Unitary Violation in a Model with A4 Flavor Symmetry

David Vanegas Forero[1]

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[1] In collaboration with S. Morisi, J. C. Romão and J. W. F. Valle

WHAT IS THE MODEL?

Defining the Model

The Babu-Ma-Valle model (BMV) (Phys.Lett.B **552** 207 (2003))
is based on the leptonic ***Higgs doublet model of ν-masses with an A4 flavor symmetry***, with two improvements:

- A4 is broken at very high scale.
- ***Supersymmetry is added*** with explicit soft breaking terms which also breaks A_4.

where A_4 is a discrete non-Abelian group of even permutations of 4 objects, it has $\underline{1}, \underline{1}', \underline{1}''$ and $\underline{3}$ irreducible representation (irrep) and it is the smallest finite group with triplet irrep.

The Decomposition property of the product is:

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}$$

The usual quark $\hat{Q}_i = (\hat{u}_i, \hat{d}_i)$, lepton $\hat{L}_i = (\hat{\nu}_i, \hat{e}_i)$, and Higgs $\hat{\phi}_i$ transforms under A_4 as follows:

	$\hat{Q}$	$\hat{L}$	$\hat{u}_1^c, \hat{d}_1^c, \hat{e}_1^c$	$\hat{u}_2^c, \hat{d}_2^c, \hat{e}_2^c$	$\hat{u}_3^c, \hat{d}_3^c, \hat{e}_3^c$	$\hat{\phi}_{1,2}$
A_4	3	3	1	1'	1"	1
Z_3	1	1	ω^2	ω^2	ω^2	1

Then the following heavy quark, lepton, and Higgs **superfields are added**:

	$\hat{U}$	$\hat{U}^c$	$\hat{D}$	$\hat{D}^c$	$\hat{E}$	$\hat{E}^c$	$\hat{N}^c$	$\hat{\chi}$
A_4	3	3	3	3	3	3	3	3
Z_3	1	1	1	1	1	1	1	ω

which are all $SU(2)$ singlets and with $\omega = \exp i2\pi/3$.

Introduction to the BMV model

Superpotential

$$\hat{W} =$$

$$M_U \hat{U}_i \hat{U}_i^c + f_u \hat{Q}_i \hat{U}_i^c \hat{\phi}_2 + h^u_{ijk} \hat{U}_i \hat{u}_i^c \hat{\chi}_k$$

$$+ M_D \hat{D}_i \hat{D}_i^c + f_d \hat{Q}_i \hat{D}_i^c \hat{\phi}_2 + h^d_{ijk} \hat{D}_i \hat{d}_i^c \hat{\chi}_k$$

$$+ M_E \hat{E}_i \hat{E}_i^c + f_e \hat{L}_i \hat{E}_i^c \hat{\phi}_1 + h^e_{ijk} \hat{E}_i \hat{e}_j^c \hat{\chi}_k$$

$$+ \frac{1}{2} M_N \hat{N}_i \hat{N}_i^c + f_N \hat{L}_i \hat{N}_i^c \hat{\phi}_2 + \mu \hat{\phi}_1 \hat{\phi}_2$$

$$+ \frac{1}{2} M_\chi \hat{\chi}_i \hat{\chi}_i + h_\chi \hat{\chi}_1 \hat{\chi}_2 \hat{\chi}_3$$

$$\mathcal{M}_{eE} = \begin{bmatrix} 0 & f_e v_1 \\ U_\omega \, \mathrm{Diag}\{\sqrt{3} h_i^e u\} & M_E \end{bmatrix}$$

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle = u, \quad v_1 = \langle \phi_1^0 \rangle$$

$$\mathcal{M}_{\nu N} = \begin{bmatrix} 0 & f_N v_2 U_\omega \\ f_N v_2 U_\omega^T & M_N \end{bmatrix}$$

$$v_2 = \langle \phi_2^0 \rangle$$

Introduction to the BMV model

Superpotential

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$$\mathcal{M}_{eE} = \begin{bmatrix} 0 & 0 & 0 & f_e v_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & f_e v_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_e v_1 \\ h_1^e\, u & h_2^e\, u & h_3^e\, u & M_E & 0 & 0 \\ h_1^e\, u & h_2^e\, u\,\omega & h_3^e\, u\,\omega^2 & 0 & M_E & 0 \\ h_1^e\, u & h_2^e\, u\,\omega^2 & h_3^e\, u\,\omega & 0 & 0 & M_E \end{bmatrix} \equiv \begin{bmatrix} 0 & X_1^D \\ X_2 & Y^D \end{bmatrix}$$

