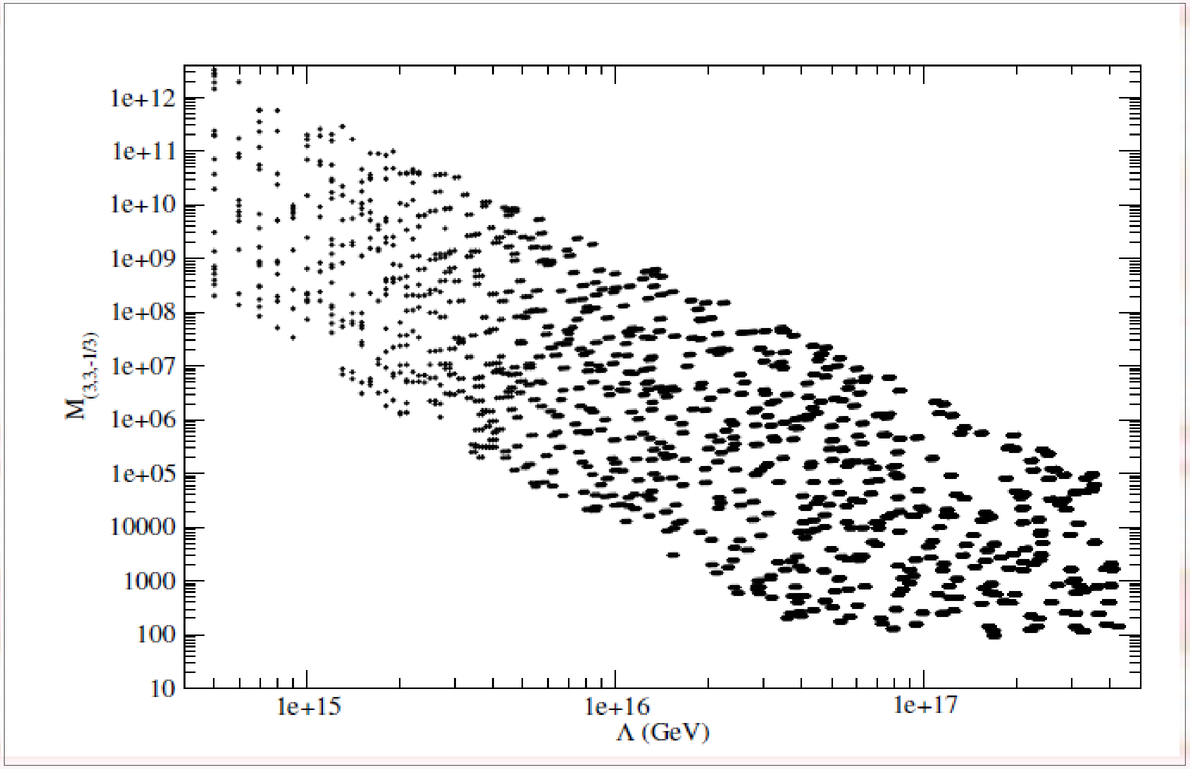
$$\alpha^{-1}(M_Z)=128.91\pm0.02$$

$$\sin^2(\theta_w)(M_Z)=0.23116\pm0.00013$$

$$B=0.718\pm0.003$$

$$B_{12}\ln\left(\frac{\Lambda}{M_Z}\right)=185.0\pm0.2$$

$$r_i=\frac{\ln\left(\frac{\Lambda}{M_I}\right)}{\ln\left(\frac{\Lambda}{M_Z}\right)}$$



Neutrino Sector (i)

The neutrino Lagrangean is given by

$$-L_Y=(M_R)_{ij}24_i24_j+\lambda_{ij}Tr(24_i24_j24_H)+(\Gamma_\nu^1)_{ij}\overline{5}_i24_j5_H+(\Gamma_\nu^2)_{ij}\overline{5}_i24_j45_H+H.c.$$

In the case of three adjoint fermionic representation, the mass of the fermions, ρ_8 , ρ_3 and ρ_0 is given by:

$$M_{\rho_0}=\frac{1}{4}\left((M_R)_{ij}-\frac{\lambda_{ij}}{\sqrt{30}}v_{24}\right)$$

$$M_{\rho_3}=\frac{1}{4}\left((M_R)_{ij}-\frac{3\lambda_{ij}}{\sqrt{30}}v_{24}\right)$$

$$M_{\rho_8}=\frac{1}{4}\left((M_R)_{ij}+\frac{2\lambda_{ij}}{\sqrt{30}}v_{24}\right)$$

$$v_{24}=\frac{\sqrt{3}M_{GUT}}{\sqrt{5\pi\alpha_{GUT}}}$$

Neutrino Sector (ii)

