69^{th} Scottish Universities Summer School in Physics

St.Andrews

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BSM phenomenology and EWSB (BSM)

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1 Extra dimensions: warm up

The idea behind extra dimensional theories is to extend space time to include more than 3 spacial dimensions. As we can see with our own eyes, the world has only 3 space dimensions, thus the extra ones must be hidden: this can be done either by compactifying them (making them smaller than our microscope's resolution), or assume that the particles we are made of are bounded to live on a 4-dimensional subspace (brane).

Another choice is the metric of the space: while the 4 visible dimensions are basically flat (Minkowski metric), the extra ones may be curved. Let's start with the simplest case of a compact flat extra dimension: what are the physical implications of its existence?

1.1 A 5D scalar field

The action is simply extended to

$$\mathcal{S}_s = \int d^5x \; (\partial^M \Phi)^\dagger \partial_M \Phi - M^2 \Phi^\dagger \phi \,, \tag{1.1}$$

where $M = \mu, 5$ labels the 5 directions in space-time, and

$$\Phi = \Phi(x^\mu, x_5)\,. \tag{1.2}$$

From the above action we can derive the usual Klein-Gordon equation of motion:

$$-\partial^M \partial_M \Phi - M\Phi = -\partial^\mu \partial_\mu \Phi + \partial_5^2 \Phi - M\Phi = 0\,. \tag{1.3}$$

The simplest compact space is a circle, i.e. a space where we impose periodic conditions on the fields:

$$\Phi(x^\mu, x_5 + 2\pi R) = e^{i\alpha_\Phi} \Phi(x^\mu, x_5)\,; \tag{1.4}$$

in general a non-zero phase α_Φ (Scherk-Schwarz phase) may be imposed, for simplicity here we will only consider periodic fields and we will set $\alpha_\Phi = 0$. If we want to go to momentum space, along the visible directions the usual Fourier transform applies; on the other hand, along x_5 we need to Fourier expand in a series of functions (the domain of the function of x_5 is finite!):

$$\Phi(x^\mu, x_5) = \int \frac{d^4p}{(2\pi)^4} \sum_n f_n(x_5)\varphi_n(p^\mu)\,; \tag{1.5}$$

where p^μ is the usual 4D momentum, f_n is a complete set of functions on the compact extra space (*wave functions*), and the "coefficients" $\varphi_n(p^\mu)$ can be interpreted as 4D fields (*Kaluza Klein modes*). Plugging this expansion in the equation of motion, we obtain a set of equations for f_n:

$$(p^2 - M^2)f_n - \partial_5^2 f_n = 0 \tag{1.6}$$

whose solutions are

$$\sin\sqrt{p^2 - M^2}x_5\,, \qquad \cos\sqrt{p^2 - M^2}x_5\,. \tag{1.7}$$

The periodicity implies that

$$\sqrt{p^2 - M^2} = n/R \tag{1.8}$$

where n is positive integer. If we interpret p^2 as the 4D mass of the 4D field

$$p^2 = m_n^2 = \frac{n^2}{R^2} + M^2 = n^2 m_{KK}^2 + M^2\,, \quad m_{KK} = 1/R\,. \tag{1.9}$$

The complete expansion of the field is then (where we have properly normalised the wave functions f_n)

$$\Phi(p^\mu, x^5) = \frac{1}{2\pi R}\varphi_0 + \sum_{n=1}^{\infty} \frac{\cos nx_5/R}{\pi R}\varphi_{n,c} + \sum_{n=1}^{\infty} \frac{\sin nx_5/R}{\pi R}\varphi_{n,s}\,, \tag{1.10}$$

with effective 4D action

$$\mathcal{S}_s = \int d^4x\; (\partial^\mu\varphi_0)^\dagger\partial_\mu\varphi_0 + \sum_n (\partial^\mu\varphi_{n,c/s})^\dagger\partial_\mu\varphi_{n,c/s} - (M^2 + n^2 m_{KK}^2)\varphi_{n,c/s}^\dagger\varphi_{n,c/s}\,. \tag{1.11}$$

1.2 Orbifold

Starting from the circle, more spaces can be defined by using the symmetries of the circle itself: one can in fact identify points mapped one into the other by such symmetry. For instance, the circle is invariant under a mirror symmetry with respect to any diameter:

$x_5 \to -x_5$. If a circle is defined for $x_5 \subset [-\pi R, \pi R)$, then the mirror symmetry identifies positive and negative points. The resulting space (the interval) is defined on $x_5 \subset [0, \pi R]$.

