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BSM phenomenology and EWSB (BSM)

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1 Supersymmetry: a fermion-boson symmetry!

Supersymmetry is a symmetry that relates fermions and bosons to each other. It is useful to address the naturalness problem because it can associate the BEH scalar with a fermion: as the two partners share the same physical properties, the chiral symmetry which protects the fermion mass will also protect the scalar partner mass!

Let's consider s spin-0 quantum state $|s>$: supersymmetry can be thought of in terms of an operator Q, which transforms the scalar state into a spin-1/2 state $|f>$:

$$Q|s>=|f> \; . \tag{1.1}$$

In order for the equation to respect rotational invariance, the operator Q must carry spin-1/2, thus it is a fermionic operator. The minimal spin-1/2 representation is a Weyl fermion, i.e. a 2-component chiral fermion. One can however construct supersymmetric theories with any number of chiral generators Q^i (extended supersymmetry). Q is a fermion, therefore it will respect anti-commutation relations:

$$\{Q,\bar{Q}\} = -2\sigma^\mu p_\mu \,, \quad \{Q,Q\} = 0\,, \quad \{\bar{Q},\bar{Q}\} = 0\,. \tag{1.2}$$

Furthermore, being a spin-1/2 object, it has the following commutation properties with the position and momentum operators:

$$[Q,p_\mu] = 0\,, \qquad [\bar{Q},p_\mu] = 0\,. \tag{1.3}$$

So, we can define a closed algebra including the usual Poincaré algebra, extended by the addition of the fermionic operator Q. In this sense supersymmetry is an extension of space-time symmetries!

1.1 How to construct a supersymmetric quantum field theory?

The most straightforward way would be to write down a theory containing a scalar field φ and a chiral fermion χ, corresponding to the two related quantum states, and then derive the transformation properties of the fields under the operator Q. This procedure, however, turns out to be quite lengthy!

A shortcut is offered by the previous observation that the operator Q can be formally included into the Poincaré algebra: this lead to the introduction of superfields! The idea is that one can think of extend space-time by adding two extra co-ordinates, corresponding to the operator Q. Such co-ordinates must anti-commute with each other, thus they are spinors: the spinor co-ordinate θ (and $\bar{\theta}$). A field living in the superspace $\{x^\mu, \theta\}$, a superfield, is therefore a function of x^μ, θ and $\bar{\theta}$:

$$S(x^\mu, \theta, \bar{\theta})\,. \tag{1.4}$$

Now, θ is an anti-commuting 2-component, thus powers of θ^n with $n > 2$ must vanish: this is due to the fact that for more than 2 spinors, at least two of them must have their spin aligned and such configuration is forbidden by Pauli's exclusion principle! This means that the superfield Φ can be expanded in a finite series in powers of the super-coordinate θ. The most general expansion reads:

$$S(x^\mu, \theta, \bar{\theta}) = a + \theta\xi + \bar{\theta}\bar{\chi} + \theta\theta b + \bar{\theta}\bar{\theta} c + \bar{\theta}\bar{\sigma}^\mu\theta v_\mu + \bar{\theta}\bar{\theta}\theta\eta + \theta\theta\bar{\theta}\bar{\zeta} + \theta\theta\bar{\theta}\bar{\theta} d\,. \tag{1.5}$$

where a, b, c, and d are scalars; ξ, χ, η and ζ are chiral fermions and v_μ is a vector. However, one should define some more minimal representations of the supersymmetric algebra, i.e. superfields that have less independent components than the general expansion. This selection is similar to the definition of spins: even though a spinor in 4 dimensions has 4 components, the minimal representation is a 2-component (chiral) Weyl fermion! The minimal superfield is the *chiral superfield* Φ, defined as

$$\Phi(y^\mu, \theta) = \varphi(y^\mu) + \sqrt{2}\,\theta\chi(y^\mu) + \theta\theta F(y^\mu)\,, \tag{1.6}$$

where $y^\mu = x^\mu + i\bar{\theta}\bar{\sigma}^\mu\theta$. Note that Ψ only depends on $\bar{\theta}$ implicitly via y^μ. The definition of Ψ can be formally extracted from the supersymmetric transformation properties of superfields, however the formalities are beyond the our scopes. Note also that Ψ contains a scalar field φ, a 2-components spinor χ, and an extra field F, whose function will be clear shortly.

The next step is to write an action for the superfield: in addition to the integral over the usual space-time, we need to integrate over the super-coordinate θ. There are, in this case, two possible ways of integrating:

$$\int d^2\theta\,, \qquad \text{and} \quad \int d^2\theta d^2\bar{\theta}\,, \tag{1.7}$$

where $\bar{\theta}$ is the hermitian conjugate of θ. Another consequence of the fermionic nature of θ is that the only non-vanishing integrals are:

$$\int d^2\theta\ \theta\theta = 1\,, \quad \int d^2\theta d^2\bar{\theta}\ \theta\theta\bar{\theta}\bar{\theta} = 1\,. \tag{1.8}$$

As you can always expand any function of θ and $\bar{\theta}$ in a finite series of terms, the integral definition is such that $\int d^2\theta$ selects the term of the expansion proportional to $\theta\theta$, i.e.

$$\int d^2\theta S(x^\mu, \theta, \bar{\theta}) = b + \bar{\theta}\bar{\zeta} + \bar{\theta}\bar{\theta}d\,, \tag{1.9}$$

while

$$\int d^2\theta d^2\bar{\theta} S(x^\mu, \theta, \bar{\theta}) = d\,. \tag{1.10}$$

Finally, we need to define a supersymmetric action, which contains an integration over the super-coordinates and is invariant under supersymmetric transformations. There are two possibilities, and they are both important in the definition of supersymmetric theories. On one hand, one can integrate over the whole superspace any superfield:

$$\mathcal{S}_1 = \int d^4x \int d^2\theta d^2\bar{\theta}\, S(x^\mu, \theta, \bar{\theta})\,. \tag{1.11}$$

The second possibility is to integrate over $d^2\theta$ a chiral superfield after setting $\bar{\theta} = 0$, i.e.

$$\mathcal{S}_2 = \int d^4x \int d^2\theta\, \Phi(x^\mu, \theta)\,. \tag{1.12}$$

Supersymmetric action for a chiral superfield

Let's consider a chiral superfield Φ. The full expansion reads

$$\Phi(y^\mu, \theta) = \varphi + \sqrt{2}\theta\chi + \theta\theta F - i\bar{\theta}\bar{\sigma}^\mu\theta\partial_\mu\varphi + \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^\mu\partial_\mu\chi - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial^\mu\partial_\mu\varphi\,, \tag{1.13}$$

where all the fields φ, χ and F are intended to be functions of x^μ.

Let's first use $\mathcal{S}_1$. The first attempt is to integrate over a single chiral superfield:

$$\int d^4x \int d^2\theta d^2\bar{\theta}\, \Phi(y^\mu, \theta) = -\int d^4x\, \frac{1}{4}\partial^\mu\partial_\mu\varphi \;= 0\,, \tag{1.14}$$

because the $\theta\theta\bar{\theta}\bar{\theta}$ term is a total derivative! If we try

$$\int d^4x \int d^2\theta d^2\bar{\theta}\, \Phi^\dagger\Phi = \int d^4x\, (\partial^\mu\varphi)^\dagger\partial_\mu\varphi - i\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi + F^*F\,. \tag{1.15}$$

This looks like the kinetic term for a scalar and a fermion. The extra field F does not have any derivative, thus it is not a dynamic field (auxiliary field), and it can be easily integrated out.

To use $\mathcal{S}_2$, we need chiral superfields: after setting $\bar{\theta} = 0$, their expansion simplifies

$$\Phi(x^\mu, \theta) = \varphi + \sqrt{2}\,\theta\chi + \theta\theta F\,. \tag{1.16}$$

The most general action will therefore be

$$\int d^4x \int d^2\theta \, \frac{1}{2}\mu\Phi\Phi + \frac{1}{3}y\Phi\Phi\Phi \,, \tag{1.17}$$

where we have kept only the normalisable interaction (and neglected a linear term). The integral selects the terms in the expansion proportional to $\theta\theta$. There are two possibilities: either we take an F component from one superfield and the scalar ones from the remaining ones, or we select a fermion from two superfields, and scalars from the remaining ones.

$$\int d^4x \int d^2\theta \, \frac{1}{2}\mu\Phi\Phi + \frac{1}{3}y\Phi\Phi\Phi + h.h. = \int d^4x \; F\left(\mu\varphi + y\varphi^2\right) - \left(\frac{1}{2}\mu + y\varphi\right)\chi\chi + h.c. \tag{1.18}$$

Putting together this term with the kinetic term in Eq. 1.15, we can integrate out the auxiliary field F by use of the equations of motion:

$$F^* + \left(\mu\varphi + y\varphi^2\right) = 0 \,. \tag{1.19}$$

Therefore

$$F^*F + F\left(\mu\varphi + y\varphi^2\right) + F^*\left(\mu\varphi + y\varphi^2\right)^* = -\left|\mu\varphi + y\varphi^2\right|^2 = -F^*F \,. \tag{1.20}$$

Note that both the complex scalar φ and the chiral fermion have mass $m_\varphi = m_\chi = \mu$.

General expressions

If we have N chiral superfields Φ_i, the most general action can be written as

$$\mathcal{S} = \int d^4x \left\{ \int d^2\theta d^2\bar{\theta} \, \Phi_i^\dagger \Phi_i + \int d^2\theta W(\Phi_i) + h.c. \right\} \,, \tag{1.21}$$

where the *superpotential* W is a function of the superfields (if renormalisable, it can contain up to trilinear terms). Besides the kinetic terms for scalars and fermions, the action will contain the following interactions:

$$\mathcal{S}_W = \int d^4x \; -\sum_i \left|\frac{\partial W(\varphi)}{\partial\varphi_i}\right|^2 - \frac{1}{2}\sum_{i,j} \frac{\partial^2 W(\varphi)}{\partial\varphi_i\partial\varphi_j}\chi^i\chi^j \; + h.c.. \tag{1.22}$$

2 Supersymmetric Standard Model

2.1 Naturalness in supersymmetry: the top loop

Let's first consider the supersymmetric version of the top Yukawa:

$$\mathcal{S}_{top} = \int d^4x \; - y_t \chi_Q \varphi_H \chi_{t_R} + h.c. \,. \tag{2.23}$$

There are 3 relevant fields: the BEH field, the left-handed top contained in the doublet Q and the right handed top t_R. A supersymmetric version, must contain 3 chiral superfields which have the same quantum numbers as the SM fields:

$$\begin{aligned}\Phi_H &= \varphi_H + \sqrt{2}\theta\chi_H + \theta\theta F_H\,, &(2.24)\\ \Phi_Q &= \varphi_Q + \sqrt{2}\theta\chi_Q + \theta\theta F_Q\,, &(2.25)\\ \Phi_{t_R} &= \varphi_{t_R} + \sqrt{2}\theta\chi_{t_R} + \theta\theta F_{t_R}\,. &(2.26)\end{aligned}$$

The most general superpotential one can write down is:

$$W = y_t \Phi_Q \Phi_H \Phi_{t_R}\,. \tag{2.27}$$

From the results above, the supersymmetric Yukawa interactions will be:

$$\begin{aligned}\mathcal{S}_{susy-top} = \int d^4x\ & - y_t\left(\varphi_H\chi_Q\chi_{t_R} + \varphi_Q\chi_H\chi_{t_R} + \varphi_{t_R}\chi_Q\chi_H + h.c.\right) + \\ & -y_t^2\left(\varphi_Q^*\varphi_H^*\ \varphi_Q\varphi_H + \varphi_H^*\varphi_{t_R}^*\ \varphi_H\varphi_{t_R} + \varphi_Q^*\varphi_{t_R}^*\ \varphi_Q\varphi_{t_R}\right)\,. \end{aligned}\tag{2.28}$$

The BEH field has two additional 4-scalar interactions with φ_Q and φ_{t_R}. Such interactions, will contribute to the loop corrections to the Higgs mass. Each loop will contribute (here we assign a mass m to the scalar top partners):

$$\begin{aligned}-i\delta m_\phi^2 &= -3iy_t^2 \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2} \\ &= \frac{3y_t^2}{16\pi^4} 2\pi^2 i \int k_E^3 dk_E \frac{1}{-k_E^2 - m^2} \\ &= -\frac{3y_t^2}{16\pi^2}\int dk_E^2 \frac{k_E^2}{k_E^2+m^2} \\ &= -\frac{3y_t^2}{16\pi^2}\int_0^{\Lambda^2} dk_E^2\left(1 - \frac{m^2}{k_E^2+m^2}\right) \\ &= -\frac{3y_t^2}{16\pi^2}\left(\Lambda^2 - m^2\log\frac{\Lambda^2+m^2}{m^2}\right)\,. \end{aligned}\tag{2.29}$$

Summing the contribution of the two scalar tops, the quadratically divergent term cancels out, and we are left with log divergent terms

$$\delta m_\phi^2 = -\frac{3y_t^2}{16\pi^2}\left(m_Q^2\log\frac{\Lambda^2}{m_Q^2} + m_{t_R}^2\log\frac{\Lambda^2}{m_{t_R}^2}\right)\,. \tag{2.30}$$

3 The Minimal Supersymmetric Standard Model (MSSM)

In a minimal supersymmetric version of the Standard Model, besides supersymmetric gauge interactions that we have not described here, we need to promote each SM field to a superfield

	label	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	3 B	L	$(-1)^{3(B-L)}$
r-h electron	e_R	**1**	**1**	1	0	-1	$-$
l-h leptons	L	**1**	**2**	-1/2	0	1	$-$
r-h up quark	u_R	**3**	**1**	-2/3	-1	0	$-$
r-h down quark	d_R	**3**	**1**	1/3	-1	0	$-$
l-h quarks	Q	**3**	**2**	1/6	1	0	$-$
Higgs (up)	H_u	**1**	**2**	1/2	0	0	$+$
Higgs (down)	H_d	**1**	**2**	-1/2	0	0	$+$

Table 1: Chiral superfield content of the MSSM. For completeness, we also add their Baryon and Lepton numbers B and L.

(listed in Table 1). Note the presence of two Higgs doublets. If we had only one Higgs, say H_u, we would only be able to write Yukawa interactions for up quarks:

$$W_u = y_u \Phi_{H_u} \Phi_Q \Phi_{u_R} \,. \tag{3.31}$$

The other Yukawa in the SM would be written in terms of φ^*, which is contained in Φ^* (which is not a chiral superfield!). Thus, we need to introduce a second Higgs doublet, with opposite sign hypercharge:

$$W_d = y_d \Phi_{H_d} \Phi_Q \Phi_{d_R} + y_e \Phi_{H_d} \Phi_L \Phi_{e_R} \,. \tag{3.32}$$

We can also add a bilinear in the two Higgs superfields:

$$W_H = \mu \Phi_{H_u} \Phi_{H_d} \,, \tag{3.33}$$

which will generate a mass for the two Higgs scalars $m_{H_u} = m_{H_d} = \mu$.

There is another reason why two Higgses are needed: in the SM, a complete generation of fermions is anomaly free. In supersymmetry, the Higgs superfield will contain a new fermion doublet, the superpartner of the Higgs. The presence of a single higgsino would generate anomalies: the role of the second Higgs is therefore to cancel such anomalies.

3.1 Troubleshooting 1: unwanted superpotential terms

In addition to the Yukawa interactions, the superfield content in Table 1 allow for many more "dangerous" terms to be added.

For instance, one can add an operator made of 3 quark singlets:

$$\Phi_{u_R} \Phi_{d_R} \Phi_{d_R} \,; \tag{3.34}$$

an operator of this kind would be forbidden in the SM because it contains 3 fermions.

Also, Φ_L and Φ_{H_d} have exactly the same quantum numbers, thus they can be interchanged:

$$\Phi_{H_d} \Phi_{H_d} \Phi_{e_R} \,, \quad \dots \tag{3.35}$$

What symmetries of the SM are violated by such superpotential terms?

The first kind will violate Baryon number (the operator has net baryon number -1, like an anti-neutron); the second kind violates lepton number. Such terms are very dangerous because, among other things, can mediate the proton decay. Recall that both Baryon and Lepton number conservation are an accidental consequence of the matter content of the SM! In supersymmetric extensions of the SM, such accident does not occur.

R-parity and Dark Matter

The solution is to impose Baryon and Lepton number conservation by hand. We therefore impose a Z_2 parity on the superfields, defined as:

$$P_M = (-1)^{3(B-L)}\,; \tag{3.36}$$

it is called *matter parity*, and it is defined in terms of $B - L$ because this combination is anomaly free in the SM. Requiring the superpotential to be even under matter parity eliminates all the unwanted terms.

We can further elaborate: any action term must contain an even number of fermions, so we can redefine matter parity by adding an extra "-1" for fermions, without modifying the interaction terms:

$$P_R = (-1)^{3(B-L)+2s}\,, \tag{3.37}$$

where s is the spin of the field; this is called *R-parity*. Note that now scalars and bosons in the same superfield have opposite R-parity; furthermore, all SM states (matter fermions and scalar Higgses) have R-parity +1, while the supersymetric partners (squarks, sleptons and higgsinos) have R-parity -1. This implies that the lightest supersymmetric particle is stable, because it cannot decay into SM states only!

Can it play the role of Dark Matter?

3.2 Troubleshooting 2: supersymmetry cannot be exact!

Another problem is that supersymmetry is not an exact symmetry, because it would predict that SM states and their partners have the same mass (we are pretty confident that there are no scalar electrons around!).

One way to break supersymmetry without spoiling its nice properties (mainly the cancellation of divergences), is to add only "mass terms", i.e. couplings with a positive mass dimension: the reason behind is that at high energies, well above the supersymmetry breaking mass scales, supersymmetry is restored, thus the divergences still cancel out! This principle is called *soft supersymmetry breaking*. We should also be careful not to violate R-parity! The allowed terms are therefore:

- Higgs mass terms: $-m^2_{H_u}\varphi^*_{H_u}\varphi_{H_u} - m^2_{H_d}\varphi^*_{H_d}\varphi_{H_d}$;

- scalar quark and lepton masses: $-\tilde{m}^2_Q\varphi^*_Q\varphi_Q - \tilde{m}^2_{t_R}\varphi^*_{t_R}\varphi_{t_R} + \ldots$;

- trilinear scalar couplings (in the same form as Yukawa couplings): $A\varphi_{H_u}\varphi_Q\varphi_{t_R} + \ldots$;
- gaugino masses (masses for the fermion partners of gauge bosons).

Note that a huge number of soft supersymmetry breaking terms can be added to the MSSM (more that 120!). In order to study the phenomenology, one needs to make simplifying assumptions or develop a mechanism of supersymmetry breaking!

4 Exercises

1) Starting with the superfield content in Table 1, write the most general superpotential (including matter parity violation terms). Which symmetries of the SM are broken by each term?

2) R-parity renders the lightest supersymmetric partner stable. Can you list all the particles in the MSSM that may be candidates for Dark Matter?

3) Consider the top scalar partner φ_{t_R}. From what we discussed today in class, can you guess the decay modes of such state in an exactly supersymmetric MSSM? What changes if we add soft supersymmetry breaking terms?

4) Consider the SM extended with a new Dirac fermion T with mass M, and that couples to the SM Higgs scalar via the interaction

$$\frac{\lambda}{M}\phi^\dagger\phi\,\bar{T}T\,.$$

Calculate the contribution of this interaction to the Higgs mass. For what value of the coupling λ can such contribution cancel the divergent contribution of the top loop?