Higgs Potential **V** of an **S**$_3$ Model or **V**itaminas y **S**abores

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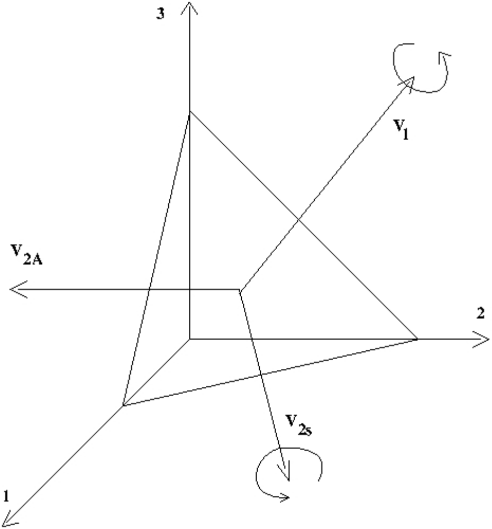
How do we choose a flavour symmetry?

- Find the smallest possible flavour symmetry suggested by the data
- Follow it to the end

The S_3 symmetry group: permutations of 3 objects.



<u>Permutations</u> <u>Rotations</u>

$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \Longleftrightarrow$ a rotation of 120° around the invariant vector $\mathbf{V}_1$

$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \Longleftrightarrow$ a rotation of 180° around the invariant vector $\mathbf{V}_{2S}$

The irreps of the group are:

- 1 dimension: $\mathbf{1}_A$, $\mathbf{1}_S$
- 2 dimensions: $\mathbf{2}$

The direct products between irreps are:

- $\mathbf{1}_S \otimes \mathbf{1}_S = \mathbf{1}_S$
- $\mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_S$
- $\mathbf{1}_A \otimes \mathbf{1}_S = \mathbf{1}_A$
- $\mathbf{1}_S \otimes \mathbf{2} = \mathbf{2}$
- $\mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}$
- $\mathbf{2} \otimes \mathbf{2} = \mathbf{2} \oplus \mathbf{1}_S \oplus \mathbf{1}_A$

The tensor product of two doublets:

$$\mathbf{p_D} = \begin{pmatrix} p_{D1} \\ p_{D2} \end{pmatrix} \qquad \textit{and} \qquad \mathbf{q}_D = \begin{pmatrix} q_{D1} \\ q_{D2} \end{pmatrix}$$

we have two singlets, r_S and r_A, and one doublet $\mathbf{r}_D$, where:

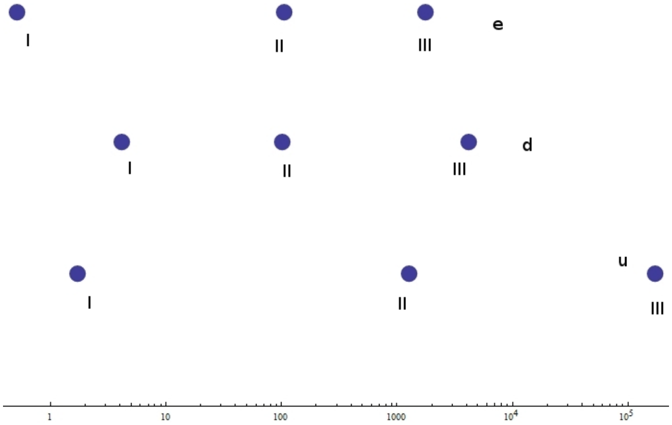
$r_S = p_{D1}q_{D1} + p_{D2}q_{D2}$ is invariant, $r_A = p_{D1}q_{D2} - p_{D2}q_{D1}$ is not invariant

and

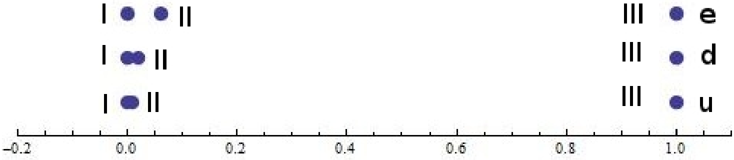
$$\mathbf{r}_D = \begin{pmatrix} p_{D1}q_{D2} + p_{D2}q_{D1} \\ p_{D1}q_{D1} - p_{D2}q_{D2} \end{pmatrix}$$

is invariant.

