# Lorentz and CPT violation

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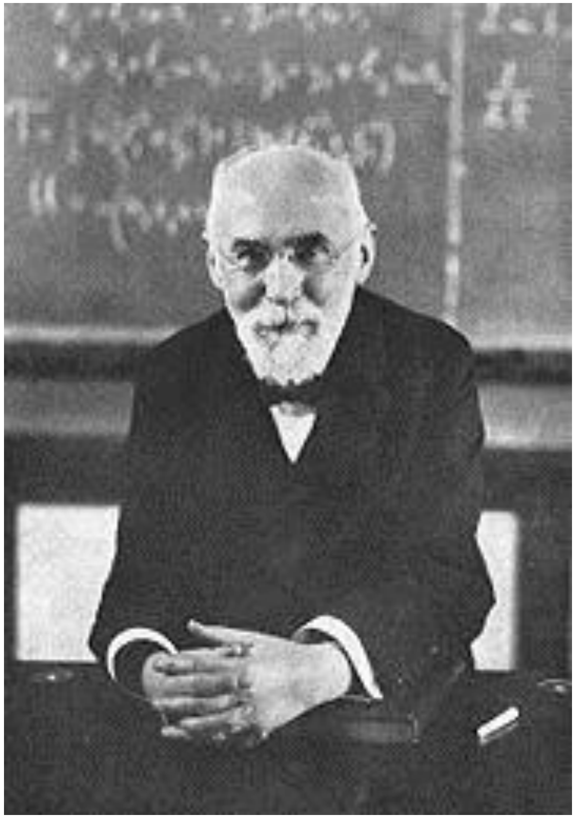
# Outline

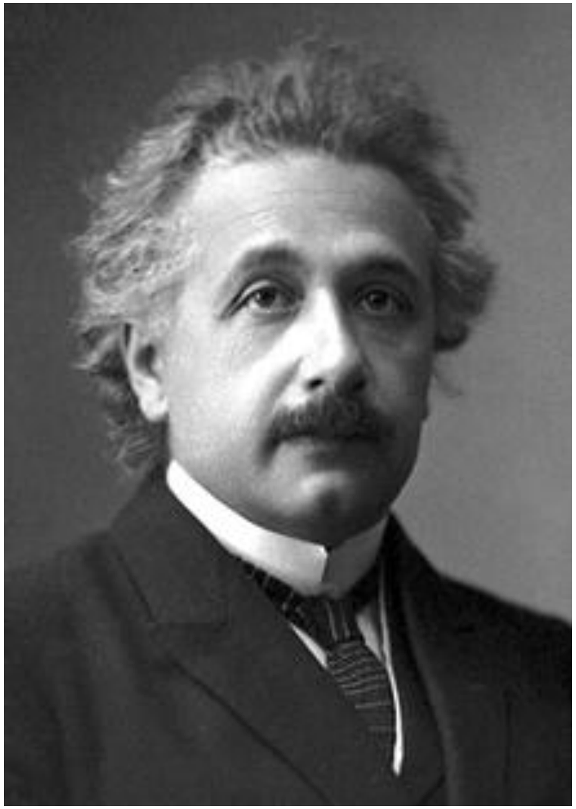
- Introduction
- CPT and Lorentz invariance violation
- Models with Lorentz Invariance violation (LIV)
- Kinematic frameworks
- Effective field Theory
- Phenomenology
- Tests of LIV
- Conclusions

# Pioneers of Lorentz symmetry



Hendrik Antoon Lorentz

*(1853-1928)*



Albert Einstein

*(1879-1955)*

# Introduction

Lorentz symmetry is a fundamental ingredient of both quantum field theory and General Relativity.

In the last two decades, there has been growing interest in the possibility that Lorentz symmetry may not be exact.

Reasons:

1: Many candidate theories of quantum gravity involve LIV as a possible effect.

*(For example, string theory, non-commutative geometry, loop quantum gravity…)*

2: Development of low-energy effective field theories with LIV has prompted much interest in experimental testing of Lorentz and CPT symmetry.

# CPT and Lorentz violation

Relation between Lorentz invariance and CPT invariance:

CPT theorem: Any Lorentz-invariant local quantum field theory with Hermitian Hamiltonian must have CPT symmetry

Schwinger ’51, Lüders ’54, Bell ’54, Pauli ’55, Jost ’57

“anti-CPT theorem”: An interacting theory that violates CPT necessarily violates Lorentz invariance.

Greenberg ’02

It is possible to have Lorentz violation without CPT violation!

# Fundamental models with Lorentz Invariance Violation (LIV)

1. Spontaneous symmetry breaking with LIV
2. Cosmologically varying scalars
3. Noncommutative geometry
4. LIV from topology

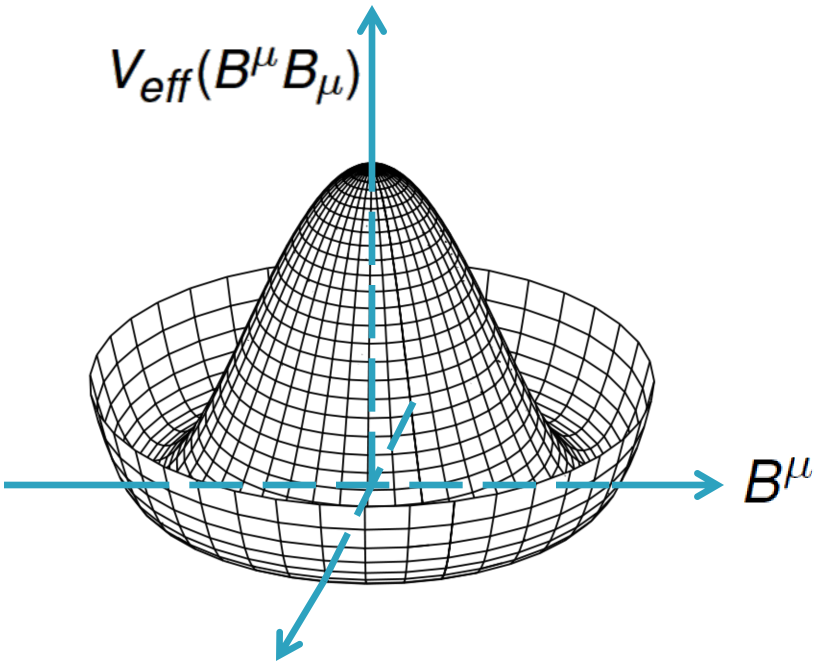
# 1. Spontaneous symmetry breaking with LIV

Suppose $\mathcal{L}_I \supset \lambda B^\mu \bar{\psi}\gamma_5\gamma_\mu\psi$

if $\langle B^\mu \rangle = b^\mu \neq 0$

$$\Downarrow$$

$$\mathcal{L} \supset \lambda b^\mu \bar{\psi}\gamma_5\gamma_\mu\psi$$




*Possible examples:*

- "bumblebee models" Kostelecky, Samuel '89
- string field theory Kostelecky, R. P. '91
- fermion condensation Tomboulis et.al. '02

## More general :

$$\mathcal{L}_I \supset \lambda m_P^{-k} T \cdot \bar{\psi} \Gamma (i\partial)^k \chi$$

If tensor T acquires v.e.v., $\mathcal{L}_I$ generates contribution to the fermion inverse propagator

$$\Delta K(p) = \lambda m_P^{-k} \langle T \rangle \cdot \Gamma p^k$$

that breaks Lorentz invariance.

# 2. Cosmologically varying scalars

*Idea:* gradient of scalar selects preferred direction

Example: $\mathcal{L}_I \supset a(x) F\tilde{F}$

$a(x)$: cosmologically varying coupling (axion?)

Integration by parts: $\mathcal{L}_I' \supset -2(\partial^\mu a) A^\nu \tilde{F}_{\mu\nu}$

Slow variation of $a(x)$ : $k^\mu = 2\partial^\mu a \simeq \text{const.}$

$$\mathcal{L}_I' \supset -k^\mu A^\nu \tilde{F}_{\mu\nu}$$

Kostelecky,Lehnert,Perry '03; Arkani-Hamed et.al. '03

# 3. Noncommuting geometry

Consider spacetime with noncommuting coordinates:

Connes et.al. '98

$$[x_\alpha, x_\beta] = i\frac{1}{\Lambda_{\mathrm{NC}}}\Theta_{\alpha\beta}$$

$\Theta_{\alpha\beta}$ is a tensor of O(1), $\Lambda_{NC}$ noncommutative energy scale.