The Yukawa interactions written in terms of the ρ_0 and ρ_3 are then given by

Pavel et al, Phys.Rev. D78 (2008) 015013

$$-L_Y = (\Gamma_\nu^1)_{ij} l_i^T i\,\sigma_2\, \rho_{3j} H_1 + \frac{3}{2\sqrt{15}} (\Gamma_\nu^1)_{ij} l_i^T i\,\sigma_2\, \rho_{0j} H_1 + \frac{\sqrt{15}}{2} (\Gamma_\nu^2)_{ij} l_i^T i\,\sigma_2\, \rho_{0j} H_2 - 3 (\Gamma_\nu^2)_{ij} l_i^T i\,\sigma_2\, \rho_{3j} H_2$$

The light effective neutrino masses are generate through type-I and type-III Seesaw

Minkowski, Phys.Lett. B67 (1977) 421
Yanagida, Conf.Proc. C7902131 (1979) 95

ρ_0 ⟶ Type-I Seesaw

ρ_3 ⟶ Type-III Seesaw

Kannike, Zhuridov , JHEP 1107 (2011) 102

Without the effects of the ρ_8 the light neutrino mass matrix for type-I and type-III seesaw is given by the standard formula,

$$m_\nu = -m_D M_{\rho_0}^{-1} m_D^T - m_D M_{\rho_3}^{-1} m_D^T$$

$m_D \ll M_R$

Neutrino Sector (iii)

Doing the follow redefinition of the Higgs doublets
$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} H \\ H' \end{pmatrix}$$

where just the Higgs field H gets a vev $\frac{v_0}{\sqrt{2}} = 174\, GeV$ and $\tan(\alpha) = \frac{v_{45}}{v_5}$

The neutrino mass matrix obtained after integrating out the fields responsible for the seesaw mechanism, ρ_0 and ρ_3, is given by

$$m_{ij}^{\nu} = (h_{i1} M_{\rho_3}^{-1} h_{j1} + h_{i2} M_{\rho_0}^{-1} h_{j2}) v_0^2$$

where

$$h_{i1} = \frac{1}{2\sqrt{2} v_0} ((\Gamma_\nu^1)_{ij} v_5 - 3(\Gamma_\nu^2)_{ij} v_{45})$$

$$h_{i2} = \frac{\sqrt{15}}{2\sqrt{2} v_0} ((\Gamma_\nu^1)_{ij} \frac{v_5}{5} + (\Gamma_\nu^2)_{ij} v_{45})$$

CFTP

TÉCNICO LISBOA

Effective Neutrino Textures: one 24

$$|M_R| \neq 0$$

$$m_\nu^{1\times 24} = \begin{pmatrix} * & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}^{A} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & 0 \end{pmatrix}^{P_{12}\,A\,P_{12}^T}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}^{B} \quad \begin{pmatrix} * & 0 & * \\ 0 & 0 & 0 \\ * & 0 & * \end{pmatrix}^{P_{12}\,B\,P_{12}^T}$$

Effective Neutrino Textures: two 24

$|M_R| \neq 0$

$$m_\nu^{2\times 24} = \underbrace{\begin{pmatrix} * & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{A} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & 0 \end{pmatrix}^{P_{12}\,A\,P_{12}^T}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}^{B} \quad \begin{pmatrix} * & 0 & * \\ 0 & 0 & 0 \\ * & 0 & * \end{pmatrix}^{P_{12}\,B\,P_{12}^T}$$

$$\begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}^{C} \quad \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}^{P_{12}\,C\,P_{12}^T}$$

$$\begin{pmatrix} 0 & * & * \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix}^{D} \quad \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & 0 \end{pmatrix}^{P_{12}\,D\,P_{12}^T}$$

Effective Neutrino Textures: three 24

$$|M_R| \neq 0$$

$$m_\nu^{3\times 24} = \begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}^{C} \quad \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}^{P_{12}\, C\, P_{12}^T}$$

$$\begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix}^{P_{132}\, \mathrm{NNI}\, P_{132}^T} \quad \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & 0 \end{pmatrix}^{P_{13}\, \mathrm{NNI}\, P_{13}^T}$$

$$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}^{E} \quad \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}^{P_{12}\, E\, P_{12}^T}$$

Effective Neutrino Textures: $|M_R|=0$

$$M_R = \begin{pmatrix} * & 0 \\ 0 & 0 \end{pmatrix}$$

$$M_R = \begin{pmatrix} 0 & 0 \\ 0 & * \end{pmatrix}$$

$$m_D = \begin{pmatrix} 0 & * \\ * & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} * & * \\ 0 & 0 \\ 0 & * \end{pmatrix} \begin{pmatrix} 0 & 0 \\ * & * \\ 0 & * \end{pmatrix} \begin{pmatrix} * & 0 \\ 0 & * \\ * & * \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ * & * \\ * & 0 \end{pmatrix} \begin{pmatrix} * & * \\ 0 & 0 \\ * & 0 \end{pmatrix} \begin{pmatrix} 0 & * \\ * & 0 \\ * & * \end{pmatrix} \begin{pmatrix} * & 0 \\ 0 & * \\ 0 & 0 \end{pmatrix}$$

