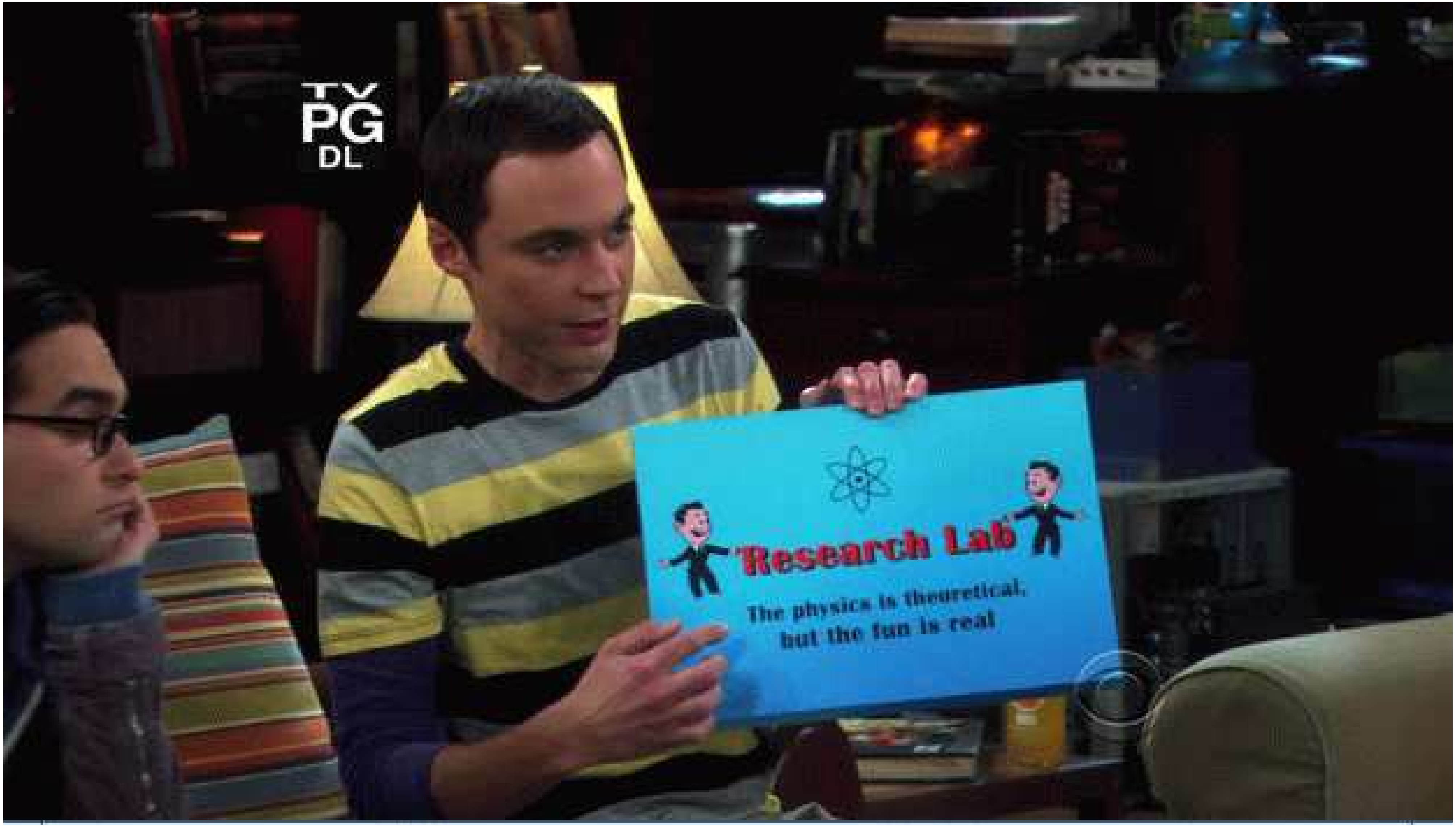


Sheldon: "Research Lab" is more than a game:
The physics is theoretical, but the fun is real!

Electroweak and Theory

(SM and MSSM)

Sven Heinemeyer, IFCA (CSIC, Santander)

St. Andrews, 08/2012

1. Higgs and Electroweak in The Standard Model (I)

2. Higgs and Electroweak in The Standard Model (II)

3. Higgs and Electroweak in the MSSM

Electroweak and Theory (I):

Higgs and Electroweak in the SM (I)

Sven Heinemeyer, IFCA (CSIC, Santander)

St. Andrews, 08/2012

1. The Standard Model and the Higgs boson
2. Top Quark physics in (B)SM
3. Electroweak Precision Observables

1. The Standard Model and the Higgs boson

Current status of knowledge: the Standard Model (SM)

1. family: quarks: d, u leptons: e^-, ν_e (neutrino)

2. family: quarks: s, c leptons: μ^-, ν_μ (neutrino)

3. family: quarks: b, t leptons: τ^-, ν_τ (neutrino)

In total:

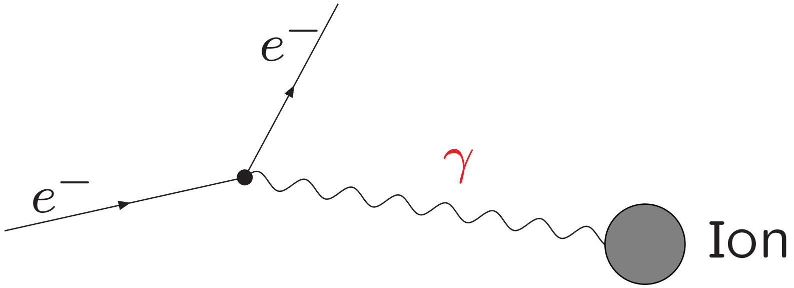
6 quarks and 6 leptons

The heavier particles (2. and 3. family) decay in very short time into the lighter particles (1. family)
Example:

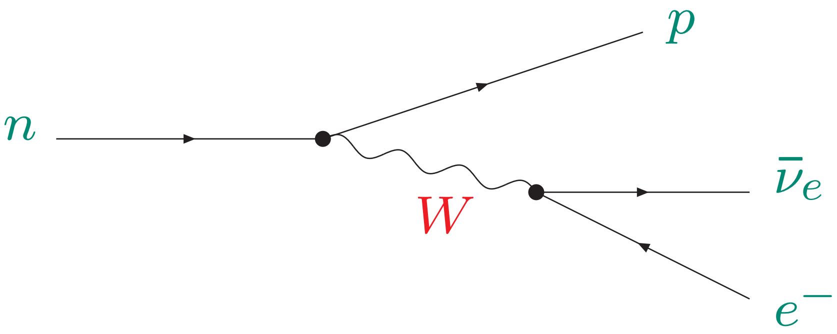
$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

Forces and force particles (I):

1. electromagnetic force: photon: γ

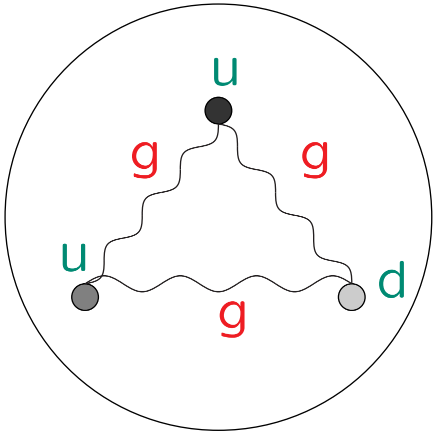


2. weak force: W^+, W^-, Z^0

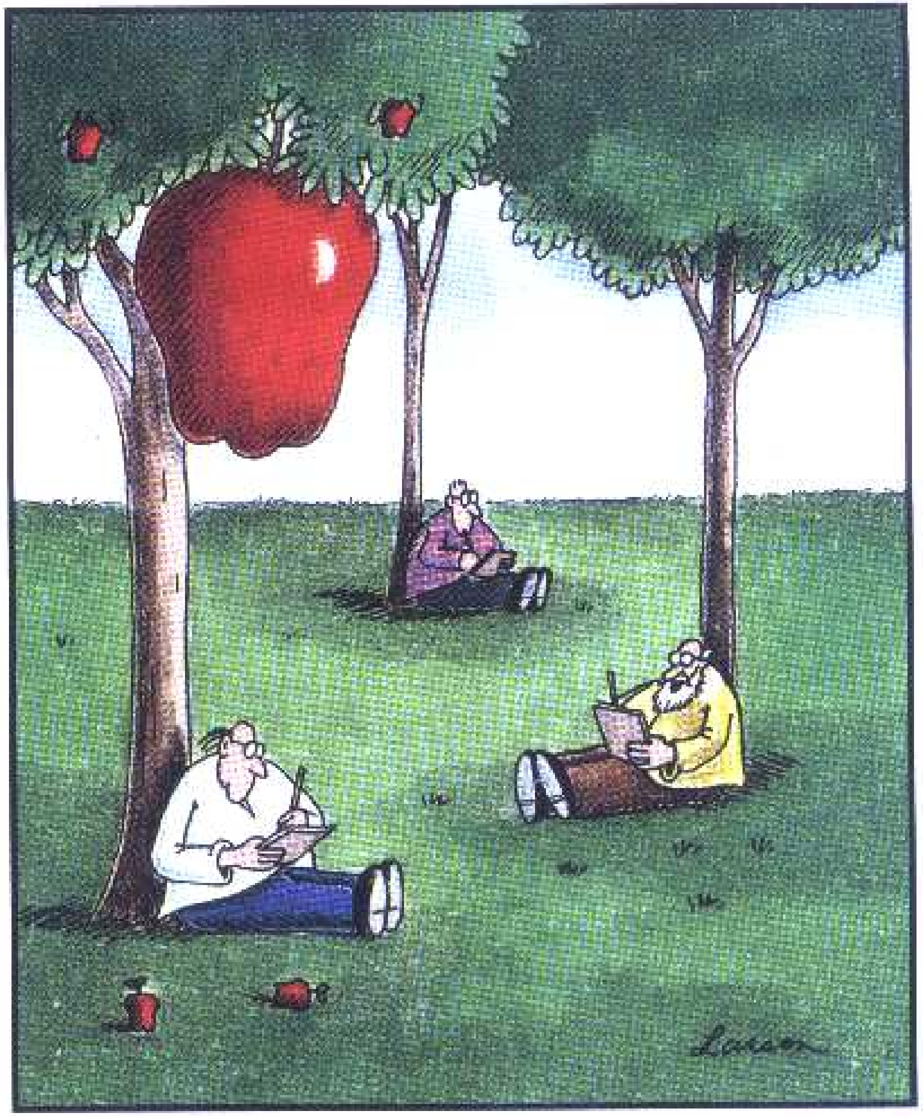


Forces and force particles (II):

3. strong force: gluon: g



4. gravitational force: graviton(?)



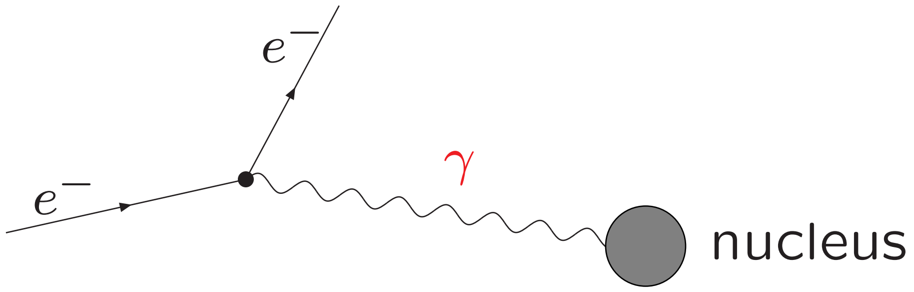
"Nothing yet. ... How about you, Newton?"