Lorentz invariance manifestly broken, so the size of $\Lambda_{NC}$ is constrained by Lorentz tests.

Deformed gauge field theories can be constructed.

UV/IR mixing problem has been pointed out, which makes low-energy expansion problematic. Possible solution by supersymmetry.

Minwalla et.al. '00

It is possible to re-express resulting field theory in terms of mSME, by use of the Seiberg-Witten map. It expresses the non-commutative fields in terms of ordinary gauge fields. For non-commutative QED this yields the following Lorentz-violating expression, at lowest nontrivial order in $1/\Lambda_{NC}$:

Seiberg,Witten '99; Carroll et.al. '01

$$
\begin{aligned}
S = {} & \frac{i}{2}\bar{\psi}\gamma^{\mu}\overleftrightarrow{D}_{\mu}\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& + \frac{q}{8}\frac{\theta^{\alpha\beta}}{\Lambda_{\rm NC}}\Big[-iF_{\alpha\beta}\bar{\psi}\gamma^{\mu}\overleftrightarrow{D}_{\mu}\psi + 2iF_{\alpha\mu}\bar{\psi}\gamma^{\mu}\overleftrightarrow{D}_{\beta}\psi \\
& \qquad + 2mF_{\alpha\beta}\bar{\psi}\psi - 4F_{\alpha\mu}F_{\beta\nu}F^{\mu\nu} + F_{\alpha\beta}F_{\mu\nu}F^{\mu\nu}\Big]
\end{aligned}
$$

# 4. LIV from topology

Consider spacetime with one compact but large dimension, radius R.

Vacuum fluctuations along this dimension have periodic boundary conditions.

- preferred direction in vacuum
- calculation applied to electrodynamics yields: Klinkhamer '00

$$k^{\mu} A^{\nu} \tilde{F}_{\mu\nu}$$

# Models with approximate Lorentz invariance at low energy

**Example:** *Horava-Lifshitz gravity*

Horava '09

Based on anisotropic scaling:

$$\vec{x} \to b\vec{x}, \qquad t \to b^z t$$

as well as a "detailed balance" condition. The action reads, for z=3:

$$S = \int dt\, d^3x \sqrt{g} N \left\{ \frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4} C_{ij}C^{ij} + \frac{\kappa^2 \mu}{2w^2} \epsilon^{ijk} R_{il} \nabla_j R^l_k \right.$$
$$\left. - \frac{\kappa^2 \mu^2}{8} R_{ij}R^{ij} + \frac{\kappa^2\mu^2}{8(1-3\lambda} \left( \frac{1-4\lambda}{4} R^2 + \Lambda_W R - 3\Lambda_W^2 \right) \right\}$$

with $C^{ij}$ equal to the Cotton tensor

$$C^{ij} = \epsilon^{ikl} \nabla_k \left( R^j_l - \frac{1}{4} R \delta^j_l \right)$$

At short distances, S is dominated by its highest dimension terms.

In this model, the graviton has 2 transverse polarizations with the highly non-relativistic dispersion relation:

$$\omega^2 = \frac{\gamma^4}{4} (\mathbf{k}^2)^3$$

At long distances, relevant deformations by operators of lower dimensions will become important, in addition to the RG flows of the dimensionless couplings. As it turns out, S flows in the IR towards the Einstein-Hilbert action, with the (emergent) light speed given by

$$c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1 - 3\lambda}}$$

and the effective Newton and cosmological constants given by

$$G_N = \frac{\kappa^2}{32\pi c}, \qquad \Lambda = \frac{3}{2}\Lambda_W$$

# Kinematic frameworks

## 1. Modified dispersion relations

Postulate that the Lorentz violating effects modify the usual relativistic dispersion relation

$$E^2 = p^2 + m^2 \qquad \text{by} \qquad E^2 = F(p, m)$$

It is natural to expand

$$E^2 = m^2 + p^2 + m_{\mathrm{pl}} f_i^{(1)} p^i + f_{ij}^{(2)} p^i p^j + \frac{f_{ijk}^{(3)}}{m_{\mathrm{pl}}} p^i p^j p^k + \ldots$$

with dimensionless coefficients $f^{(n)}$.

The coefficients $f^{(n)}$, while arbitrary, are presumably such that Lorentz violation is a small effect. The order n of the first nonzero coefficient depends on the underlying fundamental theory.

Much of the relevant literature assumes rotational invariance, and assumes the dispersion relation

$$E^2 = m^2 + p^2 + m_{pl} f^{(1)} |p| + f^{(2)} p^2 + \frac{f^{(3)}_{ijk}}{m_{pl}} |p|^3 + \ldots$$

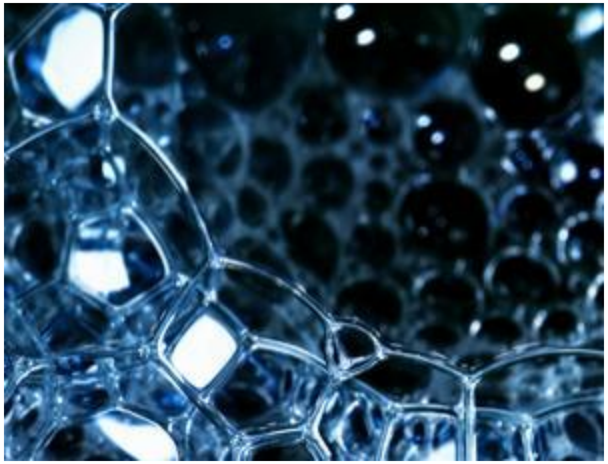
The coefficients depend on the particle species.

It has been pointed out that the terms with odd powers of *p* have problems with coordinate invariance, causality and positivity. Lehnert. ‘04

It has been suggested that Stochastic or foamy spacetime structure can lead to modifications of spacetime structure that modify over time.

Ng, van Dam ’94 ’00, Shiokawa ‘00, Dowker et.al. ‘04

In such frameworks the particle dispersion is taken to fluctuate according to a model-dependent probability distribution.



# 2. Robertson-Mansouri-Sexl framework

Here it is assumed there is a preferred frame with isotropic speed of light. The Lorentz transformation to other frames is generalized to incorporate changes from the conventional boosts:

Robertson '49 Mansouri, Sexl '77

$$
\begin{aligned}
t' &= a^{-1}(t - \vec{\epsilon}\cdot\vec{x}) \\
\vec{x}' &= d^{-1}\vec{x} - (d^{-1} - b^{-1})\frac{\vec{v}(\vec{v}\cdot\vec{x})}{v^2} - a^{-1}\vec{v}t
\end{aligned}
$$

with a, b, c, d, **ε** functions of the relative speed v. Without Lorentz violation and Einstein clock synchronization we have

$$a = b^{-1} = \sqrt{1 - v^2}, \quad d = 1, \quad \vec{\epsilon} = \vec{v}.$$

The RMS framework can be incorporated in the SME.

Modifying the values of the parameters results in a variable speed of light, assuming experiments that use a fixed set of rods and clocks.

The RMS framework can be incorporated in the Standard Model Extension.

## 3. The $c^2$ and THεμ framework

Lagrangian model that considers motion of test particle in EM field. Limiting speed of particles is considered to be 1, but speed of light $c \neq 1$.

Lightman, Lee '73, Will '01

This framework can be incorporated in SME.

# 4. Doubly Special Relativity

Amelino-Camelia ’01, Magueijo, Smolin ‘03

Here it is assumed that the Lorentz transformations act such that c *as well as* an energy scale $E_{DSR}$ are invariant. The physical energy/momentum are taken to be given by

$$E = \frac{\epsilon}{1 + \lambda_{\mathrm{DSR}}\epsilon}, \qquad p = \frac{\pi}{1 + \lambda_{\mathrm{DSR}}\pi}, \qquad \lambda_{\mathrm{DSR}} = E_{\mathrm{DSR}}^{-1}$$

in terms of the pseudo energy/momentum ε, π, which transform normally under Lorentz boosts. The dispersion relation becomes

$$E^2 - p^2 = \frac{m^2(1 - \lambda_{\mathrm{DSR}}E)^2}{(1 - \lambda_{\mathrm{DSR}}m)^2}$$

DSR can be incorporated in the SME. Kostelecky Mewes ‘09

The physical meaning of the quantities E and p, and of DSR itself, has been questioned.

