Three Neutrino Oscillations in the Earth: An AnalyticTreatment

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J. C. D'Olivo
ICN-UNAM
México

Motivation

✓Neutrino oscillations experiments can be interpreted in a 3ν framework, where the flavor eigenstates are linear combinations of the mass eigenstates.

✓The recent measurement of θ_{13} has open the door to CP violation searches in the leptonic sector.

✓Assuming a non vanishing phase, CP violation in neutrino oscillations can exists only if the mixing angles and the squared mass differences are all different from zero.

✓In most oscillation experiments neutrinos propagate considerable distances in matter.

✓An accurate description of matter neutrino oscillations is an important ingredient in the analysis of the data.

✓For an arbitrary density profile the evolution equation for the flavor amplitudes admits no exact solution, even in the 2n case.

✓Numerical integrations have been extensively used to examine the phenomenon.

✓Analytic solutions still useful.

Help to save the CPU time.

Give significant insight into the physics of the problem.

Better understanding of the dependence of the oscillations on the neutrino parameters and the properties of the medium.

$$|\psi(t)\rangle = \sum_\alpha \psi_\alpha(t)\,|\nu_\alpha\rangle, \qquad (\alpha = e, \mu, \tau)$$

$$\begin{pmatrix} \psi_e(t) \\ \psi_\mu(t) \\ \psi_\tau(t) \end{pmatrix} = \mathcal{U}(t, t_0) \begin{pmatrix} \psi_e(t_0) \\ \psi_\mu(t_0) \\ \psi_\tau(t_0) \end{pmatrix}$$

Neutrino state

$$L \simeq t - t_0$$

Distance from the production point

Evolution operator $(\hbar = c = 1)$

$$i\frac{\partial}{\partial t}\mathcal{U}(t, t_0) = H(t)\,\mathcal{U}(t, t_0), \qquad \mathcal{U}(t_0, t_0) = I, \qquad \mathcal{U}^\dagger = \mathcal{U}^{-1}$$

$$H(t) = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta_{21} & 0 \\ 0 & 0 & \Delta_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} V(t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{2E} = \frac{m_i^2 - m_j^2}{2E} \qquad (i, j = 1, 2, 3)$$

$$V(t) = V_e(t) - V_\mu(t) = \sqrt{2}\,G_F N_e(t) \qquad (V_\mu(t) = V_\tau(t))$$

Mixing Matrix

$$U = \mathcal{O}_{23}\,\Gamma\,\mathcal{O}_{13}\mathcal{O}_{12}\Gamma^* \qquad (\mathcal{O}_{12}\Gamma^* = \Gamma^*\mathcal{O}_{12})$$

$$\mathcal{O}_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \mathcal{O}_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}$$

$$\mathcal{O}_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \qquad \Gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}$$

$$c_{ij} = \cos\theta_{ij}, \quad s_{ij} = \sin\theta_{ij}$$

Vacuum

Medium

$$U_m(t)H(t)U_m^\dagger(t) = H_{\mathrm{D}}(t) = \begin{pmatrix} \mathcal{E}_1(t) & 0 & 0 \\ 0 & \mathcal{E}_2(t) & 0 \\ 0 & 0 & \mathcal{E}_3(t) \end{pmatrix}$$

Energy eigenvalues in matter

$$U_m(t) = \mathcal{O}_{23}\Gamma\mathcal{O}_m(t)\Gamma^*, \qquad \mathcal{O}_m^T(t) = \mathcal{O}_m^{-1}(t)$$

(T :Transpose)

$$\mathcal{O}_m(t) \to \mathcal{O}_{13}\mathcal{O}_{12} \quad \text{for} \quad V(t) \to 0$$

$$H = H_1 + H_2$$

$$\mathcal{U}(t, t_0) = \mathcal{U}_1(t, t_0)\,\mathcal{U}'(t, t_0)$$

$$i\,\frac{\partial \mathcal{U}_1}{\partial t} = H_1\,\mathcal{U}_1 \qquad\qquad i\,\frac{\partial \mathcal{U}'}{\partial t} = H'\,\mathcal{U}'$$

$$H' = \mathcal{U}_1^\dagger\,H_2\,\mathcal{U}_1$$

$$[\mathcal{U}_1, H_2] = 0 \quad \forall\, t \qquad \Rightarrow \qquad \mathcal{U}' = \mathcal{U}_2$$

$$\mathcal{U}(t, t_0) = \mathcal{U}_1(t, t_0)\,\mathcal{U}_2(t, t_0)$$

Adiabatic Basis

$$H(t)|\nu_i^m(t)\rangle = \mathcal{E}_i(t)|\nu_i^m(t)\rangle$$

$$|\nu_i^m(t)\rangle = \sum_\alpha U_{\alpha i}^m(t)\,|\nu_\alpha\rangle$$

$$\mathcal{U}(t,t_0) = \widetilde{U}_m(t)\mathcal{U}_{\rm a}(t,t_0)\widetilde{U}_m^\dagger(t_0)$$

