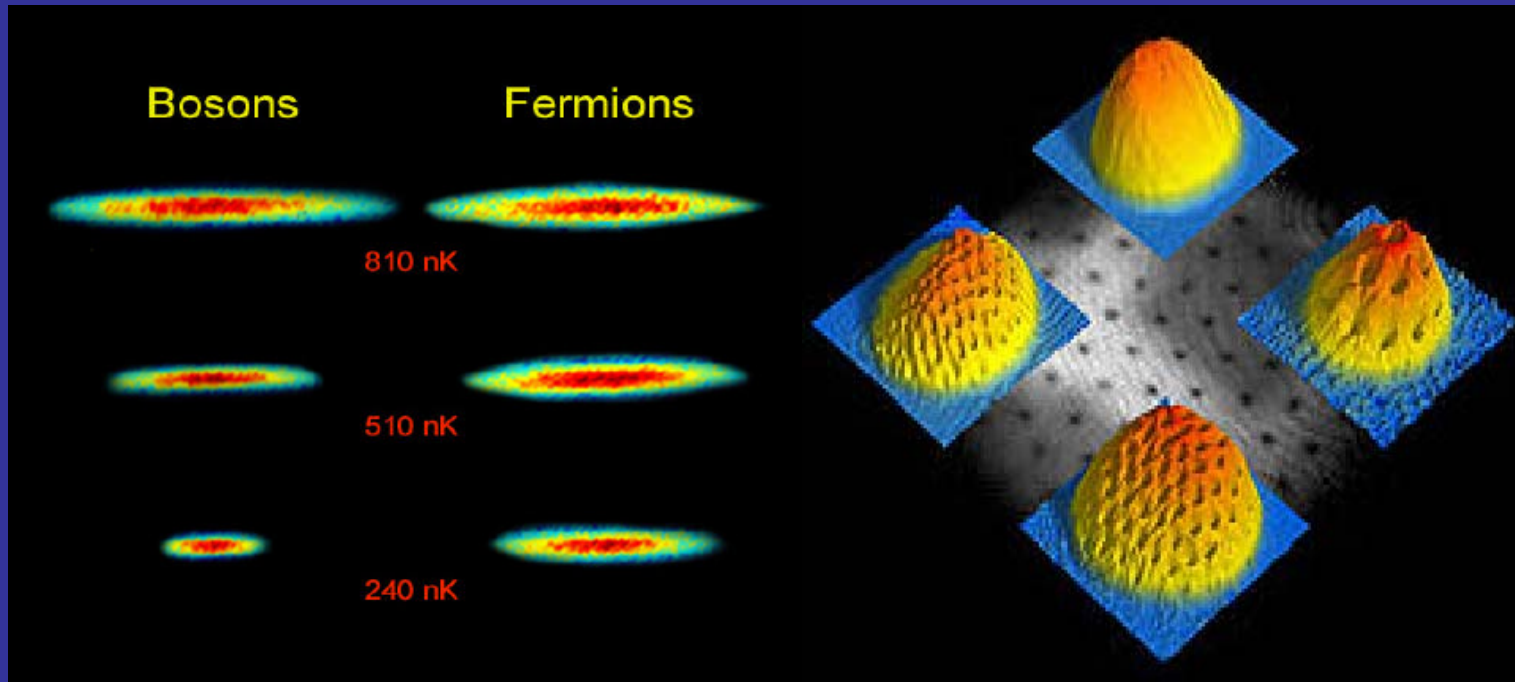


# *Between Bose-Einstein condensate and fermionic superfluid*

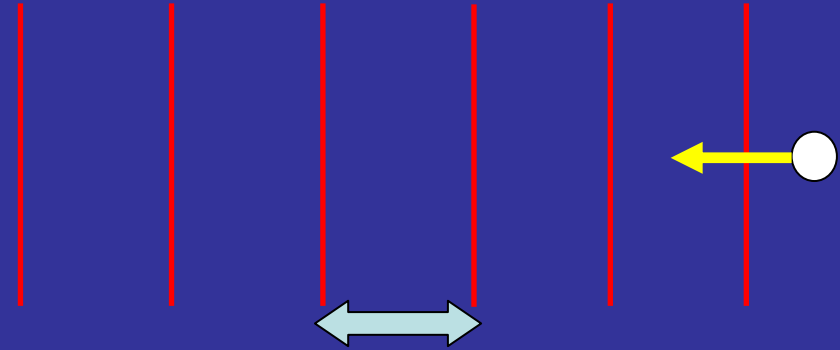
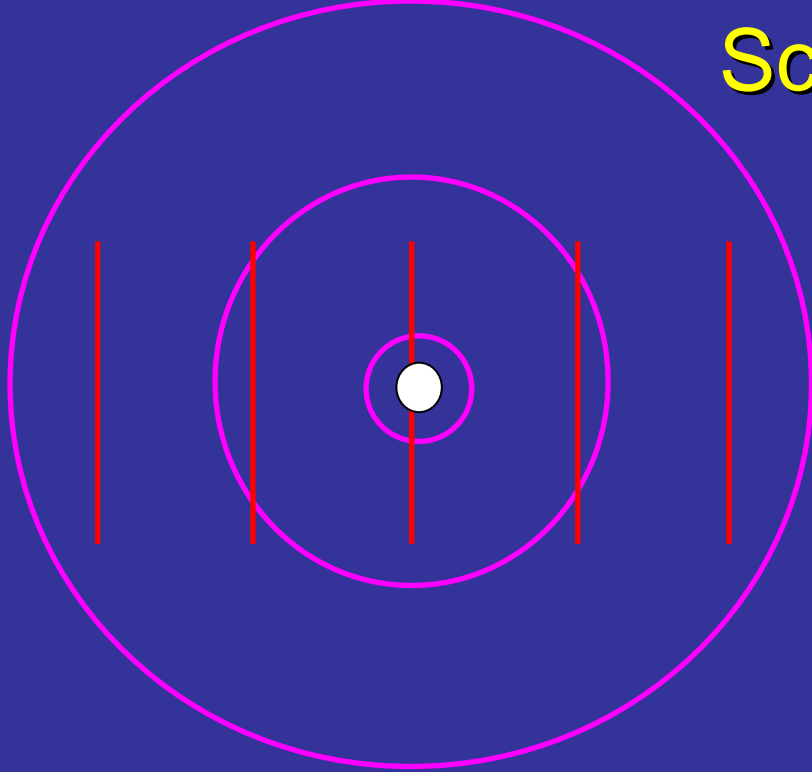


Współpraca: Aurel Bulgac – University of Washington (Seattle),  
Joaquin E. Drut (PhD student) – University of Washington (Seattle),  
Gabriel Wlazłowski (PhD student) – Warsaw University of Technology

# Outline

- **BCS-BEC crossover. Universality of the unitary regime.**
- **Dilute neutron matter and ultra cold atomic gases.**
- **Equation of state for the uniform Fermi gas in the unitary regime. Critical temperature.**
- **Measurements of the entropy and the critical temperature in a harmonic trap: experiment vs. theory.**

# Scattering at low energies (s-wave scattering)



$$\lambda = \frac{2\pi}{k} \gg R$$

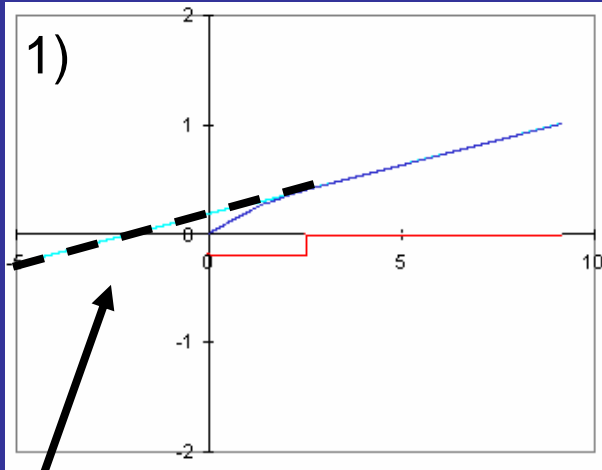
$R$  - radius of the interaction potential

$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} + f \frac{e^{ikr}}{r}; \quad f - \text{scattering amplitude}$$

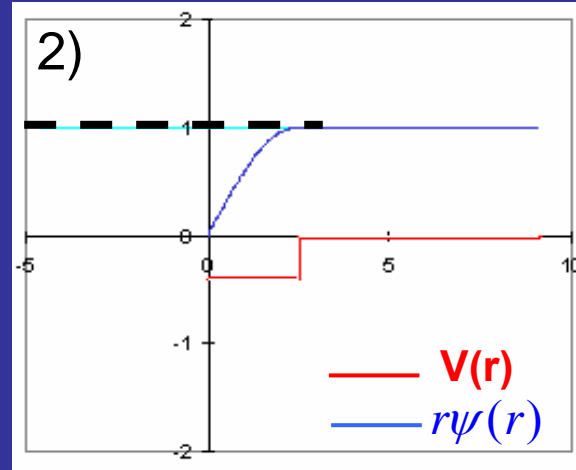
$$f = \frac{1}{-ik - \frac{1}{a} + \frac{1}{2}r_0k^2}, \quad a - \text{scattering length, } r_0 - \text{effective range}$$

If  $k \rightarrow 0$  then the interaction is determined by the scattering length alone.

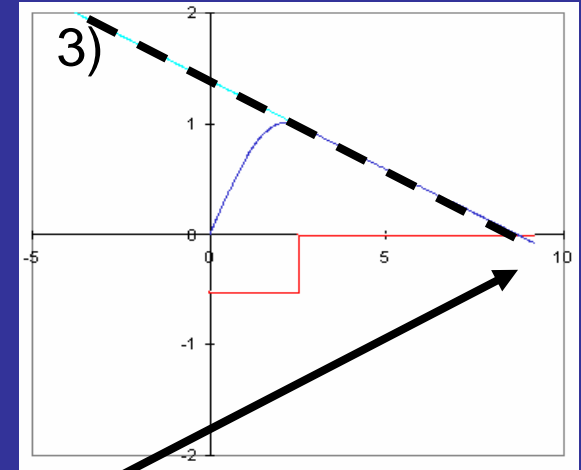
two-particle wave function for small  $r \geq R$  (range of the potential):  $r\psi(r) \propto (r - a)$



1)  $a < 0$  there is no bound state



2)  $a = \pm\infty$



3)  $a > 0$  a bound state exists

**Fermi gas:**  $n$  - number density,  $a$  - scattering length

**What is the energy of the dilute Fermi gas?**

$$E(k_F a) = ?$$

↑  
( $k_F r_0 \ll 1$ )

$$\varepsilon_F = \frac{\hbar^2 k_F^2}{2m}; \quad n = \frac{k_F^3}{3\pi^2} \text{ - particle density}$$

$$\frac{E}{E_{FG}} = 1 + \frac{10}{9\pi} (k_F a) \left[ 1 + \frac{6}{35\pi} (k_F a) (11 - 2\ln 2) + \dots \right] + \text{pairing}$$

$$E_{FG} = \frac{3}{5} \varepsilon_F N \text{ - Energy of the noninteracting Fermi gas}$$

**Perturbation series**

(works if:  $|k_F a| < 1$ )

# ➤ What is the unitary regime?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$$n r_0^3 \ll 1$$

$$n |a|^3 \gg 1$$

$n$  - particle density  
 $a$  - scattering length  
 $r_0$  - effective range

$$i.e. r_0 \rightarrow 0, a \rightarrow \pm\infty$$

**NONPERTURBATIVE  
REGIME**

The only scale:

$$E_{FG}/N = \frac{3}{5} \varepsilon_F$$

**System is dilute but  
strongly interacting!**

**UNIVERSALITY:**

$$E(T) = \xi \left( \frac{T}{\varepsilon_F} \right) E_{FG}$$

**QUESTIONS:**

What is the shape of  $\xi(T/\varepsilon_F)$ ?

What is the critical temperature for the superfluid-to-normal transition?

...

Expected phases of a two species dilute Fermi system  
BCS-BEC crossover

T

Strong interaction  
UNITARY REGIME

EASY!

EASY!

weak interaction

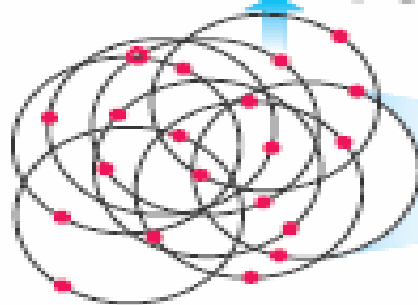
weak interactions

BCS Superfluid

?

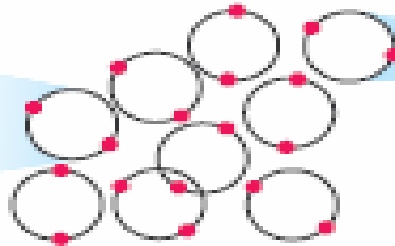
Molecular BEC and  
Atomic+Molecular  
Superfluids

1/a



$a < 0$

no 2-body bound state



$a > 0$

shallow 2-body bound state

Bose molecule

# A little bit of history

Bertsch Many-Body X challenge, Seattle, 1999

*What are the ground state properties of the many-body system composed of spin  $\frac{1}{2}$  fermions interacting via a zero-range, infinite scattering-length contact interaction.*

Why? Besides pure theoretical curiosity, this problem is relevant to neutron stars!

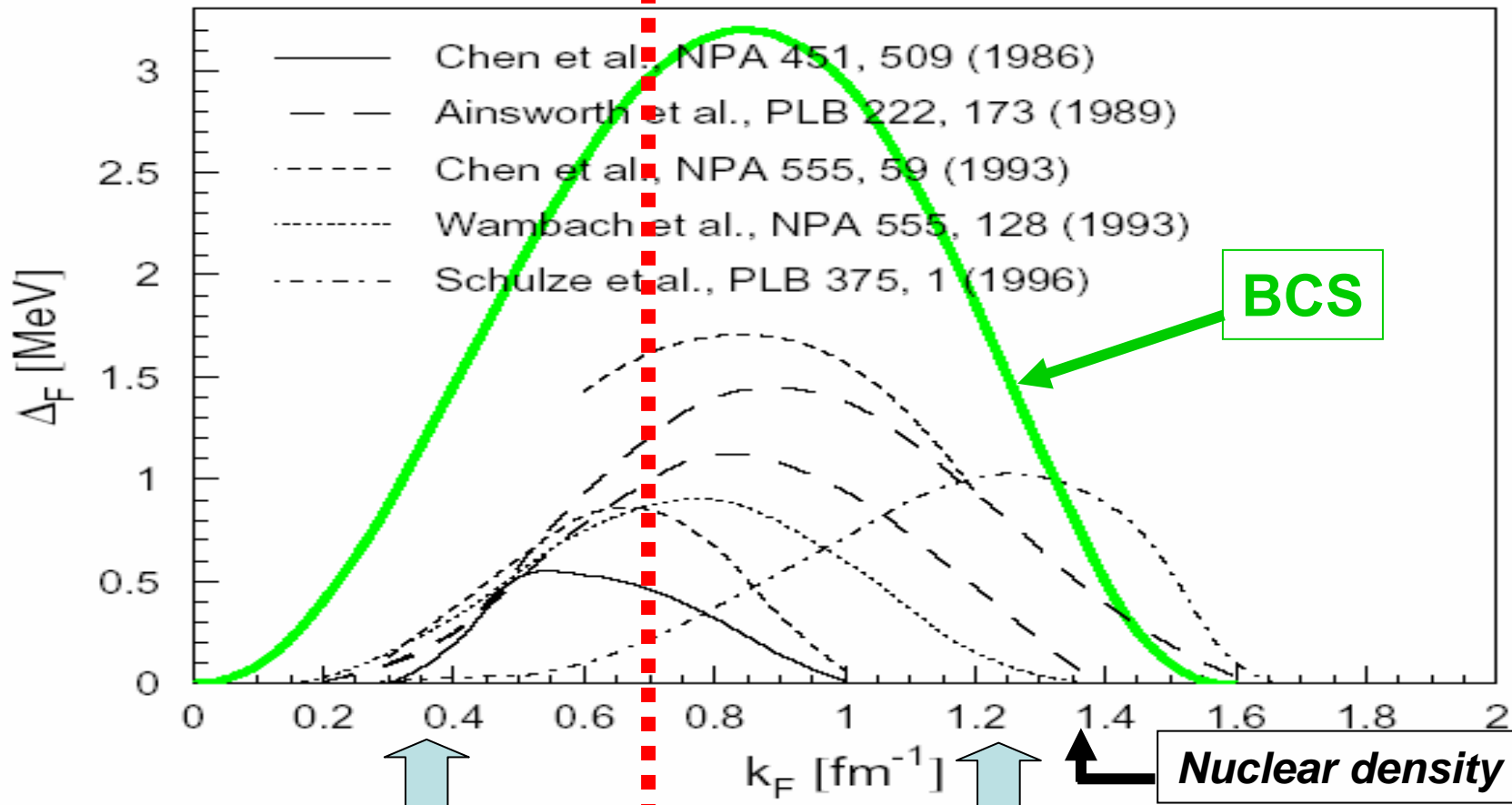
In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not! A number of people argued that under such conditions fermionic matter is unstable.

- *systems of bosons are unstable (Efimov effect)*
- *systems of three or more fermion species are unstable (Efimov effect)*
- Baker (winner of the MBX challenge) concluded that the system is stable. See also Heiselberg (entry to the same competition)
- Carlson et al (2003) Fixed-Node Green Function Monte Carlo and Astrakharchik et al. (2004) FN-DMC provided the best theoretical estimates for the ground state energy of such systems:  $\xi(T=0) \approx 0.44$
- Thomas' Duke group (2002) demonstrated experimentally that such systems are (meta)stable.

# Neutron matter

Neutron-neutron scattering  
 Scattering length:  $a \approx -18.5 \text{ fm}$   
 Effective range:  $r_0 \approx 2.8 \text{ fm}$

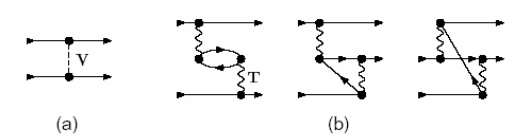
## s-wave pairing gap in infinite neutron matter with realistic NN-interactions



Dilute matter: only  $a, r_0$  matter

Details of the n-n potential matter

“Screening effects” are significant!

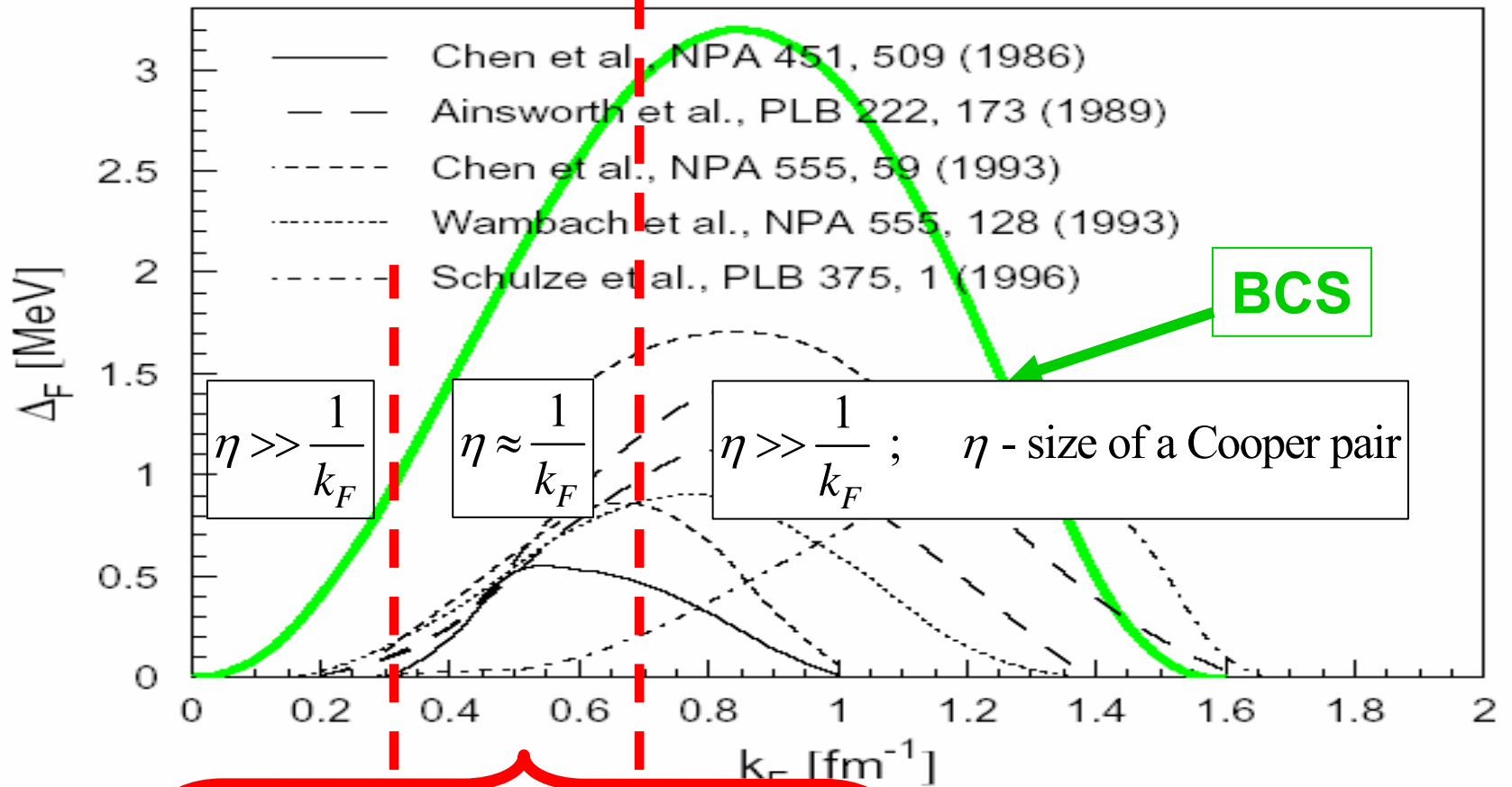




# Neutron matter

Neutron-neutron scattering  
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## s-wave pairing gap in infinite neutron matter with realistic NN-interactions



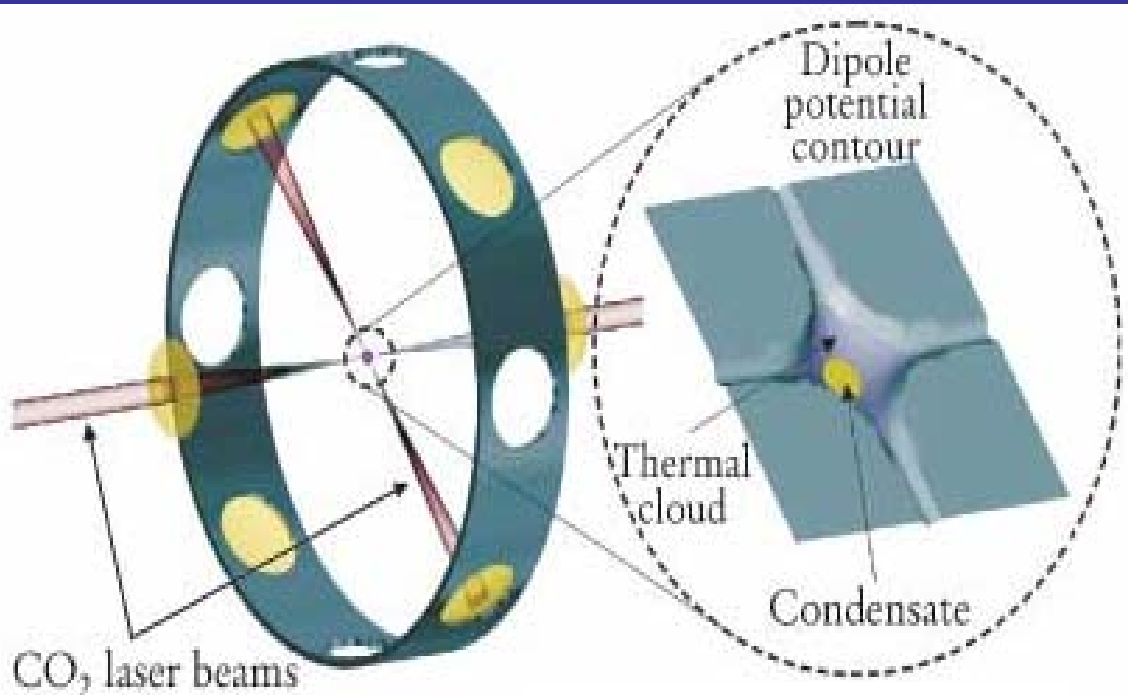
$n \approx 0.001 - 0.01 \text{ fm}^{-3}$   
 $k_F \approx 0.3 - 0.7 \text{ fm}^{-1}$

**Close to the unitary limit**

# Ultra cold atomic gases

In dilute atomic systems experimenters can control nowadays almost anything:

- The number of atoms in the trap: typically about  $10^5$ - $10^6$  atoms divided 50-50 among the lowest two hyperfine states.
- The density of atoms
- Mixtures of various atoms
- The temperature of the atomic cloud
- The strength of this interaction is fully tunable!



Physics Today, v54, 20 (2001)

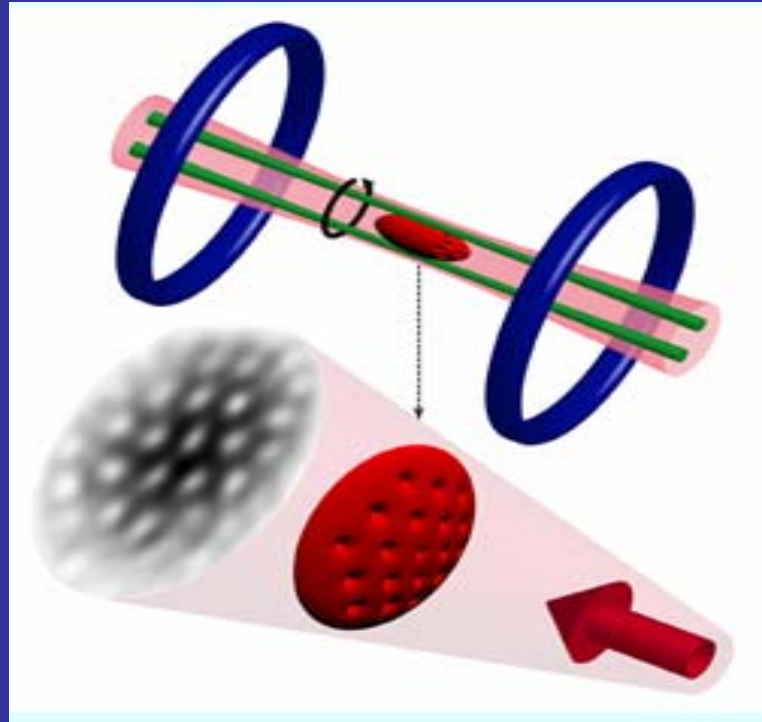
## Who does experiments?

- Jin's group at Boulder
- Grimm's group in Innsbruck
- Thomas' group at Duke
- Ketterle's group at MIT
- Salomon's group in Paris
- Hulet's group at Rice

# Evidence for fermionic superfluidity: vortices!

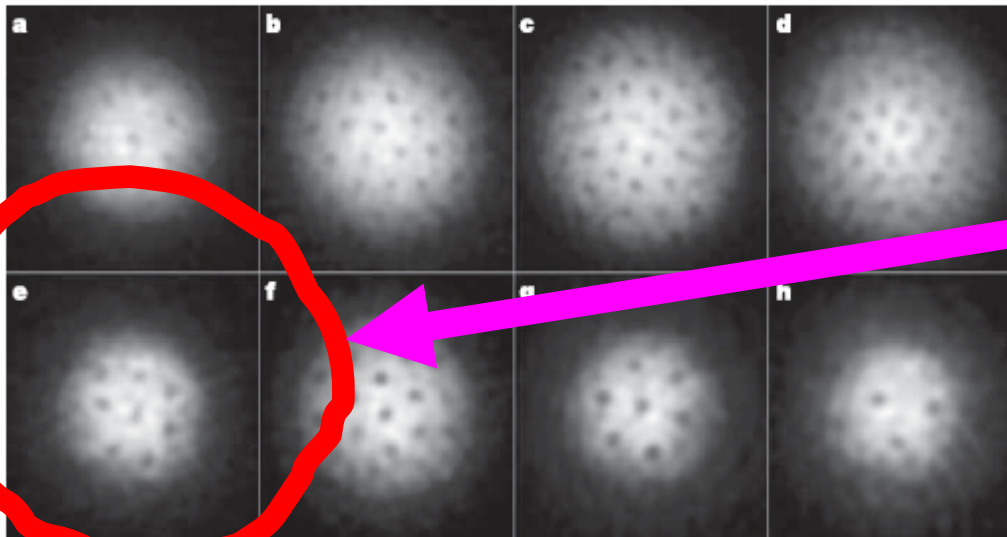
system of fermionic  ${}^6\text{Li}$  atoms

Feshbach resonance:  
B=834G



BEC side:  
 $a > 0$

BCS side:  
 $a < 0$



UNITARY REGIME

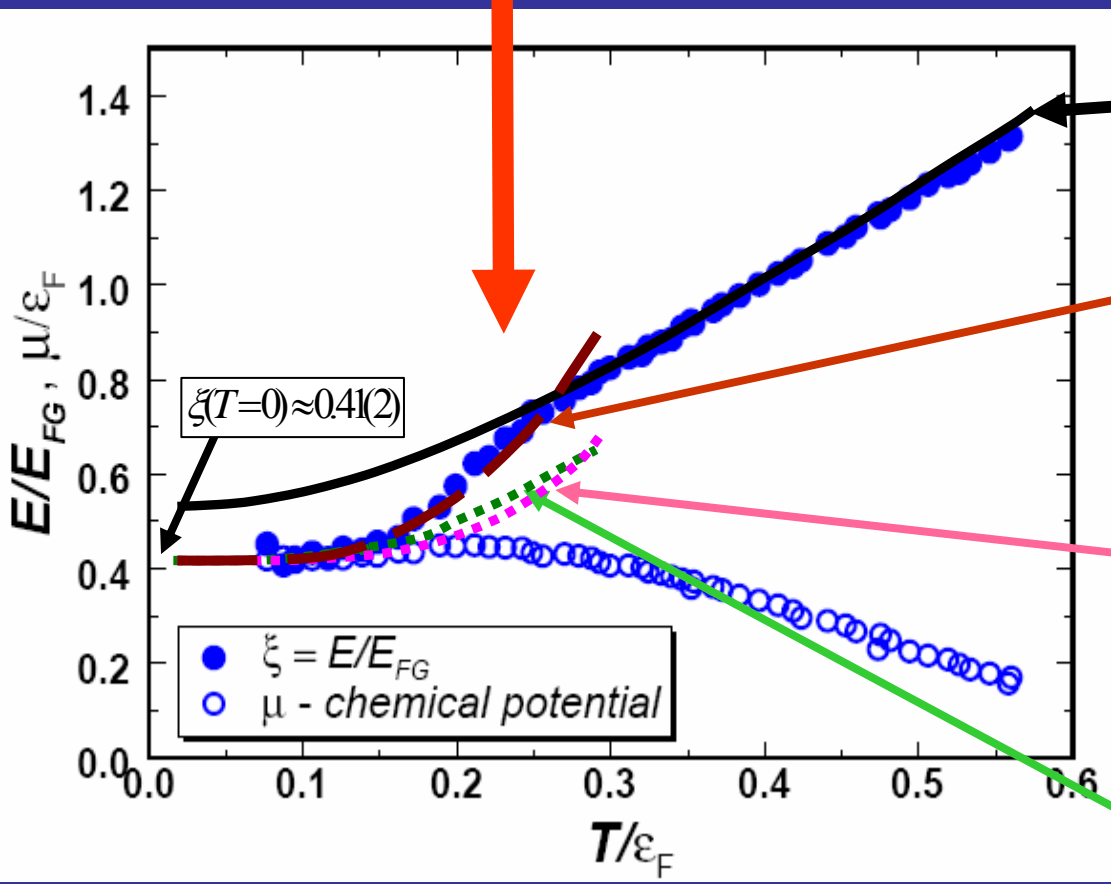
M.W. Zwierlein *et al.*,  
Nature, 435, 1047 (2005)

Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) or 500 ms (b–h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the

magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (a), 766 G (b), 792 G (c), 812 G (d), 833 G (e), 843 G (f), 853 G (g) and 863 G (h). The field of view of each image is  $880 \mu\text{m} \times 880 \mu\text{m}$ .