# Effective Field Theory

What is the most suitable dynamical framework for describing LIV?

**Criteria:**

1. *Observer coordinate independence:* physics independent of observer coordinate transformation
2. *Realism:* must incorporate known physics
3. *Generality:* most general possible formulation, to maximize reach

# The Standard Model Extension

Colladay, Kostelecky ‘97

Effective Field Theory incorporating:

1. Standard Model coupled to General Relativity;
2. Any scalar term formed by contracting operators for Lorentz violation with coefficients controlling size of the effects.
3. Possibly additional requirements like
   - gauge invariance,
   - locality,
   - stability,
   - renormalizability.

The SME includes, in principle, terms of any mass dimension (starting at dim 3).

Imposing power counting renormalizability limits one to terms of dimension $\leq 4$. This is usually referred to as the minimal SME (mSME).

The mSME has a finite number of LV parameters, while the number of LV parameters in the full SME is in principle unlimited.

The SME leads not only to breaking of Lorentz symmetry, but also to that of CPT, for about half of its terms.

Example: *free fermion sector of SME:*

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\Gamma^{\mu}\partial_{\mu} - M)\psi \\ M &= m + \not{a} - \not{b}\gamma_5 + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu} \\ \Gamma^{\mu} &= \gamma^{\mu} + c^{\mu\nu}\gamma_{\nu} - d^{\mu\nu}\gamma^{\nu}\gamma_5\end{aligned}$$

A separate set of coefficients exists for every elementary particle.

As the SME is to be considered an effective field theory, one can relax the requirement of renormalizability. This means, that the coefficients of the mSME become generalized to higher mass dimensions.

For instance:

$$
\begin{aligned}
c^{\mu\nu} &\rightarrow \hat{c}^{\mu\nu} \\
&= i^{d-4} \sum_{d=4}^{\infty} c^{(d)\mu\nu\alpha_1 \ldots \alpha_{(d-4)}} \partial_{\alpha_1} \ldots \partial_{\alpha_{(d-4)}}
\end{aligned}
$$

The higher dimensional coefficients are naturally suppressed at low energies.

# Construction of the mSME

## 1. SU(3)*SU(2)*U(1) Standard Model

Leptons: $L_A = \begin{pmatrix} \nu_A \\ l_A \end{pmatrix}_L, \quad R_A = (l_A)_R$

Quarks: $Q_A = \begin{pmatrix} u_A \\ d_A \end{pmatrix}_L, \quad U_A = (u_A)_R, \quad D_A = (d_A)_R$

$$l_A = (e, \mu, \tau), \quad \nu_A = (\nu_e, \nu_\mu, \nu_\tau), \quad u_A = (u, c, t), \quad d_A = (d, s, b).$$

Gauge fields: $G_\mu, \quad W_\mu, \quad B_\mu$

Higgs doublet: $\phi$

Gauge couplings: $g_3, \quad g, \quad g'$

Yukawa couplings: $G_L, \quad G_U, \quad G_D$

$$\mathcal{L}_{\text{lepton}} = \frac{1}{2} i \bar{L}_A \gamma^\mu \overleftrightarrow{D}_\mu L_A + \frac{1}{2} i \bar{R}_A \gamma^\mu \overleftrightarrow{D}_\mu R_A$$

$$\mathcal{L}_{\text{quark}} = \frac{1}{2} i \bar{Q}_A \gamma^\mu \overleftrightarrow{D}_\mu Q_A + \frac{1}{2} i \bar{U}_A \gamma^\mu \overleftrightarrow{D}_\mu U_A + \frac{1}{2} i \bar{D}_A \gamma^\mu \overleftrightarrow{D}_\mu U_A$$

$$\mathcal{L}_{\text{Yukawa}} = -(G_L)_{AB} \bar{L}_A \phi R_B - (G_U)_{AB} \bar{Q}_A \phi^c U_B - (G_D)_{AB} \bar{Q}_A \phi D_B$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \frac{\lambda}{6} (\phi^\dagger \phi)^2$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{2} \mathrm{Tr}(G_{\mu\nu} G^{\mu\nu}) - \frac{1}{2} \mathrm{Tr}(W_{\mu\nu} W^{\mu\nu}) - \frac{1}{4} (B_{\mu\nu} B^{\mu\nu})$$

## 2. mSME Lagrangian

### a. Fermions

$$\mathcal{L}_{\text{lepton}}^{\text{CPT-even}} = \tfrac{1}{2} i (c_L)_{\mu\nu\, AB} \bar{L}_A \gamma^\mu \overleftrightarrow{D}^\nu L_B + \tfrac{1}{2} i (c_R)_{\mu\nu\, AB} \bar{R}_A \gamma^\mu \overleftrightarrow{D}^\nu R_B$$

$$\mathcal{L}_{\text{lepton}}^{\text{CPT-odd}} = -(a_L)_{\mu\, AB} \bar{L}_A \gamma^\mu L_B - (a_R)_{\mu\, AB} \bar{R}_A \gamma^\mu R_B$$

$$\begin{aligned}\mathcal{L}_{\text{quark}}^{\text{CPT-even}} &= \tfrac{1}{2} i (c_Q)_{\mu\nu\, AB} \bar{Q}_A \gamma^\mu \overleftrightarrow{D}^\nu Q_B + \tfrac{1}{2} i (c_U)_{\mu\nu\, AB} \bar{U}_A \gamma^\mu \overleftrightarrow{D}^\nu U_B \\ &\quad + \tfrac{1}{2} i (c_D)_{\mu\nu\, AB} \bar{D}_A \gamma^\mu \overleftrightarrow{D}^\nu D_B \\ \mathcal{L}_{\text{quark}}^{\text{CPT-odd}} &= -(a_Q)_{\mu\, AB} \bar{L}_A \gamma^\mu Q_B - (a_R)_{\mu\, AB} \bar{U}_A \gamma^\mu U_B \\ &\quad - (a_D)_{\mu\, AB} \bar{D}_A \gamma^\mu D_B\end{aligned}$$

## *b. Higgs sector*

$$\mathcal{L}^{\text{CPT-even}}_{\text{Higgs}} = \frac{1}{2}(k_{\phi\phi})^{\mu\nu}(D_\mu\phi)^\dagger D_\nu\phi - \frac{1}{2}(k_{\phi B})^{\mu\nu}\phi^\dagger\phi B_{\mu\nu} - \frac{1}{2}(k_{\phi W})^{\mu\nu}\phi^\dagger W_{\mu\nu}\phi$$

$$\mathcal{L}^{\text{CPT-even}}_{\text{Yukawa}} = -\frac{1}{2}(H_L)_{\mu\nu\,AB}\bar{L}_A\phi\sigma^{\mu\nu}R_B - \frac{1}{2}(H_U)_{\mu\nu\,AB}\bar{Q}_A\phi\sigma^{\mu\nu}U_B - \frac{1}{2}(H_D)_{\mu\nu\,AB}\bar{Q}_A\phi\sigma^{\mu\nu}D_B$$

## *c. Gauge sector*

$$\mathcal{L}^{\text{CPT-even}}_{\text{gauge}} = -\frac{1}{2}(k_G)^{\kappa\lambda\mu\nu}\text{Tr}(G_{\kappa\lambda}G_{\mu\nu}) - \frac{1}{2}(k_W)^{\kappa\lambda\mu\nu}\text{Tr}(W_{\kappa\lambda}W_{\mu\nu}) - \frac{1}{2}(k_B)^{\kappa\lambda\mu\nu}B_{\kappa\lambda}B_{\mu\nu}$$

$$\mathcal{L}^{\text{CPT-odd}}_{\text{gauge}} = (k_3)_\kappa\epsilon^{\kappa\lambda\mu\nu}\text{Tr}(G_\lambda G_{\mu\nu} + \frac{2}{3}ig_3G_\lambda G_\mu G_\nu) + (k_2)_\kappa\epsilon^{\kappa\lambda\mu\nu}\text{Tr}(W_\lambda W_{\mu\nu} + \frac{2}{3}ig_3W_\lambda W_\mu W_\nu) + (k_1)_\kappa\epsilon^{\kappa\lambda\mu\nu}B_\lambda B_{\mu\nu} + (k_0)_\kappa B^\kappa$$