$$\widetilde{U}_m(t) = \mathcal{O}_{23}\Gamma\mathcal{O}_m(t) \qquad \left(U_m(t) = \widetilde{U}_m(t)\Gamma^*\right)$$

$$i\frac{\partial}{\partial t}\mathcal{U}_{\rm a}(t,t_0) = H_{\rm a}(t)\mathcal{U}_{\rm a}(t,t_0)$$

$$H_{\rm a}(t) = H_{\rm D}(t) - i\mathcal{O}_m^T(t)\dot{\mathcal{O}}_m(t)$$

$$\mathcal{U}_{\rm a}(t,t_0) = \mathcal{P}(t,t_0)\mathcal{U}'_{\rm a}(t,t_0)$$

Adiabatic evolution

$$\mathcal{P}(t,t_0) = \exp\left(-i\int_{t_0}^t dt' H_{\rm D}(t')\right) = \begin{pmatrix} e^{-i\alpha_1(t)} & 0 & 0 \\ 0 & e^{-i\alpha_2(t)} & 0 \\ 0 & 0 & e^{-i\alpha_3(t)} \end{pmatrix}$$

$$\alpha_j(t) = \int_{t_0}^t dt'\mathcal{E}_j(t') \qquad j = 1,2,3$$

Non adiabatic corrections

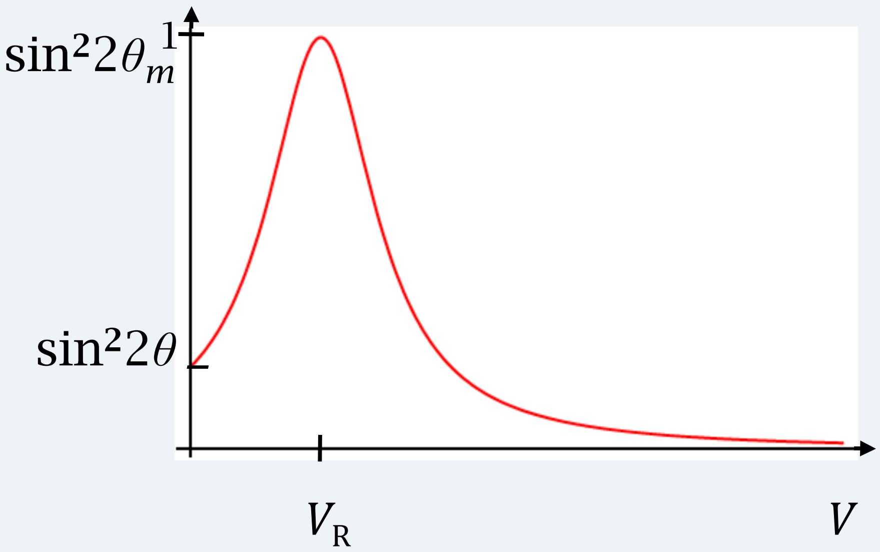
$$i\frac{\partial}{\partial t}\mathcal{U}'_{\rm a}(t,t_0) = H'_{\rm a}(t)\mathcal{U}'_{\rm a}(t,t_0)$$

$$H'_{\rm a}(t) = \mathcal{P}^\dagger(t,t_0)\left[-i\mathcal{O}_m^T(t)\dot{\mathcal{O}}_m(t)\right]\mathcal{P}(t,t_0)$$

$$\Delta m^2_{21} \ll |\Delta m^2_{31}| \quad \Longrightarrow \quad \mathcal{O}_m(t) \simeq \mathcal{O}^m_{13}(t)\mathcal{O}^m_{12}(t)$$

$$\mathcal{O}^m_{12}(t) = \begin{pmatrix} c^m_{12}(t) & s^m_{12}(t) & 0 \\ -s^m_{12}(t) & c^m_{12}(t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathcal{O}^m_{13}(t) = \begin{pmatrix} c^m_{13}(t) & 0 & s^m_{13}(t) \\ 0 & 1 & 0 \\ -s^m_{13}(t) & 0 & c^m_{13}(t) \end{pmatrix}$$

$$s^m_{ij}(t) = \sin\theta^m_{ij}(t) \qquad c^m_{ij}(t) = \cos\theta^m_{ij}(t)$$



$$\sin 2\theta^m_{12}(t) = \frac{\Delta_{21}}{\Delta^m_{21}(t)} \sin 2\theta_{12}$$

$$\sin 2\theta^m_{13}(t) = \frac{\Delta_{31} - \Delta_{21} s^2_{12}}{\Delta^m_{31}(t)} \sin 2\theta_{13}$$