Singular Seesaw

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & 0 & A \\ 0 & 0 & 0 & B & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & B & 0 & 0 & 0 \\ A & 0 & 0 & 0 & C \end{pmatrix}$$

Allen et al, Mod.Phys.Lett. A6 (1991) 1967-1976
Chun et al, Phys.Rev. D58 (1998) 093003
Chikira et al, Eur.Phys.J. C16 (2000) 701-705

$$m_L^{new} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B \\ 0 & 0 & 0 & 0 \\ 0 & B & 0 & 0 \end{pmatrix} \qquad m_D^{new} = \begin{pmatrix} A \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad M_R^{new} = C$$

$$m_\nu = m_L^{new} - m_D^{new} \left(M_R^{new}\right)^{-1} \left(m_D^{new}\right)^T$$

Charged lepton sector (i)

$$m_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l' & 0 & B_l \\ 0 & B_l' & C_l \end{pmatrix}$$

$$\epsilon_a^\ell \equiv \frac{|A_\ell'| - |A_\ell|}{|A_\ell'| + |A_\ell|} \qquad \epsilon_b^\ell \equiv \frac{|B_\ell'| - |B_\ell|}{|B_\ell'| + |B_\ell|}$$

$$\varepsilon_\ell \equiv \sqrt{\frac{(\epsilon_a^\ell)^2 + (\epsilon_b^\ell)^2}{2}}$$

$$K_l = diag\left(e^{ik_1}, e^{ik_2}, 1\right)$$

$$h_l = m_l m_l^\dagger \qquad O_l^T h_l O_l = diag\left(m_e^2, m_\mu^2, m_\tau^2\right)$$

For $\varepsilon_{a,b}^\ell$ small $O_\ell \approx \begin{pmatrix} 1 & -\sqrt{\frac{m_e}{m_\mu}}\left(1 - \epsilon_a^\ell - \frac{m_\mu}{m_\tau}\epsilon_b^\ell\right) & \sqrt{\frac{m_e m_\mu^2}{m_\tau^3}}\left(1 + \epsilon_b^\ell - \epsilon_a^\ell\right) \\ \sqrt{\frac{m_e}{m_\mu}}\left(1 - \epsilon_a^\ell - \frac{m_e}{m_\tau}\epsilon_b^\ell\right) & 1 & \sqrt{\frac{m_\mu}{m_\tau}}\left(1 - \epsilon_b^\ell\right) \\ -\sqrt{\frac{m_e}{m_\tau}}\left(1 - \epsilon_a^\ell - \epsilon_b^\ell\right) & -\sqrt{\frac{m_\mu}{m_\tau}}\left(1 - \epsilon_b^\ell + \frac{m_e}{m_\mu}\epsilon_a^\ell\right) & 1 \end{pmatrix}$

Branco, Mota, Phys.Lett. B280 (1992) 109-112
Branco, Costa, C.S., Phys.Lett. B690 (2010) 62-67

Neutrino sector (i)

$$m_\nu^a = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu e^{i\varphi} \end{pmatrix}$$

and

$$m_\nu^b = \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}$$

$$P_{12} m_\nu^a P_{12}^T$$

$$U_\nu^T m_\nu U_\nu = diag(m_1, m_2, m_3)$$

The Pontcorvo-Maki-Nakagawa-Sakata matrix: $U_{PMNS} = O_l^T K_l^{\rightarrow} P_g U_\nu$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{ij} = \sin(\theta_{ij}), c_{ij} = \cos(\theta_{ij})$$

δ Dirac phase

$\alpha_{1,} \alpha_2$ Majorana phases

Experimental Data

Forero, Tortola, Valle, Phys.Rev. D86 (2012) 073012

$\Delta m_{ij}^2 = m_i^2 - m_j^2$

parameter	1σ range	3σ range
Δm_{21}^2 $[10^{-5}$ eV$]$	7.43–7.81	7.12–8.20
$\lvert\Delta m_{31}^2\rvert$ $[10^{-3}$ eV$]$	2.46 – 2.61 2.37 – 2.50	2.31 – 2.74 2.21 – 2.64
$\sin^2\theta_{12}$	0.303-0.336	0.27-0.37
$\sin^2\theta_{23}$	0.400-0.461 & 0.573–0.635 0.569–0.626	0.36–0.68 0.37–0.67
$\sin^2\theta_{13}$	0.0218–0.0275 0.0223–0.0276	0.017–0.033
δ	$0 - 2\pi$	$0 - 2\pi$