SM: Quantum field theory ⇒ interaction: exchange of field quanta

Construction principle of the SM: gauge invariance

Example: Quantum electro-dynamics (QED)

field quanta: photon A_μ



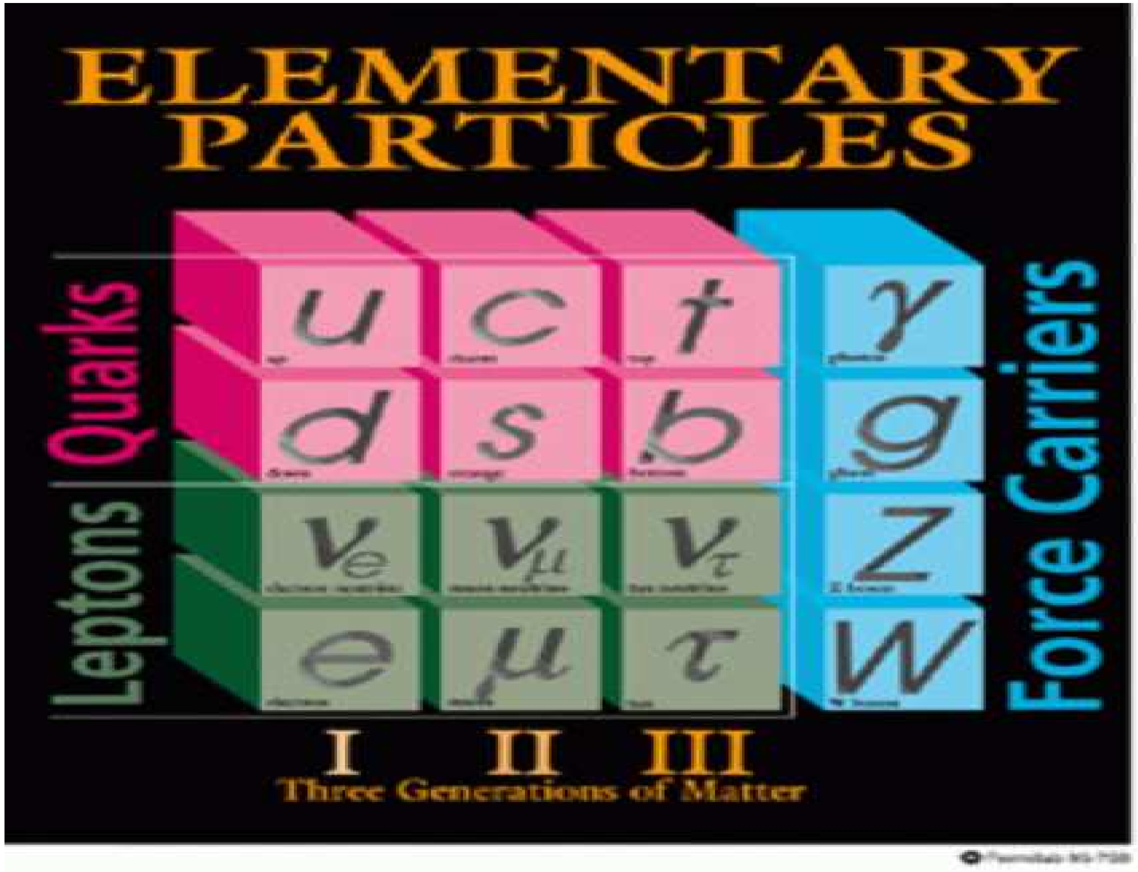
$\mathcal{L}_{\mathrm{QED}}$ invariant under gauge transformation:

$\Psi \to e^{i\,e\,\lambda(x)}\Psi, \; A_\mu \to A_\mu + \partial_\mu \lambda(x)$

mass term for photon: $m^2 A^\mu A_\mu$ not gauge invariant

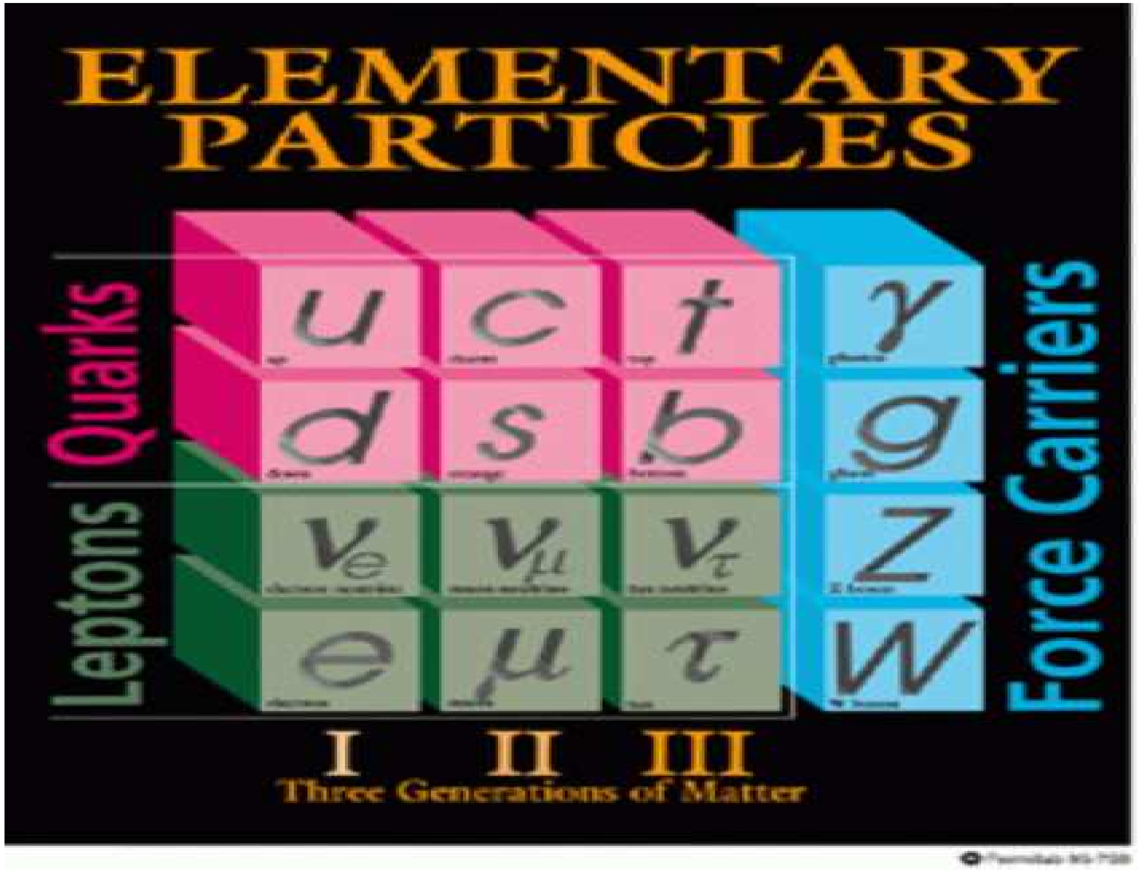
⇒ A_μ is massless gauge field

Current status of knowledge: the Standard Model (SM)



⇒ all particles experimentally seen

Current status of knowledge: the Standard Model (SM)



⇒ all particles experimentally seen

⇒ but it predicts massless gauge bosons . . .

Problem:

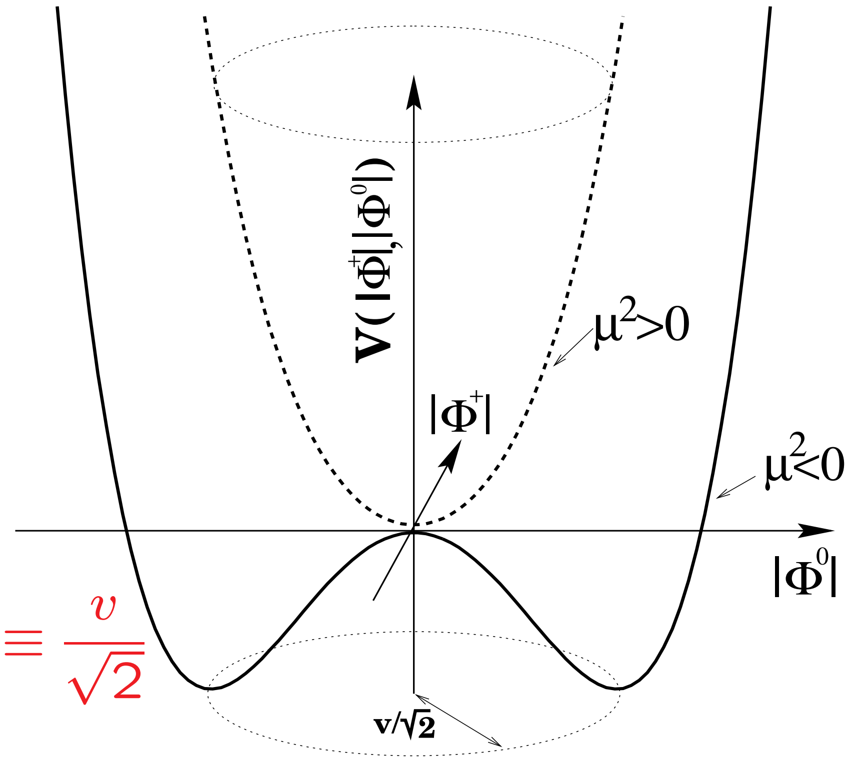
Gauge fields Z, W^+, W^- are massive

explicite mass terms in the Lagrangian $\Leftrightarrow$ breaking of gauge invariance

Solution: Higgs mechanism

scalar field postulated, mass terms from coupling to Higgs field

Higgs sector in the Standard Model:



Scalar SU(2) doublet: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

Higgs potential:

$$V(\phi) = \mu^2 \left|\Phi^\dagger \Phi\right| + \lambda \left|\Phi^\dagger \Phi\right|^2, \quad \lambda > 0$$

$\mu^2 < 0$: Spontaneous symmetry breaking

minimum of potential at $|\langle\Phi_0\rangle| = \sqrt{\dfrac{-\mu^2}{2\,\lambda}} \equiv \dfrac{v}{\sqrt{2}}$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \quad \text{(unitary gauge)}$$

H: elementary scalar field, Higgs boson

Lagrange density:

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} = \quad & (D_\mu \Phi)^\dagger (D^\mu \Phi) \\ & - g_d \bar{Q}_L \Phi d_R \; - \; g_u \bar{Q}_L \Phi_c u_R \\ & - V(\Phi) \end{aligned}$$

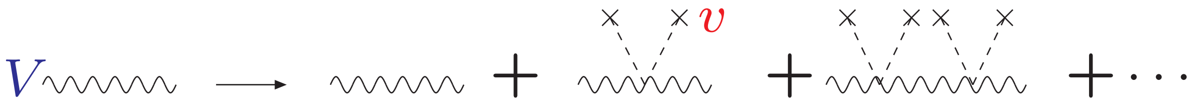
with

$$\begin{aligned} iD_\mu &= i\partial_\mu - g_2 \vec{I}\vec{W}_\mu - g_1 Y B_\mu \\ \Phi_c &= i\sigma_2 \Phi^* \qquad\qquad Q_L \sim \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \; \Phi \sim \begin{pmatrix} 0 \\ v \end{pmatrix}, \; \Phi_c \sim \begin{pmatrix} v \\ 0 \end{pmatrix} \end{aligned}$$