Low resonance

High resonance

$$V_\ell^{\rm R} = V(t_\ell) = \frac{\Delta_{21}}{c_{13}^2}\cos 2\theta_{12}$$

$$V_h^{\rm R} = V(t_h) = \left(\Delta_{31} - \Delta_{21} s_{12}^2\right)\cos 2\theta_{13}$$

$$\Delta_{21}^m = c_{13}^2\sqrt{(V - V_\ell^{\rm R})^2 + (V_\ell^{\rm R}\tan 2\theta_{12})^2}$$

$$\Delta_{32}^m = \sqrt{(V - V_h^{\rm R})^2 + (V_h^{\rm R}\tan 2\theta_{13})^2}$$

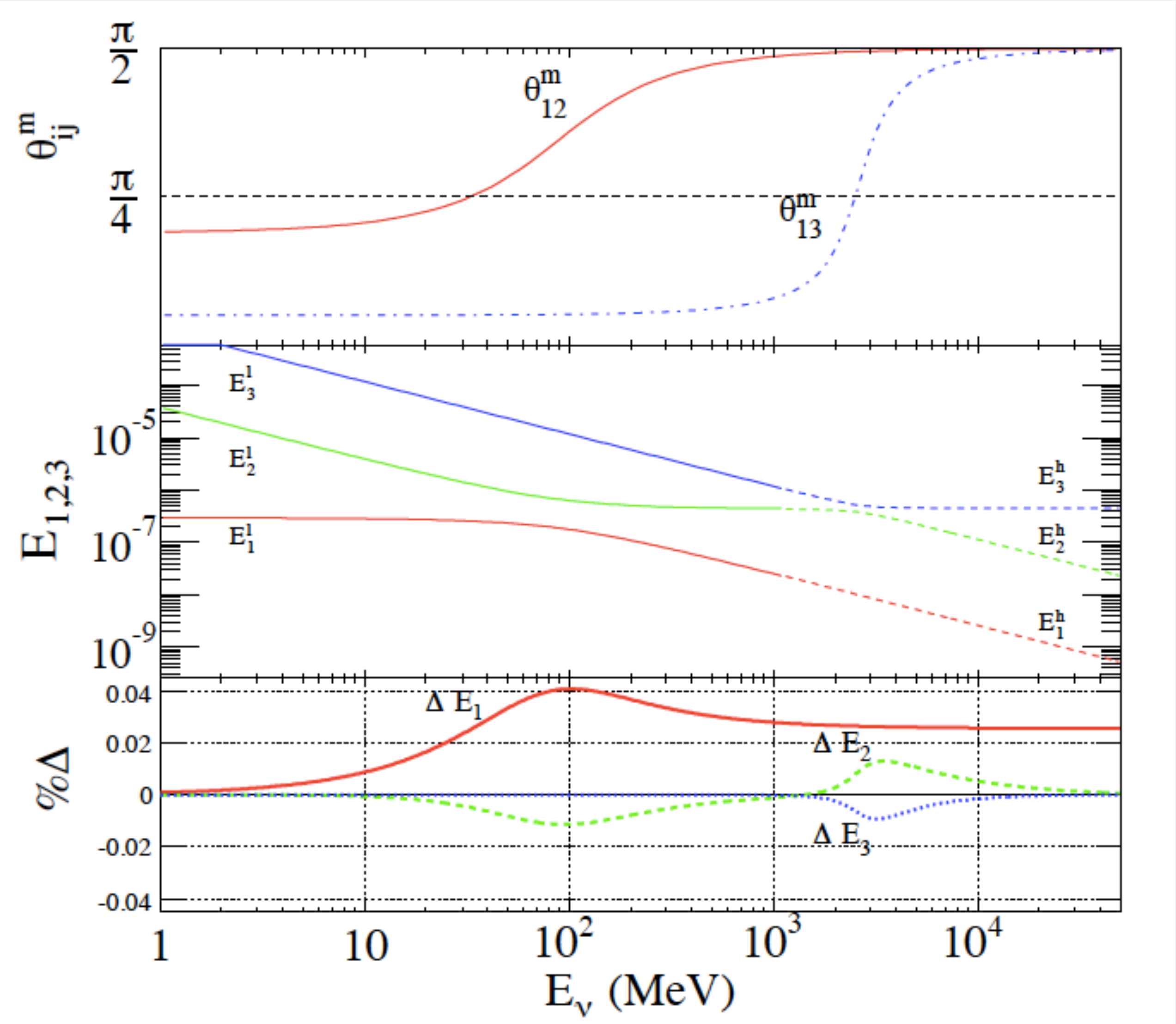
$$\mathcal{E}_2 - \mathcal{E}_1 \simeq \Delta_{21}^m(t), \qquad t \sim t_\ell$$