On the fields, the orbifold projection means that each field must satisfy:

$$\Phi(p^\mu, -x_5) = \pm\Phi(p^\mu, x_5)\,. \tag{1.12}$$

Each field is characterised by a sign choice; the wave functions that do not respect the transformation properties are then removed.

$$\Phi^+ = \frac{1}{2\pi R}\varphi_0 + \sum_{n=1}^{\infty} \frac{\cos n x_5/R}{\pi R}\varphi_{n,c}\,; \tag{1.13}$$

$$\Phi^- = \sum_{n=1}^{\infty} \frac{\sin n x_5/R}{\pi R}\varphi_{n,s}\,. \tag{1.14}$$

Note that the massless $n = 0$ mode is only present for Φ^+; both choices have a tower of massive states with the same mass but different wave functions.

1.3 A 5D vector (gauge) field

The action can be written as (for an abelian gauge group):

$$\begin{aligned} \mathcal{S}_{gauge} &= \int d^5x\ -\frac{1}{4}F_{MN}F^{MN} = \\ &= \int d^5x\ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}F_{\mu 5}F^{\mu}_{5} = \\ &= \int d^5x\ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu A_5 \partial^\mu A_5 + \frac{1}{2}\partial_5 A^\mu \partial_5 A^\mu - \partial_\mu A_5 \partial_5 A^\mu\,. \end{aligned} \tag{1.15}$$

The $\mu 5$ term generate a mixing between the 4D vector components A_μ and the 4D scalar term A_5: this is similar to the mixing we obtain in the SM between the massive vectors and the Goldstone components of the Higgs field. To simplify the equations, we can add a "gauge fixing" term to the action, which is a total derivative that can cancel out the mixing term and decouple the vector and the scalar. The extra dimensional R_ξ gauge fixing term is then:

$$\mathcal{S}_{GF} = \int d^5x\ -\frac{1}{2\xi}\left(\partial_\mu A^\mu - \xi \partial_5 A^5\right)^2\,. \tag{1.16}$$

$$\mathcal{S}_{gauge+GF} = \int d^5x\ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}\partial_5 A^\mu \partial_5 A^\mu + \tag{1.17}$$

$$+\frac{1}{2}\partial_\mu A_5 \partial^\mu A_5 - \frac{\xi}{2}(\partial_5 A^5)^2\,. \tag{1.18}$$

Vector

The equation of motion for the vector part is

$$\partial^\mu F_{\mu\nu} + \frac{1}{\xi}\partial_\nu\partial^\mu A_\mu - \partial_5^2 A_\nu = 0\,. \tag{1.19}$$

We can Fourier transform and expand the field as before

$$A_\mu(p^\mu, x_5) = \sum_n f_n A_\mu^n\,, \tag{1.20}$$

and, assuming that the 4D fields A_μ^n satisfy the usual 4D equation of motion in ξ gauge for a massive state,

$$\partial^\mu F_{\mu\nu} + \frac{1}{\xi}\partial_\nu\partial^\mu A_\mu = -p^2 A_\nu\,. \tag{1.21}$$

we have the following equation for the wave functions

$$(p^2 + \partial_5^2) f_n = 0\,, \tag{1.22}$$

which is the same as in the scalar case (but with $M = 0$). The final KK expansion is therefore analogous to the scalar one. The spectrum contain one massless gauge bosons (thus in 4D gauge symmetries are respected), and a tower of massive states. Where do the massive state get the longitudinal polarisation, as there is no Higgs field here?

Scalar (and the extra dimension "Higgs" mechanism)

The equation of motion for the A_5 scalar reads:

$$(\partial_\mu\partial^\mu - \xi\partial_5^2) A_5 = 0\,, \tag{1.23}$$

which is similar to the one for a 5D scalar, with the exception of the parameter ξ. After the usual Fourier expansion, the equation for the wave functions is:

$$\left(\frac{p^2}{\xi} + \partial_5^2\right) f_n = 0\,, \tag{1.24}$$

thus the expansion is the same as above, except for the substitution $p^2 \to \frac{p^2}{\xi}$.

The masses will therefore be

$$m_n^2 = \xi n^2 m_{KK}^2\,, \tag{1.25}$$

which look like the masses of a Goldstone boson in the "Higgs" mechanism. Note that the only mode whose mass is independent on ξ is the zero mode $n = 0$. What we learn, therefore, is that the massive modes of the scalar polarisation A_5 are the Goldstone bosons eaten up by the massive vectors! The only physical mode is the $n = 0$ mode (that corresponds to the massless vector).