Logarithmic plot of fundamental known fermion masses



Plot of fundamental fermion mass ratios



Fundamental fermions normalized by the heaviest of each type

Suggests $\mathbf{2} \oplus \mathbf{1}$ structure

The Lagrangian $\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}$

$$\begin{aligned}
\mathcal{L}_{Y_D} &= -Y_1^d \overline{Q}_I H_S d_{IR} - Y_3^d \overline{Q}_3 H_S d_{3R} \\
&\quad -Y_2^d [\, \overline{Q}_I \kappa_{IJ} H_1 d_{JR} + \overline{Q}_I \eta_{IJ} H_2 d_{JR} \,] \\
&\quad -Y_4^d \overline{Q}_3 H_I d_{IR} - Y_5^d \overline{Q}_I H_I d_{3R} + \text{ h.c.}, \\
\mathcal{L}_{Y_U} &= -Y_1^u \overline{Q}_I (i\sigma_2) H_S^* u_{IR} - Y_3^u \overline{Q}_3 (i\sigma_2) H_S^* u_{3R} \\
&\quad -Y_2^u [\, \overline{Q}_I \kappa_{IJ} (i\sigma_2) H_1^* u_{JR} + \eta \overline{Q}_I \eta_{IJ} (i\sigma_2) H_2^* u_{JR} \,] \\
&\quad -Y_4^u \overline{Q}_3 (i\sigma_2) H_I^* u_{IR} - Y_5^u \overline{Q}_I (i\sigma_2) H_I^* u_{3R} + \text{ h.c.}, \\
\mathcal{L}_{Y_E} &= -Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} \\
&\quad -Y_2^e [\, \overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR} \,] \\
&\quad -Y_4^e \overline{L}_3 H_I e_{IR} - Y_5^e \overline{L}_I H_I e_{3R} + \text{ h.c.}, \\
\mathcal{L}_{Y_\nu} &= -Y_1^\nu \overline{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (i\sigma_2) H_S^* \nu_{3R} \\
&\quad -Y_2^\nu [\, \overline{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR} \,] \\
&\quad -Y_4^\nu \overline{L}_3 (i\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \overline{L}_I (i\sigma_2) H_I^* \nu_{3R} + \text{ h.c.},
\end{aligned}$$

and

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Quarks

Numerical study of quarks showed compatibility with data (Kubo, Mondragón, Mondragón, Rodríguez-Jáuregui, 2003)

FCNC's in quark sector are suppressed (Teshima, 2012)

A general study of the parameterization of the quark mass matrices and numerical analysis with recent data shows very good agreement with data (F. González, A. Mondragón, U. Saldaña, L. Velasco, 2012)

Leptons

- In the leptonic sector we add a Z_2 symmetry
- FCNC's are strongly suppressed by the $S_3 \times Z_2$ symmetry and the strong mass hierarchy of the charged leptons
- Predictions for neutrino masses and mixings
- S_3 gives $\neq \Theta_{13}$

A. Mondragón, M. Mondragón, E. Peinado, 2007,2008

- Compatible with recent data

A. Mondragón, M. Mondragón, F. González, 2012

Multi-Higgs models and flavour symmetries

- Interesting work has been done on stability of multi-Higgs models with flavour symmetries
- Also in their properties concerning CP violation
- Of immediate interest to us: adding discrete symmetries can imply continuous symmetries ⇒ Goldstone bosons

Lavoura et al, 1994; Barroso et al; 2004, 2006, 2007, Branco et al, 2005; Ferreira et al, 2005, 2010,2011; Pilaftsis 2011

The Higgs potential in the S_3 flavour model

Some references of works with an S_3 invariant Higgs potential...

- S. Pakvasa and H. Sugawara, Phys. Lett. 73B, 61 (1978)
- E. Derman, Phys. Rev. D19, 317 (1979)
- D. Wyler, Phys. Rev. D19, 330 (1979)
- R. Yahalom, Phys. Rev. D29, 536 (1984)
- Y. Koide, Phys. Rev. D60, 077301 (1999)
- J. Kubo et al, Phys. Rev. D70, 036007 (2004)
- S. Chen et al, Phys. Rev. D70, 073008 (2004)
- O. Félix-Beltrán, M.M., et al, J.Phys.Conf.Ser. 171, 012028 (2009)
- D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)
- G. Bhattacharyya et al, Phys. Rev. D83, 011701 (2011)
- There are many more, I apologize for those not included.

