Lectures on

QCD

(with particular emphasis on applications to hadron colliders)

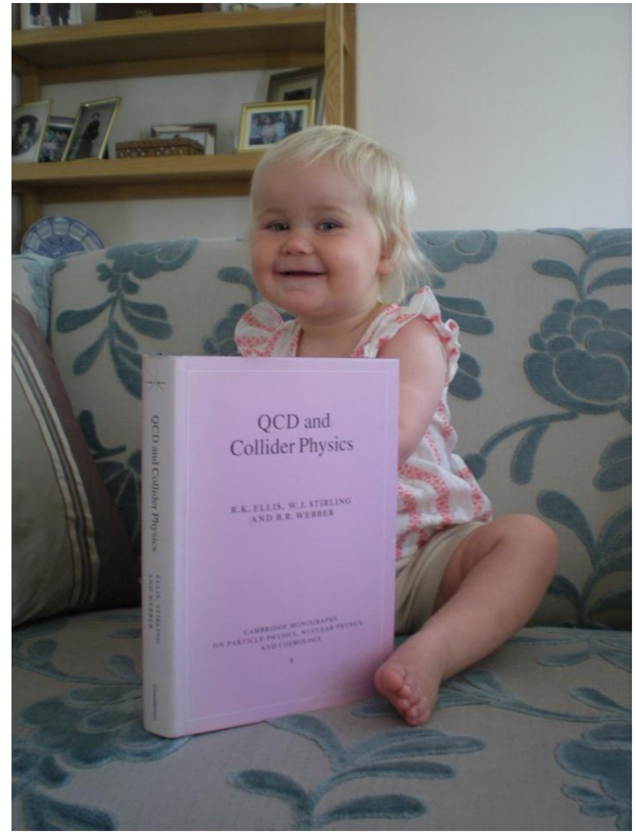
James Stirling
Cambridge University

SUSSP, St Andrews, August 2012

references

“QCD and Collider Physics”

RK Ellis, WJ Stirling, BR Webber
Cambridge University Press (1996)



also

“Handbook of Perturbative QCD”

G Sterman et al (CTEQ Collaboration)
www.phys.psu.edu/~cteq/#Handbook

… and

“Hard Interactions of Quarks and Gluons: a Primer for LHC Physics ”

JM Campbell, JW Huston, WJ Stirling (CSH)

arxiv.org/abs/hep-ph/0611148

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REVIEW ARTICLE

Hard Interactions of Quarks and Gluons: a Primer for LHC Physics

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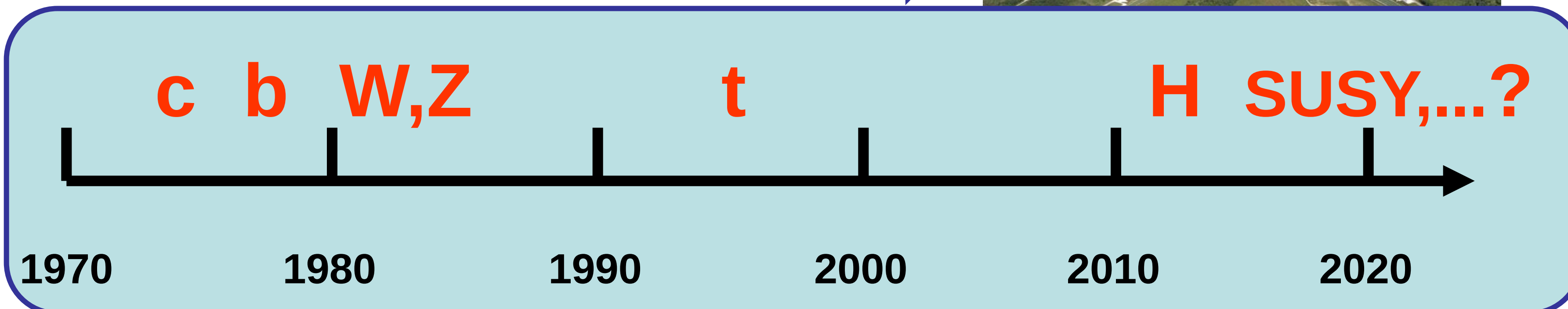
Abstract. In this review article, we will develop the perturbative framework for the calculation of hard scattering processes. We will undertake to provide both a reasonably rigorous development of the formalism of hard scattering of quarks and gluons as well as an intuitive understanding of the physics behind the scattering. We will emphasize the role of logarithmic corrections as well as power counting in α_s in order to understand the behaviors of hard scattering processes. We will include “rules of thumb” as well as “official recommendations”, and where possible will seek to dispel some myths. We will also discuss the impact of soft processes on the measurements of hard scattering processes. Experiences that have been gained at the Fermilab Tevatron will be recounted and, where appropriate, extrapolated to the LHC.

1. Introduction

Scattering processes at high energy hadron colliders can be classified as either hard or soft. Quantum Chromodynamics (QCD) is the underlying theory for all such processes, but the approach and level of understanding is very different for the two cases. For hard processes, e.g. Higgs or high p_T jet production, the rates and event properties

past and present proton/(anti)proton colliders…

Tevatron (1987 → 2011)
Fermilab
proton-antiproton collisions
$\sqrt{s}$ = 1.8, 1.96 TeV



$Sp\bar{p}S$ (1981 → 1990)
CERN
proton-antiproton collisions
$\sqrt{s}$= 540, 630 GeV

LHC (2009 →)
CERN
proton-proton and heavy ion collisions
$\sqrt{s}$ = 7, 8, 14, ... TeV

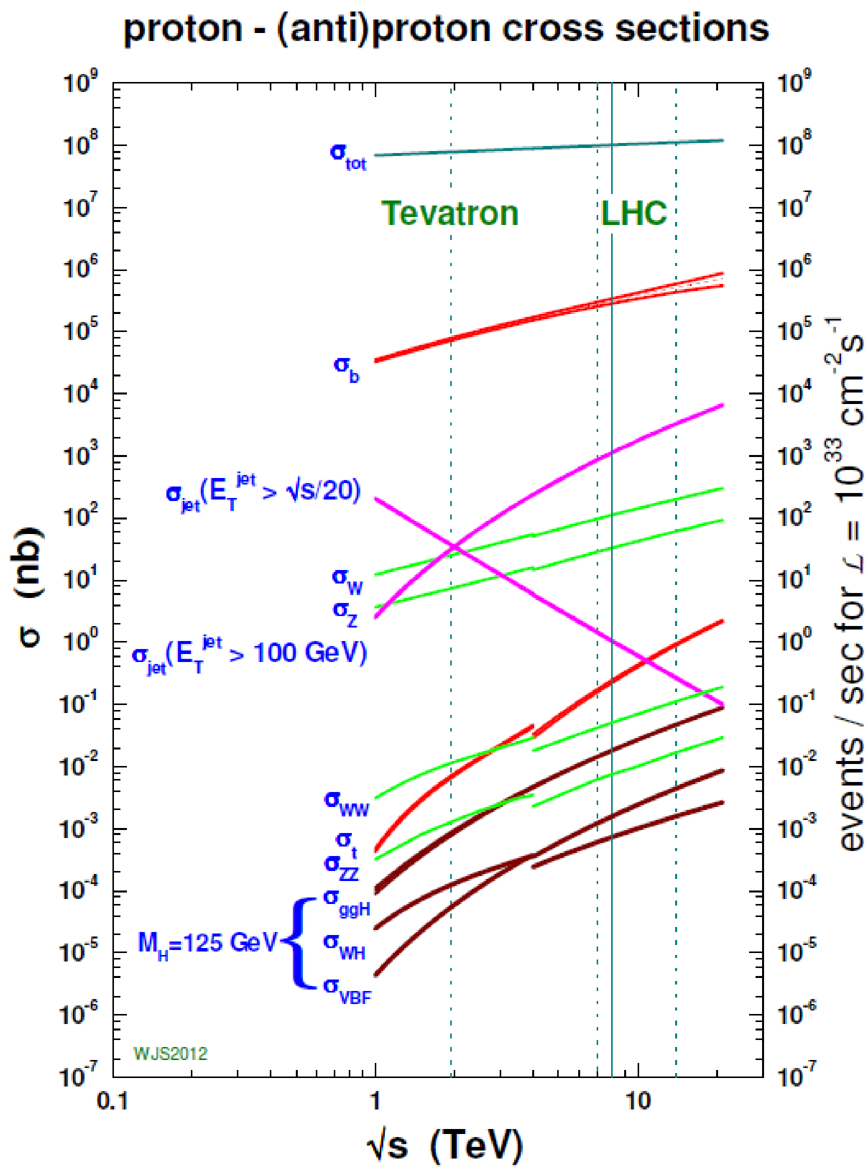
What can we calculate?

Scattering processes at high energy hadron colliders can be classified as either **HARD** or **SOFT**

Quantum Chromodynamics (QCD) is the underlying theory for **all** such processes, but the approach (and the level of understanding) is very different for the two cases

For **HARD** processes, e.g. W or high-E_T jet production, the rates and event properties can be predicted with some precision using perturbation theory

For **SOFT** processes, e.g. the total cross section or diffractive processes, the rates and properties are dominated by non-perturbative QCD effects, which are much less well understood



Outline

- Basics I: introduction to QCD
  - Motivation, Lagrangian, Feynman rules
  - the running coupling α_S : theory and measurement
  - general structure of the QCD perturbation series
- Basics II: partons and Deep Inelastic Scattering
  - basic parton model ideas for DIS
  - scaling violation & DGLAP
  - parton distribution functions
- QCD and hadron colliders
  - hard scattering & basic kinematics
  - the Drell-Yan process in the parton model
  - factorisation
  - parton luminosity functions
- QCD phenomenology at the Tevatron and LHC
  - leading-order calculations
  - beyond leading order: higher-order perturbative QCD corrections
  - resummation
  - some examples of precision QCD phenomenology at the LHC
  - beyond perturbation theory
    - *parton showering models, Monte Carlo tools* ***(→ see Mike Seymour's lectures!)***
    - double parton scattering
    - central exclusive production

1

introduction to QCD

- Motivation, Lagrangian, Feynman rules
- the running coupling α_S : theory and measurement
- general structure of the QCD perturbation series

Quantum Chromodynamics

- a Yang-Mills gauge theory with SU(3) symmetry

Rationale – evidence that quarks come in 3 colours

- $\Delta^{++}\uparrow = (u\uparrow u\uparrow u\uparrow)$ requires additional (≥3) internal degrees of freedom to satisfy Fermi-Dirac statistics

- cross sections and decay rates, e.g. $\sigma(e^+e^- \rightarrow \text{hadrons}) \propto N_c$ and $\Gamma(\pi^0 \rightarrow \gamma\gamma) \propto N_c^2$, imply $N_c = 3.0 \pm \ldots$

Thus, put quarks in triplets, $\psi_i^q = (q,q,q)$, and require invariance under local SU(3) transformations

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_q \overline{\psi}^q_i (i\gamma^\mu D_{\mu ij} - m_q \delta_{ij})\psi^q_j + \mathcal{L}_{\text{gauge fixing}} + \mathcal{L}_{\text{ghost}}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$

$$D_{\mu ij} = \delta_{ij}\partial_\mu - i g_s T^a_{ij} A^a_\mu$$

where

- g_s is the QCD coupling constant
- f^{abc} are the structure constants of SU(3): $[T^a, T^b] = i f^{abc} T^c$ $(a,b,c = 1,\ldots,8)$
- A_μ^a are the 8 *gluon* fields
- T_{ij}^a are 8 'colour matrices', i.e. generators of the SU(3) transformation acting on the fundamental (triplet) representation: $$T^a_{ij} = \frac{1}{2}\lambda^{(a)}_{ij}$$

Gell-Mann 3×3 matrices, see ESW

- this corresponds to the normalisation

$$\mathrm{Tr}(T^a T^b) = T^a_{ij} T^b_{ji} = T_F \delta^{ab} = \frac{1}{2}\delta^{ab}$$

- other colour identities include

$$\begin{aligned} (T^a T^a)_{ij} &= C_F \delta_{ij},\ C_F = (N^2-1)/2N = 4/3 \\ f^{acd} f^{bcd} &= C_A \delta^{ab},\ C_A = N = 3 \end{aligned}$$

- the QCD Lagrangian is invariant under local SU(3) transformations:

$$\psi \longrightarrow \exp\left(i \sum_{a=1}^{8} T^a \alpha^a(x)\right) \psi$$

$$A^a_\mu \longrightarrow A^a_\mu - \frac{1}{g_s}\partial_\mu \alpha^a - \sum_{b,c=1}^{8} f^{abc} \alpha^b A^c_\mu$$

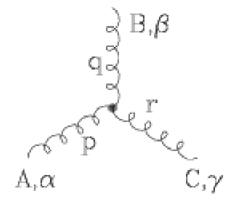
and from the Lagrangian the Feynman rules can be derived

Feynman Rules for QCD (covariant gauge)

$$\delta^{AB}\left[-g^{\alpha\beta}+(1-\lambda)\frac{p^{\alpha}p^{\beta}}{p^{2}+i\epsilon}\right]\frac{i}{p^{2}+i\epsilon}$$

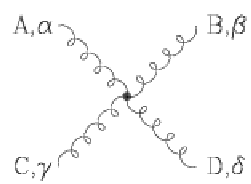
$$\delta^{AB}\frac{i}{(p^{2}+i\epsilon)}$$

$$\delta^{ab}\frac{i}{(\not{p}-m+i\epsilon)_{ji}}$$

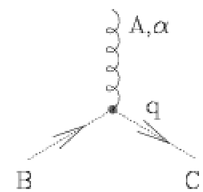


$$-g\, f^{ABC}\left[(p-q)^{\gamma}g^{\alpha\beta}+(q-r)^{\alpha}g^{\beta\gamma}+(r-p)^{\beta}g^{\gamma\alpha}\right]$$

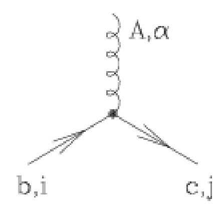
(all momenta incoming, $p+q+r = 0$)



$$-ig^{2}\, f^{XAC}f^{XBD}\left[g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}\right]$$
$$-ig^{2}\, f^{XAD}f^{XBC}\left[g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}\right]$$
$$-ig^{2}\, f^{XAB}f^{XCD}\left[g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}\right]$$



$$g\, f^{ABC}q^{\alpha}$$

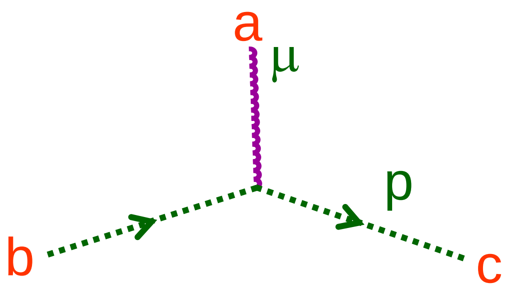


$$-ig\,(t^{A})_{cb}\,(\gamma^{\alpha})_{ji}$$

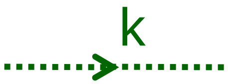
- Note: gauge fixing – to quantise the theory and reduce the number of degrees of freedom of the gauge fields, need to introduce a gauge fixing term:

$$\Delta\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\alpha}\sum_a(\partial^\mu A^a_\mu)^2 \quad\Rightarrow\quad P^{\mu\nu}(k) = -g^{\mu\nu}+(1-\alpha)\frac{k^\mu k^\nu}{k^2+i\epsilon}$$

- these are *covariant gauges*, and additional *ghost fields* are required….



$$g_s\ f^{abc}\ p^\mu$$



$$\frac{i}{k^2+i\epsilon}\ \delta^{ab}$$

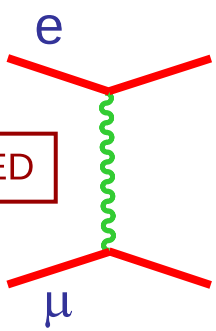
or

$$\Delta\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\alpha}\sum_a(n^\mu A^a_\mu)^2 \quad\Rightarrow\quad P^{\mu\nu}(k) = -g^{\mu\nu}+\frac{n^\mu k^\nu + n^\nu k^\mu}{n\cdot k}-(n^2+\alpha k^2)\frac{k^\mu k^\nu}{(n\cdot k)^2}$$

- these are *non-covariant ("axial") gauges*… no ghosts required!

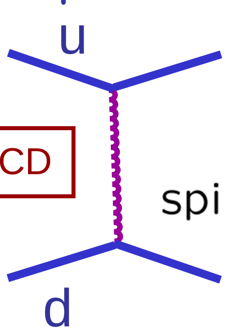
sample QCD calculation

QED



$$\sum_{\text{spins}} |\mathcal{M}|^2 = 8\, e^4 \frac{s^2+u^2}{t^2}$$

QCD



$$\sum_{\text{spins, colours}} |\mathcal{M}|^2 = 8\, g_s^4 \frac{s^2+u^2}{t^2} \times \text{CF}$$

→ dijet cross section at LHC

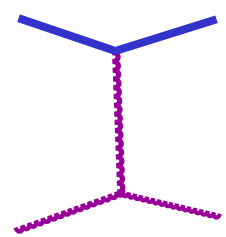
$$\begin{aligned}
\text{CF}_{\mathcal{M}} &= T^a_{ij} T^a_{kl} \\
\text{CF}_{|\mathcal{M}|^2} &= T^a_{ij} T^a_{kl} \, (T^b_{ij} T^b_{kl})^* \\
&= T^a_{ij} T^b_{ji} T^a_{kl} T^b_{lk} \\
&= \text{Tr}(T^a T^b)\ \text{Tr}(T^a T^b) \\
&= \frac{1}{2}\delta^{ab}\, \frac{1}{2}\delta^{ab} = 2
\end{aligned}$$

spin and colour averaging:

QED: $\times$ 1/4

QCD: $\times$ 1/4 $\times$ 1/3 $\times$ 1/3 = $\times$ 1/36

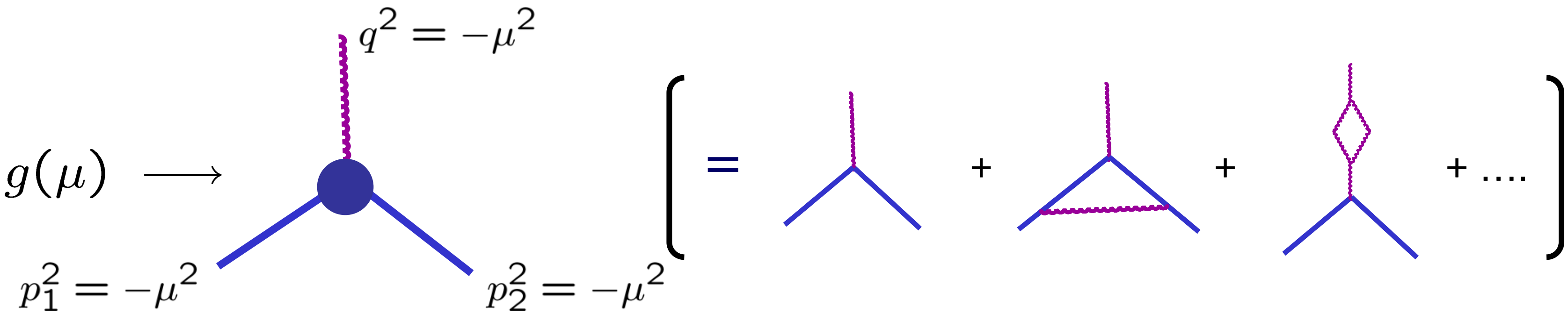
exercise:



CF = 12

renormalised coupling constants

- when we renormalise the coupling constant in a QFT, we introduce a renormalisation scale, μ



$$g(\mu) = g_0 + g_0^3 \left[b \ln \frac{M}{\mu} + c \right] + \ldots$$

$$\int^M \frac{d^4k}{k^4}$$

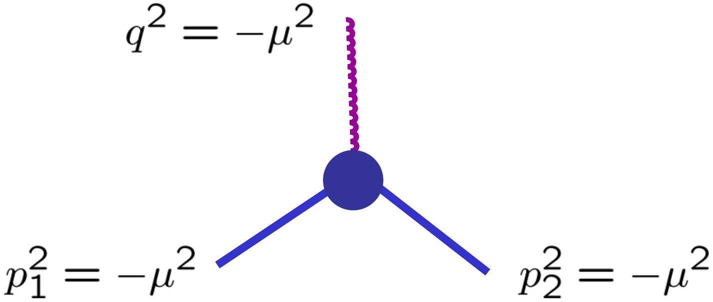
bare coupling

u/v cut-off for divergent loop integrals

- … and because there are additional diagrams (interactions) in QCD, the *b,c,…* coefficients in QCD and QED will be different

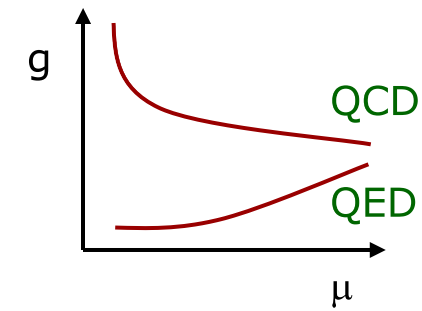
- how does *g(μ) depend on μ* ?

the QCD running coupling



$$\mu \frac{\partial}{\partial \mu} g(\mu) = -b g_0^3 + \mathcal{O}(g_0^5)$$
$$= -b g^3(\mu) + \mathcal{O}(g^5(\mu))$$

- and by explicit calculation



$$b_{\rm QED} = -\frac{1}{12\pi^2} < 0$$
$$b_{\rm QCD} = \frac{1}{16\pi^2}\left(11 - \frac{2}{3} n_f\right) > 0$$

- formally

$$\mu \frac{\partial}{\partial \mu} g(\mu) = -\beta(g(\mu))\, g(\mu)$$

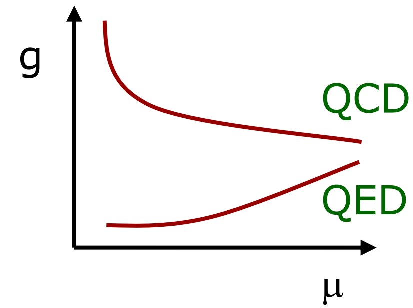
the β function →

$$\beta(g) = \beta_0 \frac{g^2}{16\pi^2} + \beta_1 \left(\frac{g^2}{16\pi^2}\right)^2 + \ldots$$

- in principle, can solve the differential equation in terms of *$g(\mu_0)$*, to be determined from experiment

- explicit leading order solution

$$g^2(\mu) = \frac{g^2(\mu_0)}{1 + g^2(\mu_0) b \ln(\mu^2/\mu_0^2)}$$



- QED

$$\mu_0 \sim m_e$$

$$\alpha(\mu_0) \equiv \alpha_{\text{em}} = \frac{1}{137 \cdot \ldots}$$

$$\alpha = \frac{g^2}{4\pi}$$

- QCD

$$\mu_0 = M_Z$$

$$\alpha_S(M_Z) = 0.118$$

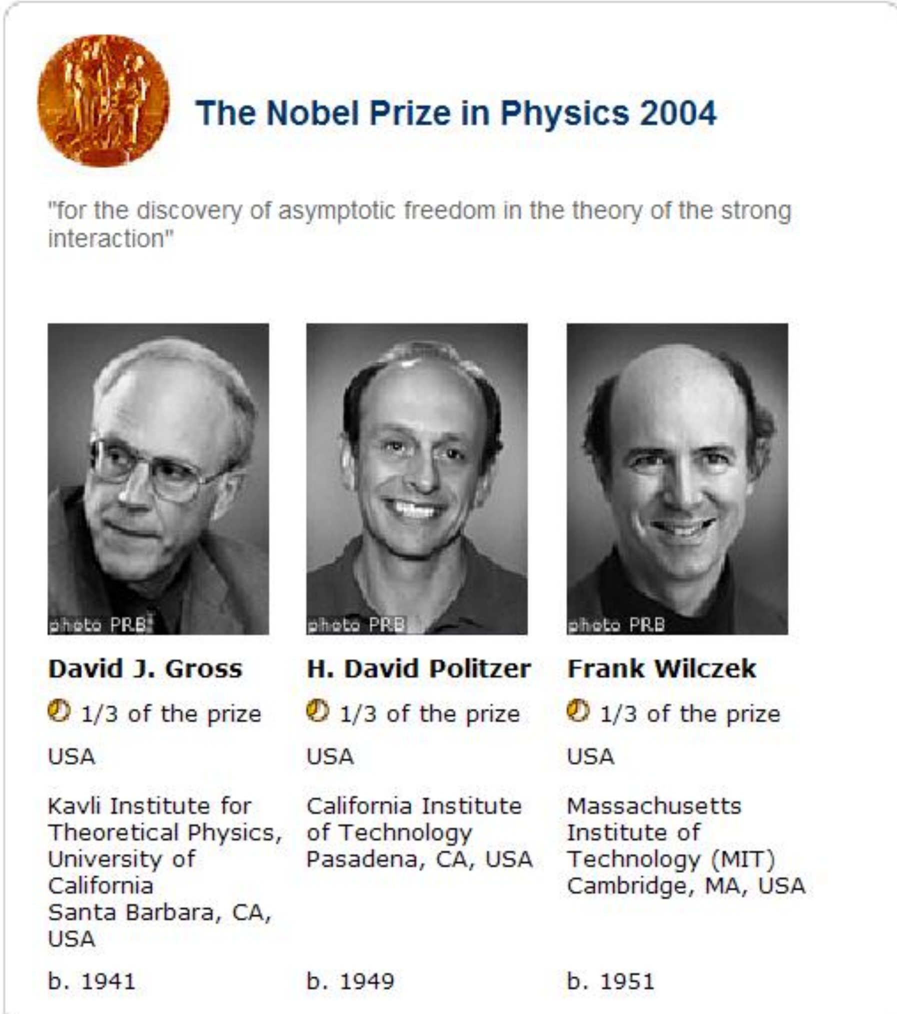
from experiment

or (historically)

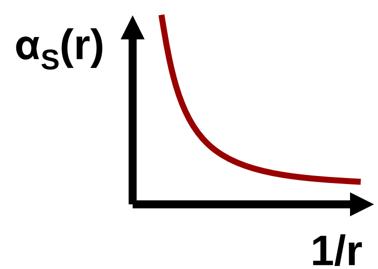
$$g_S^2(\mu) = \frac{1}{b \ln(\mu^2/\Lambda_{\text{QCD}}^2)}$$

where $\Lambda_{\text{QCD}}^2 = \mu_0^2 \, \mathrm{e}^{-1/bg^2(\mu_0)}$

Asymptotic Freedom



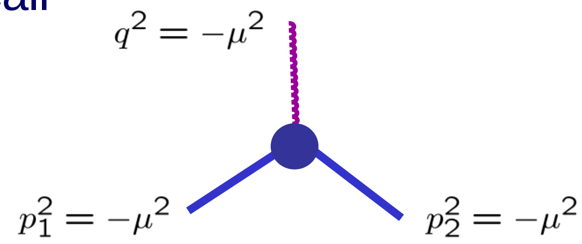
“What this year's Laureates discovered was something that, at first sight, seemed completely contradictory. The interpretation of their mathematical result was that the closer the quarks are to each other, the *weaker* is the 'colour charge'. When the quarks are really close to each other, the force is so weak that they behave almost as free particles. This phenomenon is called ‘asymptotic freedom’. The converse is true when the quarks move apart: the force becomes stronger when the distance increases.”



beyond leading order

- many QCD cross sections are nowadays measured to high accuracy, therefore need to take into account higher order (HO) corrections, e.g. $1 + c\,\alpha_S$, including in the definition of α_S

- recall



$$g(\mu) = g_0 + g_0^3 \left[b \ln \frac{M}{\mu} + c \right] + \ldots$$

this represents a particular convention; we could have defined another coupling, *g'* , by say replacing $\mu \to 2\mu$ or by using the ggg vertex:

$$\begin{aligned} g'(\mu) \equiv g(2\mu) &= g_0 + g_0^3 \left(b \ln(M/2\mu) + c \right) + \ldots \\ &= g_0 + g_0^3 \left(b \ln(M/\mu) + \{c - b \ln 2\} \right) + \ldots \\ &= g(\mu) \left[1 - b \ln 2 \; g^2(\mu) + \ldots \right] \end{aligned}$$

- in general, the coupling constants in 2 different schemes will be related by:

$$g'(\mu) = g(\mu) \left[1 + \kappa g^2(\mu) + \ldots \right]$$

minimal subtraction renormalisation schemes

- instead of regularising integrals like $\int d^4k/k^4$ with a u/v cut-off M, reduce the number of dimensions to $N < 4$; introduce $\varepsilon = 2 - N/2$

$$\frac{d^4k}{(2\pi)^4} \longrightarrow \frac{d^{4-2\epsilon}k}{(2\pi)^{4-2\epsilon}}\,(\mu^2)^\epsilon$$

with log M divergences then replaced by $1/\varepsilon$ poles

- MS prescription: when calculating a divergent scattering amplitude beyond leading order, subtract off the $1/\varepsilon$ poles and replace g_0 by the renormalised coupling $g(\mu)$

- but notice that the poles always appear in the combination

$$\frac{1}{\epsilon} - \gamma_E + \ln(4\pi)$$

- … so instead subtract off this combination; this is the modified minimal subtraction ($\overline{\text{MS}}$) scheme, widely used in practical pQCD calculations

QCD coupling beyond leading order

$$\frac{\mu^2}{\alpha_S(\mu^2)}\frac{\partial\alpha_S(\mu^2)}{\partial\mu^2} = -\frac{\alpha_S(\mu^2)}{4\pi}\beta_0 - (\frac{\alpha_S(\mu^2)}{4\pi})^2\beta_1 - (\frac{\alpha_S(\mu^2)}{4\pi})^3\beta_2 + \ldots$$

$$\beta_0 = 11 - \frac{2}{3}n_f\,, \qquad \beta_1 = 102 - \frac{38}{3}n_f\,, \qquad \beta_2 = \ldots \quad \text{(see ESW)}$$

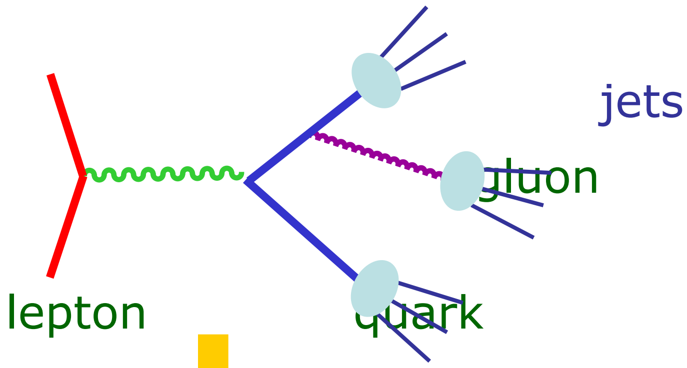
- the LO $\to$ NLO solution is

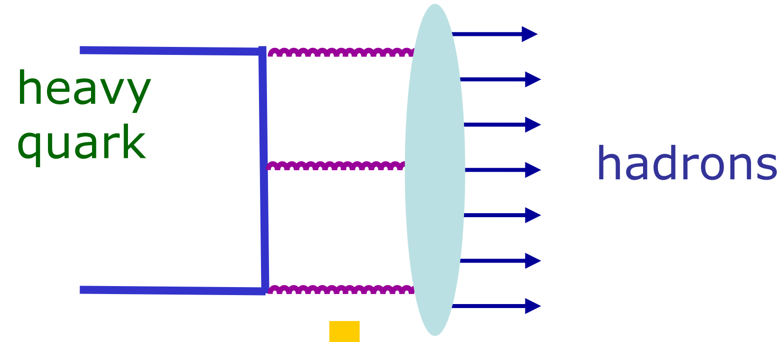
$$\frac{1}{\alpha_S(\mu^2)} - \frac{1}{\alpha_S(\mu_0^2)} + b' \ln\left(\frac{\alpha_S(\mu^2)(1+b'\alpha_S(\mu_0^2))}{\alpha_S(\mu_0^2)(1+b'\alpha_S(\mu^2))}\right) = b \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

$$b = \frac{\beta_0}{16\pi^2}, \qquad b' = \frac{\beta_1}{\beta_0}$$

- so for LO/NLO/NNLO pQCD phenomenology we need to include β_0, β_1, β_2 in the definition of $\boldsymbol{\alpha}_S$
- see e.g. ESW for treatment of non-zero quark masses etc

techniques for α_S measurements





$$\frac{\Gamma^{\Upsilon\to \text{hads}}}{\Gamma^{\Upsilon\to\mu\mu}} = \frac{10(\pi^2-9)}{9\pi}\frac{\alpha_S^3}{\alpha^2}\left[1+\mathcal{O}(\alpha_S)\right]$$

$$R_{e^+e^-}, R_Z, R_\tau = R_0\left[1+\frac{\alpha_S}{\pi}+\ldots\right]$$

$$\frac{1}{\sigma}\frac{d\sigma}{dT}, f_3, \ldots = A\alpha_S + B\alpha_S^2 + \ldots$$

$$Q^2\frac{\partial D^h(z,Q^2)}{\partial Q^2} = \alpha_S D^h \otimes P + \ldots$$

e$^+$e$^-$

$$Q^2\frac{\partial F_i(x,Q^2)}{\partial Q^2} = \alpha_S F_i \otimes P + \ldots$$

$$\int dx\, F_i(x,Q^2) = A + B\alpha_S + \ldots$$

$$\sigma^{(2+1)\,\text{jet}} = A\alpha_S + B\alpha_S^2 + \ldots$$

DIS

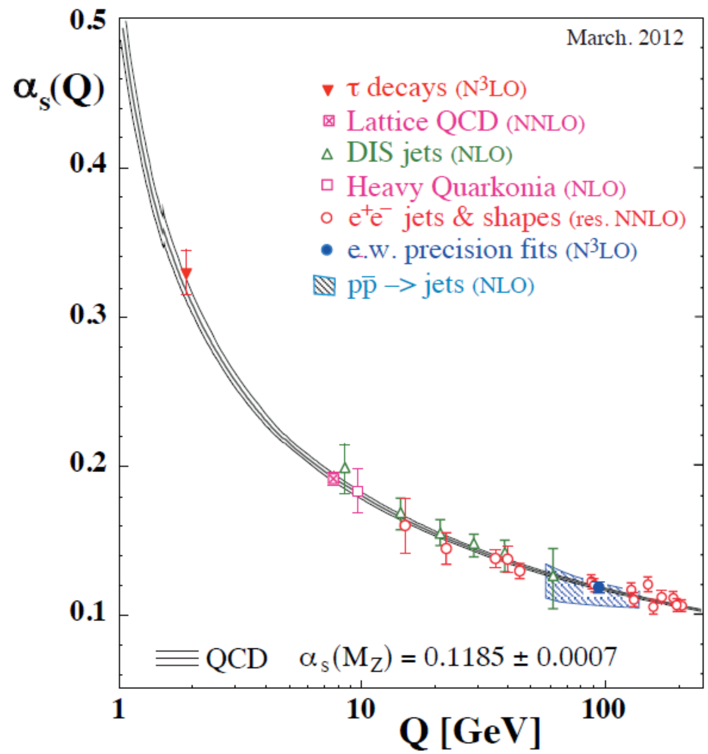
theoretical issues:

- HO pQCD corrections ($\to$ scale dependence)
- Q^{-2n} power corrections ($\to$ higher twist, hadronisation)

S. Bethke 2012*

α_s measurements and world average

Process	$\alpha_s(M_{Z^0})$	excl. mean $\alpha_s(M_{Z^0})$	std. dev.
τ-decays	0.1197 ± 0.0016	0.1183 ± 0.0008	0.8
Lattice QCD	0.1186 ± 0.0008	0.1183 ± 0.0011	0.2
DIS [F_2]	0.1148 ± 0.0024	0.1188 ± 0.0011	1.5
e^+e^- [jets & shps]	0.1172 ± 0.0037	0.1185 ± 0.0007	0.3
ew. prec. data]	0.1193 ± 0.0028	0.1184 ± 0.0007	0.3



$\alpha_s(M_Z) = 0.1185 \pm 0.0007$

Note: difficult for hadron colliders to be competitive!

*Nucl. Phys. Proc. Suppl. 222-224 (2012) 94-100

note the "shrinking error" effect…

- from the basic (LO) definition

$$\frac{\delta\alpha_S(Q^2)}{\alpha_S(Q^2)} \approx \frac{\alpha_S(Q^2)}{\alpha_S(M_Z^2)}\,\frac{\delta\alpha_S(M_Z^2)}{\alpha_S(M_Z^2)}$$
$$> \frac{\delta\alpha_S(M_Z^2)}{\alpha_S(M_Z^2)} \qquad \text{for} \quad Q^2 < M_Z^2$$

- therefore a precise measurement of the coupling at a small scale Q can given improved precision on the fundamental parameter $\alpha_S(M_Z^2)$

- however, the small-scale determination may be more "contaminated" by power corrections or other non-perturbative effects

general structure of a QCD perturbation series

- choose a renormalisation scheme (e.g. MSbar)
- calculate cross section to some order (e.g. NLO)

$$\sigma(P) \;=\; A\,\alpha_S^N(\mu) + \alpha_S^{N+1}(\mu)\;\left[B + \frac{NAb}{2\pi}\ln\frac{\mu}{P}\right] + \ldots$$

physical variable(s)

process dependent coefficients depending on *P*

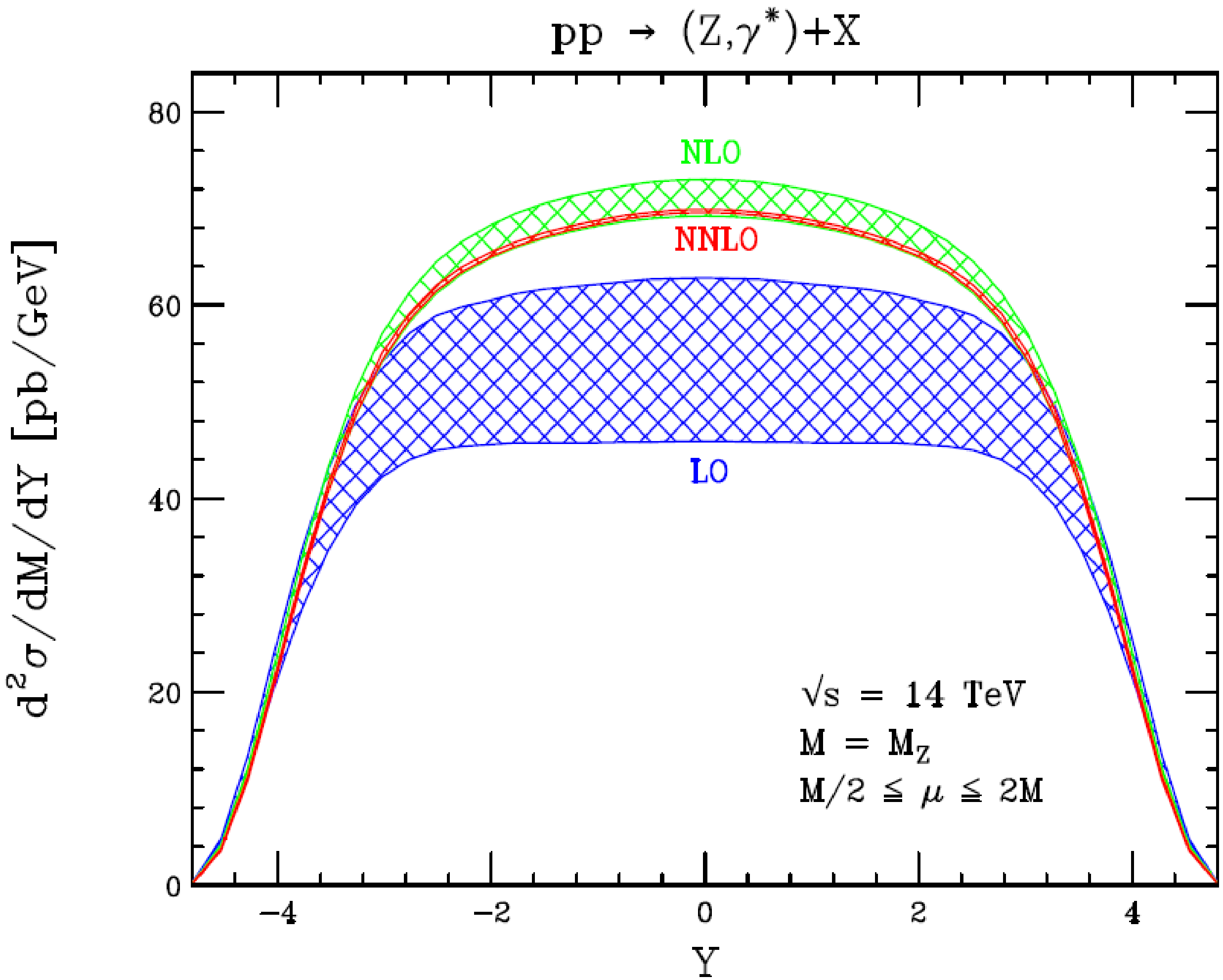
renormalisation scale

- note $d\sigma/d\mu=0$ "to all orders", but in practice $d\sigma^{(N+n)}/d\mu = O((N+n)\,\alpha_S^{N+n+1})$

- can try to help convergence by using a "physical scale choice", $\mu \sim P$, e.g. $\mu = M_Z$ or $\mu = E_T^{jet}$ at LHC

- what if there is a wide range of *P*'s in the process, e.g. W + multijet production at hadron colliders?

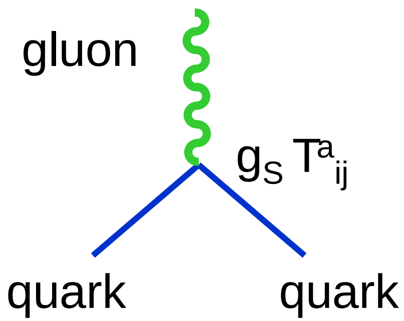
the higher the order in perturbation theory, the weaker the scale dependence …

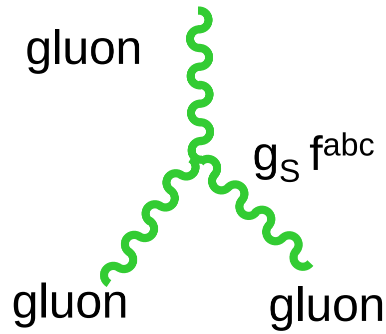


Anastasiou, Dixon, Melnikov, Petriello, 2004

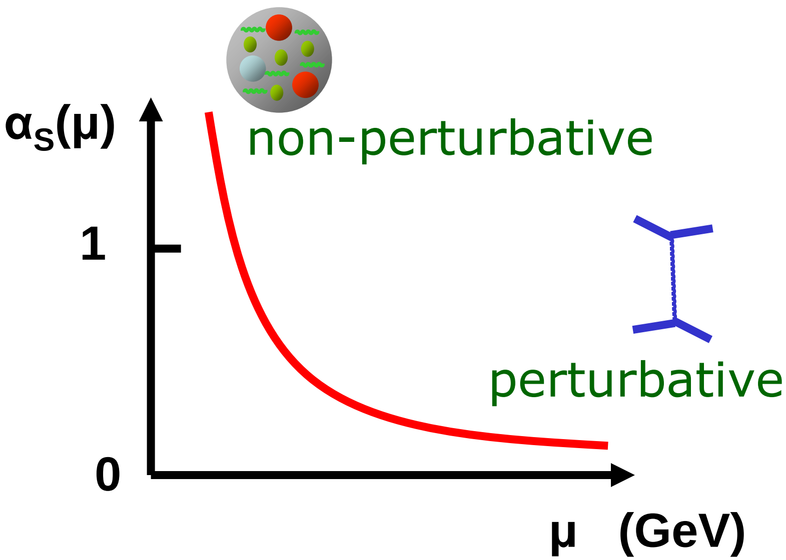
- only scale variation uncertainty shown
- central values calculated for a *fixed* set PDFs with a *fixed* value of $\alpha_S(M_Z^2)$

Basics of QCD - Summary





$\alpha_S = g_S^2/4\pi$



- renormalisation of the coupling

$$\frac{\mu^2}{\alpha_s(\mu^2)}\frac{\partial\alpha_s(\mu^2)}{\partial\mu^2} = -\frac{\alpha_s(\mu^2)}{4\pi}\beta_0 - (\frac{\alpha_s(\mu^2)}{4\pi})^2\beta_1 - (\frac{\alpha_s(\mu^2)}{4\pi})^3\beta_2 + \ldots$$

$$\beta_0 = 11 - \frac{2}{3}n_f\,, \qquad \beta_1 = 102 - \frac{38}{3}n_f\,, \qquad \ldots$$

- colour matrix algebra

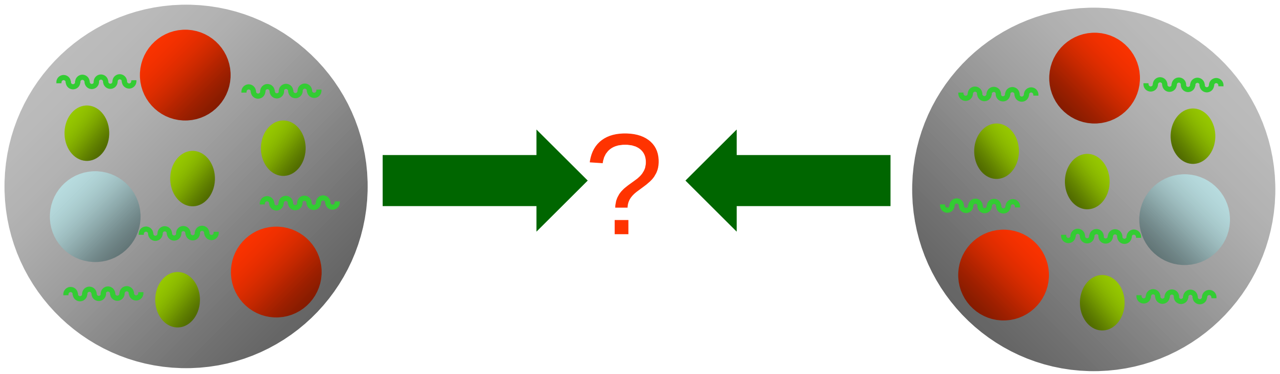
$$[T^a, T^b] = if^{abc}T^c\,, (T^aT^a)_{ij} = C_F\delta_{ij} = 4/3\delta_{ij}\,, \mathrm{Tr}(T^aT^b) = T_F\delta^{ab} = 1/2\delta^{ab}\,, \ldots$$

2

Partons and Deep Inelastic Scattering

- basic parton model ideas for DIS
- scaling violation & DGLAP
- parton distribution functions

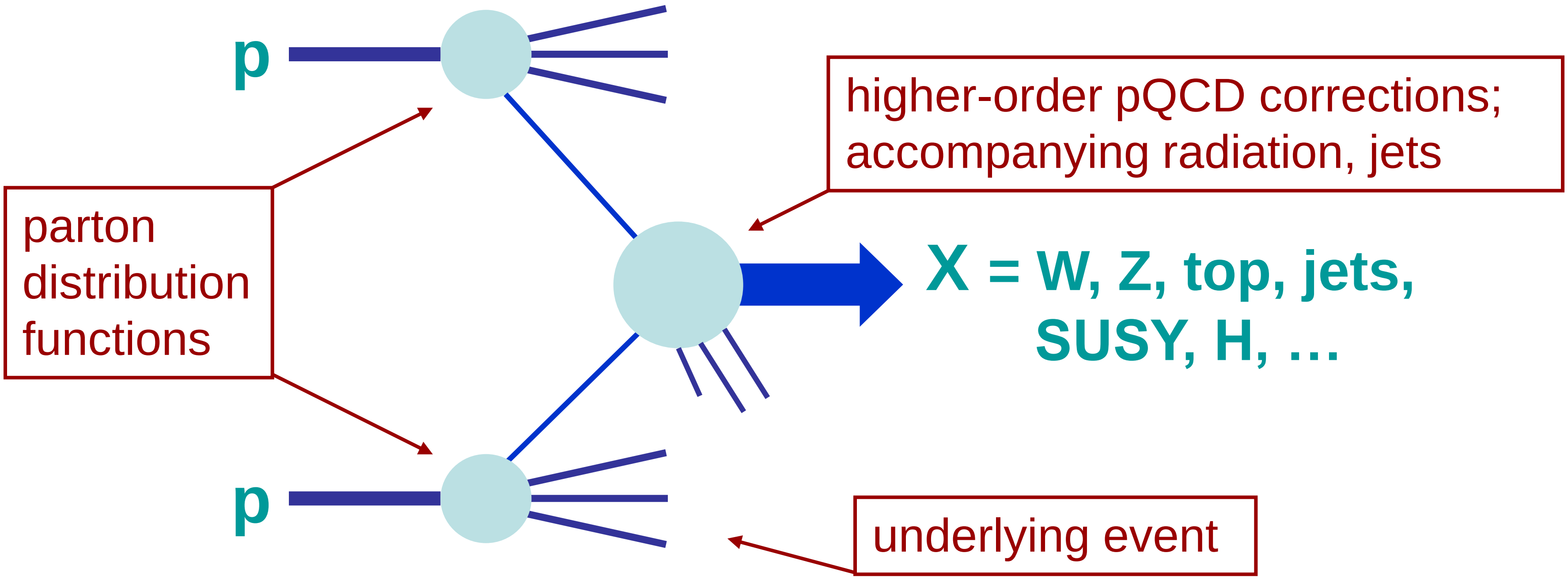
protons are not fundamental particles – what happens when they collide?



Most of the time – nothing of much interest, the protons break up and the final state consists of many low energy particles (pions, kaons, photons, neutrons, ….)

But, occasionally, a parton (quark or gluon) from each proton can undergo a ‘hard scattering’

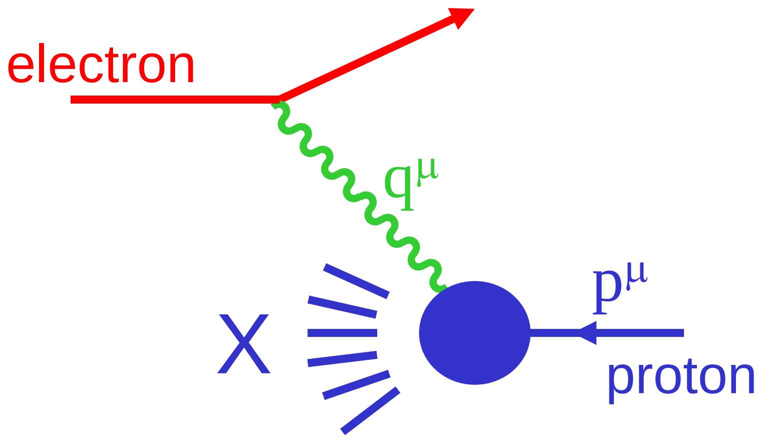
hard scattering in hadron-hadron collisions



for inclusive production, the basic calculational framework is provided by the QCD FACTORISATION THEOREM:

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a(x_1, \mu_F^2) \, f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \to X} \left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2} \right)$$

deep inelastic scattering



- variables

$Q^2 = -q^2$

$x = Q^2/2p\cdot q$ (Bjorken x)

$(\, y = Q^2/x\, s \,)$

- resolution

$$\lambda = \frac{h}{Q} = \frac{2\times 10^{-16}\,\mathrm{m\ GeV}}{Q}$$

at HERA, $Q^2 < 10^5\ \mathrm{GeV}^2$

$\Rightarrow \lambda > 10^{-18}\ \mathrm{m} = r_p/1000$

- inelasticity

$$x = \frac{Q^2}{Q^2 + M_X^2 - M_p^2}$$

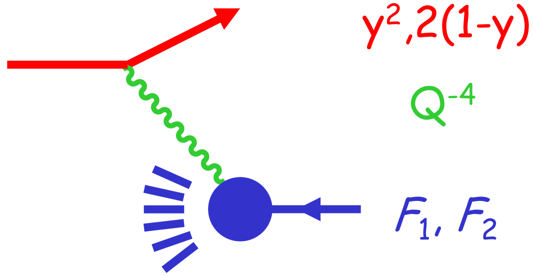
$\Rightarrow 0 < x \leq 1$

structure functions

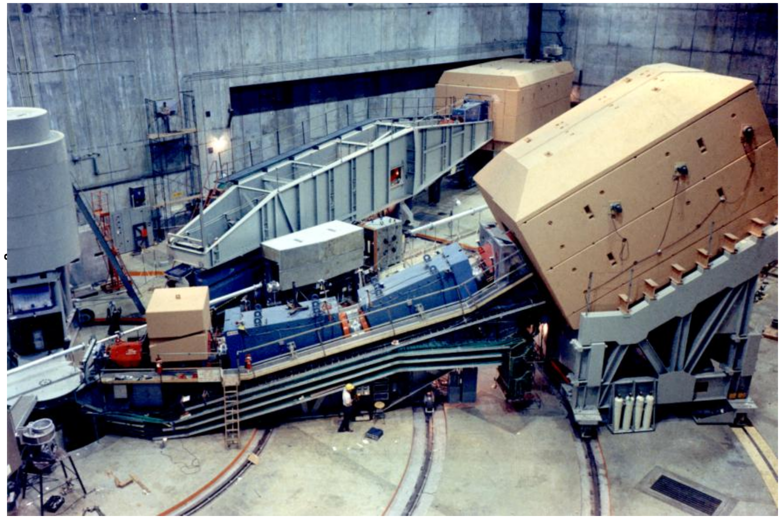
- in general, we can write

$$\frac{d\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[y^2 F_1 + 2(1-y)x^{-1}F_2\right]$$

where the $F_i(x,Q^2)$ are called structure functions



SLAC, ~1970



- experimentally, for $Q^2 > 1\ GeV^2$
  - $F_i(x,Q^2) \rightarrow F_i(x)$ "scaling"
  - $F_2(x) \approx 2\,x\,F_1(x)$

Bjorken 1968

toy model

- suppose that the electron scatters off a pointlike, ~massless, spin ½ particle *a* of charge e_a moving collinear with the parent proton with four-momentum $p_a^\mu = \xi p^\mu$
- calculate the scattering cross section $ea \to ea$

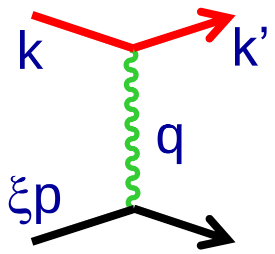
$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = 2\, e^4\, e_a^2\, \frac{s^2+u^2}{t^2}$$

$$\frac{d\sigma^{ea\to ea}}{dt} = \frac{e^4 e_a^2}{8\pi s^2}\,\frac{s^2+u^2}{t^2}$$

$$\frac{d\sigma^{ea\to ea}}{dQ^2} = \frac{2\pi\alpha^2 e_a^2}{Q^4}\left[1+(1-y)^2\right]$$

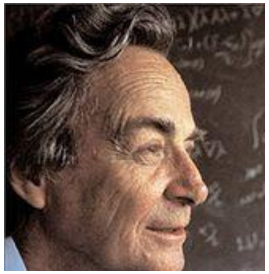
$$\frac{d\sigma}{dx\, dQ^2} = \frac{2\pi\alpha^2}{Q^4}\left[y^2+2(1-y)\right]\, e_a^2\delta(x-\xi)$$

$$\Rightarrow \quad F_2 = x e_a^2 \delta(x-\xi) = 2xF_1$$



- Exercise: show that if *a* has spin-zero, then $F_1 = 0$

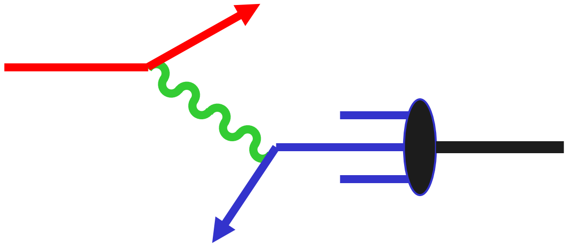
the parton model (Feynman 1969)



- photon scatters incoherently off massless, pointlike, spin-1/2 quarks

infinite momentum frame

- probability that a quark carries fraction ξ of parent proton's momentum is $q(\xi)$, $(0 < \xi < 1)$



$$F_2(x) = \sum_{q,\bar{q}} \int_0^1 d\xi \; e_q^2 \, \xi \, q(\xi) \, \delta(x - \xi) = \sum_{q,\bar{q}} e_q^2 \, x \, q(x)$$

$$= \frac{4}{9} x u(x) + \frac{1}{9} x d(x) + \frac{1}{9} x s(x) + \ldots$$

- the functions u(x), d(x), s(x), … are called parton distribution functions (PDFs) - they encode information about the proton's deep structure

extracting PDFs from experiment

- different beams (e,μ,ν,…) & targets (H,D,Fe,…) measure different combinations of quark PDFs
- thus the individual q(x) can be extracted from a set of structure function measurements
- gluon not measured directly, but carries about 1/2 of the proton's momentum

$$F_2^{ep} = \frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s}) + \ldots$$

$$F_2^{en} = \frac{1}{9}(u+\bar{u}) + \frac{4}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s}) + \ldots$$

$$F_2^{\nu p} = 2\left[d + s + \bar{u} + \ldots\right]$$

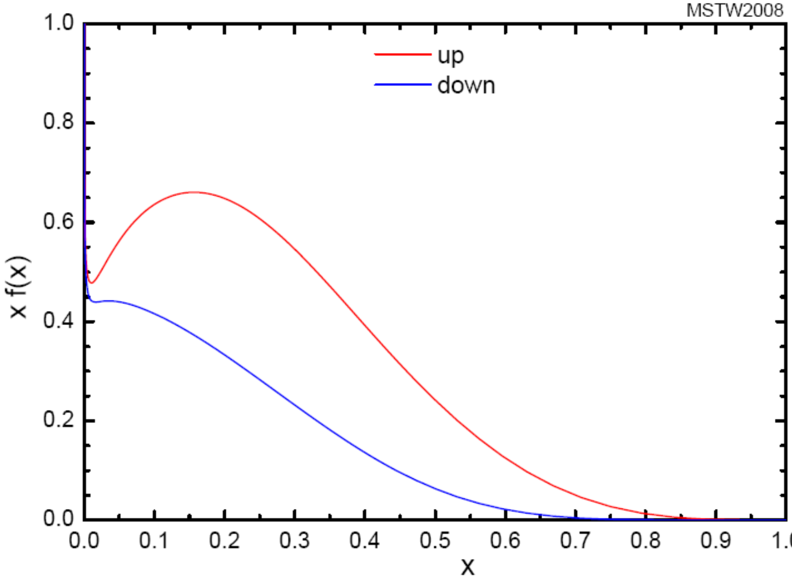
$$F_2^{\nu n} = 2\left[u + \bar{d} + \bar{s} + \ldots\right]$$

$$s = \bar{s} = \frac{5}{6} F_2^{\nu N} - 3F_2^{eN}$$

$$\sum_q \int_0^1 dx\, x\left(q(x) + \bar{q}(x)\right) = 0.55$$

quarks as partons!

- and, indeed, up quark and down quark ‘partons’ are observed in the proton, and their distribution functions measured…..

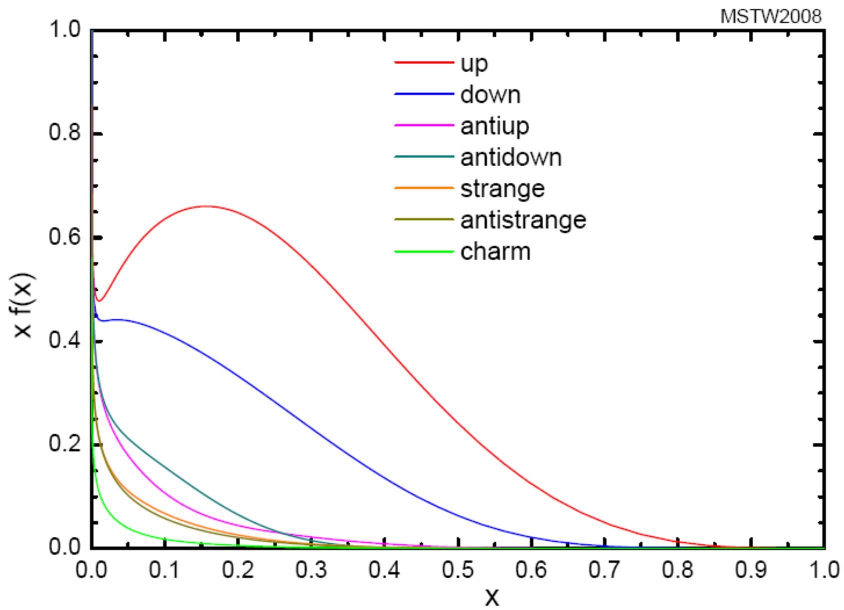


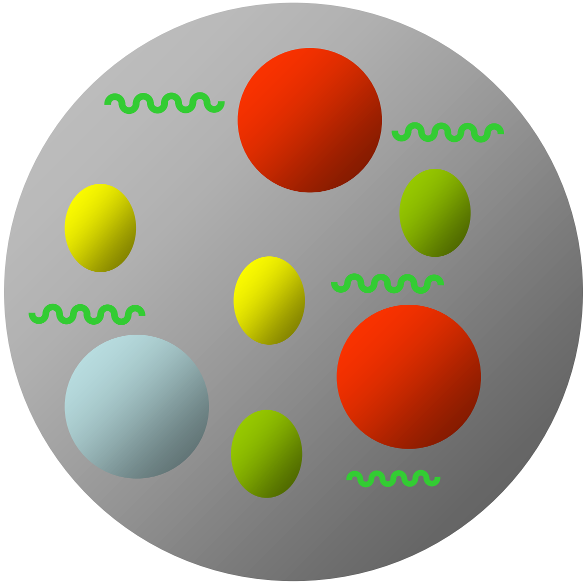
- however, they only appear to carry about 30% of the proton’s momentum – what carries the remainder?!

- answer: a ‘sea’ of quark and antiquark pairs (up, down, strange, charm, …) and gluons

sea quarks and gluons…

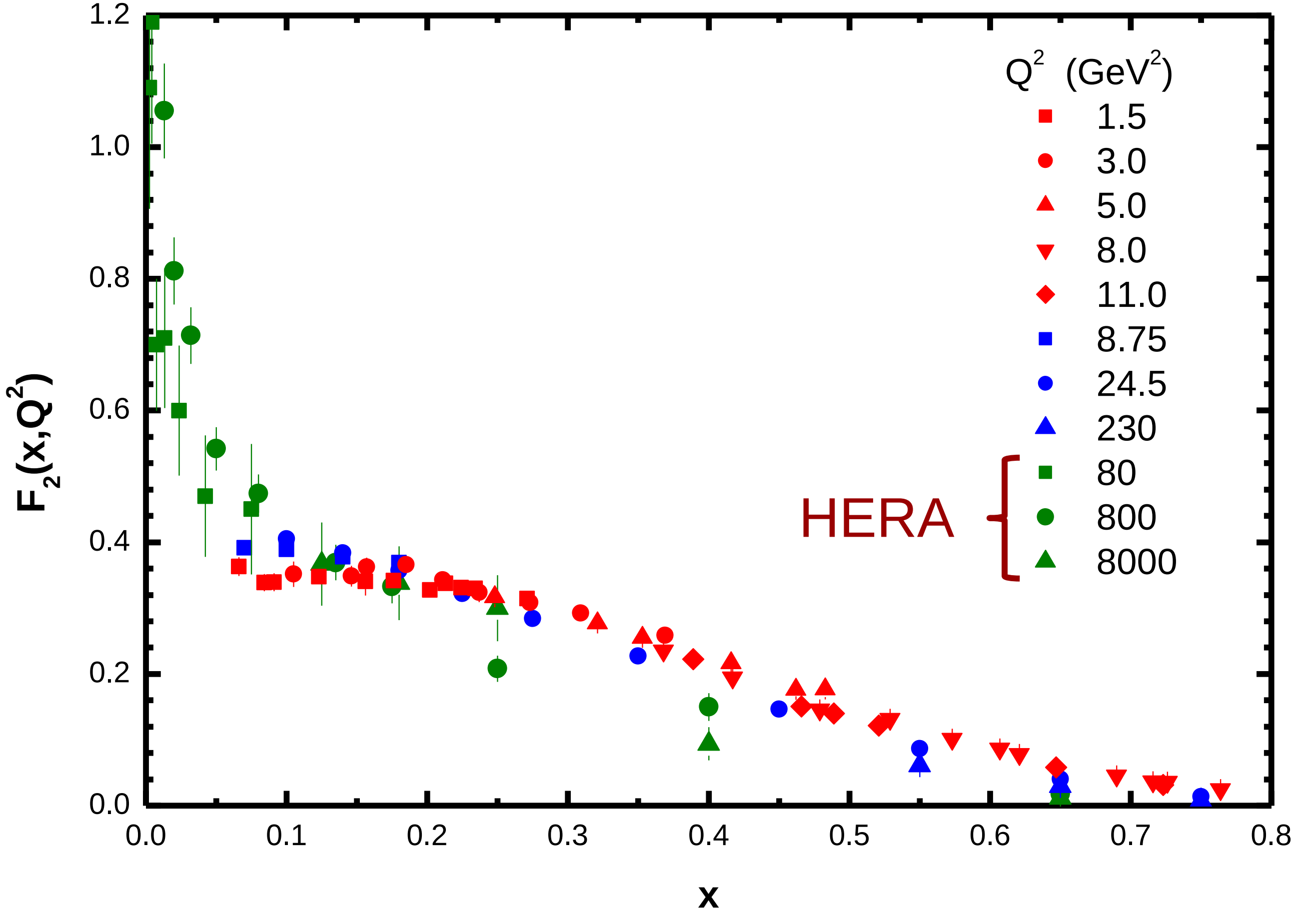
- the strong force field inside the proton causes quark-antiquark pairs to fluctuate out of the vacuum, and become candidate partons



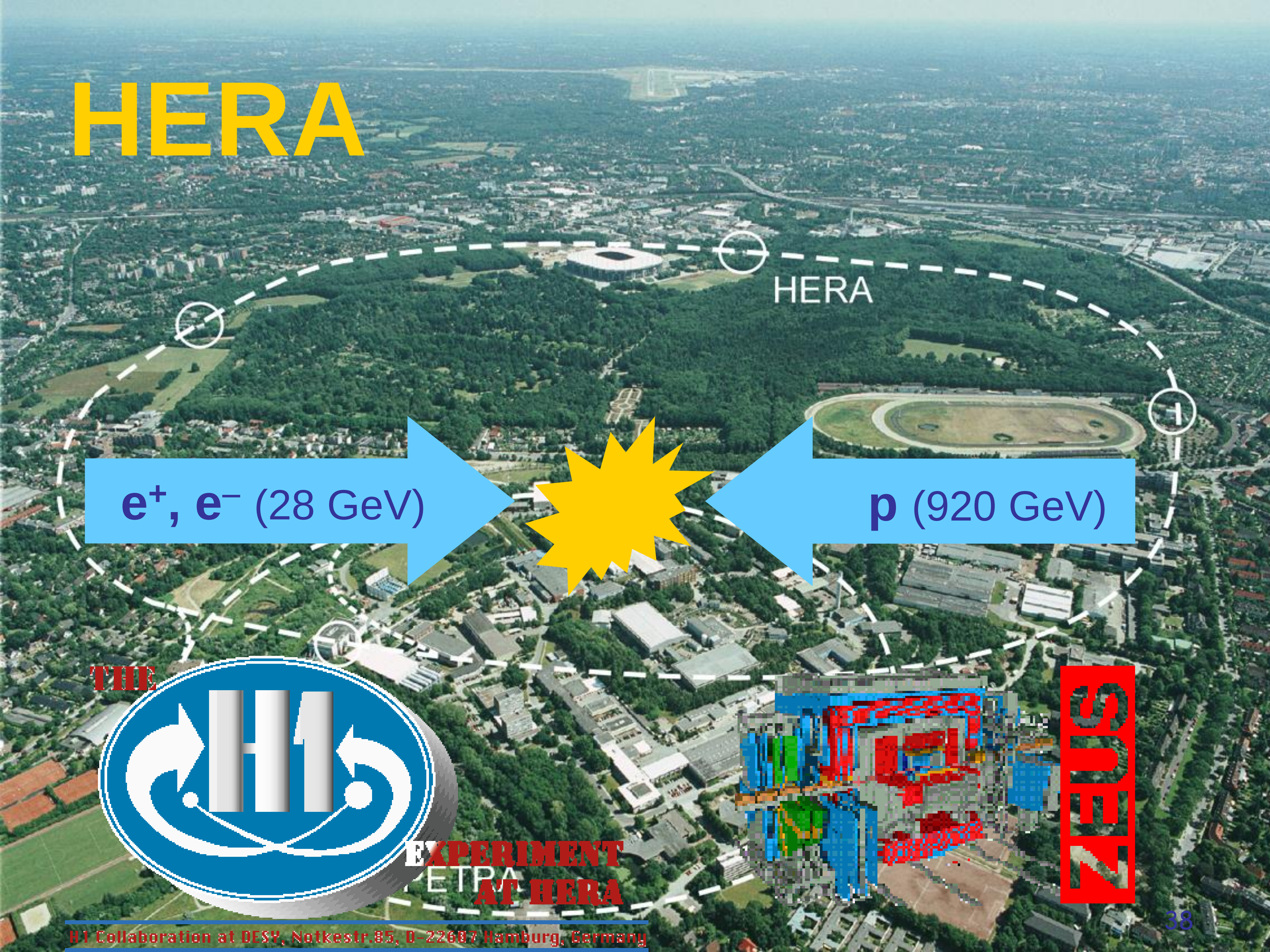


- but valence (u,d) quarks and sea quarks still only account for about 50% of the momentum; the rest is carried by gluons

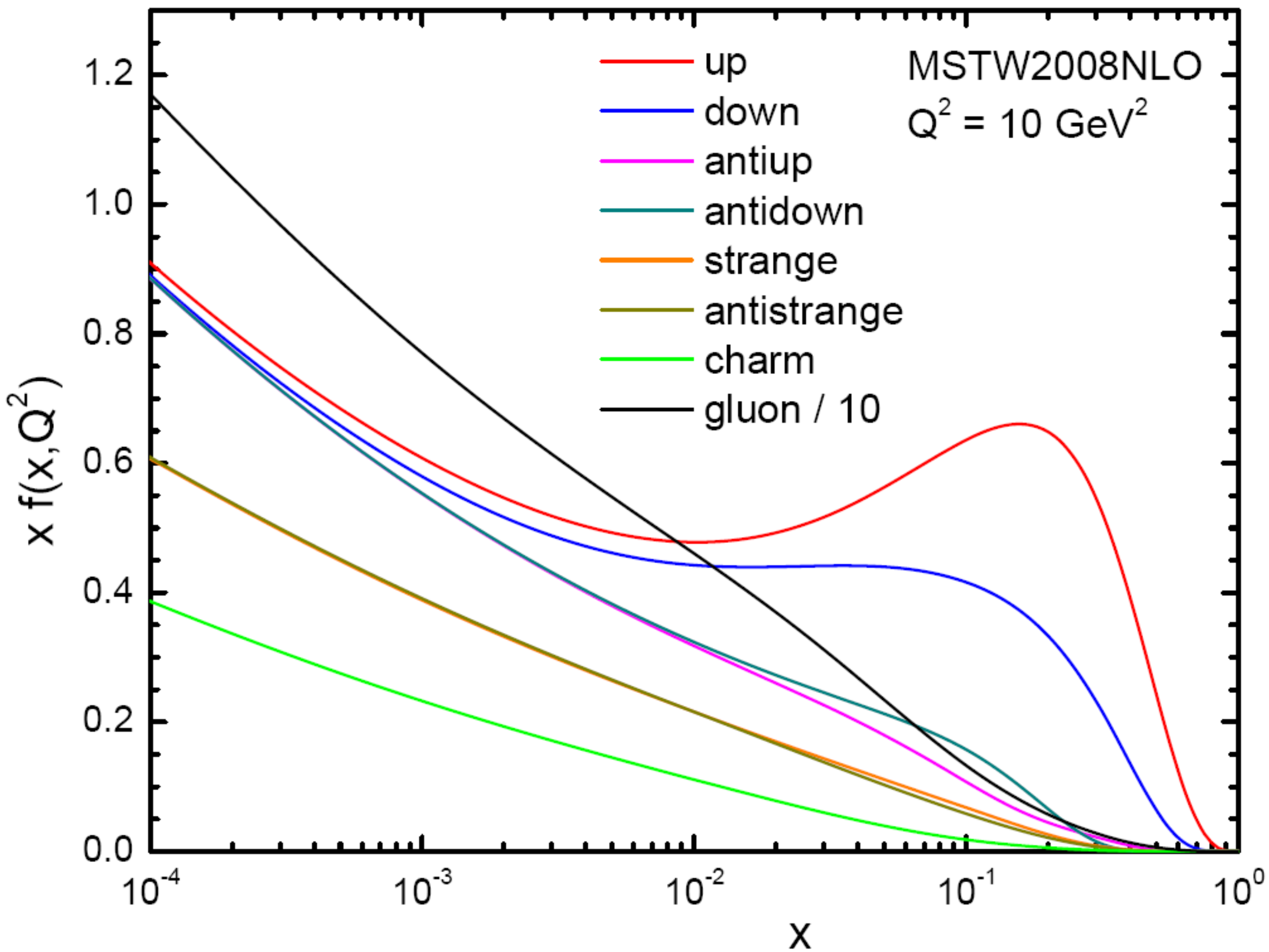
40 years of Deep Inelastic Scattering



HERA


HERA
e^+, e^- (28 GeV)
p (920 GeV)
PETRA
THE H1 EXPERIMENT AT HERA
H1 Collaboration at DESY, Notkestr.85, D-22607 Hamburg, Germany
ZEUS

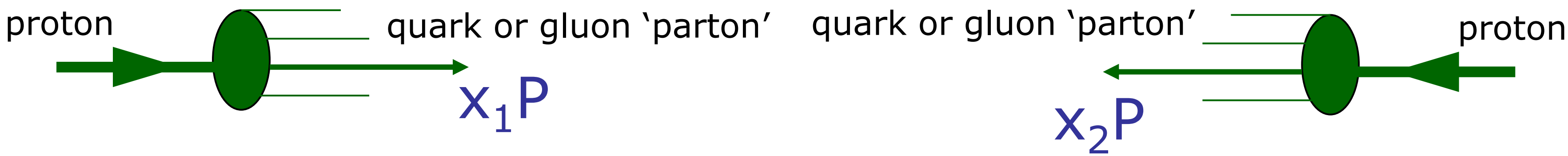
(MSTW) parton distribution functions



partons = valence quarks + sea quarks + gluons

* MSTW = Martin, S, Thorne, Watt

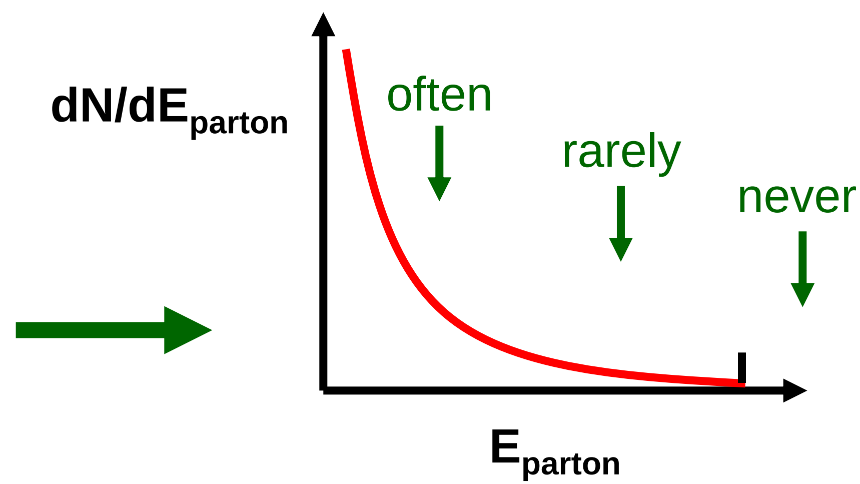
…and so in proton-proton collisions



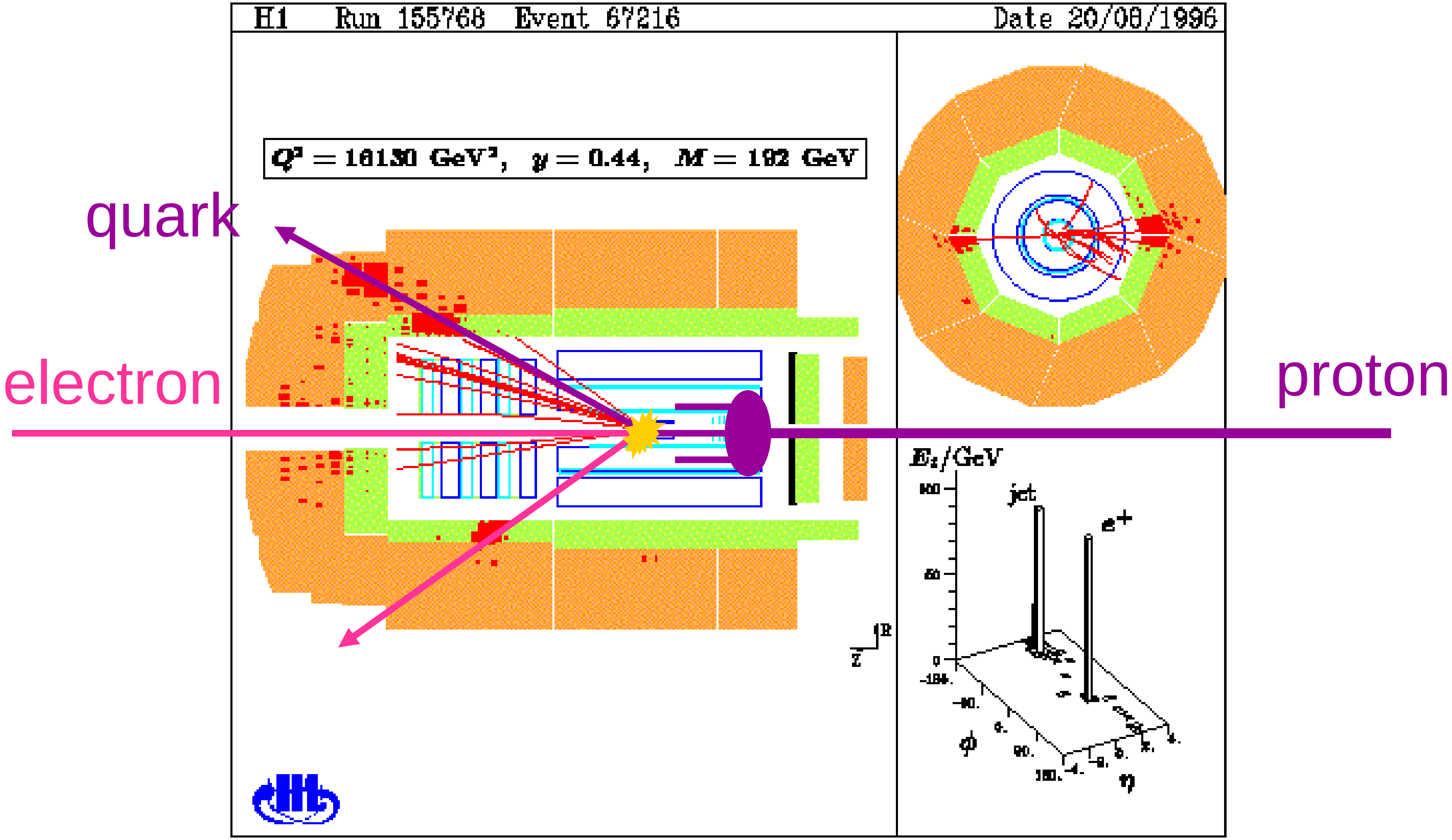
$\Rightarrow$ $E_{parton} = \sqrt{(x_1 x_2)}\, E_{collider} \leq E_{collider}$

relativistic kinematics

this collision energy distribution is just a convolution of the two parton probability distribution functions $f(x_1)*f(x_2)$