Gauge invariant coupling to gauge fields

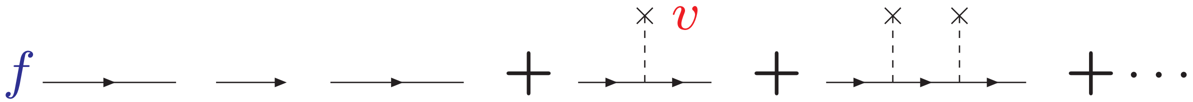
⇒ mass terms for gauge bosons and fermions

1.) $VV\Phi\Phi$ coupling:



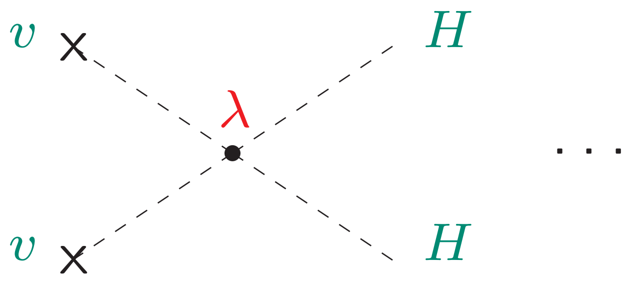
$$\frac{1}{q^2} \to \frac{1}{q^2} + \sum_j \frac{1}{q^2}\left[\left(\frac{gv}{\sqrt{2}}\right)^2 \frac{1}{q^2}\right]^j = \frac{1}{q^2 - M^2} \;:\; M^2 = g^2\frac{v^2}{2} \qquad \Rightarrow M \propto g$$

2.) fermion mass terms: Yukawa couplings:



$$\frac{1}{\not{q}} \to \frac{1}{\not{q}} + \sum_j \frac{1}{\not{q}}\left[\frac{g_f v}{\sqrt{2}}\frac{1}{\not{q}}\right]^j = \frac{1}{\not{q} - m_f} \;:\; m_f = g_f\frac{v}{\sqrt{2}} \qquad \Rightarrow m_f \propto g_f$$

3.) mass of the Higgs boson: self coupling

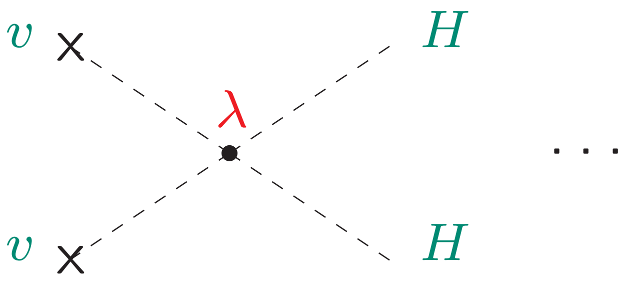


$$\lambda = M_H^2/v$$

$M_H = v\sqrt{\lambda}$ free parameter

→ last unknown(??) parameter of the SM

3.) mass of the Higgs boson: self coupling



$$\lambda = M_H^2/v$$

$M_H = v\sqrt{\lambda}$ free parameter

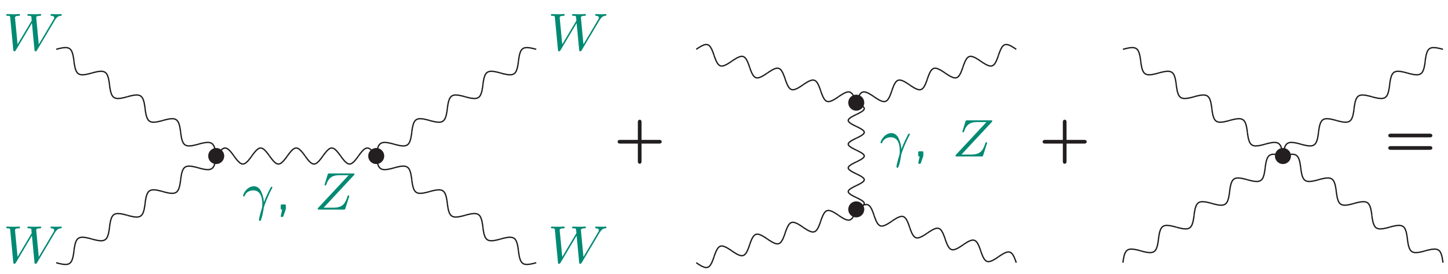
→ last unknown(??) parameter of the SM

⇒ establish Higgs mechanism ≡ find the Higgs ⊕ measure its couplings

Another effect of the Higgs field:

Scattering of longitudinal W bosons: $W_L W_L \to W_L W_L$

$\mathcal{M}_V =$

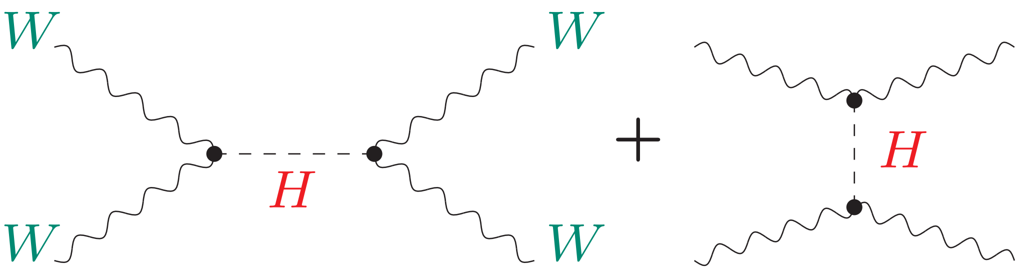


$= -g^2 \frac{E^2}{M_W^2} + \mathcal{O}(1)$ for $E \to \infty$

$\Rightarrow$ violation of unitarity

Contribution of a scalar particle with couplings prop. to the mass:

$\mathcal{M}_S =$



$= g_{WWH}^2 \frac{E^2}{M_W^4} + \mathcal{O}(1)$ for $E \to \infty$

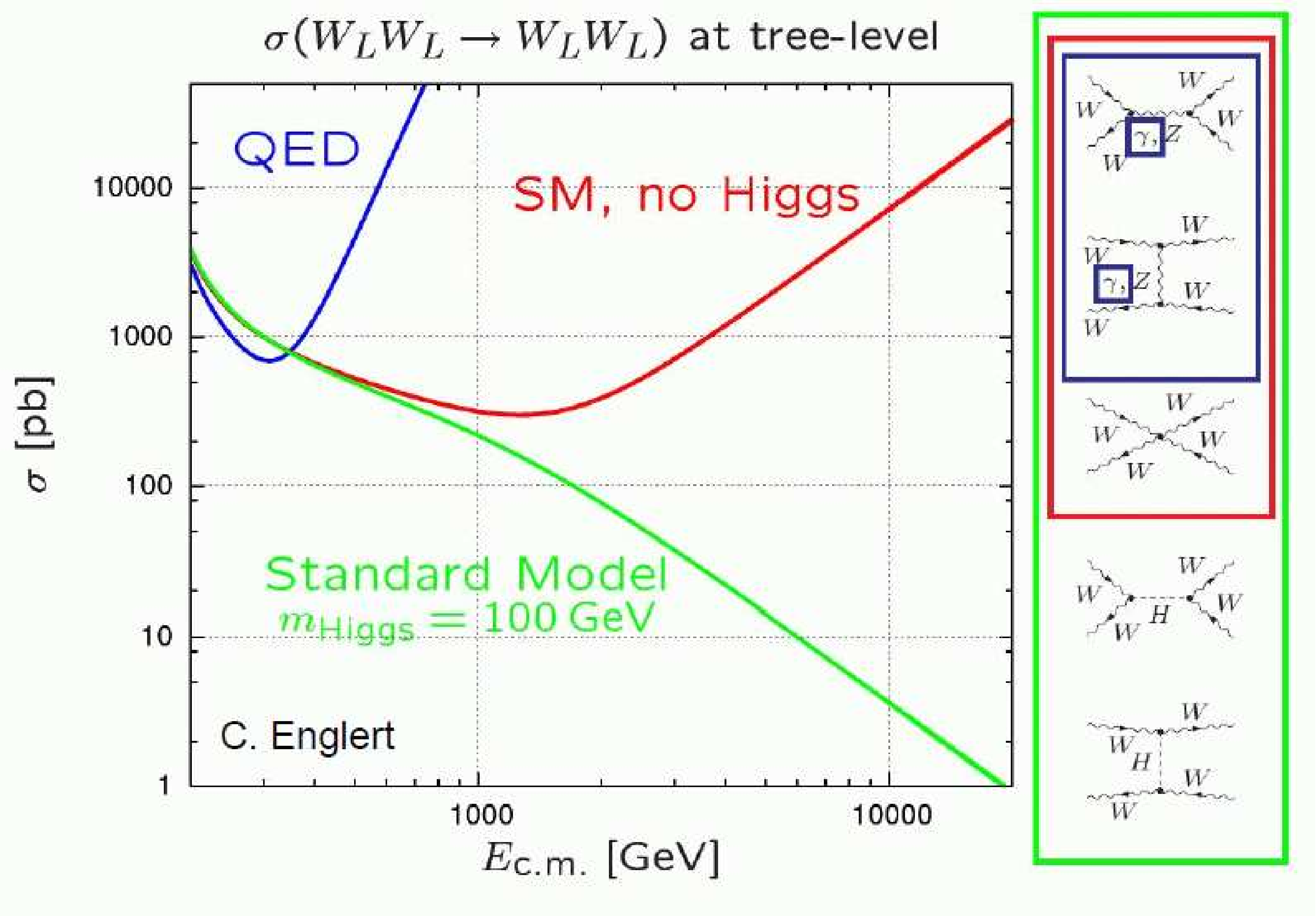
$$\mathcal{M}_{\rm tot} = \mathcal{M}_V + \mathcal{M}_S = \frac{E^2}{M_W^4}\left(g_{WWH}^2 - g^2\, M_W^2\right) + \ldots$$

$\Rightarrow$ compensation of terms with bad high-energy behavior for

$$g_{WWH} = g\, M_W$$

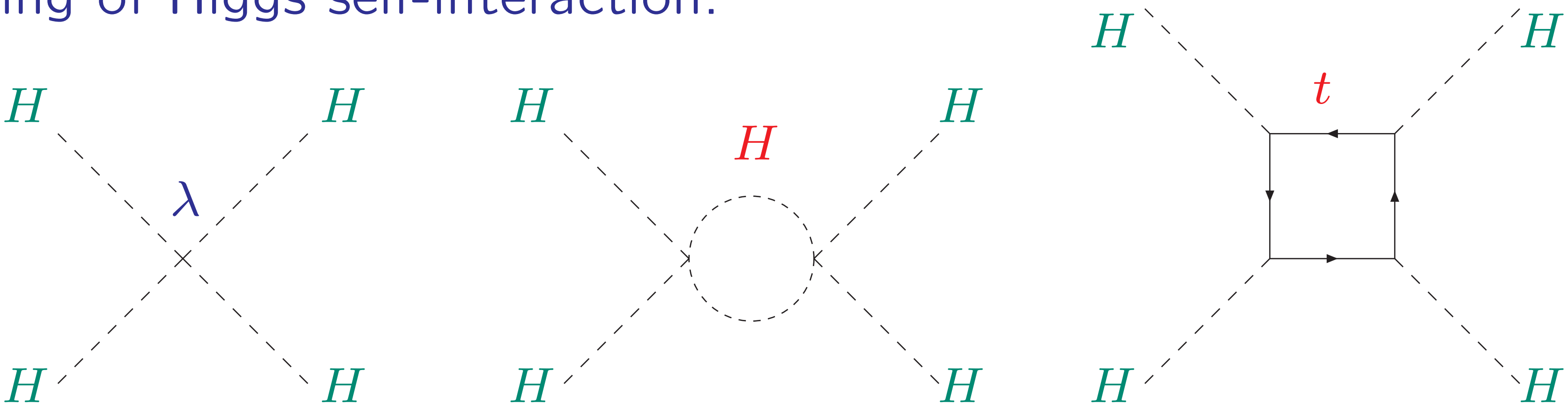
Cross section with/without the Higgs:

[*taken from M. Schumacher '12 / C. Englert*]



What else do we know about the Higgs boson?