Charged lepton masses @ Mz scale: running charged lepton masses from PDG'10 to Mz in the Msbar scheme using RGE's for QCD @ 1-loop level

Koide et al, Phys. Rev. D57 (1998) 3986
Xing et al, Phys rev D 77 (2008) 113016

$m_e(Mz) = 0.486661305 \pm 0.000000056$ MeV

$m_\mu(Mz) = 102.728989 \pm 0.000013$ MeV

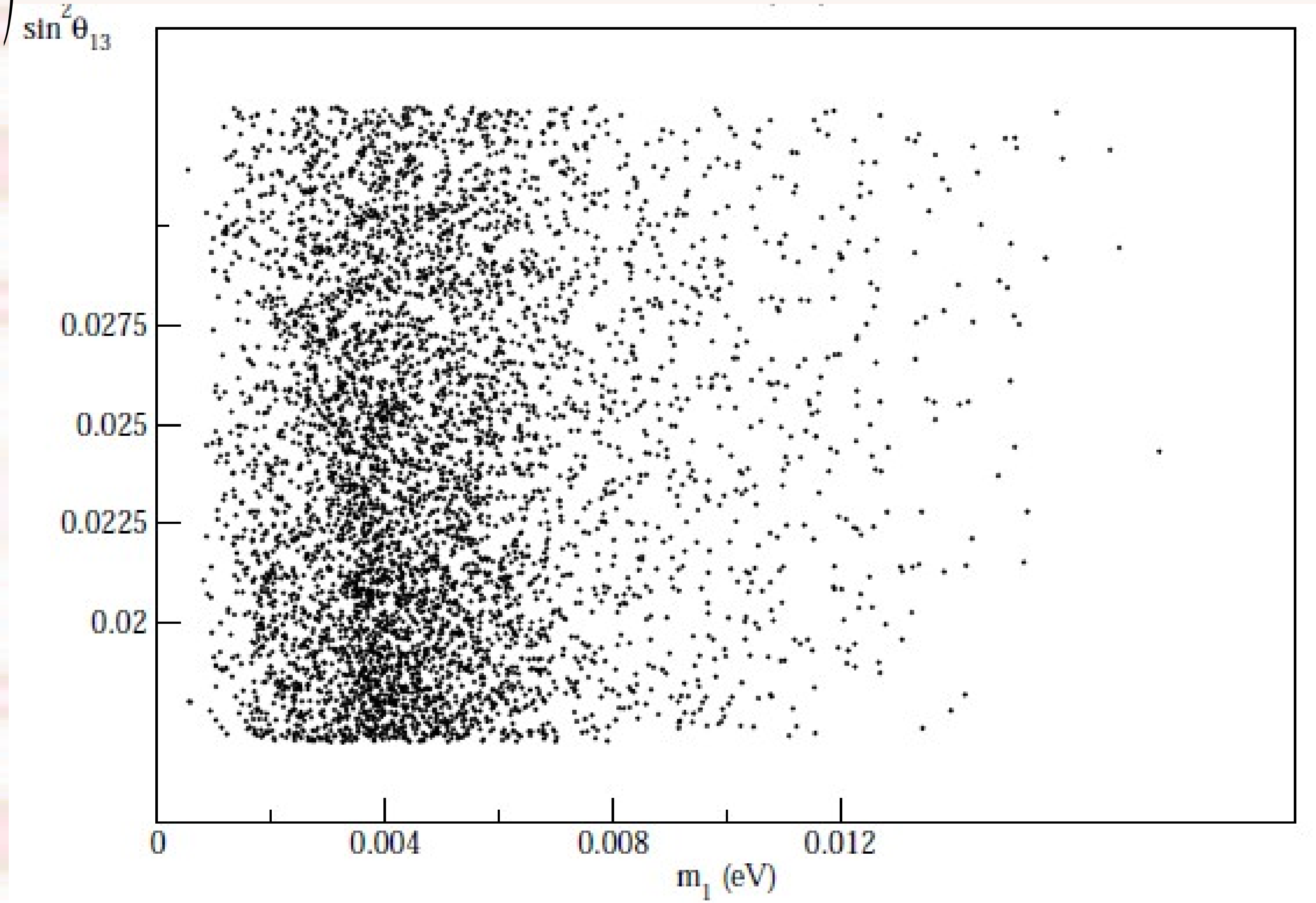
$m_\tau(Mz) = 1746.28 \pm 0.16$ MeV