# 3. *Inclusion of Gravity*

## Example: lepton sector

Standard model Lagrangian density coupled to gravity:

$$\mathcal{L}_{\text{lepton}} = \tfrac{1}{2} i e\, e^{\mu}{}_{a} \bar{L}_A \gamma^a \overleftrightarrow{D}_{\mu} L_A + \tfrac{1}{2} i e\, e^{\mu}{}_{a} \bar{R}_A \gamma^a \overleftrightarrow{D}_{\mu} R_A$$

$e^{\mu}{}_{a}$ : vierbein, used to convert local Lorentz indices to spacetime indices: $b_{\mu} = e_{\mu} b_a$

Flat-space LIV sectors can be coupled to gravity using vierbein, for example:

$$\mathcal{L}^{\text{CPT-even}}_{\text{lepton}} = -\tfrac{1}{2} i (c_L)_{\mu\nu\, AB}\, e e^{\mu}{}_{a} \bar{L}_A \gamma^a \overleftrightarrow{D}^{\nu} L_B - \tfrac{1}{2} i (c_R)_{\mu\nu\, AB}\, e e^{\mu}{}_{a} \bar{R}_A \gamma^a \overleftrightarrow{D}^{\nu} R_B$$

## *Pure gravity sector:*

LIV Lagrangian terms are built of the vierbein, spin connection and derivatives. They can be converted to curvature and torsion. Minimal sector:

$$\mathcal{L}^{\mathrm{LV}}_{e,\omega} = e(k_T)^{\lambda\mu\nu} T_{\lambda\mu\nu} + e(k_R)^{\kappa\lambda\mu\nu} R_{\kappa\lambda\mu\nu} + e(k_{TT})^{\alpha\beta\gamma\lambda\mu\nu} T_{\alpha\beta\gamma} T_{\lambda\mu\nu} + e(k_{DT})^{\kappa\lambda\mu\nu} D_\kappa T_{\lambda\mu\nu}$$

Riemannian limit of minimal SME gravity sector:

$$S_{e,\omega,\Lambda} = \frac{1}{2\kappa}\int d^4x\, e\left[(1-u)R - 2\Lambda + s^{\mu\nu}R_{\mu\nu} + t^{\kappa\lambda\mu\nu}R_{\kappa\lambda\mu\nu}\right]$$

# Energy scaling of SME coefficients

The assumption that Lorentz breaking originates at high energy (UV) scale, by spontaneous symmetry breaking or otherwise, makes insight in energy scaling of coefficients desirable.

*Renormalization group studies:*

1. mSME renormalizable (at least) to first order at one loop. Coefficients pertaining to QED run logarithmically: no natural suppression with power of energy scale Kostelecky et.al. '02
   - Scalar field model with Planck scale LV cutoff yields percent-level LIV at low energy

   Collins et.al. '04

   Observed strong bounds on LV ⇨ "naturalness" problem

2. Of dimension 5 operators pertaining to QED and to Standard model. Various types of terms:

   a) Terms that transmute into lower-dimensional terms multiplied by power of UV cutoff: extremely strong bounds. Supersymmetry eliminates most of them.

   b) Terms that grow with energy (UV-enhanced): modification of dispersion relations

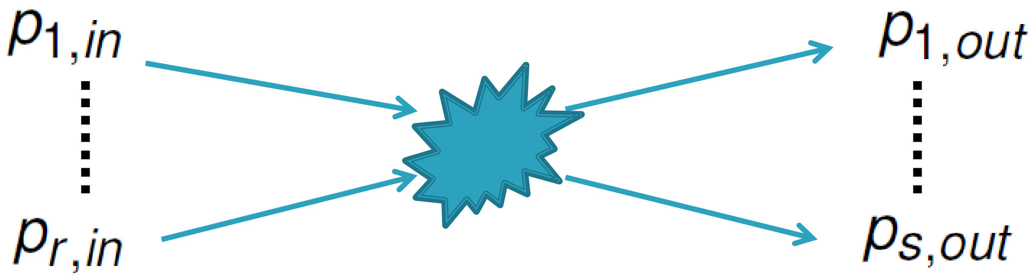
   c) “Soft” (non-enhanced) interactions not growing with energy

Myers,Pospelov ’03; GrootNibbelink, Pospelov ‘05; Bolokhov, Pospelov ‘07

# Phenomenology

## 1. *Free particles: modified dispersion relations*

Modified dispersion relations imply: $E(\vec{p}) = \sqrt{m^2 + \vec{p}^2} + \delta E_{LIV}(\vec{p})$

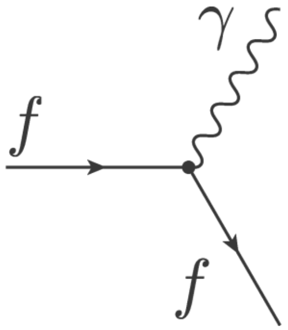



Modified dispersion relation implies shifted reaction thresholds:

- Normally allowed processes may be forbidden
- Normally forbidden processes may be allowed in certain regions of phase space

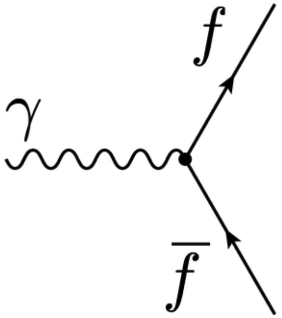
# Examples:

## 1. Vacuum Cherenkov radiation:




Nonobservance in LEP electrons leads to bounds on SME parameters in QED sector . Altschul. '10

## 2. Photon decay:




Nonobservance in Tevatron photons leads to bounds on SME parameters. Hohensee et.al. '09

# 2. Mesons

Meson systems have long provided tests for CP and CPT.

Also provide test for $a_\mu$ coefficients in SME.

Schrödinger equation:

$$i\partial_t \Psi = \Lambda \Psi$$

$\Psi$: 2-component neutral meson/antimeson (K, D, $B_d$, $B_s$) wave function. $\Lambda = M - i\,\Gamma/2$: effective $2 \times 2$ Hamiltonian, with eigenvalues:

$$\lambda_S \equiv m_S - \frac{i}{2}\gamma_S \quad , \qquad \lambda_L \equiv m_L - \frac{i}{2}\gamma_L$$

Can show simple relation with SME coefficients $a_\mu$

$$\Delta\Lambda \approx \beta^\mu \Delta a_\mu \quad , \quad \Delta\Lambda \equiv \Lambda_{11} - \Lambda_{22}, \quad \beta^\mu \equiv \gamma(1, \vec{\beta})$$

It is useful and common to introduce dimensionless parameter ξ parametrizing CPT violation:

$$\xi = \Delta\Lambda / \Delta\lambda \approx 2\delta$$

Note that ξ depends explicitly on meson four-velocity!

Sensitivities obtained:

- $10^{-17}$ to $10^{-20}$ GeV for $\Delta a_\mu$ in K system KLOE '08; KTeV '01
- $10^{-15}$ GeV for $\Delta a_\mu$ in D system FOCUS '02
- $10^{-15}$ GeV for $\Delta a_\mu$ in $B_d$ system BaBar '07

# 3. Neutrinos

SME leads to many possible observable consequences in neutrino sector.

*Example:* Neutrino oscillations caused by Lorentz violation. Yields very precise tests of LIV.