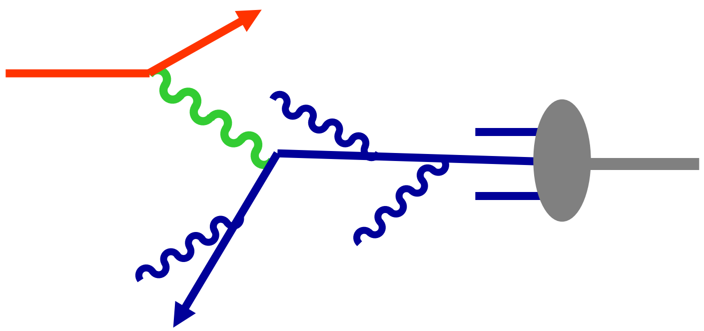


a deep inelastic scattering event at HERA

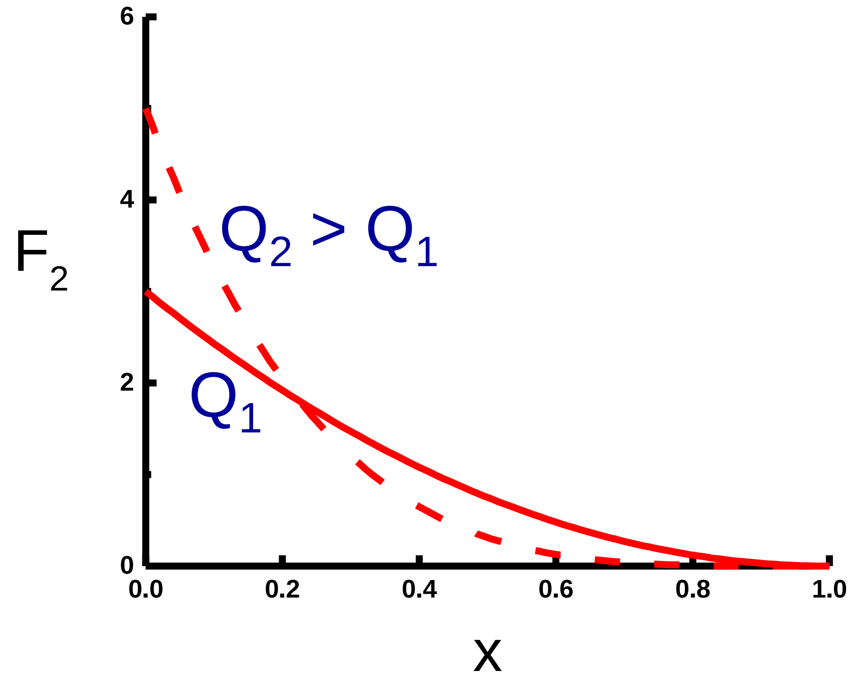


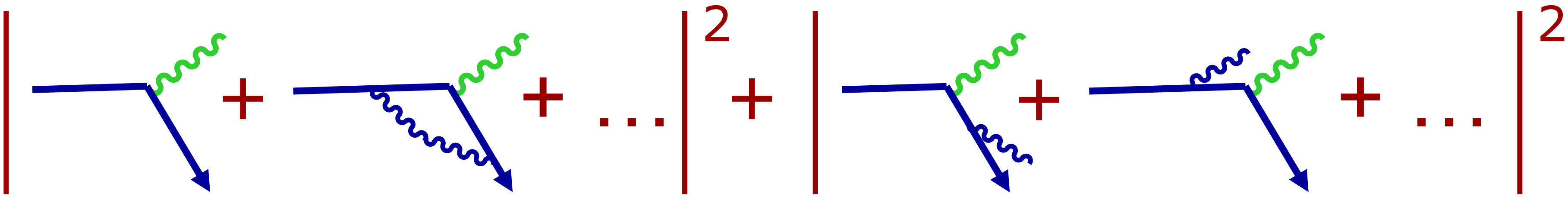
scaling violations and QCD

The structure function data exhibit systematic violations of Bjorken scaling:



quarks emit gluons!





$$\Rightarrow \quad \hat{F}_2 = e_q^2\,\delta(1-x) + e_q^2 \frac{\alpha_S}{2\pi} x \left[P(x) \ln(Q^2/\kappa^2) + C(x) \right]$$

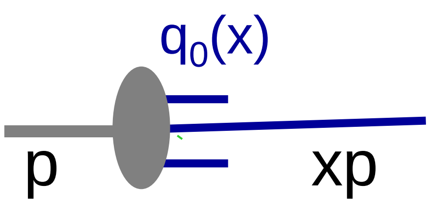
where the logarithm comes from ('collinear singularity') and

$$\int_0^{\sim Q^2} \frac{dk_T^2}{k_T^2} \rightarrow \int_{\kappa^2}^{\sim Q^2} \frac{dk_T^2}{k_T^2} \rightarrow \ln(Q^2/\kappa^2)$$

$$P(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right]$$

$$\int_0^1 dx = \frac{f(x)}{(1-x)_+} = \int_0^1 \frac{f(x)-f(1)}{1-x}$$

then convolute with a 'bare' quark distribution in the proton:



$$F_2(x, Q^2) = x \sum_q e_q^2 \Big[q_0(x) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{dy}{y} q_0(y) \left\{ P(x/y) \ln(Q^2/\kappa^2) + C(x/y) \right\} \Big]$$

next, factorise the collinear divergence into a 'renormalised' quark distribution, by introducing the factorisation scale μ^2 :

$$q(x,\mu^2) = q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y} q_0(y)\left\{P(x/y)\ln(\mu^2/\kappa^2) + \overline{C}(x/y)\right\}$$

then

$$\frac{1}{x}F_2(x,Q^2) = x\sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y,\mu^2) \left\{\delta(1-\frac{x}{y}) + \frac{\alpha_s}{2\pi}\left(P(x/y)\ln(Q^2/\mu^2) + C_q(x/y)\right)\right\}$$

finite, by construction

note arbitrariness of $C_q = C - \overline{C}$ → 'factorisation scheme dependence'

we can choose $\overline{C}$ such that $C_q = 0$, the DIS scheme, or use dimensional regularisation and remove the poles at N=4, the $\overline{\text{MS}}$ scheme, with $C_q \neq 0$

$q(x,\mu^2)$ is not calculable in perturbation theory,* but its scale (μ^2) dependence is:

$$\mu^2\frac{\partial}{\partial\mu^2}q(x,\mu^2) = \frac{\alpha_S(\mu^2)}{2\pi}\int_x^1 \frac{dy}{y} q(y,\mu^2)P(x/y)$$

Dokshitzer
Gribov
Lipatov
Altarelli
Parisi

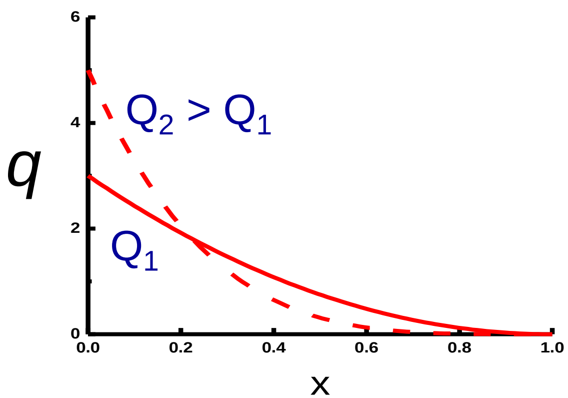
*lattice QCD?

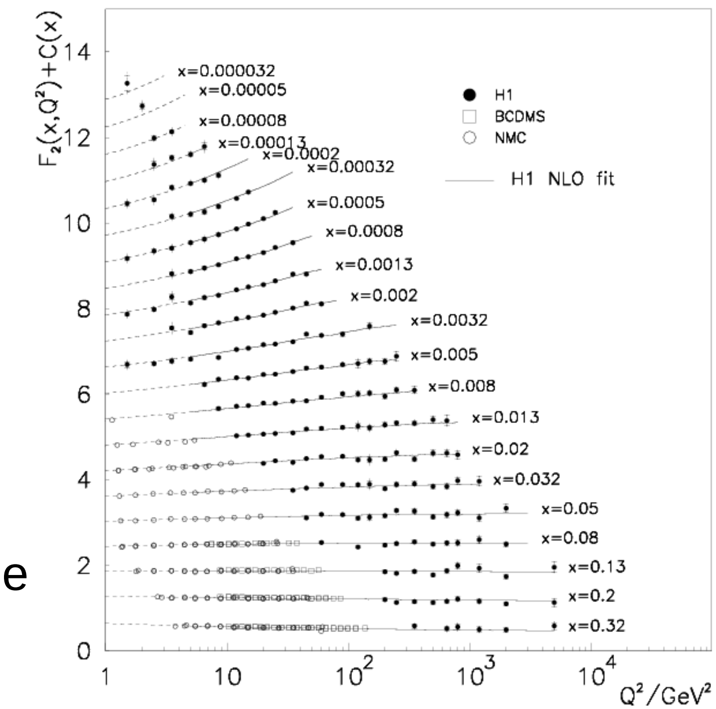
note that we are free to choose $\mu^2 = Q^2$ in which case

$$\frac{1}{x}F_2(x,Q^2) = x\sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y,Q^2)\left\{\delta(1-\frac{x}{y}) + \frac{\alpha_s}{2\pi}C_q(x/y)\right\}$$

coefficient function, see ESW QCD book

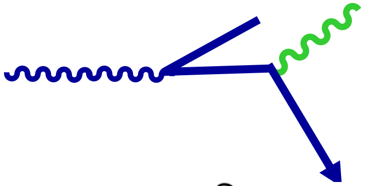
… and thus the scaling violations of the structure function follow those of $q(x,Q^2)$ predicted by the DGLAP equation:





the rate of change of F_2 is proportional to α_S (DGLAP), hence structure function data can be used to measure the strong coupling!

however, we must also include the gluon contribution



$$\frac{1}{x}F_2(x,Q^2) = x\sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y,Q^2)\left\{\delta(1-\frac{x}{y}) + \frac{\alpha_s(Q^2)}{2\pi}C_q(x/y)\right\}$$
$$+ \; x\sum_q e_q^2 \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q^2)C_g(x/y)$$

coefficient functions
- see ESW QCD book

… and with additional terms in the DGLAP equations

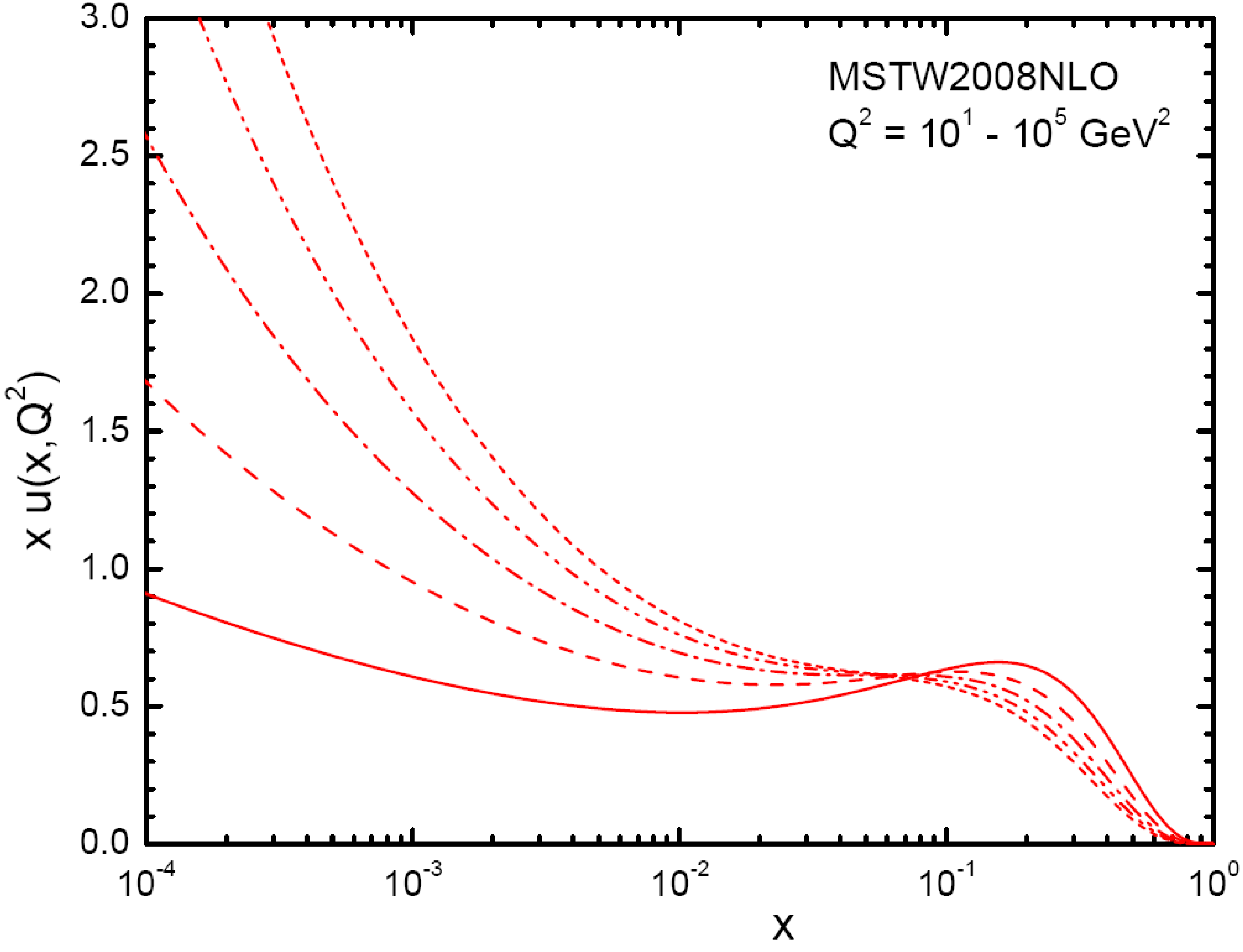
$$\mu^2\frac{\partial q_i(x,\mu^2)}{\partial\mu^2} = \frac{\alpha_S(\mu^2)}{2\pi}(P^{qq} * q_i + 2n_f P^{qg} * g)$$
$$\mu^2\frac{\partial g(x,\mu^2)}{\partial\mu^2} = \frac{\alpha_S(\mu^2)}{2\pi}(P^{gq} * \sum_i q_i + P^{gg} * g)$$

$q_i = u, \bar{u}, d, \bar{d}, \ldots$
$* =$ convolution integral

note that at small (large) x, the gluon (quark) contribution dominates the evolution of the quark distributions, and therefore of F_2

$$P^{qq} = \frac{4}{3}(\frac{1+x^2}{1-x})_+$$
$$P^{qg} = \frac{1}{2}(x^2+(1-x)^2)$$
$$P^{gq} = \frac{4}{3}\left(\frac{1+(1-x)^2}{x}\right)$$
$$P^{gg} = 6\left(\frac{1-x}{x} + x(1-x) + (\frac{x}{1-x})_+\right) - \left(\frac{1}{2}+\frac{n_f}{3}\right)\delta(1-x)$$

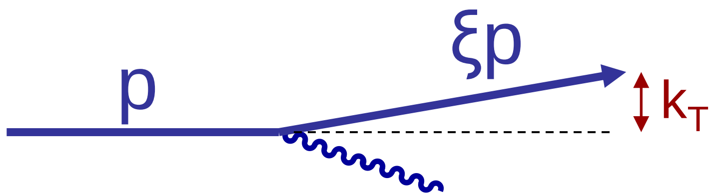
splitting functions


MSTW2008NLO
Q2 = 10^1 - 10^5 GeV^2
x u(x,Q2)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
10^-4
10^-3
10^-2
10^-1
10^0
x

DGLAP evolution: physical picture

Altarelli, Parisi (1977)

- a fast-moving quark loses momentum by emitting a gluon:

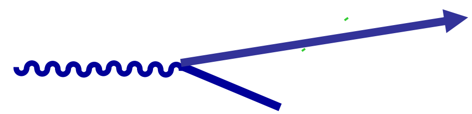


$$d\mathcal{P} \simeq \frac{\alpha_S(k_T^2)}{2\pi}\,\frac{dk_T^2}{k_T^2}\,P^{qq}(\xi)\,d\xi$$

- … with phase space $k_T^2 < O(Q^2)$, hence

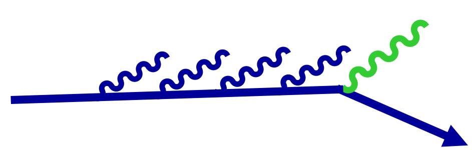
$$d\mathcal{P} \simeq \frac{\alpha_S}{2\pi}\,\ln Q^2\,P^{qq}(\xi)\,d\xi$$

- similarly for other splittings



$$\sim\ P^{gq}$$

- the combination of all such probabilistic splittings correctly generates the leading-logarithm approximation to the all-orders in pQCD solution of the DGLAP equations



basis of parton shower Monte Carlos!

beyond lowest order in pQCD

going to higher orders in pQCD is straightforward in principle, since the above structure for F_2 and for DGLAP generalises in a straightforward way:

$$\frac{\partial \mathbf{q_i(x,Q^2)}}{\partial \log \mathbf{Q^2}} = \frac{\alpha_\mathbf{S}}{\mathbf{2}\pi}\int_\mathbf{x}^\mathbf{1}\frac{\mathbf{dy}}{\mathbf{y}}\Big\{\mathbf{P_{q_iq_j}(y,\alpha_S)\ q_j(\frac{x}{y},Q^2)} + \mathbf{P_{q_ig}(y,\alpha_S)\ g(\frac{x}{y},Q^2)}\Big\}$$

$$\frac{\partial \mathbf{g(x,Q^2)}}{\partial \log \mathbf{Q^2}} = \frac{\alpha_\mathbf{S}}{\mathbf{2}\pi}\int_\mathbf{x}^\mathbf{1}\frac{\mathbf{dy}}{\mathbf{y}}\Big\{\mathbf{P_{gq_j}(y,\alpha_S)\ q_j(\frac{x}{y},Q^2)} + \mathbf{P_{gg}(y,\alpha_S)\ g(\frac{x}{y},Q^2)}\Big\}$$

1972-77	1977-80	2004

DGLAP: $$P(x,\alpha_S) = P^{(0)} + \alpha_S P^{(1)}(x) + \alpha_S^2 P^{(2)}(x) + \ldots$$

see above — see book — very complicated!

The calculation of the complete set of $P^{(2)}$ splitting functions by Moch, Vermaseren and Vogt (hep-ph/0403192,0404111) completed the calculational tools for a consistent NNLO pQCD treatment of Tevatron & LHC hard-scattering cross sections!

- and for the structure functions…

$$\frac{1}{x}F_2(x,Q^2) = x\sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y,Q^2)\left\{\delta(1-\frac{x}{y}) + \frac{\alpha_s(Q^2)}{2\pi} C_q^{(1)}(x/y)\right\}$$
$$x\sum_q e_q^2 \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} g(y,Q^2) C_g^{(1)}(x/y) + \mathcal{O}(\alpha_s^2(Q^2))$$

… where up to and including the $O(\alpha_S^3)$ coefficient functions are known

- terminology:
  - LO: $P^{(0)}$
  - NLO: $P^{(0,1)}$ and $C^{(1)}$
  - NNLO: $P^{(0,1,2)}$ and $C^{(1,2)}$

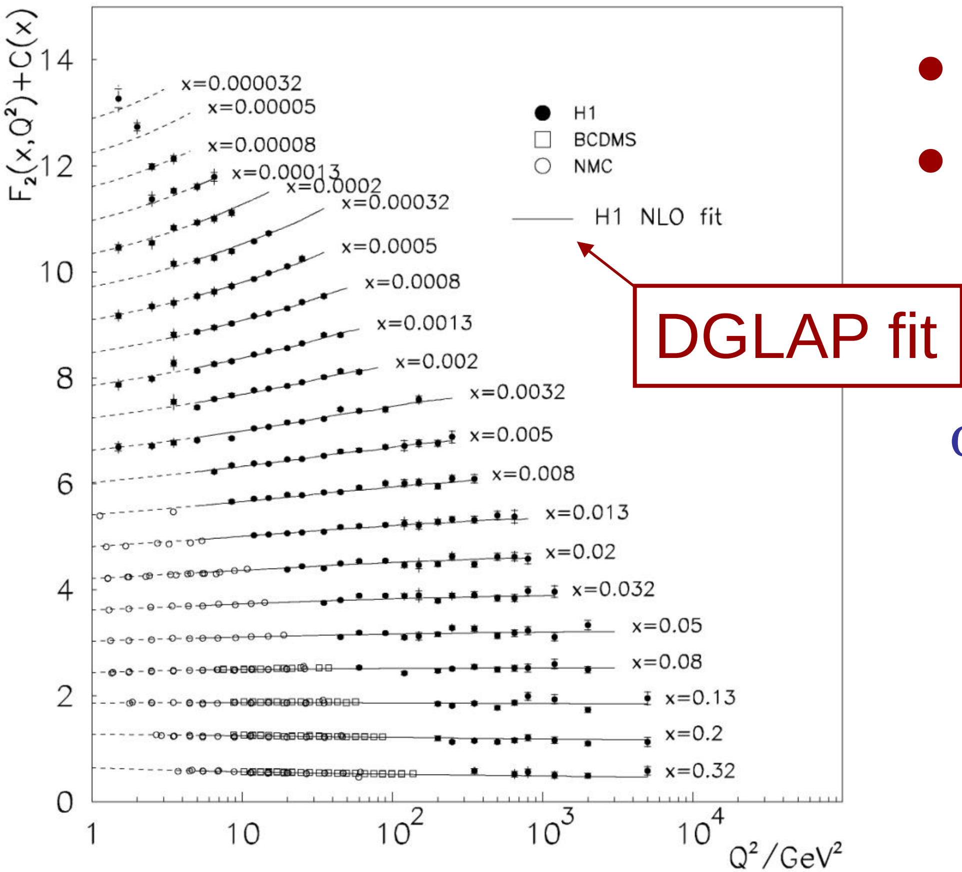
- the more pQCD orders are included, the weaker the dependence on the (unphysical) factorisation scale, μ_F^2

– and also the (unphysical) renormalisation scale, μ_R^2 ; note above has $\mu_R^2 = Q^2$

testing QCD

structure function data

from H1, BCDMS, NMC



- precision test of QCD
- measurement of the strong coupling:

$$\alpha_S^{NNLO}(M_Z) = 0.1171 \pm 0.0014$$

(MSTW 2008, from global fit)

how PDFs are obtained*

- choose a factorisation scheme (e.g. MSbar), an order in perturbation theory (LO, NLO, NNLO) and a 'starting scale' Q_0 where pQCD applies (e.g. 1-2 GeV)
- parametrise the quark and gluon distributions at $Q_{0,}$, e.g.

$$f_i(x,Q_0^2) = A_i x^{a_i}[1 + b_i\sqrt{x} + c_i x](1-x)^{d_i}$$

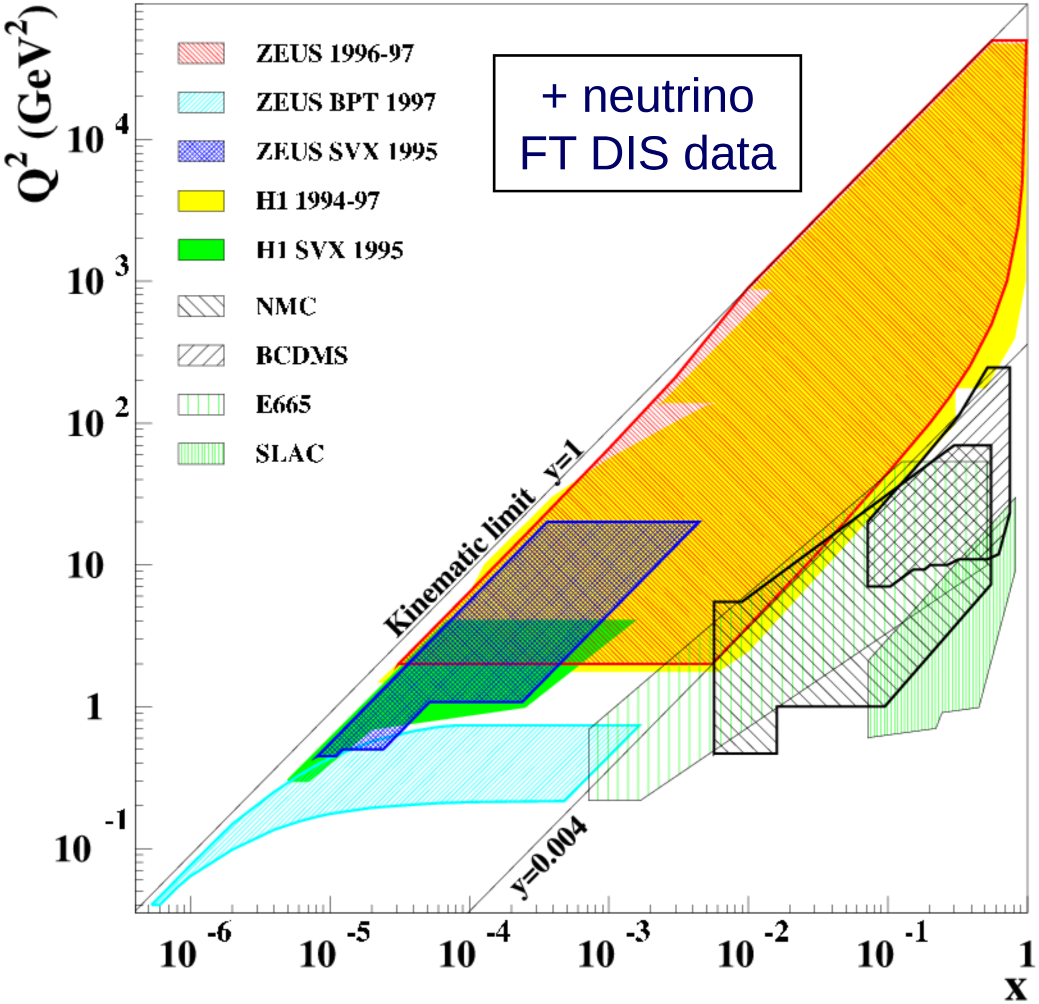
- solve DGLAP equations to obtain the PDFs at any x and scale $Q > Q_0$; fit data for parameters $\{A_i, a_i, \ldots \alpha_S\}$
- approximate the exact solutions (e.g. interpolation grids, expansions in polynomials etc) for ease of use; thus the output 'global fits' are available 'off the shelf", e.g.

```
SUBROUTINE PDF(X,Q,U,UBAR,D,DBAR,…,BBAR,GLU)
```

input | output

*traditional method

summary of DIS data



Note: must impose cuts on DIS data to ensure validity of leading-twist DGLAP formalism in analyses to determine PDFs, typically:

$Q^2 > 2 - 4$ GeV^2

$W^2 = (1-x)/x\, Q^2 > 10 - 15$ GeV^2

examples of data sets used in fits*

Data set	$N_{\text{pts.}}$
H1 MB 99 e^+p NC	8
H1 MB 97 e^+p NC	64
H1 low Q^2 96–97 e^+p NC	80
H1 high Q^2 98–99 e^-p NC	126
H1 high Q^2 99–00 e^+p NC	147
ZEUS SVX 95 e^+p NC	30
ZEUS 96–97 e^+p NC	144
ZEUS 98–99 e^-p NC	92
ZEUS 99–00 e^+p NC	90
H1 99–00 e^+p CC	28
ZEUS 99–00 e^+p CC	30
H1/ZEUS $e^\pm p$ F_2^{charm}	83
H1 99–00 e^+p incl. jets	24
ZEUS 96–97 e^+p incl. jets	30
ZEUS 98–00 $e^\pm p$ incl. jets	30
DØ II $p\bar{p}$ incl. jets	110
CDF II $p\bar{p}$ incl. jets	76
CDF II $W \to l\nu$ asym.	22
DØ II $W \to l\nu$ asym.	10
DØ II Z rap.	28
CDF II Z rap.	29

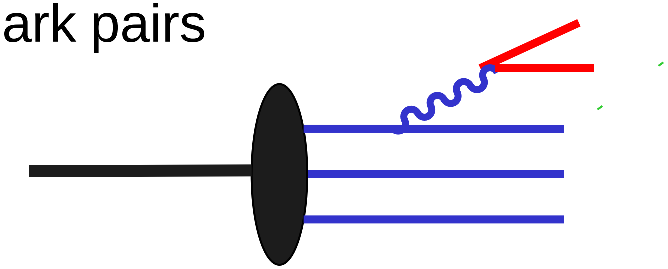
Data set	$N_{\text{pts.}}$
BCDMS μp F_2	163
BCDMS μd F_2	151
NMC μp F_2	123
NMC μd F_2	123
NMC $\mu n/\mu p$	148
E665 μp F_2	53
E665 μd F_2	53
SLAC ep F_2	37
SLAC ed F_2	38
NMC/BCDMS/SLAC F_L	31
E866/NuSea pp DY	184
E866/NuSea pd/pp DY	15
NuTeV νN F_2	53
CHORUS νN F_2	42
NuTeV νN xF_3	45
CHORUS νN xF_3	33
CCFR $\nu N \to \mu\mu X$	86
NuTeV $\nu N \to \mu\mu X$	84
All data sets	**2743**

- Red = New w.r.t. MRST 2006 fit.

*MSTW2008

the asymmetric sea

- the sea presumably arises when ‘primordial‘ valence quarks emit gluons which in turn split into quark-antiquark pairs, with suppressed splitting into heavier quark pairs

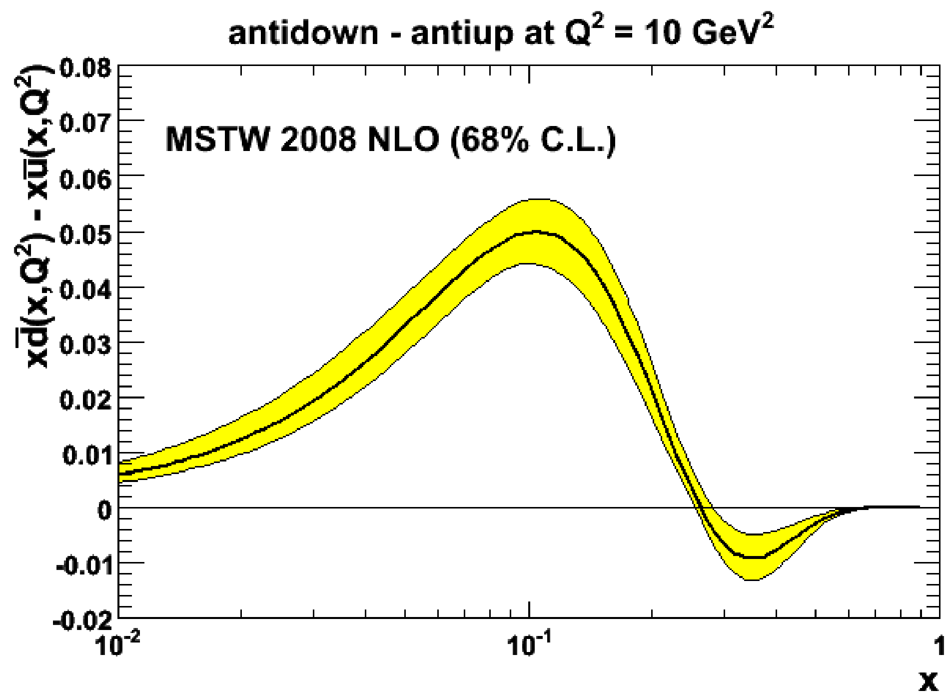


- so we naively expect

$$\bar{u} \approx \bar{d} > \bar{s} > \bar{c} > \ldots$$

- u_{sea}, d_{sea}, s obtained from fits to data
- c,b from pQCD, g → Q $\bar{Q}$

The ratio of Drell-Yan cross sections (see later) for $pp,pn \rightarrow \mu^+\mu^- + X$ provides a measure of the difference between the *u* and *d* sea quark distributions



strange

earliest PDF fits had SU(3) symmetry: $s(x,Q_0^2) = \bar{s}(x,Q_0^2) = \bar{u}(x,Q_0^2) = \bar{d}(x,Q_0^2)$

later relaxed to include (constant) strange suppression (cf. fragmentation):

$$s(x,Q_0^2) = \bar{s}(x,Q_0^2) = \frac{\kappa}{2}\left[\bar{u}(x,Q_0^2) + \bar{d}(x,Q_0^2)\right]$$

with $\kappa = 0.4 - 0.5$

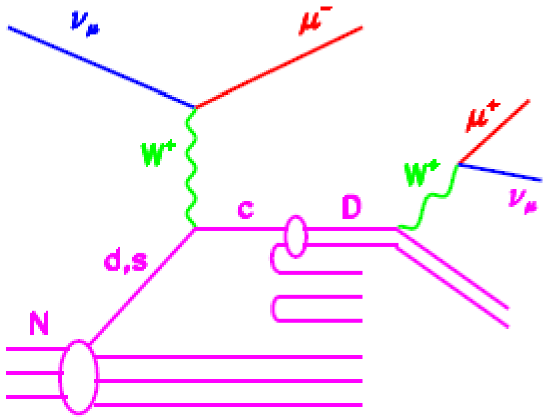
nowadays, dimuon production in νN DIS (CCFR, NuTeV) allows 'direct' determination:

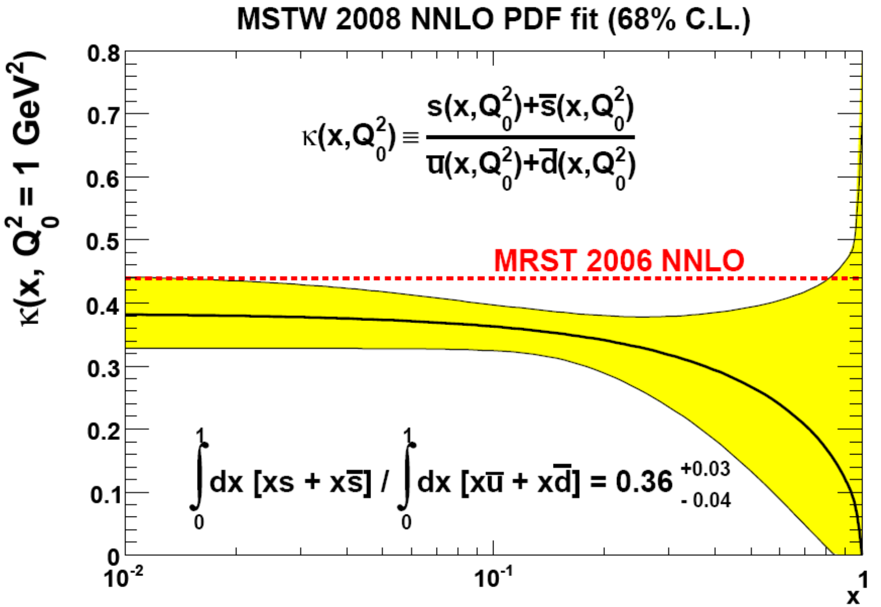
$$\frac{d\sigma}{dxdy}\left(\nu_\mu(\bar{\nu}_\mu)N \to \mu^+\mu^- X\right) = B_c\,\mathcal{N}\mathcal{A}\,\frac{d\sigma}{dxdy}\left(\nu_\mu s(\bar{\nu}_\mu \bar{s}) \to c\mu^-(\bar{c}\mu^+)X\right)$$

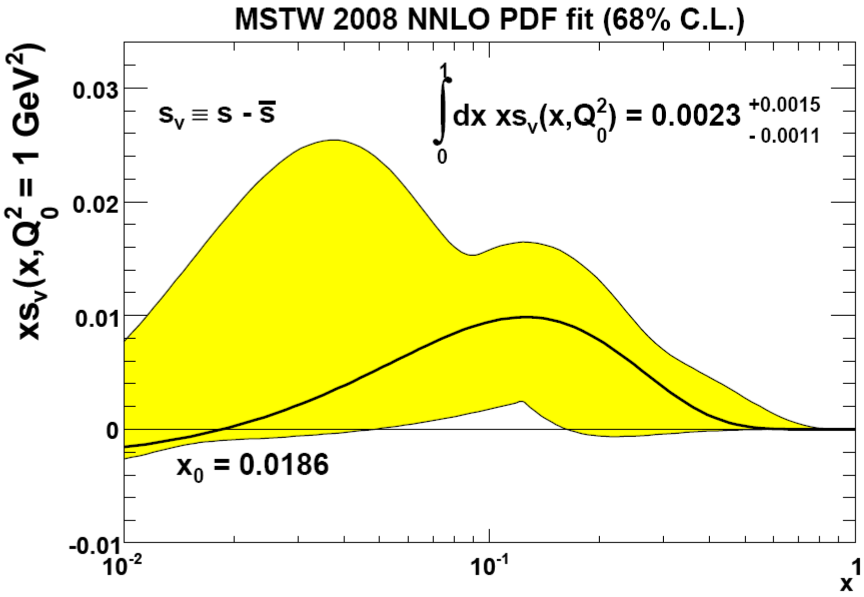
in the range $0.01 < x < 0.4$

data seem to prefer $s(x,Q_0^2) - \bar{s}(x,Q_0^2) \neq 0$

theoretical explanation?!