In what sense are we asking: Which is the **most general** S_3-invariant Higgs potential?

- It has the **highest** level of flavour symmetry.
- It has the **highest** arbitrariness without breaking the flavour symmetry.
- Crucial to phenomenology ⇒ consistency is essential

Two essential things to work it out were:

The tensorial products between irreps:

To carefully carry the weak ($SU(2)_L$) index.

Follow the symmetry...

1. Find out all the **l.i.** S_3-invariant terms for **2** and **4** scalar fields.

$n = 2$:

- $\mathbf{1_S} \otimes \mathbf{1_S}$
- $[\mathbf{2} \otimes \mathbf{2}]_\mathbf{S}$

$n = 4$:

- $\mathbf{1_S} \otimes \mathbf{1_S} \otimes \mathbf{1_S} \otimes \mathbf{1_S}$
- $[(\mathbf{1_S} \otimes \mathbf{2}) \otimes (\mathbf{1_S} \otimes \mathbf{2})]_\mathbf{S}$
- $[(\mathbf{1_S} \otimes \mathbf{2}) \otimes (\mathbf{2} \otimes \mathbf{2})_\mathbf{2}]_\mathbf{S}$
- $(\mathbf{2} \otimes \mathbf{2})_\mathbf{A} \otimes (\mathbf{2} \otimes \mathbf{2})_\mathbf{A}$
- $(\mathbf{2} \otimes \mathbf{2})_\mathbf{S} \otimes (\mathbf{2} \otimes \mathbf{2})_\mathbf{S}$
- $[(\mathbf{2} \otimes \mathbf{2})_\mathbf{2} \otimes (\mathbf{2} \otimes \mathbf{2})_\mathbf{2}]_\mathbf{S}$

2. Take an explicit convention for the whole theory (Yukawa Lagrangian and Higgs potential) of where to place the symmetric and antisymmetric doublet components.

$$H_D = \begin{pmatrix} H_{DA} \\ H_{DS} \end{pmatrix}$$

$$\begin{aligned}(f_{DA}, f_{DS})^T \otimes (g_{DA}, g_{DS})^T = {} & \frac{1}{\sqrt{2}}(f_{DA}g_{DA} + f_{DS}g_{DS})_{\mathbf{1}_S} \\ & \oplus \frac{1}{\sqrt{2}}(f_{DA}g_{DS} - f_{DS}g_{DA})_{\mathbf{1}_A}\end{aligned}$$

$$\oplus \frac{1}{\sqrt{2}} \begin{pmatrix} f_{DA}g_{DS} + f_{DS}g_{DA} \\ f_{DA}g_{DA} - f_{DS}g_{DS} \end{pmatrix}_{\mathbf{2}}$$

3. For each S_3-invariant term make all the different independent weak indices contractions.

4. Assign the same self-coupling parameter for each different contraction of the same S_3-invariant term.

We label the three Higgs doublets H_1, H_2 and H_S in terms of their real and imaginary parts as

$$H_1 = \left(\begin{array}{c} \phi_1 + i\phi_2 \\ \phi_7 + i\phi_{10} \end{array} \right), \; H_2 = \left(\begin{array}{c} \phi_3 + i\phi_4 \\ \phi_8 + i\phi_{11} \end{array} \right), \; H_S = \left(\begin{array}{c} \phi_5 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{array} \right),$$

where S is the flavour index for the S_3 Higgs field singlet.