At leading order, LIV in neutrino sector described by effective two-component Hamiltonian acting on neutrino-antineutrino state vector:

$$(h_{\text{eff}})_{ab} = |\vec{p}|\delta_{ab}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2|\vec{p}|}\begin{pmatrix} (\widetilde{m}^2)_{ab} & 0 \\ 0 & (\widetilde{m}^2)^*_{ab} \end{pmatrix}$$
$$+\frac{1}{|\vec{p}|}\begin{pmatrix} [(a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu]_{ab} & -i\sqrt{2}p_\mu(\epsilon_+)_\nu[(g^{\mu\nu\sigma}p_\sigma - H^{\mu\nu})\mathcal{C}]_{ab} \\ i\sqrt{2}p_\mu(\epsilon_+)^*_\nu[(g^{\mu\nu\sigma}p_\sigma + H^{\mu\nu})\mathcal{C}]^*_{ab} & [-(a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu]^*_{ab} \end{pmatrix}$$

## Potential signals:

- Oscillations with unusual energy dependences (oscillation length may grow rather than shrink with energy)

- Anisotropies arising from breakdown of rotational invariance: sidereal variations in observed fluxes

Many bounds on SME parameters in the neutrino sector have been deduced by analysis of LSND, MiniBooNe and MINOS (and other) data.

Models have been proposed that reproduce current observations and may help resolve the LSND anomaly.

# *4. QED sector*

Sharpest laboratory tests in systems with predominant interactions described by QED.

Write QED sector of mSME lagrangian as:

$$\mathcal{L}_{QED} = \mathcal{L}_0 + \mathcal{L}_{int}$$

with $\mathcal{L}_0$ the usual QED lagrangian describing fermions and photons, $\mathcal{L}_{int}$ LIV interactions. For photons + single fermion:

$$\begin{aligned}\mathcal{L}_{int} = &-a_\mu \bar{\psi}\gamma^\mu\psi - b_\mu\bar{\psi}\gamma_5\gamma^\mu\psi + ic_{\mu\nu}\bar{\psi}\gamma^\mu D^\nu\psi \\ &+ id_{\mu\nu}\bar{\psi}\gamma_5\gamma^\mu D^\nu\psi - \tfrac{1}{2}H_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi \\ &- \tfrac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \tfrac{1}{2}(k_{AF})^\kappa\epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu}\end{aligned}$$

follows from Standard Model SME lagrangian upon EW symmetry breaking and mass generation. The SME treats protons & neutrons as fundamental constituents.

Effective Hamiltonian can be constructed using perturbation theory for small LV, such that

$$i\partial_t \chi = \hat{H}\chi$$

Non-relativistic regime: use Foldy-Wouthuysen approach and make field redefinitions; one finds for massive fermion

$$\begin{aligned}\hat{H}_{\text{pert}} = {} & a_\mu \gamma^0 \gamma^\mu - b_\mu \gamma_5 \gamma^0 \gamma^\mu - c_{00} m \gamma^0 - i(c_{0j} + c_{j0}) D^j \\ & + i(c_{00} D_j - c_{jk} D^k) \gamma^0 \gamma^j - d_{j0} m \gamma_5 \gamma^j + i((d_{0j} + d_{j0}) D^j \gamma_5 \\ & + i(d_{00} D_j - d_{jk} D^k) \gamma^0 \gamma_5 \gamma^j + \tfrac{1}{2} H_{\mu\nu} \gamma^0 \sigma^{\mu\nu}\end{aligned}$$

This expression assumes fixed nonrotating axes. Usual convention: sun-centered frame using celestial equatorial coordinates, denoted by uppercase X, Y, Z, T.

Using rotating, earth-fixed laboratory axes implies using an appropriate mapping. For instance, for the combination

$$\tilde{b}_j^e \equiv b_j^e - m d_{j0}^e - \frac{1}{2}\epsilon_{jkl} H_{kl}^e$$

one finds

$$\tilde{b}_1^e = \tilde{b}_X^e \cos\chi \cos\Omega t + \tilde{b}_Y^e \cos\chi \sin\Omega t - \tilde{b}_Z^e \sin\chi$$
$$\tilde{b}_2^e = -\tilde{b}_X^e \sin\Omega t + \tilde{b}_Y^e \cos\Omega t$$
$$\tilde{b}_3^e = \tilde{b}_X^e \sin\chi \cos\Omega t + \tilde{b}_Y^e \sin\chi \sin\Omega t + \tilde{b}_Z^e \cos\chi$$

The earth's rotation axes is along $Z$, the angle $\chi$ is between the $j=3$ lab axis and $Z$ axis.

$\Omega$ is the angular frequency corresponding to a sidereal day:

$$\Omega \approx \frac{2\pi}{23\text{h } 56\text{m}}$$

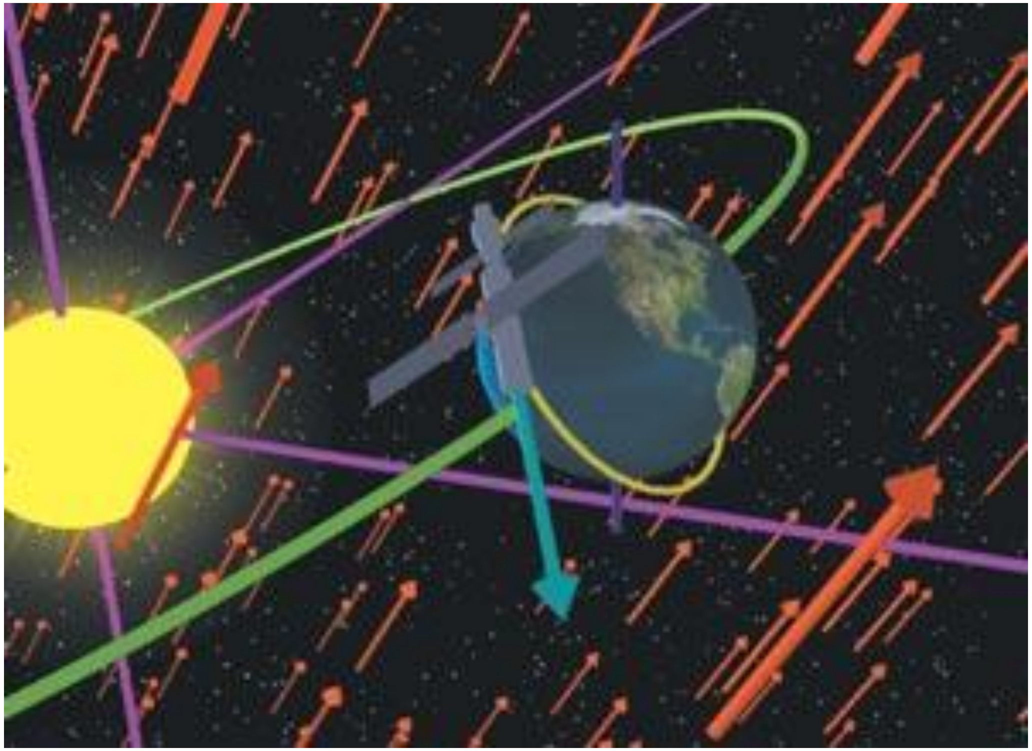


Illustration: sidereal variations

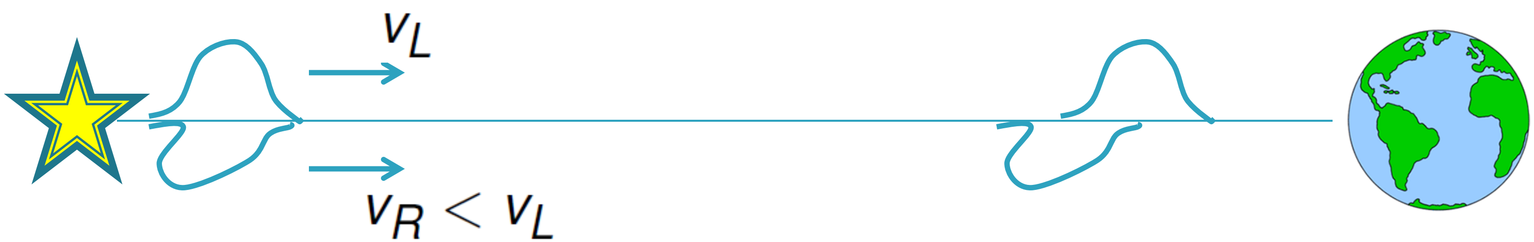
(illustration: PhysicsWorld)

# 5. *Astrophysical tests*

Some of the most stringest bounds on LIV parameters come from astrophysical tests.

*Example:* Spectropolarimetry of cosmological sources

LIV vacuum can lead to birefringence:




Polarization at emission $\neq$ observed polarization

Cosmological sources with known polarization can be used to verify model-dependent polarization changes

# Experimental tests

Sensitivity to Lorentz/CPT violation stems from ability to detect anomalous energy shifts in various systems. Experiments most effective when all energy levels are scrutinized for possible anomalous shifts.

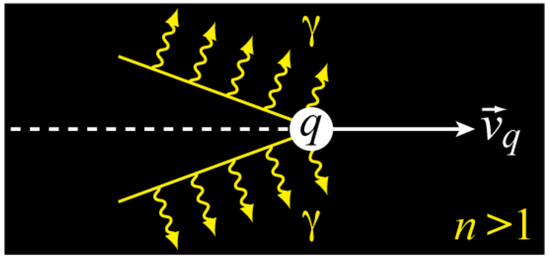
In past decade a number of new Lorentz/CPT signatures have been identified in addition to known tests.

*Two types of lab tests*:

1. Lorentz tests: sidereal time variations in energy levels
2. CPT tests: difference in particle/antiparticle energy levels

# Photons

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \frac{1}{2}(k_{AF})^{\kappa}\epsilon_{\kappa\lambda\mu\nu}A^{\lambda}F^{\mu\nu}$$




## $k_{AF}$ term:

- CPT violating
- timelike $k_{AF}$ gives rise to potential instability
- Leads to *birefringence:* cosmological sources with known polarization permit searching for energy-dependent polarization changes either from distant sources or from CMB ⇨ $|(k_{AF})^{\mu}| \leq 10^{-42}$ GeV Carrol, Field'97
- Gives rise to vacuum Cerenkov radiation Lehnert, R. P. '04

# $k_F$ term:

- CPT even
- Gives rise to vacuum Cerenkov radiation

Use analogy with dielectrics: Kostelecky, Mewes '02

$$\begin{pmatrix} \vec{D} \\ \vec{H} \end{pmatrix} = \begin{pmatrix} 1+\kappa_{DE} & \kappa_{DB} \\ \kappa_{HE} & 1+\kappa_{HB} \end{pmatrix} \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix}$$

$$(\kappa_{DE})^{jk} = -2(k_F)^{0j0k}$$
$$(\kappa_{HB})^{jk} = \frac{1}{2}\epsilon^{jkq}\epsilon^{krs}(k_F)^{pqrs}$$
$$(\kappa_{DB})^{jk} = -(\kappa_{HE})^{jk} = \epsilon^{kpq}(k_F)^{0jpq}$$

Modified Maxwell equations:

$$\vec{\nabla} \times \vec{H} - \partial_t \vec{D} = 0$$
$$\vec{\nabla} \cdot \vec{D} = 0$$
$$\vec{\nabla} \times \vec{E} + \partial_t \vec{B} = 0$$
$$\vec{\nabla} \cdot \vec{B} = 0$$

Constraints on the linear combinations:

$$\tilde{\kappa}_{tr} = \frac{1}{3}(\kappa_{DE})^{kk}$$

$$(\tilde{\kappa}_{e+})^{jk} = \frac{1}{2}(\kappa_{DE} + \kappa_{HB})^{jk}$$

$$(\tilde{\kappa}_{e-})^{jk} = \frac{1}{2}(\kappa_{DE} - \kappa_{HB})^{jk} - \delta^{jk}\tilde{\kappa}_{tr}$$

$$(\tilde{\kappa}_{o+})^{jk} = \frac{1}{2}(\kappa_{DB} + \kappa_{HE})^{jk}$$

$$(\tilde{\kappa}_{o-})^{jk} = \frac{1}{2}(\kappa_{DB} - \kappa_{HE})^{jk}$$

$\tilde{\kappa}_{tr}$ : bound by a variety of lab experiments. Best current bounds from LEP data up to $O(10^{-15})$ Hohensee et.al '09, Altschul '09

Best astrophysical bound (absence of vacuum Cerenkov radiation in cosmic rays) yields $O(10^{-19})$ bound Klinkhamer, Risse '09

$\tilde{\kappa}_{e-}$ and $\tilde{\kappa}_{o+}$ (8 coefficients): bounded by cavity experiments up to $O(10^{-17})$ and $O(10^{-12})$, studying sidereal effects in optical or microwave cavities Herrmann et.al. '07; Mueller et.al. '07; ....... and by an experiment studying sidereal effects in Compton edge photons.

Bocquet et.al. '10

Best astrophysical bound (absence of vacuum Cerenkov radiation in cosmic rays) yields $O(10^{-18})$ bound Klinkhamer, Risse '09

$\tilde{\kappa}_{e+}$ and $\tilde{\kappa}_{o-}$ (10 coefficients): lead to birefringence: strongly bound by cosmological measurements ⇨ $|(k_F)^{\alpha\beta\gamma\delta}| \leq 2 \times 10^{-32}$

Kostelecky, Mewes '01, '06

Complete updated list of bounds:

V.A. Kostelecky and N. Russell, arXiv: 0801.0287 [hep-ph]

# Penning traps

Used recently in experiments with electrons and positrons.

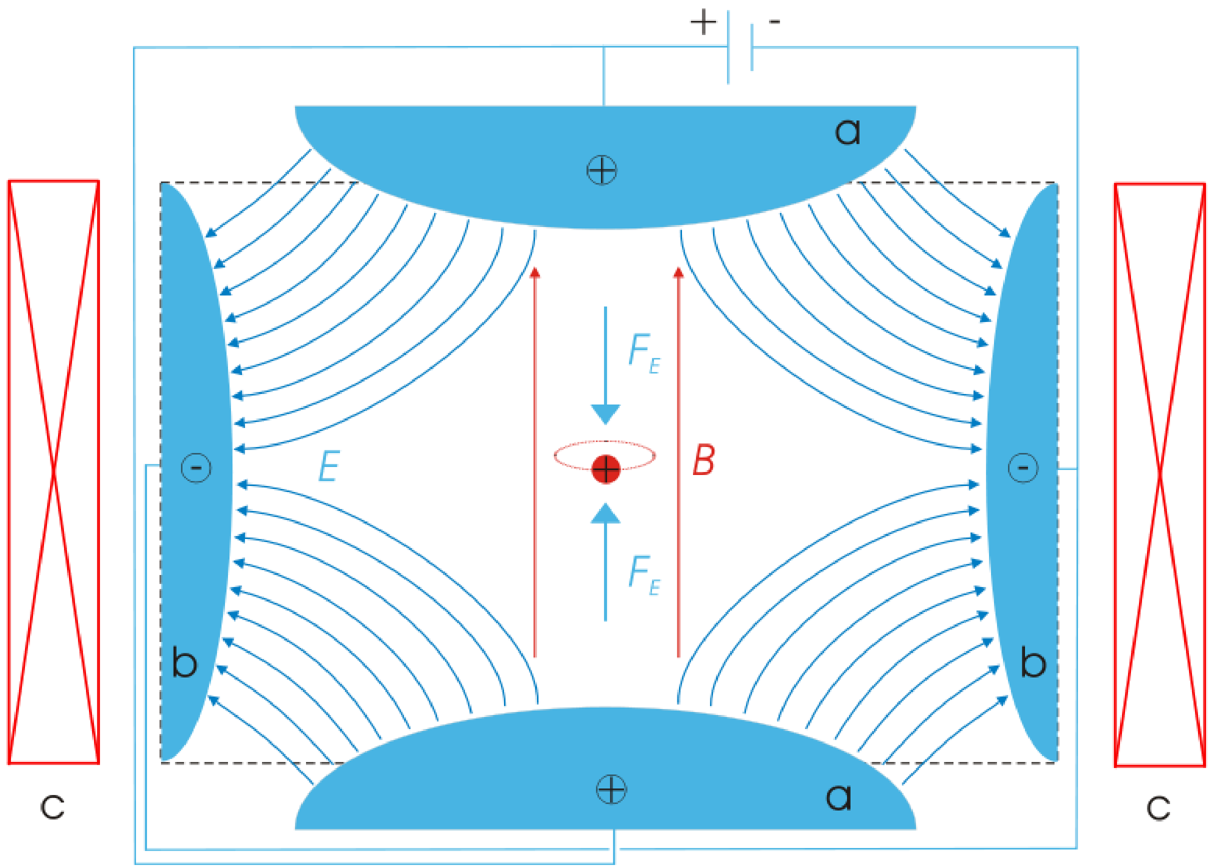
High precision measurements of anomaly frequency $\omega_a$ and cyclotron frequency $\omega_c$ of trapped particles. One can show at lowest order in the mSME Bluhm et.al. '98

$$\omega_c^{e^-} = (1 - c_{00}^e - c_{XX}^e - c_{YY}^e)\omega_c^{e,0}$$

$$\omega_a^{e^\pm} = \omega_a^{e,0} \pm 2b_Z^e + 2d_{Z0}^e m_e + 2H_{XY}^e$$

Comparison of anomaly frequency for electron / positron yields the bound : Dehmelt et.al. '99

$$|\vec{b}^e| \lesssim 3 \times 10^{-25}\,\mathrm{GeV}$$



+
-
a
⊕
b
⊖
E
$F_E$
B
$F_E$
c

# Clock comparison experiments

Classic Hughes-Drever experiments: spectroscopic tests of isotropy of mass and space. Hughes et.al. '60; Drever '61