$$\mathcal{E}_3 - \mathcal{E}_1 \simeq \Delta_{31}^m(t), \qquad t \sim t_h$$

$$\mathcal{E}_1(t) \simeq \tfrac{1}{2}\Big[\left(\Delta_{21} + V c_{13}^2\right) - \Delta_{21}^m(t)\Big]$$

$$\mathcal{E}_2(t) \simeq \tfrac{1}{2}\Big[\left(\Delta_{31} + \Delta_{21} c_{12}^2 + V s_{13}^2\right) + (\Delta_{21}^m - \Delta_{32}^m)(t)\Big]$$

$$\mathcal{E}_3(t) \simeq \tfrac{1}{2}\Big[\left(\Delta_{31} + \Delta_{21} s_{12}^2 + V\right) + \Delta_{32}^m(t)\Big]$$



$V = 4.54 \times 10^{-4}\ \mathrm{eV}^2/\mathrm{GeV}$ (Earth's core)

$\Delta m^2_{21} = 7.59 \times 10^{-5}\ \mathrm{eV}^2, \quad |\Delta m^2_{31}| = 2.35 \times 10^{-3}\ \mathrm{eV}^2$

$\theta_{12} = 34°, \quad \theta_{13} = 8.9°, \quad \delta = 0$

$$H'_{\mathrm{a}}(t) = \underbrace{\begin{pmatrix} 0 & \varrho_l(t) & 0 \\ \varrho_l^*(t) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{H_\ell(t)} + \underbrace{\begin{pmatrix} 0 & 0 & \varrho'_h(t) \\ 0 & 0 & \varrho_h(t) \\ \varrho_h'^*(t) & \varrho_h^*(t) & 0 \end{pmatrix}}_{H_h(t)}$$

$$\begin{aligned} \varrho_l(t) &= -i\,\dot\theta_{12}^m \exp[-i\phi_{21}(t,t_0)] \\ \varrho_h(t) &= -i\,\dot\theta_{13}^m s_{12}^m \exp[-i\phi_{32}(t,t_0)] \\ \varrho'_h(t) &= -i\,\dot\theta_{13}^m c_{12}^m \exp[-i\phi_{31}(t,t_0)] \end{aligned}$$

$$\boxed{\phi_{ij}(t_2,t_1) = \int_{t_1}^{t_2} dt\,\left(\mathcal{E}_i(t) - \mathcal{E}_j(t)\right)}$$

$$\dot\theta_{12}^m(t) = \frac{\sin^2 2\theta_{12}^m(t)}{2\sin 2\theta_{12}}\,\frac{c_{13}^2\dot V(t)}{\Delta_{21}}$$

$$\dot\theta_{13}^m(t) = \frac{\sin^2 2\theta_{13}^m(t)}{2\sin 2\theta_{13}}\,\frac{\dot V(t)}{\Delta_{31} - \Delta_{21}s_{12}^2}$$

$\mathcal{U}_\ell(t,t_0)$: Evolution operator corresponding to $H_\ell(t)$

$\mathcal{U}_h(t,t_0)$: Evolution operator corresponding to $H_h(t)$

- High density region

$$\theta_{12}^m(t) \simeq \frac{\pi}{2}\,, \qquad \dot{\theta}_{12}^m(t) \simeq 0 \qquad \Longrightarrow \qquad H_\ell(t) \simeq 0\,, \qquad \mathcal{U}_\ell(t,t_0) \simeq I$$

$$H_h(t) \neq 0\,, \qquad \mathcal{U}_h(t,t_0) \neq I$$

- Low density region

$$\theta_{13}^m(t) \simeq \theta_{13}\,, \qquad \dot{\theta}_{13}^m(t) \simeq 0 \qquad \Longrightarrow \qquad H_h(t) \simeq 0\,, \qquad \mathcal{U}_h(t,t_0) \simeq I$$

$$H_\ell(t) \neq 0\,, \qquad \mathcal{U}_\ell(t,t_0) \neq I$$

Decreasing potential

$$[\mathcal{U}_\ell(t,t_0), \mathcal{H}_h(t)] \simeq 0 \qquad \forall\, t$$

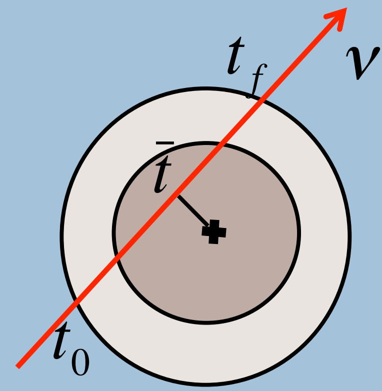
$$\mathcal{U}_a'(t,t_0) \simeq \mathcal{U}_\ell(t,t_0)\,\mathcal{U}_h(t,t_0)$$