The potential is:

$$\begin{aligned} V \;=\;& \mu_D^2\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) + \mu_S^2\left(H_S^\dagger H_S\right) + a\left(H_S^\dagger H_S\right)^2 \\ &+ b\left(H_S^\dagger H_S\right)\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) + c\left(H_1^\dagger H_1 + H_2^\dagger H_2\right)^2 \\ &+ d\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)^2 + e f_{ijk}\left(\left(H_S^\dagger H_i\right)\left(H_j^\dagger H_k\right) + h.c.\right) \\ &+ f\left\{\left(H_S^\dagger H_1\right)\left(H_1^\dagger H_S\right) + \left(H_S^\dagger H_2\right)\left(H_2^\dagger H_S\right)\right\} \\ &+ g\left\{\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \left(H_1^\dagger H_2 + H_2^\dagger H_1\right)^2\right\} \\ &+ h\Big\{\left(H_S^\dagger H_1\right)\left(H_S^\dagger H_1\right) + \left(H_S^\dagger H_2\right)\left(H_S^\dagger H_2\right) \\ &+ \left(H_1^\dagger H_S\right)\left(H_1^\dagger H_S\right) + \left(H_2^\dagger H_S\right)\left(H_2^\dagger H_S\right)\Big\}, \end{aligned}$$

where $f_{112} = f_{121} = f_{211} = -f_{222} = 1$; 1,2 indices of the flavour doublets

We define

$$\begin{aligned} x_1 &= H_1^\dagger H_1, & x_4 &= \mathcal{R}\left(H_1^\dagger H_2\right), & x_7 &= \mathcal{I}\left(H_1^\dagger H_2\right), \\ x_2 &= H_2^\dagger H_2, & x_5 &= \mathcal{R}\left(H_2^\dagger H_S\right), & x_8 &= \mathcal{I}\left(H_1^\dagger H_S\right), \\ x_3 &= H_S^\dagger H_S, & x_6 &= \mathcal{R}\left(H_1^\dagger H_S\right), & x_9 &= \mathcal{I}\left(H_2^\dagger H_S\right) \quad , \end{aligned}$$

then the potential is

$$\begin{aligned} V \;=\; & \mu_D^2 (x_1 + x_2) + \mu_S^2 x_3 + a x_3^2 + b (x_1 + x_2) x_3 + c (x_1 + x_2)^2 \\ & -4 d x_7^2 + 2e\left[(x_1 - x_2) x_6 + 2 x_4 x_5\right] + f\left(x_5^2 + x_6^2 + x_8^2 + x_9^2\right) \\ & +g\left[(x_1 + x_2)^2 + 4 x_4^2\right] + 2h\left(x_5^2 + x_6^2 - x_8^2 - x_9^2\right). \end{aligned}$$

Potential with $e = 0$:

$$\begin{aligned} V &= \mu_D^2 (x_1 + x_2) + \mu_S^2 x_3 + a x_3^2 + b (x_1 + x_2) x_3 + c (x_1 + x_2)^2 \\ &\quad -4 d x_7^2 + f \left(x_5^2 + x_6^2 + x_8^2 + x_9^2\right) \\ &\qquad + g \left[(x_1 + x_2)^2 + 4 x_4^2\right] + 2h \left(x_5^2 + x_6^2 - x_8^2 - x_9^2\right) . \end{aligned}$$

Has an accidental $S2'$ symmetry (Pakvasa and Sugawara 1978).

The minimum has a rotational symmetry in the v_1, v_2 plane, around v_3.

Has an extra Goldstone boson

Most S_3 symmetric potential

$$
\begin{aligned}
V_{H_S\oplus H_D} = \mu_0^2 x_3 + \mu_1^2(x_1 + x_2) + a x_3^2 + b x_3(x_1 + x_2) + c(x_1 + x_2 \\
-4 d x_7^2 + g[(x_1 - x_2)^2 + 4x_4^2] + f(x_5^2 + x_6^2 + x_8^2 + x_9^2) + 2h(x_5^2 + x_6^2 - x_8^2 - x \\
+2e[2x_4 x_6 + x_5(x_1 - x_2
\end{aligned}
$$

At the price of loosing arbitrariness in $SU(2)$

probably not a good idea...

