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BSM phenomenology and EWSB (BSM)

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1 Why do we need to go Beyond the Standard Model (BSM)?

The Standard Model of Particle Physics (SM for brevity) was proposed in the '60 and '70 by Sheldon Glashow, Stephen Weinberg and Abdus Salam to describe, in terms of fundamental degrees of freedom (or particles), the theory of electroweak interactions first proposed by E. Fermi in the '30. The effective 4-fermion interactions in Fermi's theory are replaced by the presence of massive gauge bosons, the charged $W^{\pm}$ and the neutral Z: while the W is required to explain the beta decay of atoms, the Z is a prediction of the gauge structure introduced in the model and its discovery in 1984 was a crucial validation of the theory. The model consists of the following sectors:

- $SU(3)_c$ gauge group (colour), that describes strong interactions (QCD);
- $SU(2)_L \times U(1)_Y$ that describes electroweak interactions, it is spontaneously broken to an exact $U(1)_{em}$ gauge group (QED);
- the Brout-Englert-Higgs sector, that describes the breaking of the electroweak symmetry (EWSB) by means of a scalar field that develops a vacuum expectation value;
- fermionic matter: the model contains 3 generations of fermions, each one consisting of two quarks (that describe the baryonic matter, like pions and nucleons), a charged lepton (electron) and a neutrino (predicted by W.Pauli);
- Yukawa interactions between fermions and the Higgs sector that give mass to the fermions and are responsible for flavour physics.

This apparently complex structure is based on symmetries! Gauge symmetries describe the behaviour and interactions of vector bosons, the force carriers. The generation of masses is based on a mechanism of spontaneous symmetry breaking: the theory is invariant under the $SU(2)_L \times U(1)_Y$ gauge symmetry, however the vacuum of the scalar BEH field is not invariant! The field content of each generation of fermions is fixed by the cancellation of gauge anomalies in the theory. The presence of 3 generations is responsible for a set of accidental flavour symmetries which are partially broken by the Yukawa interactions; a leftover is Baryon symmetry that prohibits the decay of the proton.

The least knows sector of the theory is the EWSB sector, i.e. the physics associated with the BEH field. In the Standard Model, this sector is described in terms of a complex scalar doublet of $SU(2)_L$. After the EWSB, 3 of the degrees of freedom of this field are eaten by the massive gauge bosons $W^\pm$ and Z providing them with a longitudinal polarisation, while the fourth is a physical scalar, the Higgs boson. The Higgs boson has eluded all experimental efforts to discover it, until July 2012 when the LHC collaborations CMS and ATLAS announced the discovery of a new resonance at a mass of 125 GeV, that has similar properties as the Higgs boson. This discovery seems to complete the validation of the Standard Model, as all of the predicted particles and interactions have now been observed. So, why do we still need to talk about New Physics?

1.1 Evidences and hints of New Physics (BSM)

There are several evidences and hints that seem to suggest the presence of New Physics: they can be roughly divided into two classes. On one hand, there are experimental results that cannot be explained within the Standard Model. On the other, purely theoretical considerations does not allow us to accept the SM as the ultimate theory. Here it follows a brief list:

- **EXP: neutrino masses.** The observation and patterns of neutrino oscillations suggest that at least two neutrino species have mass, and from cosmological observations we know that the mass scale is very small (sub-eV). The masses can be added by simply extending the SM with 3 right-handed singlet neutrinos. See-saw mechanisms would hint that the mass of the new states is at the $10^{3\div13}$ GeV.

- **EXP: Dark Matter in the Universe.** 23% of the total mass of our universe is made of non-barionic and non-luminous matter, which is unaccounted for in the SM. This observation is also supported by astrophysical observations: rotation curves of disk galaxies, gravitational microlensing of galaxy clusters and large structure formation models. A particle interpretation would suggest a weakly interacting particle with a mass of $\sim 100 \div 1000$ GeV. This new particle must be neutral and stable (on cosmological time scales).