After block diagonalization:

$$\mathcal{M}_e \approx U_\omega \, \mathrm{Diag}\{m_i\} U_\omega^\dagger$$

where, for $f_e v_1 \ll h_i\, u \ll M_E$:

$$\tilde{m}_i^2 \simeq \frac{3 f_e^2 v_1^2}{M_E^2} \frac{h_i^{e\,2} u^2}{1 + 3(h_i^e u)^2 / M_E^2} \qquad U_\omega = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

As coming from the $d = 5$ Weinberg operator, the see-saw masses are:

$$\frac{f_N^2}{M_N}\lambda_{ij}\,\nu_i\nu_j\,\phi_2^0\phi_2^0 \quad \Rightarrow \quad m_{ij} = \lambda_{ij}\, m_0$$

where $\lambda_{ee} = \lambda_{\mu\tau} = \lambda_{\tau\mu} = 1$ and all the others λ's zero, which is valid at some *large scale*.

At *low scale*, fixed to first order, we have:

$$\lambda_{ij} = \begin{bmatrix} 1 + 2\delta_{ee} & \delta_{e\mu} + \delta_{e\tau} & \delta_{e\mu} + \delta_{e\tau} \\ \delta_{e\mu} + \delta_{e\tau} & 2\delta_{\mu\tau} & 1 + \delta_{\mu\mu} + \delta_{\tau\tau} \\ \delta_{e\mu} + \delta_{e\tau} & 1 + \delta_{\mu\mu} + \delta_{\tau\tau} & 2\delta_{\mu\tau} \end{bmatrix}$$

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At *low scale*, fixed to first order, we have:

$$\lambda_{ij} = \begin{bmatrix} 1+\delta_0+2\delta+2\delta' & \delta'' & \delta'' \\ \delta'' & \delta & 1+\delta_0+\delta \\ \delta'' & 1+\delta_0+\delta & \delta \end{bmatrix}$$

$\mu - \tau$ invariant matrix is diagonalized by:

$$U_\nu(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & -1/\sqrt{2} \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & 1/\sqrt{2} \end{bmatrix},$$

with the eigenvalues:

$$\lambda_1 = 1 + \delta_0 + 2\delta + \delta' - \sqrt{\delta'^2 + 2\delta''^2}$$
$$\lambda_2 = 1 + \delta_0 + 2\delta + \delta' + \sqrt{\delta'^2 + 2\delta''^2}$$
$$\lambda_3 = -1 - \delta_0$$

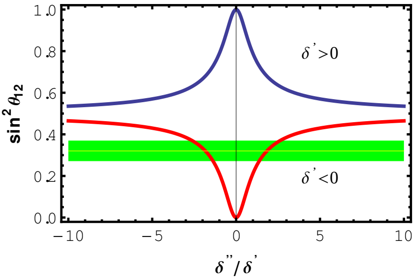
Lepton mixing: $K = U_\omega^\dagger . U_\nu(\theta)$

Neutrino prediction in the BMV

From the lepton mixing, we obtain:

$$\tan^2\theta_{12} = \frac{\delta''^2}{\delta''^2 + \delta'^2 - \delta'\sqrt{\delta'^2 + 2\delta''^2}}$$

$$\sin^2\theta_{13} = 0 \quad \tan^2\theta_{23} = 1 \Rightarrow \text{maximal}$$



and for the masses:

$$\Delta m^2_{31} \simeq \Delta m^2_{32} \simeq 4\delta m_0^2$$
$$\Delta m^2_{21} \simeq 4\sqrt{\delta'^2 + 2\delta''^2}\, m_0^2$$