- Typically use hyperfine or Zeeman transitions

- Test of Lorentz/CPT in neutron using $^3He/^{129}Xe$ gas maser yields $|\tilde{b}^n_J| \lesssim 10^{-31}$ GeV for $J = X, Y$ Bear et.al. '00

- Bound on Lorentz/CPT in proton sector using H maser

$$|\tilde{b}^p_Z| \lesssim 2 \times 10^{-27} \text{ GeV}$$

Phillips et.al '01

- Advantageous to carry out clock comparison experiments in space:
    - Additional sensitivity to J = T, Z components
    - Much faster (16 times) data acquisition
    - New types of signals

# Hydrogen and antihydrogen

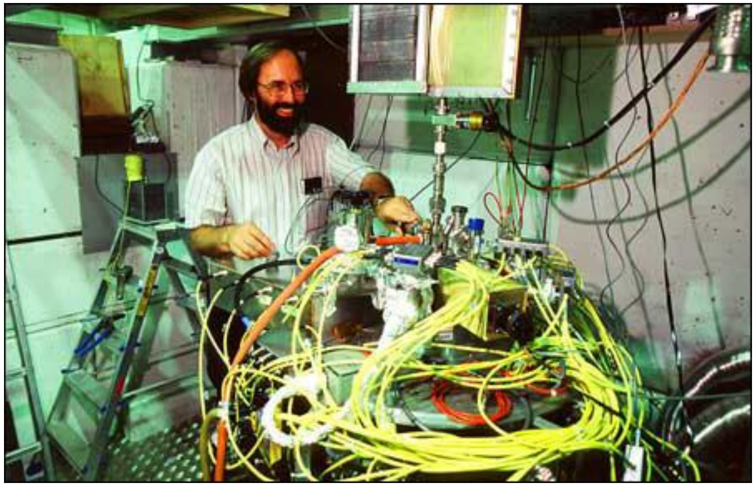
(Anti)Hydrogen is simplest (anti)atom: possibility for clean Lorentz/CPT tests involving protons/electrons.

Current most stringent Lorentz/CPT test for proton: hydrogen maser using double resonance technique searching for sidereal Zeeman variations: $|\tilde{b}_J^p| \lesssim 10^{-27}$ GeV Phillips et.al. '01

Experiments underway at CERN:

• ALPHA and ATRAP: intend to make high precision spectroscopic measurements of 1S-2S transitions in H and anti-H: frequency comparison at level of $10^{-18}$. Inclusion of magnetic field provides leading order sensitivity to Lorentz/CPT.

• ASACUSA: intend to analyze ground state Zeeman hyperfine transitions: direct stringent CPT test



G. Gabrielse (Harvard)

ALPHA and ASACUSA teams (CERN)

# Muon experiments

Muonium experiments: frequencies of ground-state Zeeman hyperfine transitions in strong magnetic fields, looking for sidereal variations: $|\tilde{b}^{\mu}_{J}| \leq 2 \times 10^{-23}\,\mathrm{GeV}$ Hughes et.al. '01

Analysis of relativistic g−2 experiments using positive muons with large boost parameter at Brookhaven yields: Bennett et.al. '08

$$\tilde{b}^{\mu^+}_{\perp} = \sqrt{(\tilde{b}^{\mu^+}_{X})^2 + (\tilde{b}^{\mu^+}_{Y})^2} < 1.4 \times 10^{-24}\,\mathrm{GeV}$$

$$\tilde{b}^{\mu^-}_{\perp} = \sqrt{(\tilde{b}^{\mu^-}_{X})^2 + (\tilde{b}^{\mu^-}_{Y})^2} < 2.6 \times 10^{-24}\,\mathrm{GeV}$$

$$\tilde{b}^{\mu^\pm}_{J} \equiv \pm\frac{b_J}{\gamma} + m_\mu d_{J0} + \tfrac{1}{2}\epsilon_{JKL}H_{KL}, \qquad (J = X, Y, Z)$$

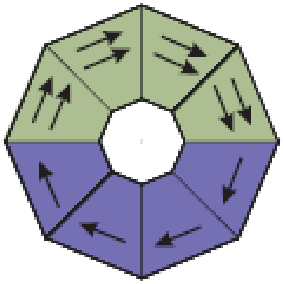
# Spin polarized torsion pendulum

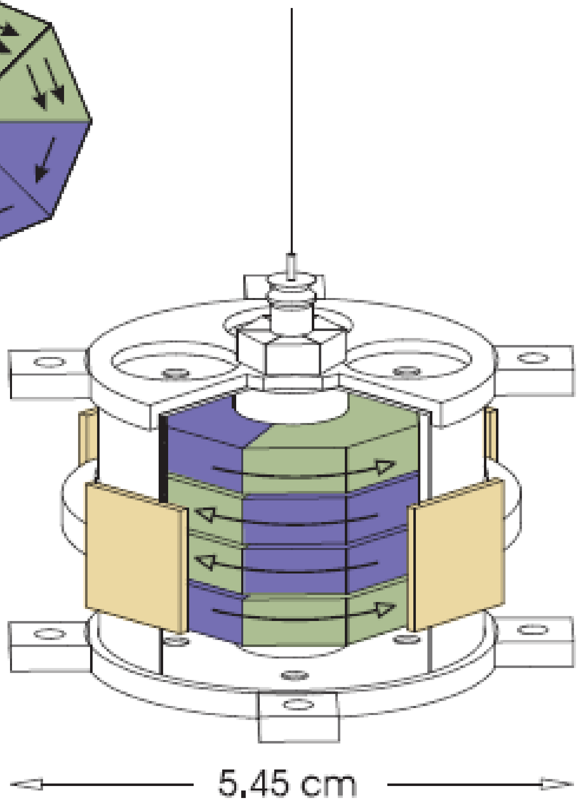
Experiments with spin polarized torsion pendula at the University of Washington provide current sharpest bounds on Lorentz/CPT violation in electron sector: huge number of electron spins ($8 \times 10^{22}$) with negligible magnetic field.