- **EXP: Baryon asymmetry in the Universe.** The Universe is populated by baryons, however the number of anti-baryons is very scarce. To explain this, the mechanism of baryogenesis has been proposed: it requires 3 conditions, namely that baryon symmetry is broken (by anomalies in the SM), that the model violates CP and that there is a non-thermal process (the EW phase transition in the SM). However, in the SM, the amount of CP violation in the quark sector is not enough to explain the baryon density at present days by many orders of magnitude.

- **TH/EXP: gauge coupling unification.** Running the SM gauge couplings at high energies, their values tend to converge to the same value at $\sim 10^{15\div17}$ GeV. A new unified gauge theory (GUT) may be present at such energy scales.

- **TH: quantum gravity.** Classical gravity, well described by general relativity, should break down at energy scales close to the Planck mass $M_{Pl} \sim 10^{19}$ GeV, where quantum effects may arise. The SM is necessarily invalidated at such energy. This scale can be lowered in models where the fundamental Planck scale is lower (for instance, models with extra dimensions).

- **TH: Higgs mass (electroweak scale) instability.** The Higgs mass is sensitive to quadratically divergent radiative corrections, thus it is unstable. In physical terms, new states that couple to the Higgs will generate corrections to its mass which are proportional to the mass of the new state. As the mass of the Higgs cannot exceed ~ 1 TeV (or is 125 GeV), such states must be at the TeV scale in order to avoid large cancellations with the tree level mass (naturalness argument).

- **Dark Energy (?),** ...

From this non-exhaustive list, you can see that only a particle candidate for Dark Matter and the naturalness argument hint for New Physics, i.e. new particles beyond the Standard Model, at the TeV scale. The other cases can also be lowered to the TeV scale: TeV scale see saw in the case of neutrino masses, accelerated unification or gravity in extra dimensions (TeV scale Black Holes); however, this is not required! In these lectures we will be interested in the naturalness argument which involves the Higgs sector of the Standard Model (and, sometimes, Dark Matter). Before starting our journey, it is important to point out what the naturalness argument really is: it is a theoretical prejudice against the Standard Model as the fundamental theory of particle physics. In fact, the Standard Model is renormalisable, thus divergences can be simply reabsorbed in tree level parameters (the Higgs mass in this case) and, no matter how large they are, they leave no trace in observable quantities! The naturalness argument bases its power on the assumption that the SM is an effective model valid up to a cut-off, i.e. a high energy scale above which the model must be replaced by a more fundamental theory. And, we do know that above the Planck mass a theory of quantum gravity must replace the SM! One may conclude that a natural Higgs boson must have a mass close to the Planck scale (and thus goes the W and the Z bosons), unless a protection mechanism is at work. Protection mechanisms in particle physics are called **symmetries**. A central point in these lectures will be the role of symmetries in models of physics Beyond the Standard Model!

2 The EWSB sector, and the role of symmetries

In the SM, the electroweak symmetry breaking (EWSB) is *described* by the Brout-Englert-Higgs scalar particle. Such particle is a complex scalar, which transforms as a doublet under the SU(2)$_L$ symmetry and has hypercharge 1/2 (and no colour). The action for the BEH scalar is:

$$\mathcal{S}_{BEH} = \int d^4x \, (D^\mu \phi)^\dagger D_\mu \phi - m_\phi^2 \phi^\dagger \phi - \frac{\lambda}{2} (\phi^\dagger \phi)^2 \, ; \tag{2.1}$$

where

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi_0 \end{pmatrix} \tag{2.2}$$

is the doublet in components, the covariant derivative D_μ contains the gauge interactions

$$D_\mu \phi = \left(\partial_\mu - ig \sum_{a=1}^{3} W_\mu^a t^a - ig' \frac{1}{2} B_\mu \right) \phi \, . \tag{2.3}$$

You should be already very familiar with the fact that, for $m_\phi^2 < 0$, the potential for the BEH field in Eq. 2.1 has minima with $\langle \phi \rangle \neq 0$. In fact, the equation of motion for a static ϕ (i.e. such that $\partial_\mu \phi = 0$) is:

$$\left(-m_\phi^2 - \lambda \phi^\dagger \phi \right) \phi = 0 \, . \tag{2.4}$$

The solutions are $\langle \phi \rangle = 0$, which is a maximum, and $\langle \phi^\dagger \phi \rangle = -\frac{m_\phi^2}{\lambda}$ which is a minimum. The theory is invariant under SU(2)$_L \times$U(1)$_Y$ gauge symmetries (in fact the lagrangian is invariant as well as the minimum conditions), thus we can use SU(2)$_L \times$U(1)$_Y$ transformations to write the solution as

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \, , \quad v^2 = -2 \frac{m_\phi^2}{\lambda} \, . \tag{2.5}$$

This solution is NOT invariant under the gauge symmetries, thus, when plugged back into Eq. 2.1, it will give mass to 3 of the gauge bosons:

$$m_{W^\pm}^2 = \frac{g^2}{4} v^2 \, , \qquad m_Z^2 = \frac{g^2 + g'^2}{4} v^2 \, . \tag{2.6}$$

Now, 3 of the degrees of freedom in the BEH field are eaten up by the massive $W^\pm$ and Z, and one remains as a massive physical scalar:

$$\phi = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \, , \tag{2.7}$$

where the field h in this expansion is the Higgs boson, and it has a mass

$$m_h^2 = -2m_\phi^2 = \lambda v^2\,. \tag{2.8}$$

Why is this a *description* of EWSB? The mechanism relies on the fact that the mass has the wrong sign, i.e. $m_\phi^2 < 0$, however there is no explanation of what the origin of such negative mass is! Explaining the origin of the negative mass square can be addressed in BSM models!

2.1 Symmetries: exposed and hidden ones.

Let's have another look at the BEH action:

$$\mathcal{S}_{BEH} = \int d^4x\, (D^\mu\phi)^\dagger D_\mu\phi - m_\phi^2\phi^\dagger\phi - \frac{\lambda}{2}(\phi^\dagger\phi)^2\,. \tag{2.9}$$

Can you list all the symmetries this action is invariant under?
There are 3 kinds if symmetries:

1) symmetries of space time: Poincaré (rotations, translations, Lorentz boosts) and C, P and T.

2) gauge (local) symmetries: $SU(2)_L \times U(1)_Y$ (the colour $SU(3)_c$ plays no role here).

3) accidental (global) symmetries.

4) discrete symmetries.

The action in Eq. 2.9 contains a very important hidden global symmetry: the custodial symmetry. Let's for a moment ignore gauge interactions: if we do so, the action only depends on the combination $\phi^\dagger\phi$ (and $(\partial^\mu\phi^\dagger)\partial_\mu\phi$). This element can be written as

$$\phi^\dagger\phi = \varphi^-\varphi^+ + \varphi_0^*\varphi_0 = \frac{1}{2}\left(\varphi_1^2 + \varphi_2^2 + \varphi_3^3 + \varphi_4^2\right) \tag{2.10}$$

where $\varphi^+ = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}$ and $\varphi_0 = \frac{\varphi_3 + i\varphi_4}{\sqrt{2}}$ are decomposed in terms of real and imaginary parts. If we define a real 4-component field

$$\tilde{\Phi} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix}, \tag{2.11}$$

the action in Eq. 2.9 can be re-written as

$$\mathcal{S}_{EBH} = \int d^4x\, \frac{1}{2}(\partial^\mu\tilde{\Phi}^T)\partial_\mu\tilde{\Phi} - \frac{1}{2}m_\phi^2\tilde{\Phi}^T\tilde{\Phi} - \frac{\lambda}{4}(\tilde{\Phi}^T\tilde{\Phi})^2\,. \tag{2.12}$$

Now, it is evident that the action is invariant under an orthogonal rotation of the 4-component real vector $\tilde{\Phi}$, i.e. the action is invariant under O(4) $\sim$ SU(2)×SU(2)! This hidden global symmetry is the so-called custodial symmetry!

The SU(2)×SU(2) structure becomes evident if we rewrite the BEH field in the following way:

$$\Phi = \begin{pmatrix} \varphi_0^* & \varphi^+ \\ -\varphi^- & \varphi_0 \end{pmatrix} . \tag{2.13}$$

This field transforms as a bi-doublet under SU(2)×SU(2), i.e.

$$\Phi \to U_L \Phi U_R^\dagger$$

where $U_{L/R}$ are SU(2) transformations. The action in Eq. 2.9 can now be re-written as

$$\mathcal{S}_{EBH} = \int d^4x \; \mathrm{Tr}(\partial^\mu \Phi^\dagger)\partial_\mu \Phi - m_\phi^2 \mathrm{Tr}\Phi^\dagger\Phi - \frac{\lambda}{2}(\mathrm{Tr}\Phi^\dagger\Phi)^2 \, , \tag{2.14}$$

where $\mathrm{Tr}\Phi^\dagger\Phi$ is the matricial trace. In this notation, the vacuum solution can be written as

$$\langle\Phi\rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} . \tag{2.15}$$

This vacuum is invariant under a SU(2) symmetry, defined as the symmetry for which $U_L = U_R$! In other words, the Higgs vacuum is breaking SU(2) ×SU(2) $\to$ SU(2)! The preserved SU(2) global symmetry is very important in the SM: in fact, it protects the relative values of the W and Z mass against radiative corrections. This allows to define a parameter

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} \, , \qquad \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \, . \tag{2.16}$$

At three level, ρ is exactly 1. As the custodial symmetry is not an exact symmetry in the SM, radiative corrections tend to spoil the equality, however, due to the custodial symmetry, the corrections to ρ are of the order of 10^{-3}! Keeping such corrections small in models of new physics is a big challenge!

2.2 Radiative stability.

The Higgs mass (and thus the electroweak scale) suffers from divergent radiative corrections. To better understand this statement, let's closely look at the contribution from the top quark. In the SM, fermion masses are generated via Yukawa interactions. For the top

$$\mathcal{L}_{top} = y_t \bar{Q} \phi^\dagger t_R + h.c. \tag{2.17}$$

where $\bar{Q} = (\bar{t}_L, \bar{b}_L)$ is the left-handed SU(2) doublet and t_R the right-handed singlet. This interaction will generate a top mass $m_{top} = \frac{y_t v}{\sqrt{2}}$.

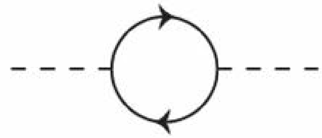

Figure 1: Top loop contribution to the Higgs mass.

The Yukawa interaction will also contribute to the ϕ mass via the loop diagram in Figure 1:

$$\begin{aligned}
-i\delta m_\phi^2 &= -3(iy_t)^2 \int \frac{d^4k}{(2\pi)^4} \, \mathrm{Tr} \frac{ik^\mu\gamma_\mu}{k^2}\frac{1+\gamma^5}{2}\frac{ik^\nu\gamma_\nu}{k^2}\frac{1-\gamma^5}{2} \\
&= -\frac{3y_t^2}{2(2\pi)^4}\int d^4k \, \frac{\mathrm{Tr}k^\mu\gamma_\mu \, k^\nu\gamma_\nu \, (1-\gamma^5)}{k^4} \\
&= -\frac{3y_t^2}{2(2\pi)^4}\int d^4k \, 4\frac{1}{k^2} \\
&= \frac{3y_t^2}{8\pi^4} \, i2\pi^2 \int k_E dk_E \\
&= i\frac{3y_t^2}{8\pi^2}\int dk_E^2
\end{aligned} \tag{2.18}$$

The last integral is clearly divergent. One way to regularise the divergence is to add a hard cut off to the integral, i.e. $\int dk_E^2 \to \int_0^{\Lambda^2} dk_E^2 = \Lambda^2$. Thus, the corrected ϕ mass would be

$$m_\phi^2 \to m_\phi^2 - \frac{3y_t^2}{8\pi^2}\Lambda^2 \, . \tag{2.19}$$

Why is the scalar mass so sensitive to the cut off of the theory? The ultimate reason is related so symmetries! Did you know that the responsible is the spin, i.e. rotation symmetry? In fact, for particles with non-zero spin, like spin-1 vector bosons and spin-$\frac{1}{2}$ fermions, the mass is protected against divergent corrections.

Spin 1

Spin 1 particles have 3 possible spin configurations. In the case of a massless vector boson, the lagrangian is invariant under gauge symmetries, and one can use a gauge transformation to remove one of the degrees of freedom. Physically, this corresponds to the fact that the photon has only 2 circular polarisations, but no longitudinal modes. Adding a non-zero mass to the vector, gauge symmetries are broken and the vector regains its lost polarisation. Thus there is a fundamental difference between massless and massive vector bosons.

Even for massive vectors, loop corrections to the masses are protected. In fact, for large momenta running in the loop (near the cut off), the mass of the vector can be neglected, thus a gauge symmetry is restored. The restored gauge symmetry makes the loop correction vanish near the cut off.

Spin 1/2

Spin 1/2 particles have two spin configurations. For fermions, a mass term connects left-handed to right-handed chiralities. For massless fermions, the two polarisations are independent, thus they can be considered as physically distinct fields. This fact increases the number of symmetries of the system!

Again, in the limit of small mass, the two polarisation decouple and the loop correction must vanish!

Spin 0

No such argument applies to scalar fields, which have a single spin configuration!

3 Symmetries and New Physics

The naturalness argument requires the presence of a symmetry (protection mechanism) to shield the Higgs mass from quadratically divergent loop corrections. We have seen that masses can be protected by the presence of spin, or more specifically by chirality in the case of fermions and gauge symmetries in the case of vectors. Ultimately, every new physics scenario tries to apply these two symmetries on the Higgs boson!

- **Spin-0 related to spin-1/2**: is there a symmetry that relates a scalar to a spin-1/2 particle? If so, the chirality will protect the fermion mass, and our new symmetry will project the protection on the scalar!

    a) **Supersymmetry**: it extends the Poincaré algebra to include a symmetry between particles with different spins!

- **Replace a spin-0 with spin-1/2**: can we trade the scalar Higgs for fermions? QCD does it: it is a theory based on quarks and vector gluons, however, due to the strong interacting regime, fermions form mesons which are scalars!

    b) **Technicolour**: the Higgs emerges as a composite state (meson) of new fermions bound together by a new strong interaction.

- **Replace a spin-0 with a spin-1 (vector)**: can we use gauge symmetries to protect the Higgs, i.e. embed the BEH scalar into a gauge boson?

c) **Extra dimensions**: in models with extra spacial dimensions, a vector boson has more then 3 polarisations. The extra polarisations appear, from the 4-dimensional point of view, as scalars, however they are secretly part of a vector and thus protected by extra dimensional gauge symmetries.

- **A special spin-0, global symmetries**: can global symmetries have any role in protecting the Higgs boson?

d) **Little Higgs models**: the Higgs arises as the Goldstone boson of a spontaneously broken global symmetry. The global symmetry is not exact, thus the Higgs develops a mass, however the model can be engineered to have finite one-loop corrections. It does not work beyond one loop!

4 Exercises

1) List all the symmetries of the Higgs sector of the SM, i.e. of the action in Eq. 2.1.

2) Consider the action in Eq. 2.14.

2a) Check that it is equivalent to the action in Eq. 2.1.

2b) Show that the SU(2) that generates the transformation U_L is the gauged $\mathrm{SU}(2)_L$ of the SM. Show also that the gauged $\mathrm{U}(1)_Y$ is the U(1) subgroup of U_R, generated by the diagonal generator

$$t^3 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}.$$

3) Consider a scalar field φ with action

$$\mathcal{S} = \int d^4x \, \frac{1}{2}(\partial^\mu \varphi)\partial_\mu \varphi - \frac{1}{2}m^2\varphi^2 + \frac{1}{3}\mu\varphi^3 - \frac{1}{4}\lambda\varphi^4 \, .$$

List all the symmetries of this action. Under which condition will φ develop a non-vanishing minimum? How many massless scalars (goldstone bosons) would you expect?

4) Consider a coloured scalar field ϕ_t, singlet under $\mathrm{SU}(2)_L$ and with mass m, that couples with the SM BEH field with coupling

$$-y_t^2 \phi_t^\dagger \phi_t \, \phi^\dagger \phi \, .$$

Calculate the loop corrections to the ϕ mass coming from this interaction. To regularise the divergent loop, apply a hard cut off, and expand the result up to logarithmic divergent terms (i.e. terms proportional to $\log \Lambda$).