Gauge invariance

The 5D action is invariant under a generalised gauge transformation:

$$A_M \to A_M + ig\partial_M \alpha(x^\mu, x_5)\,. \tag{1.26}$$

The local gauge parameter α must satisfy the same properties as the gauge field A_M, thus it also is a periodic function of x_5. We can therefore Fourier expand both A_μ and α, and write down 4D gauge transformations for each KK mode:

$$A^n_\mu \quad \to \quad A^n_\mu + ig\partial_\mu \alpha^n(x^\mu)\,. \tag{1.27}$$

Naively, we would expect the presence of an infinite number of gauge groups, however, as shown in the mass spectrum, the extra polarisation "spontaneously breaks" the gauge invariance associated with the massive modes; only the 4D gauge invariance of the massless mode is (explicitly) preserved.

Caveat: the Fourier expansion of the gauge transformation properties is a bit naive, one should really consider the gauge transformation on 5D fields!

Orbifold

We can now extend the analysis to orbifolds. As before, the field must be associated with a parity under the orbifold symmetry. However, the parities of the A_μ and A_5 components are related to each other by the fact that they belong to a vector! So if the orbifold symmetry is $x_5 \to -x_5$ (change sign to the 5th component of the vector but not to the other 4), the parity assignment for the 5D vector must be

$$A_\mu(-x_5) = \pm A_\mu(x_5)\,, \qquad A_5(-x_5) = \mp A_5(x_5)\,, \tag{1.28}$$

in other words their parity must be opposite!

For a + vector, the scalar is -: in this case, the vector contains a massless zero mode and massive vectors (with cos wave function), while the scalars only contain a tower of Goldstone bosons (with wave function sin).

For a - vector, the scalar is -: now the vectors only contain a tower of massive states (sin), while the scalars contain a physical massless scalar and a tower of Goldstone bosons (cos).

Note also that for - vectors, the 4D gauge symmetry is broken, as signalled by the absence of a massless vector in the KK expansion! However, a massless scalar is present!

1.4 A 5D fermion

The Dirac Gamma matrices must be generalised to 5D, i.e. we need to define a set of 5 (not 4) anticommuting matrices. The natural choice is to promote γ^5 to the role of the gamma matrix for the 5th direction. The minimal spinor is now a 4-component one, and it is not possible to define chiral projections. The action is

$$\mathcal{S}_f = \int d^5x \; i\bar{\Psi}\Gamma^M \partial_M \Psi - m\bar{\Psi}\Psi \,, \tag{1.29}$$

where the 5D fermion can be described in terms of 2 2-component Weyl fermions:

$$\Psi = \left(\begin{array}{c} \chi \\ \bar{\eta} \end{array} \right) . \tag{1.30}$$

In terms of Weyl fermions, the action reads

$$\mathcal{S}_f = \int d^5x \; - i\bar{\chi}\bar{\sigma}^\mu \partial_\mu \chi - i\eta\sigma^\mu\partial_\mu\bar{\eta} - \bar{\chi}\partial_5\bar{\eta} + \eta\partial_5\chi + m(\bar{\chi}\bar{\eta} + \eta\chi) \,; \tag{1.31}$$

from which we can derive the following equations of motion

$$-i\bar{\sigma}^\mu\partial_\mu\chi - \partial_5\bar{\eta} + m\bar{\eta} \;\; = \;\; 0 \,, \tag{1.32}$$

$$-i\sigma^\mu\partial_\mu\bar{\eta} + \partial_5\chi + m\chi \;\; = \;\; 0 \,. \tag{1.33}$$

The KK decomposition is in the form

$$\chi = \sum_n g_n(x_5)\chi_n(x^\mu) \,, \quad \bar{\eta} = \sum_n f_n(x_5)\bar{\eta}(x^\mu) \,, \tag{1.34}$$

where χ_n and $\bar{\eta}_n$ are usual 4D Weyl spinors.

The usual procedure can be followed: we can plug the expansions in the equations of motion, use the 4D equations of motion to replace derivatives with the 4D momenta and combine the two equations. We obtain that both f_n and g_n must satisfy the same equations of motion as a massive scalar field!

Note that on a circle, both chiral fields η and χ have a massless mode! In order to have a massless spectrum that corresponds to the SM fermions, we need to remove one or the other in order to have a chiral spectrum!

Orbifold

The orbifold symmetry changes sign to x_5: in order for the kinetic term to be invariant, the parities of χ and $\bar{\eta}$ must be opposite! This implies that only one of the two chiralities will have a zero model.

The massive modes of the two chiralities will be combined to form a massive Dirac fermion. The orbifold is thus an essential ingredient for Model Building!

Note also that the mass term is forbidden exactly for the same reason.

Odd mass terms

Another possibility is to assume that the mass term is odd under the orbifold symmetry: this is not entirely inconsistent, because the fundamental domain of the orbifold is an interval where the mass is uniform. So, let's force the presence of a mass term!

The most obvious problem we encounter is that the mass term would like to couple the two zero modes to form a Dirac fermion of mass m, however one of the two chiralities is removed.

If we remove the η chirality, the equations of motion for the zero mode reduce to:

$$\partial_5 g_0 + m g_0 = 0\,, \qquad g_0(x_5) \sim e^{-m x_5}\,. \tag{1.35}$$

The wave function of a left-handed mode, therefore, is exponentially localised toward the $x_5 = 0$ boundary of the space (for $m > 0$)

For right-handed zero modes

$$-\partial_5 f_0 + m f_0 = 0\,, \qquad f_0(x_5) \sim e^{+m x_5}\,; \tag{1.36}$$

thus it is localised toward the other boundary.

This trick allows us to localise the massless modes toward one or the other boundary.

2 First model: Gauge-Higgs Unification in flat space

Because of the chiral SM fermions, we need to use an orbifold. Our goal is to build a model where the Higgs is the A_5 of a bulk gauge boson; in order to have couplings between the Higgs and the electroweak gauge bosons, the SU(2)×U(1) gauge bosons and the Higgs should be unified into a single gauge group G, with the following features:

- G must contain at least $3_{SU(2)} + 1_{U(1)} + 4_H = 8$ generators;
- at the level of zero modes, only SU(2)×U(1) must survive, i.e. the orbifold must break $G \to$SU(2)×U(1);
- breaking a gauge group corresponds to assign a parity + for the unbroken generators, and - for the broken ones. This must be done in a consistent way, i.e. a gauge boson can be mapped into itself up to a gauge transformation:

  $$A_\mu(-x_5) = U A_\mu(x_5) U^\dagger\,,$$

  where U is a gauge transformation of G. In particular, this preserves the rank of the original group G and the rank of the preserved gauge group;
- at zero mode level, a doublet of SU(2) with non-zero hypercharge should survive.

An attractive possibility is to use SU(3): it has rank 2 (like SU(2)×U(1)), 8 generators and it can be broken to SU(2)×U(1) with

$$U = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{2.1}$$

The parity assignments of the gauge components will therefore be:

$$\begin{pmatrix} + & + & - \\ + & + & - \\ - & - & + \end{pmatrix} ; \tag{2.2}$$

The 2×2 block corresponds to SU(2) generators, the + in the lower corner to a U(1) generators, finally the 4 components with parity - will provide the Higgs candidate, as they transform like a doublet under SU(2). The extra polarisation will have opposite parities.

Spectrum

The spectrum of vector bosons will contain SU(2) gauge bosons $W^\pm$ and W^3, which contain a zero mode and a tower of massive modes; a U(1) gauge boson BB with same spectrum as the SU(2) ones; two charged gauge bosons, with the same quantum numbers as the Higgs, $C^\pm$ and $D^\pm$, that have no zero mode and just a tower of massive modes. They are embedded in the SU(3) structure as:

$$A_\mu = \begin{pmatrix} \frac{1}{2}W^3_\mu - \frac{1}{\sqrt{12}}B_\mu & \frac{1}{\sqrt{2}}W^+ & \frac{1}{\sqrt{2}}C^+ \\ \frac{1}{\sqrt{2}}W^- & -\frac{1}{2}W^3_\mu - \frac{1}{\sqrt{12}}B_\mu & \frac{1}{\sqrt{2}}D^+ \\ \frac{1}{\sqrt{2}}C^- & \frac{1}{\sqrt{2}}D^- & \frac{2}{\sqrt{12}}B_\mu \end{pmatrix} . \tag{2.3}$$

The scalar sector will only contain a massless doublet of SU(2), that well play the role of the Higgs, embedded in SU(3) as:

$$A_5 = \begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{2}}\phi^+ \\ 0 & 0 & \frac{1}{\sqrt{2}}\phi_0 \\ \frac{1}{\sqrt{2}}\phi^- & \frac{1}{\sqrt{2}}\phi_0^* & 0 \end{pmatrix} . \tag{2.4}$$

At three level, the Higgs will not have any potential, because it can only come from the gauge boson action:

$$\mathcal{S} = \int d^5x \; -\frac{1}{2}\mathrm{Tr}\, F_{MN}F^{MN} , \qquad F_{MN} = \partial_M A_N - \partial_N A_M + g(A_M A_N - A_N A_M) . \tag{2.5}$$

No A_5^2 nor A_5^4 terms are present in this action! So, the potential for the Higgs is generated at one loop. We expect it to be **finite**, because the tree level action does not contain a counterterm either for the mass or quartic coupling! Note that this is true at all perturbation orders!

Potential issues

- the Higgs field is a gauge boson, so it couples to all particles with strength g. What about fermion masses? To obtain masses below m_W, we can use the mass trick to localise the light quarks towards the two boundaries of the space, in order to reduce the overlap to the Higgs.

- How about the top mass? This is a crucial issue, as the localisation can only suppress the couplings with respect to g. One may use gauge group factors to enhance the coupling.

- how about the Higgs mass? The potential is one-loop generated, so the mass should be rather small. The precise value depends on the details.

2.1 The Higgs potential

The Higgs potential is generated completely at one loop. Only the zero mode will be sensitive to the eventual negative mass, thus the vacuum solution must be independent on the extra coordinate x_5. This implies that no tree level mixing with the heavy gauge bosons will be generated! The reason is that modes with different mass have orthogonal wave functions.

The Hosotani mechanism

Let's assume that the Higgs does develop a VEV that breaks SU(2)×U(1) → U(1): the vacuum will have the SU(3) embedding

$$\langle A_5 \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}}\langle \phi_0 \rangle \\ 0 & \frac{1}{\sqrt{2}}\langle \phi_0 \rangle & 0 \end{pmatrix} . \tag{2.6}$$

It is always possible to find a gauge transformation $\Omega(x_5)$ such that

$$\langle A_5' \rangle = \Omega(x_5) \langle A_5 \rangle \Omega^\dagger(x_5) = 0\,; \tag{2.7}$$

so that the Higgs VEV disappears from the action. Expanding at leading order in the gauge transformation parameter:

$$\langle A_5 \rangle + i\partial_5(\alpha x_5) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{pmatrix} = 0\,, \qquad \alpha = \frac{1}{\sqrt{2}} \langle \phi_0 \rangle\,. \tag{2.8}$$

The same transformation must be applied to the gauge vectors:

$$A_\mu' = \Omega(x_5) A_\mu \Omega^\dagger(x_5)\,. \tag{2.9}$$

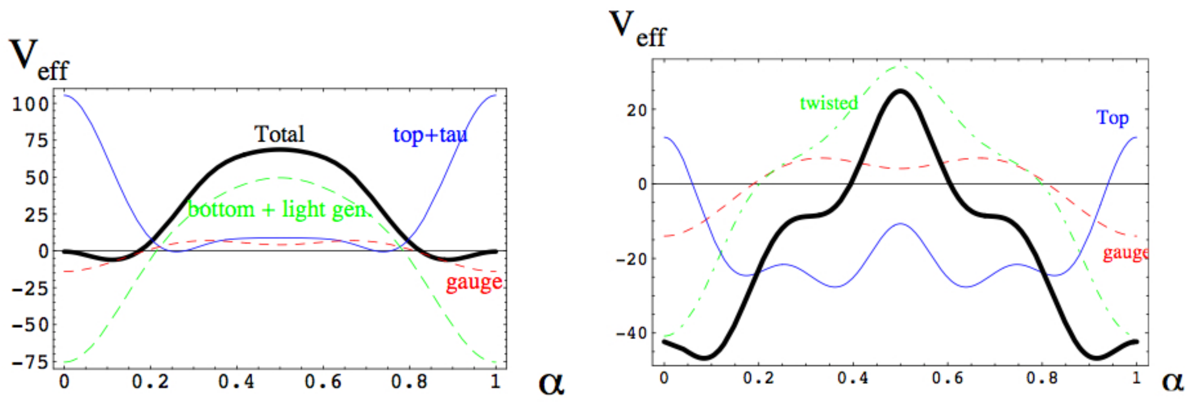

Figure 1: One-loop Higgs potential in two variations of the model, distinguished by the representations of SU(3) the SM fermions are embedded in. [hep-ph/0510366]

What does it change in the theory? The action is invariant, however the periodicity condition on the field A'_μ is different from before:

$$\begin{aligned} A'_\mu(x_5 + 2\pi R) &= \Omega(x_5 + 2\pi R) A_\mu(x_5) \Omega^\dagger(x_5 + 2\pi R) \\ &= \Omega(x_5 + 2\pi R)\Omega^\dagger(x_5) A'_\mu(x_5) \Omega(x_5) \Omega^\dagger(x_5 + 2\pi R)\,. \end{aligned} \tag{2.10}$$

Working out the algebra, we find that this is equivalent to imposing different Scherk-Schwarz phases on the different components of the SU(3) adjoint. Note that the gauge transformation is equivalent to a SS phase only in the orbifolds that do allow for SS phases.

The spectrum for the $W^\pm$ bosons will not be modified by the gauge transformation, and it will depend on the parameter α, which is in turn related to the Higgs VEV:

$$m_n^{W^\pm} = \frac{n+\alpha}{R}\,, \qquad m_{W^\pm} = \frac{\alpha}{R}. \tag{2.11}$$

The numerical value of α will therefore determine the relation between the SM W mass and the KK mass $m_{KK} = 1/R$.

Numerical results

The calculation of the potential is rather complex: as we know the spectrum as a function of the Higgs VEV α, we can use the Weinberg-Coleman potential:

$$V_{eff}(\alpha) = \pm \frac{1}{2} \sum_i \int \frac{d^4 p}{(2\pi)^4} \log[p^2 + M_i^2(\alpha)]\,. \tag{2.12}$$

The results, in two variations of the model, are shown in Figure 1: it's interesting that the contribution of the gauge bosons (red/dashed) and of light fermions (green/dot-dashed)

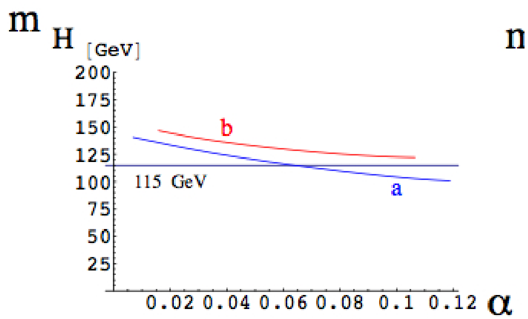

Figure 2: Higgs mass as a function of the Higgs VEV α for two variations of the model. [hep-ph/0510366]

have minima at $\alpha \sim \langle \varphi_0 \rangle = 0$, while it is the contribution of the top loops (blue/solid) that generates a non trivial vacuum. From the potential we can also calculate the Higgs mass, which is proportional to the second derivative of the potential. The results are shown in Figure 2. The two curved correspond to two versions of the model, and they are obtained by scanning over some free parameters of the model (the masses that control the fermion localisation). Like in supersymmetry, a fairly light Higgs is preferred; furthermore, $m_h = 125$ GeV can be easily obtain for small values of $\alpha = m_W/m_{KK}$.

3 Second model: Gauge-Higgs Unification in warped space, or a composite Higgs

A warped extra dimension (or Randall-Sundrum space) has been widely studied, because it can fairly easily generate hierarchies between mass scales. In models of Gauge-Higgs, it offers two main advantages: it automatically enhances both the Higgs and the top mass.

The difference between flat and warped space is the metric: the simple Minkowsky metric in flat space is replaced by

$$ds^2 = e^{-2x_5/R} dx^\mu dx_\mu - dx_5^2\,, \quad x_5 \subset [0, L]\,. \tag{3.1}$$

This metric has an interesting property, conformal invariance, which is more evident if we rewrite it in terms of $z = Re^{x_5/R}$:

$$ds^2 = \left(\frac{R}{z}\right)^2 (dx^\mu dx_\mu - dz^2)\,, \quad z \subset [R, R' = Re^{L/R}]\,. \tag{3.2}$$

An increase in the value of $z \to \xi z$ can be compensated by an analogous rescaling of $x_\mu \to \xi x_\mu$ to leave ds^2 invariant: thus moving along the extra co-ordinate corresponds to a rescaling of the size (and therefore of the energy) of physical systems. One can chose the two scales in the metric so that $R^{-1} \sim M_{Pl}$ and $(R')^{-1} \sim 1$ TeV: moving from the boundary at $z = R$

(Planck scale) to the $z = R'$ one (TeV brane) will rescale energy scales from the Planck scale down to the TeV. Note that the length of the interval is $L = R \log R'/R$.