Results

$$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$$

NH

$m_1 = [0.353, 20.884] \times 10^{-3}$ eV



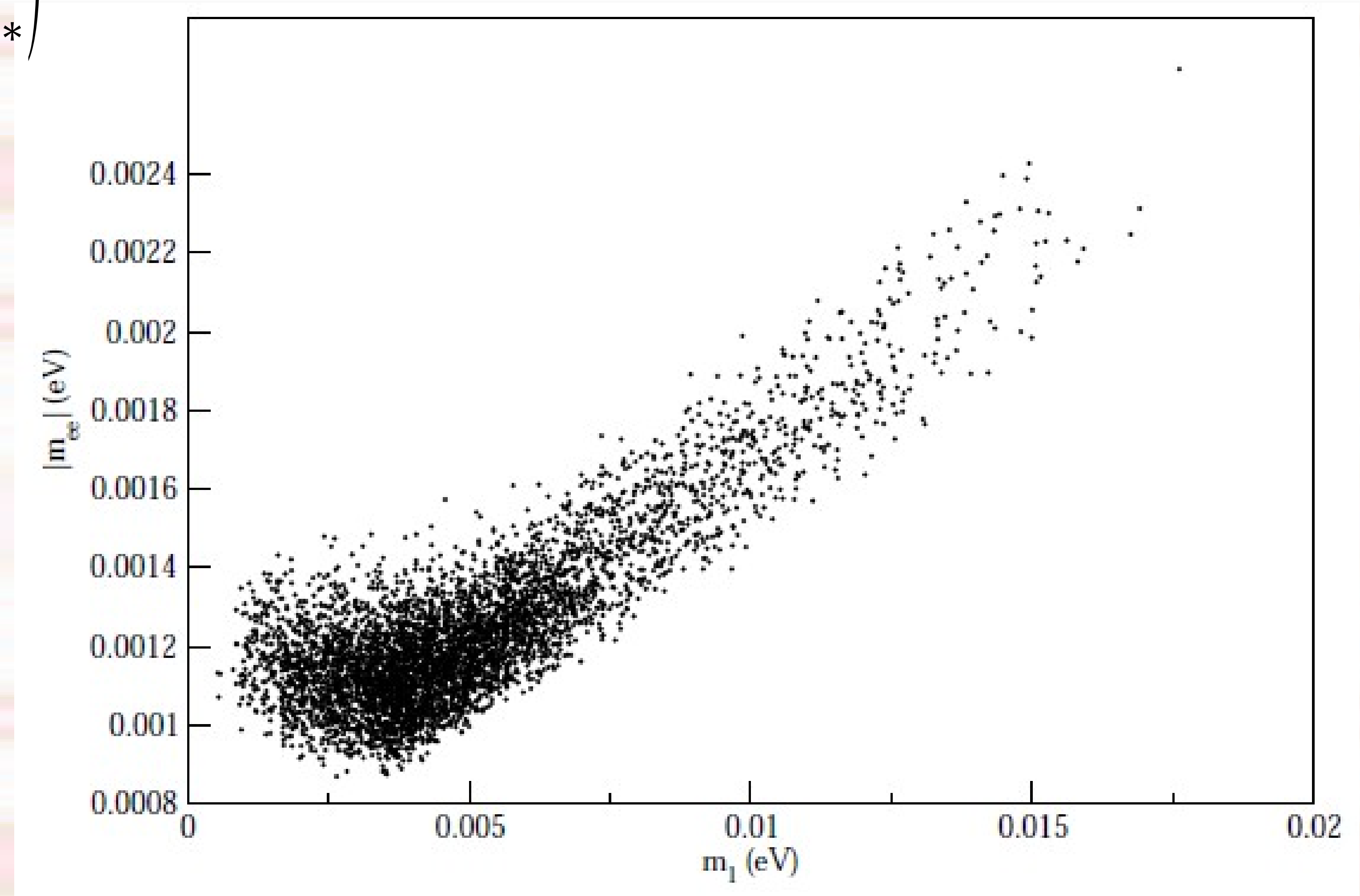
data at 3σ

Results

$$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$$

$m_1 = [0.353, 20.884] \times 10^{-3}$ eV

NH

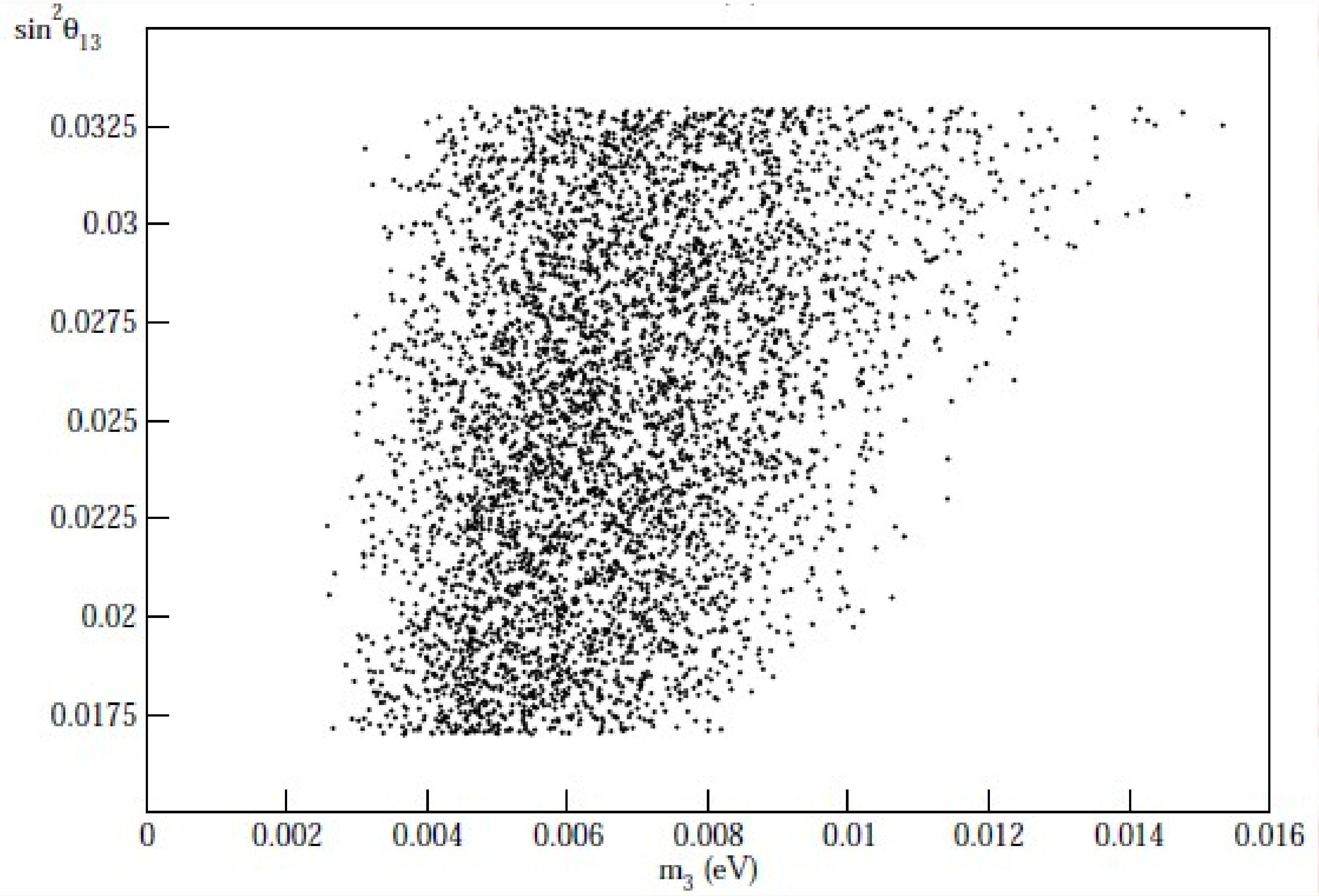


Results