Current neutrino parameters values

The 3×3 mixing matrix:

$V_1 = \omega(\eta_{23})\,\omega(\eta_{13})\,\omega(\eta_{12})$ with $\eta_{ab} = |\eta_{ab}| \exp(i\phi_{ab})$,

Which is equivalent to PDG parameterization:

$$V_1(\theta_{ij}) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\, e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\, e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\, e^{-i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\, e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\, e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \mathrm{diag}\left(1, \exp\left(i\frac{\alpha_{21}}{2}\right), \exp\left(i\frac{\alpha_{31}}{2}\right)\right)$$

used in oscillation analysis.

The oscillation parameters at 3σ C.L are:

D. V. Forero et al. (PRD **86** (2012))

Parameter	Δm^2_{21} $[10^{-5} eV^2]$	Δm^2_{31} $[10^{-3} eV^2]$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$
NH	7.12 – 8.20	2.31 – 2.74	0.27 – 0.37	0.36 – 0.68	0.017 – 0.033
IH		–(2.21 – 2.64)		0.37 – 0.67	

HOW ACOMODATE THE ν DATA?

UNITARITY VIOLATION IDEA

Relaxing some conditions

We know $K = U_\omega^\dagger U_\nu(\theta)$

where $U_\omega^\dagger U_\omega = I$.

But if M_E scale tends to TeV scale then, in principle, U_ω could be no longer unitary therefore, LFV would be enhanced.

So, the new strategy could be:

Diagonalize the Charged lepton sector $U_{eE}^\dagger \left(\mathcal{M}_{eE}\mathcal{M}_{eE}^\dagger\right) U_{eE}$ adding NLO terms, using the Schechter-Valle procedure
Schechter, Valle (PRD **25** (1982))

At the end, $U_\omega^c = (U_{eE})_{3\times 3}$ where c means adding NLO terms.

Introduction to the BMV model

Charge Sector: Dirac mass matrix linking (e_i, E_i) to (e_i^c, E_i^c)

$$\mathcal{M}_{eE} = \begin{bmatrix} 0 & 0 & 0 & f_e v_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & f_e v_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_e v_1 \\ h_1^e\, u & h_2^e\, u & h_3^e\, u & M_E & 0 & 0 \\ h_1^e\, u & h_2^e\, u\,\omega & h_3^e\, u\,\omega^2 & 0 & M_E & 0 \\ h_1^e\, u & h_2^e\, u\,\omega^2 & h_3^e\, u\,\omega & 0 & 0 & M_E \end{bmatrix} \equiv \begin{bmatrix} 0 & X_1^D \\ X_2 & Y^D \end{bmatrix}$$

$f_e v_1 \ll h_i\, u \ll M_E$: $\rightarrow$ $f_e v_1 \ll h_i\, u < M_E$

$$\mathcal{M}_e \approx U_\omega \,\mathrm{Diag}\{m_i\} U_\omega^\dagger \rightarrow U_\omega^c \,\mathrm{Diag}\{m_i\} U_\omega^{c\,\dagger}$$

and, $U_\omega \rightarrow$ $U_\omega^c = \delta . U_\omega$

$$\tilde{m}_i^2 \simeq \frac{3 f_e^2 v_1^2}{M_E^2} \frac{h_i^{e\,2} u^2}{1 + 3 (h_i^e u)^2 / M_E^2} + \mathrm{NLO}$$

$$\mathcal{M}_{eE} = \begin{bmatrix} 0 & 0 & 0 & f_e v_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & f_e v_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_e v_1 \\ h_1^e\, u & h_2^e\, u & h_3^e\, u & M_E & 0 & 0 \\ h_1^e\, u & h_2^e\, u\,\omega & h_3^e\, u\,\omega^2 & 0 & M_E & 0 \\ h_1^e\, u & h_2^e\, u\,\omega^2 & h_3^e\, u\,\omega & 0 & 0 & M_E \end{bmatrix} \equiv \begin{bmatrix} 0 & X_1^D \\ X_2 & Y^D \end{bmatrix}$$