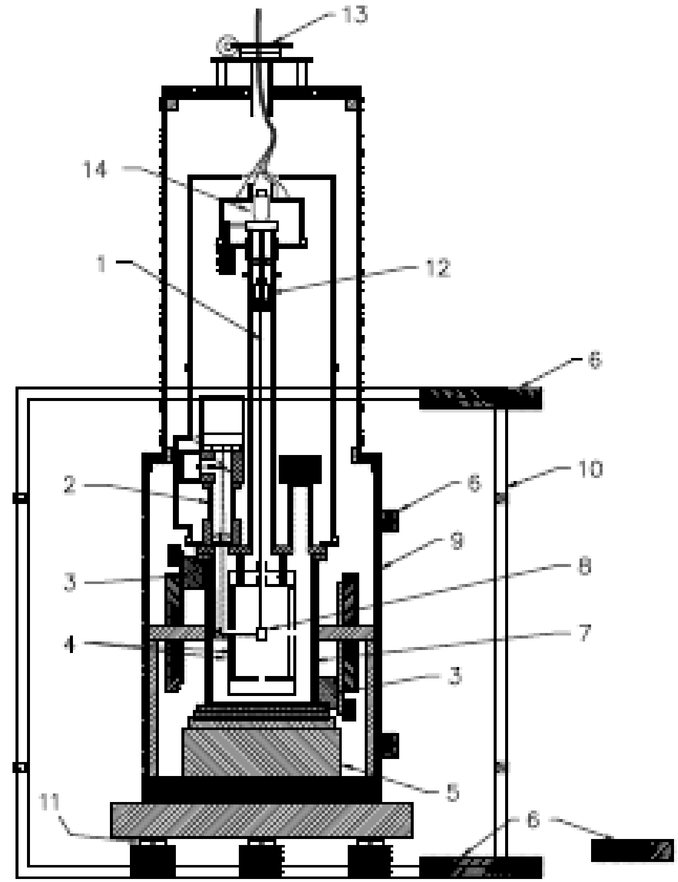
Obtained bounds:

Heckel et.al. '08

$$|\tilde{b}^e_J| \lesssim 10^{-31}\,\text{GeV} \;\; \text{for} \;\; J = X, Y, \qquad |\tilde{b}^e_Z| \lesssim 10^{-30}\,\text{GeV}$$




5.45 cm


13
14
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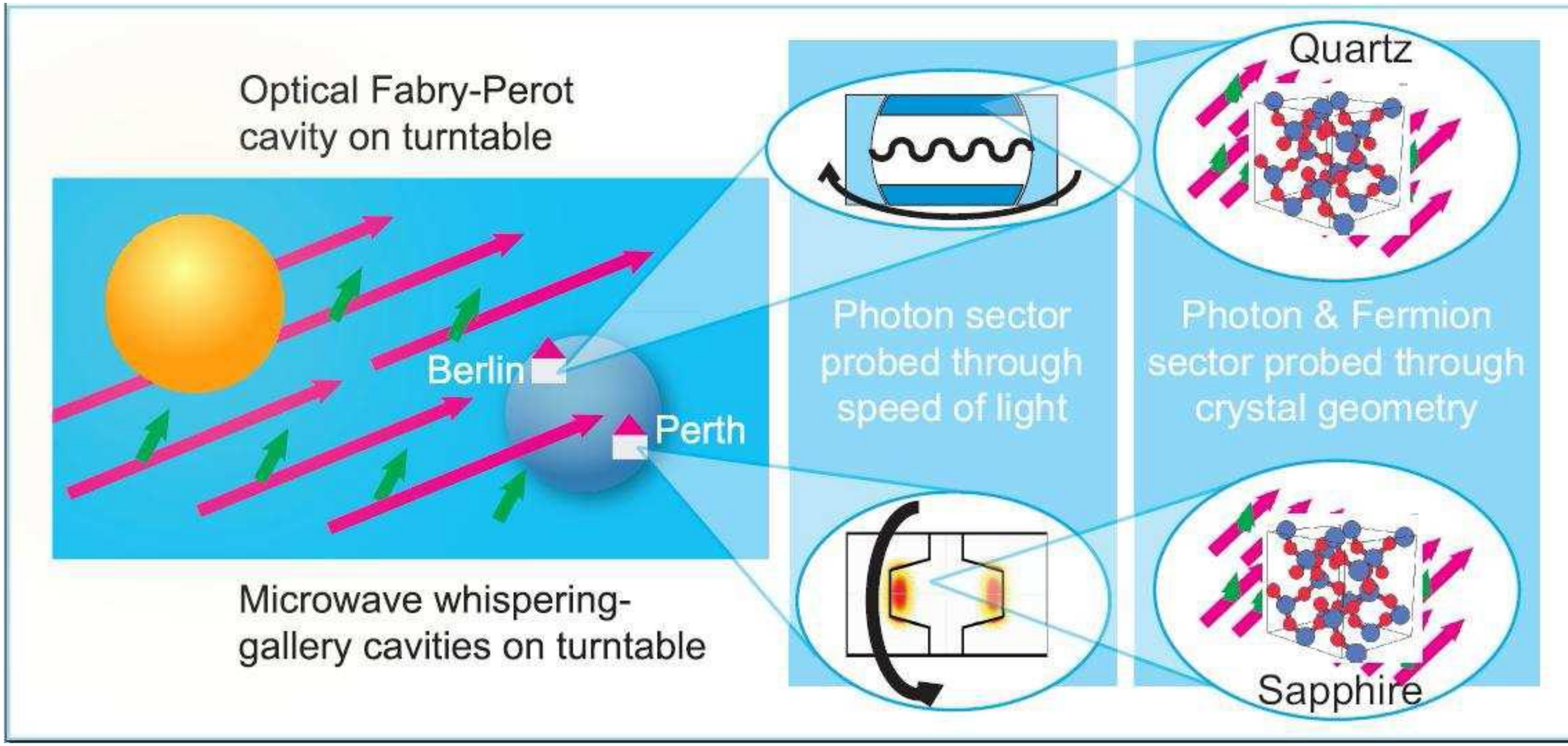
11
6

# Optical resonators

Relativity tests have been done based on data from Michelson-Morley experiments using optical (Fabry-Perot) or microwave resonators.

They provide the most stringent laboratory bounds on a variety of mSME coefficients in the electron and photon sector:

$$|c^{e}_{IJ}| \lesssim 10^{-16}$$
$$|\tilde{\kappa}^{IJ}_{o+}| \lesssim 10^{-11} - 10^{-16}$$
$$|\tilde{\kappa}^{IJ}_{e-}| \lesssim 10^{-15} - 10^{-17}$$


Optical Fabry-Perot
cavity on turntable
Berlin
Perth
Microwave whispering-
gallery cavities on turntable
Photon sector
probed through
speed of light
Photon & Fermion
sector probed through
crystal geometry
Quartz
Sapphire

# Bounds on higher dimensional LV operators

Much less work has been done on bounding higher dimensional operators.

Laboratory experiments are concerned with low energies, thus best suited for mSME. Higher-dimension operators scale with energy, giving an a-priori advantage to astrophysical tests.

## *Higher dimensional operators in photon sector*

Consider SME photon Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F^{\kappa\lambda}(\hat{k}_F)_{\kappa\lambda\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon_{\kappa\lambda\mu\nu}A^{\lambda}(\hat{k}_{AF})^{\kappa}F^{\mu\nu}$$

The full SME photon Lagrangian can be obtained by expanding

Kostelecky, Mewes '07

$$(\hat{k}_F)^{\kappa\lambda\mu\nu} = \sum_{d=2,4,6\ldots} (k_F^{(d)})^{\kappa\lambda\mu\nu\alpha_1\ldots\alpha_{(d-4)}} \partial_{\alpha_1} \ldots \partial_{\alpha_{(d-4)}}$$

$$(\hat{k}_{AF})_\kappa = \sum_{d=1,3,5\ldots} (k_{AF}^{(d)})_\kappa{}^{\alpha_1\ldots\alpha_{(d-3)}} \partial_{\alpha_1} \ldots \partial_{\alpha_{(d-3)}}$$

$(\hat{k}_{AF})_\kappa, (\hat{k}_F)^{\kappa\lambda\mu\nu}$ : constant coefficients of dimension 4−d

Bounds can be obtained from analyzing:

1. polarization changes due to birefringence in CMB radiation:
   - various $k_{AF}^{(5)}$ coefficients: O($10^{-19}$ GeV$^{-1}$)
   - various $k_F^{(6)}$ coefficients: O($10^{-9}$ GeV$^{-2}$) Kostelecky, Mewes '07
2. Dispersion relations (time of flight differences) in GRB's
   - various $k_F^{(6)}$ coefficients: O($10^{-22}$ GeV$^{-2}$)

Kostelecky, Mewes '07; MAGIC, Ellis et.al. '07

# Conclusions

- Fundamental theories may allow for Lorentz-invariance violation (LIV), typically at the Planck scale.
  - Makes sense to look for LIV as a testing ground for new physics
- Many testing schemes exists, kinematical as well as effective field theories
- The Standard Model Extension offers a comprehensive parametrization of Lorentz and CPT violation at low energy, allowing for systematic experimental testing.