

# Electroweak and Theory (SM and MSSM)

*Sven Heinemeyer, IFCA (CSIC, Santander)*

St. Andrews, 08/2012

1. Higgs and Electroweak in The Standard Model (I)
2. Higgs and Electroweak in The Standard Model (II)
3. Higgs and Electroweak in the MSSM

# Electroweak and Theory (I): Higgs and Electroweak in the SM (I)

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1. The Standard Model and the Higgs boson
2. Top Quark physics in (B)SM
3. Electroweak Precision Observables

# 1. The Standard Model and the Higgs boson

## Current status of knowledge: the Standard Model (SM)

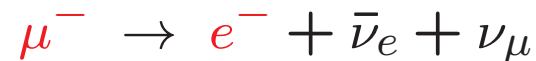
- |            |                |  |
|------------|----------------|--|
| 1. family: | quarks: $d, u$ | leptons: $e^-, \nu_e$ (neutrino)       |
| 2. family: | quarks: $s, c$ | leptons: $\mu^-, \nu_\mu$ (neutrino)   |
| 3. family: | quarks: $b, t$ | leptons: $\tau^-, \nu_\tau$ (neutrino) |

In total:

**6 quarks and 6 leptons**

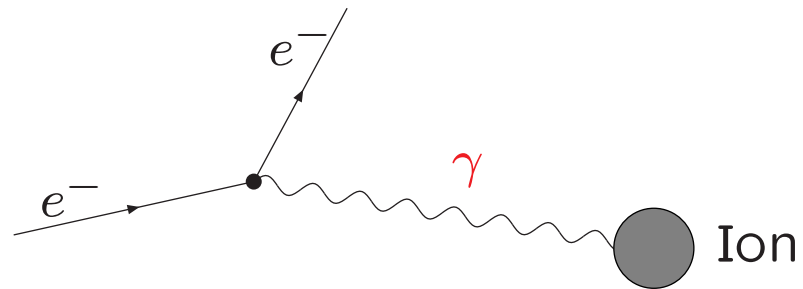
The heavier particles (2. and 3. family) decay in very short time into the lighter particles (1. family)

Example:

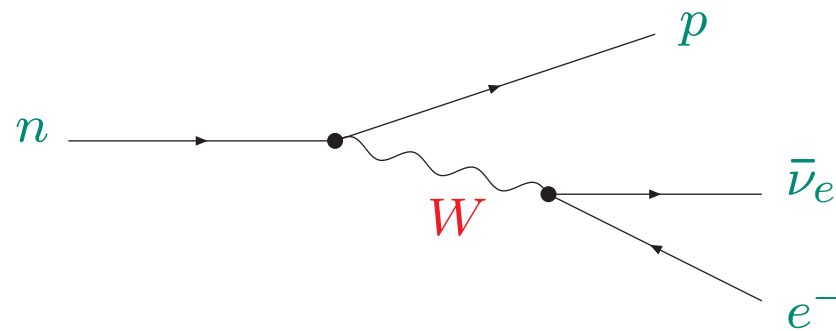


## Forces and force particles (I):

### 1. electromagnetic force: photon: $\gamma$

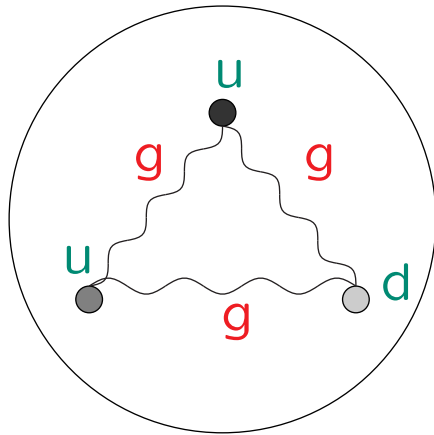


### 2. weak force: $W^+$ , $W^-$ , $Z^0$

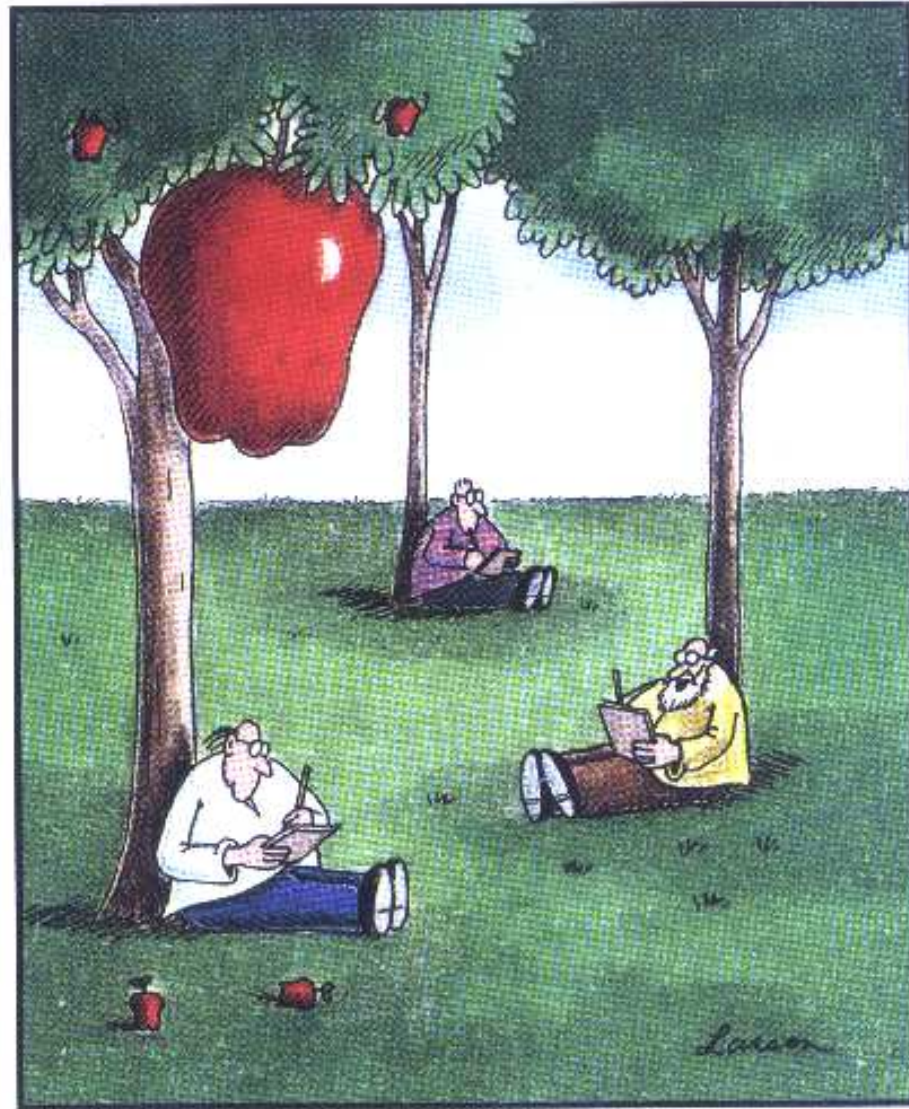


## Forces and force particles (II):

### 3. strong force: gluon: $g$



### 4. gravitational force: graviton(?)



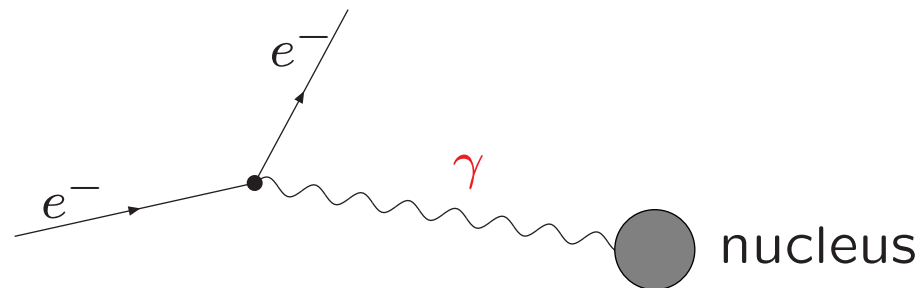
"Nothing yet. ... How about you, Newton?"

SM: Quantum field theory  $\Rightarrow$  interaction: exchange of field quanta

Construction principle of the SM: gauge invariance

### Example: Quantum electro-dynamics (QED)

field quanta: photon  $A_\mu$



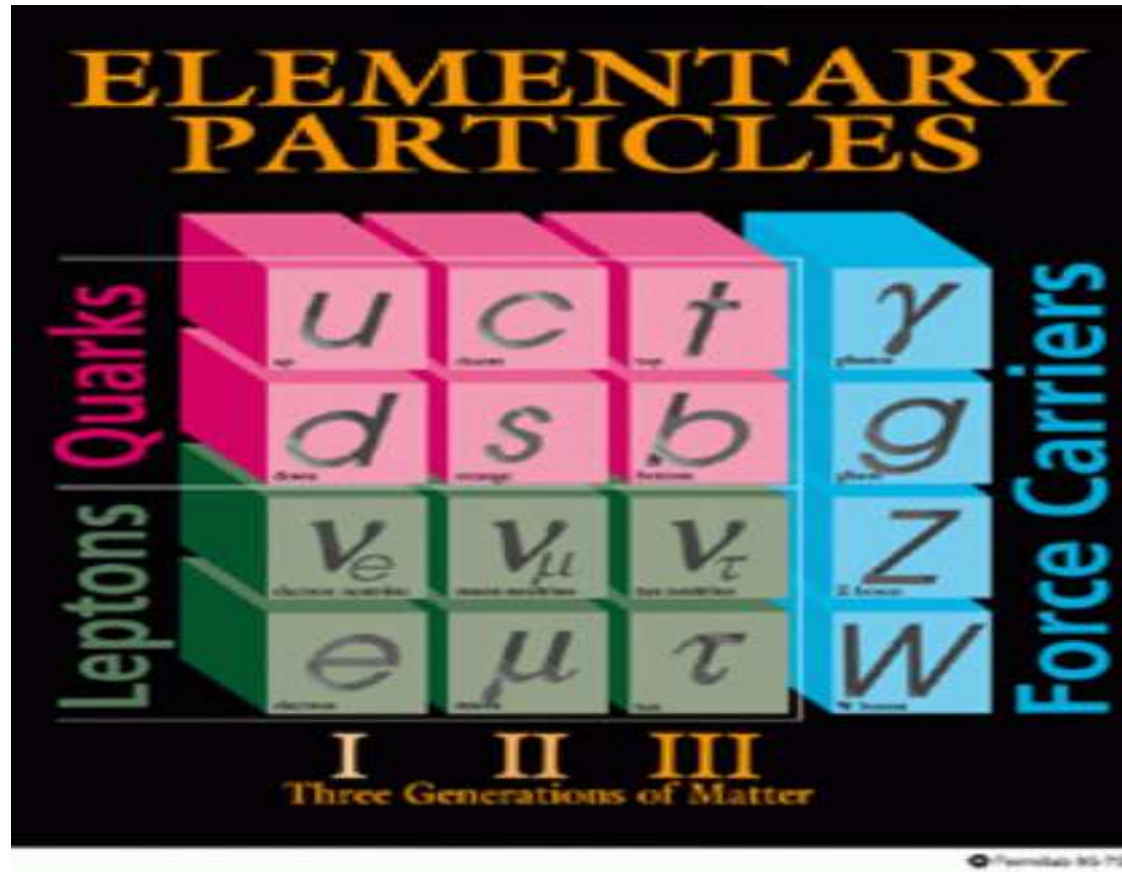
$\mathcal{L}_{\text{QED}}$  invariant under gauge transformation:

$$\Psi \rightarrow e^{ie\lambda(x)}\Psi, \quad A_\mu \rightarrow A_\mu + \partial_\mu\lambda(x)$$

mass term for photon:  $m^2 A^\mu A_\mu$  not gauge invariant

$\Rightarrow A_\mu$  is massless gauge field

## Current status of knowledge: the Standard Model (SM)



⇒ all particles experimentally seen

⇒ but it predicts massless gauge bosons ...

## Problem:

Gauge fields  $Z$ ,  $W^+$ ,  $W^-$  are **massive**

explicit mass terms in the Lagrangian  $\Leftrightarrow$  breaking of gauge invariance

## Solution: Higgs mechanism

scalar field postulated, mass terms from coupling to Higgs field

## Higgs sector in the Standard Model:

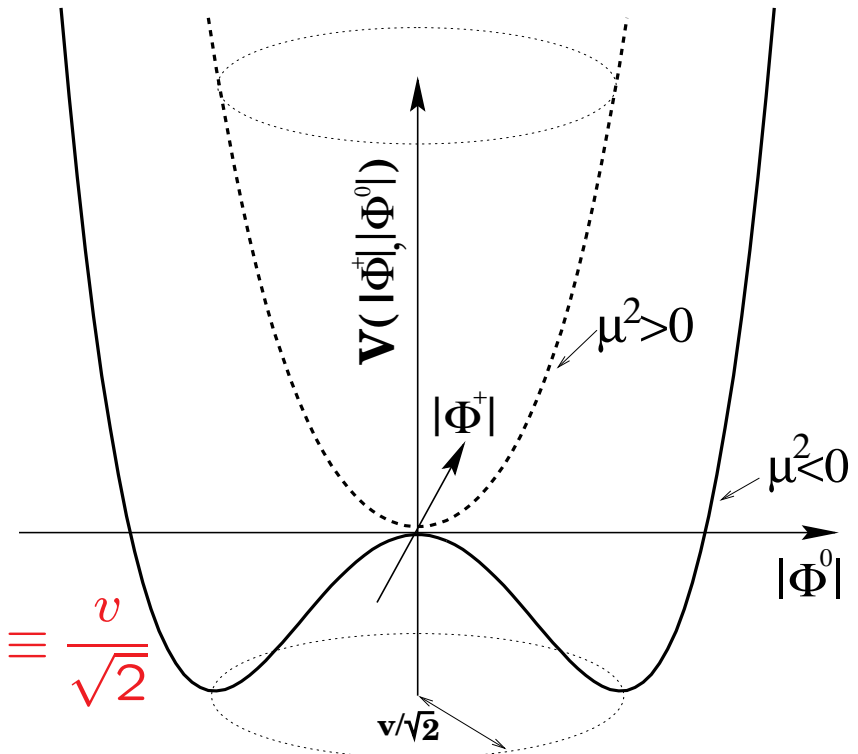
Scalar SU(2) doublet:  $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

Higgs potential:

$$V(\phi) = \mu^2 |\Phi^\dagger \Phi| + \lambda |\Phi^\dagger \Phi|^2, \quad \lambda > 0$$

$\mu^2 < 0$ : Spontaneous symmetry breaking

minimum of potential at  $|\langle \Phi_0 \rangle| = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$





$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \quad (\text{unitary gauge})$$

$H$ : elementary scalar field, Higgs boson

Lagrange density:

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} = & (D_\mu \Phi)^\dagger (D^\mu \Phi) \\ & - g_d \bar{Q}_L \Phi d_R - g_u \bar{Q}_L \Phi_c u_R \\ & - V(\Phi) \end{aligned}$$

with

$$\begin{aligned} iD_\mu &= i\partial_\mu - g_2 \vec{I} \vec{W}_\mu - g_1 Y B_\mu \\ \Phi_c &= i\sigma_2 \Phi^* \quad Q_L \sim \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \Phi \sim \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \