Flavor physics

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Yesterday...

- What is HEP and model building
- The SM, and its flavor structure
- CKM matrix
- FCNCs

Today we will talk about counting parameters and how to determine the flavor parameters

Parameter counting

How many parameters we have?

How many parameters are physical?

- "Unphysical" parameters are those that can be set to zero by a basis rotation
- General theorem

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken})$$

  - $N(\text{Phys})$, number of physical parameters
  - $N(\text{tot})$, total number of parameters
  - $N(\text{broken})$, number of broken generators

Example: Zeeman effect

A hydrogen atom with weak magnetic field

- The magnetic field add one new physical parameter, B

$$V(r) = \frac{-e^2}{r} \quad \Rightarrow \quad V(r) = \frac{-e^2}{r} + B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

- But there are 3 total new parameters
- The magnetic field breaks explicitly: $SO(3) \to SO(2)$
- 2 broken generators, can be “used” to define the z axis

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken}) \quad \Rightarrow \quad 1 = 3 - 2$$

Back to the flavor sector

Without the Yukawa interactions, a model with N copies of the same field has a $U(N)$ global symmetry

- It is just the symmetry of the kinetic term

$$\mathcal{L} = \bar{\psi}_i D_\mu \gamma^\mu \psi_i, \qquad i = 1, 2, ..., N$$

- $U(N)$ is the general rotation in N dimensional complex space
- $U(N) = SU(N) \times U(1)$ and it has N^2 generators

Two generation SM

First example, two generation SM

- Two Yukawa matrices: Y^D, Y^U, $N_{Total} = 16$
- Global symmetries of the kinetic terms: $U(2)_Q \times U(2)_D \times U(2)_U$, 12 generators
- Exact accidental symmetries: $U(1)_B$, 1 generator
- Broken generators due to the Yukawa: $N_{Broken} = 12 - 1 = 11$
- Physical parameters: $N_{Physical} = 16 - 11 = 5$. They are the 4 quarks masses and the Cabibbo angle

The SM flavor sector

Back to the SM with three generations. Do it yourself

- Total parameters (in Yukawas): $N_T =$
- Symmetry generators of kinetic terms: $N_G =$
- Unbroken global generators: $N_U =$
- Broken generators: $N_B =$
- Physical parameters: $N_P =$

The SM flavor sector

Back to the SM with three generations. Do it yourself

- Total parameters (in Yukawas): $N_T = 2 \times 18 = 36$
- Symmetry generators of kinetic terms: $N_G = 3 \times 9 = 27$
- Unbroken global generators: $N_U = 1$
- Broken generators: $N_B = 27 - 1 = 26$
- Physical parameters: $N_P = 36 - 26 = 10$
- 6 quark masses, 3 mixing angles and one CPV phase

Remark: The broken generators are 17 Im and 9 Re. We have 18 real and 18 imaginary to “start with” so the physical ones are $18 - 17 = 1$ and $18 - 9 = 9$

Homework

Consider a model with the same gauge symmetry and SSB as in the SM. The fermions, however, are

$$Q_L(3,2)_{1/6},\ \ S_L(3,1)_{-1/3},\ \ U_R(3,1)_{2/3},\ \ D_R(3,1)_{-1/3},\ \ S_R(3,1)_{-1/3}$$

- What is the spectrum of this model? That is, what are the quarks after SSB. Note that you can also have “bare masses” in this model, for example, $m_{ss}\bar{s}_L s_R$.
- How many physical parameters there are, and what are they?
- Are there W exchange flavor changing interactions?
- Is CP a good symmetry?
- Are there tree level FCNCs in this model?

The CKM matrix

The flavor parameters

- The 6 masses. We kind of know them. There is a lot to discuss, but I will not do it in these lectures
- The CKM matrix has 4 parameters
  - 3 mixing angles (the orthogonal part of the mixing)
  - One phase (CP violating)
- We will concentrate on trying to find ways to determine the CKM three mixing angles and one phase. Here we will get into some details

The CKM matrix

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \overline{U_L} V \gamma^\mu D_L W_\mu^+ + \text{h.c.}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- CKM is unitary

$$\sum V_{ij} V_{ik}^* = \delta_{jk}$$

- Experimentally, $V \sim 1$. Off diagonal terms are small
- Many ways to parametrize the matrix

CKM parametrization

- The standard parametrization

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

  where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$
- In general there are 5 entries that carry a phase
- Experimentally: (will explain later how these measurements were done)

$$|V| \approx \begin{pmatrix} 0.97383 & 0.2272 & 3.96 \times 10^{-3} \\ 0.2271 & 0.97296 & 4.221 \times 10^{-2} \\ 8.14 \times 10^{-3} & 4.161 \times 10^{-2} & 0.99910 \end{pmatrix}$$

The Wolfenstein parametrization

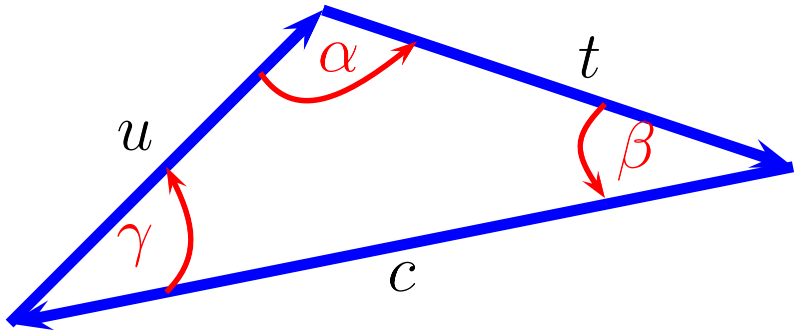
- Since $V \sim 1$ it is useful to expand it

$$V \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- One small parameter $\lambda \sim 0.2$, and three (A, ρ, η) that are roughly $O(1)$
- As always, be careful (unitarity...)
- Note that to this order only V_{13} and V_{31} have a phase

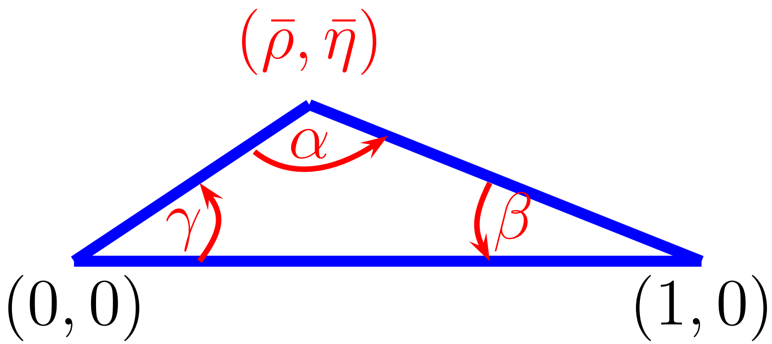
The unitarity triangle

A geometrical presentation of $V_{ub}^* V_{ud} + V_{tb}^* V_{td} + V_{cb}^* V_{cd} = 0$



Rescale by the c size and rotated

$$A\lambda^3 \left[(\rho + i\eta) + (1 - \rho - i\eta) + (-1) \right] = 0$$



CKM determination

CKM determination

- Basic idea: Measure the 4 parameters in many different ways. Any inconstancy is a signal of NP
- Problems: Experimental errors and theoretical errors
- Have to be smart...
  - Smart theory to reduce the errors
  - Smart experiment to reduce the errors
- There are cases where both errors are very small

Classifications

Two classifications:

- Parameters
  - Sides of the UT (magnitudes of CKM elements)
  - Angles of the UT (relative phases between CKM elements)
  - Combination of those
- Amplitudes
  - Tree (mostly SM)
  - Loop (SM and maybe also NP)
  - Combination of those

Experimental issues

Just very brief

- Many times we look at very small rates or small asymmetries (we like to probe small couplings). Statistics is needed
- Very important to get the PID (like K/π separation)
- Flavor tagging: is it a B or a $\bar{B}$
- CP properties: the detector is made of matter

Theoretical uncertainties

Always: QCD

- We calculate with quark, but we measure hadrons
- The strong interaction is strong. No perturbation theory. Really a problem

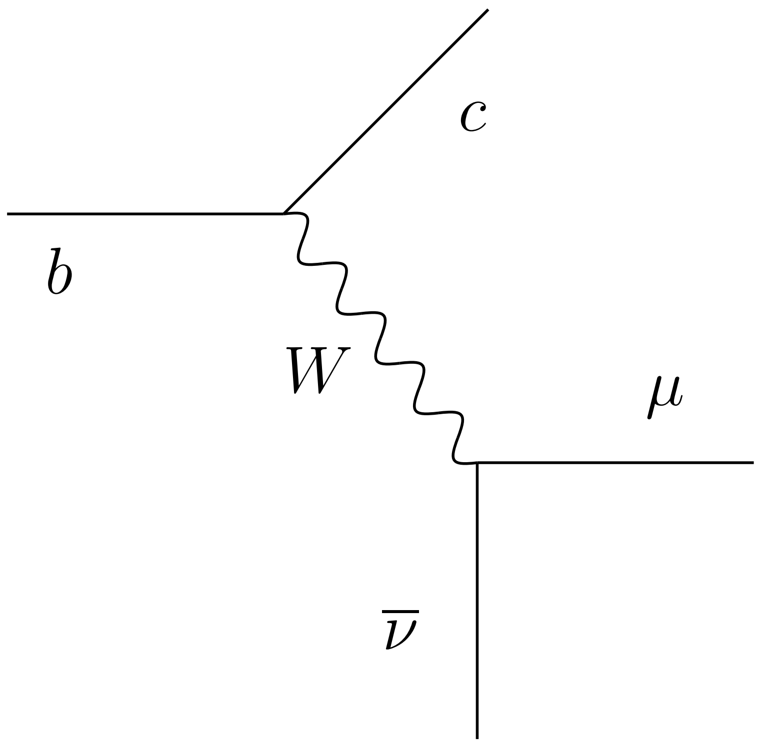
Solutions:

- We use some approximate symmetries, for example: isospin, flavor SU(3), HQS
- There are cases where one can construct observables where the hadronic physics cancels

Measuring sides

Tree level decays are sensitive to absolute values of CKM element

$$\Gamma(B \to X_c \mu \nu) \propto |V_{cb}|^2$$



Measuring sides: problems

Not so simple...

$$\Gamma(b \to c\mu\nu) \propto m_b^5 |V_{cb}|^2$$

- Because the b is heavy, $m_b \gg \Lambda_{QCD}$ we can expand in Λ_{QCD}/m_b and we get

$$\Gamma(b \to c\mu\nu) \approx \Gamma(B \to X_c\mu\nu)$$

- Not easy to get m_b the mass of the b quark. We use HQS and use m_B, the B meson mass
- Using symmetries, and expanding around them we can get rather accurate determination

Always: Look for a process where we have sensitivity, and work our way around QCD

An aside: what is m_B?

$$m_B = ?$$

Other sides

Similar issues with other tree level decays

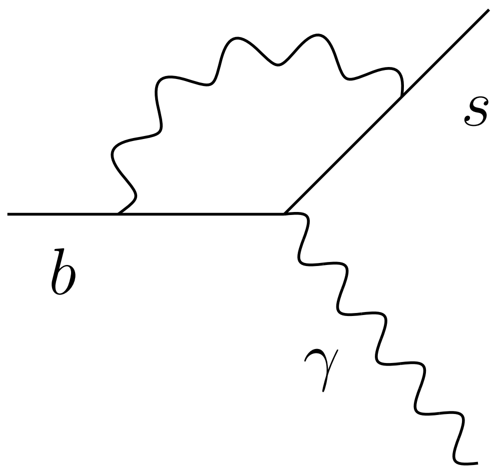
- β-decay, $d \to ue\bar{\nu} \propto V_{ud}$; Isospin
- K-decay, $s \to ue\bar{\nu} \propto V_{us}$; Isospin and SU(3)
- D-decay, $c \to qe\bar{\nu} \propto V_{cq}\ q = d, s$; HQS
- B decays can be used also for V_{ub}. Harder
- Not easy with top. Cannot tag the final flavor, low statistics

Loop decays

- We have sensitivity to magnitude of CKM elements in loops
- More sensitive to V_{tq} that is harder to get in tree level decays
- But at the same time it may be modified by new heavy particles
- This is a general argument. NP is likely to include “heavy” particles, that can affect loop processes much more than tree level decays

Loop: example

$$A(b \to s\gamma) \propto \sum V_{ib} V_{is}^*$$



What is $\sum V_{ib} V_{is}^*$?

GIM Mechanism

what we really have is

$$A(b \to s\gamma) \propto \sum V_{ib} V_{is}^* f(m_i)$$

- Because the CKM is unitary, the m_i independent term in f vanishes
- Must be proportional to the mass (in fact, m_i^2) so the heavy fermion in the loop is dominant
- In Kaon decay this gives m_c^2/m_W^2 extra suppression. Numerically not important for b decays
- CKM unitarity and tree level Z exchange are related. (Is the diagram divergent?)

Meson mixing

Two level system

Two level system in QM. $|1\rangle$ and $|2\rangle$ are energy E.S.

$$|f_1\rangle = \frac{|1\rangle + |2\rangle}{\sqrt{2}}, \qquad |f_2\rangle = \frac{|1\rangle - |2\rangle}{\sqrt{2}},$$

The time evolution

$$|f_1\rangle(t) = \exp\left[i\Delta E t/2\right]|1\rangle + \exp\left[-i\Delta E t/2\right]|2\rangle$$

The probability to measure flavor f_i at time t is

$$|\langle f_1|f_1\rangle|^2 = \frac{1+\cos\Delta E t}{2} \qquad |\langle f_1|f_2\rangle|^2 = \frac{1-\cos\Delta E t}{2}$$

- Oscillations with frequency ΔE
- The relevant parameter is $x \equiv \Delta E t$

Meson mixing

For relativistic case

- $E \to m$. Roughly,

$$K_{S,L} = \frac{K \pm \bar{K}}{\sqrt{2}}$$

- "Measurement" is done by the decay

The probability to measure flavor f_i at time t is

$$|\langle f_1|f_1\rangle|^2 = \frac{1+\cos\Delta mt}{2} \qquad |\langle f_1|f_2\rangle|^2 = \frac{1-\cos\Delta mt}{2}$$

- Oscillations with frequency Δm
- The relevant time scale is $x \equiv \Delta m/\Gamma$

Calculations of Δm

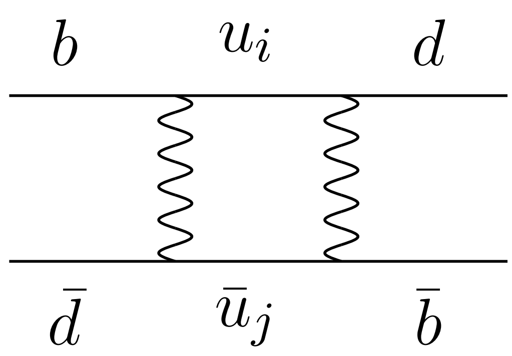
- There are 4 neutral mesons: $K(\bar{s}d)$, $B(\bar{b}d)$, $B_s(\bar{b}s)$, $D(c\bar{u})$
  - Why not charged mesons?
  - Why not the neutral pion?
  - Why not the K^*
  - Why not T mesons?
- The two flavor eigenstate B and $\bar{B}$ mix via the weak interactions. It is an FCNC process $m_{weak} = A(B \to \bar{B})$
- In the SM it is a loop process, and it gives an effect that is much smaller than the mass

$$M = \begin{pmatrix} m_B & m_{weak} \\ m_{weak} & m_B \end{pmatrix} \quad \Rightarrow \quad M_{H,L} = m_B \pm m_{weak}/2$$

$$\Rightarrow \quad \Delta M = m_{weak}$$

The box diagram

- In the SM the mixing is giving by the box diagram



- The result is

$$\Delta M \propto \sum_{i,j} V_{is} V_{id}^* V_{js} V_{jd}^* f(m_i, m_j)$$

- To leading order $f \sim m_i^2/m_W^2$ so for K mixing m_c^2/m_W^2 suppression

Meson mixing: remarks

- Mixing can be used to determine magnitude of CKM elements. The heavy fermion is the dominant one. For example B mixing is used to get $|V_{td}|$
- There are still hadronic uncertainties. We calculate at the quark level and we need the meson. Lattice QCD is very useful here
- My treatment was very simplistic, there are more effects
- Each meson has its own set of approximations

Meson mixing

In general we have also width difference between the two eigenstates. They are due to common final states.

$$x \equiv \frac{\Delta m}{\Gamma} \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma}$$

K	$x \sim 1$	$y \sim 1$
D	$x \sim 10^{-2}$	$y \sim 10^{-2}$
B_d	$x \sim 1$	$y \sim 10^{-2}$
B_s	$x \sim 10$	$y \sim 10^{-1}$

Mixing measurements

How this is done?

- Need the flavor of the initial state. Usually the mesons are pair produced
  - Same side tagging ($D^* \to D\pi$)
  - Other side tagging (semileptonic B decays)
- The final flavor
  - Use time dependent (easier for highly boosted mesons)
  - Use time integrated signals
  - The final state may not be a flavor eigenstate, but we still can have oscillations as long as it is not a mass eigenstate

CPV

What is CP

- A symmetry between a particle and its anti-particle
- CP is violated if we have

$$\Gamma(A \to B) \neq \Gamma(\bar{A} \to \bar{B})$$

- It is a very small effect in Nature, and thus sensitive to NP
- In the SM it is closely related to flavor
- We do not discuss the strong CP problem that is not directly related to flavor
- We also do not discuss the need for CP for baryogenesis

How to find CPV

It is not easy to detect CPV

- Always need interference of two (or more) amplitudes
- CPT implies that the total widths of a particles and it anti-particles are the same, so we need at least two modes with CPV
- To see CPV we need 2 amplitudes with different weak and strong phases

All these phases

- Weak phase (CP-odd phase)
  - Phase in $\mathcal{L}$
  - In the SM they are only in the weak part so they are called weak phases

$$CP(Ae^{i\phi}) = Ae^{-i\phi}$$

Strong phase

- Strong phase (CP-even phase). Do not change under CP

$$CP(Ae^{i\delta}) = Ae^{i\delta}$$

  - Due to time evolution

$$\psi(t) = e^{iHt}\psi(0)$$

  - They are also due to intermediate real states, and have to do with “rescattering” of hadrons
  - Such strong phases are very hard to calculate

Why we need the two phases?

Intuitive argument

- If we have only one $|A|^2 = |\bar{A}|^2$
- Two but with a different of only weak phase

$$\left|A + be^{i\phi}\right|^2 = \left|A + be^{-i\phi}\right|^2$$

- When both are not zero it is not the same (do it for HW!)

CPV remarks

- The basic idea is to find processes where we can measure CPV
- In some cases they are clean so we get sensitivity to the phases of the UT (or of the CKM matrix)
- We can be sensitive to the CP phase without measuring CP violation
- Triple products and EDMs are also probes of CPV. I will not talk about that
- So far CPV was only found in meson decays, K_L, B_d and $B^{\pm}$, and we will concentrate on that

The three classes of CPV

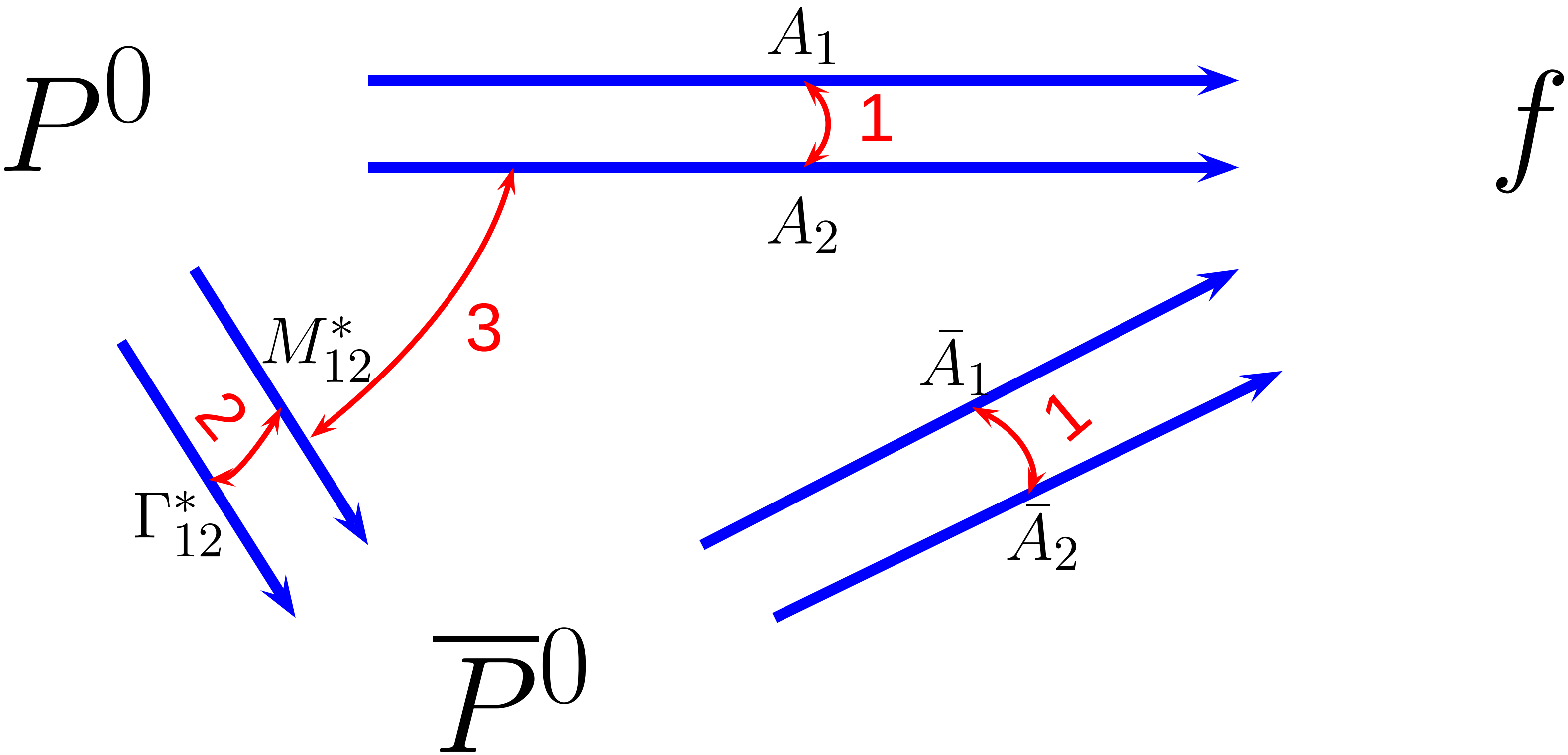
We need to find processes where we have two interfering amplitudes

- Two decay amplitudes
- Two oscillation amplitudes
- One decay and one oscillation amplitudes

Where the phases are coming from?

- Weak phases from the decay or mixing amplitudes (SM or NP)
- Strong phase is the time evolution (mixing) or the rescattering (decay)

The 3 classes



- 1: Decay   2: Mixing   3: Mixing and decay

Type 1: CPV in decay

Two decay amplitudes

$$|A(B \to f)| \neq |A(\bar{B} \to \bar{f})|$$

- The way to measure it is via

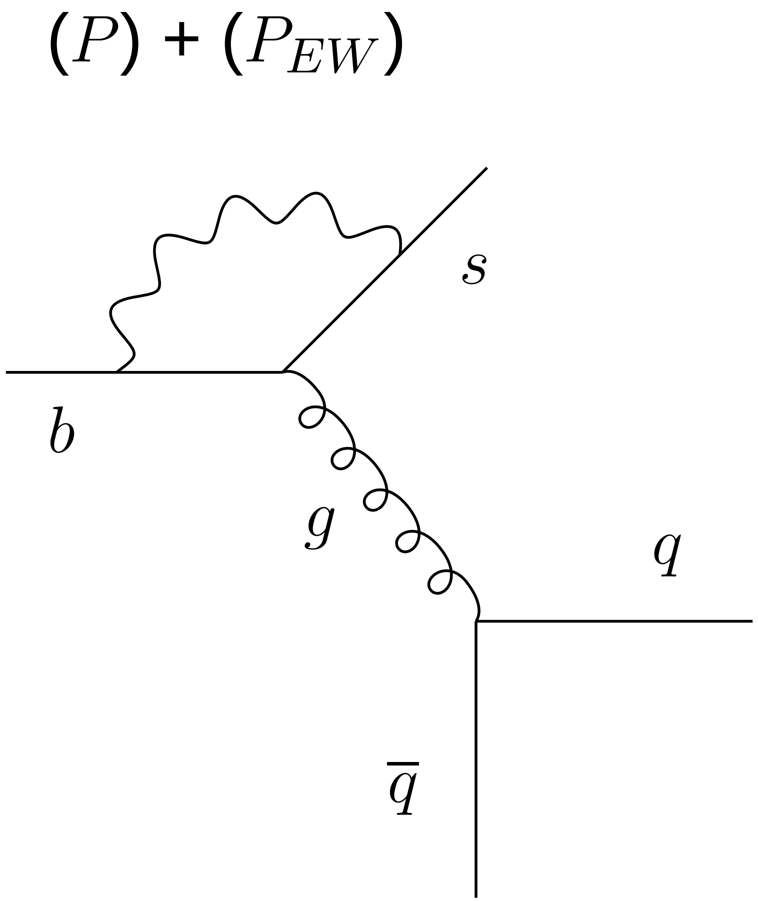
$$a_{CP} \equiv \frac{\Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f)}{\Gamma(\bar{B} \to \bar{f}) + \Gamma(B \to f)} = \frac{|\bar{A}/A|^2 - 1}{|\bar{A}/A|^2 + 1}$$

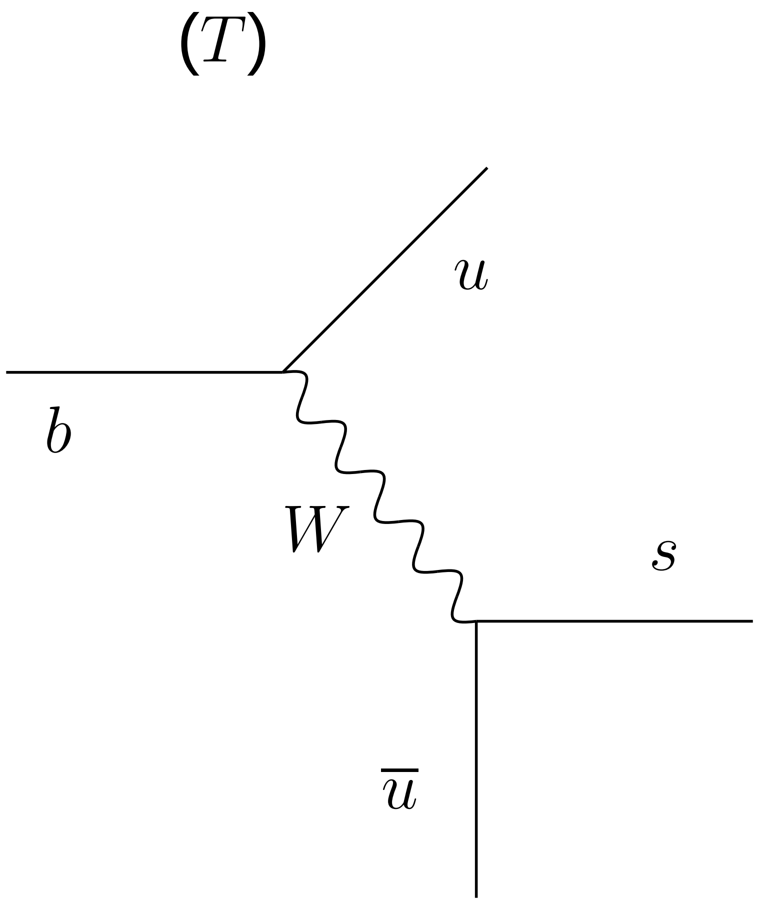
- If we write $A = A\left(1 + r \exp[i(\phi + \delta)]\right)$

$$a_{CP} = r \sin\phi \sin\delta$$

- We like r, δ and ϕ to be large
- Work for decays of both charged and neutral hadrons

CPV in decay, example: $B \to K\pi$





P is a loop amplitude, but due to CKM factors $P/T \sim 3$. We also have a strong phase difference

One more example: $B \to DK$

- A bit more “sophisticated” example of CPV in decay
- Theoretically by far the cleanest measurement of any CKM parameter

Mixing formalism with CPV

When there is CPV the mixing formalism is more complicated. Diagonalizing the Hamiltonian we get

$$B_{H,L} = p|B\rangle \pm q|\bar{B}\rangle$$

- In general B_H and B_L are not orthogonal
- This is because they are "resonances" not asymptotic states. Open system
- The condition for the non orthogonality is CPV

2: CPV in mixing

The second kind of CPV is when it is pure in the mixing

$$|q| \neq |p| \qquad (B_{H,L} = p|B\rangle \pm q|\bar{B}\rangle)$$

We measure it by semileptonic asymmetries

- It was measured in

$$\frac{\Gamma(K_L \to \pi\ell^+\nu) - \Gamma(K_L \to \pi\ell^-\bar{\nu})}{\Gamma(K_L \to \pi\ell^+\nu) + \Gamma(K_L \to \pi\ell^-\bar{\nu})} = (3.32 \pm 0.06) \times 10^{-3}$$

- This is so far the only way we can define the electron microscopically!

3: CPV in interference mixing & decay

Interference between decay and mixing amplitudes

$$A(B \to f_{CP}) \qquad A(B \to \bar{B} \to f_{CP})$$

- Best with one decay amplitude
- Very useful when f is a CP eigenstate
- In that case $|A(B \to f_{CP})| = |A(\bar{B} \to f_{CP})|$

Some definitions

$$\lambda \equiv \frac{q}{p}\frac{\bar{A}}{A}$$

In the case of a CP final state

- $\lambda \neq \pm 1 \;\Rightarrow$ CPV
  - $|\lambda| \neq 1$ because $|A| \neq |\bar{A}|$. CPV in decay
  - $|\lambda| \neq 1$ because $|q| \neq |p|$. CPV in mixing
  - The cleanest case $|\lambda| \approx 1$ and $Im(\lambda) \neq 0$. Interference between mixing and decay
- We can have several classes at the same time
- In the clean cases we have one dominant source

Formalism

B at $t = 0$ compared to a $\bar{B}$ and let them evolve

$$a_{CP}(t) \equiv \frac{\Gamma(B(t) \to f) - \Gamma(\bar{B}(t) \to f)}{\Gamma(B(t) \to f) + \Gamma(\bar{B}(t) \to f)}$$

Consider the case where $|\lambda| = 1$

$$A_{CP}(t) = -Im\lambda \sin \Delta m t$$

- We know Δm so we can measure $Im\lambda$
- $Im\lambda$ is the phase between mixing and decay amplitudes
- When we have only one dominant decay amplitude all the hadronic physics cancel (YES!!!)
- In some cases this phase is $O(1)$

Example: $B \to \psi K_S$

Reminder ψ is a $\bar{c}c$, K_S is s and d

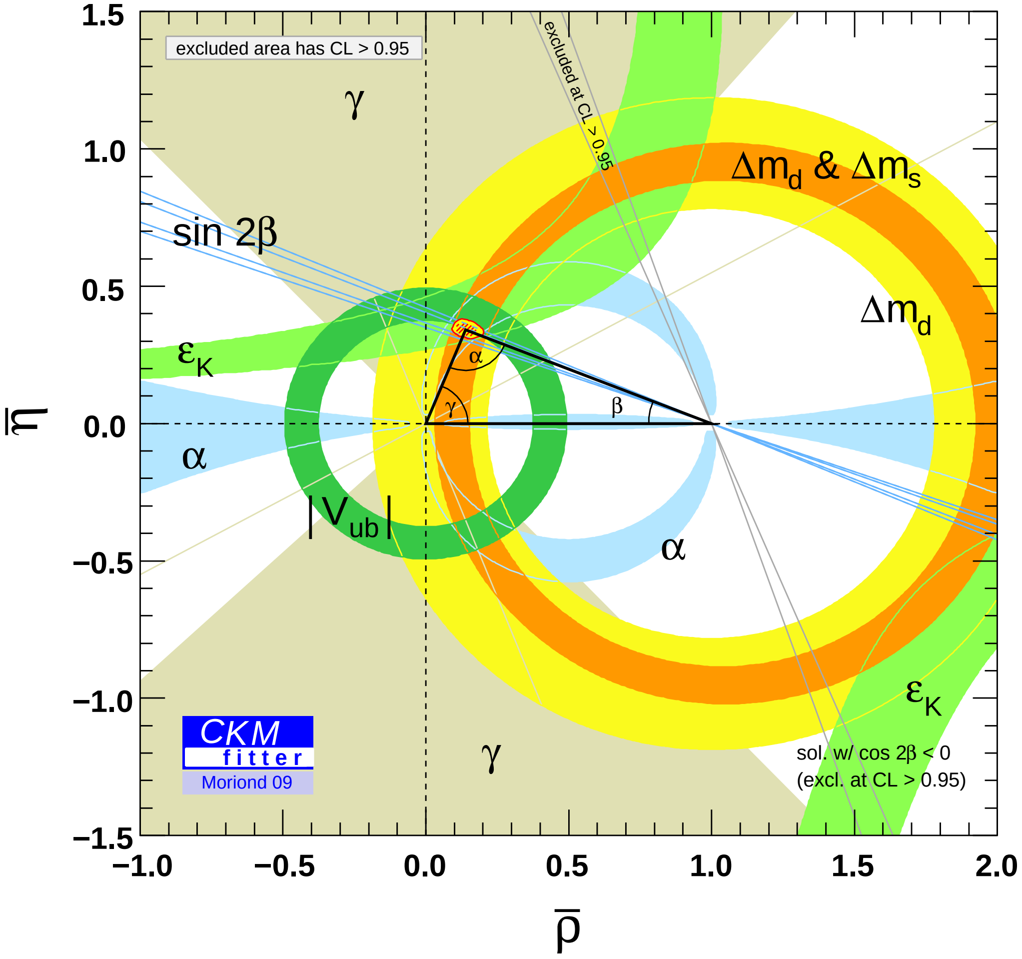
- One decay amplitude, tree level $A \propto V_{cb}V_{cs}^*$. In the standard parametrization it is real
- Very important: $|A| = |\bar{A}|$ to a very good approximation.
- In the standard parametrization $q/p = \exp(2i\beta)$ to a very good approximation
- We then get

$$Im\lambda = Im\left[\frac{q}{p}\frac{\bar{A}}{A}\right] = \sin 2\beta$$

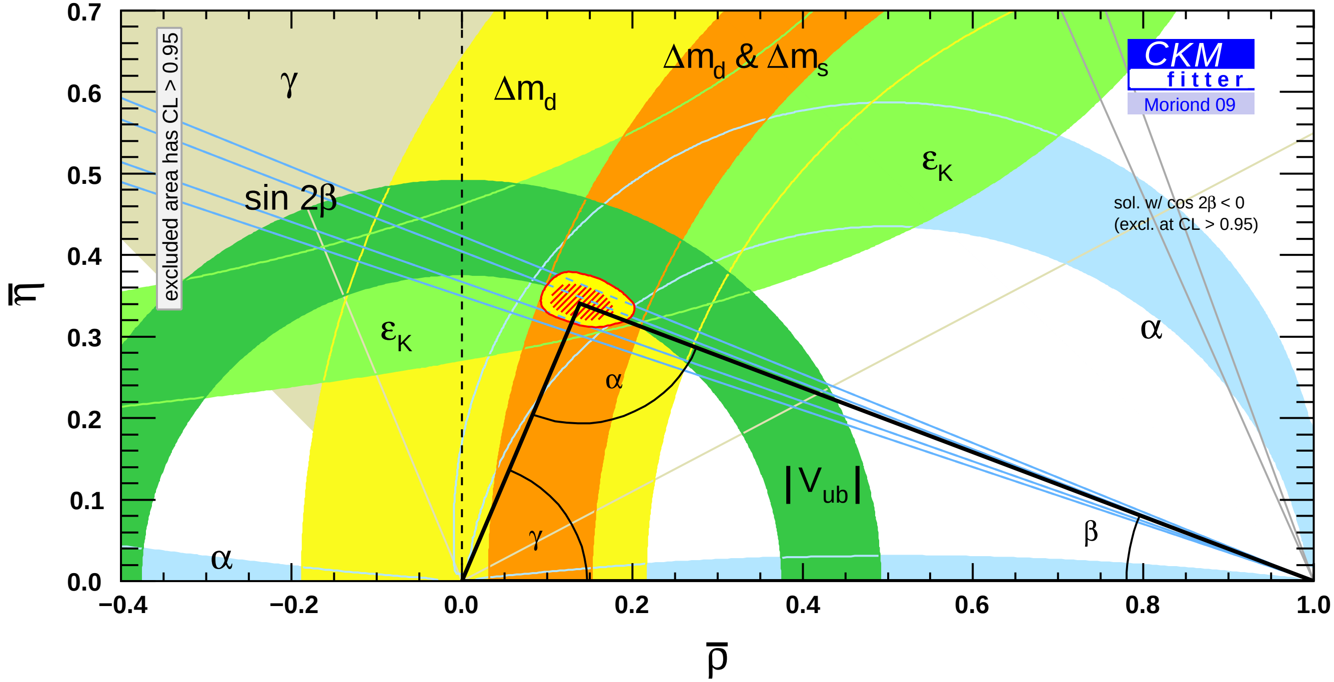
- For HW do some other decays: D^+D^-, $\pi^+\pi^-$, ϕK_S and $B_s \to \psi\phi$ (Ignore the subtleties)

Instead of summary

All together now



Zoom in



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$\bar{\eta}$
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excluded area has CL > 0.95
γ
Δm_d
Δm_d & Δm_s
CKM fitter
Moriond 09
sin 2β
ε_K
ε_K
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
α
α
γ
β
$|V_{ub}|$

The NP flavor problem

The flavor problems

- “Problem” is not a problem. It is a hint for something more fundamental
- The SM flavor problems
  - Why there are 3 generations?
  - Why the mass ratios and mixing angles are small and hierarchical?
- The NP flavor problem is different

The SM is not perfect...

- We know the SM does not describe gravity
- At what scale it breaks down?

We parametrize the NP scale as the denominator of an effective higher dimension operator. The weak scale is roughly

$$\mathcal{L}_{\rm eff} = \frac{\mu\, e \nu \bar\nu}{\Lambda_W^2} \Rightarrow \Lambda_W \sim 100~{\rm GeV}$$

- The effective scale is roughly the masses of the new fields times unknown couplings
- Flavor bounds give $\Lambda \gtrsim 10^4~{\rm TeV}$

Flavor and the hierarchy problem

There is tension:

- The hierarchy problem $\Rightarrow \; \Lambda \sim 1$ TeV
- Flavor bounds $\Rightarrow \; \Lambda \gtrsim 10^4$ TeV

Any TeV scale NP has to deal with the flavor bounds

$$\Downarrow$$

Such NP cannot have a generic flavor structure

Flavor is mainly an input to model building, not an output