Increasing potential

$$[\mathcal{U}_h(t,t_0), \mathcal{H}_\ell(t)] \simeq 0 \qquad \forall\, t$$

$$\mathcal{U}_a'(t,t_0) \simeq \mathcal{U}_h(t,t_0)\,\mathcal{U}_\ell(t,t_0)$$

Symmetric Potential



$$\bar{t} = (t_f + t_0)/2$$

- $t_0 \le t \le \bar{t}$ increasing potential $\Longrightarrow$ $\mathcal{U}'_{\rm a}(\bar{t}, t_0) \simeq \mathcal{U}_h(\bar{t}, t_0)\mathcal{U}_\ell(\bar{t}, t_0)$
- $\bar{t} \le t \le t_f$ decreasing potential $\Longrightarrow$ $\mathcal{U}'_{\rm a}(t_f, \bar{t}) \simeq \mathcal{U}_\ell(t_f, \bar{t})\mathcal{U}_h(t_f, \bar{t})$

$$\begin{aligned} \mathcal{U}'_{\rm a}(t_f, t_0) &= \mathcal{U}'_{\rm a}(t_f, \bar{t})\mathcal{U}'_{\rm a}(\bar{t}, t_0) \\ &\simeq \mathcal{U}_\ell(t_f, \bar{t})\mathcal{U}_h(t_f, t_0)\mathcal{U}_\ell(\bar{t}, t_0) \end{aligned}$$

General solution for a neutrino with a given E

$$\boxed{V(2\bar{t} - t) = V(t)} \Longrightarrow \mathcal{U}_\ell(t_f, \bar{t}) = \mathcal{P}^\dagger\, \mathcal{U}_\ell^T(\bar{t}, t_0)\, \mathcal{P}$$

$$\mathcal{P}\,\mathcal{U}'_{\rm a}(t_f,t_0)=\mathcal{U}_\ell^T(\bar t,t_0)\mathcal{P}\,\mathcal{U}_h(t_f,t_0)\,\mathcal{U}_\ell(\bar t,t_0)$$

$$\mathcal{P}=\mathcal{P}(t_f,t_0)=e^{-i\alpha_2}\mathcal{P}_{12}\mathcal{P}_{32}$$

$$\begin{aligned}\phi_{ij} &= \phi_{ij}(t_f,t_0)=2\bar\phi_{ij}\\ \bar\phi_{ij} &= \phi_{ij}(\bar t,t_0)\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{12} &= \begin{pmatrix} e^{-i\phi_{12}} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\\ \mathcal{P}_{32} &= \begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & e^{-i\phi_{32}}\end{pmatrix}\end{aligned}$$

$$\boxed{\mathcal{U}_{\rm a}(t_f,t_0)\simeq\left[\overline{\mathcal{P}}_{12}\,\mathcal{U}_\ell(\bar t,t_0)\right]^T\left[\mathcal{P}_{32}\,\mathcal{U}_h(t_f,t_0)\right]\left[\overline{\mathcal{P}}_{12}\,\mathcal{U}_\ell(\bar t,t_0)\right]}$$

$$\left(\text{For } \dot\theta_{13}^m\neq 0\,,\quad \theta_{12}^m\simeq\frac{\pi}{2}\ \&\ c_{12}^m\simeq 0 \quad\Longrightarrow\quad \rho'_h(t)\simeq 0\ \ \forall\, t\right)$$

Magnus Expansion

$$\mathcal{U}(t,t_0) = \exp[\Omega(t,t_0)]$$

$$\Omega(t,t_0) = \sum_{n=1}^{\infty} \Omega^{(n)}(t,t_0)$$

$\Omega^{(n)}$: Sum of integrals of n-fold nested commutarors or H(t)

$$\Omega^{(n)\dagger}(t,t_0) = -\Omega^{(n)}(t,t_0)$$

Unitarity preserved order by order

$$\begin{aligned}
\Omega^{(1)}(t,t_0) &= -\frac{i}{\hbar}\int_{t_0}^{t} dt_1 H(t_1) \\
\Omega^{(2)}(t,t_0) &= -\frac{1}{2\hbar^2}\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 [H(t_1), H(t_2)]
\end{aligned}$$

S. Blanes, F. Casas, J. A. Oteo & J. Ross, Phys. Rep. 470. 151 (2009).

To second order

$$\mathcal{U}_{\ell,h}(t,t_0) = \exp\left[\Omega_{\ell,h}(t,t_0)\right]$$

$$\Omega_{\ell,h}(t,t_0) \simeq \Omega^{(1)}_{\ell,h}(t,t_0) + \Omega^{(2)}_{\ell,h}(t,t_0)$$

$$\overline{\mathcal{P}}_{12}\,\mathcal{U}_h(t_f,t_0) = \begin{pmatrix} u_{11}\,e^{i\bar{\phi}_{21}} & u_{12} & 0 \\ -u^*_{21}\,e^{i\bar{\phi}_{12}} & u^*_{11} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\overline{\mathcal{P}}_{13}\,\mathcal{U}_\ell(\bar{t},t_0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & v_{11} & v_{12}\,e^{-i\bar{\phi}_{32}} \\ 0 & -v^*_{12}\,e^{-i\bar{\phi}_{32}} & v^*_{11}\,e^{-i\phi_{32}} \end{pmatrix}$$

$(\rho'_h(t) \simeq 0 \;\; \forall t)$

$$\begin{aligned} u_{11} &= \cos\xi_\ell - i\,\frac{\sin\xi_\ell}{\xi_\ell}\,\xi^{(2)}_\ell \\ u_{12} &= i\,\frac{\sin\xi_\ell}{\xi_\ell}\,\xi^{(1)}_\ell \end{aligned}$$

$$\begin{aligned} v_{11} &= \cos\xi_h - i\,\frac{\sin\xi_h}{\xi_h}\,\xi^{(2)}_h \\ v_{12} &= i\,\frac{\sin\xi_h}{\xi_h}\,\xi^{(1)}_h \end{aligned}$$

$$\xi_\ell^{(1)} = i\int_{t_0}^{\bar{t}} dt' \dot{\theta}_{12}^m(t') \exp[-i\phi_{21}(t',\bar{t})]$$

$$\xi_\ell^{(2)} = \int_{t_0}^{t_f} dt' \int_{t_0}^{t'} dt'' \dot{\theta}_{12}^m(t')\dot{\theta}_{12}^m(t'') \sin[\phi_{21}(t'',t')]$$

$$\xi_h^{(1)} = 2\int_{t_0}^{\bar{t}} dt' \dot{\theta}_{13}^m(t') \sin[\phi_{32}(t',\bar{t})]$$

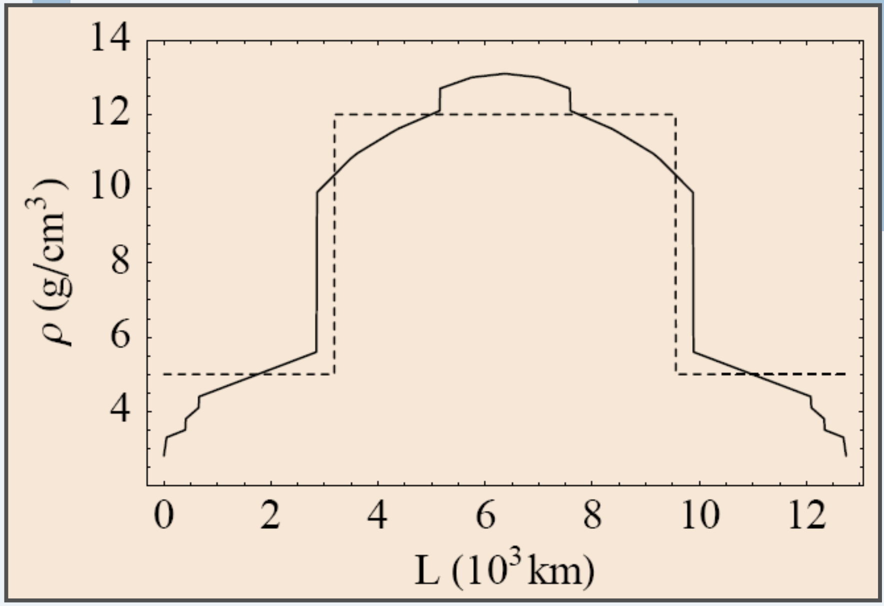
$$\xi_h^{(2)} = \int_{t_0}^{t_f} dt' \int_{t_0}^{t'} dt'' \dot{\theta}_{13}^m(t')\dot{\theta}_{13}^m(t'') \sin[\phi_{32}(t'',t')]$$

For any potential V(t)

$$\xi_{\ell,h} = \sqrt{|\xi_{\ell,h}^{(1)}|^2 + (\xi_{\ell,h}^{(2)})^2}$$

Earth Potential

Mantle-Core-Mantle Model



$$N_e(r) = N_A \begin{cases} 5.95\ \mathrm{cm}^{-3}, & r \leq R_\oplus/2 \\ 2.48\ \mathrm{cm}^{-3}, & R_\oplus/2 < r \leq R_\oplus \end{cases}$$

$$|\psi(t_0)\rangle = |\nu_\mu\rangle$$

$$\psi_\mu(t_0) = 1, \qquad \psi_e(t_0) = \psi_\tau(t_0) = 0$$

$$P(\nu_\mu \to \nu_e) = |\psi_e(t_f)|^2$$

$$\psi_e(t_f) = \sum_i \widetilde{U}^m_{e1}(t_f)\psi^m_i(t_f) \qquad \left(\widetilde{U}^m_{e1}(t_f) = \widetilde{U}^m_{e1}(t_0)\right)$$

$$
\begin{aligned}
\psi_1^m(t_f) &= a_{11}\,\widetilde{U}_{\mu 1}^{m*}(t_0) + a_{12}\,\widetilde{U}_{\mu 2}^{m*}(t_0) + a_{13}\,\widetilde{U}_{\mu 3}^{m*}(t_0) \\
\psi_2^m(t_f) &= a_{21}\,\widetilde{U}_{\mu 1}^{m*}(t_0) + a_{22}\,\widetilde{U}_{\mu 2}^{m*}(t_0) + a_{23}\,\widetilde{U}_{\mu 3}^{m*}(t_0) \\
\psi_3^m(t_f) &= a_{31}\widetilde{U}_{\mu 1}^{m*}(t_0) + a_{32}\,\widetilde{U}_{\mu 2}^{m*}(t_0) + a_{33}\,\widetilde{U}_{\mu 3}^{m*}(t_0)
\end{aligned}
$$

$$a_{11} = \left(u_{11}^2 + u_{12}^{*2}\,v_{11}\right)e^{i\phi_{21}} \qquad a_{13} = -u_{12}^*\,v_{12}\,e^{i\bar{\phi}_{21}}\,e^{-i\bar{\phi}_{32}}$$

$$a_{12} = a_{21} = \left(u_{11}u_{12} - u_{11}^* u_{12}^* v_{11}\right)e^{i\bar{\phi}_{21}}$$

$$a_{22} = u_{12}^2 + u_{11}^{*2}\,v_{11} \qquad a_{23} = u_{11}^* v_{12}\,e^{-i\bar{\phi}_{32}}$$

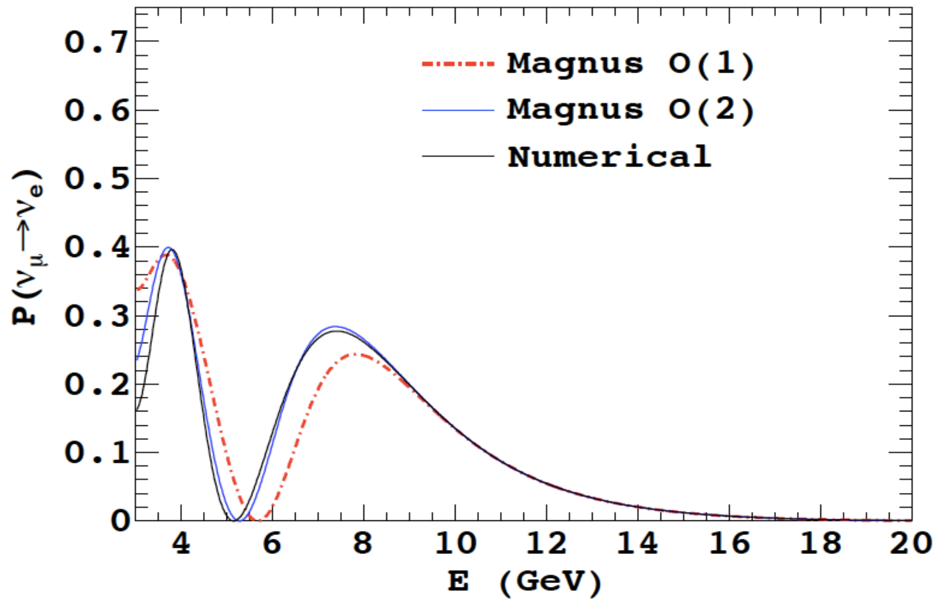
$$a_{31} = u_{12}^* v_{12}^*\,e^{i\bar{\phi}_{21}}\,e^{-i\bar{\phi}_{32}} \qquad a_{32} = -u_{11}^*\,v_{12}^*\,e^{-i\bar{\phi}_{32}}$$

$$a_{33} = v_{11}^* e^{-i\phi_{32}}$$

High Energy(E >1GeV)