MSTW 2008 NNLO PDF fit (68% C.L.)
$\kappa(x, Q_0^2) \equiv \frac{s(x,Q_0^2)+\bar{s}(x,Q_0^2)}{\bar{u}(x,Q_0^2)+\bar{d}(x,Q_0^2)}$
MRST 2006 NNLO
$\int_0^1 dx\,[xs + x\bar{s}] \,/\, \int_0^1 dx\,[x\bar{u} + x\bar{d}] = 0.36^{+0.03}_{-0.04}$
$\kappa(x,\ Q_0^2 = 1\ \mathrm{GeV}^2)$
x


MSTW 2008 NNLO PDF fit (68% C.L.)
$s_v \equiv s - \bar{s}$
$\int_0^1 dx\ xs_v(x,Q_0^2) = 0.0023^{+0.0015}_{-0.0011}$
$x_0 = 0.0186$
$xs_v(x,Q_0^2 = 1\ \mathrm{GeV}^2)$
x

MSTW

charm, bottom

considered sufficiently massive to allow pQCD treatment: $g \to Q\overline{Q}$

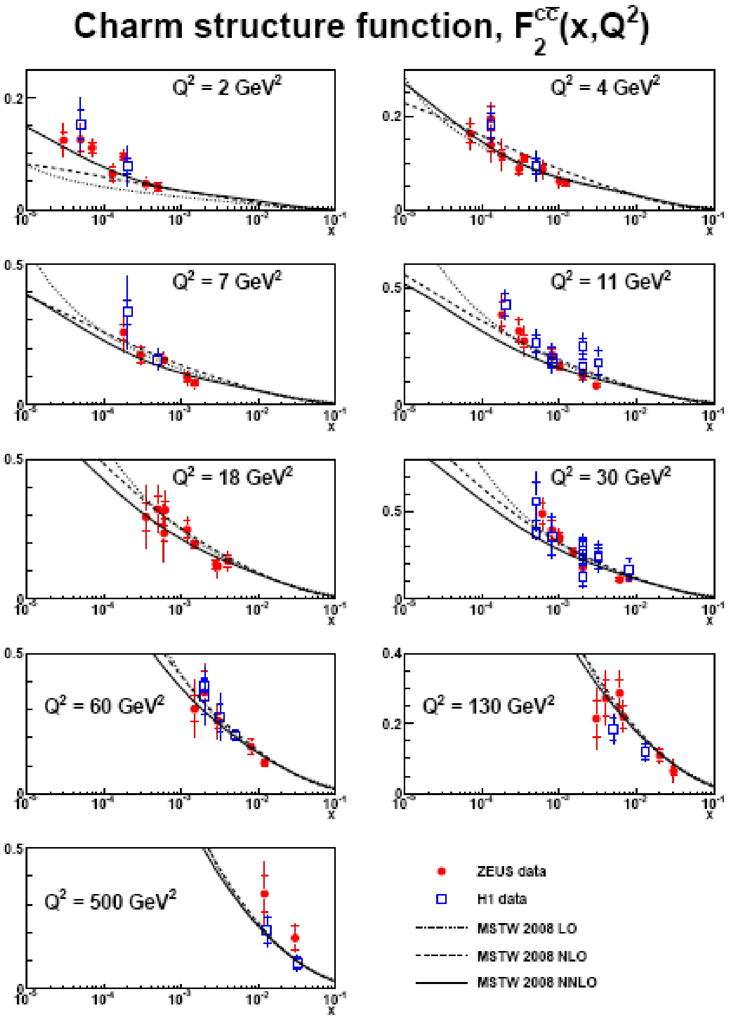
distinguish two regimes:
(i) $Q^2 \sim m_H^2$ include full $\boldsymbol{m_H}$ dependence to get correct threshold behaviour
(ii) $Q^2 \gg m_H^2$ treat as ~massless partons to resum $\boldsymbol{\alpha_S^n \log^n(Q^2/m_H^2)}$ via DGLAP

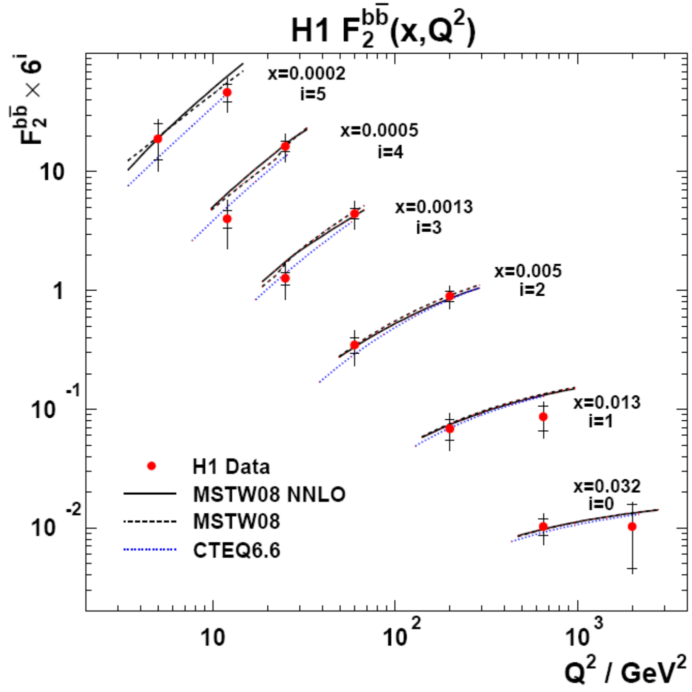
FFNS: OK for (i) only **ZM-VFNS:** OK for (ii) only

consistent **GM**(=general mass)-**VFNS** now available (e.g. ACOT(χ), Roberts-Thorne, ...) which interpolates smoothly between the two regimes

Note: definition of these is tricky and non-unique (ambiguity in assignment of $\boldsymbol{O(m_H^2/Q^2)}$ contributions), and the implementation of improved treatment (e.g. in going from MRST2006 to MSTW 2008) can have a sizeable effect on light partons

charm and bottom structure functions





- MSTW 2008 uses *fixed* values of m_c = 1.4 GeV and m_b = 4.75 GeV in a GM-VFNS

- the sensitivity of the fit to these values, and impact on LHC cross sections, is discussed in MSTW, arXiv:1006.2753

the PDF industry

- many groups now extracting PDFs from 'global' data analyses (MSTW, CTEQ, NNPDF, HERAPDF, AKBM, GJR, …)

- broad agreement, but differences due to
  - choice of data sets (including cuts, corrections and weighting) and treatment of data errors
  - definition of 'PDF uncertainties'
  - treatment of heavy quarks (s,c,b), FFNS, ZM-VFNS, GM-VFNS,
  - treatment of α_S (fitted or fixed)
  - parametric form at Q_0
  - (hidden) theoretical assumptions (if any) about flavour symmetries, $x \to 0,1$ behaviour, etc.

HERA-DIS
FT-DIS
Drell-Yan
Tevatron jets
Tevatron W,Z
other
LHC
new

... and all now with NLO and NNLO* sets

*not 'true' NNLO fits when collider inclusive jet data are included, since NNLO pQCD corrections not yet known

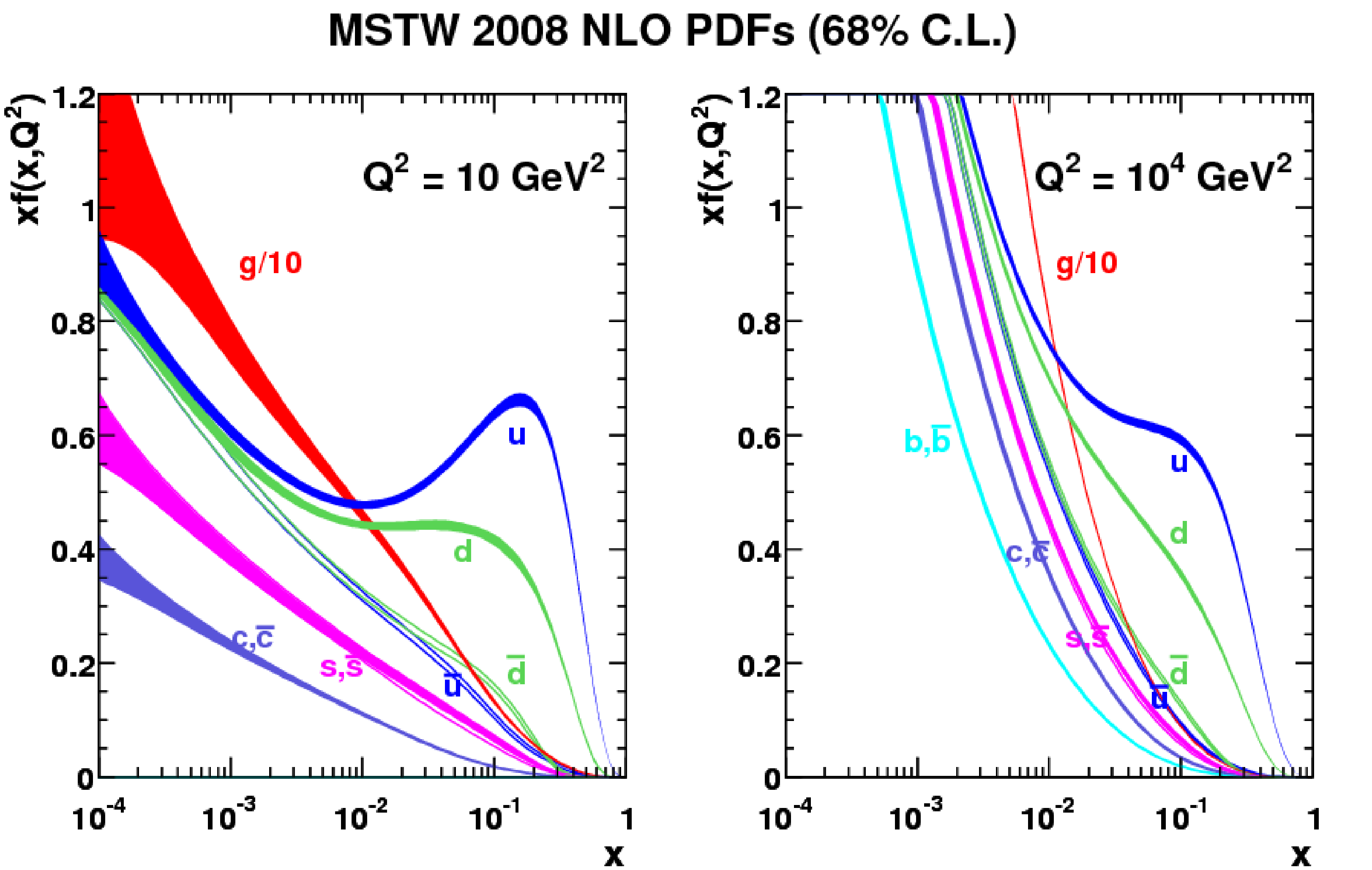
recent global or quasi-global PDF fits

PDFs	authors	arXiv
AB(K)M	S. Alekhin, J. Blümlein, S. Klein, S. Moch, and others	1202.2281,1105.5349, 1007.3657, 0908.3128, …
CT(EQ)	H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. Nadolsky, J. Pumplin, C.-P. Yuan, and others	1007.2241, 1004.4624, 0910.4183, 0904.2424, 0802.0007, …
(G)JR	M. Glück, P. Jimenez-Delgado, E. Reya, and others	1011.6259,1006.5890, 0909.1711, 0810.4274, …
HERAPDF	H1 and ZEUS collaborations	1012.1438,1006.4471, 0906.1108, …
MSTW	A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt	1007.2624, 1006.2753, 0905.3531, 0901.0002, …
NNPDF	R. Ball, L. Del Debbio, S. Forte, A. Guffanti, J. Latorre, J. Rojo, M. Ubiali, and others	1207.1303, 1110.2483, 1108.1758, 1107.2652, 1102.3182, 1101.1300, 1012.0836, 1005.0397, …

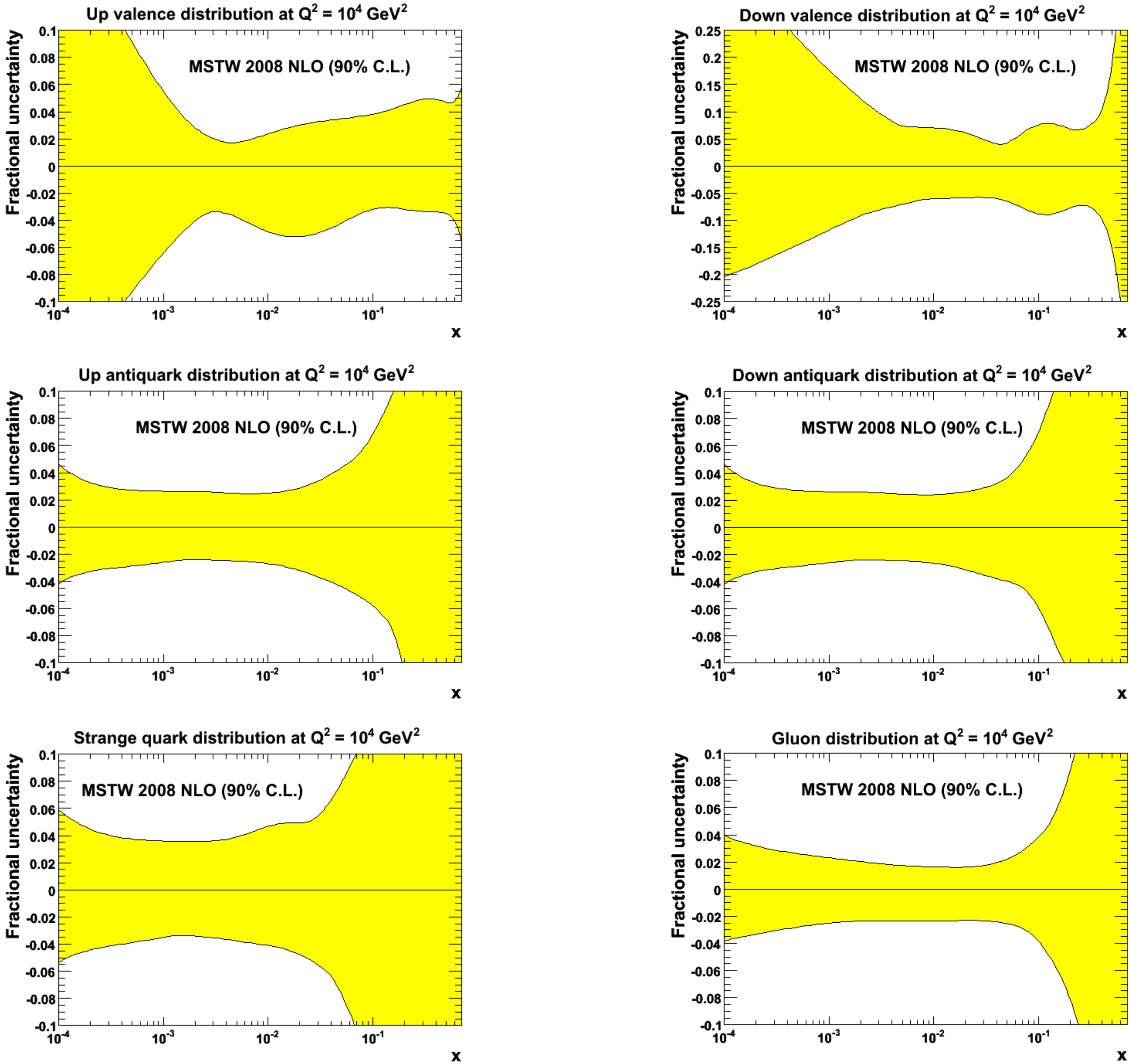
PDF uncertainties

- most global fitting groups produce ‘PDFs with errors’
- typically, 30-40 ‘error’ sets based on a ‘best fit’ set to reflect $\pm 1\sigma$ variation of all the parameters* $\{A_i, a_i, \ldots, \alpha_S\}$ inherent in the fit
- these reflect the uncertainties on the data used in the global fit (e.g. $\delta F_2 \approx \pm 3\% \rightarrow \delta u \approx \pm 3\%$)
- however, there are also systematic PDF uncertainties reflecting theoretical assumptions/prejudices in the way the global fit is set up and performed

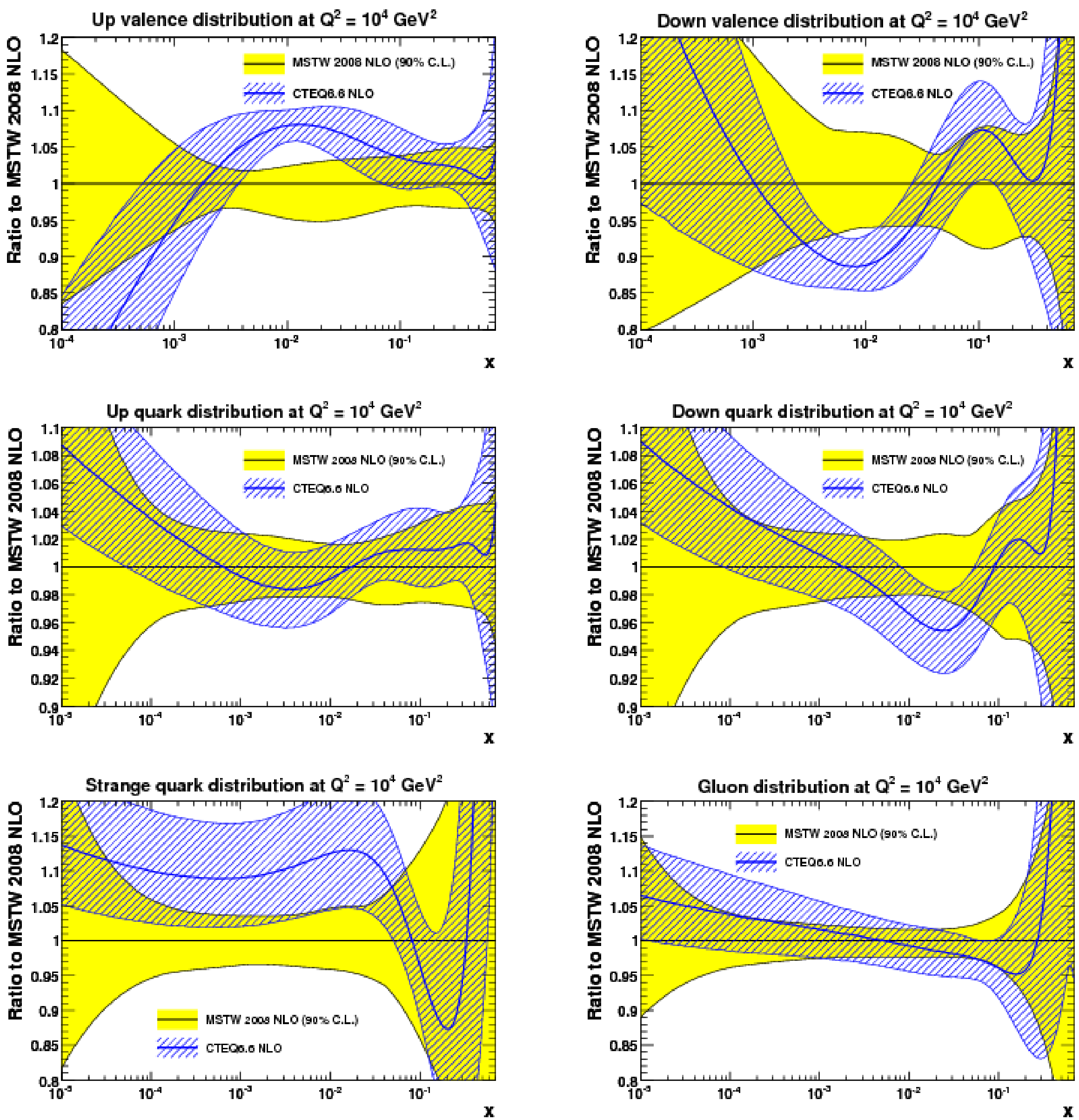
* e.g. $f_i(x, Q_0^2) = A_i x^{a_i}[1 + b_i\sqrt{x} + c_i x](1-x)^{d_i}$



MSTW 2008 NLO PDFs (68% C.L.)
xf(x,Q^2)
Q^2 = 10 GeV^2
g/10
u
d
$\bar{d}$
$\bar{u}$
s,$\bar{s}$
c,$\bar{c}$
Q^2 = 10^4 GeV^2
b,$\bar{b}$
x


Up valence distribution at Q2 = 104 GeV2
MSTW 2008 NLO (90% C.L.)
Fractional uncertainty
x
Down valence distribution at Q2 = 104 GeV2
MSTW 2008 NLO (90% C.L.)
Fractional uncertainty
x
Up antiquark distribution at Q2 = 104 GeV2
MSTW 2008 NLO (90% C.L.)
Fractional uncertainty
x
Down antiquark distribution at Q2 = 104 GeV2
MSTW 2008 NLO (90% C.L.)
Fractional uncertainty
x
Strange quark distribution at Q2 = 104 GeV2
MSTW 2008 NLO (90% C.L.)
Fractional uncertainty
x
Gluon distribution at Q2 = 104 GeV2
MSTW 2008 NLO (90% C.L.)
Fractional uncertainty
x

MSTW2008(NLO) vs. CTEQ6.6



Note:

CTEQ error bands comparable with MSTW 90%cl set (different definition of tolerance)

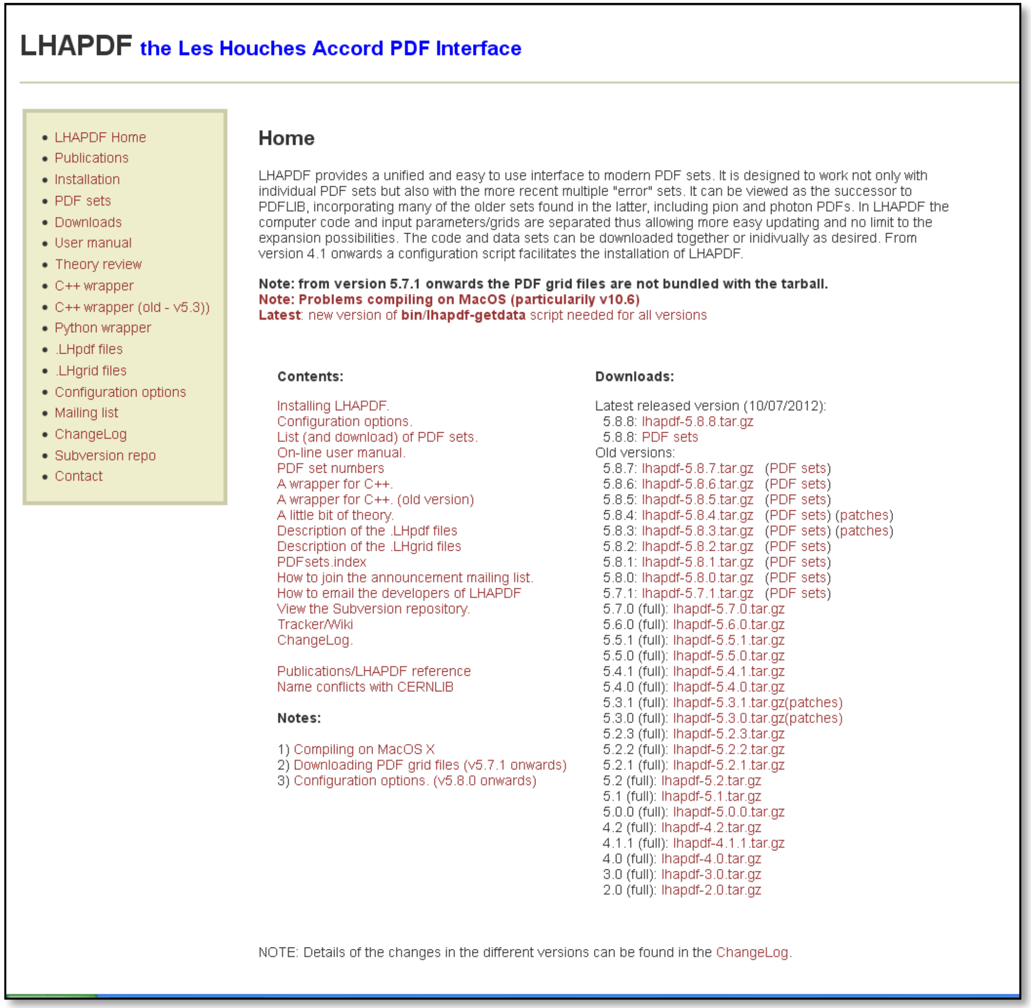
CTEQ light quarks and gluons slightly larger at small x because of imposition of positivity on gluon at Q_0^2

where to find parton distributions

LHAPDF interface at lhapdf.hepforge.org

- access to code for MSTW, CTEQ, NNPDF etc

- see also hepdata.cedar.ac.uk/pdfs for online PDF plotting


LHAPDF the Les Houches Accord PDF Interface

- LHAPDF Home
- Publications
- Installation
- PDF sets
- Downloads
- User manual
- Theory review
- C++ wrapper
- C++ wrapper (old - v5.3))
- Python wrapper
- .LHpdf files
- .LHgrid files
- Configuration options
- Mailing list
- ChangeLog
- Subversion repo
- Contact

Home

LHAPDF provides a unified and easy to use interface to modern PDF sets. It is designed to work not only with individual PDF sets but also with the more recent multiple "error" sets. It can be viewed as the successor to PDFLIB, incorporating many of the older sets found in the latter, including pion and photon PDFs. In LHAPDF the computer code and input parameters/grids are separated thus allowing more easy updating and no limit to the expansion possibilities. The code and data sets can be downloaded together or inidivually as desired. From version 4.1 onwards a configuration script facilitates the installation of LHAPDF.

Note: from version 5.7.1 onwards the PDF grid files are not bundled with the tarball.
Note: Problems compiling on MacOS (particularily v10.6)
Latest: new version of **bin/lhapdf-getdata** script needed for all versions

Contents:

Installing LHAPDF.
Configuration options.
List (and download) of PDF sets.
On-line user manual.
PDF set numbers
A wrapper for C++.
A wrapper for C++. (old version)
A little bit of theory.
Description of the .LHpdf files
Description of the .LHgrid files
PDFsets.index
How to join the announcement mailing list.
How to email the developers of LHAPDF
View the Subversion repository.
Tracker/Wiki
ChangeLog.

Publications/LHAPDF reference
Name conflicts with CERNLIB

Notes:

1) Compiling on MacOS X
2) Downloading PDF grid files (v5.7.1 onwards)
3) Configuration options. (v5.8.0 onwards)

Downloads:

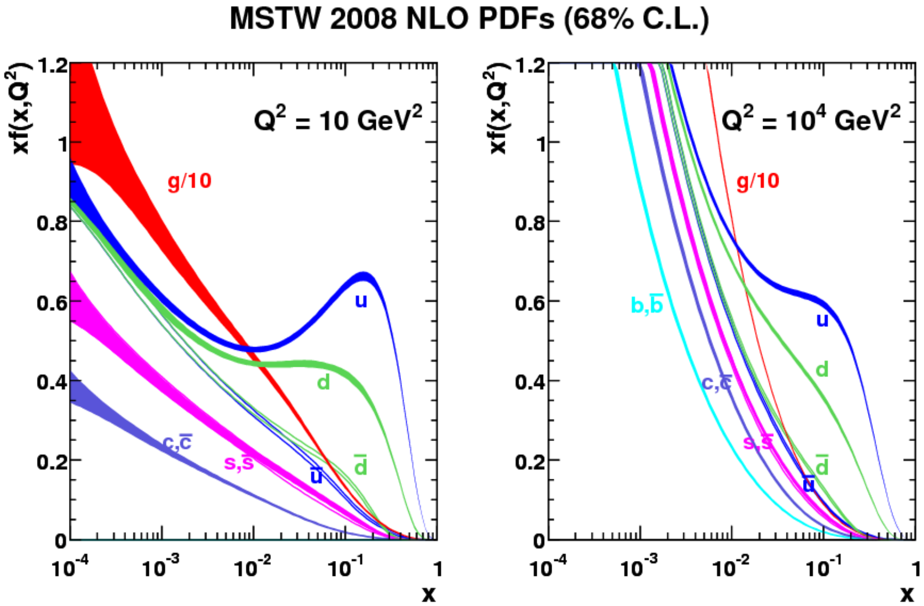
Latest released version (10/07/2012):
5.8.8: lhapdf-5.8.8.tar.gz
5.8.8: PDF sets
Old versions:
5.8.7: lhapdf-5.8.7.tar.gz (PDF sets)
5.8.6: lhapdf-5.8.6.tar.gz (PDF sets)
5.8.5: lhapdf-5.8.5.tar.gz (PDF sets)
5.8.4: lhapdf-5.8.4.tar.gz (PDF sets) (patches)
5.8.3: lhapdf-5.8.3.tar.gz (PDF sets) (patches)
5.8.2: lhapdf-5.8.2.tar.gz (PDF sets)
5.8.1: lhapdf-5.8.1.tar.gz (PDF sets)
5.8.0: lhapdf-5.8.0.tar.gz (PDF sets)
5.7.1: lhapdf-5.7.1.tar.gz (PDF sets)
5.7.0 (full): lhapdf-5.7.0.tar.gz
5.6.0 (full): lhapdf-5.6.0.tar.gz
5.5.1 (full): lhapdf-5.5.1.tar.gz
5.5.0 (full): lhapdf-5.5.0.tar.gz
5.4.1 (full): lhapdf-5.4.1.tar.gz
5.4.0 (full): lhapdf-5.4.0.tar.gz
5.3.1 (full): lhapdf-5.3.1.tar.gz(patches)
5.3.0 (full): lhapdf-5.3.0.tar.gz(patches)
5.2.3 (full): lhapdf-5.2.3.tar.gz
5.2.2 (full): lhapdf-5.2.2.tar.gz
5.2.1 (full): lhapdf-5.2.1.tar.gz
5.2 (full): lhapdf-5.2.tar.gz
5.1 (full): lhapdf-5.1.tar.gz
5.0.0 (full): lhapdf-5.0.0.tar.gz
4.2 (full): lhapdf-4.2.tar.gz
4.1.1 (full): lhapdf-4.1.1.tar.gz
4.0 (full): lhapdf-4.0.tar.gz
3.0 (full): lhapdf-3.0.tar.gz
2.0 (full): lhapdf-2.0.tar.gz

NOTE: Details of the changes in the different versions can be found in the ChangeLog.

what we have learned

- the proton consists of pointlike ‘partons’: valence (uud) quarks, gluons, and a sea of quark—antiquark pairs

- the sea has interesting quark flavour structure, some of which is not understood, i.e. heavier quarks are less likely, but why anti-u ≠ anti-d?



- the small-x partons are predominantly gluons, and they play an important role in LHC physics (see next part)

- the observed scale (Q^2) dependence of the distributions is beautifully described by the QCD theory

- we know the distributions to few % accuracy over most of the x range

3

QCD and Hadron Colliders

- hard scattering & basic kinematics
- the Drell-Yan process in the parton model
- factorisation
- parton luminosity functions

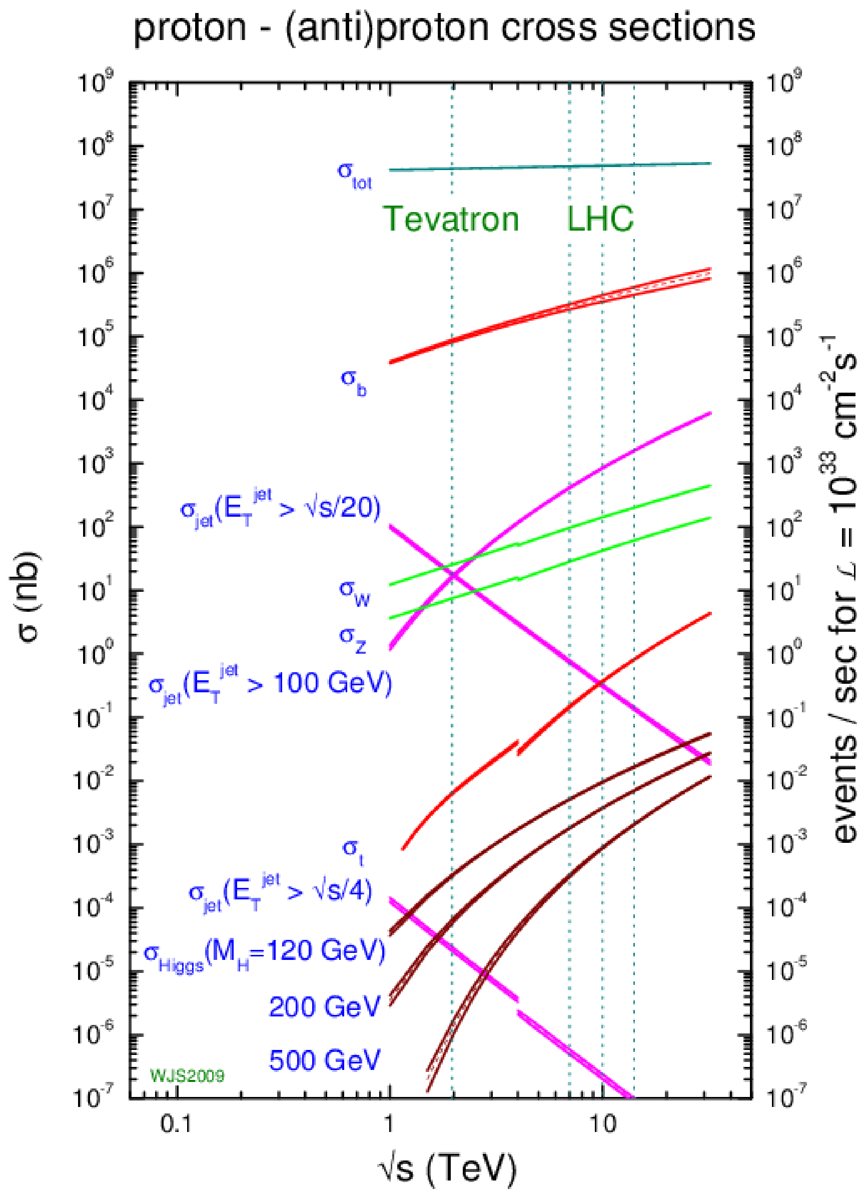
What can we calculate?

Scattering processes at high energy hadron colliders can be classified as either **HARD** or **SOFT**

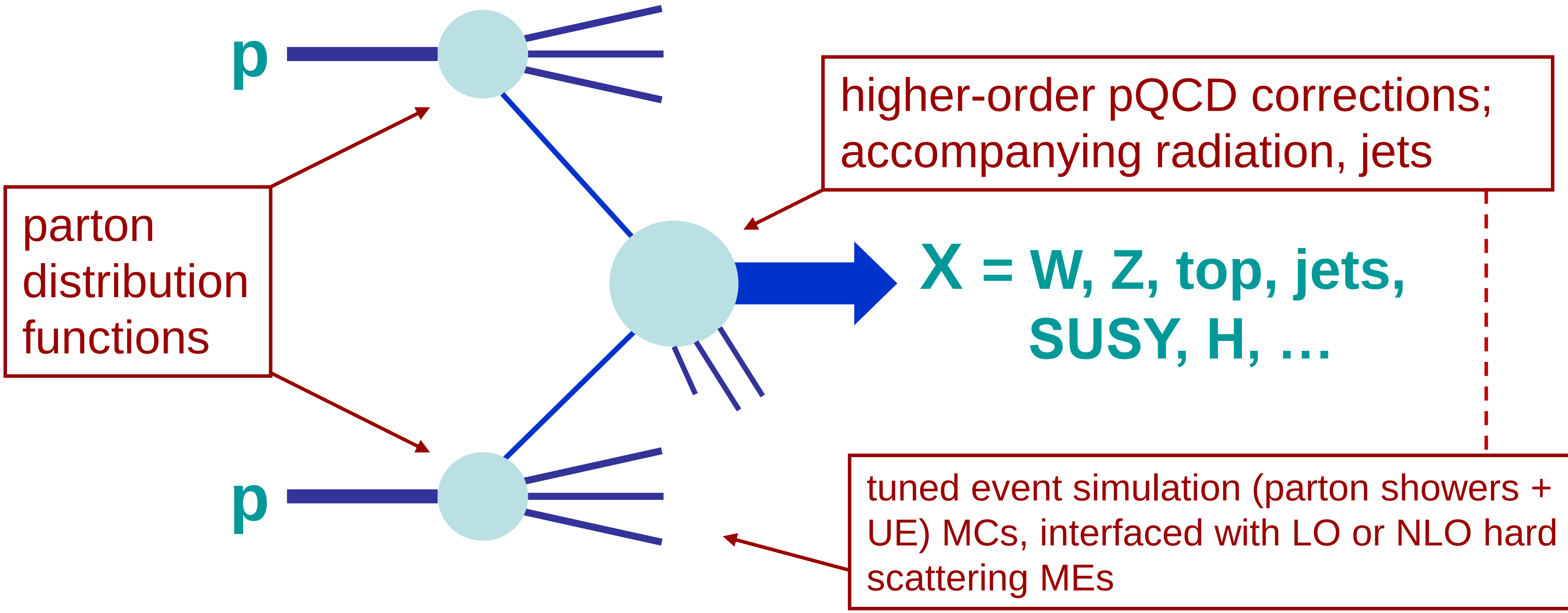
Quantum Chromodynamics (QCD) is the underlying theory for **all** such processes, but the approach (and the level of understanding) is very different for the two cases

For **HARD** processes, e.g. W or high-E_T jet production, the rates and event properties can be predicted with some precision using perturbation theory

For **SOFT** processes, e.g. the total cross section or diffractive processes, the rates and properties are dominated by non-perturbative QCD effects, which are much less well understood



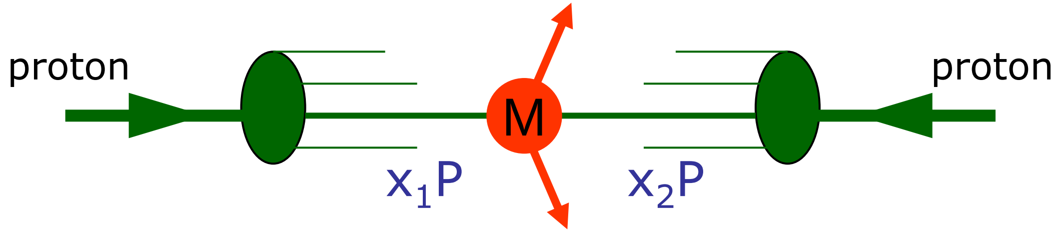
hard scattering in hadron-hadron collisions



for inclusive production, the basic calculational framework is provided by the QCD FACTORISATION THEOREM:

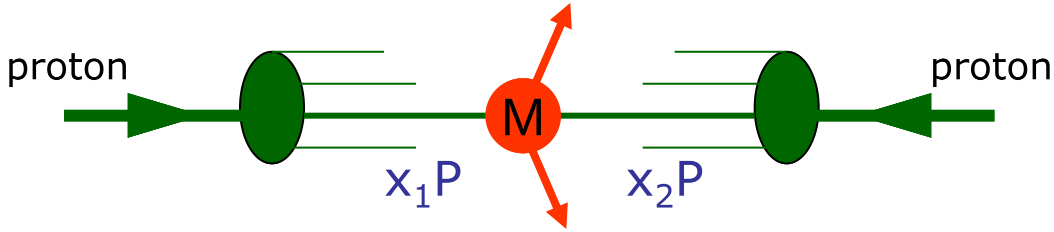
$$\begin{aligned}\sigma_X &= \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1,\mu_F^2)\, f_b(x_2,\mu_F^2) \\ &\times\ \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)\end{aligned}$$

kinematics

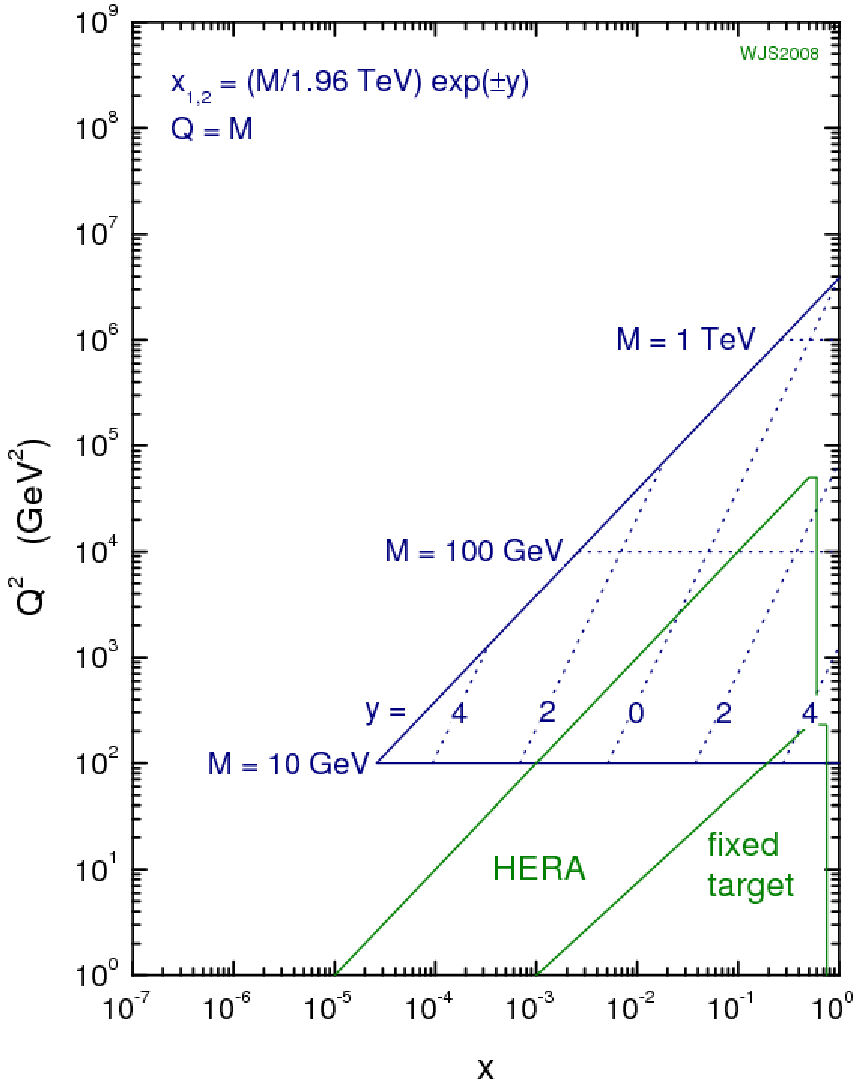


- collision energy: $\sqrt{s}$
- parton momenta:
$$p_1^\mu = x_1\,\sqrt{s}/2\,(1,0,0,1)$$
$$p_2^\mu = x_2\,\sqrt{s}/2\,(1,0,0,-1)$$
- invariant mass: $M^2 = (p_1+p_2)^2 \equiv \hat{s} = x_1 x_2 s$
- rapidity: $y = \frac{1}{2}\,\log\frac{E+p_z}{E-p_z} = \frac{1}{2}\,\log\frac{x_1}{x_2} \Rightarrow \frac{x_1}{x_2} = \mathrm{e}^{2y}$

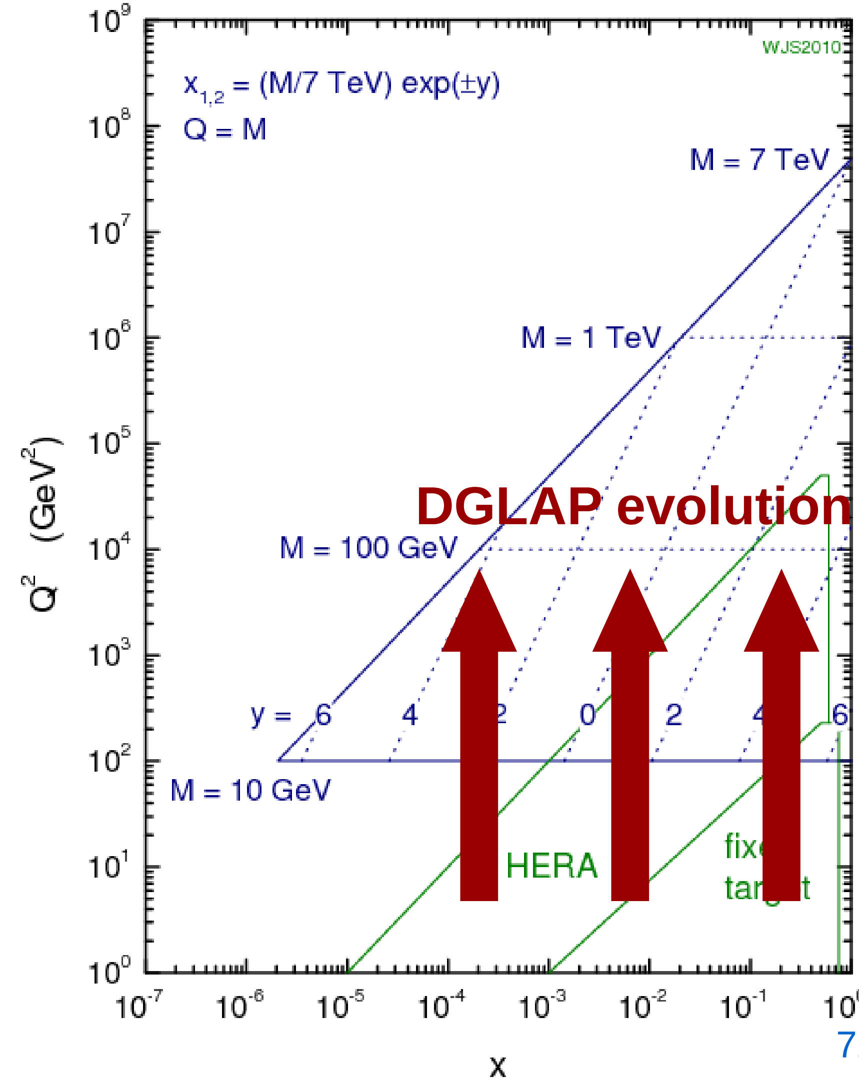
$$x_1 = \frac{M}{\sqrt{s}}\,\mathrm{e}^{y}\;,\quad x_2 = \frac{M}{\sqrt{s}}\,\mathrm{e}^{-y}$$



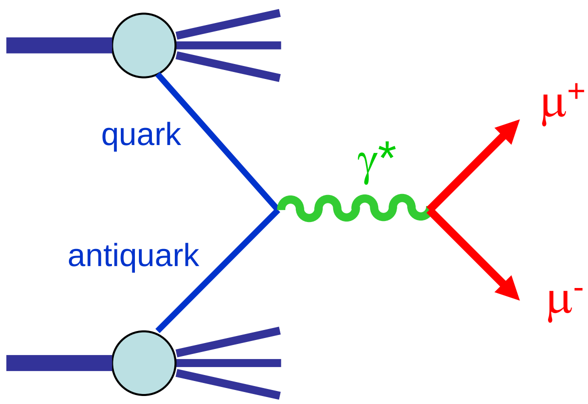
Tevatron parton kinematics



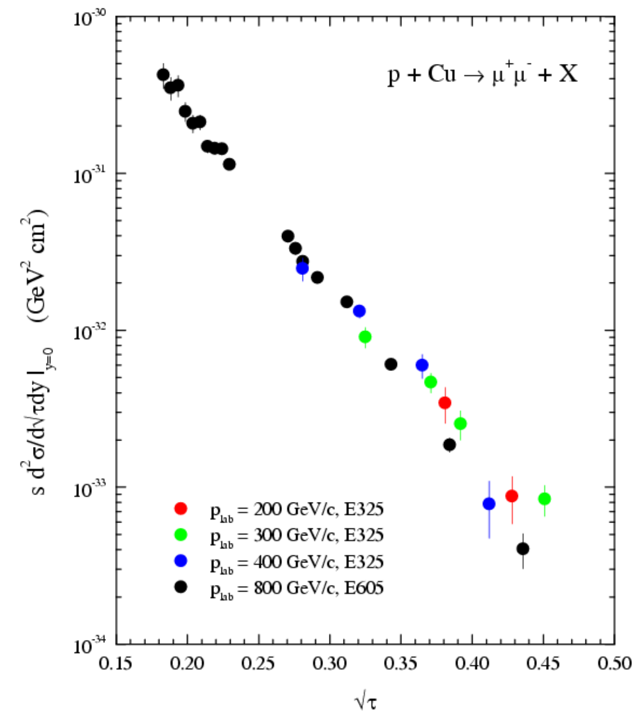
7 TeV LHC parton kinematics



early history: the Drell-Yan process



$\tau = M_{\mu\mu}^2/s$



$$\frac{d^2\sigma}{dM^2} = \frac{4\pi\alpha^2}{3M^4}\int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau)\sum_a e_a^2 f_a(x_1) f_{\bar{a}}(x_2)$$

$$= \frac{4\pi\alpha^2}{3M^4}\mathcal{F}(\tau) \qquad \text{(scaling)}$$

"The full range of processes of the type $A + B \rightarrow \mu^+\mu^- + X$ with incident p, π, K, γ etc affords the interesting possibility of comparing their parton and antiparton structures" (Drell and Yan, 1970)

(nowadays) … and to study the scattering of quarks and gluons, and how such scattering creates **new particles**

Volume 123B, number 1,2 PHYSICS LETTERS

jets! (1981)

OBSERVATION OF JETS IN HIGH TRANSVERSE ENERGY EVENTS AT THE CERN PROTON ANTIPROTON COLLIDER

UA1 Collaboration, CERN, Geneva, Switzerland

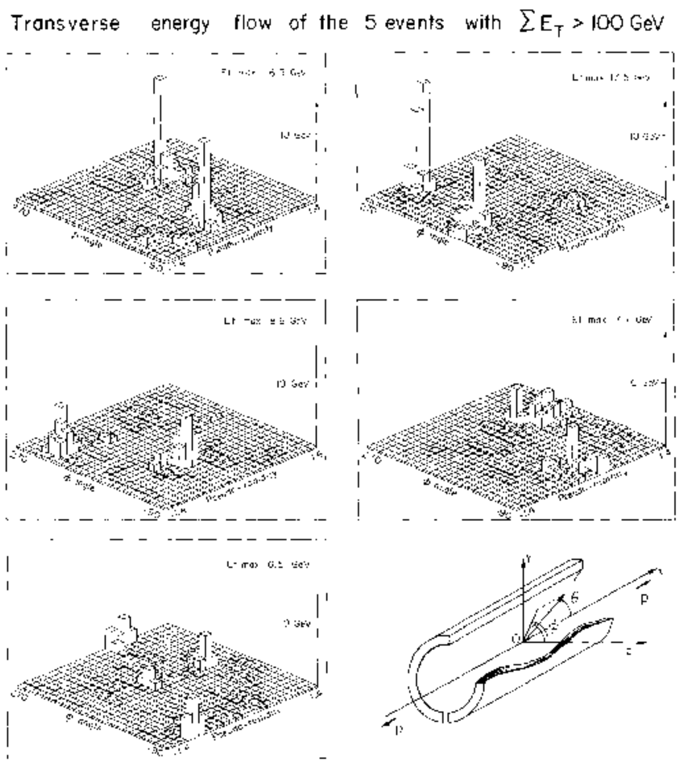
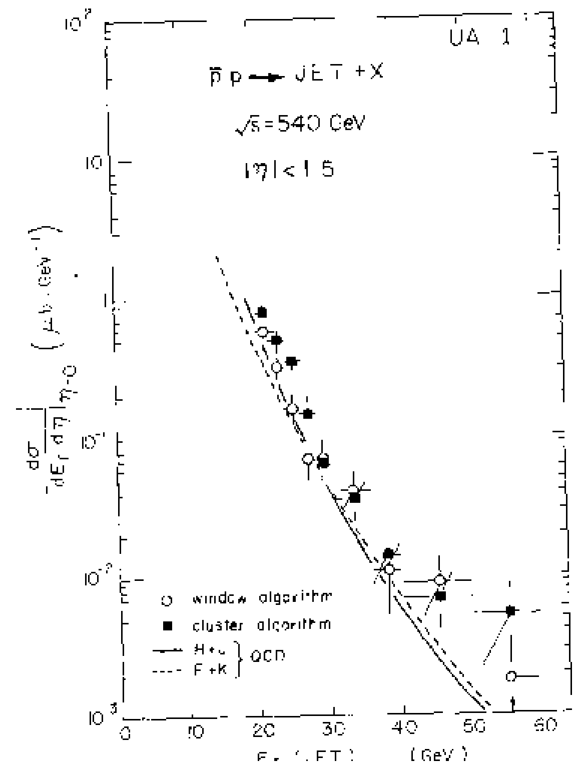
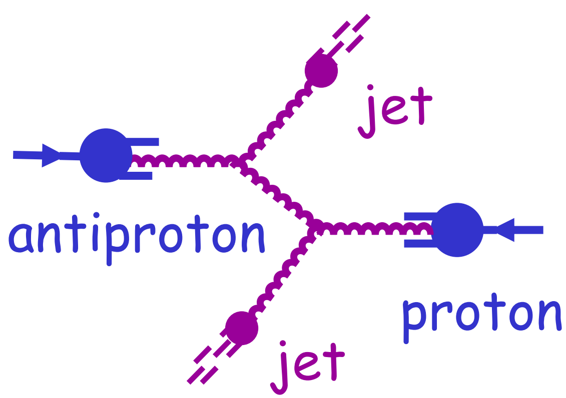


Fig. 3. Distribution of transverse energy versus azimuth ϕ and pseudo-rapidity η, for the five events with the highest ΣE_T

1

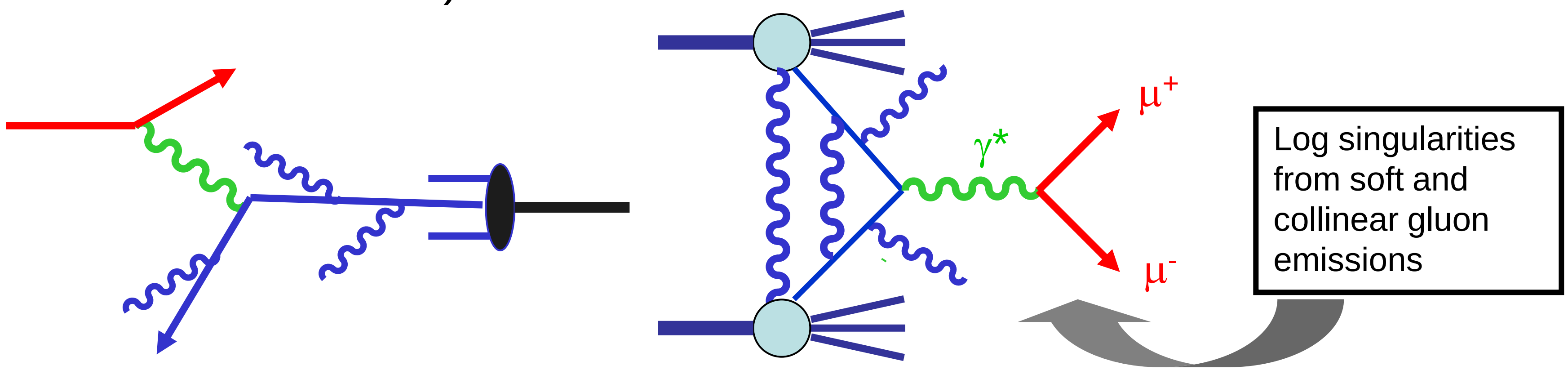




e.g. two gluons scattering at wide angle

factorisation

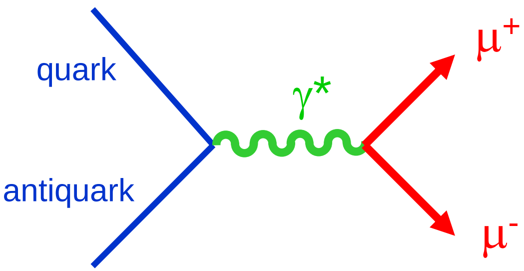
- the factorisation of ‘hard scattering’ cross sections into products of parton distributions was experimentally confirmed and theoretically plausible
- however, it was not at all obvious in QCD (i.e. with quark–gluon interactions included)



- in QCD, for any hard, inclusive process, the soft, nonperturbative structure of the proton can be factored out & confined to universal measurable parton distribution functions $f_a(x,\mu_F^2)$ Collins, Soper, Sterman (1982-5)

and evolution of $f_a(x,\mu_F^2)$ in factorisation scale calculable using the DGLAP equations, as we have seen earlier

Drell-Yan in more detail



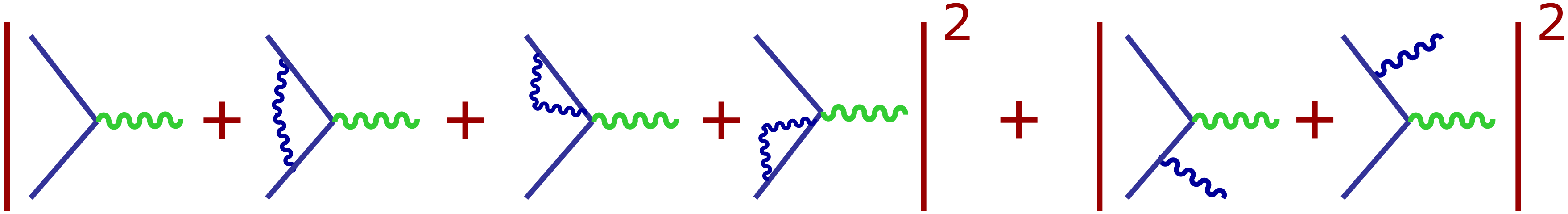
$$\frac{d\hat{\sigma}}{dM^2}^{q\bar{q}\to l^+l^-} = \frac{4\pi\alpha^2}{3N_c M^2}\, e_q^2\, \delta(\hat{s} - M^2)\,, \qquad \hat{s} = x_1 x_2 s$$

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{3N_c M^4}\,\tau \sum_q e_q^2 \int_0^1 dx_1 dx_2\, \delta(x_1 x_2 - \tau)\; q(x_1)\bar{q}(x_2) + (q \leftrightarrow \bar{q})$$

$$\Rightarrow M^4 \frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{3N_c}\,\tau\, \mathcal{F}(\tau)$$ scaling!

also $$\frac{d\sigma}{dM^2 dy} = \frac{4\pi\alpha^2}{3N_c M^4}\,\tau \sum_q e_q^2\; q(x_1)\bar{q}(x_2) + (q \leftrightarrow \bar{q})\,, \qquad \tau = M^2/s$$

beyond leading order …



$$d\hat{\sigma} = \hat{\sigma}_0 \left[\delta(x_1 x_2 - \tau) + \frac{\alpha_s}{2\pi} \frac{\theta(x_1 x_2 - \tau)}{x_1 x_2} \left\{ f_q\left(\frac{\tau}{x_1 x_2}\right) + P\left(\frac{\tau}{x_1 x_2}\right) \ln \frac{M^2}{\kappa_1^2} + P\left(\frac{\tau}{x_1 x_2}\right) \ln \frac{M^2}{\kappa_2^2} \right\} \right]$$

Note:

- collinear divergences, with same coefficients of logs as in DIS: *P(x)*
- finite correction: $f_q(x)$
- introduce a factorisation scale, as before:

$$\ln(M^2/\kappa^2) = \ln(M^2/\mu^2) + \ln(\mu^2/\kappa^2)$$

- then fold the parton-level cross section with $q_0(x_1)$ and $q_0(x_2)$, and with the same 'renormalised' distributions as before*, we obtain

$$d\sigma = \int_0^1 dx_1 dx_2 q(x_1,\mu^2)\bar{q}(x_2,\mu^2)\hat{\sigma}_0 \left[\delta(x_1 x_2 - \tau) + \frac{\alpha_s}{2\pi}\frac{1}{x_1 x_2}\left\{ 2P\left(\frac{\tau}{x_1 x_2}\right) \ln\frac{M^2}{\mu^2} + f_q\left(\frac{\tau}{x_1 x_2}\right) - 2\overline{C}\left(\frac{\tau}{x_1 x_2}\right)\right\} + \mathcal{O}(\alpha_s^2) \right]$$

finite

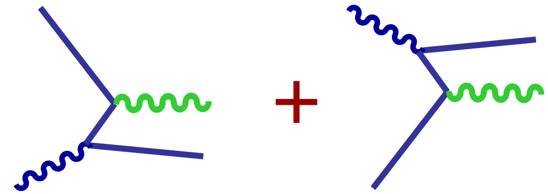
- the standard scale choice is *μ=M*

Altarelli et al
Kubar et al
1978-80

$$^{*}q(x,\mu^2) = q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y} q_0(y)\left\{P(x/y)\ln(\mu^2/\kappa^2) + \overline{C}(x/y)\right\}$$

Note:

- the full calculation at $O(\alpha_S)$ also includes

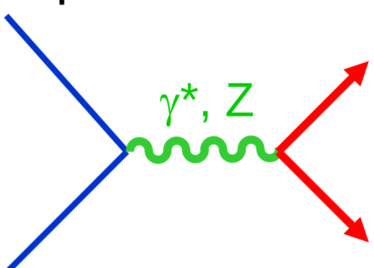


- which gives rise to $\alpha_S\, q * g$ terms in the cross section (see ESW book)

- the (finite) correction is sometimes called the 'K-factor', it is generally large and positive

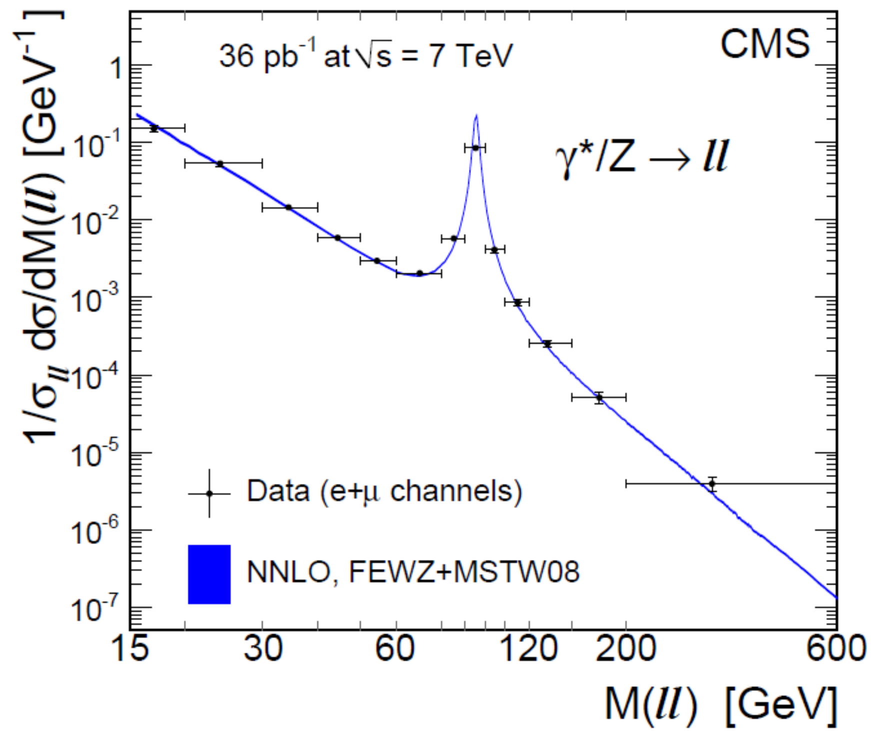
- … and is factorisation scheme/scale dependent (to compensate the scheme dependence of the PDFs)

- need also to include Z exchange for high-mass production



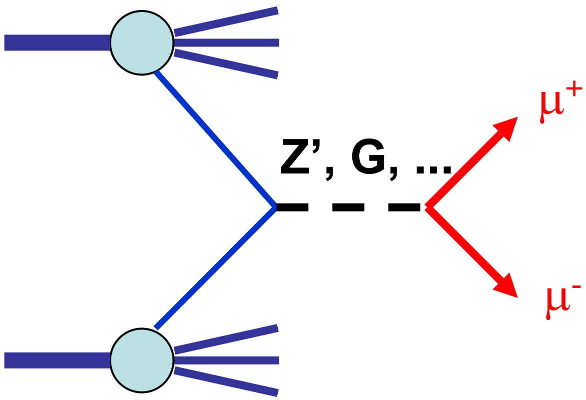
- NNLO corrections now also known

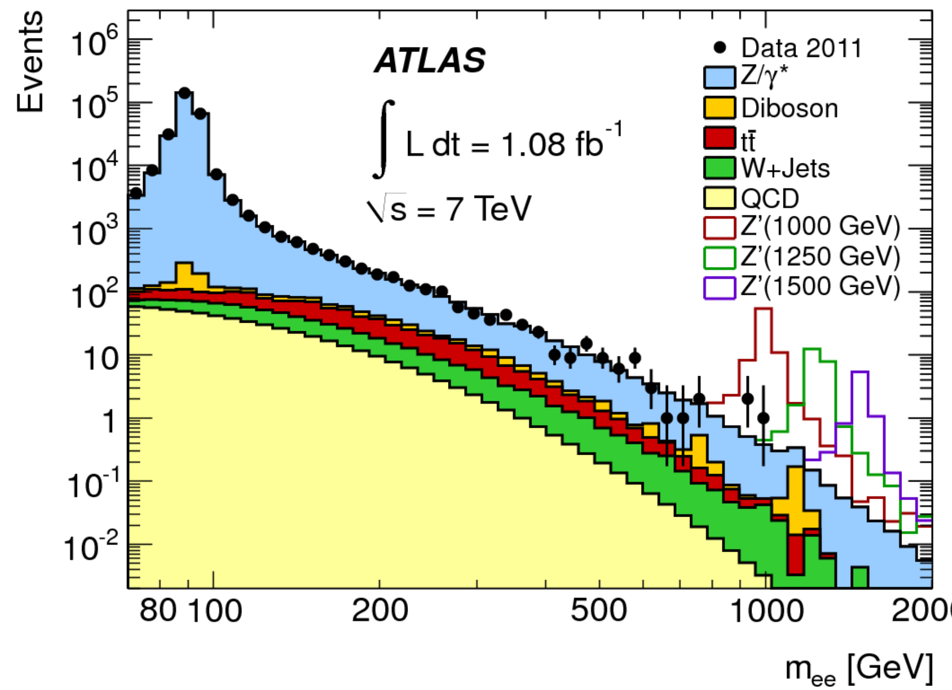
Drell-Yan phenomenology at LHC



Drell-Yan as a probe of new physics

Large Extra Dimension (and other New Physics) models have new resonances that could contribute to Drell-Yan





⇒ need to understand the SM contribution to high precision!

ATLAS 2011 data

Summary: the QCD **factorization theorem** for hard-scattering (short-distance) inclusive processes

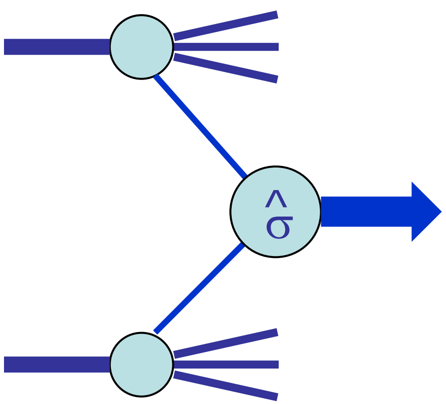
$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2)\, f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

where X=W, Z, H, high-E_T jets, SUSY sparticles, black hole, …, and Q is the 'hard scale' (e.g. = M_X), usually $\mu_F = \mu_R = Q$, and $\hat{\sigma}$ is known …

- to some fixed order in pQCD, e.g. high-E_T jets

$$\hat{\sigma} = A\alpha_S^2 + B\alpha_s^3$$

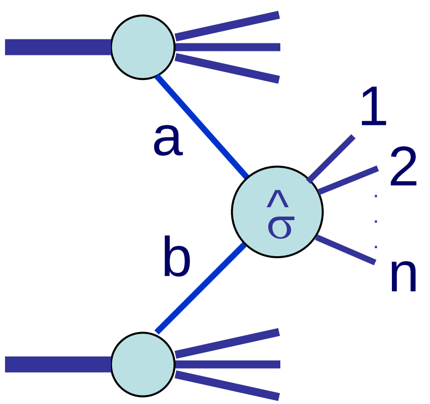
- or in some leading logarithm approximation (LL, NLL, …) to all orders via resummation



hard scattering cross section master formula

$$\sigma = \sum_{a,b=q,g} \int_0^1 dx_a dx_b\, f_a(x_a,\mu^2) f_b(x_b,\mu^2) \times \frac{1}{2\hat{s}} \int_{\text{cuts}} \prod_{i=1,n} \frac{d^3 p_i}{2E_i (2\pi)^3} (2\pi)^4 \delta^{(4)}(p_a + p_b - \sum_i p_i) \overline{\sum} |M^{ab\to 1,\dots,n}|^2$$

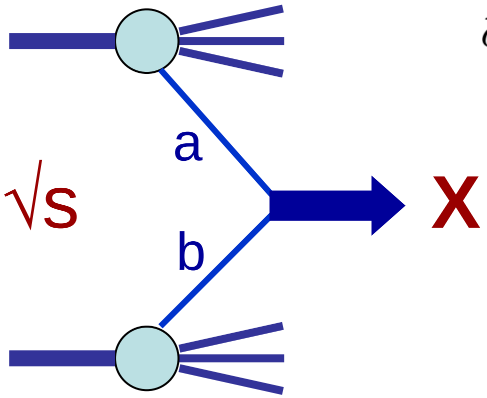
where $\hat{s} = (p_a + p_b)^2 = x_a x_b s$



- impose cuts on final state energies, angles, etc. as required
- μ^2 ? You choose!
- maximum 3n-2 integrations (fewer for differential distributions); in practice, generally use Monte Carlo techniques

parton luminosity functions

- a quick and easy way to assess the mass and collider energy dependence of production cross sections, and to compare different PDF sets



$$\hat\sigma_{ab\to X} = C_X \delta(\hat s - M_X^2)$$

$$\sigma_X = \int_0^1 dx_a dx_b\, f_a(x_a, M_X^2) f_b(x_b, M_X^2)\, C_X\, \delta(x_a x_b - \tau)$$

$$\equiv C_X \left[\frac{1}{s}\frac{\partial \mathcal{L}_{ab}}{\partial \tau}\right] \qquad (\tau = M_X^2/s)$$

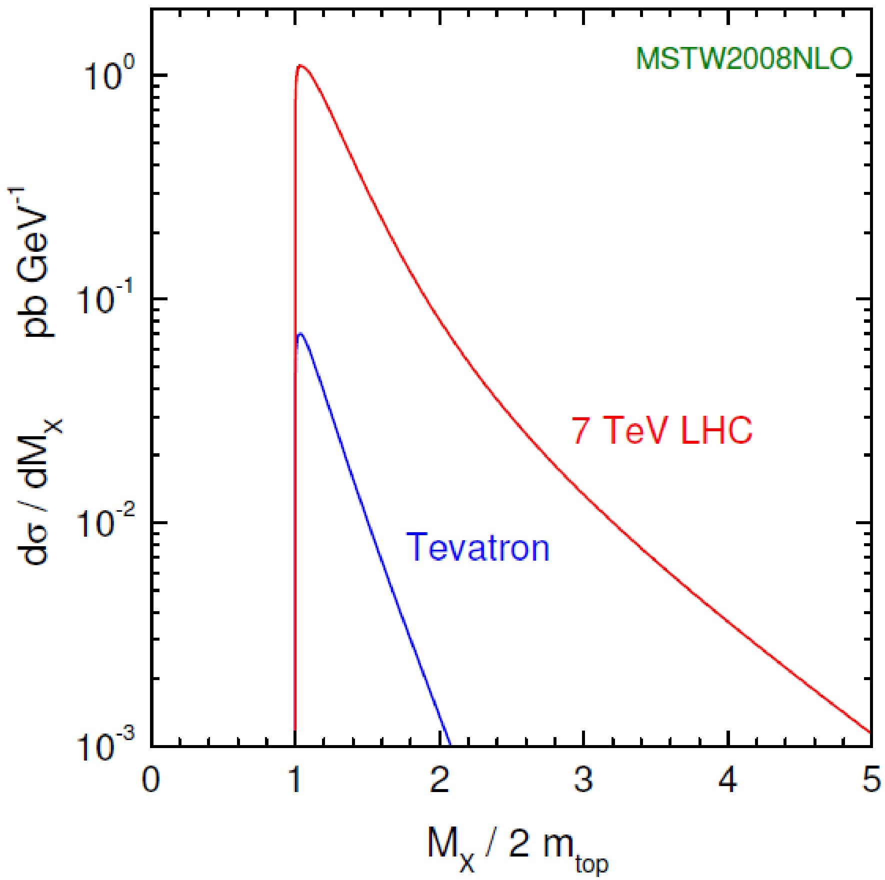
$$\frac{\partial \mathcal{L}_{ab}}{\partial \tau} = \int_0^1 dx_a dx_b\, f_a(x_a, M_X^2) f_b(x_b, M_X^2)\, \delta(x_a x_b - \tau)$$

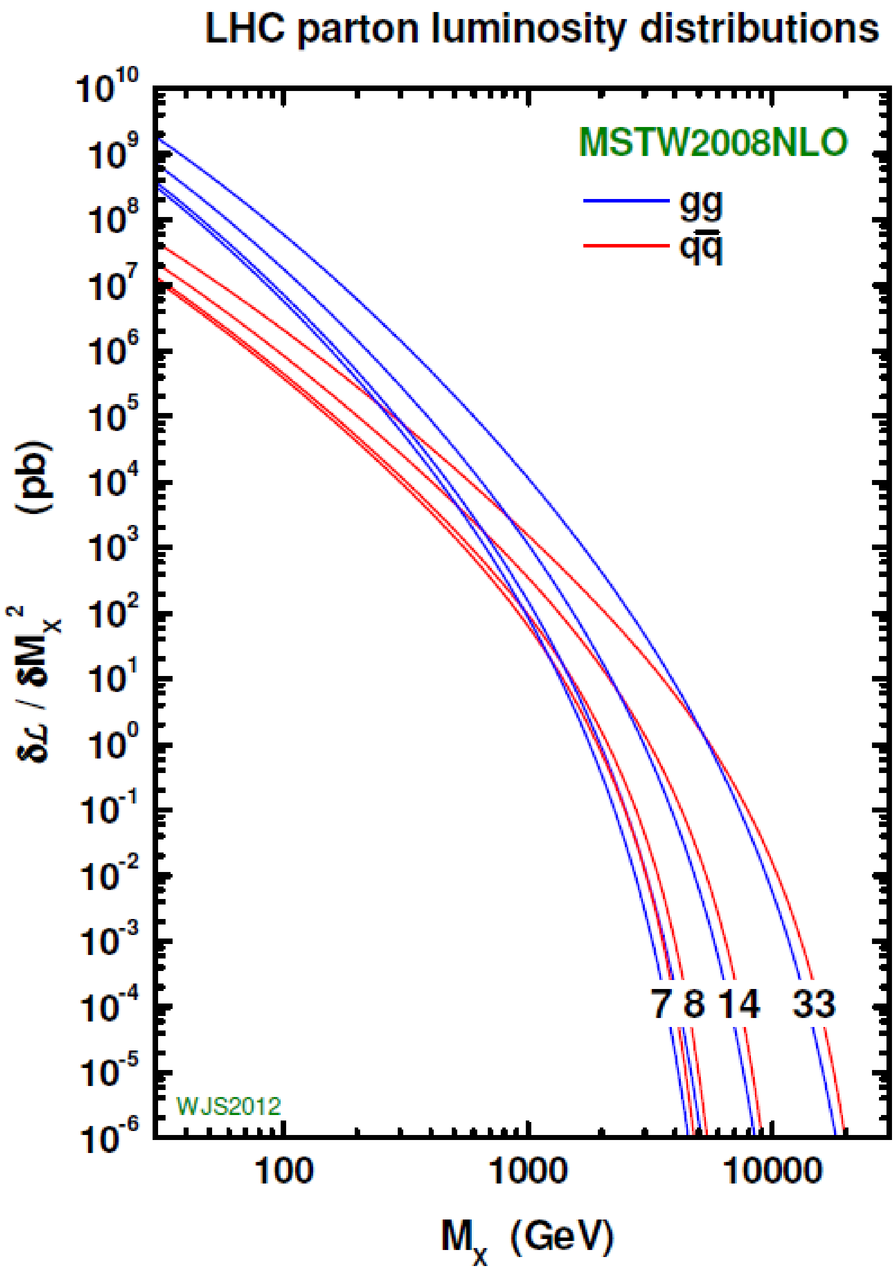
narrow resonance production, e.g. Higgs

- i.e. all the mass and energy dependence is contained in the **X**-independent parton luminosity function in [ ]
- useful combinations are $ab = gg, \sum_q q\bar q, \ldots$
- and also useful for assessing the uncertainty on cross sections due to uncertainties in the PDFs

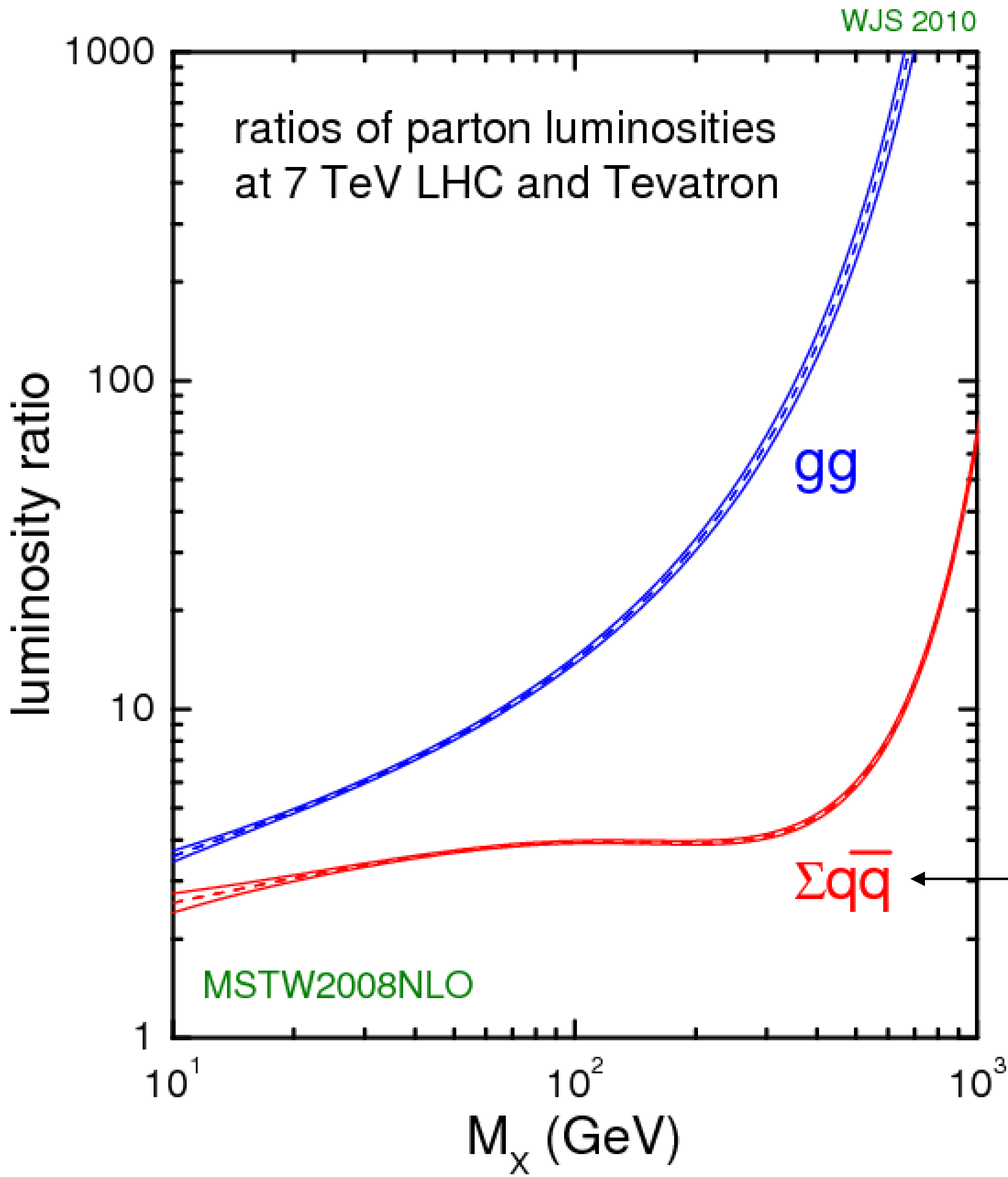
- even when **X** is not a resonance, can still use the luminosity function concept, by identifying a 'typical' value of M_X

- e.g. $t\bar{t}$ production...

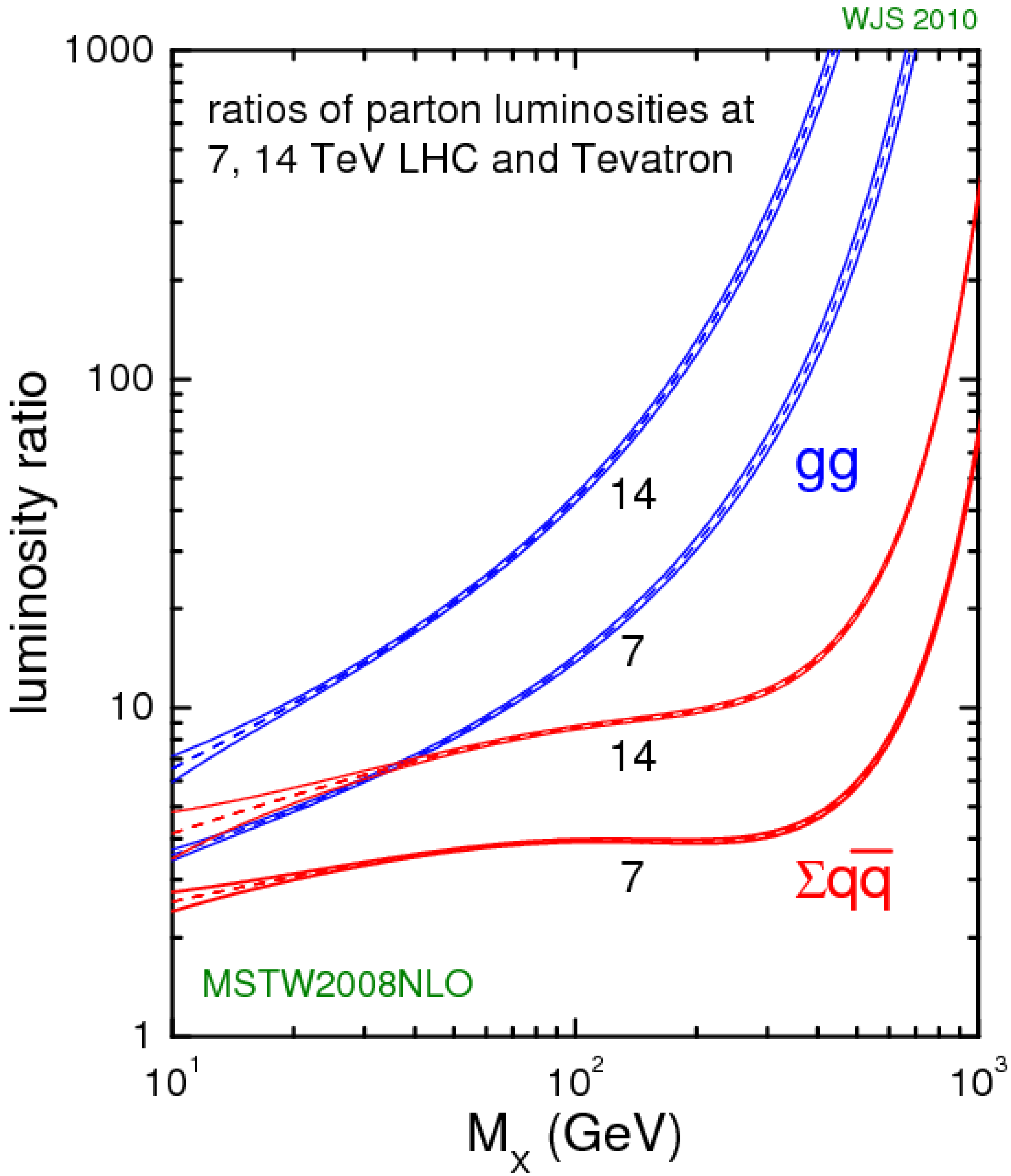




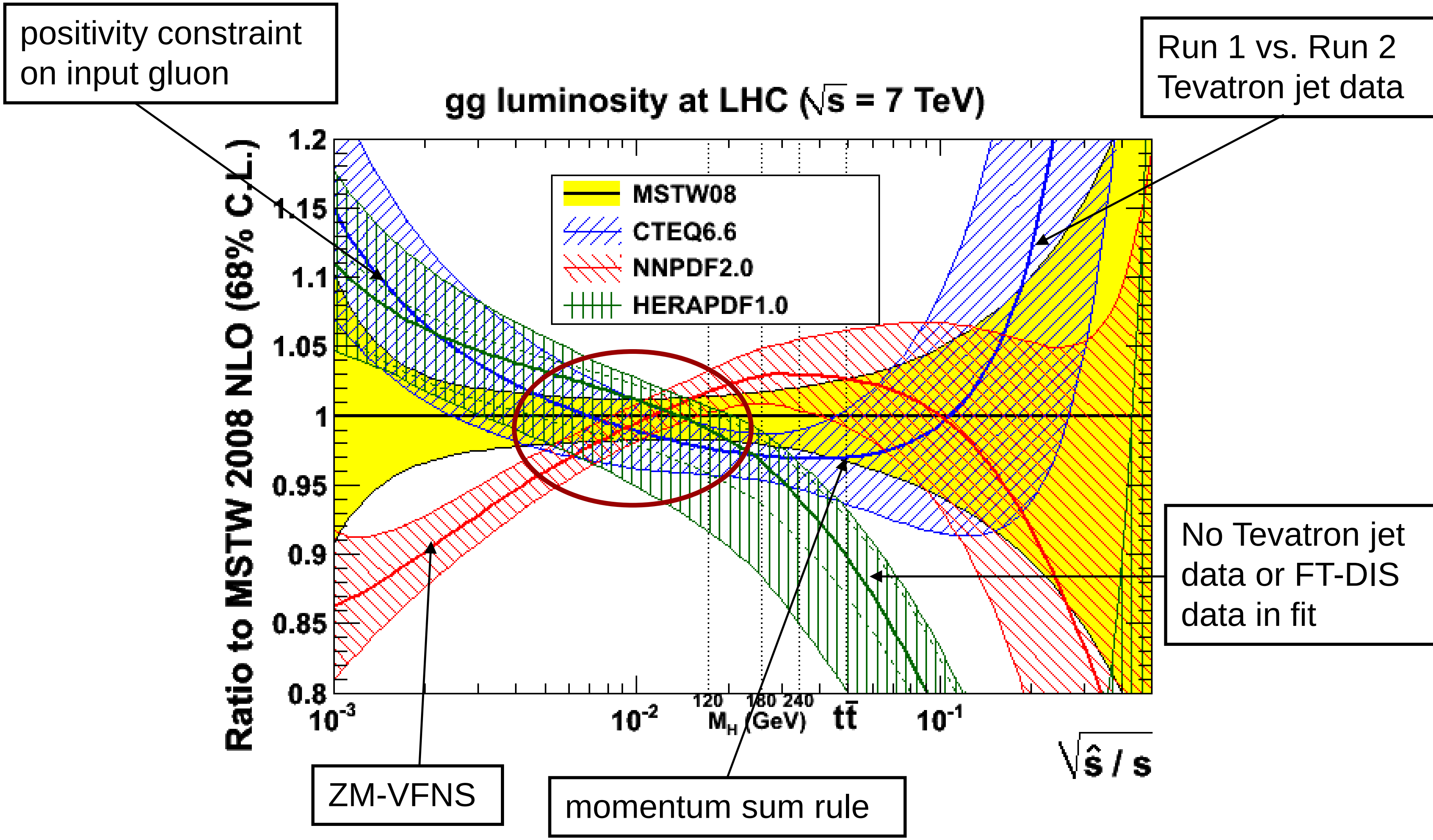
... more plots like this at www.hep.phy.cam.ac.uk/~wjs/plots/plots.html


WJS 2010
ratios of parton luminosities
at 7 TeV LHC and Tevatron
gg
$\Sigma q\bar{q}$
MSTW2008NLO
luminosity ratio
1000
100
10
1
10^1
10^2
10^3
M_X (GeV)

$\frac{ss}{vs}$


WJS 2010
ratios of parton luminosities at
7, 14 TeV LHC and Tevatron
luminosity ratio
1000
100
10
1
14
7
gg
14
7
Σq$\bar{q}$
MSTW2008NLO
10^1
10^2
10^3
M_X (GeV)

parton luminosity* comparisons



$$* \quad \frac{\partial \mathcal{L}_{ab}}{\partial \tau} = \int_0^1 dx_a dx_b \, f_a(x_a, M_X^2) f_b(x_b, M_X^2) \, \delta(x_a x_b - \tau)$$

4

QCD phenomenology at hadron colliders

- leading-order calculations
- beyond leading order: higher-order perturbative QCD corrections
- benchmark cross sections
- beyond perturbation theory

precision phenomenology

- *Benchmarking*
  - inclusive SM quantities (V, jets, top,…), calculated to the highest precision available (e.g. NNLO, NNLL,...)

- *Backgrounds*
  - new physics generally results in some combination of multijets, multileptons, missing E_T
  - therefore, we need to know SM cross sections {V,VV,bb,tt,H,…} + jets to high precision → `wish lists'
  - ratios can be useful

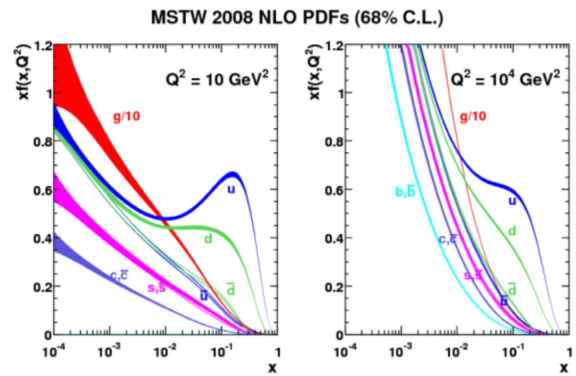
Note: $V = \gamma^*, Z, W^{\pm}$

tools for precision phenomenology at hadron colliders

- The key theoretical tool is the QCD *factorisation theorem:*

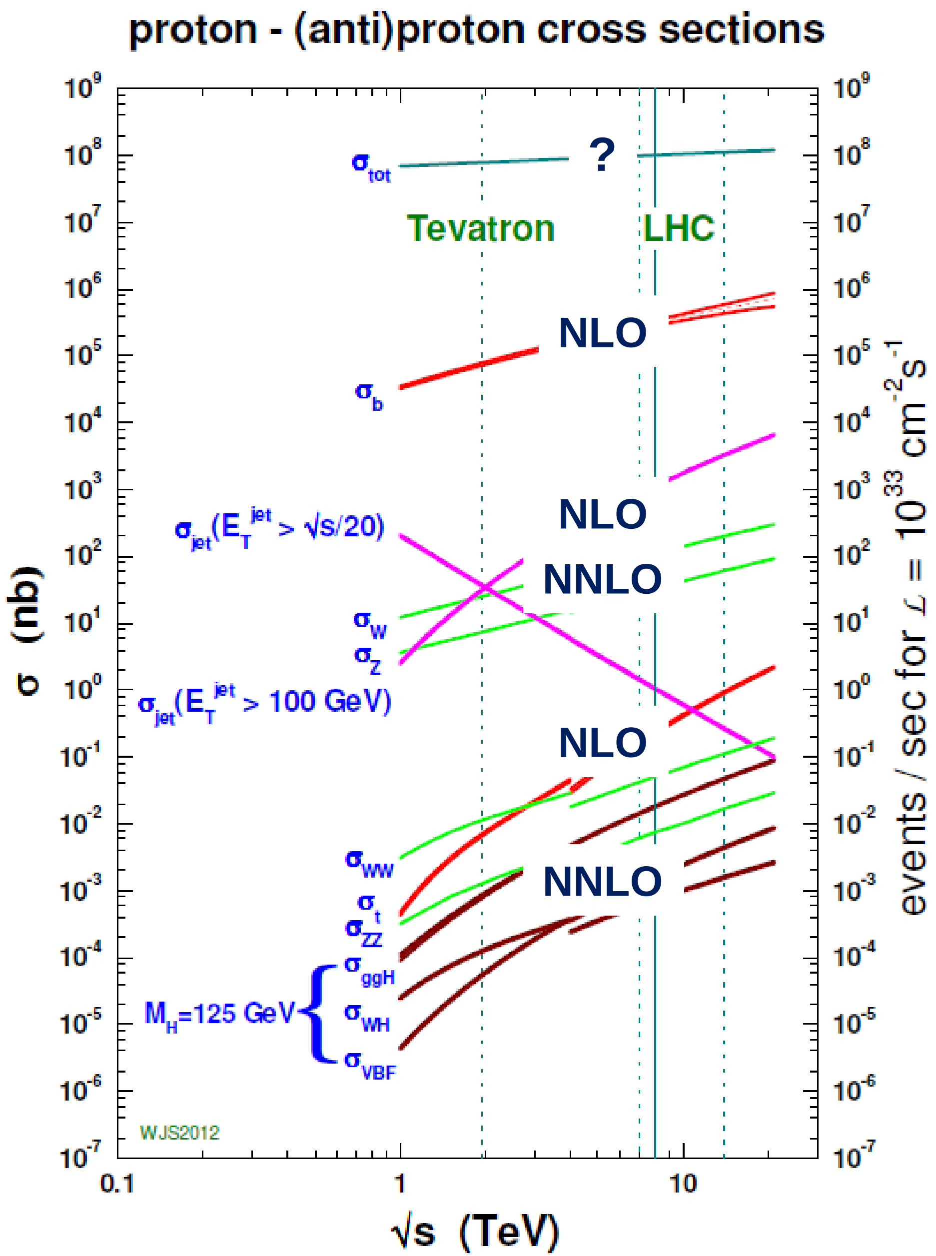
$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a(x_1, \mu_F^2) \, f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

- precision SM tests require detailed knowledge of
  - perturbative corrections to the hard scattering cross sections (both EW and QCD)
  - the parton structure of the proton, as encoded in the parton distribution functions (PDFs)
  - accurate modeling of the 'underlying event', e.g. parton showers + tuned UE MCs, interfaced with LO or NLO hard scattering MEs



- the precision we can ultimately achieve is highly process dependent – it can vary from O(few %) (super-inclusive quantities like $\sigma_{tot}(Z)$) to O(100%) (multiparton production processes known only at LO in pQCD)

how precise in practice?



Why are higher-order corrections *so* important for precision predictions?

survey of pQCD calculations

- focus first on fixed-order calculations:

$$d\sigma = A(\{P\})\, \alpha_S(\mu^2)^N\, [\, 1 + C_1(\{P\}, \mu^2)\, \alpha_S(\mu^2) + C_2(\{P\},\mu^2)\, \alpha_S(\mu^2)^2 + \ldots\,]$$

… where {P} refers to the kinematic variables for the particular process. For hadron colliders there will also be PDFs and dependence on the factorisation scale.

- thus LO (A only), NLO (A, C_1 only), NNLO (A,C_1,C_2 only) etc,

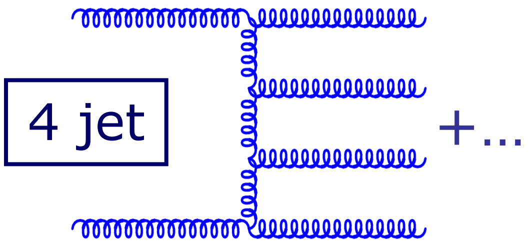
- note that in some cases the coefficients may contain large logarithms **L** of ratios of kinematic variables, and it may be possible to identify and resum these to all orders using a leading log approximation, e.g.

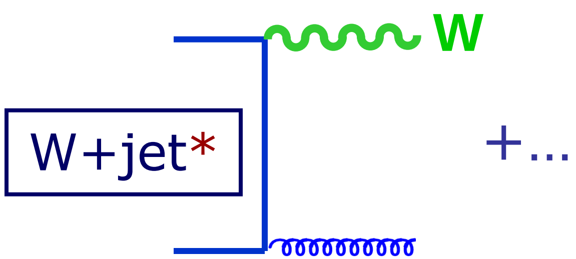
$$d\sigma = A\, \alpha_S^N\, [\, 1 + (c_{11}\, L + c_{10})\, \alpha_S + (c_{22}\, L^2 + c_{21}\, L + c_{20})\, \alpha_S^2 + \ldots\,]$$
$$\sim\; A\, \alpha_S^N\; \exp(c_{11}\, L\, \alpha_S + c_{21}\, L\, \alpha_S^2)\; \times [\, 1 + c_{10}\, \alpha_S + c_{20}\, \alpha_S^2 + \ldots\,]$$

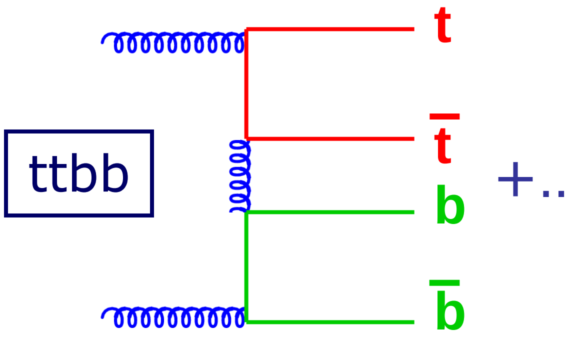
where e.g. $L = \log(M/q_T)$, $\log(1/x)$, $\log(1-T)$, … >> 1, thus LL, NLL, NNLL, etc.

leading order calculations

- scattering amplitudes for $2 \to N$ processes calculated at tree-level (i.e. no loops)
- automated codes for arbitrary matrix element generation (MADGRAPH, COMPHEP, HELAS, …) – very powerful (e.g. SM + MSSM) but can be slow and cumbersome; more streamlined packages based on *recursion* (ALPGEN, HELAC, …)
- jet = parton, but 'easy' to interface to hadronisation MCs
- uncertainties in normalisation (e.g. from large scale dependence $\alpha_S(\mu^2)^N$) and distributions, therefore not good for precision analyses

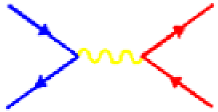






*Note: LO contribution to σ(W+jet) but NLO contribution to σ_{tot}(W)!

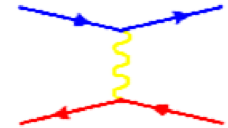
High Energy Physics
Illinois



The MadGraph homepage

UCL UIUC Launchpad

by the MG/ME Development team



Generate processes online using MadGraph 5

To improve our web services we request that you register. Registration is quick and free. You may register for a password by clicking here.
Please note the correct reference for MadGraph 5, JHEP 1106(2011)128, arXiv:1106.0522 [hep-ph].
You can still use **MadGraph 4** here.

Code can be generated either by:

I. Fill the form:

Model: SM | Model descriptions

Input Process: | Examples/format

Example: p p > w+ j j QED=3, w+ > l+ vl

p and j definitions: p=j=d u s c d~ u~ s~ c~ g

sum over leptons: l+ = e+, mu+; l- = e-, mu-; vl = ve, vm, vt; vl~ = ve~, vm~, vt~

Submit

II. Upload the proc_card.dat

Process card examples

proc_card format MadGraph 5

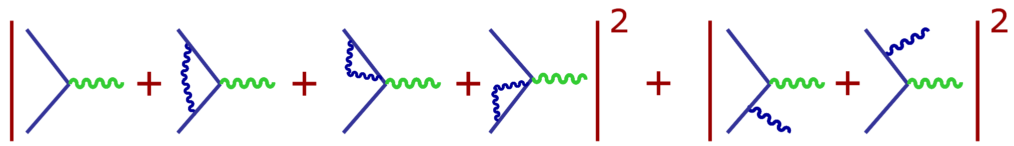
Browse... and send it to the server.

III. Upload the full banner (all cards are uploaded as the "current" ones)

Browse... and send it to the server.

next-to-leading order calculations

- the NLO contributions correspond to an additional real gluon in final state and a virtual gluon in loop correction, i.e. $d\sigma_V^{(N)} + d\sigma_R^{(N+1)}$ for a $2 \rightarrow$ N process at LO, e.g.



- the LO prediction is stabilised, in particular by reducing the (renormalisation and factorisation) scale dependence
- jet structure begins to emerge
- much recent progress, including automation (see below)
- ... and now can interface with parton shower MC (e.g. MC@NLO, POWHEG, ...)
- the NLO corrections are now known for essentially all processes of phenomenological interest at the Tevatron and LHC

recall

general structure of a QCD perturbation series

- choose a renormalisation scheme (e.g. $\overline{\text{MS}}$)
- calculate cross section to some order (e.g. NLO)

$$\sigma(P) = A\,\alpha_S^N(\mu) + \alpha_S^{N+1}(\mu)\left[B + \frac{NAb}{2\pi}\ln\frac{\mu}{P}\right] + \ldots$$

physical variable(s)

process dependent coefficients depending on *P*

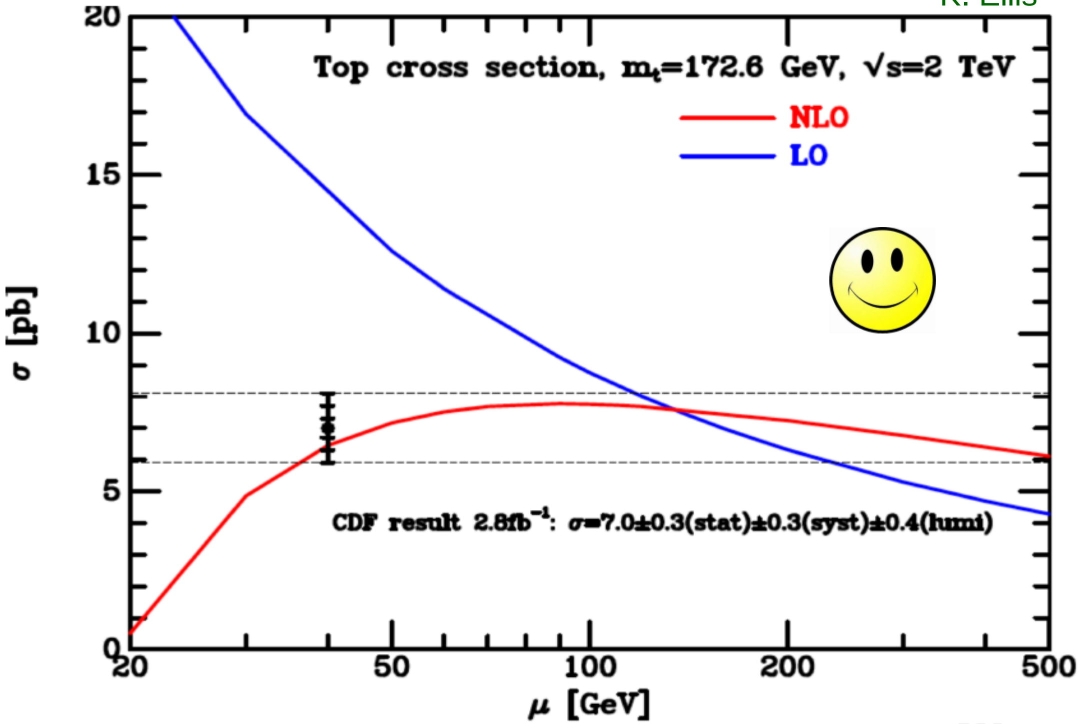
renormalisation scale

- note $d\sigma/d\mu=0$ "to all orders", but in practice $d\sigma^{(N+n)}/d\mu = O((N+n)\alpha_S^{N+n+1})$ → as many orders as possible!

- can try to help convergence by using a "physical scale choice", $\mu \sim P$, e.g. $\mu = M_Z$ or $\mu = E_T^{jet}$

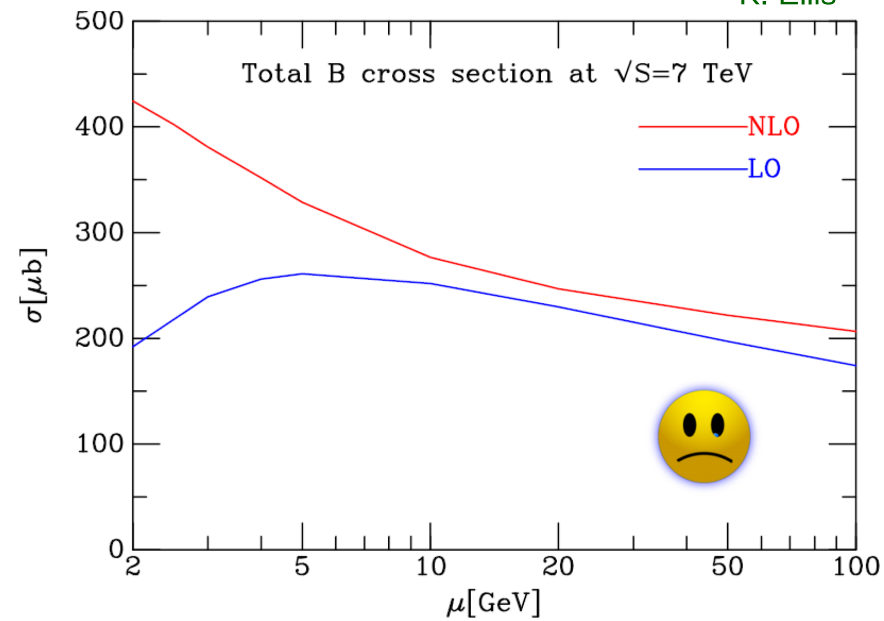
- what if there is a wide range of *P*'s in the process, e.g. W + n jets? – see below

K. Ellis



Top at Tevatron

K. Ellis



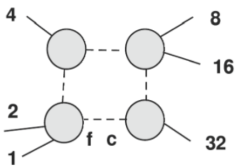
Bottom at LHC

reason: new processes open up at NLO!

recent developments at NLO

- traditional methods based on Feynman diagrams, then reduction to known (scalar box, triangle, bubble and tadpole) integrals

- … and new methods based on unitarity and on-shell recursion: assemble loop-diagrams from individual tree-level diagrams
  - basic idea: Bern, Dixon, Kosower 1993
  - cuts with respect to on-shell complex loop momenta: Cachazo, Britto, Feng 2004



  - tensor reduction scheme: Ossola, Pittau, Papadopoulos 2006
  - integrating the OPP procedure with unitarity: Ellis, Giele, Kunszt 2008
  - D-dimensional unitarity: Giele, Kunszt, Melnikov 2008
  - …

- … and the appearance of automated programmes for one-loop, multi-leg amplitudes, either based on
  - traditional or numerical Feynman approaches (Golem, …)
  - unitarity/recursion (BlackHat, CutTools, Rocket, …)

some recent NLO results…*

• pp → W+3j	[Rocket: Ellis, Melnikov & Zanderighi]	[unitarity]
• pp → W+3j	[BlackHat: Berger et al]	[unitarity]
• pp → tt bb	[Bredenstein et al]	[traditional]
• pp → tt bb	[HELAC-NLO: Bevilacqua et al]	[unitarity]
• pp → qq → 4b	[Golem: Binoth et al]	[traditional]
• pp → tt+2j	[HELAC-NLO: Bevilacqua et al]	[unitarity]
• pp → Z,γ*+3j	[BlackHat: Berger et al]	[unitarity]
• pp → W+4j	[BlackHat: Berger et al]	[unitarity]
• …		

with earlier results on V,H + 2 jets, VV,tt + 1 jet, VVV, ttH, ttZ, …

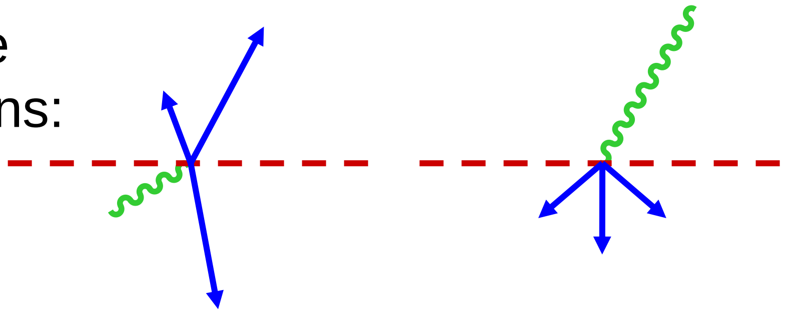
In contrast, for NNLO we still only have inclusive γ*,W,Z,H, WH (but with rapidity distributions and decays, although there is much progress on top, single jet, …) – for a recent review see M. Grazzini, indico.cern.ch/conferenceDisplay.py?confId=172986

*particularly relevant for LHC

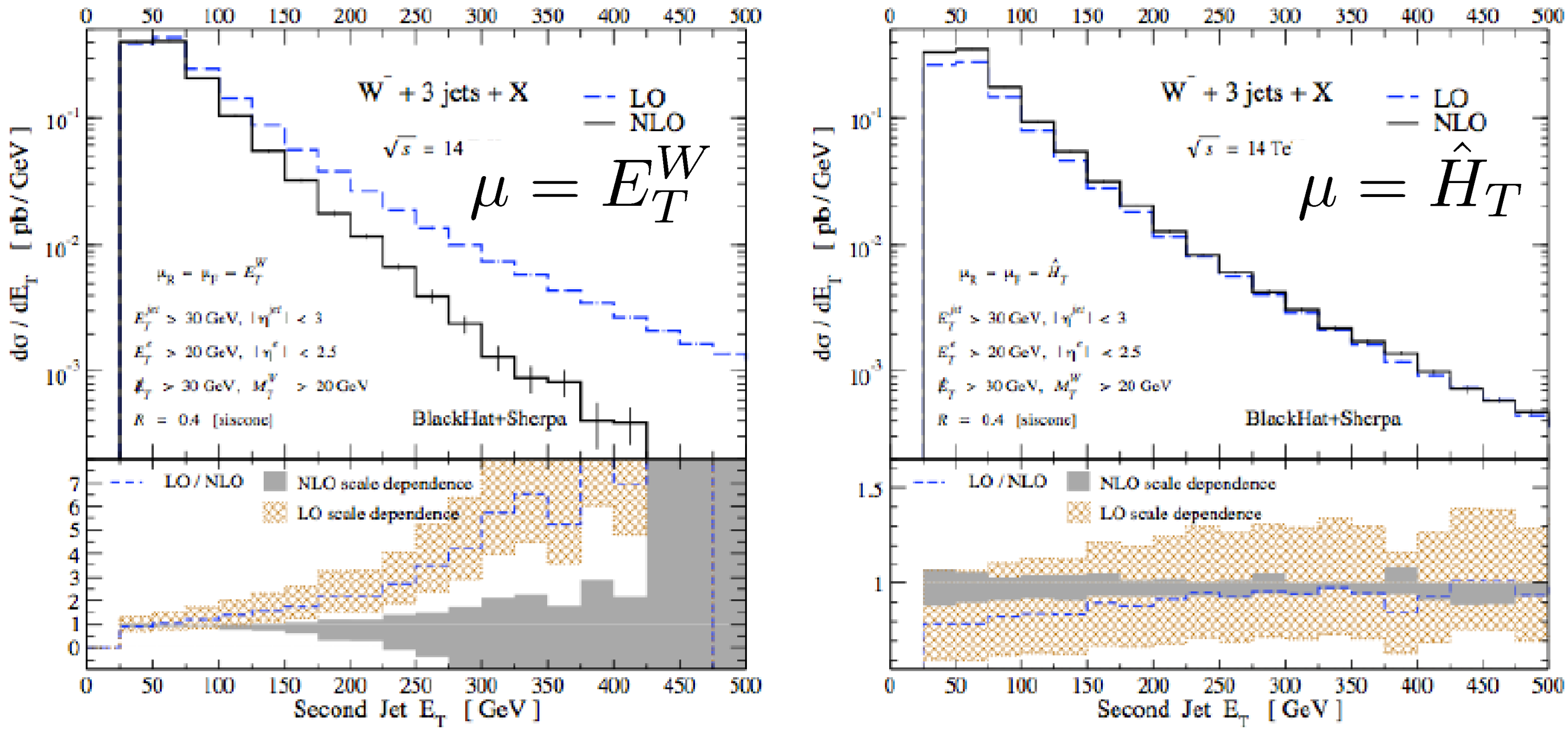
However...

in complicated processes like W + n jets, there are often many ‘reasonable’ choices of scales: $\mu = M_W,\ E_{TW},\ \langle E_{T\text{jet}} \rangle,\ H_T,\ \ldots$

‘blended’ scales like H_T can seamlessly take account of different kinematical configurations:

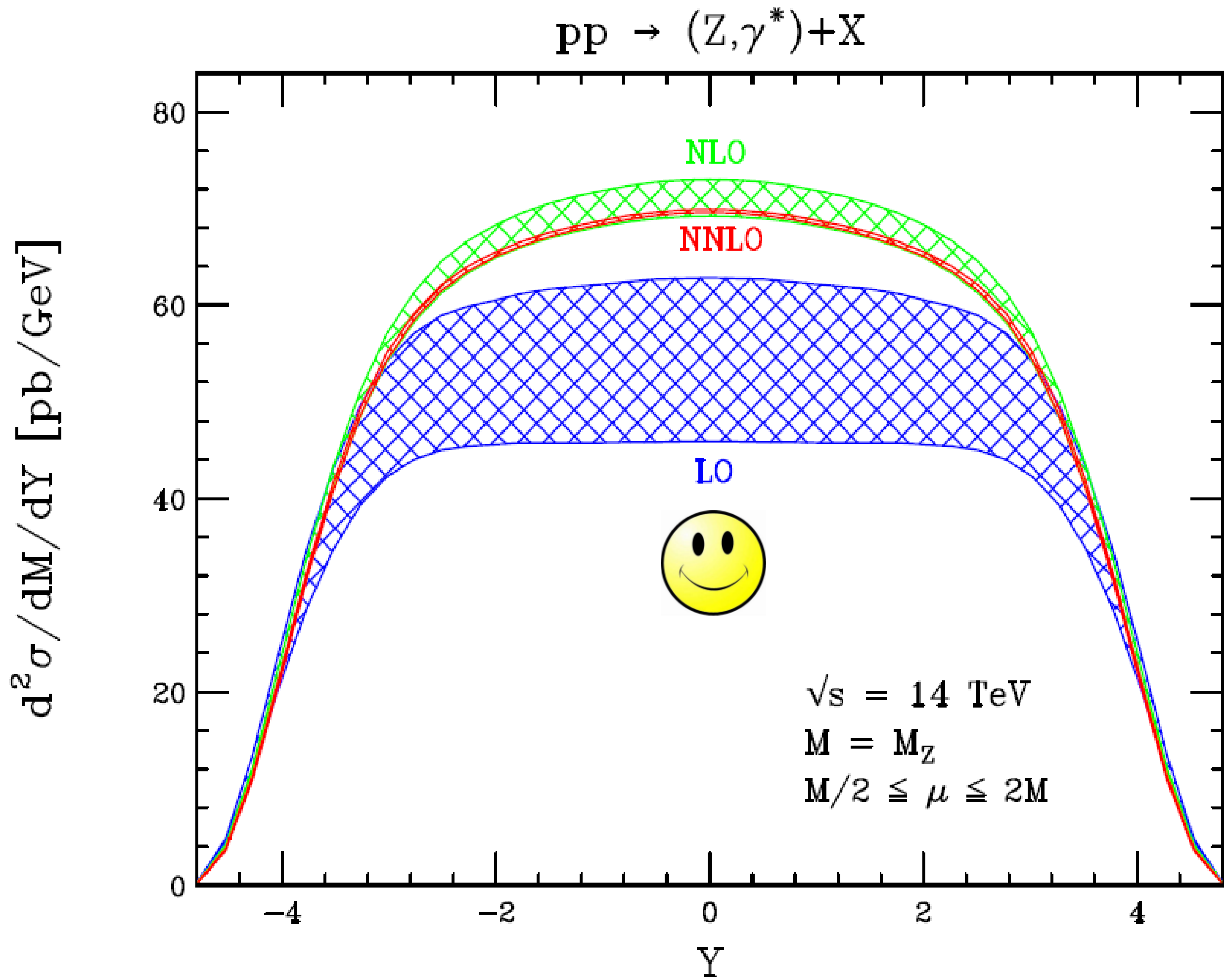


$$H_T = \sum_{i=\text{partons}} E_{Ti} + E_{Te} + E_{T\text{miss}}$$



Berger et al., arXiv:0907.1984

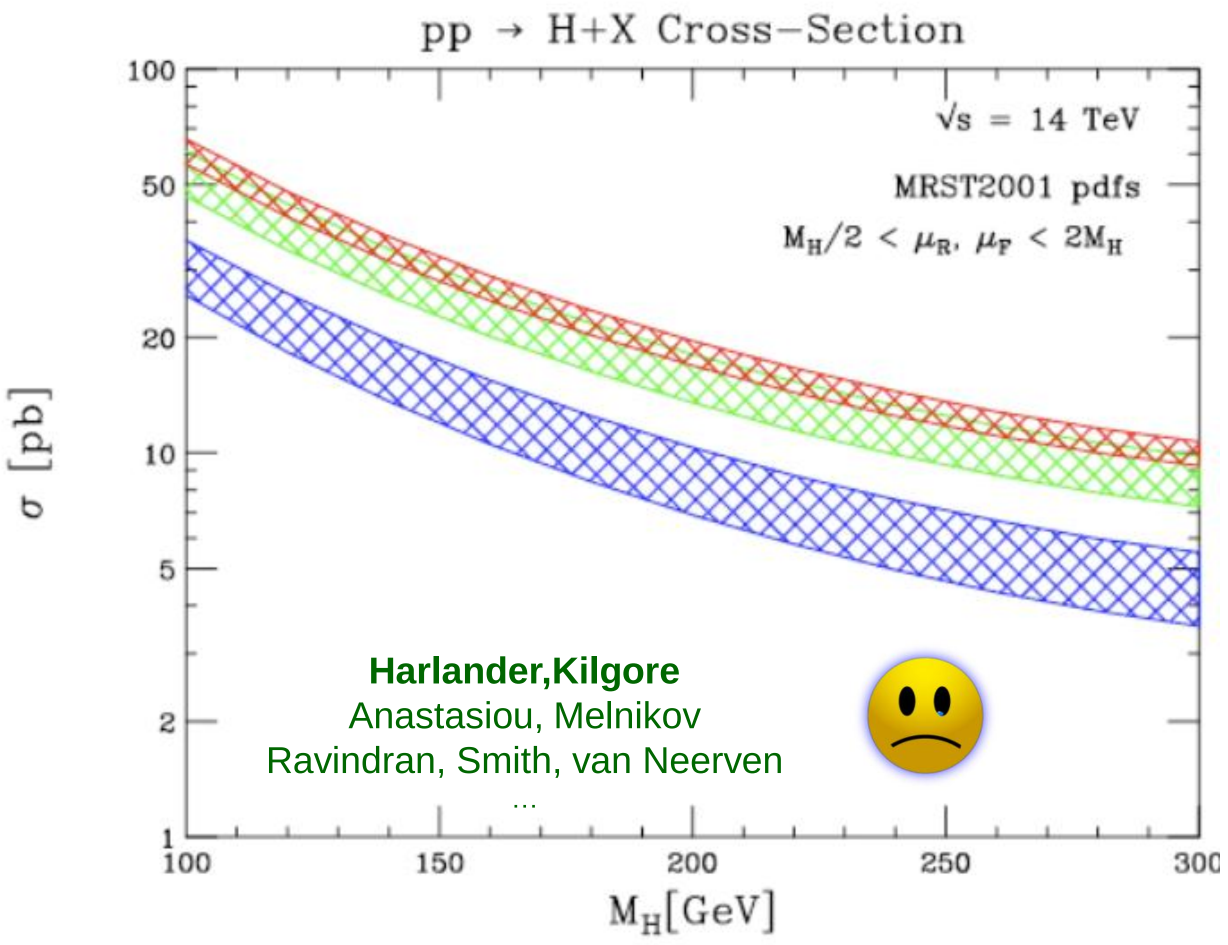
the impact of NNLO: σ(Z)



Anastasiou, Dixon, Melnikov, Petriello, 2004

- only scale variation uncertainty shown
- central values calculated for a *fixed* set PDFs with a *fixed* value of $\alpha_S(M_Z^2)$

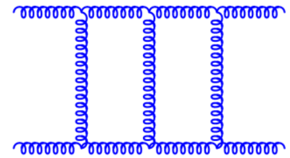
the impact of NNLO: σ(Higgs)



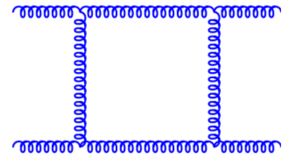
- the NNLO band is about ±10%, or ±15% if μ_R and μ_F varied independently

anatomy of a NNLO calculation: p + p $\rightarrow$ jet + X

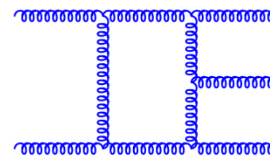
- 2 loop, 2 parton final state



- | 1 loop |^{2}, 2 parton final state

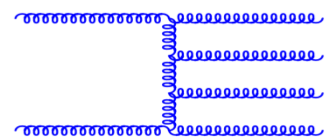


- 1 loop, 3 parton final states
  or 2 +1 final state



soft, collinear

- tree, 4 parton final states
  or 3 + 1 parton final states
  or 2 + 2 parton final state



the collinear and soft singularities exactly cancel between the N +1 and N + 1-loop contributions rapid progress in last two years [many authors]

- many 2→2 scattering processes with up to one off-shell leg now calculated at two loops

- … to be combined with the tree-level 2→4, the one-loop 2→3 and the self-interference of the one-loop 2→2 to yield physical NNLO cross sections

- the key is to identify and calculate the ‘subtraction terms’ which add and subtract to render the loop (analytically) and real emission (numerically) contributions finite

- expect progress soon!

resummation

- when $p_T \ll M$, the pQCD series contains large logarithms $\ln(M^2/p_T^2)$ at each order:

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \simeq \frac{1}{p_T^2}\left[A_1 \alpha_S \ln\frac{M^2}{p_T^2} + A_2 \alpha_S^2 \ln^3\frac{M^2}{p_T^2} + \ldots + A_n \alpha_S^n \ln^{2n-1}\frac{M^2}{p_T^2} + \ldots\right]$$

which spoils the convergence of the series when $\alpha_S \ln^2\frac{M^2}{p_T^2} \sim 1$

- fortunately, these logarithms can be *resummed* to all orders in pQCD, to generate a Sudakov form factor:

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \simeq \frac{d}{dp_T^2}\exp\left(-\frac{\alpha_S}{2\pi}C_F \ln^2\frac{M^2}{p_T^2}\right) = \frac{\alpha_S C_F}{\pi}\frac{\ln(M^2/p_T^2)}{p_T^2}\exp\left(-\frac{\alpha_S}{2\pi}C_F \ln^2\frac{M^2}{p_T^2}\right)$$

… which regulates the LO singularity at $p_T = 0$

- the effect of the form factor is visible in the (Tevatron) data

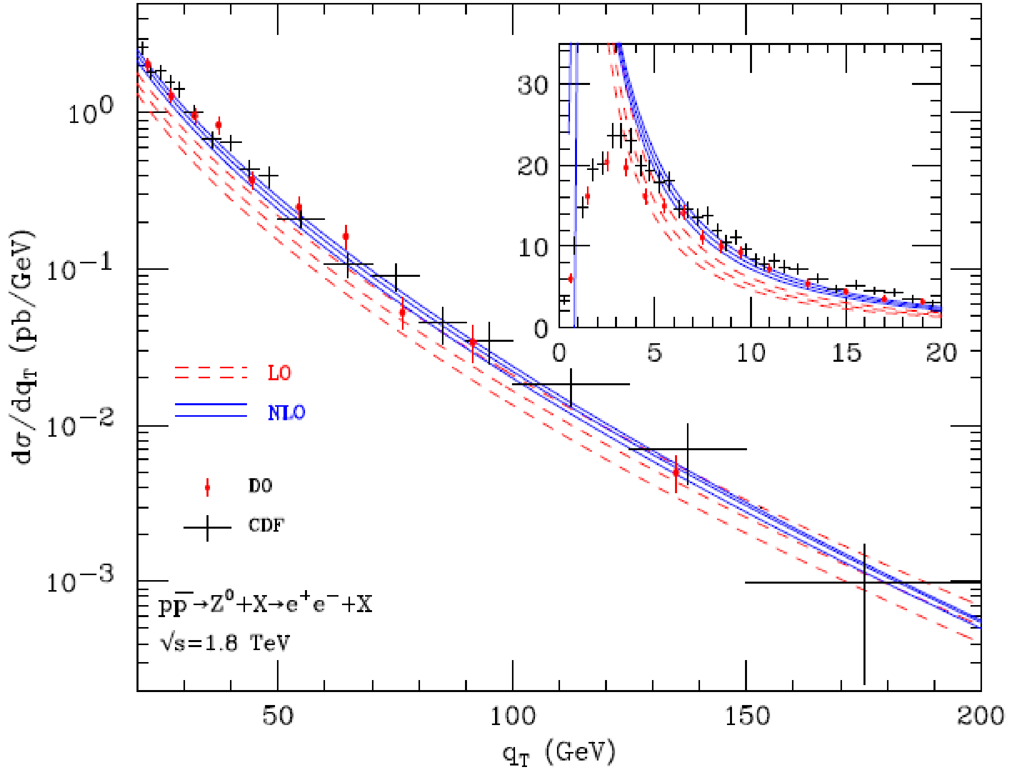
resummation contd.

Bozzi, Catani,Ferrera, de Florian, Grazzini, arXiv:0812.2862

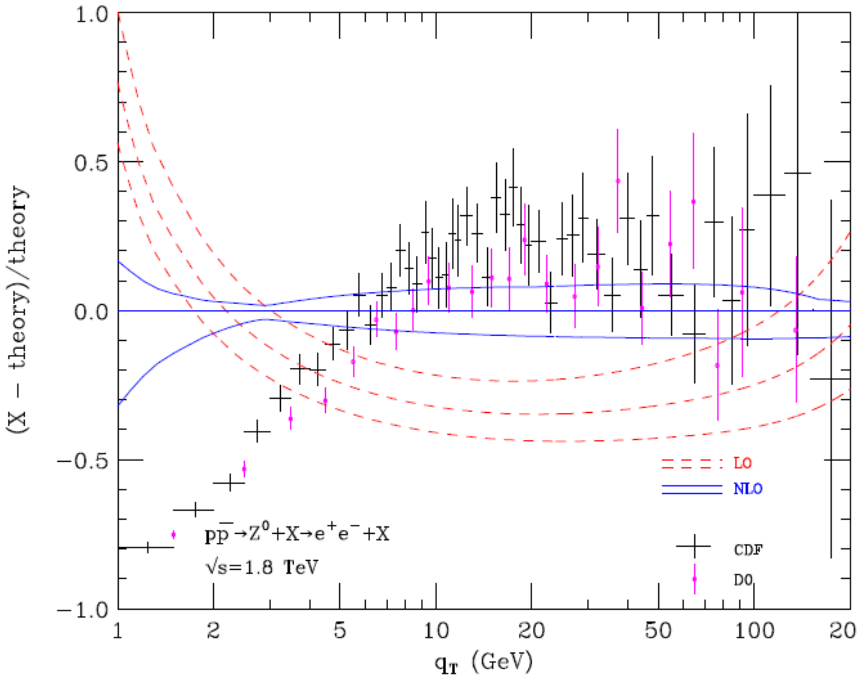
- theoretical refinements include the addition of sub-leading logarithms (e.g. NNLL) and nonperturbative contributions, and merging the resummed contributions with the fixed order (e.g. NLO) contributions appropriate for large p_T

- implementations/studies include RESBOS, Bozzi et al, Sterman et al. ...

- the resummation formalism is also valid for Higgs production at LHC via $gg \to H$ etc.

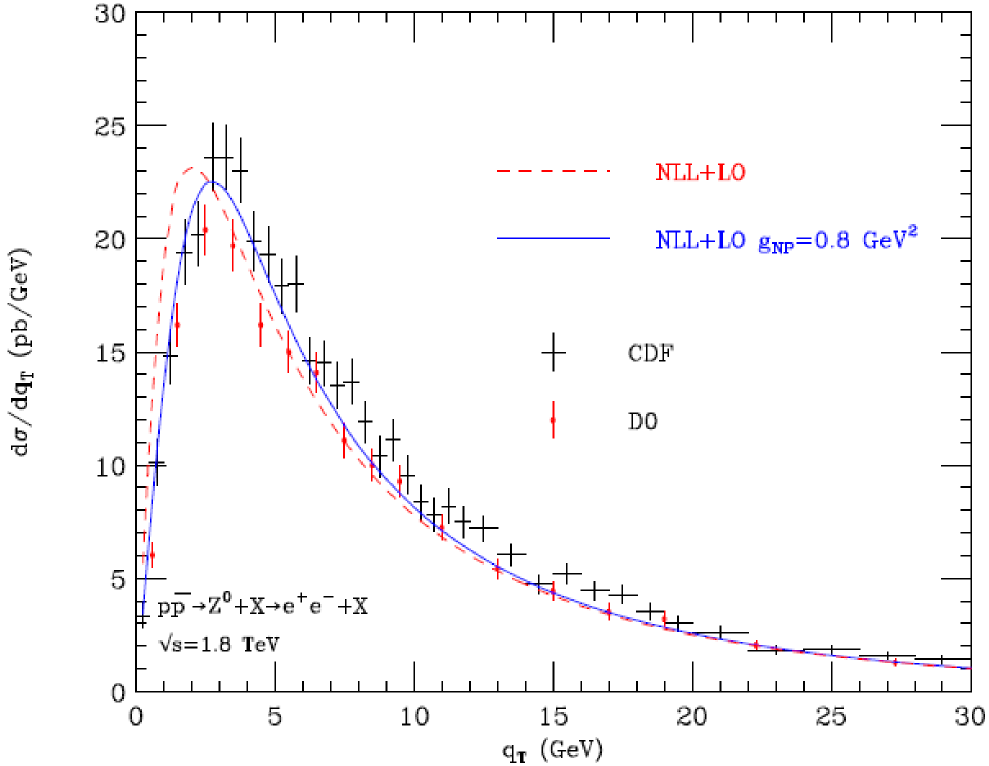


inadequacy of fixed order calculations



fixed-order pQCD calculations overshoot the data at small q_T

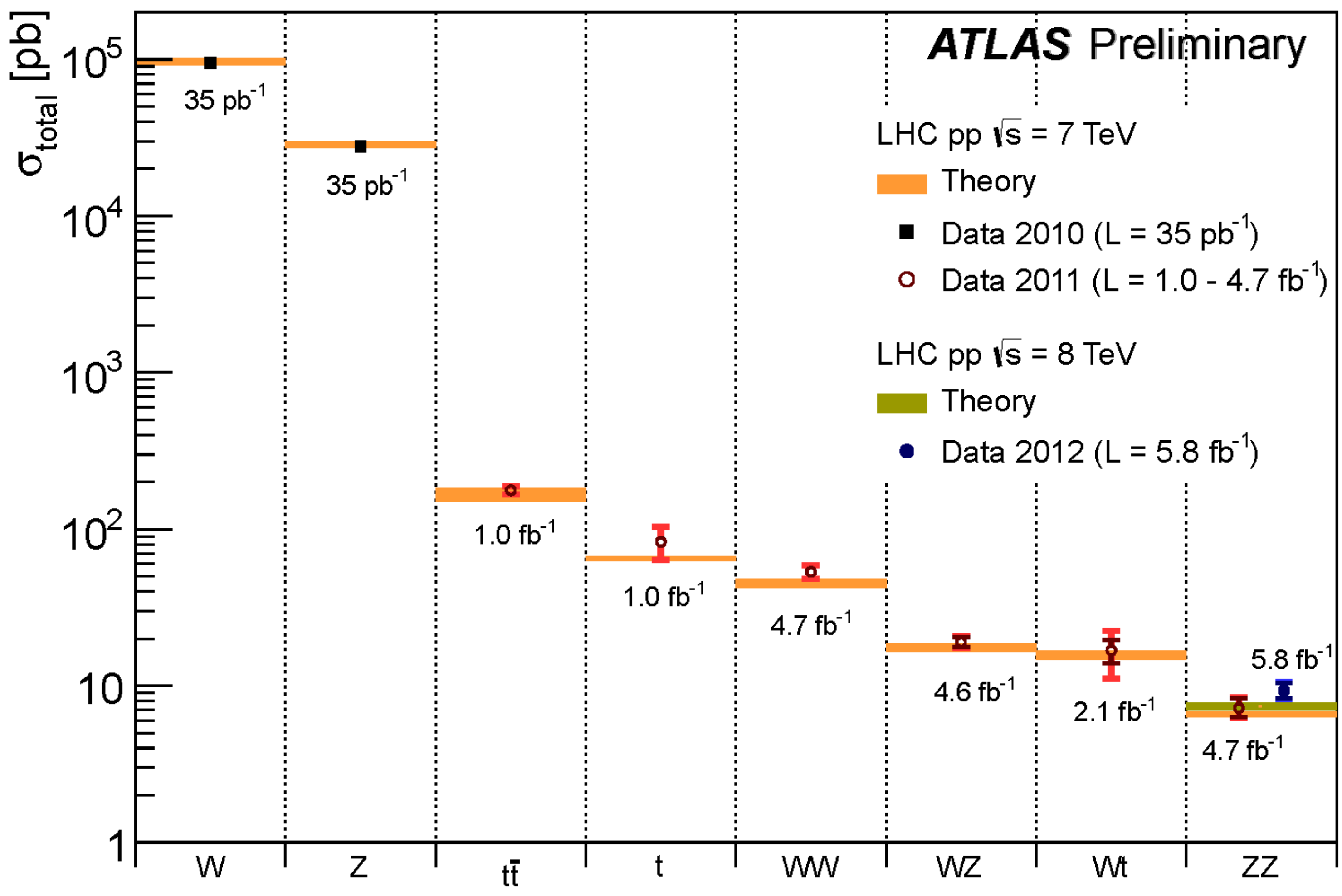
resummed (Sudakov) logs + non-perturbative ('intrinsic k_T') form factor give much better agreement with data



plots from: Bozzi, Catani, Ferrera, de Florian, Grazzini, arXiv:0812.2862

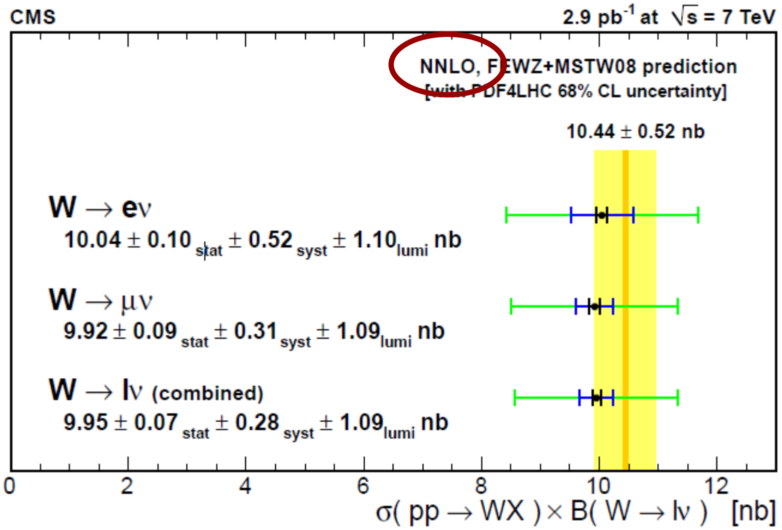
some examples of LHC precision QCD phenomenology

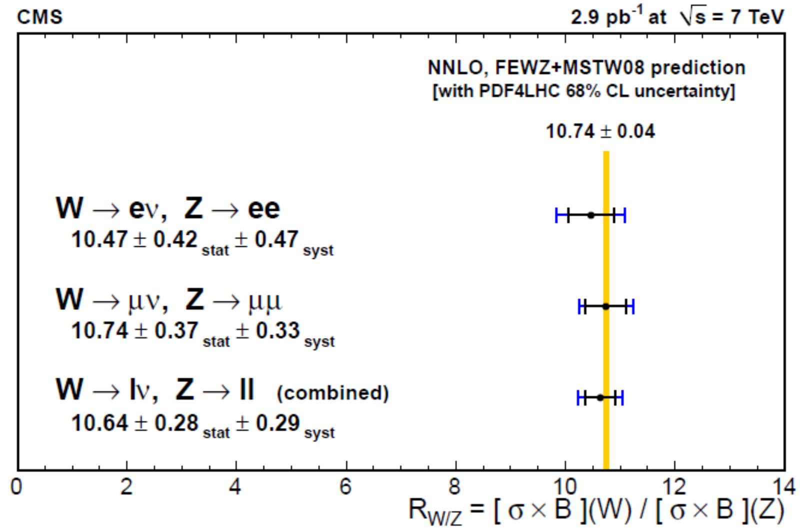
summary of electroweak and top cross sections



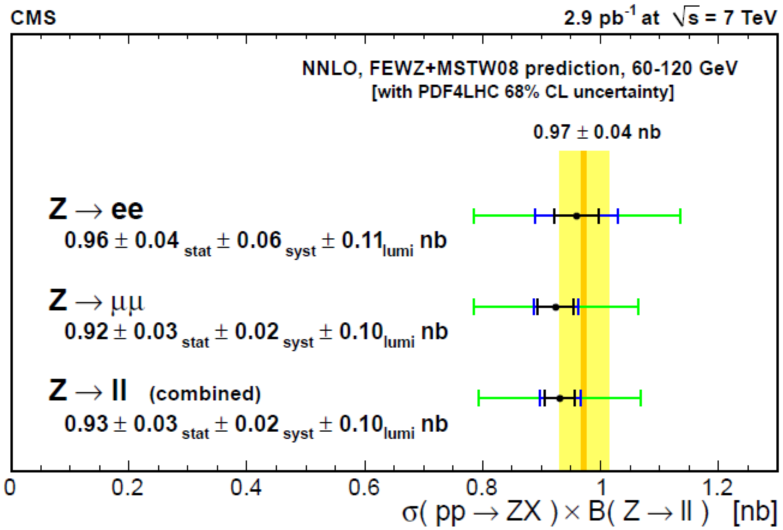
... in each case, theory is NLO or NNLO pQCD

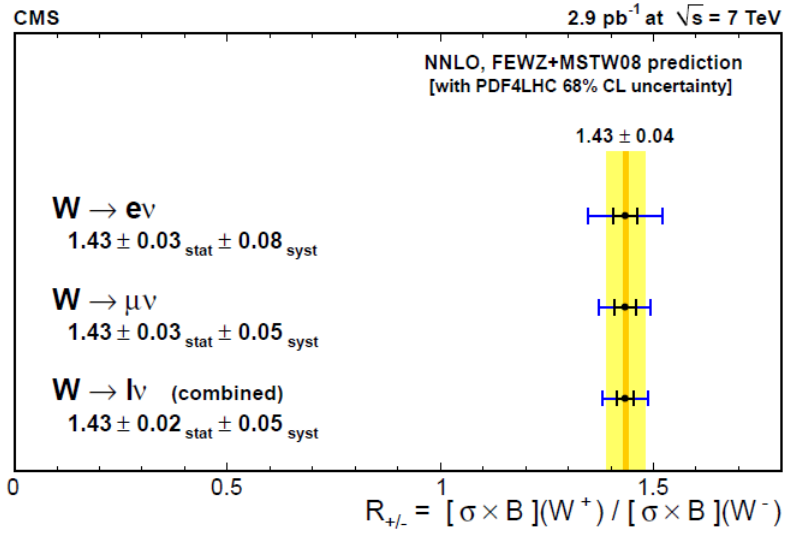
total W and Z cross sections

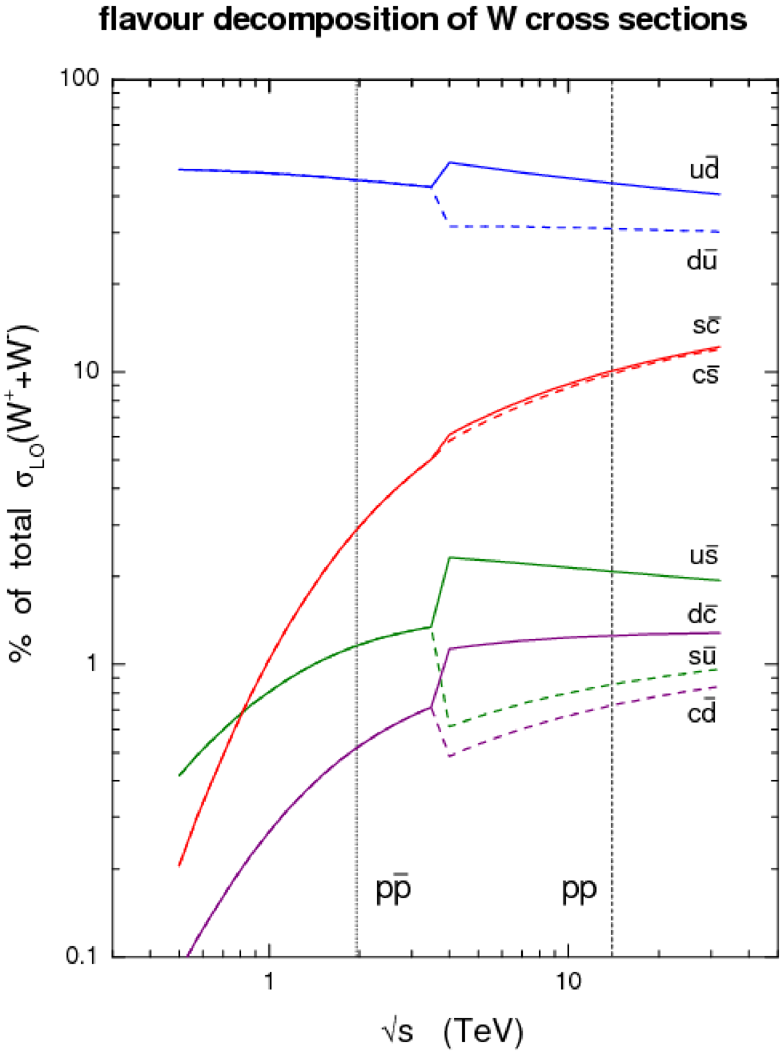


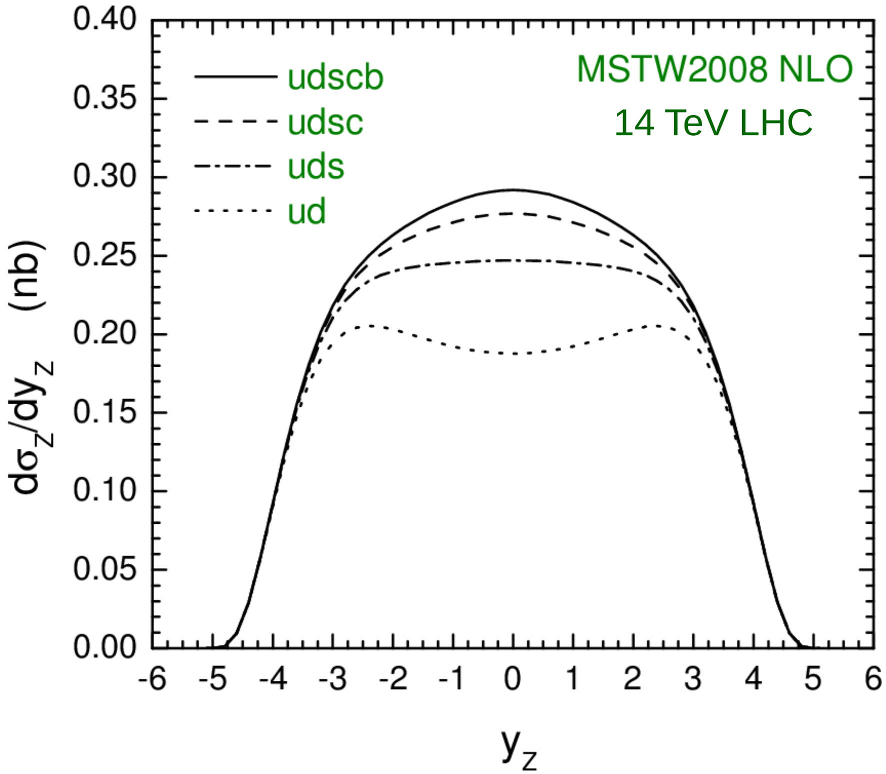


Note: pQCD corrections largely cancel in the ratios



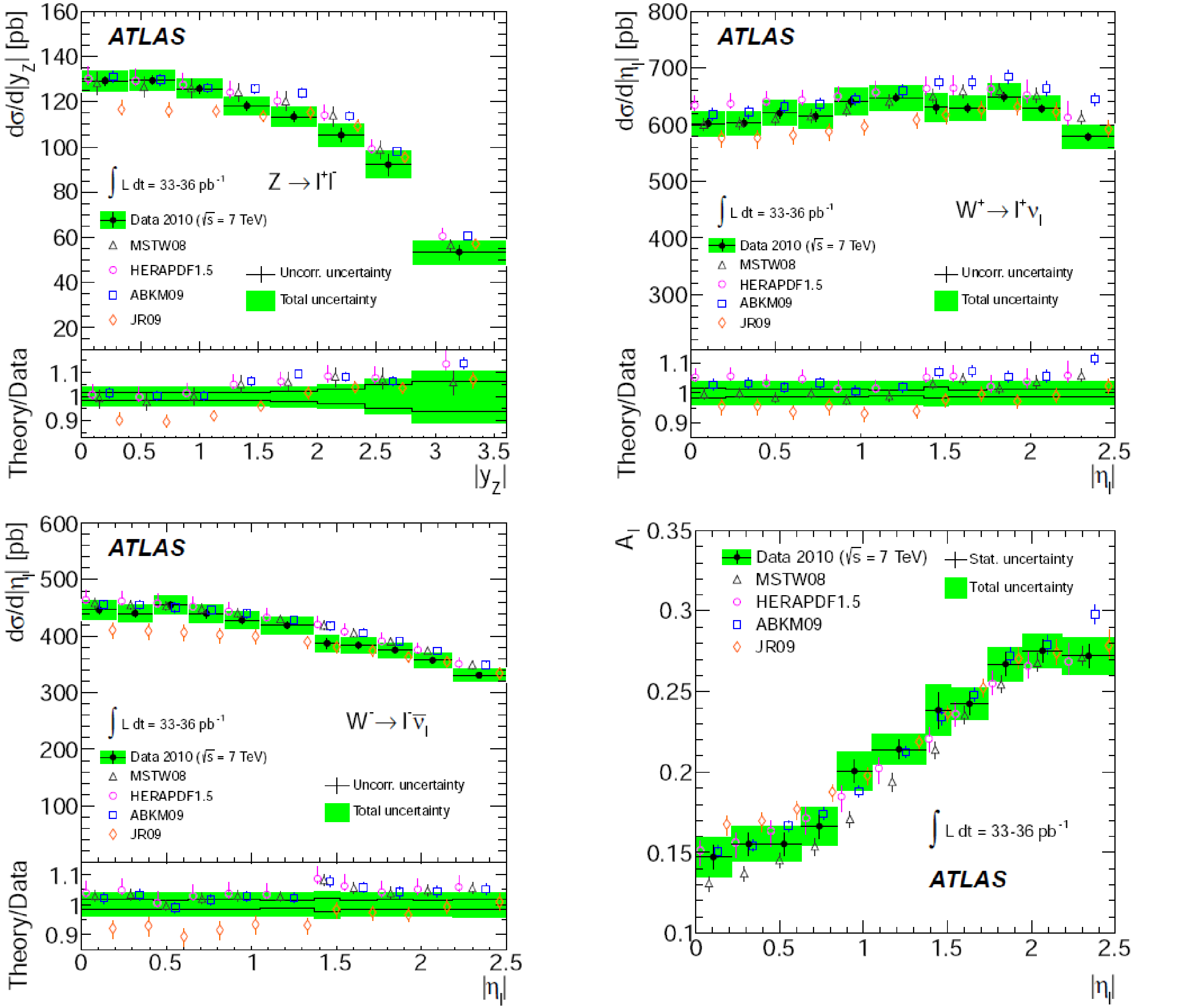




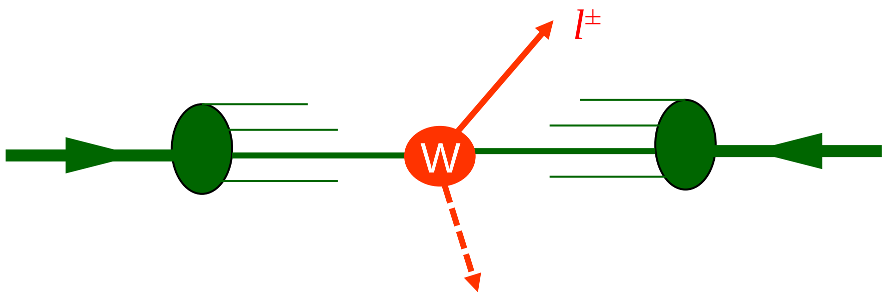


at LHC, ~30% of W and Z total cross sections involves heavy (s,c,b) quarks!

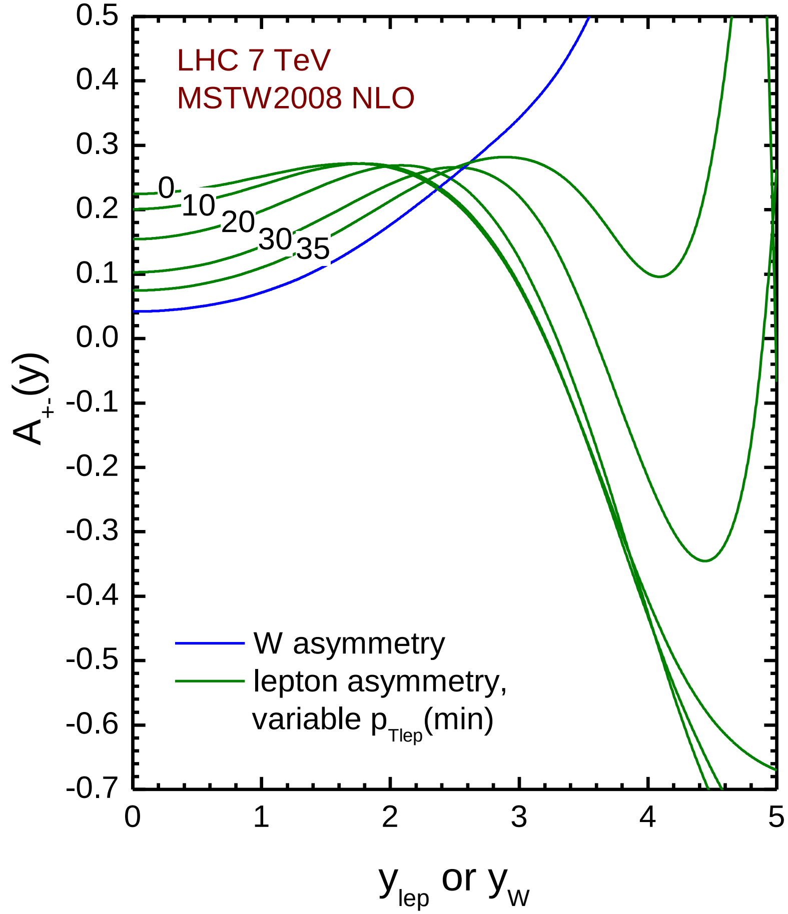
PDF discriminating power of LHC W and Z measurements



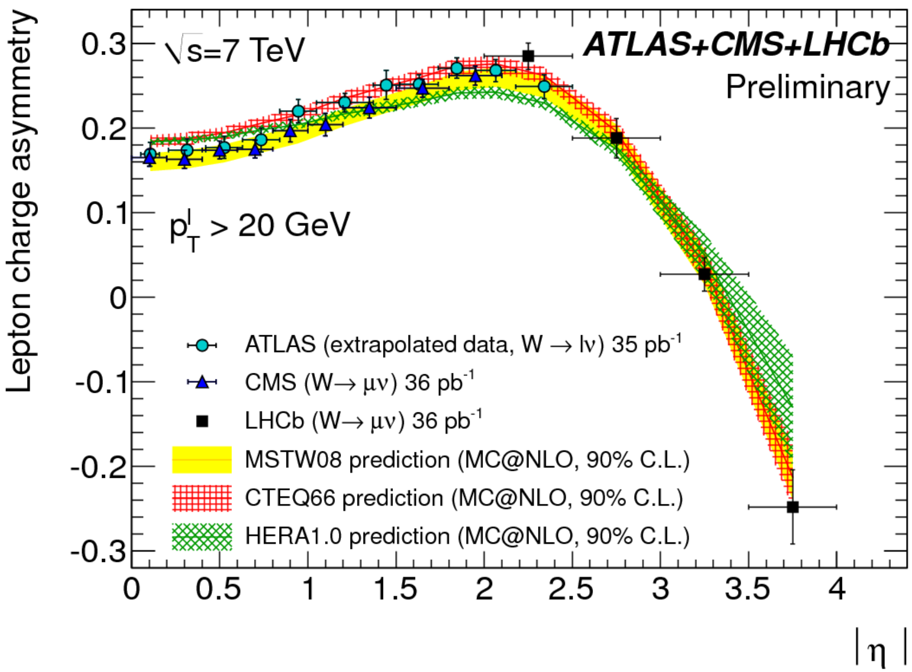
$W \rightarrow l\nu$ rapidity asymmetry

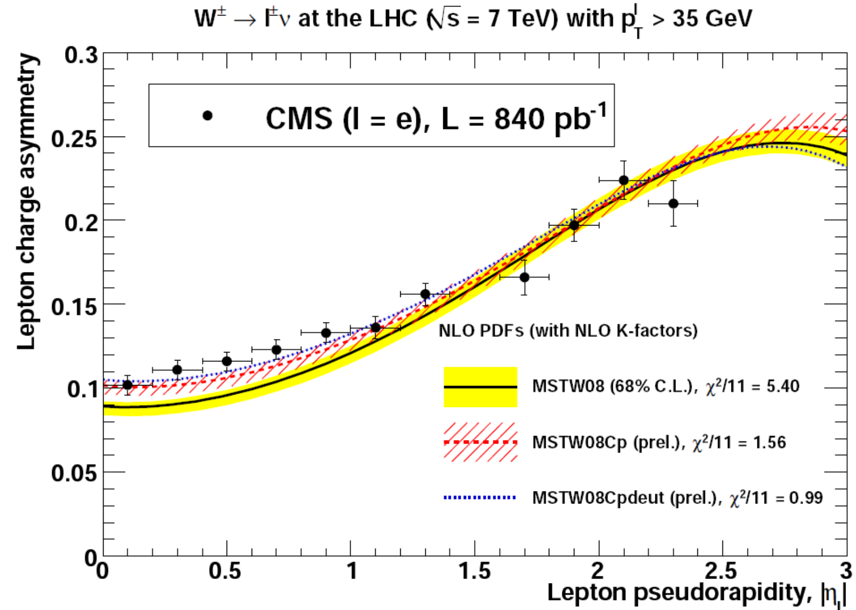


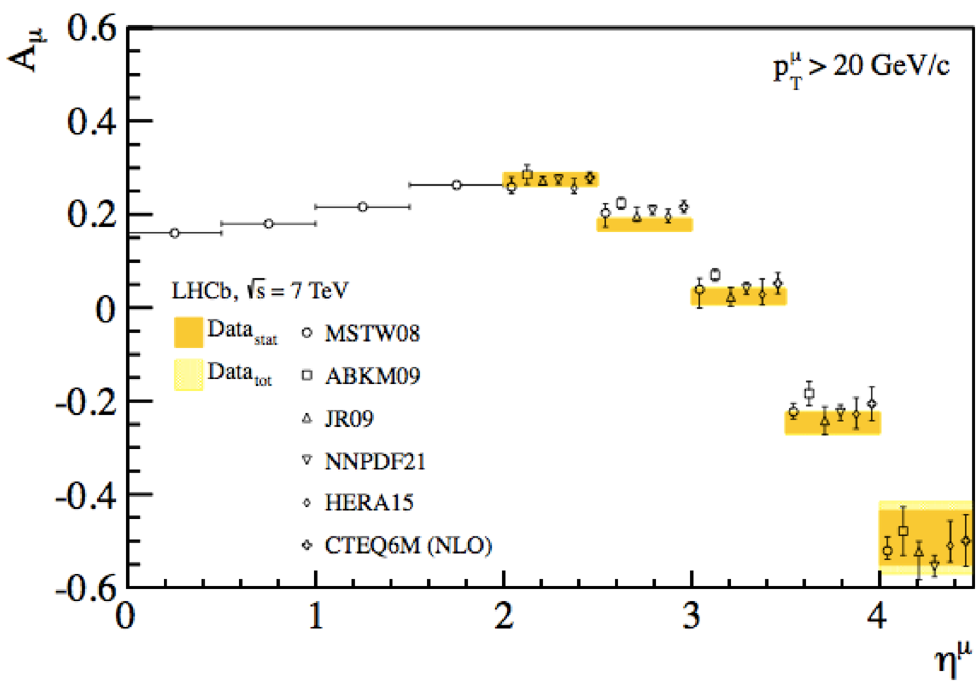
- very sensitive to pdfs
- complex interplay of u_V, d_V, Sea, V ± A decay
- lots of 7 TeV data now!



$$A_{\pm}(y_\ell) \approx \frac{S(x_1)u_V(x_2) - d_V(x_1)S(x_2)}{S(x_1)u_V(x_2) + d_V(x_1)S(x_2) + 2S(x_1)S(x_2)}.$$







the MSTW08 valence quarks need to be retuned

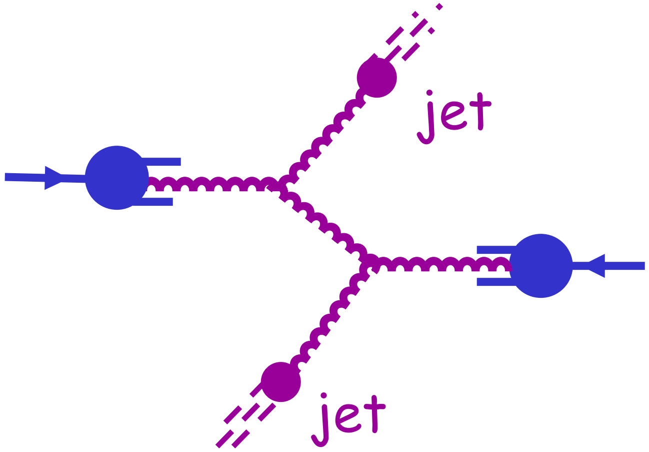
LHCb extends the reach to high rapidity

High-p_T jet production

$$E_J \frac{d\sigma}{d^3p_J} = \sum_{a,b,c,d=q,g} \int_0^1 dx_a dx_b \; f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2)$$
$$\times \delta(\hat{s} + \hat{t} + \hat{u}) \frac{1}{16\pi^2 \hat{s}} |\overline{M}^{ab\to cd}|^2$$

see ESW book

- where *ab*→*cd* represents all quark & gluon 2→2 scattering processes

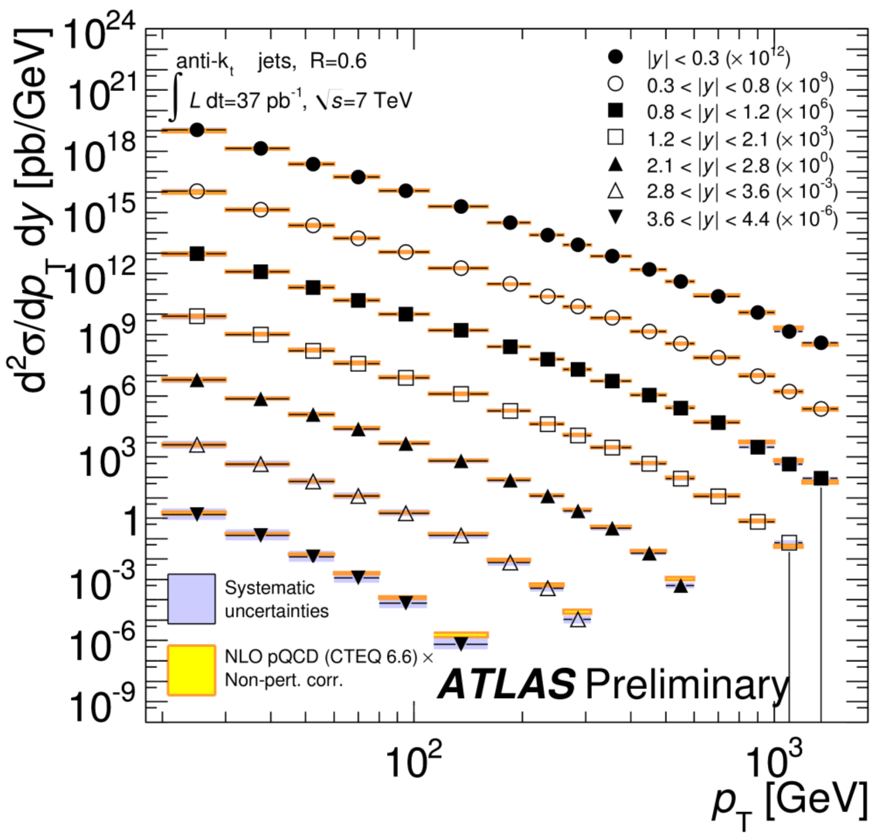


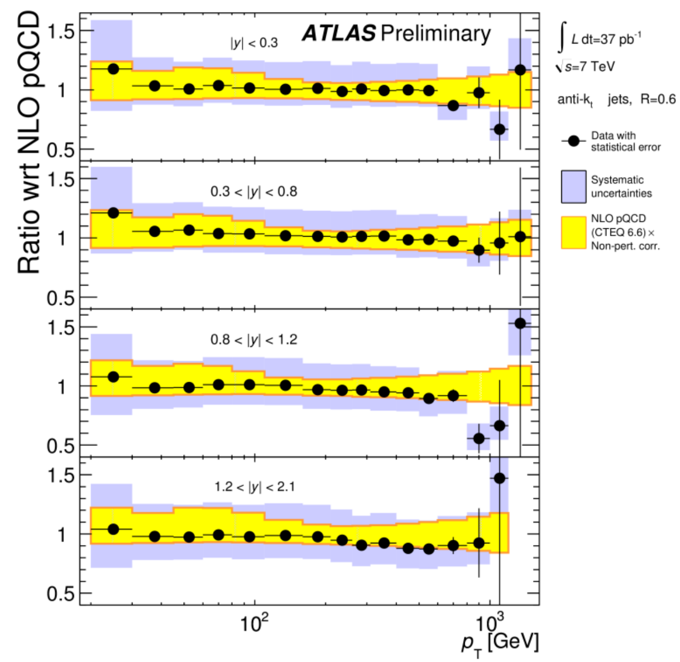
- NLO pQCD corrections also known, NNLO corrections awaited!