$a = \pm\infty$

**Superfluid to Normal Fermi Liquid Transition**



Normal Fermi Gas  
(with vertical offset, solid line)

Bogoliubov-Anderson phonons  
and quasiparticle contribution  
(dashed line)

Bogoliubov-Anderson phonons  
contribution only (dotted line)

Quasi-particle contribution only  
(dotted line)

$$E_{\text{quasi-particles}}(T) = \frac{3}{5} \varepsilon_F N \frac{5}{2} \sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}} \exp\left(-\frac{\Delta}{T}\right)$$

$$\Delta = \left(\frac{2}{e}\right)^{7/3} \varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

$$E_{\text{phonons}}(T) = \frac{3}{5} \varepsilon_F N \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}} \left(\frac{T}{\varepsilon_F}\right)^4, \quad \xi_s \approx 0.44$$

## Low temperature behaviour of a Fermi gas in the unitary regime

$$F(T) = \frac{3}{5} \varepsilon_F N \varphi \left( \frac{T}{\varepsilon_F} \right) = E - TS \quad \text{and} \quad \frac{\mu(T)}{\varepsilon_F} \approx \xi_s \approx 0.41(2) \quad \text{for } T < T_C$$

$$\mu(T) = \frac{dF(T)}{dN} = \varepsilon_F \left[ \varphi \left( \frac{T}{\varepsilon_F} \right) - \frac{2T}{5\varepsilon_F} \varphi' \left( \frac{T}{\varepsilon_F} \right) \right] \approx \varepsilon_F \xi_s$$

$$\varphi \left( \frac{T}{\varepsilon_F} \right) = \varphi_0 + \varphi_1 \left( \frac{T}{\varepsilon_F} \right)^{5/2}$$

$$E(T) = \frac{3}{5} \varepsilon_F N \left[ \xi_s + \zeta_s \left( \frac{T}{\varepsilon_F} \right)^n \right]$$

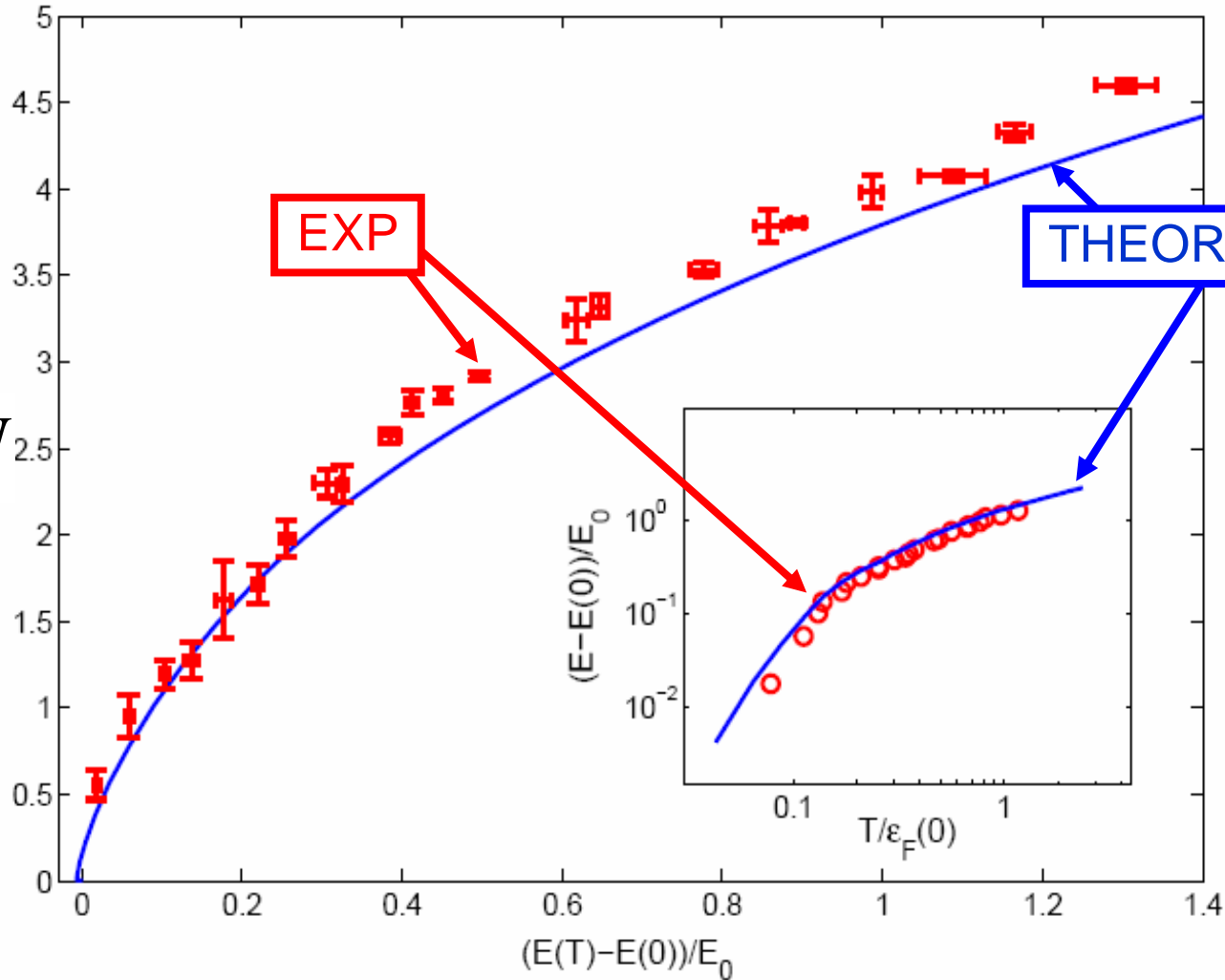
Lattice results disfavor  
either  $n \geq 3$  or  $n \leq 2$   
and suggest  $n = 2.5(0.25)$

This is the same behavior as for a gas of noninteracting (!) bosons below the condensation temperature.

## Comparison with experiment

John Thomas' group at Duke University,  
L.Luo, et al. Phys. Rev. Lett. 98, 080402, (2007)

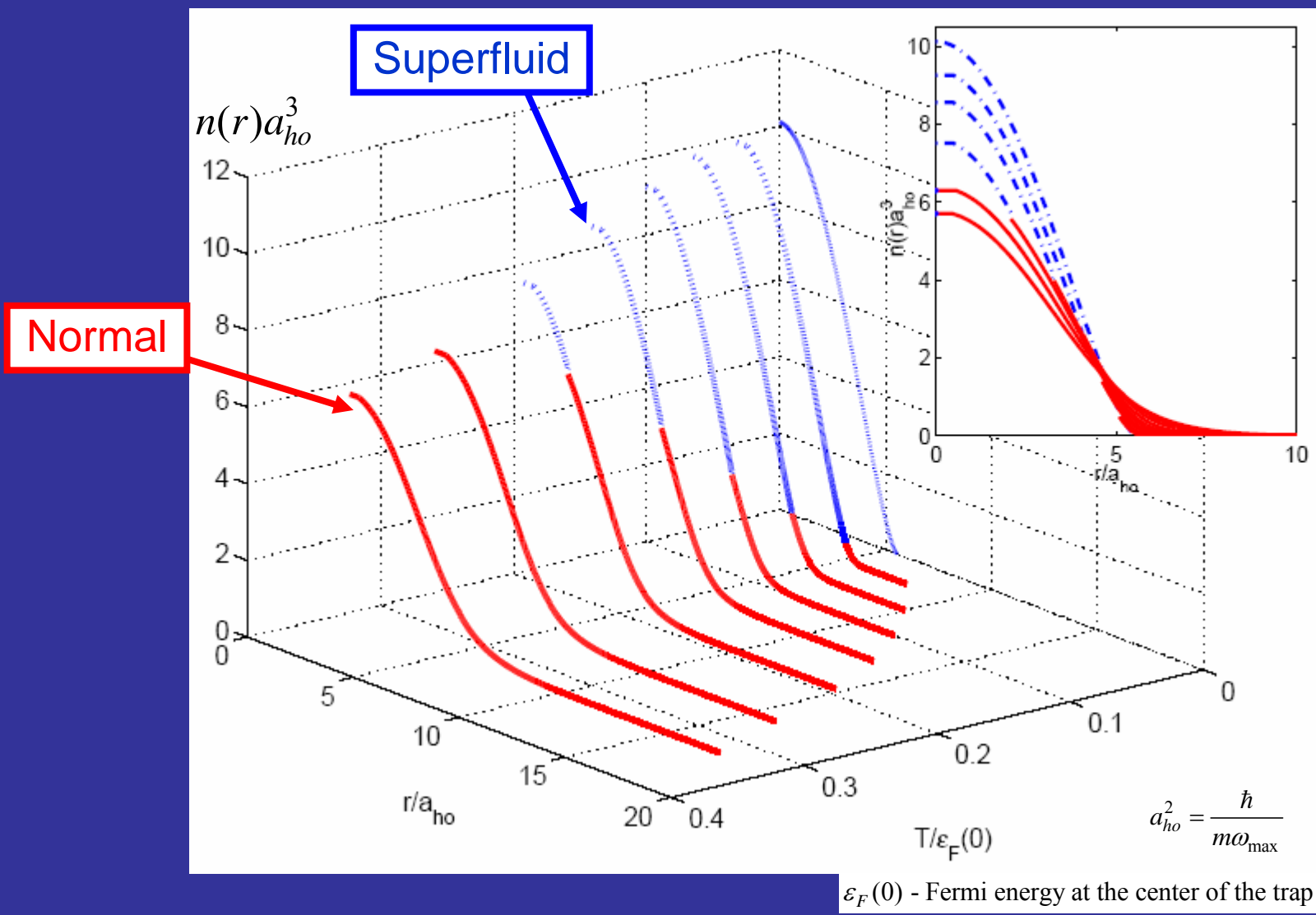
$S/k_B N$



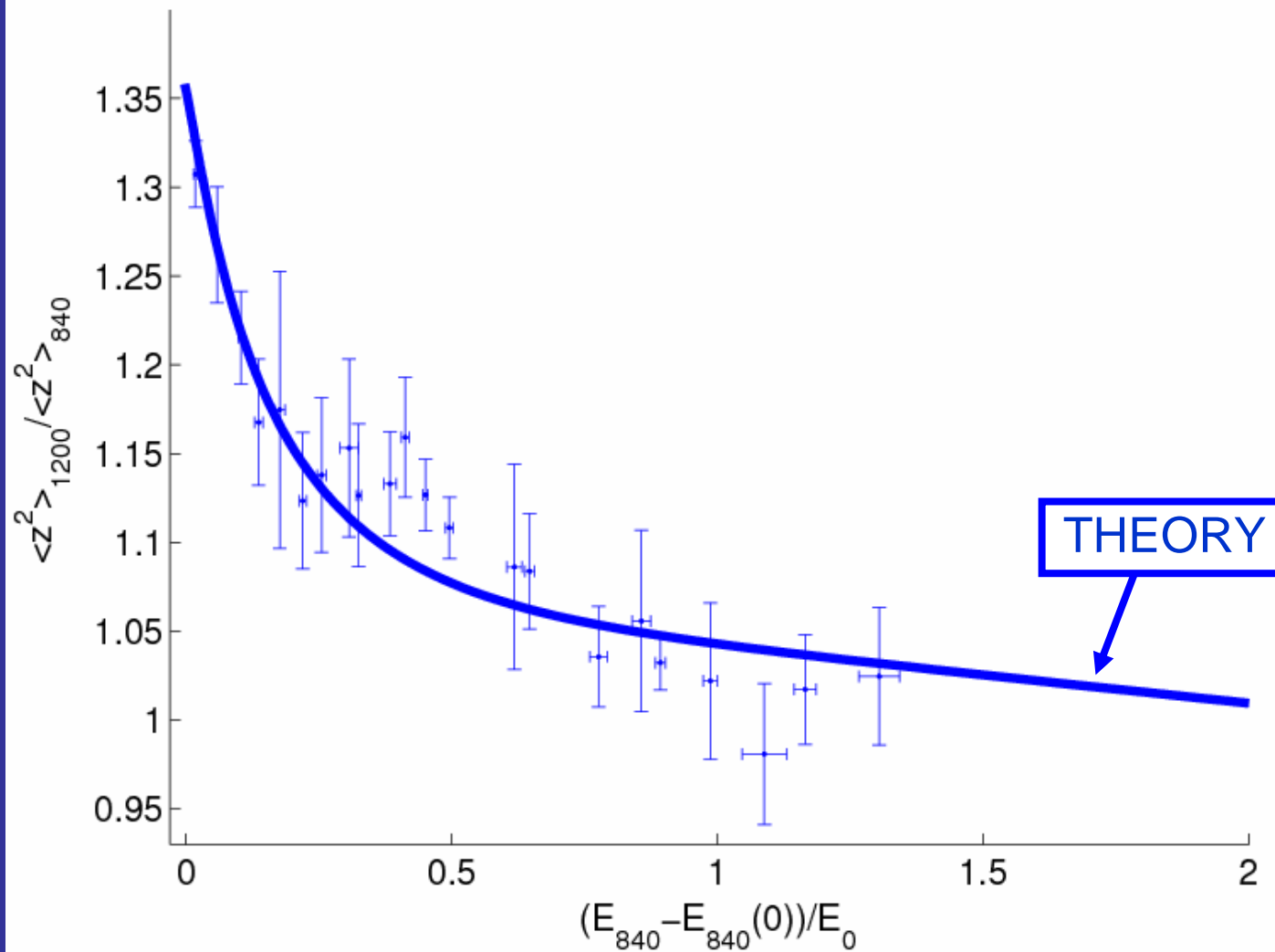
$$E_0 = N \epsilon_F^{ho}$$

$\epsilon_F(0)$  - Fermi energy  
at the center of the trap

Entropy as a function of energy (relative to the ground state) for the unitary Fermi gas in the harmonic trap. Inset: log-log plot of energy as a function of temperature.



The radial (along shortest axis) density profiles of the atomic cloud in the Duke group experiment at various temperatures.



$$E_0 = N \varepsilon_F^{ho}$$

Ratio of the mean square cloud size at B=1200G to its value at unitarity (B=840G) as a function of the energy. Experimental data are denoted by point with error bars.

$$B = 1200G \Rightarrow 1/k_F a \approx -0.75$$

$$B = 840G \Rightarrow 1/k_F a \approx 0$$



# Summary

We presented the first model-independent comparison of recent measurements of the entropy and the critical temperature, performed by the Duke group: L.Luo, et al. Phys. Rev. Lett. 98, 080402, (2007), with our recent finite temperature Monte Carlo calculations.

## EXP.

$$\left\{ \begin{array}{l} E(T_c) - E(0) \approx 0.41(5) N \varepsilon_F^{ho}, \\ S_c / N \approx 2.7(2) k_B, \\ T_c \approx 0.29(3) \varepsilon_F^{ho} \end{array} \right.$$

## THEORY

$$\left\{ \begin{array}{l} E(T_c) - E(0) \approx 0.34(2) N \varepsilon_F^{ho}, \\ S_c / N \approx 2.4(3) k_B, \\ T_c \approx 0.27(3) \varepsilon_F^{ho} \end{array} \right.$$

A.Bulgac, J.E. Drut, P. Magierski,  
Phys. Rev. Lett 99, 120401 (2007)

The results are consistent with the predicted value of the critical temperature for the uniform unitary Fermi gas:  $0.23(2) \varepsilon_F$

## Conclusions

- ✓ Fully non-perturbative calculations for a spin  $\frac{1}{2}$  many fermion system in the unitary regime at finite temperatures are feasible and apparently the system undergoes a phase transition in the bulk at  $T_c = 0.23 (2) \varepsilon_F$
- ✓ Chemical potential is constant up to the critical temperature – note similarity with Bose systems!
- ✓ Below the transition temperature, both phonons and fermionic quasiparticles contribute almost equally to the specific heat. In more than one way the system is at crossover between a Bose and Fermi systems.

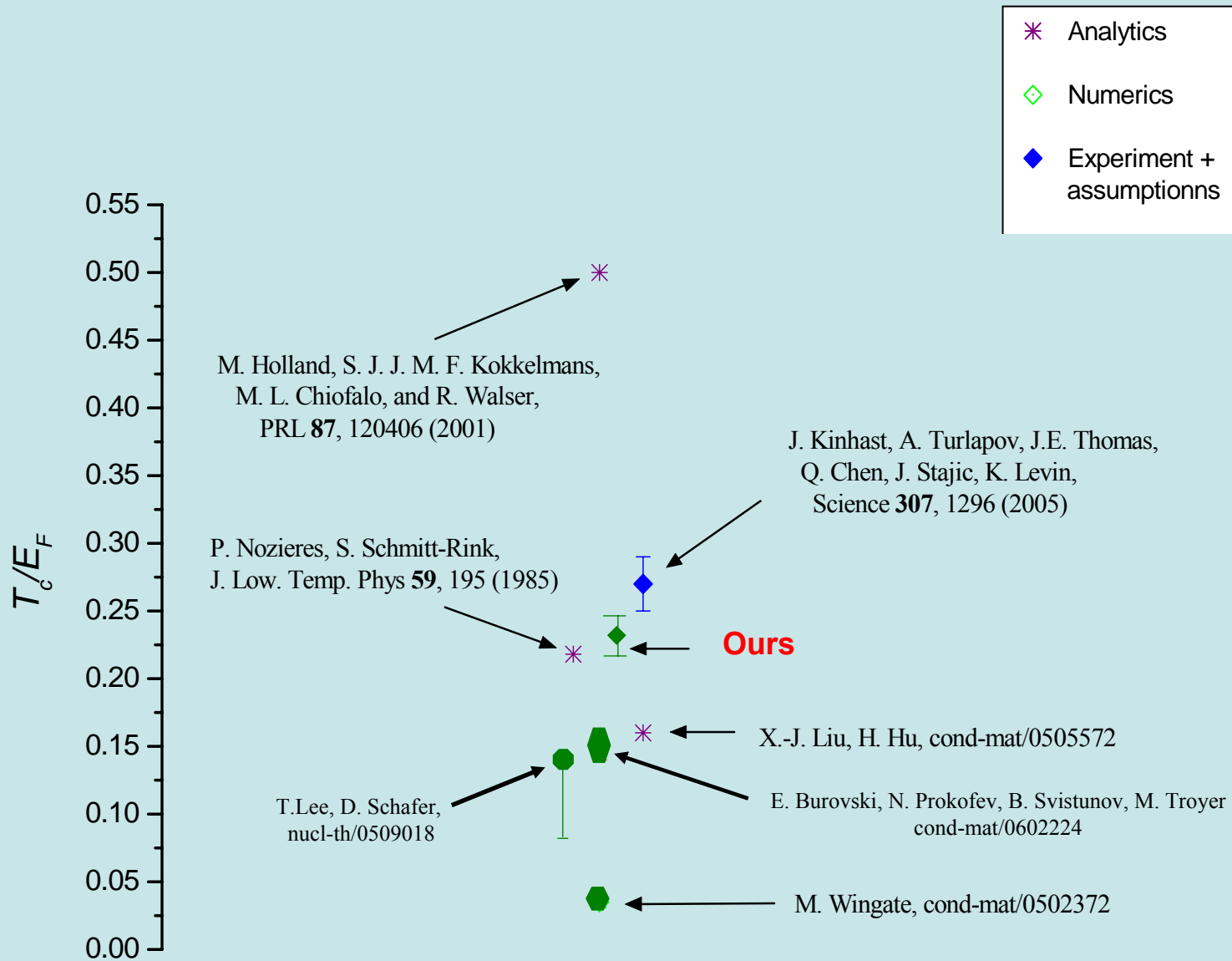
There are reasons to believe that below the critical temperature this system is a new type of fermionic superfluid, with unusual properties.

What we can learn from physics of cold Fermi atoms?

Pairing dependence on the density. Pairing field in an inhomogeneous nuclear matter.

We should try to get away from the heavily phenomenological approach which dominated nuclear pairing studies most of last 40 years and put more effort in an *ab initio* and many-body theory of pairing and be able to make reliable predictions, especially for neutron stars and nuclei far from stability. The studies of dilute atomic gases with tunable interactions could serve as an extraordinary testing ground of theories.

# Quest for unitary point critical temperature



$$E(T) \approx \frac{3}{5} \varepsilon_F N \xi_s + \frac{m_B^{3/2} \Gamma\left(\frac{3}{2}\right) \zeta\left(\frac{3}{2}\right)}{2^{1/2} \pi^2 \hbar^3} T^{5/2} V, \quad \text{if } T \ll m_B c^2$$

and fitting to lattice results  $\Rightarrow m_B \approx 3m$

- Why this value for the bosonic mass?
- Why these bosons behave like noninteracting particles?

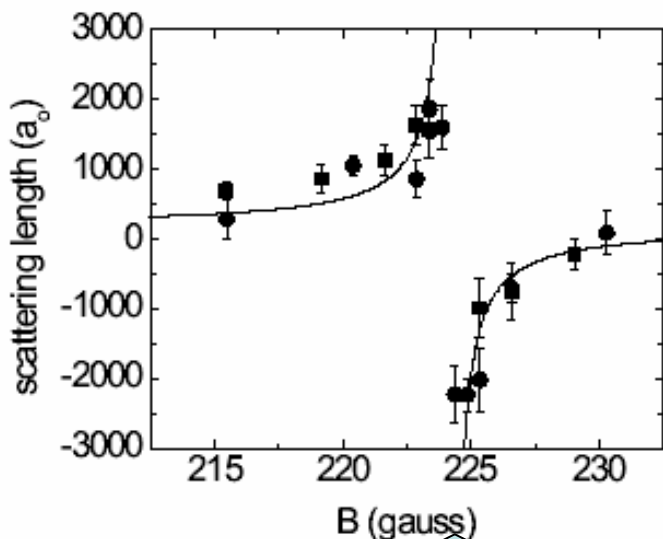
# Feshbach resonance

$$H = \frac{\vec{p}^2}{2\mu} + \sum_{i=1}^2 (V_i^{hf} + V_i^Z) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + \cancel{V^d}$$

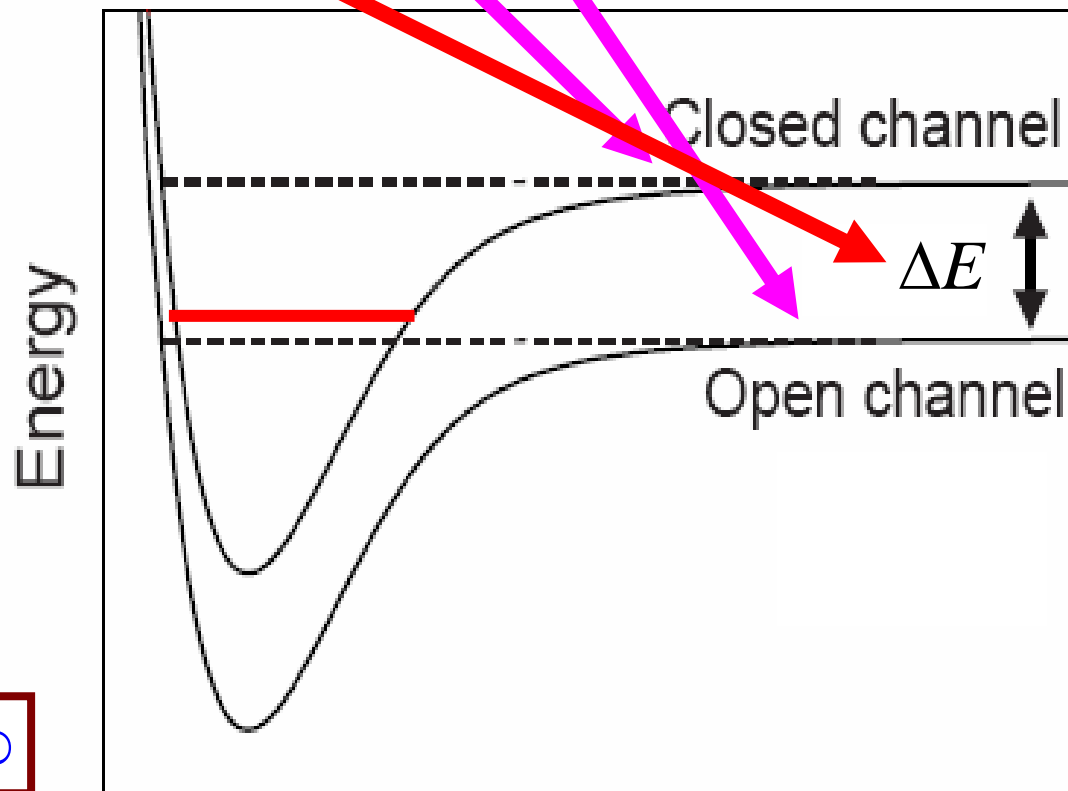
$$V^{hf} = \frac{a_{hf}}{\hbar^2} \vec{I} \cdot \vec{J}, \quad V^Z = (\gamma_e J_z - \gamma_n I_z) B$$

Tiesinga, Verhaar,  
Stoof, Phys. Rev.  
A47, 4114 (1993)

**Channel coupling**



**resonance:  $a \rightarrow \pm\infty$**



**Interatomic distance**

# Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} = \int d^3r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^\dagger(\vec{r}) \left( -\frac{\hbar^2 \Delta}{2m} \right) \hat{\psi}_s(\vec{r}) - g \int d^3r \hat{n}_\uparrow(\vec{r}) \hat{n}_\downarrow(\vec{r})$$

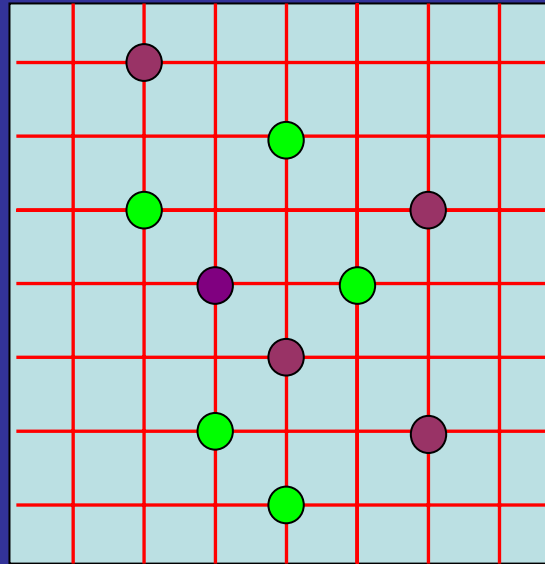
$$\hat{N} = \int d^3r (\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r})); \quad \hat{n}_s(\vec{r}) = \hat{\psi}_s^\dagger(\vec{r}) \hat{\psi}_s(\vec{r})$$

## Theoretical approach: Fermions on 3D lattice

### Coordinate space

L-limit for the spatial correlations in the system

$$k_{cut} = \frac{\pi}{\Delta x}; \quad \Delta x$$



$$Volume = L^3$$

$$lattice\ spacing = \Delta x$$

● - Spin up fermion: ↑

● - Spin down fermion: ↓

External conditions:

$T$  - temperature

$\mu$  - chemical potential

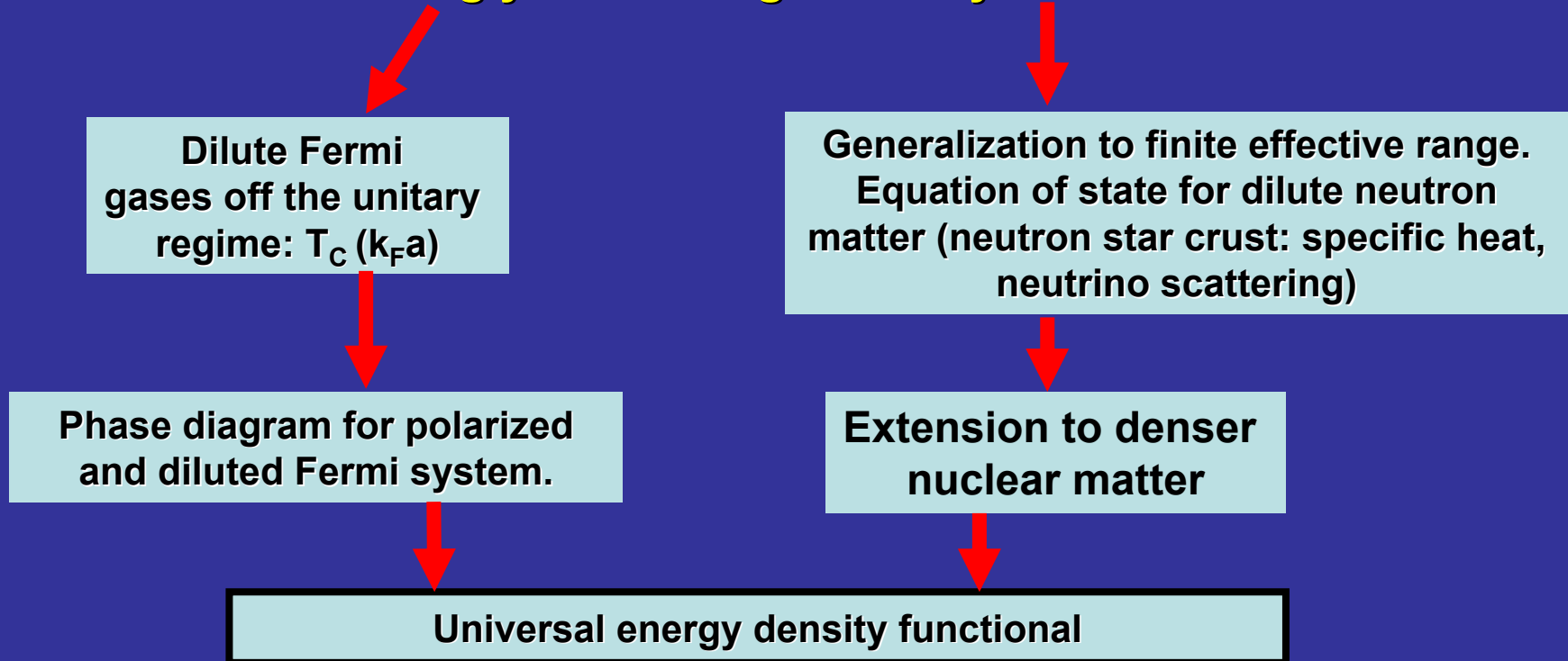
Periodic boundary conditions imposed

## More details of the calculations:

- Lattice sizes used from  $8^3 \times 257$  (high Ts) to  $8^3 \times 1732$  (low Ts),  $\langle N \rangle = 50$ , and  $6^3 \times 257$  (high Ts) to  $6^3 \times 1361$  (low Ts),  $\langle N \rangle = 30$ .
- Effective use of FFT(W) makes all imaginary time propagators diagonal (either in real space or momentum space) and there is no need to store large matrices.
- Update field configurations using the Metropolis importance sampling algorithm.
- Change randomly at a fraction of all space and time sites the signs the auxiliary fields  $\sigma(r, \tau)$  so as to maintain a running average of the acceptance rate between 0.4 and 0.6 .
- Thermalize for 50,000 – 100,000 MC steps or/and use as a start-up field configuration a  $\sigma(x, \tau)$ -field configuration from a different T
- At low temperatures use Singular Value Decomposition of the evolution operator  $U(\{\sigma\})$  to stabilize the numerics.
- Use 200,000-2,000,000  $\sigma(x, \tau)$ - field configurations for calculations
- MC correlation “time”  $\approx 150 - 200$  time steps at  $T \approx T_c$

## Open questions and future prospects:

- We have a theoretical tool that enable us to study various aspects of strongly interacting Fermi system.



## What we can learn from physics of cold Fermi atoms?

**Pairing dependence on the density. Pairing field in an inhomogeneous nuclear matter.**

We should try to get away from the heavily phenomenological approach which dominated nuclear pairing studies most of last 40 years and put more effort in an *ab initio* and many-body theory of pairing and be able to make reliable predictions, especially for neutron stars and nuclei far from stability. The studies of dilute atomic gases with tunable interactions could serve as an extraordinary testing ground of theories.



## Experiment

John Thomas' group at Duke University,  
L.Luo, et al. *Phys. Rev. Lett.* **98**, 080402, (2007)

Dilute system of fermionic  ${}^6\text{Li}$  atoms in a harmonic trap

- The number of atoms in the trap:  $N=1.3(0.2) \times 10^5$  atoms divided 50-50 among the lowest two hyperfine states.

- Fermi energy:  $\varepsilon_F^{ho} = \hbar\Omega(3N)^{1/3}$ ;  $\Omega = (\omega_x\omega_y\omega_z)^{1/3}$

$$\varepsilon_F^{ho} / k_B \approx 1\mu\text{K}$$

- Depth of the potential:  $U_0 \approx 10\varepsilon_F^{ho}$
- How they measure: energy, entropy and temperature?

$$\left. \begin{array}{l} PV = \frac{2}{3} E \\ \vec{\nabla}P = -n(\vec{r})\vec{\nabla}U \end{array} \right\} \Rightarrow N\langle U \rangle = \frac{E}{2} \text{ - virial theorem}$$

$n(\vec{r})$  - local density

↑  
Holds at unitarity and for noninteracting Fermi gas

- For the weakly interacting gas ( $B = 1200G \Rightarrow 1/k_F a \approx -0.75$ ) the energy and entropy is calculated. In this limit one can use Thomas-Fermi approach to relate the energy to the given density distribution.

The entropy can be estimated as for the noninteracting system with 1% accuracy. In practice:  $\left\langle z^2 \right\rangle_{B=1200} \Rightarrow E, S$

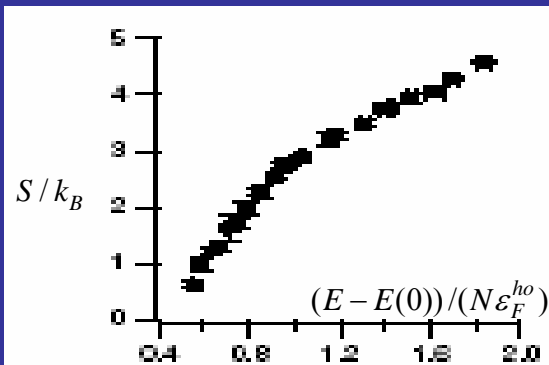
- The magnetic field is changed adiabatically ( $S = \text{const.}$ ) to the value corresponding to the unitary limit:  $B = 840G \Rightarrow 1/k_F a \approx 0$
- Relative energy in the unitary limit is calculated from virial theorem:

$$\frac{E(T_1)}{E(T_2)} = \frac{\left\langle z^2 \right\rangle_{T_1}}{\left\langle z^2 \right\rangle_{T_2}}$$

- Temperature is calculated from the identity:

$$\frac{1}{T} = \frac{\partial S}{\partial E}$$

- The plot  $S(E)$  contains a cusp related to the phase transition:



$$\left\{ \begin{array}{l} E(T_c) - E(0) \approx 0.41(5) N \epsilon_F^{ho}, \\ S_c / N \approx 2.7(2) k_B, \\ T_c \approx 0.29(3) \epsilon_F^{ho} \end{array} \right.$$

## Theory: local density approximation (LDA)

Uniform  
system

$$\Omega = F - \lambda N = \frac{3}{5} \varphi(x) \varepsilon_F N - \lambda N$$

Nonuniform  
system  
(gradient  
corrections  
neglected)

$$\Omega = \int d^3 r \left[ \frac{3}{5} \varepsilon_F(\vec{r}) \varphi(x(\vec{r})) + U(\vec{r}) - \lambda \right] n(\vec{r})$$

$$x(\vec{r}) = \frac{T}{\varepsilon_F(\vec{r})}; \quad \varepsilon_F(\vec{r}) = \frac{\hbar^2}{2m} \left[ 3\pi^2 n(\vec{r}) \right]^{2/3}$$

The overall chemical potential  $\lambda$  and the temperature  $T$  are constant throughout the system. The density profile will depend on the shape of the trap as dictated by:

$$\frac{\delta \Omega}{\delta n(\vec{r})} = \frac{\delta (F - \lambda N)}{\delta n(\vec{r})} = \mu(x(\vec{r})) + U(r) - \lambda = 0$$

Using as an input the Monte Carlo results for the uniform system and experimental data (trapping potential, number of particles), we determine the density profiles.

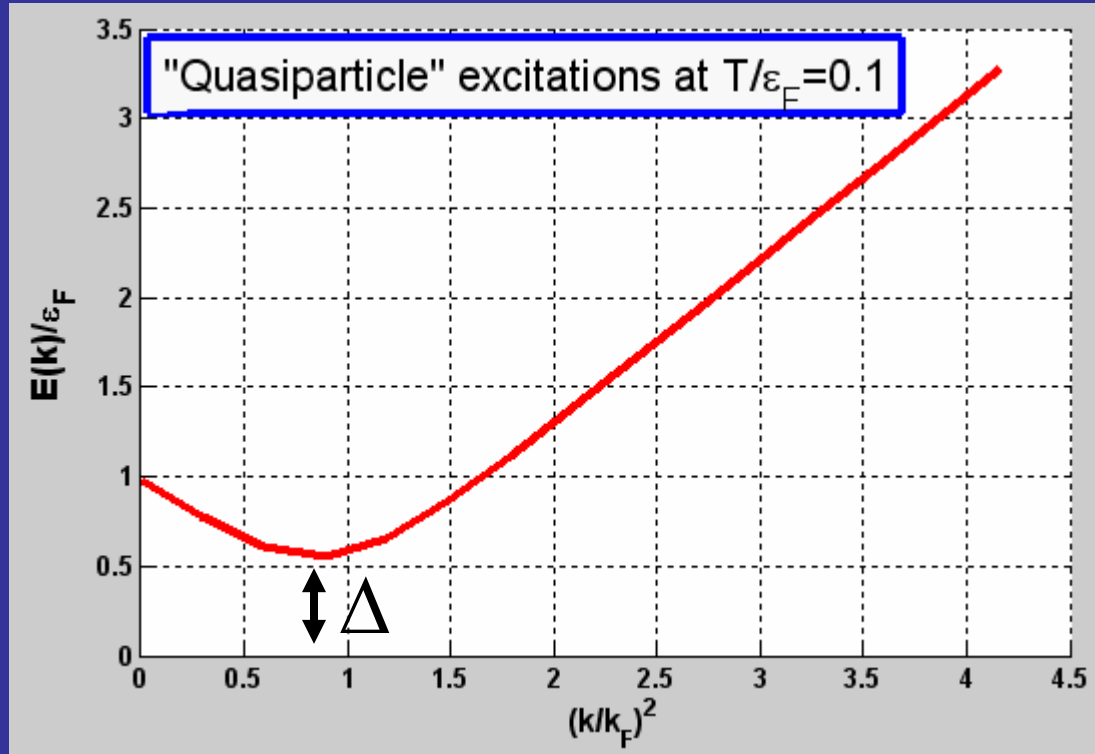
## Open questions:

- Pairing gap  $\Delta(T) = ?$

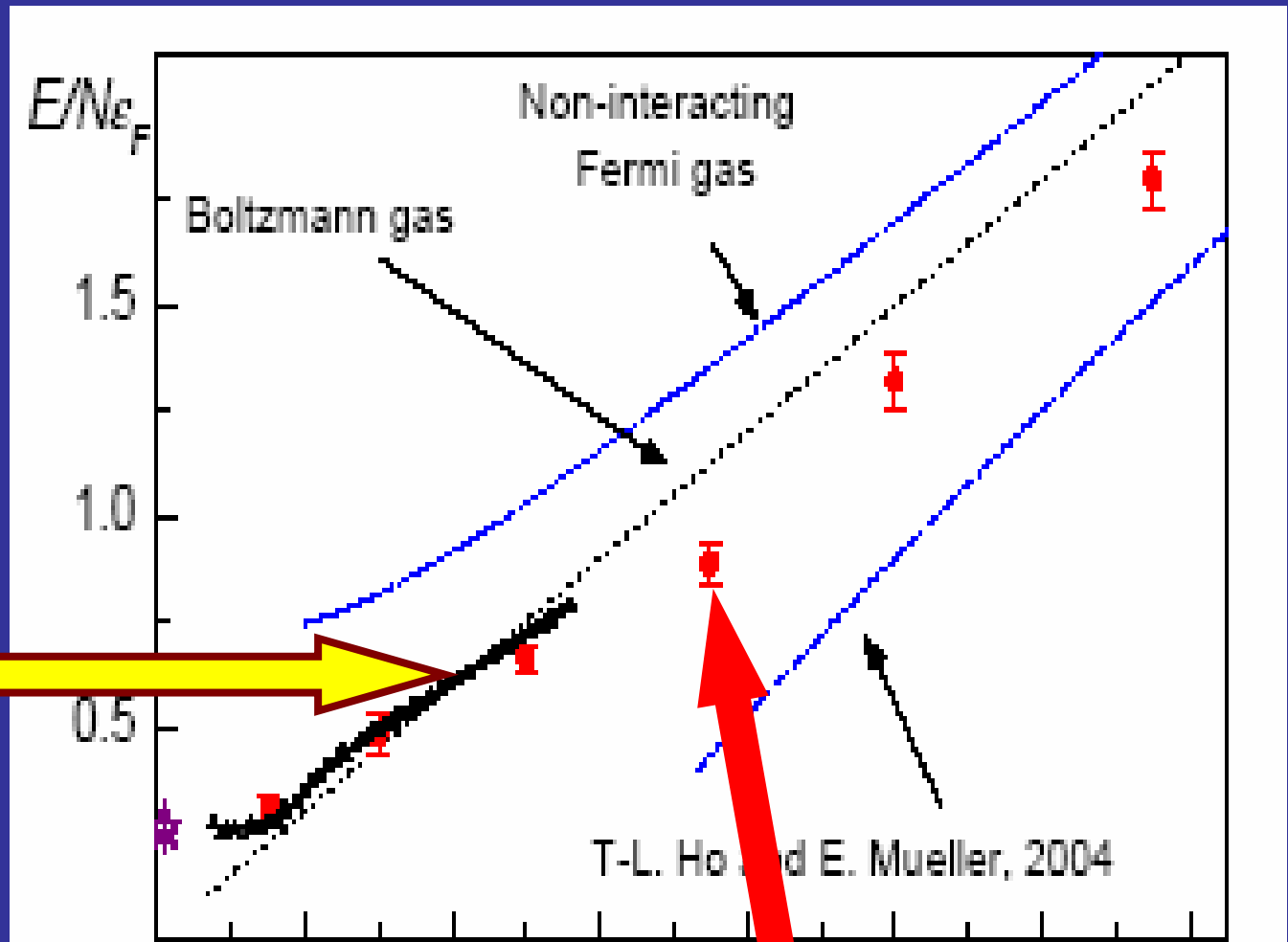
At small temperatures:  $T/\varepsilon_F \leq 0.1$ :  $\Delta \approx 0.55\varepsilon_F$

For comparison:

In atomic nuclei  $\Delta \leq 0.03\varepsilon_F$



- Energy density functional for an inhomogeneous system at finite temperatures
- Structure of the wave function in homogeneous system.



**OURS**

From: E.Burovski, N.Prokof'ev, B.Svistunov, M.Troyer, cond-mat/0602224

One of my favorite times in the academic year occurs in early spring when I give my class of extremely bright graduate students, who have mastered quantum mechanics but are otherwise unsuspecting and innocent, a take-home exam in which they are asked to deduce superfluidity from first principles. There is no doubt a special place in hell being reserved for me at this very moment for this mean trick, for the task is impossible. Superfluidity, like the fractional quantum Hall effect, is an emergent phenomenon – a low-energy collective effect of huge numbers of particles that cannot be deduced from the microscopic equations of motion in a rigorous way and that disappears completely when the system is taken apart<sup>A)</sup>. There are prototypes for superfluids, of course, and students who memorize them have taken the first step down the long road to understanding the phenomenon, but these are all approximate and in the end not deductive at all, but fits to experiment. The students feel betrayed and hurt by this experience because they have been trained to think in reductionist terms and thus to believe that everything not amenable to such thinking is unimportant. But nature is much more heartless than I am, and those students who stay in physics long enough to seriously confront the experimental record eventually come to understand that the reductionist idea is wrong a great deal of the time, and perhaps always.

**Robert B. Laughlin, Nobel Lecture, December 8, 1998**

$$\rho_2(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = \langle \hat{\psi}_{\uparrow}^{\dagger}(\vec{r}_1) \hat{\psi}_{\downarrow}^{\dagger}(\vec{r}_2) \hat{\psi}_{\downarrow}(\vec{r}_4) \hat{\psi}_{\uparrow}(\vec{r}_3) \rangle$$

$$\rho_2^P(\vec{r}) = \frac{2}{N} \int d^3r_1 d^3r_2 \rho_2(\vec{r}_1 + \vec{r}, \vec{r}_2 + \vec{r}, \vec{r}_1, \vec{r}_2)$$

$$\lim_{r \rightarrow \infty} \rho_2^P(\vec{r}) = \alpha - \text{condensate fraction}$$

## More Results...

Condensate fraction  $\alpha$ :

Order parameter for  
Off Diagonal  
Long Range Order  
(C.N. Yang)

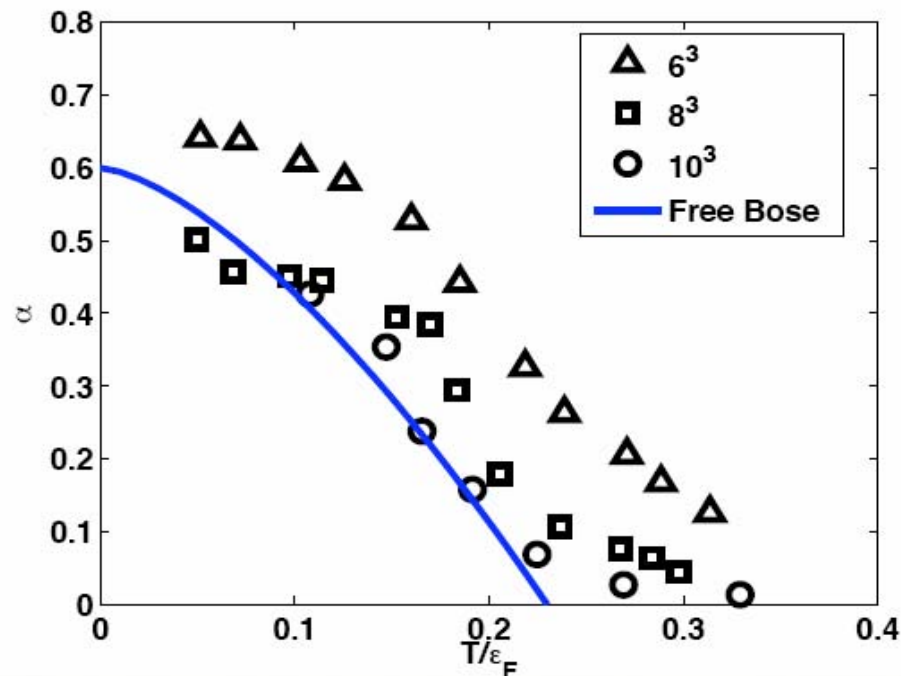
Free Bose gas-like:

$$\alpha(T) = \alpha(0) \left[ 1 - \left( \frac{T}{T_c} \right)^{3/2} \right]$$

Free :  $\alpha(0) = 1$

Unitary:  $\alpha(0) \cong 0.6$

$$T_c = 0.23(2)$$



From a talk of J.E. Drut

# Thermodynamics of the unitary Fermi gas

$$\text{ENERGY: } E(x) = \frac{3}{5} \xi(x) \varepsilon_F N; \quad x = \frac{T}{\varepsilon_F}$$

$$C_V = T \frac{\partial S}{\partial T} = \frac{\partial E}{\partial T} = \frac{3}{5} N \xi'(x) \Rightarrow S(x) = \frac{3}{5} N \int_0^x \frac{\xi'(y)}{y} dy$$

$$\text{ENTROPY/PARTICLE: } \sigma(x) = \frac{S(x)}{N} = \frac{3}{5} \int_0^x \frac{\xi'(y)}{y} dy$$

$$\text{FREE ENERGY: } F = E - TS = \frac{3}{5} \varphi(x) \varepsilon_F N$$

$$\varphi(x) = \xi(x) - x\sigma(x)$$

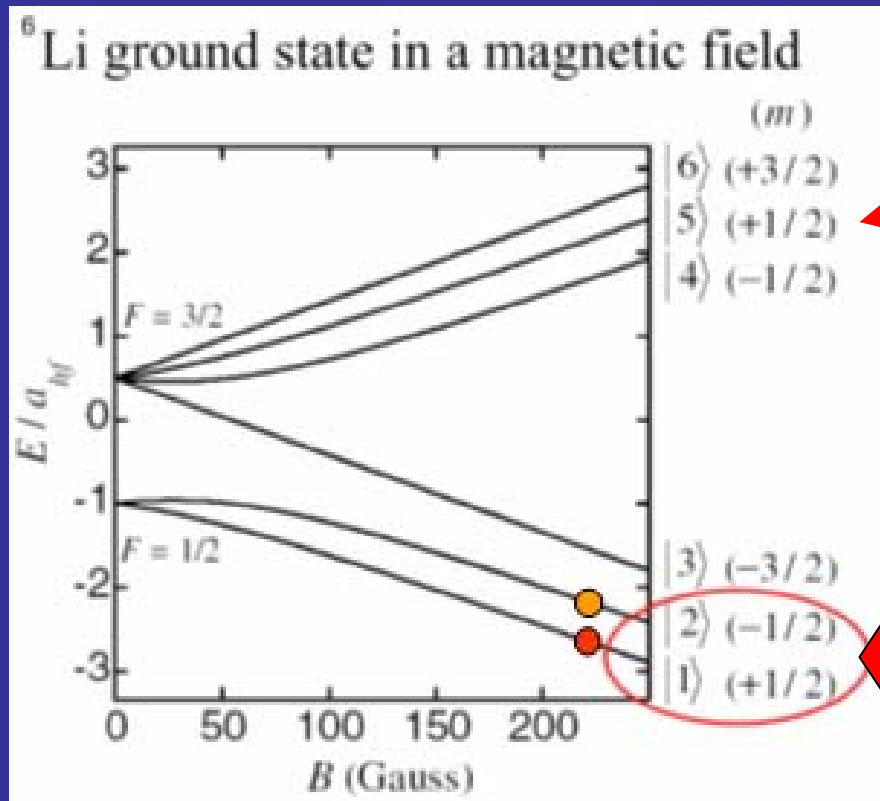
$$\text{PRESSURE: } P = -\frac{\partial E}{\partial V} = \frac{2}{5} \xi(x) \varepsilon_F \frac{N}{V}$$

$$PV = \frac{2}{3} E$$

Note the similarity to  
the ideal Fermi gas



# One fermionic atom in magnetic field



$$|F m_F\rangle$$

$$\vec{F} = \vec{I} + \vec{J}; \quad \vec{J} = \vec{L} + \vec{S}$$

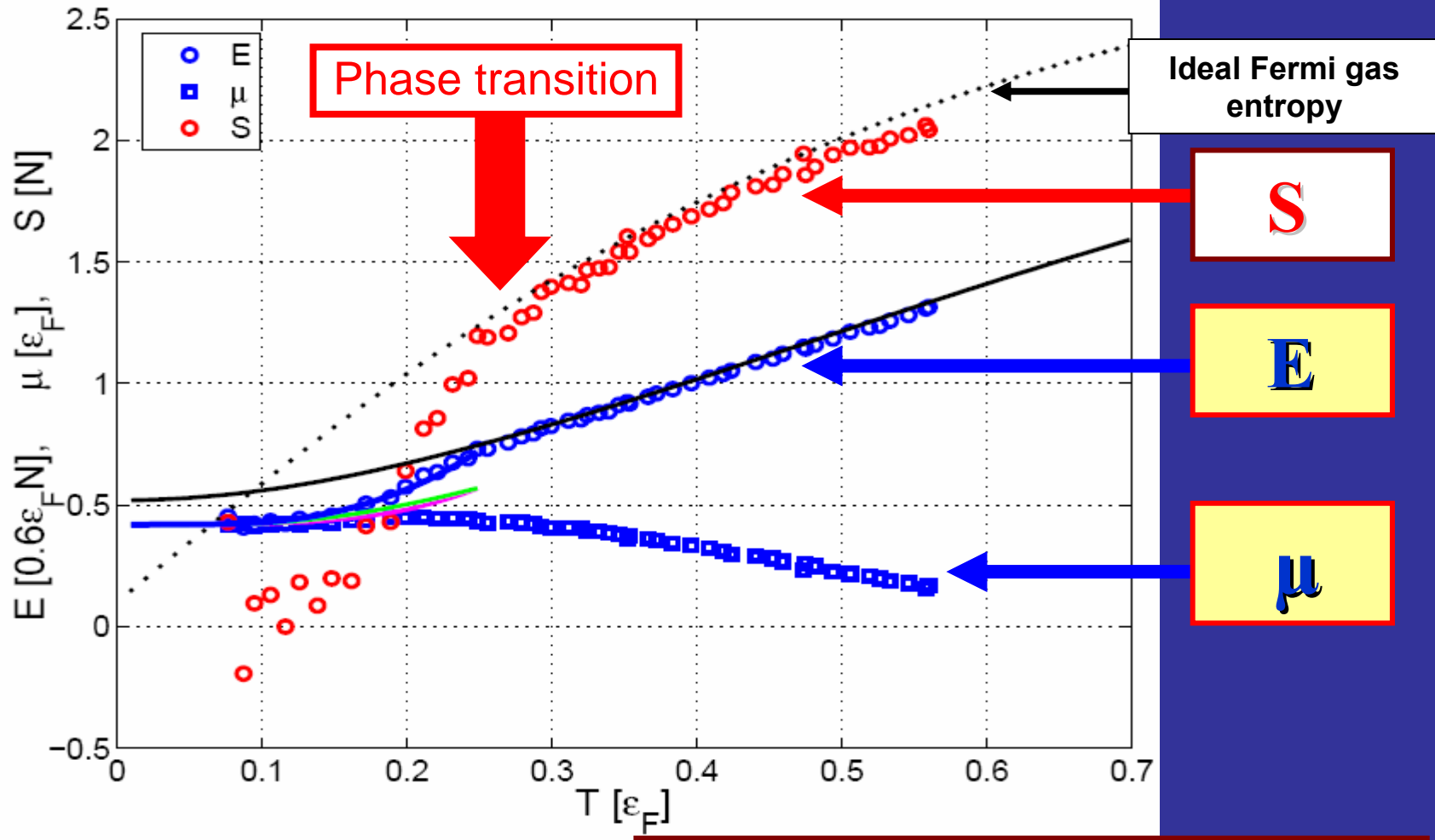
Nuclear spin

Electronic spin

Two hyperfine states are populated in the trap

Collision of two atoms: At low energies (low density of atoms) only  $L=0$  (s-wave) scattering is effective.

- Due to the high diluteness atoms in the same hyperfine state do not interact with one another.
- Atoms in different hyperfine states experience interactions only in s-wave.



$$E = \frac{3}{5} \varepsilon_F(n) N \xi \left( \frac{T}{\varepsilon_F(n)} \right)$$

$$n = \frac{N}{V} = \frac{k_F^3}{3\pi^2}, \quad \varepsilon_F(n) = \frac{\hbar^2 k_F^2}{2m}$$

$$S(T) = S(0) + \int_0^T \frac{\partial E}{\partial T} \frac{dT}{T}$$

$$S(T) = \frac{3}{5} N \int_0^{T/\varepsilon_F} dy \frac{\xi'(y)}{y}$$