$$\begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}$$

IH

$m_3 = [2.575, 15.335] \times 10^{-3}$ eV

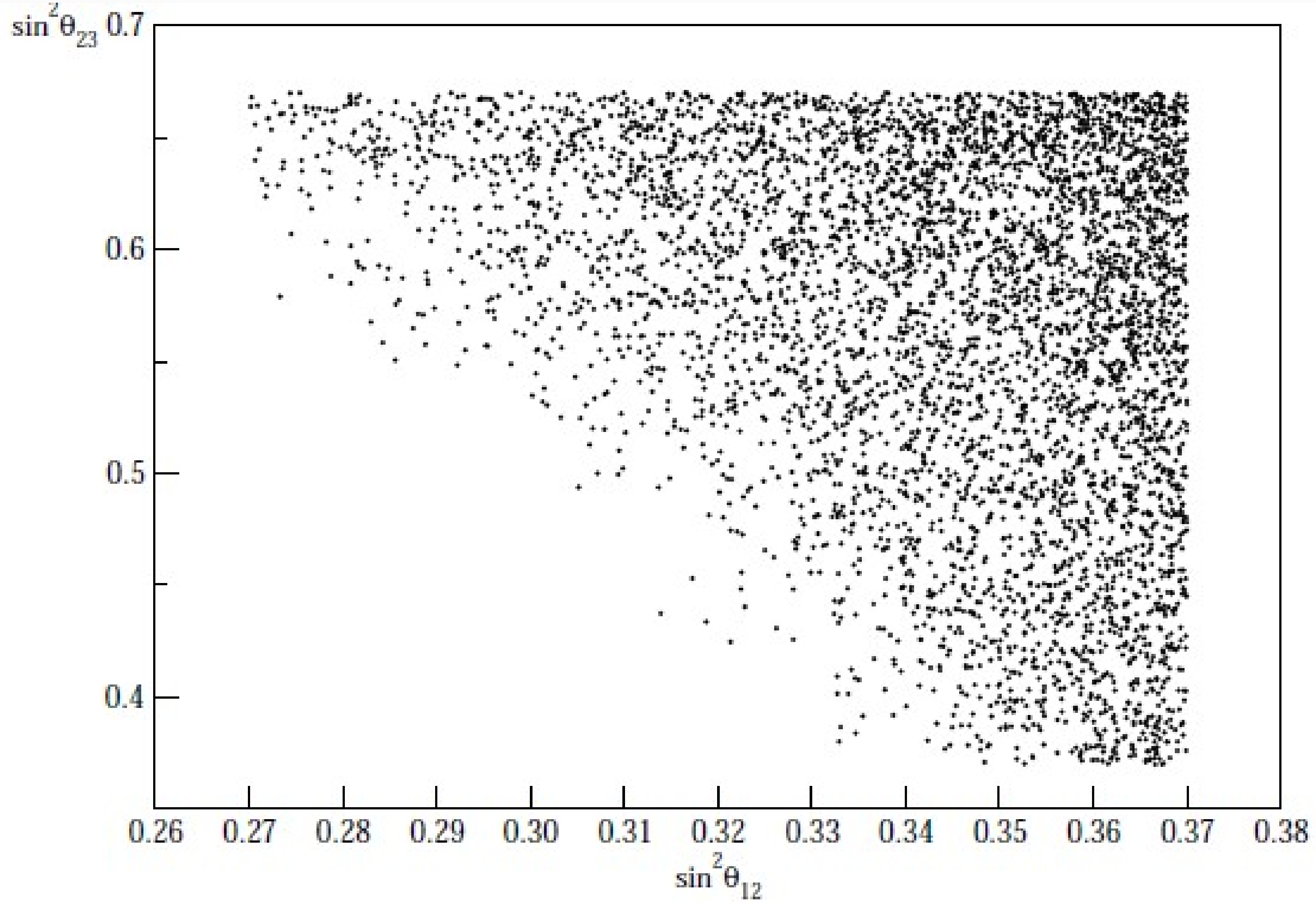


Results

$$\begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}$$

IH

$m_3=[2.575,15.335]\times10^{-3}$ eV

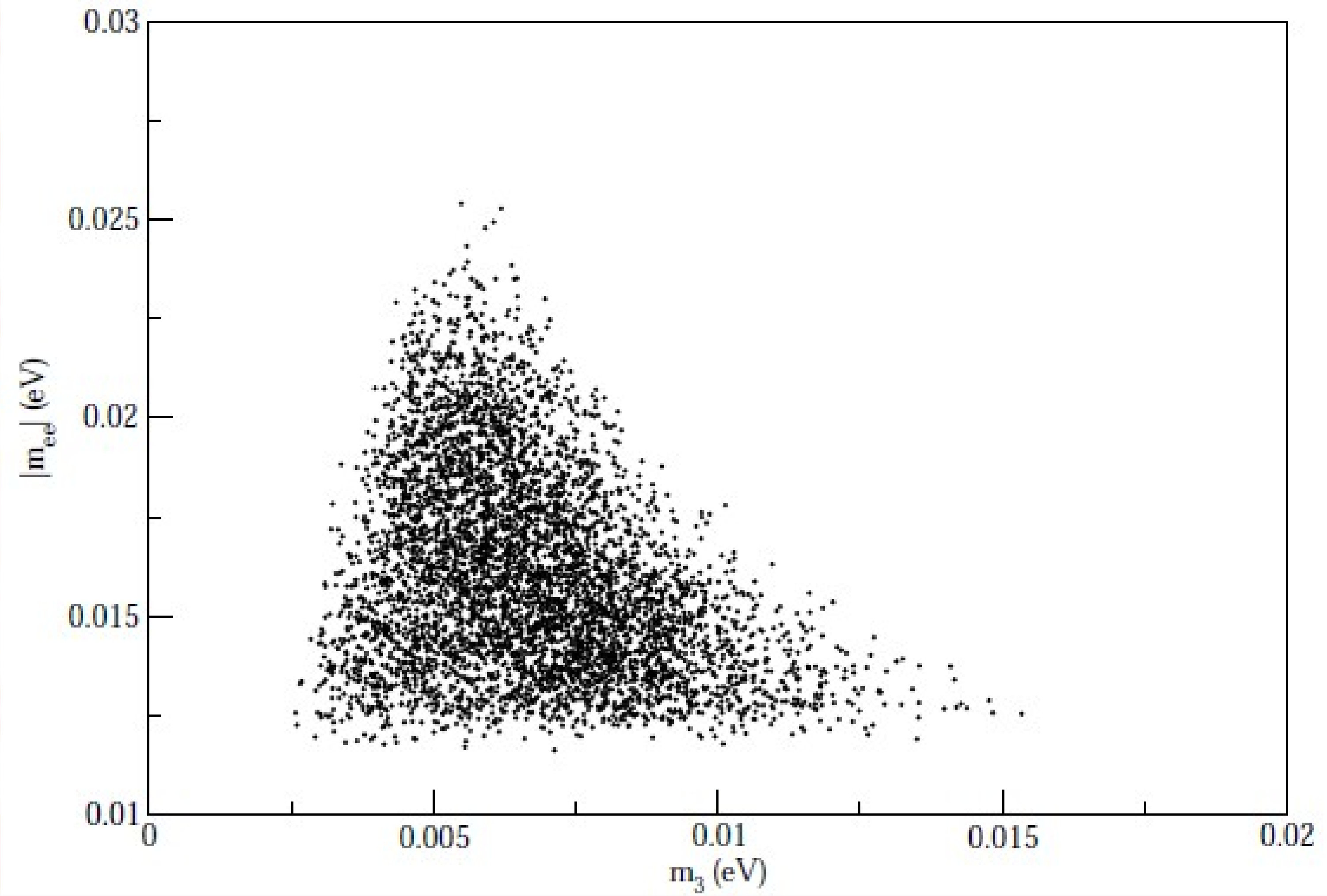


Results

$$\begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}$$

IH