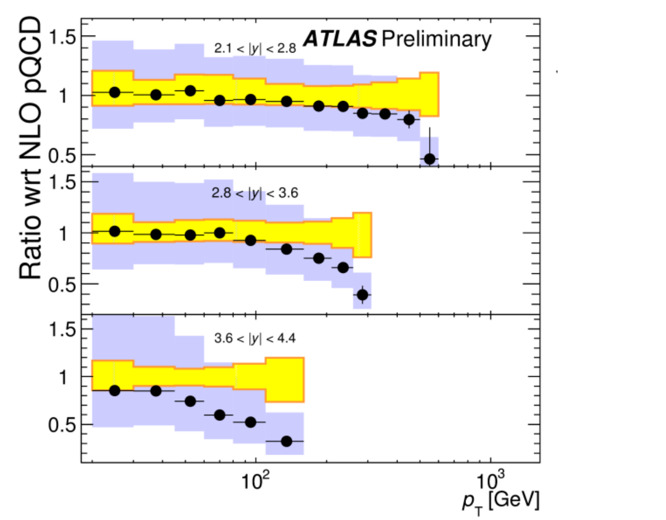
inclusive jet cross sections at LHC

cross section is measured as a function of jet p_T and rapidity up to p_T of 1.5 TeV and rapidity of 4.4

- total exp. uncertainty on cross section 50% - 10% (dominated by JES)
- good agreement with NLO pQCD predictions within experimental uncertainty

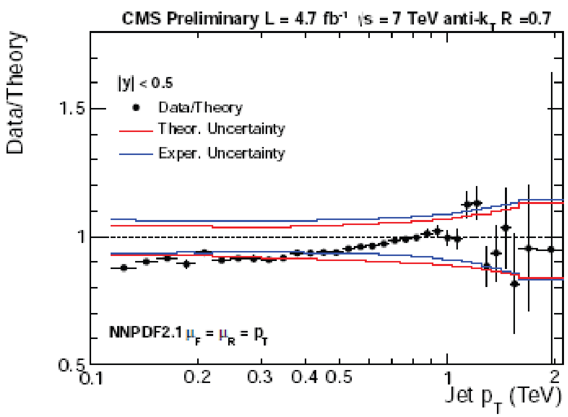


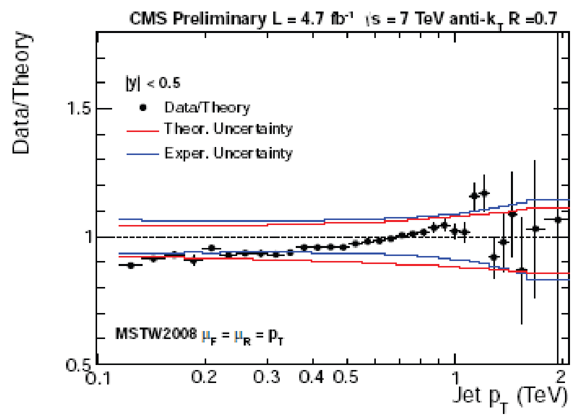


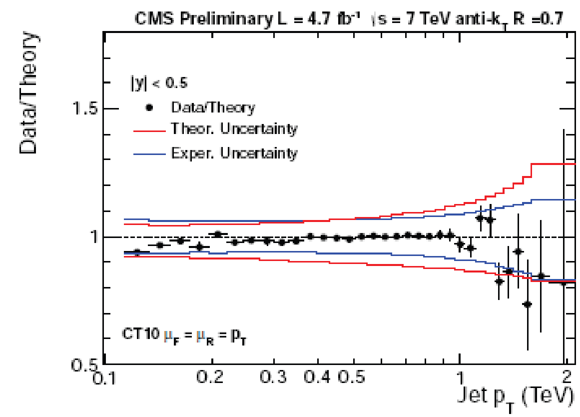


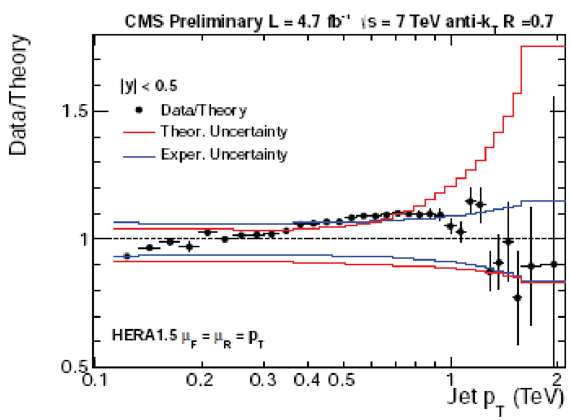
the LHC high p_T jet data are now beginning to constrain the PDFs ...

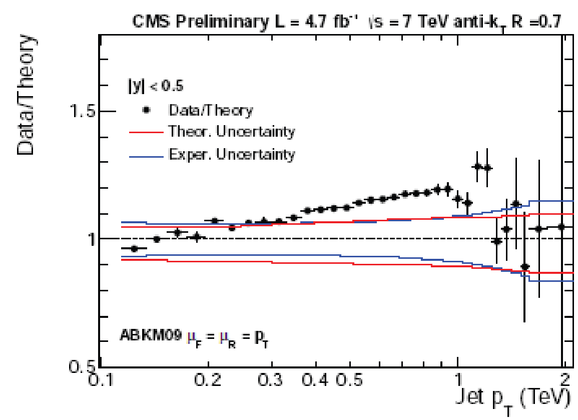
Here, CMS jet cross sections are compared to NLO QCD predictions using various PDF sets (NNPDF2.1, MSTW2008, CT10, HERAPDF1.5, ABKM09)











top quark production

$$\hat{\sigma}^{q\bar{q}\to Q\bar{Q}} = \frac{\pi\alpha_S^2\beta\rho}{27M_Q^2}(2+\rho)$$

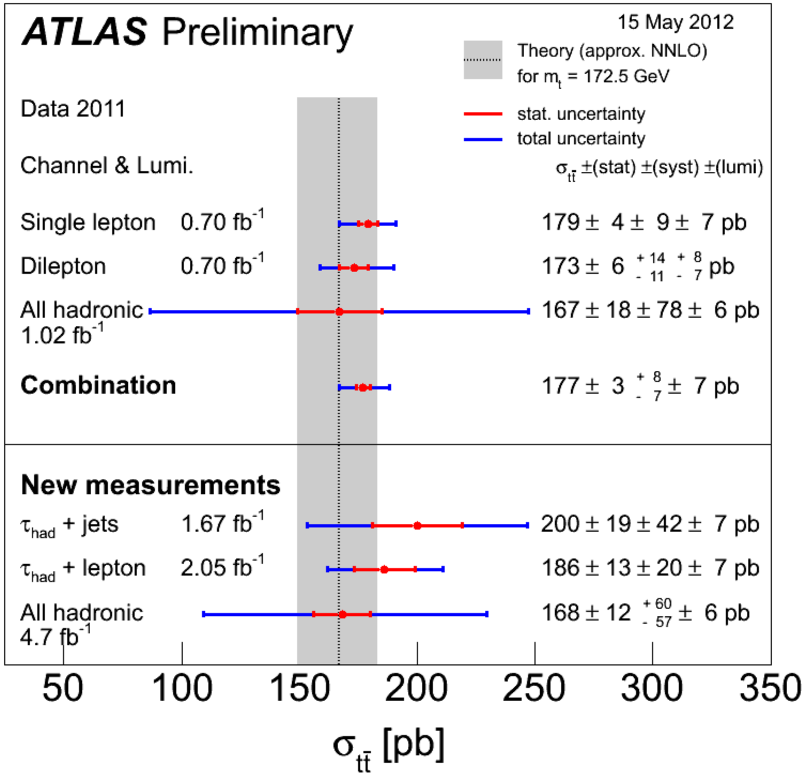
(dominates at Tevatron)

$$\hat{\sigma}^{gg\to Q\bar{Q}} = \frac{\pi\alpha_S^2\beta\rho}{192M_Q^2}[\frac{1}{\beta}(\rho^2+16\rho+16)\log\frac{1+\beta}{1-\beta} - 28 - 31\rho],$$

(dominates at LHC)

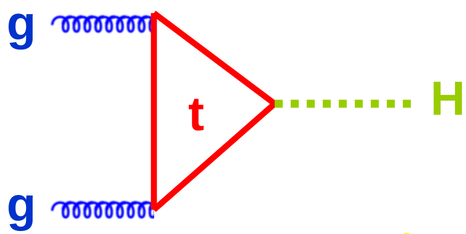
where $\rho = 4M_Q^2/\hat{s}$, $\beta = \sqrt{1-\rho}$.

- NLO known, but awaits full NNLO pQCD calculation
- NNLO & N^{n}LL "soft + virtual" approximations exist
- potential for distinguishing PDF sets (sensitivity to gluon PDF)



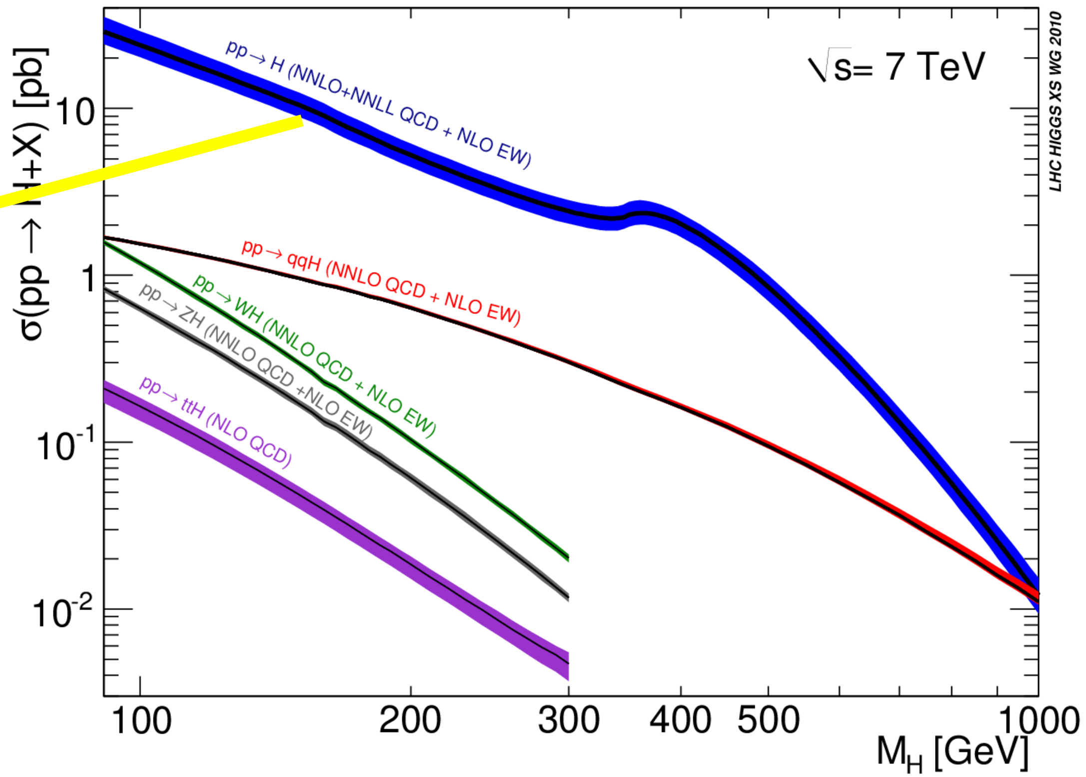
Higgs production

dominated by gluon-gluon fusion, NNLO corrections known



$$\hat{\sigma}^{gg\to H} = \frac{\alpha\alpha_S^2 M_H^2}{576\sin^2\theta_W M_W^2}\left|I\left(\frac{m_t^2}{M_H^2}\right)\right|^2$$

$$\begin{aligned} I(x) &= 3x[2+(4x-1)F(x)] \\ F(x) &= \theta(1-4x)\frac{1}{2}\left[\log\left(\frac{1+\sqrt{1-4x}}{1-\sqrt{1-4x}}\right)-i\pi\right]^2 \\ &\quad -\theta(4x-1)2\left[\sin^{-1}(1/2\sqrt{x})\right]^2 \end{aligned}$$



CERN–2011–002
17 February 2011

ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE

CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

Handbook of LHC Higgs cross sections:
1. Inclusive observables

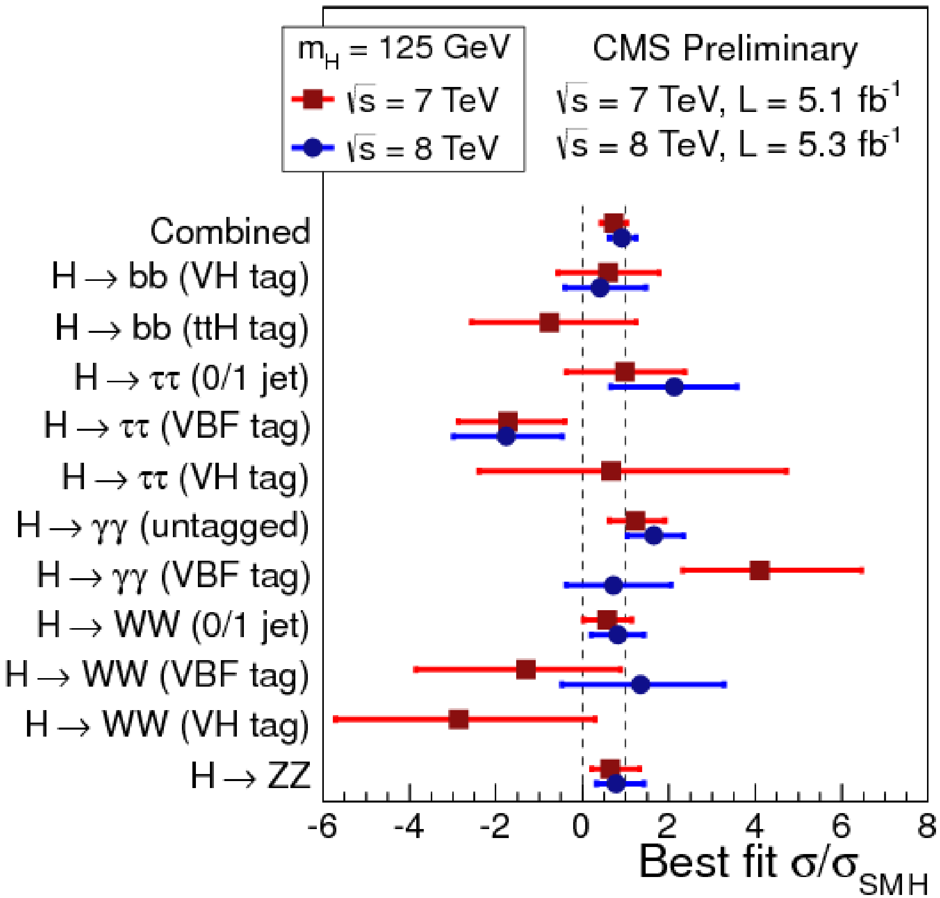
Report of the LHC Higgs Cross Section Working Group

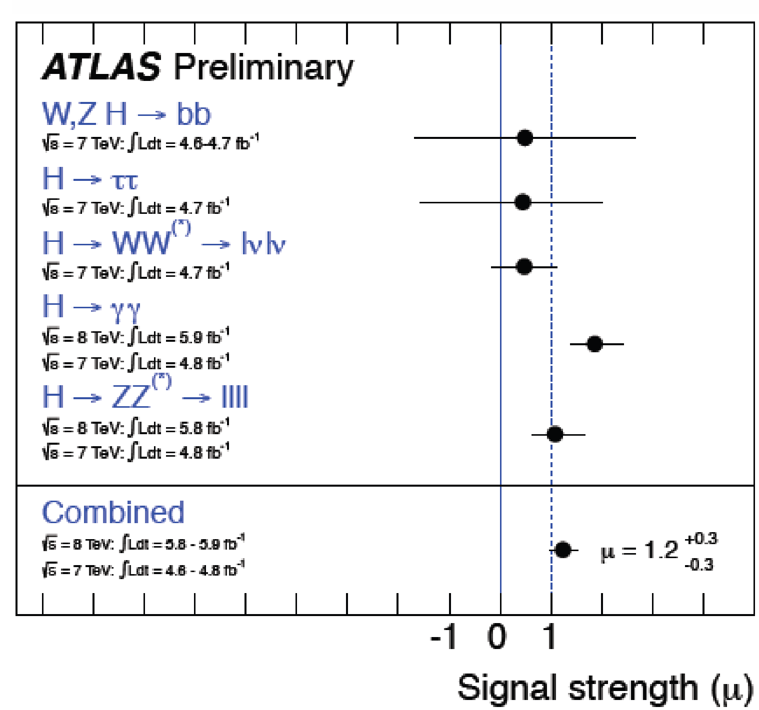
Editors: S. Dittmaier
C. Mariotti
G. Passarino
R. Tanaka

GENEVA
2011

arXiv:1101.0593

comparison of Higgs cross section measurements with SM predictions





$= \sigma/\sigma_{SMH}$

beyond perturbation theory

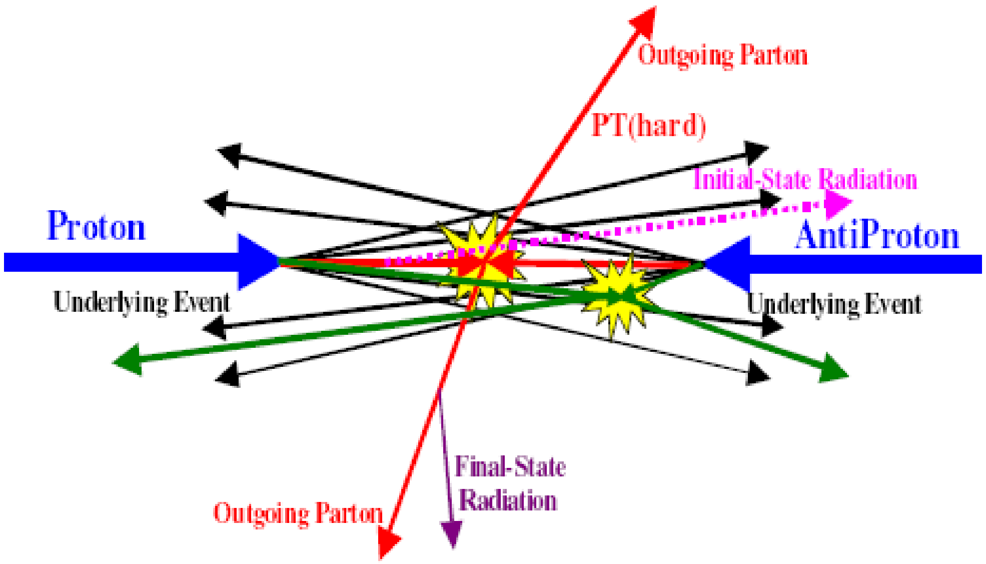
beyond perturbation theory

non-perturbative effects arise in many different ways

- emission of gluons with $k_T < Q_0$ off ‘active’ partons
- soft exchanges between partons of the same or different beam particles
- the transition from partons to hadrons in the final state
- ...

manifestations include…

- hard scattering occurs at net non-zero transverse momentum
- ‘underlying event’ additional hadronic energy



⇒ precision phenomenology requires a quantitative understanding of these effects, e.g. via models tuned to data

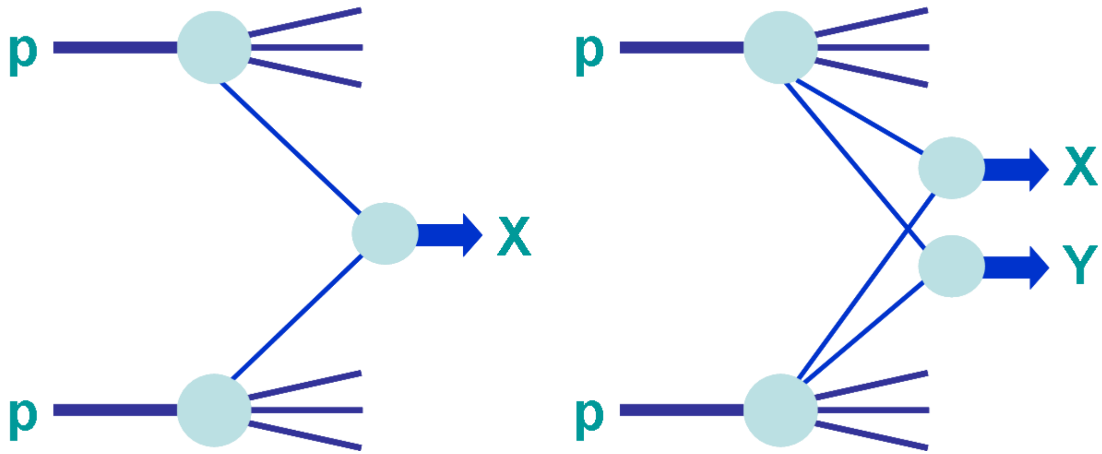
Monte Carlo Event Generators

- programs that simulates particle physics events with the same probability as they occur in nature
- widely used for signal and background estimates
- the main programs in current use are PYTHIA and HERWIG
- the simulation comprises different phases:
  - start by simulating a hard scattering process – the fundamental interaction (usually a $2 \rightarrow 2$ process but could be more complicated for particular signal/background processes)
  - this is followed by the simulation of (soft and collinear) QCD radiation using a parton shower algorithm
  - non-perturbative models are then used to simulate the hadronization of the quarks and gluons into the observed hadrons and the underlying event

➡ see 'MC Tools' lectures by Mike Seymour

finally, there are interesting QCD processes
where our theoretical understanding
is rather less developed...

single and double hard parton scattering

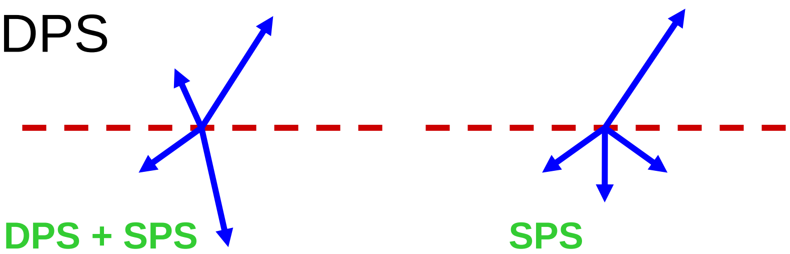


e.g. X,Y = jj,bb,W,Z,J/ψ,..

- folklore $$\sigma^{\text{DPS}}(X,Y) = \frac{m}{2}\,\frac{\sigma_X \sigma_Y}{\sigma_{\text{eff}}}$$

X,Y distinct: m=2
X,Y same: m=1

- studies of γ+3j production by CDF and D0 suggest $\sigma_{\text{eff}} \sim 15$ mb
- use shape variables as a discriminator for DPS
- however, simple factorisation hypothesis now known to be invalid
→ much recent theoretical activity, see
"Multi-Parton Interactions at the LHC", P. Bartalini et al., arXiv:1111.0469



experimental measurements of DPS

Experimental measurements more or less limited to σ_{eff} :

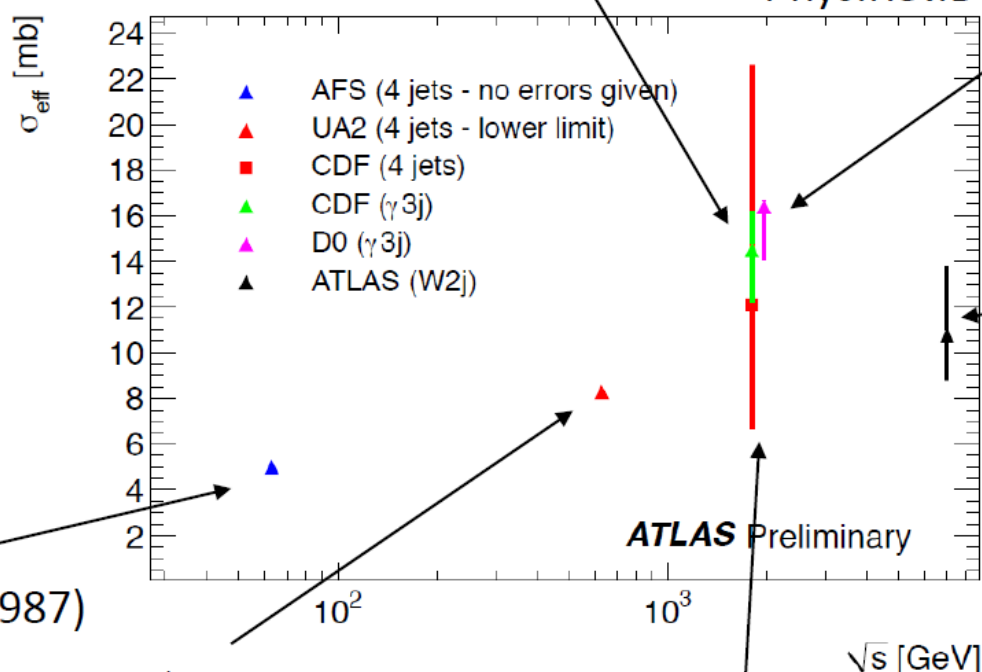
$p_T^{jet} > 6\ \mathrm{GeV},\ p_T^{\gamma} > 16\ \mathrm{GeV}$

Phys. Rev. D 56, 3811 (1997)

$15 < p_T^{jet2} < 30\ \mathrm{GeV}$

$60 < p_T^{\gamma} < 80\ \mathrm{GeV}$

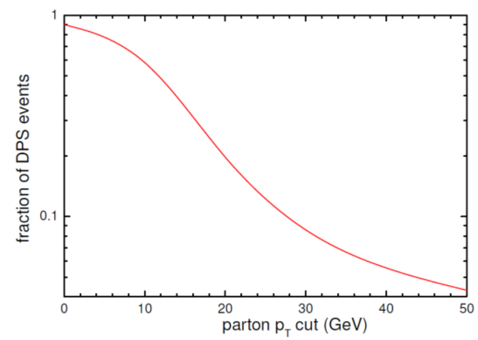
Phys.Rev.D81, 052012(2010)



$p_T^{jet} > 20\ \mathrm{GeV}$

$p_T^{lepton} > 20\ \mathrm{GeV}$

ATLAS-CONF-2011-160



$p_T^{jet} > 4\ \mathrm{GeV}$

Z. Phys. C 34, 163 (1987)

$p_T^{jet} > 15\ \mathrm{GeV}$

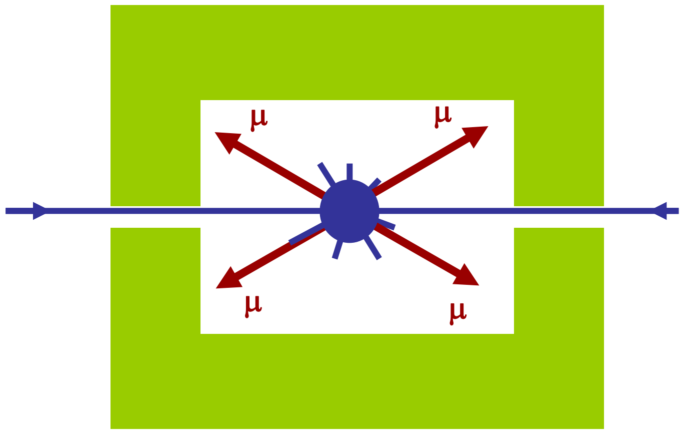
Phys. Lett. B268, 145-154 (1991)

$p_T^{jet} > 25\ \mathrm{GeV}$

Phys. Rev. D 47, 4857-4871 (1993).

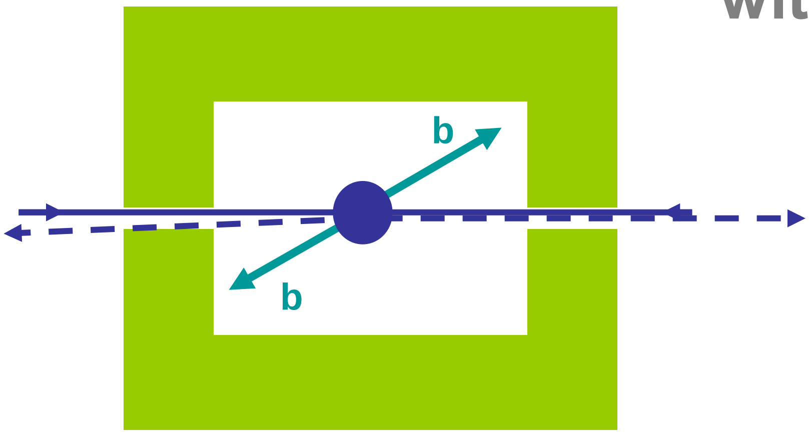
central exclusive production

compare …



- $p + p \rightarrow H + X$
  - the rate (σ_{parton}, PDFs, α_S)
  - the kinematic distribtns. ($d\sigma/dydp_T$)
  - the environment (jets, underlying event, backgrounds, …)

with …

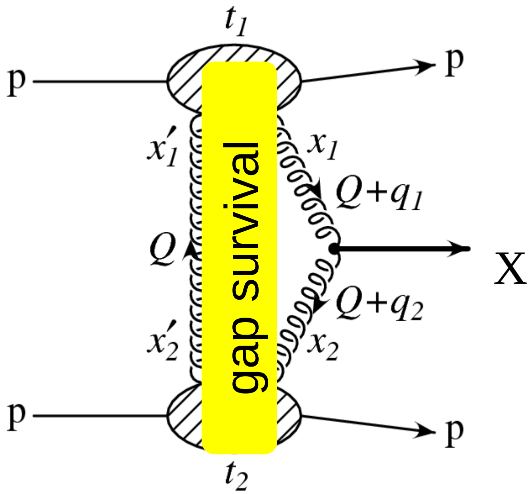


- $p + p \rightarrow p + H + p$
  - a real challenge for theory (pQCD + npQCD) and experiment (tagging forward protons, triggering, …)

central exclusive production – theory

$p + p \to p \oplus X \oplus p$

- colliding protons interact via a colour singlet exchange and remain intact: can be triggered by adding proton detectors far down the beam-pipe or by using large rapidity gaps
- a system of mass M_X is produced at the collision point, and only its decay products are present in the central detector region.
- the generic process $pp \to p + X + p$ is modeled perturbatively by the exchange of two t-channel gluons (→'Durham Model' – Khoze Martin Ryskin)



- the possibility of additional soft rescatterings filling the rapidity gaps is encoded in 'eikonal' and 'enhanced' survival factors

CEP at LHC?

$p + p \to p \oplus X \oplus p$

- in the limit that the outgoing protons scatter at zero angle, the centrally produced state X must have $J_Z^P = 0^+$ *quantum numbers* → spin-parity filter/analyser
- in certain regions of MSSM parameter space, couplings of Higgs to bb is enhanced, and CEP could be the discovery channel
- or any exotic 0^{++} state, which couples strongly to glue, is a real possibility: radions, gluinoballs, …
- in the meantime, many 'standard candle' processes at RHIC, Tevatron, LHC: X= jj, $\gamma\gamma$, J/ψ, χ_c, χ_b, $\pi\pi$, …
- example:

CDF(arXiv:0902.1271):
$$\left.\frac{d\sigma(\chi_{c0})}{dy_\chi}\right|_{y=0} = (76 \pm 14)\,\text{nb}$$

KRYSTHAL (Khoze, Ryskin, S, Harland-Lang, arXiv:1005.0695):
$$\left.\frac{d\sigma^{\text{tot}}_{\chi_c}}{dy_\chi}\right|_{y_\chi=0} \approx 60\,\text{nb}$$

Durham/St Petersburg /Cambridge (Khoze, Martin, Ryskin, S, Harland-Lang,....)

Manchester (Cox, Forshaw, Monk, Pilkington, Coughlin, ...)

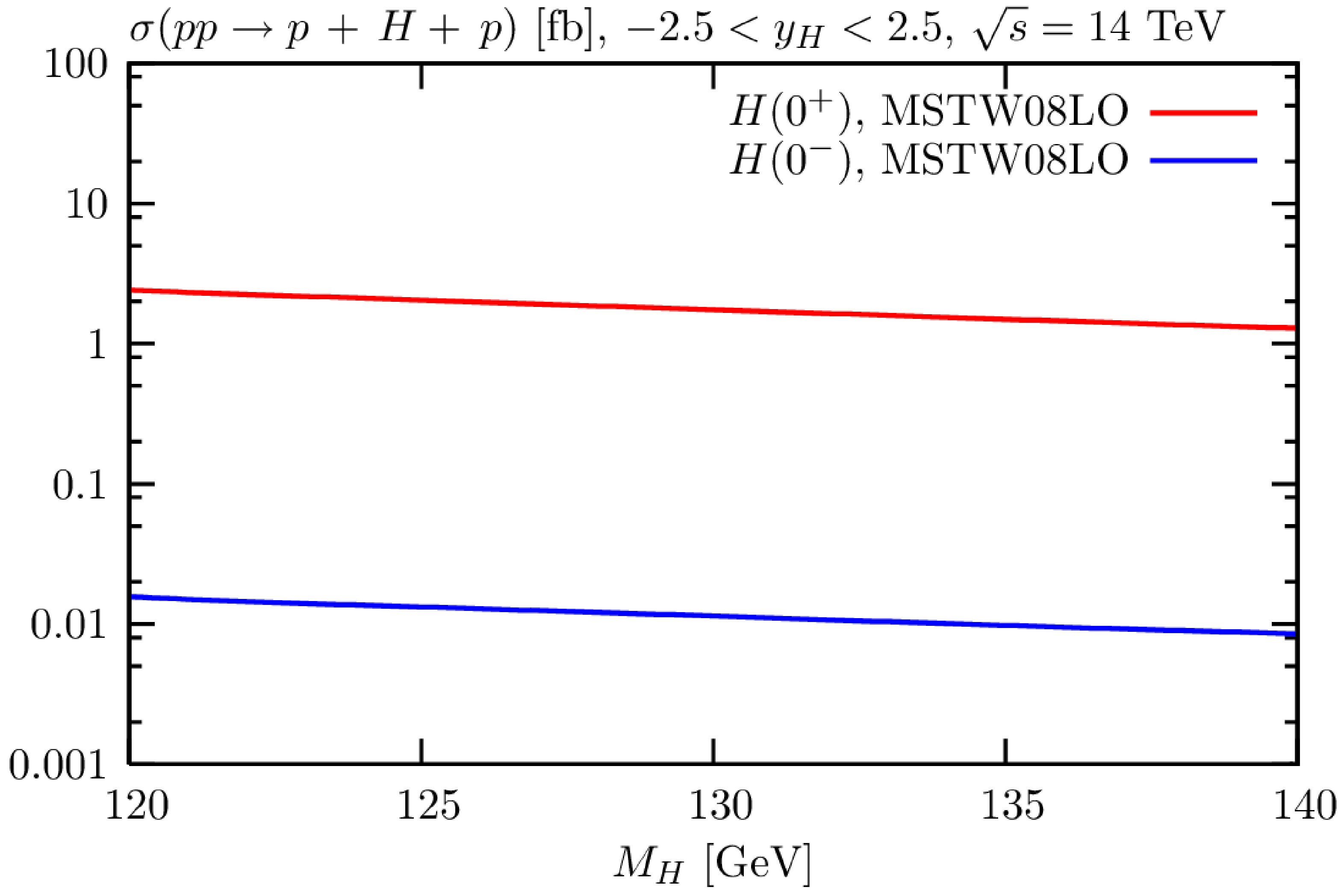
Helsinki (Orava, ...)

Saclay (Royon, ...)

Cracow (Szczurek, ...)

…

Higgs production via CEP



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summary

- QCD: non-abelian gauge field theory for the strong interaction and essential component of the Standard Model; symmetry = SU(3) and $\alpha_S(M_Z^2) = 0.1185 \pm 0.0007$

- thanks to ~ 40 years theoretical studies, supported by experimental measurements, we now know how to calculate (an important class of) proton-proton collider event rates reliably and with a high precision

- the key ingredients are the factorisation theorem and the universal parton distribution functions

- such calculations underpin searches (at the Tevatron and the LHC) for New Physics

summary contd.

- …but much work still needs to be done, in particular:
  - calculating more and more NNLO pQCD corrections (and a few missing NLO ones too)
  - better understanding of ‘scale dependence’
  - further refining the PDFs, using new LHC data
  - understanding the detailed event structure, much of which is outside the domain of pQCD and is currently simply modelled
  - extending the calculations to new types of production processes, e.g. central exclusive diffractive production, double parton scattering, ...

extra slides