$f_e v_1 \ll h_i\, u \ll M_E$: $\rightarrow$ $f_e v_1 \ll h_i\, u < M_E$

$$\mathcal{M}_e \approx U_\omega\, \text{Diag}\{m_i\} U_\omega^\dagger \rightarrow U_\omega^c\, \text{Diag}\{m_i\} U_\omega^{c\,\dagger}$$

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UNITARITY VIOLATION IDEA

Using the Schechter-Valle procedure Schechter, Valle (PRD **25** (1982))

$$U = \mathcal{U}\cdot V = \exp(iH)\cdot V \qquad H = \begin{pmatrix} 0 & S \\ S^\dagger & 0 \end{pmatrix}, \qquad V = \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix}.$$

for a general Hermitian matrix:

$$M = \begin{bmatrix} m_1 & m_2 \\ m_2^\dagger & m_3 \end{bmatrix} \Rightarrow \begin{bmatrix} (f_e v_1)^2\, I & M_E\, f_e v_1\, I \\ M_E\, f_e v_1\, I & U_\omega(\mathrm{Diag}\{3(h_i^e u)^2 + M_E^2\})U_\omega^\dagger \end{bmatrix}$$

and

$$i\,S = -m_2(m_1 - m_3)^{-1} \Rightarrow U_\omega\,\mathrm{diag}\{-M_E\, f_e v_1[(f_e v_1)^2 - 3(h_i^e u)^2 - M_E^2]^{-1}\}\, U_\omega^\dagger$$

$$S \sim U_\omega\, D\, U_\omega^\dagger \Rightarrow SS^\dagger = S^\dagger S = U_\omega\, D^2\, U_\omega^\dagger$$

$$m_{eff}^{NLO} \sim \sum_i m_i C\{S, S^\dagger, SS^\dagger\}_i$$

So, at all orders U_ω diagonalizes the effective charge lepton matrix.
Therefore the lepton mixing does not change! A_4 protect it to all orders

A SECOND TRY

Modifying the BMV model

Adding a scale singlet

Then we are going to include a singlet ($1'$ of A_4) extra scalar ζ because Its vev breaks the residual Z_2 symmetry ($\mu-\tau$ origin of U_ν):

$$+\zeta_{ii}E_iE_i^c$$

where we will parametrize the flavon scale as $\langle\zeta\rangle = \beta\, M_E$.

The new mass matrix structure:

$$\hat{Y}_D = M_E \times I + \beta M_E \times \text{Diag}\{1,\omega,\omega^2\}$$

$$M = \begin{bmatrix} m_1 & m_2 \\ m_2^\dagger & m_3 \end{bmatrix} \Rightarrow \begin{bmatrix} (f_e v_1)^2\, I & f_e v_1\, \hat{Y}_D^\dagger \\ f_e v_1\, \hat{Y}_D & U_\omega(\text{Diag}\{3(h_i^e u)^2\})U_\omega^\dagger + \hat{Y}_D\hat{Y}_D^\dagger \end{bmatrix}$$

where $U_\omega^\dagger \hat{Y}_D \hat{Y}_D^\dagger U_\omega \neq \text{Diag}\{y_i^2\}$:

$$\hat{Y}_D\hat{Y}_D^\dagger = M_E^2 \times$$
$$\times\, \text{Diag}\{(1+\beta)(1+\beta^*),(1+\beta\omega)(1+(\beta\omega)^*),(1+\beta\omega^2)(1+\beta^*(\omega^*)^2)\}$$

New neutrino prediction

Re-writing the lepton mixing $K = (U^c_\omega)^\dagger . U_\nu(\theta)$:

$$U^c_\omega = \delta . U_\omega \Rightarrow K = (U^\dagger_\omega . \delta^\dagger) . U_\nu(\theta) = U^\dagger_\omega \, [\delta^\dagger . U_\nu(\theta)]$$

$$\tan\theta_{12} = |K_{1,2}(\theta)|/|K_{1,1}(\theta)|$$
$$\sin\theta_{13} = |K_{1,3}(\theta)|$$
$$\tan\theta_{23} = |K_{2,3}(\theta)|/|K_{3,3}(\theta)|$$