$m_3=[2.575,15.335]\times 10^{-3}$ eV



Conclusions

➜ $SU(5)\times Z_4$ **model with up- and down-quark mass matrices in the** $NNI=\begin{pmatrix}0 & * & 0\\ * & 0 & *\\ 0 & * & *\end{pmatrix}$ **form, where the Higgs sector is composed by 5_H, 24_H and 45_H + three 24 fermionic representations**

✔ **Unfication**

✔ $M_e = M_d^T$

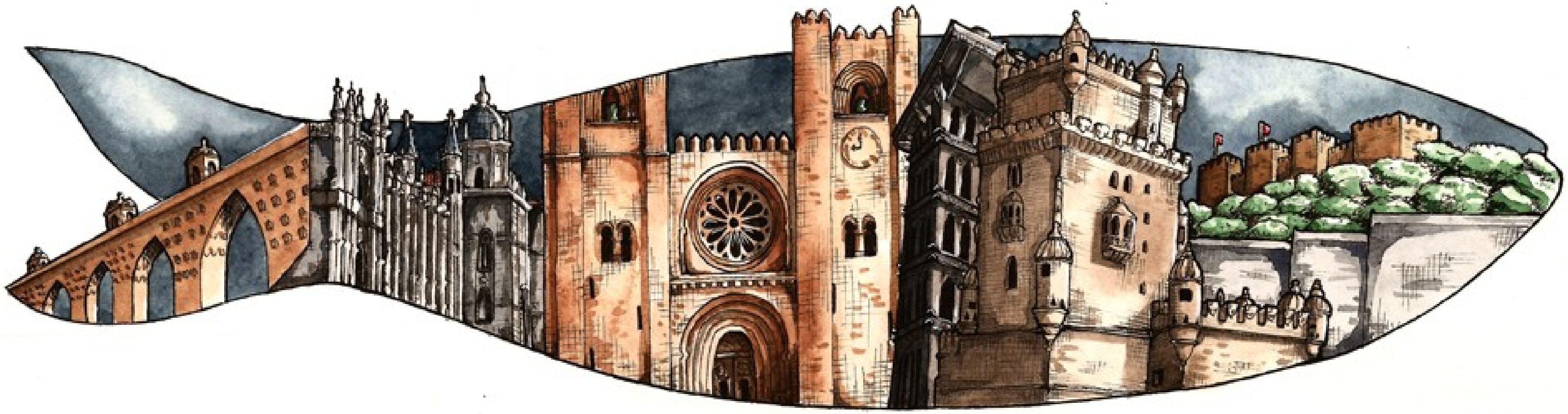
➜ **As before, only 2 possible textures for the effective neutrino mass matrix are viable**

$$\begin{pmatrix}0 & * & 0\\ * & * & *\\ 0 & * & *\end{pmatrix} \text{ NH} \qquad \begin{pmatrix}* & * & *\\ * & 0 & 0\\ * & 0 & *\end{pmatrix} \text{ IH}$$

$m_1 = [0.353, 20.884]\times 10^{-3}$ eV $\qquad m_3 = [2.575, 15.335]\times 10^{-3}$ eV

CFTP

TÉCNICO LISBOA



Thank you!