Running of Higgs self-interaction:



Renormalization group equation:

$$\frac{d\lambda}{dt} = \frac{3}{8\pi^2}\left[\lambda^2 + \lambda g_t^2 - g_t^4 + \frac{1}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right] , \qquad t = \log\left(\frac{Q^2}{v^2}\right)$$

Two conditions:

1.) avoid Landau pole (for large $\lambda \sim M_H^2$)

2.) avoid vacuum instability (for small/negative λ)

1.) avoid Landau pole (for large $\lambda \sim M_H^2$)

$$\frac{d\,\lambda}{d\,t} = \frac{3}{8\,\pi^2}\left[\lambda^2\right]$$

$$\Rightarrow \qquad \lambda(Q^2) = \frac{\lambda(v^2)}{1 - \frac{3\lambda(v^2)}{8\,\pi^2}\log\left(\frac{Q^2}{v^2}\right)}$$

$$\lambda(\Lambda) < \infty \;\Rightarrow\; M_H^2 \le \frac{8\,\pi^2\,v^2}{3\log\left(\frac{\Lambda^2}{v^2}\right)} \quad : \text{upper bound on } M_H$$

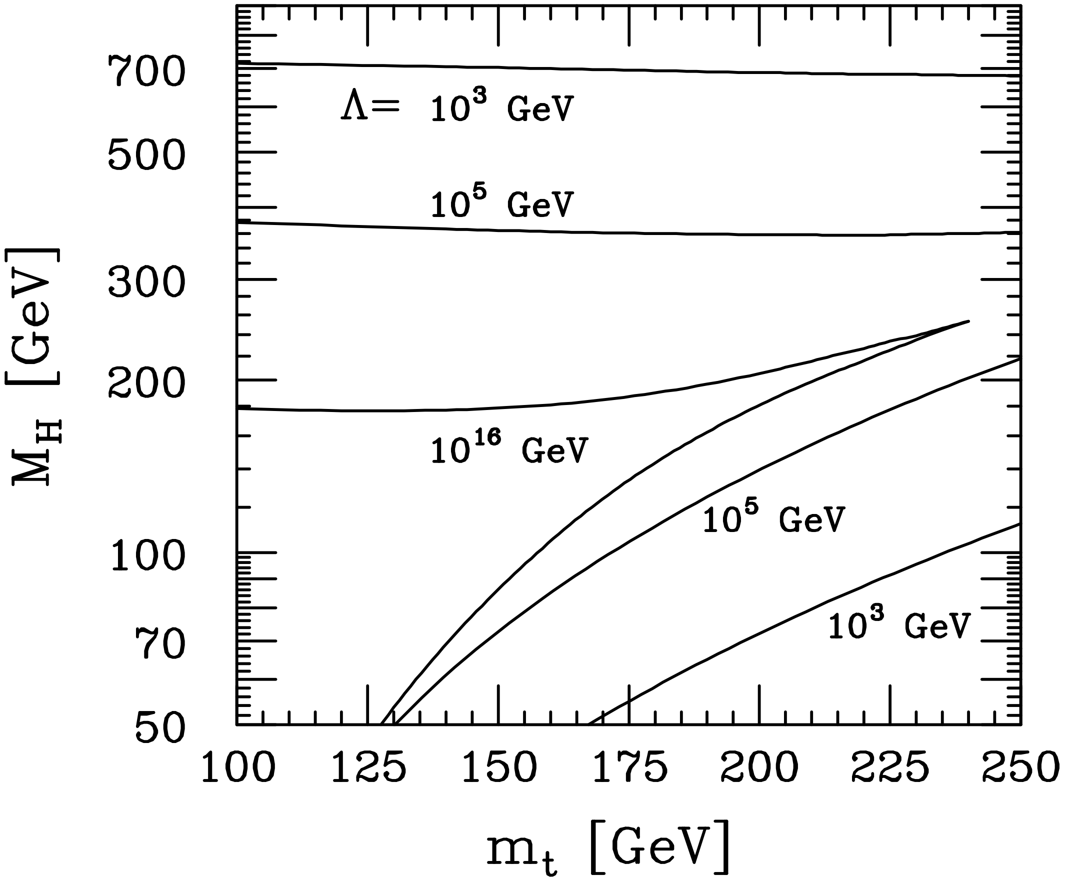
2.) avoid vacuum instability (for small/negative λ): $V(v) < V(0) \Rightarrow \lambda(\Lambda) > 0$

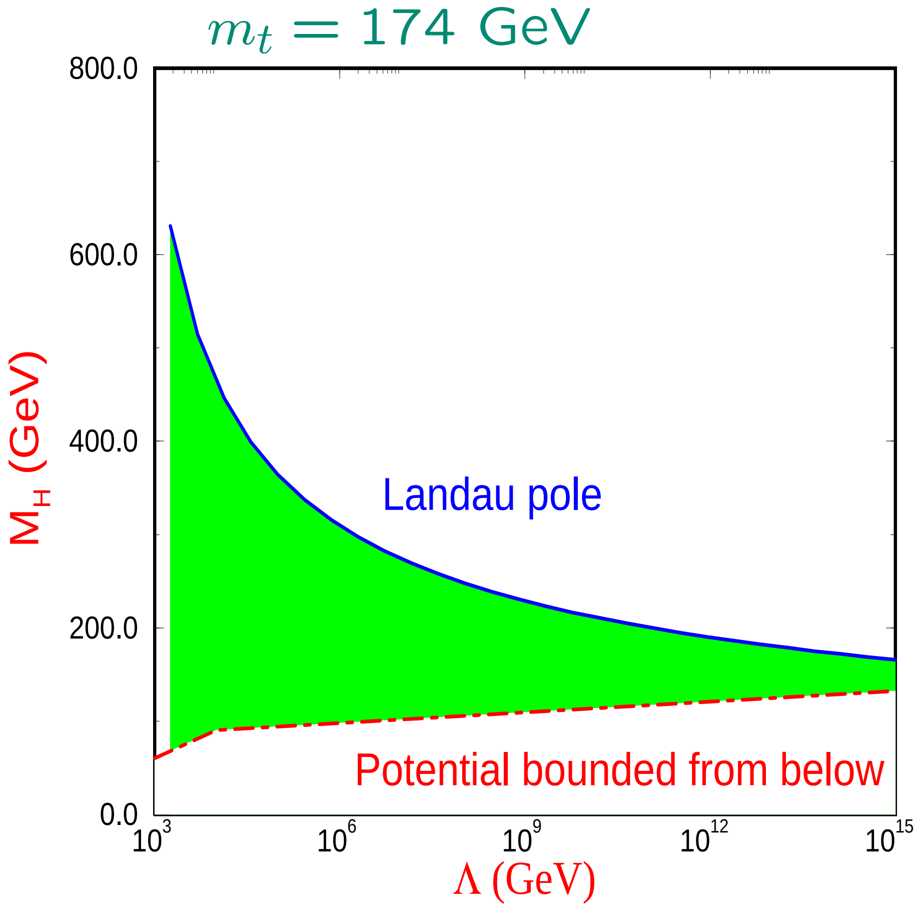
$$\frac{d\,\lambda}{d\,t} = \frac{3}{8\,\pi^2}\left[-g_t^4 + \frac{1}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$

$$\Rightarrow \qquad \lambda(Q^2) = \lambda(v^2)\frac{3}{8\,\pi^2}\left[-g_t^4 + \frac{1}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]\log\left(\frac{Q^2}{v^2}\right)$$

$$\lambda(\Lambda) > 0 \;\Rightarrow\; M_H^2 > \frac{v^2}{4\,\pi^2}\left[-g_t^4 + \frac{1}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]\log\left(\frac{\Lambda^2}{v^2}\right) : \text{lower bound}$$

Both limits combined:





Λ: scale up to which the SM is valid

$\Lambda = M_{\text{GUT}} \Rightarrow 130 \text{ GeV} \lesssim M_H \lesssim 180 \text{ GeV}$

Top quark physics in (B)SM

Top-quark mass is a fundamental parameter of the electroweak theory

By far the largest quark mass,
largest mass of all known fundamental particles

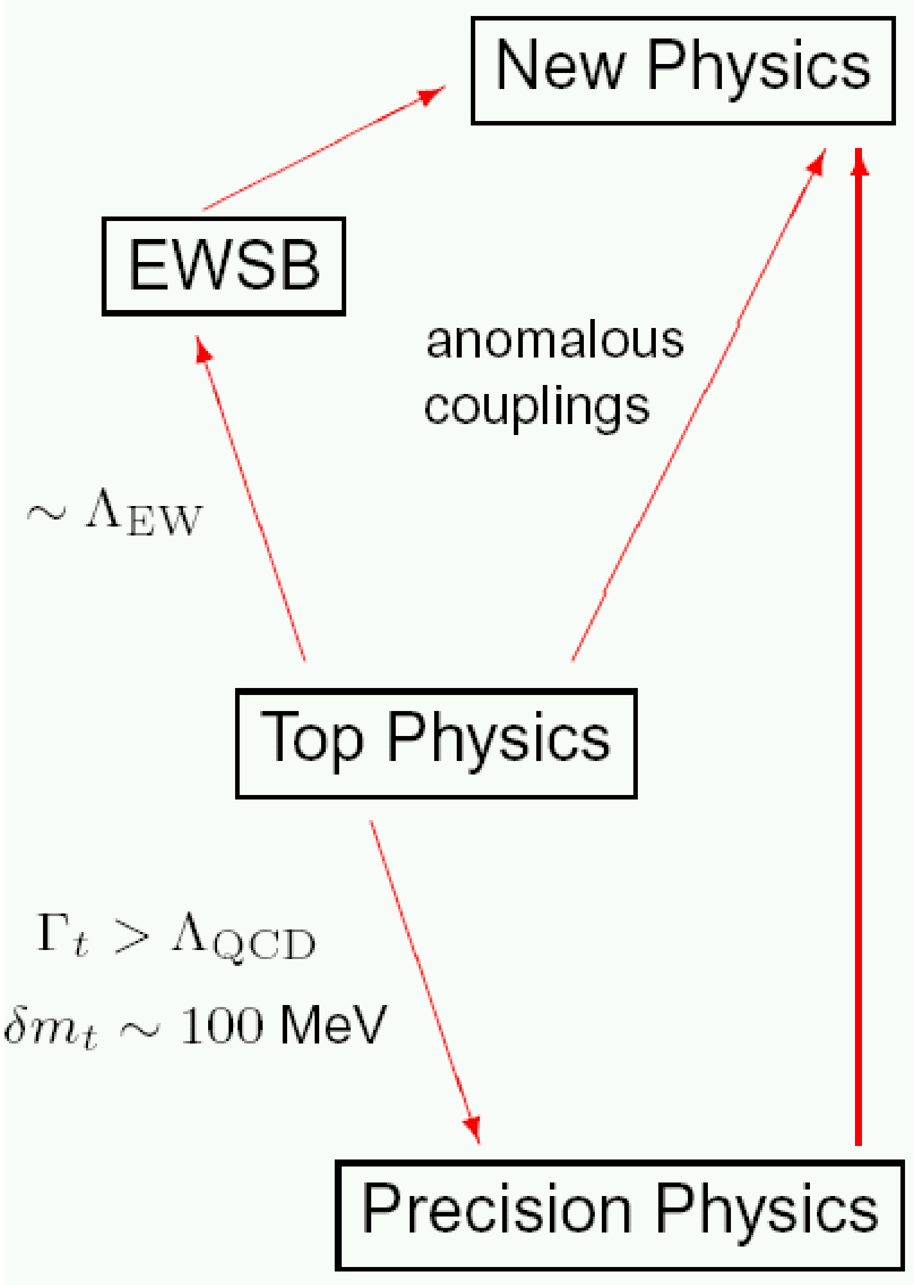
Window to new physics?

Large coupling to the Higgs boson; physics of flavor;
prediction of m_t from underlying theory?

Radiative corrections

$\Rightarrow$ non-decoupling effects proportional to powers of m_t

$\Rightarrow$ Need to know m_t very precisely in order to have sensitivity to effects of new physics



EWSB: just a heavy quark?
special role for t in EWSB?
strong constraint on any model

Precision physics:
$\delta m_t^{\rm exp}$ leading parametric uncertainty
→ could obscure new physics

SUSY: m_t crucial input parameter
drives SSB/unification

Little Higgs: heavier top

What can be done at the LHC?

What is the top mass?

Particle masses are not observables
one can only measure cross sections, decay rates, . . .

Additional problem for the top mass:

what is the mass of a colored object?

Top pole mass is not IR safe (affected by large long-distance contributions), cannot be determined to better than $\mathcal{O}(\Lambda_{\text{QCD}})$

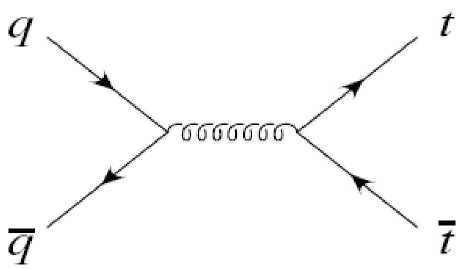
Measurement of m_t:

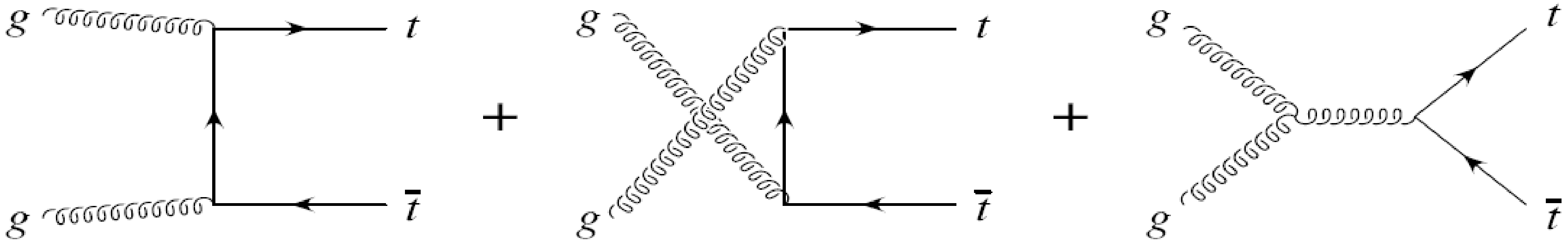
- At Tevatron, LHC:
  kinematic reconstruction, fit to invariant mass distribution
  $\Rightarrow$ “pole” mass

- At the ILC:
  mainly from threshold behavior $\Rightarrow$ threshold mass

Top quark production at the LHC:

Top production through quark antiquark annihilation and gluon gluon fusion



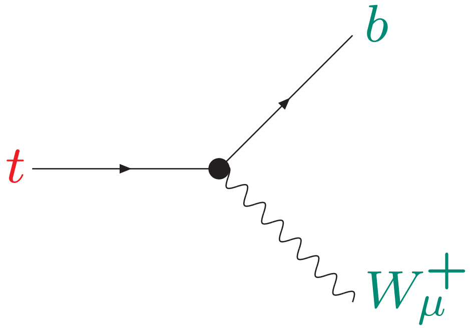


$q\bar{q} \to t\bar{t}$: 10% $\qquad$ $gg \to t\bar{t}$: 90%

$$\sigma_{\text{NLO}}^{\text{LHC}} = 830 \text{ pb } \pm 15\%$$

Top quark decays (I):

The dominant decay is $t \to W^+ b$:



$$= -i \frac{g}{2\sqrt{2}} |V_{tq}|^2 \gamma_\mu (1 - \gamma_5)$$

$$\Gamma(t \to W^+ b) = \frac{G_\mu m_t^2}{8\pi\sqrt{2}} |V_{tb}|^2 \left(1 - \frac{M_W^2}{m_t^2}\right)\left(1 - \frac{2M_W^2}{m_t^2}\right) \approx |V_{tb}|^2 \times 1.42 \text{ GeV}$$

Unitarity of CKM matrix: $|V_{tb}|^2 + |V_{cb}|^2 + |V_{ub}|^2 = 1 \Rightarrow |V_{tb}|^2 \approx 1$

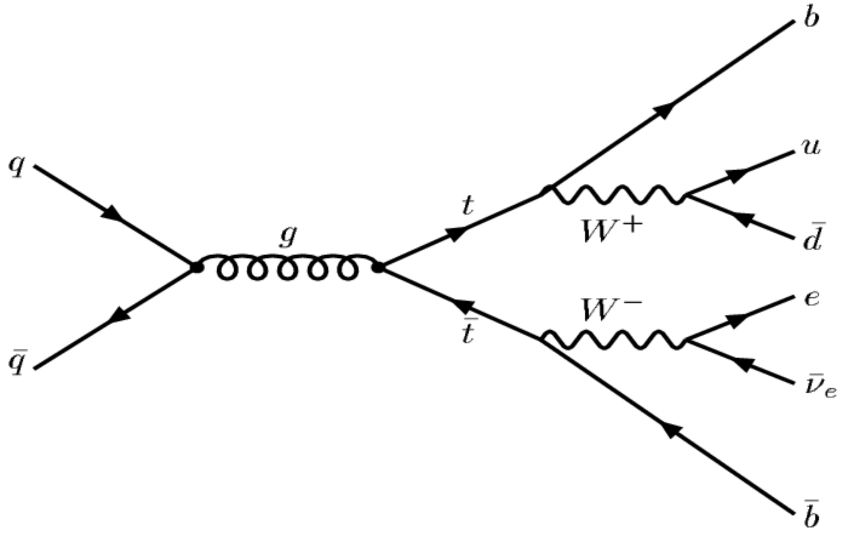
$\Rightarrow$ top quark life time $\tau_t \approx 5 \times 10^{-25}$ sec

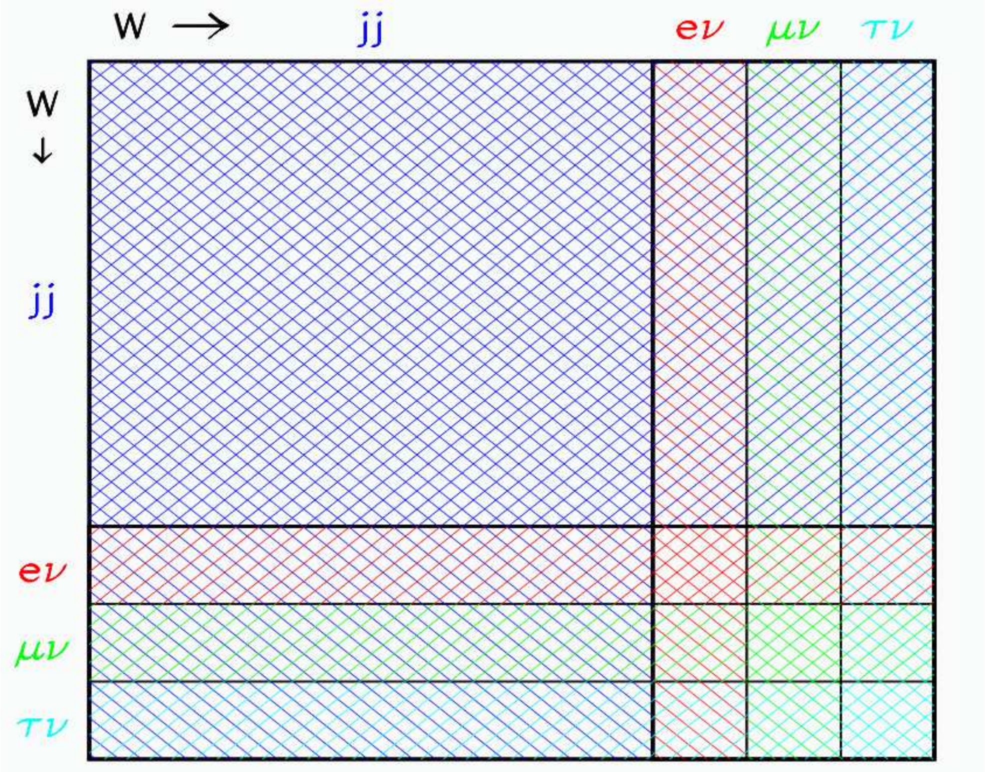
Typical QCD time scale for hadron formation: $\tau_{\text{QCD}} \approx 3 \times 10^{-24}$ sec

$\Rightarrow$ the top quark decays before it can form bound states

Top quark decays (II):

Signature depends on the WW decay modes





⇒ often semi-leptonic channels easiest

Measurement of V_{tb} (I):

Measure the ratio

$$\frac{\mathrm{BR}(t \to W^+ b)}{\mathrm{BR}(t \to W^+ q)} = \frac{|V_{tb}|^2}{|V_{tb}|^2 + |V_{cb}|^2 + |V_{ub}|^2}$$

If one assumes 3 generations then $|V_{tb}|^2 + |V_{cb}|^2 + |V_{ub}|^2 = 1$

Current Tevatron measurements:

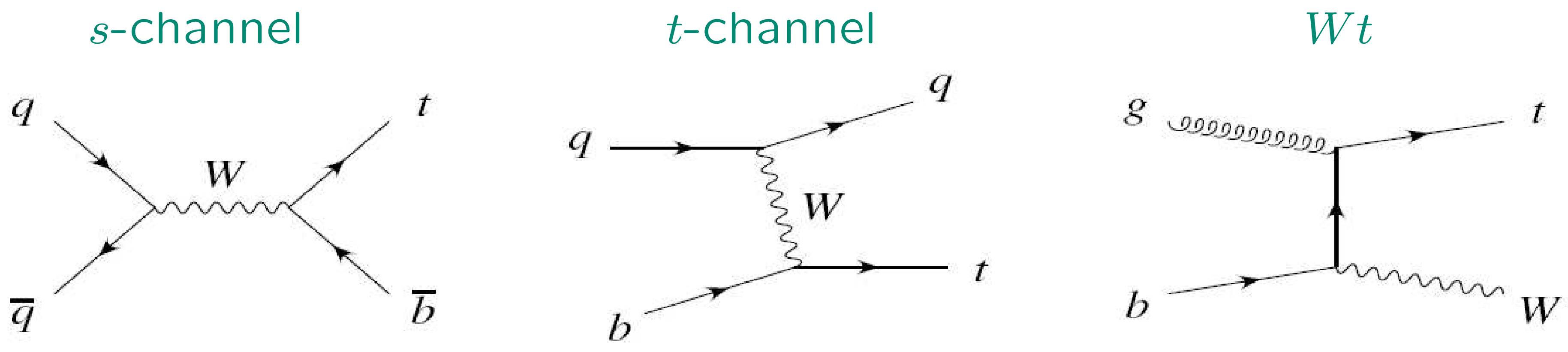
$$|V_{tb}| = \begin{cases} 1.05^{+0.10}_{-0.09} & \text{([CDF '10])} \\ 0.95^{+0.02}_{-0.02} & \text{([DØ '11])} \end{cases}$$

⇒ ATLAS and CMS are taking over now . . .

However:
assuming three generations we know $0.9990 < |V_{tb}| < 0.9993$ anyway
from unitarity of the CKM matrix

Measurement of V_{tb} (II):

Cleaner: single top production



	s-channel	t-channel	Wt
σ_t^{NLO} [pb]	~ 7	~ 153	~ 31
$\sigma_{\bar{t}}^{\text{NLO}}$ [pb]	~ 4	~ 90	~ 31

⇒ better prospects, no assumption on unitarity needed

Top quark physics at the LHC:

The top cross section is $\sigma_{tt} \approx 830$ pb

One year LHC running at low luminosity, $\sim 10\,\mathrm{fb}^{-1} \Rightarrow \mathcal{O}\left(10^7\right)$ top events

Physics goals:

- $\delta m_t = 1$ GeV with $\int \mathcal{L} = 100\,\mathrm{fb}^{-1}$
- Observation of single top production, measurement of V_{tb} with $\int \mathcal{L} = 30\,\mathrm{fb}^{-1}$
- test of quantum numbers
- measurement of rare (BSM) decay modes

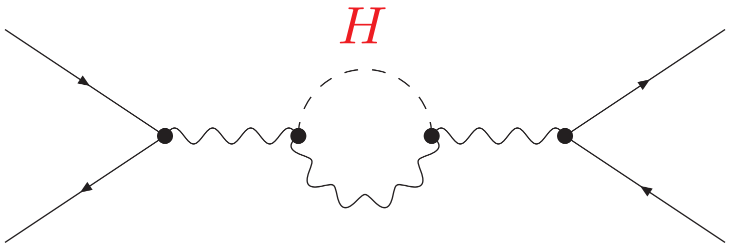
Electroweak Precision Observables (EWPO):

Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
$M_W, \sin^2\theta_{\text{eff}}, a_\mu$	$\leftrightarrow$	SM, MSSM , . . .

$\Downarrow$

Test of theory at quantum level: Sensitivity to loop corrections, e.g. H



$\Downarrow$

SM: limits on M_H

Very high accuracy of measurements and theoretical predictions needed

Example: prediction of M_W, $\sin^2\theta_{\text{eff}}$

A) Theoretical prediction for M_W in terms

of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}(1 + \Delta r)$$

$\Updownarrow$

loop corrections

Evaluate Δr from μ decay $\Rightarrow M_W$

One-loop result for M_W in the SM:

[*A. Sirlin '80*] , [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\text{loop}} = \quad \Delta\alpha \quad - \quad \frac{c_W^2}{s_W^2}\Delta\rho \quad + \quad \Delta r_{\text{rem}}(M_H)$$

$$\sim \log\frac{M_Z}{m_f} \qquad \sim m_t^2 \qquad \log(M_H/M_W)$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

Example: prediction of M_W, $\sin^2\theta_{\rm eff}$

A) Theoretical prediction for M_W in terms

of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}\,(1+\Delta r)$$

$\Updownarrow$

loop corrections

B) Effective mixing angle:

$$\sin^2\theta_{\rm eff}=\frac{1}{4\,|Q_f|}\left(1-\frac{{\rm Re}\,g_V^f}{{\rm Re}\,g_A^f}\right)$$

Higher order contributions:

$$g_V^f \to g_V^f+\Delta g_V^f, \quad g_A^f \to g_A^f+\Delta g_A^f$$

Comparison of SM prediction of M_W with direct measurements:

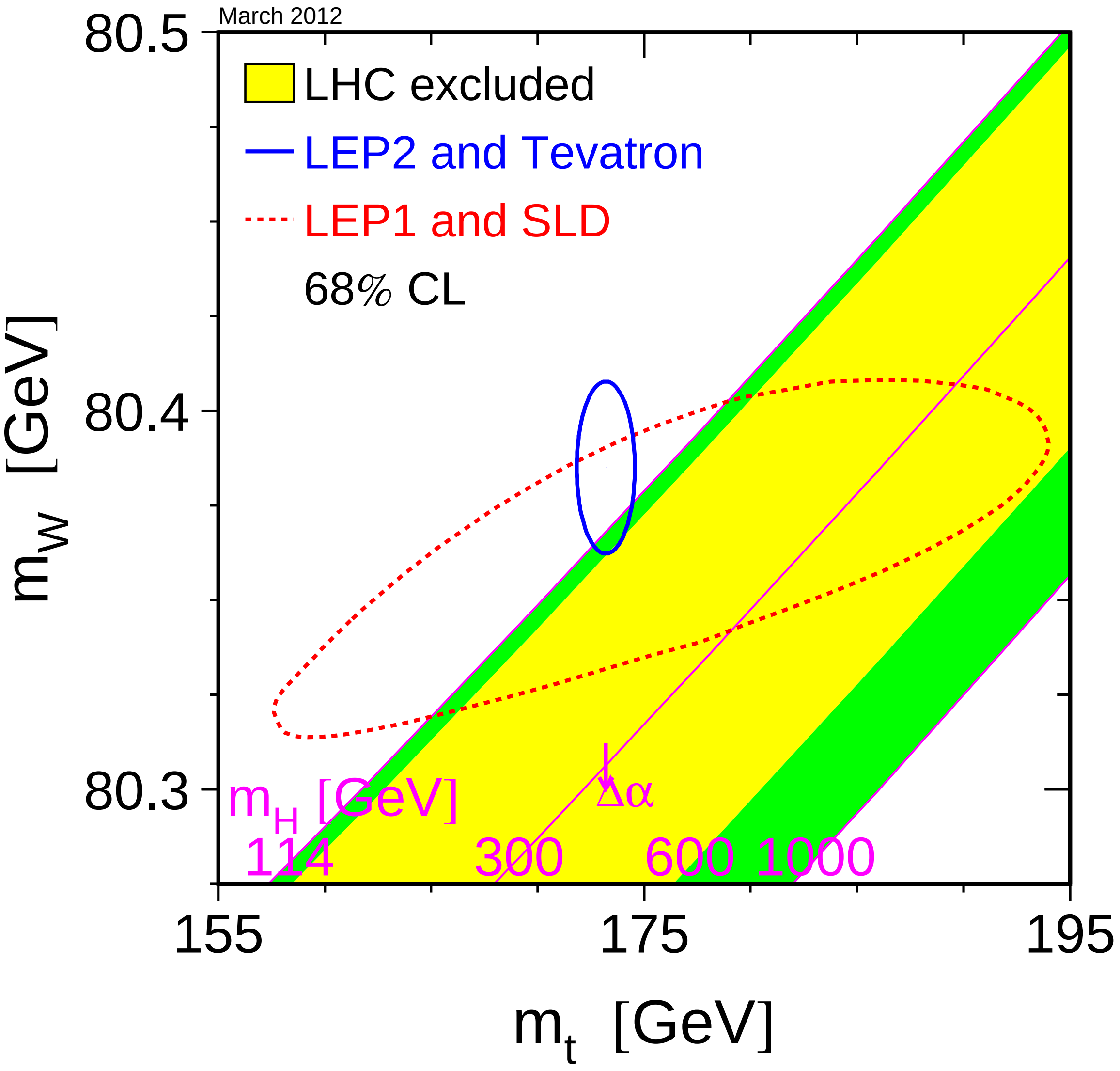
$$\Delta r = -\frac{11 g_2^2}{96\,\pi^2}\frac{s_W^2}{c_W^2}\log\left(\frac{M_H}{M_W}\right)$$

general for EWPO:

$$\Delta \sim g_2^2\left[\log\left(\frac{M_H}{M_W}\right) + g_2^2\frac{M_H^2}{M_W^2}\right]$$

leading term: $\log(M_H)$

first term $\sim M_H^2$ with g_2^4



⇒ light Higgs boson preferred

[*LEPEWWG '12*]

Results for M_H from other EWPO:

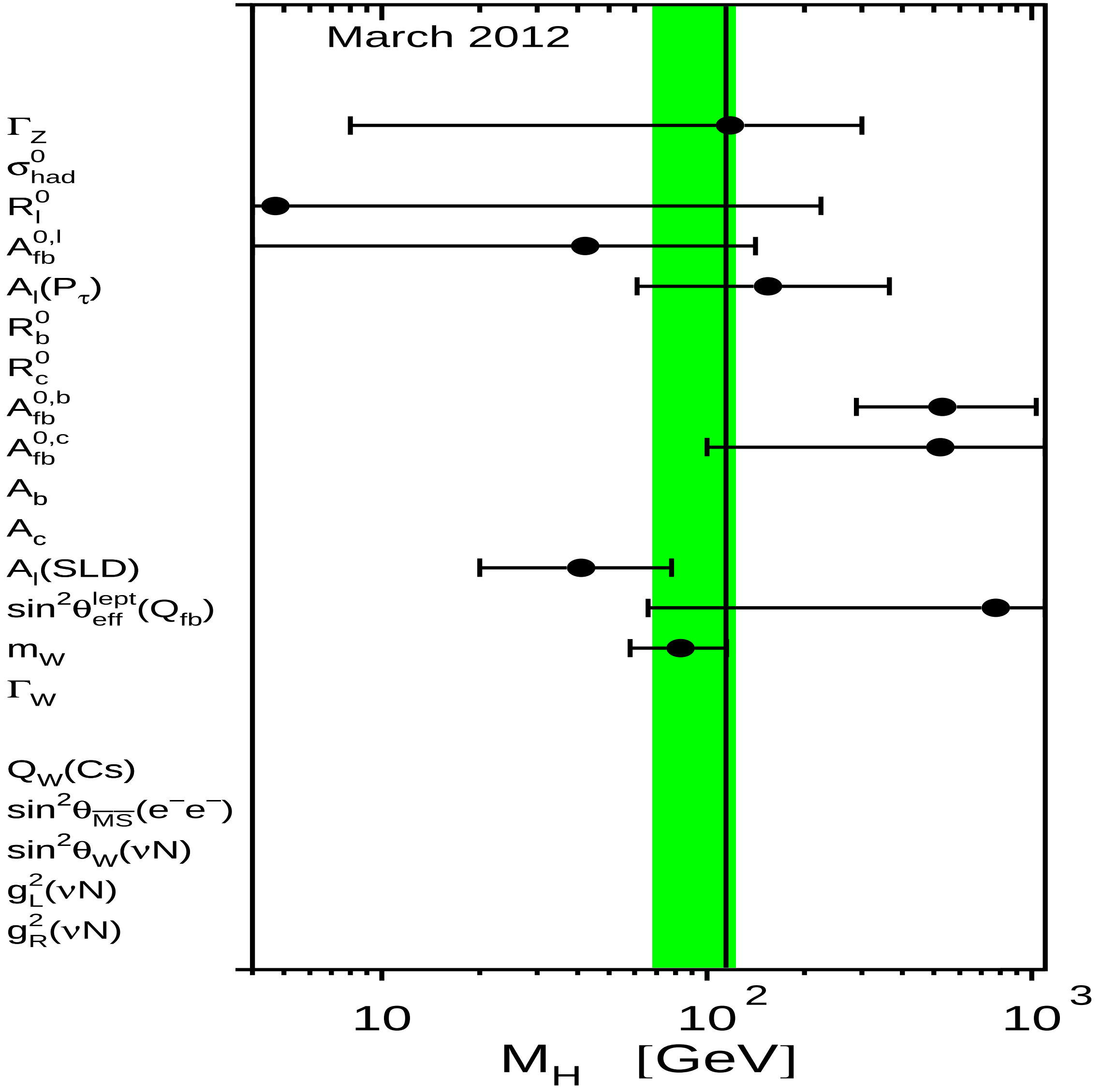
light Higgs preferred by:

M_W, A_l^{LR} (SLD)

heavier Higgs preferred by:

A_b^{FB} (LEP)

$\Rightarrow$ keeps SM alive



[LEPEWWG '12]

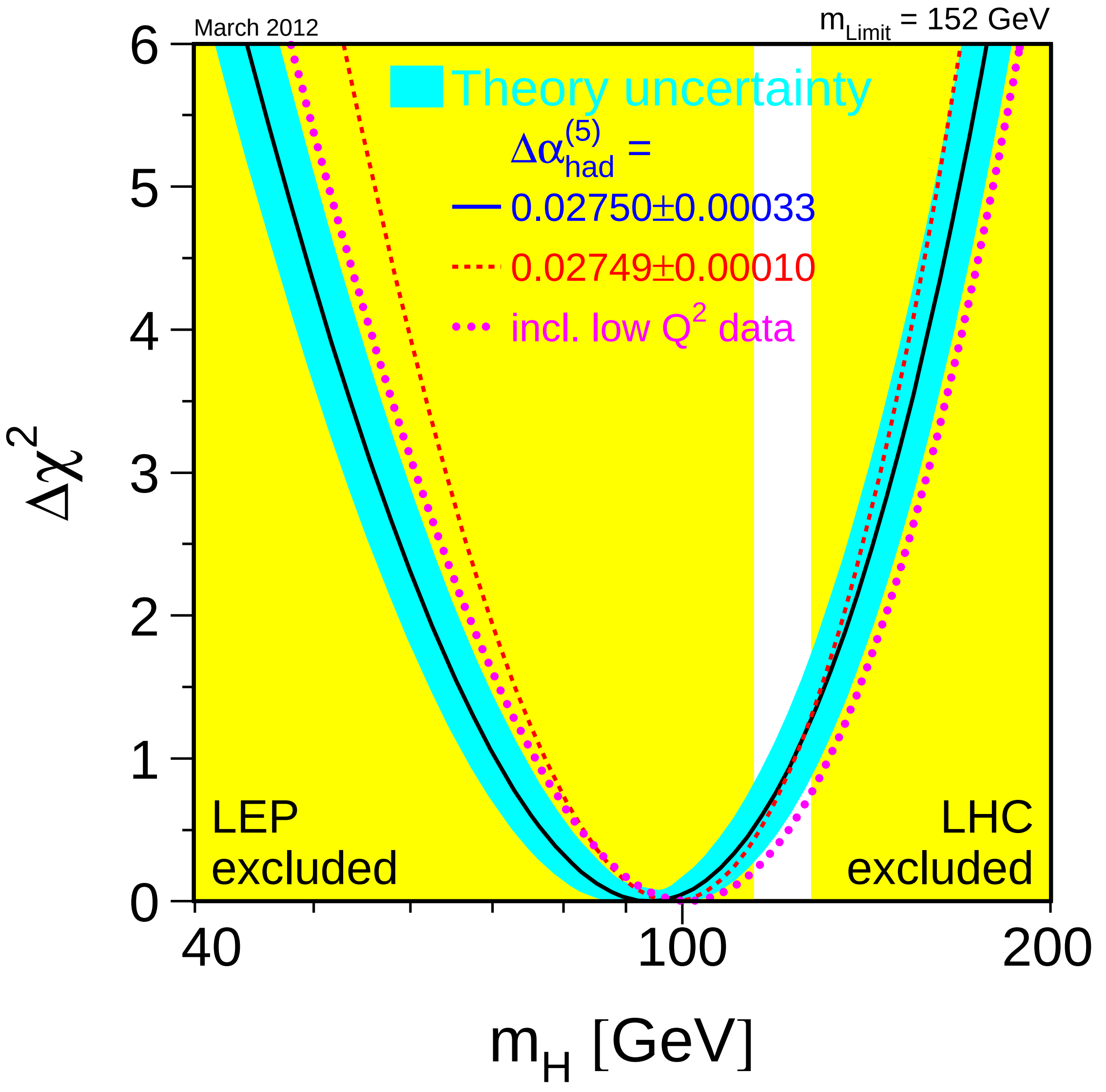
$\Rightarrow$ light Higgs boson preferred

Global fit to all SM data:

[*LEPEWWG '12*]

$\Rightarrow M_H = 94^{+29}_{-24}$ GeV

$M_H < 152$ GeV, 95% C.L.



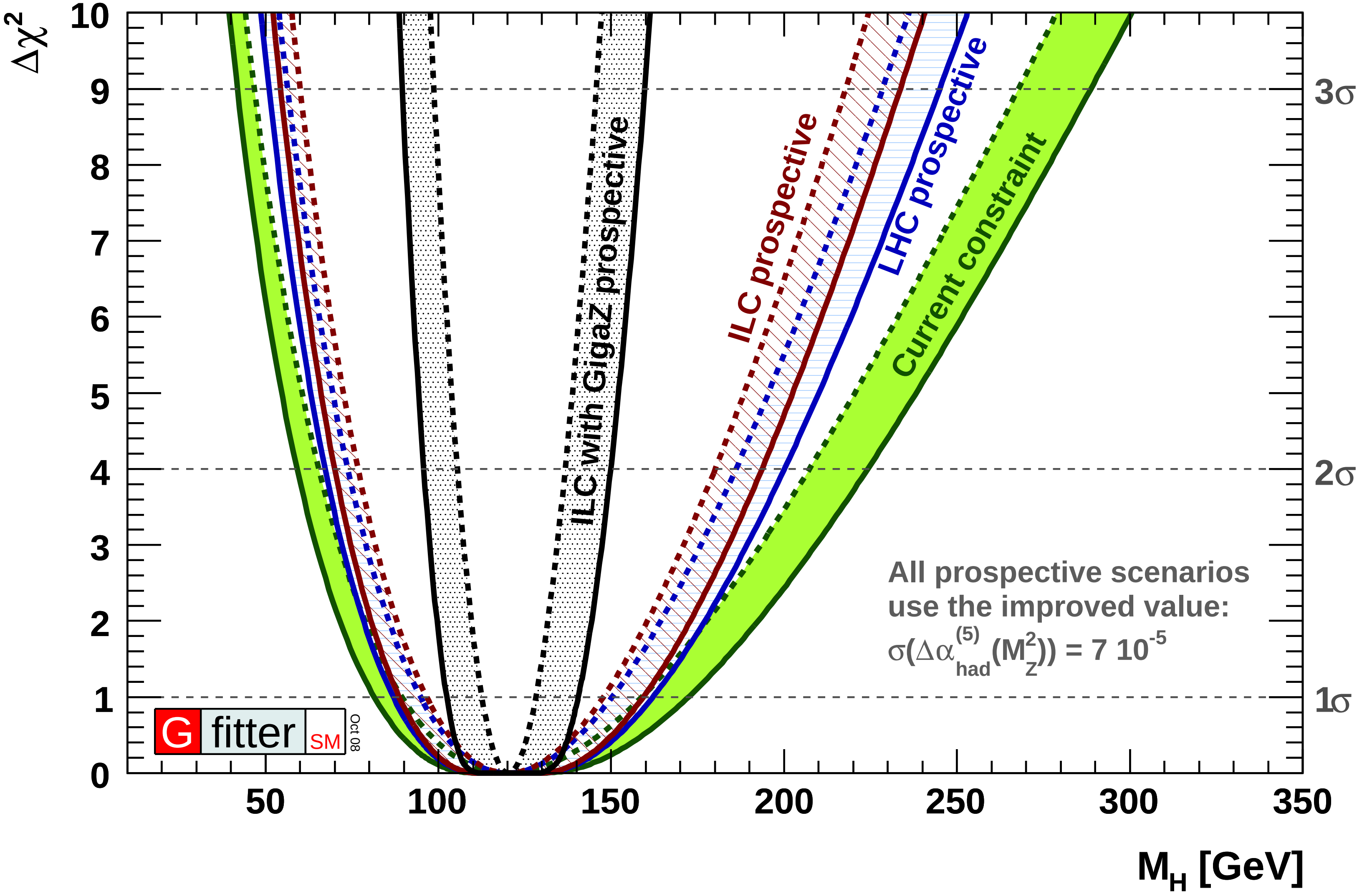
Assumption for the fit:
SM incl. Higgs boson

$\Rightarrow$ no confirmation of Higgs mechanism

$\Rightarrow$ Higgs boson in the SM must be light, $M_H \lesssim 160$ GeV

Improvement in the Blue Band plot:

[*GFitter '09*]



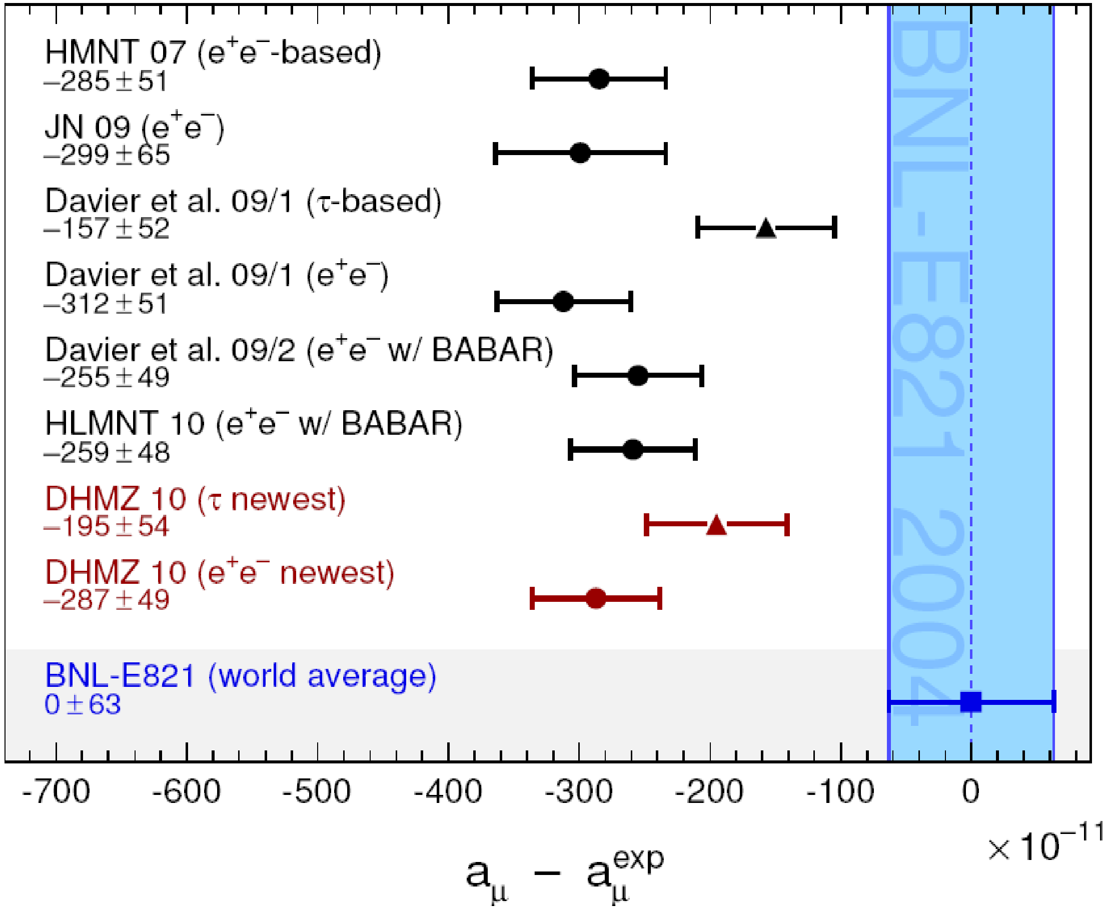
(note: artificially $M_H^{\text{SM}} = 120$ GeV)

Another EWPO: the anomalous magnetic moment of the muon

$$a_\mu \equiv (g-2)_\mu/2$$

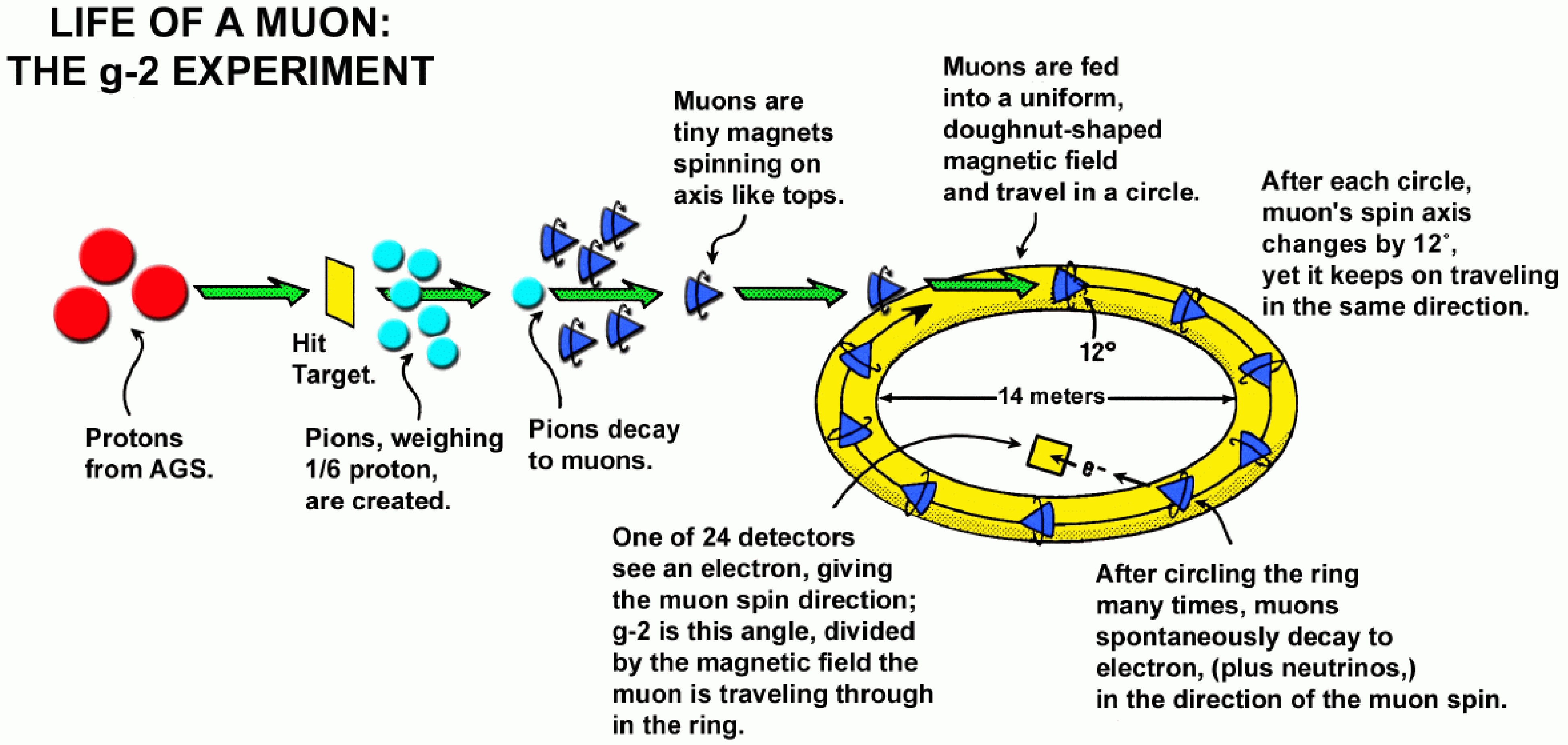
Overview about the current experimental and SM (theory) result:

[M. Davier, A. Hoecker, B. Malaescu, Z. Zhang '10] → T



$$a_\mu^{\rm exp} - a_\mu^{\rm theo,SM} \approx (28.7 \pm 8) \times 10^{-10} \quad : \; 3.6\,\sigma$$

The $(g-2)_\mu$ experiment:



LIFE OF A MUON:
THE g-2 EXPERIMENT

Protons from AGS.

Hit Target.

Pions, weighing 1/6 proton, are created.

Pions decay to muons.

Muons are tiny magnets spinning on axis like tops.

Muons are fed into a uniform, doughnut-shaped magnetic field and travel in a circle.

After each circle, muon's spin axis changes by 12°, yet it keeps on traveling in the same direction.

One of 24 detectors see an electron, giving the muon spin direction; g-2 is this angle, divided by the magnetic field the muon is traveling through in the ring.

After circling the ring many times, muons spontaneously decay to electron, (plus neutrinos,) in the direction of the muon spin.

Coupling of muon to magnetic field : $\mu - \mu - \gamma$ coupling

$$\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i}{2m_\mu}\sigma^{\mu\nu}q_\nu F_2(q^2)\right]u(p)A_\mu \qquad F_2(0) = a_\mu$$

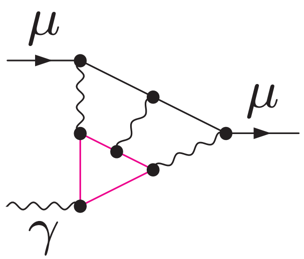
Current status of $(g-2)_\mu$:

Experiment:

- 2001 - 2006: very stable development
- final error: 6×10^{-10} , still statistically dominated

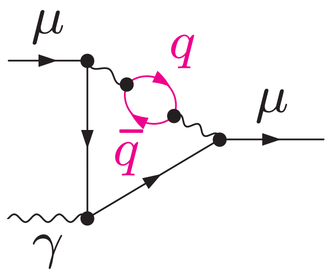
Theory:

- the light-by-light contribution:



2002: sign error discovered; since then stabilized

- the hadronic vacuum contribution:



problems with the τ data ⇒ hardly used anymore

'direct' e^+e^- data:
from CMD-II, SND, KLOE (radiative return)
⇒ agree quite well (also with old e^+e^- data)

new SM evaluations, based on new exp e^+e^- data for a_μ^{had} :

$$a_\mu(\text{Exp-SM}) = \left\{ \begin{array}{ll} \text{[HMNT '06]} & 28(8) \\ \text{[DEHZ '06]} & 28(8) \\ \text{[FJ '07]} & 29(9) \\ \text{[MRR '07]} & 29(9) \\ \text{[DH '10]} & 28.7(8.0) \end{array} \right\} \times 10^{-10}$$

better agreement between evaluations, more precise,

larger deviation from exp than ever before

⇓

3σ deviation has now been definitely established

(based on e^+e^- data)

New development for τ data:

[*F. Jegerlehner, R. Szafron '11*]

Re-evaluation of τ data: improved evaluation of ρ–γ mixing
$\Rightarrow$ shift in τ data:

Now: agreement with e^+e^- data! $\Rightarrow$ still tbc!

If correct: $\Rightarrow$ new average of all data possible . . .

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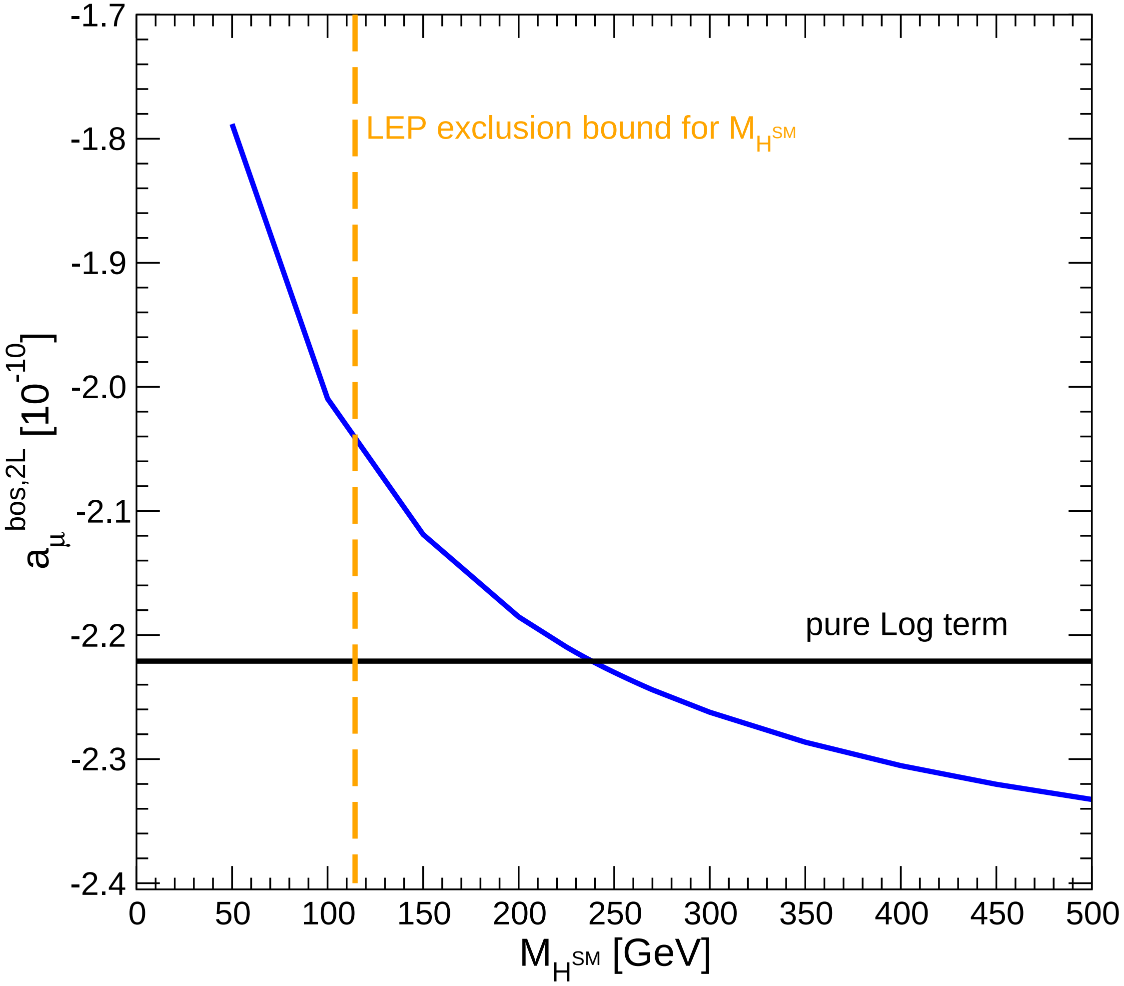
New physics needed to explain this discrepancy?

Restrictions on M_H from a_μ?

$\Rightarrow$ Higgs enters only at the two-loop level

Example for M_H dependence: [S.H., D. Stöckinger, G. Weiglein '04]



$\Rightarrow$ no restrictions on M_H (but just wait a bit . . . :-)