What we expect is:

$$\sin^2\theta_{13} \neq 0$$
$$\tan^2\theta_{23} \sim 1 \Rightarrow \text{slightly non-maximal}$$

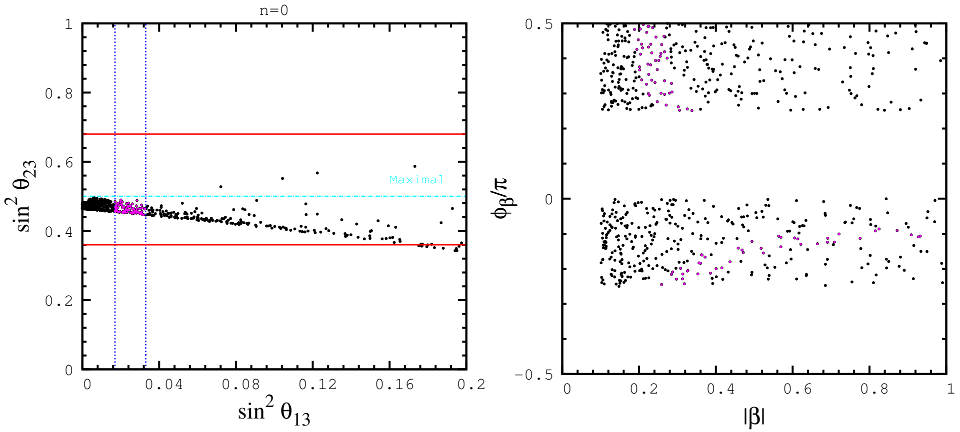
and for the masses:

$$\Delta m^2_{31} \simeq \Delta m^2_{32} \simeq 4\delta m^2_0$$
$$\Delta m^2_{21} \simeq 4\sqrt{\delta'^2 + 2\delta''^2} m^2_0$$

PUTTING SOME NUMBERS

Numerical results (preliminary)

With $0 \leq \sin^2\theta \leq 1$ and $\sin^2\theta_{12}$ at $3\,\sigma$ range, we obtained:



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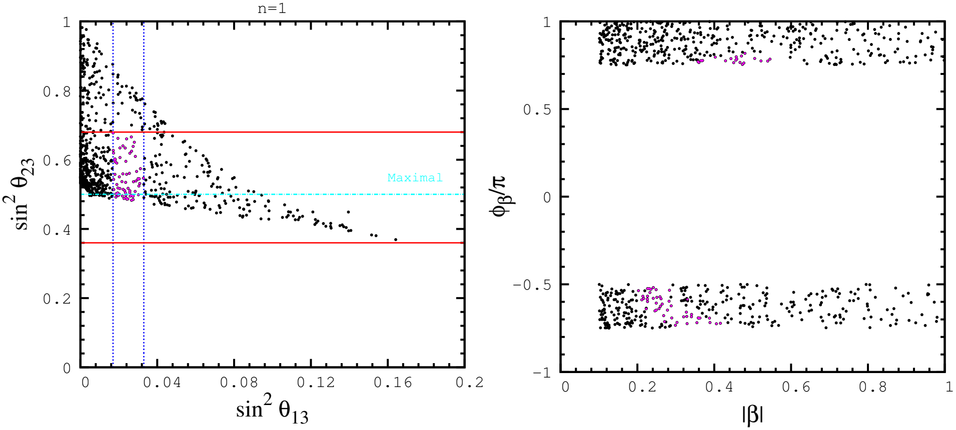

Conclusions

- We have showed adding NLO terms in the diagonalization of charged sector does not change the lepton mixing pattern.
- To break the remnant symmetry, we have introduced a scalar singlet. That result in a good description of the current neutrino mixing angles at 3 σ for some values of θ and β.
- We still have to check if the corrections in the neutrino sector (δ' and δ'') are possible of being obtained in some specific model..
- These results are preliminary. We still have to check if they are compatible with lepton flavor violating bounds (work in progress).

THANKS!!!