$$V \gg \Delta_{21}$$

$$\theta_{12}^m \simeq \frac{\pi}{2}\,, \qquad \dot{\theta}_{12}^m \simeq 0$$



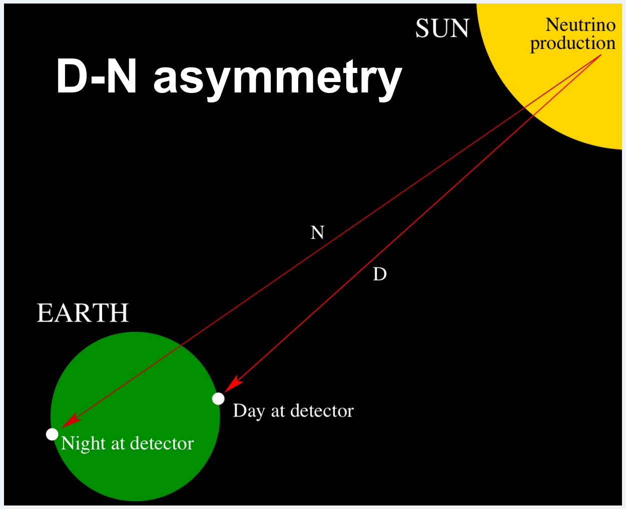
$$\xi_\ell^{(1,2)} = 0 \quad \xi_\ell = 0 \qquad \Longrightarrow \qquad u_{11} = u_{12} = 0$$

$$a_{11} = e^{i\phi_{21}}\,, \qquad a_{12} = a_{21} = a_{13} = a_{31} = 0$$

$$P(\nu_\mu \to \nu_e) \;\simeq\; s_{23}^2\Big(\cos 2\theta_{13}^m(t_0)\,\mathcal{I}m(v_{12}) - \sin 2\theta_{13}^m(t_0)\,\mathcal{I}m(v_{11}e^{i\bar{\phi}_{32}})\Big)^2$$

2ν result

D. Supanitsky, J. C. D & G. Medina Tanco, Phys. Rev. D78 045024 (2008). A. N. Ioannisian& A. Yu Smirnov. Nucl. Phys. B816, 94 (2009).



At night, neutrinos coming from the Sun reach the detector after they propagate through the Earth.

Adiabatic transformation inside the Sun.

Loss of coherence in the propagation from the Sun to the Earth.

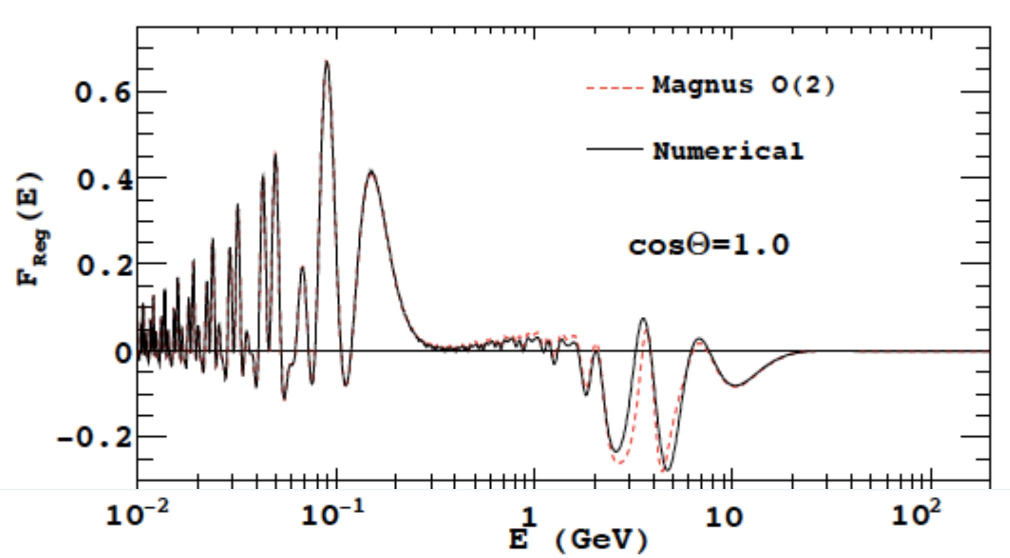
$$\overline{P}(\nu_e \to \nu_e) = \sin^2\theta + \cos 2\theta \cos^2\theta_\odot^0 - \cos 2\theta_\odot^0\, f_{reg} \qquad (\theta = \theta_{12})$$

$\theta_\odot^0$: Matter mixing angle at the production point in the solar core

$$f_{reg} = P_{2e}^{Earth} - P_{2e}^{vac}$$

$$|\langle \nu_e | \nu_2 \rangle|^2$$

$$|\langle \nu_e | \mathcal{U}(t_f, t_0) | \nu_2 \rangle|^2$$



Conclusions

✓The Magnus expansion for the evolution operator (implemented in the adiabatic basis) provides an efficient formalism to describe three neutrino oscillations in a medium with an arbitrary density profile.

✓This method takes properly into account the Earth matter effects on the transition probabilities for neutrinos with a wide interval of energies, making possible a simple (and accurate) description of such effects in the case of solar and atmospheric neutrinos.

✓The same formalism can be applied to the study of other situations of physical interest (for example, long baseline experiments).