A gauge boson in the warped space will have an action

$$\mathcal{S}_{gauge} = -\frac{1}{4} \int d^4x\, dz\, \left(\frac{R}{z}\right)^5 F_{MN} F^{MN} = -\frac{1}{4} \int d^4x\, dz\, \left(\frac{R}{z}\right) (F^{\mu\nu}_{\mu\nu} - 2 F_{\mu z} F^{\mu}_{z})\,. \quad (3.3)$$

The factors of R/z come from the metric. As in the flat case, a gauge fixing term is added to remove A_μ-A_5 mixing:

$$\mathcal{S}_{GF} = -\frac{1}{2\xi} \int d^4x\, dz\, \left(\frac{R}{z}\right) (\partial^\mu A_\mu - \xi z \partial_z (A_5/z))^2\,. \quad (3.4)$$

The equation of motion for the wave function of a vector are

$$z \partial_z \left(\frac{1}{z} \partial_z f_n\right) + m_n^2 f_n = 0\,, \quad (3.5)$$

whose solutions can be expressed in terms of Bessel functions of the first and second kind:

$$f_n = z\, (A\, J_1(m_n z) + B\, Y_1(m_n z))\,. \quad (3.6)$$

For the scalars, the equation of motion reads:

$$\partial_z \left(z \partial_z \left(\frac{A_5}{z}\right)\right) + \frac{m_n^2}{\xi} A_5 = 0\,. \quad (3.7)$$

As before, massive mode are Goldstone bosons eaten by the massive vectors, while for the zero mode

$$A_5 \sim z\,. \quad (3.8)$$

3.1 Custodial symmetry?

We may want to try constructing a SU(3) model: however this is not acceptable in warped space. The difference with respect to the flat case is that the Higgs vev depends linearly on the extra co-ordinate, thus mixing between various KK modes is possibly generated by it. In the flat case:

$$\langle A_5 \rangle W_n^+ W_m^0 \sim v \int f_n(x_5) f_m(x_5) = 0 \quad (3.9)$$

because the two wave functions are orthogonal. In the warped case, this is not true, therefore tree level corrections to the electroweak precision measurements are usually generated, in particular to the ρ parameter. In order to protect it, we need to use a gauge group that includes a custodial symmetry: SO(5).

- on the TeV brane, we can break SO(5)$\rightarrow$ SO(4): the SO(4) $\sim$ SU(2)$\times$ SU(2) contains the desired custodial symmetry (the breaking of this symmetry will be achieved via the Higgs VEV).
- the generators of SO(5) that do not belong to the unbroken subgroup SO(4) form a 4 of SO(4), like the Higgs field in the SM! These fields will play the role of the Higgs.
- on the Planck brane, we break SO(5)$\rightarrow$SU(2)$\times$U(1), so that only the SM invariance is preserved. As it is a subgroup of the unbroken SO(4), only the SU(2)$\times$U(1) gauge bosons have zero modes, as desired.
- for the scalars, only the Higgs has a zero mode.

This structure of the symmetry breaking is enough to ensure that the values of the W and Z mass respect the SM relations at tree level (thus $\rho = 1$ at tree level).

3.2 AdS/CFT

The presence of a conformal symmetry in the metric suggests a correspondence between models in warped space (anti de Sitter) and strongly interacting conformal theories in 4 dimensions. The correspondence goes as follows:

- fields and symmetries on the Planck brane correspond to the elementary sector of the theory (like the photon in QCD);
- fields in the bulk correspond to operators (bound states) of the conformal sector, the TeV brane breaks the conformal invariance and generates a mass gap (tower of meson resonances);
- symmetries in the bulk and on the TeV brane correspond to global symmetries of the strong sector (so, our strong sector is invariant under SO(5) which is spontaneoulsy broken to SO(4)).

Thus our model can be seen as the SM (the Planck brane is invariant under the SM gauge group) coupled to a conformal sector which is invariant under a global O(4) (that generated the custodial symmetry!).

The properties of all the fields depend on their localisation in the extra space: the cartoon in Figure 3 shows the typical scenario. Gauge bosons have a flat profile (due to gauge invariance), while the Higgs is moderately localised toward the TeV brane. Light fermions, like leptons, light quarks and the right-handed bottom, are localised toward the Planck brane: they correspond to mostly elementary fields, and the localisation suppresses their overlap with the Higgs. The top is localised toward the TeV brane, thus it is a mostly composite state: its localisation enhances the overlap with the Higgs, thus it makes possible to achieve masses larger than m_W, even though the coupling is of the order of the gauge couplings. The Higgs, being localised toward the TeV brane, is also a composite state! All the massive resonances are also strongly localised to the TeV brane, thus showing their composite nature.