Stationary points

The potential has three types of stationary points

- The normal minimum with the following field configuration:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_i = 0, \quad i \neq 7, 8, 9$$

- The stationary point which breaks electric charge, here two of the charged fields ϕ acquire non zero vev's :

$$\phi_7 = v'_1, \phi_8 = v'_2, \phi_9 = v'_3, \phi_1 = \alpha, \phi_3 = \beta,$$

- The *CP* breaking minimum, where two imaginary components of the neutral fields ϕ acquire non zero vev's.

$$\phi_7 = v''_1, \phi_8 = v''_2, \phi_9 = v''_3, \phi_{10} = \delta, \phi_{11} = \gamma,$$

We analyze here only the normal minimum of the most general V:

$$\phi_7 = v_1,\ \phi_8 = v_2,\ \phi_9 = v_3,\ \phi_i = 0, \quad i \neq 7, 8, 9$$

Then

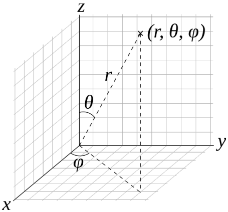
$$\begin{aligned}
0 &= \left[\mu_1^2 + (b+f+2h)\, v_3^2 + 2\,(c+g)\left(v_1^2 + v_2^2\right)\right] v_1 + 6 e v_1 v_2 v_3, \\
0 &= \left[\mu_1^2 + (b+f+2h)\, v_3^2 + 2\,(c+g)\left(v_1^2 + v_2^2\right)\right] v_2 + 3e(v_1^2 - v_2^2) v_3, \\
0 &= \left[\mu_0^2 + (b+f+2h)\left(v_1^2 + v_2^2\right) + 2 a v_3^2\right] 2 v_3 + 2e(3 v_1^2 - v_2^2) v_2.
\end{aligned}$$

v_1 and v_2 correspond to the Higgs flavour doublet, v_3 to the singlet

From the first two equations:

$$v_1^2 = 3v_2^2 \Rightarrow \tan^2\phi = 1/3$$

and the third is a cubic equation on v_3^2 in terms of the self couplings.
We express v_1, v_2 and v_3 in spherical coordinates, to simplify the analysis:



$$v_1 = r\sin\theta\cos\phi$$
$$v_2 = r\sin\theta\sin\phi$$
$$v_3 = r\cos\theta$$

θ parametrizes whether we have a minimum with:
$\sin\theta = 0$ one vev different from zero
$\cos\theta = 0$ two vev's different from zero
or the three vev's different from zero, the solution we will analyze

$v_1^2 + v_2^2 + v_3^2 = v^2$ and $\tan^2\phi = 1/3$ imply $\phi = \pi/3$, thus:

$$v_1 = \frac{v\sin\theta}{2}$$
$$v_2 = \frac{\sqrt{3}v\sin\theta}{2}$$
$$v_3 = v\cos\theta$$

$\cos\theta$ function of the potential parameters

The minimization conditions imply:

$$\begin{aligned}\mu_1^2 &= -8v_2^2(c+g) - 6ev_3v_2 - v_3^2(b+f+2h)\\ \mu_0^2 &= \frac{2}{v_3}(-4ev_2^3 - 2v_3v_2^2(b+f+2h) - av_3^3)\end{aligned}$$

Mass matrix for the neutral Higgses:

$$\mathbf{M}_S^2 = v^2 \begin{pmatrix} 9(c+g)\sin^2\theta & \frac{3\sqrt{3}}{2}(2(c+g)\sin^2\theta + 3\sqrt{3}e\sin\theta\cos\theta) & \frac{3\sqrt{3}}{2}(3e\sin^2\theta + \sqrt{3}b\sin\theta\cos\theta) \\ \frac{3\sqrt{3}}{2}(2(c+g)\sin^2\theta + 3\sqrt{3}e\sin\theta\cos\theta) & 3((c+g)\sin^2\theta - 3\sqrt{3}e\sin\theta\cos\theta) & \frac{3}{2}(3e\sin^2\theta + \sqrt{3}b\sin\theta\cos\theta) \\ \frac{3\sqrt{3}}{2}(3e\sin^2\theta + \sqrt{3}b\sin\theta\cos\theta) & \frac{3}{2}(3e\sin^2\theta + \sqrt{3}b\sin\theta\cos\theta) & -\frac{3\sqrt{3}}{8}\sin^2\theta\tan\theta + 2a\cos^2\theta \end{pmatrix}$$

$$\begin{aligned} m^2_{H^0_1} &= -ev^2|\sin(\theta)\cos(\theta)| \\ m^2_{H^0_{2,3}} &= S \pm T \end{aligned}$$

where S and T functions of the parameters, $\cos\theta$ and $\tan\theta$

Mass matrix for the charged Higgses:

$$\mathbf{M}_C^2 = \sqrt{3}/2v^2 \sin\theta\cos\theta \begin{pmatrix} -2\left(g\sqrt{3}\tan\theta + e\right) & \sqrt{3}\left(2g\sqrt{3}\tan\theta + e\right) & 3\tan\theta e \\ \sqrt{3}\left(2b\sqrt{3}\tan\theta + e\right) & -2\left(g\sqrt{3}\tan\theta + e\right) & \sqrt{3}\tan\theta e \\ 3\tan\theta e & \sqrt{3}\tan\theta e & -3\tan^2\theta e \end{pmatrix}$$

two identical matrices, with the following eigenvalues

$$\begin{aligned} m^2_{G^\pm} &= 0 \\ m^2_{H_1^\pm} &= -8gv^2\sin^2\theta + \frac{5}{9}m^2_{H_1^0} \\ m^2_{H_2^\pm} &= -2ev^2|\tan\theta| \end{aligned} \qquad (1)$$

Mass matrix for the pseudoscalar Higgses:

$$\mathbf{M}_P^2 = \frac{\sqrt{3}v^2 \sin\theta\cos\theta}{2} \cdot$$

$$\begin{pmatrix} -2\left((d+g)\sqrt{3}\tan\theta + e + \frac{\sqrt{3}}{2}h\tan\theta\right) & \sqrt{3}(2(d+g)\sqrt{3}\tan\theta + e) & \sqrt{3}(e\sqrt{3}\tan\theta + 2h) \\ 3(2(d+g)\tan\theta + e) & -6\sqrt{3}(d+g)\tan\theta - 4e - 2\sqrt{3}h\tan\theta & (e\sqrt{3}\tan\theta + 2h) \\ \sqrt{3}(e\sqrt{3}\tan\theta + 2h) & (e\sqrt{3}\tan\theta + 2h) & -3\tan^2\theta - 4\sqrt{3}h\tan\theta \end{pmatrix}$$

with the pseudoscalar Higgs mass eigenvalues given as

$$\begin{aligned} m^2_{G^0} &= 0 \\ m^2_{A_1^0} &= m^2_{H_1^\pm} - 8v^2\left(d\sin^2\theta + h\cos^2\theta\right) \\ m^2_{A_2^0} &= m^2_{H_2^\pm} - 8hv^2 \end{aligned}$$

- After the electroweak symmetry breaking we have the following massive Higgses:
- 4 charged ones
- 3 neutral ones
- 2 neutral pseudoscalar ones
- 3 Goldstone bosons to give mass to the $W^{\pm}$ and Z
- No extra unwanted Goldstone bosons

- We look at the normal minimum, i.e. no CP or charge breaking
- We analyze the most general potential: highest degree of symmetries plus highest degree of arbitriness
- For the case $e = 0$, i.e. no mixing between the singlet and doublet Higgs: We find a rotational symmetry at the minimum and an extra Goldstone boson, accidental S_2 symmetry (consistent with Sugawara and Pakvasa)
- The case with $e \neq 0$ (most general) gives a mixing between the three vev's

  seems consistent with more general quark mass matrices analysis
- We derive the mass matrices and find the eigenvalues

  crucial for phenomenology
- No extra Goldstone bosons
- Details of the minima have to be analyzed in detail

  (work in progress)

Conclusions

- The permutational symmetry S_3 with extended Higgs sector accomodates very well the quark and lepton masses, reducing the number of free parameters
- Allows a “unified” treatment of quark, lepton and Higgs sectors
- Possible to find analytical expressions for mixing matrices in terms of masses
- Gives predictions in the neutrino sector
  mixing angles in terms of masses
  in particular $\Theta_{13} \neq 0$
  and consistent with experimental data

Conclusions

- Further predictions will come from the Higgs sector
- Essential to define *consistently* the potential
- In our case: maximum degree of symmetry without loosing generality
- The normal minimum, without mixing of singlet and doublet, has an accidental S_2 symmetry
- Mixing of doublet and singlet appears consistent with more general analysis of quark masses
- No extra Goldstone bosons
- Look at the constraints that are imposed on the vev's and couplings from internal consistency and experiment
- Leptogenesis (with Arturo Alvarez) possible and consistent with above