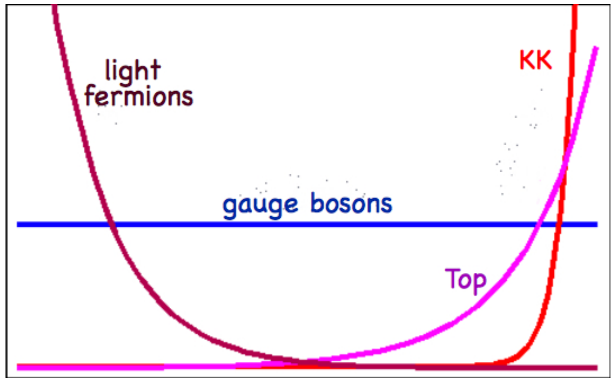

Figure 3: Wave functions in the warped model for gauge bosons, light fermions, tops and KK modes, showing their localisation: the Planck brane is on the right, the TeV brane on the left. The Higgs is also moderately localised towards the TeV brane.

3.3 Higgs potential and mass

The calculation of the Higgs potential proceeds as in the flat space, however the calculations are complicated by the presence of Bessel functions. In Figure 4 we show the predicted mass for the Higgs and the first KK resonance of various particles. The points correspond to different choices for the parameters of the model. While the KK resonances are naturally fairly heavy, above 1 TeV for fermions and above 3÷4 TeV for vectors, the Higgs mass is predicted to be light $m_H < 140$ GeV.

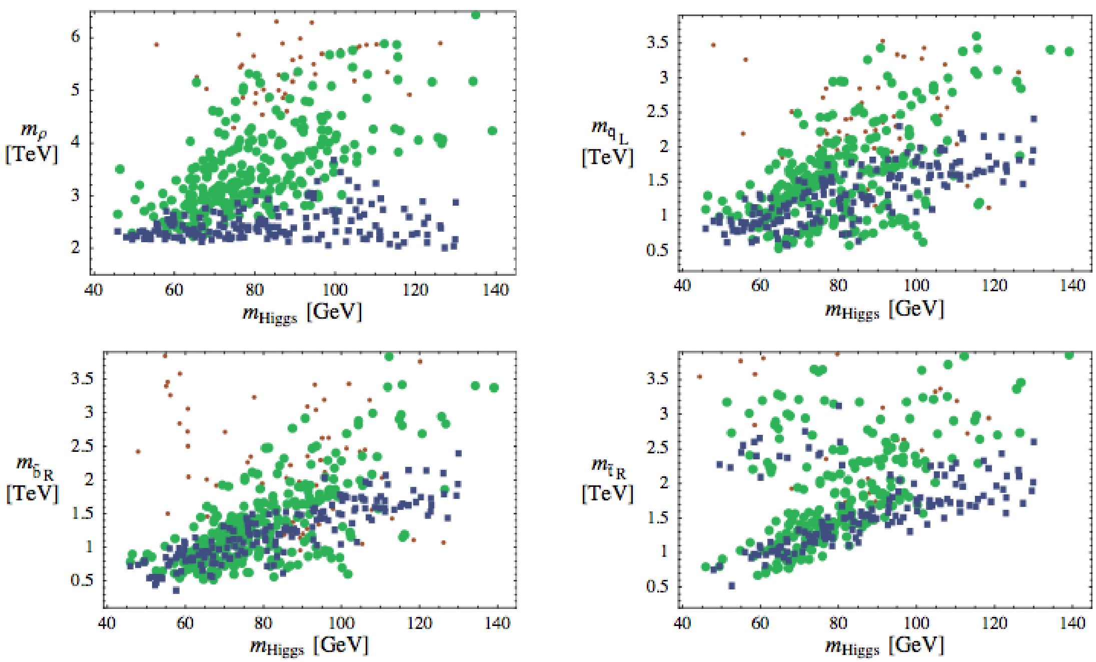

Figure 4: Masses of the first KK states for the gauge bosons m_ρ and third generation quarks: doublet q_L, singlet top t_R and singlet bottom b_R [from hep-ph/0412089]. The round green dots are preferred by electroweak precision measurements.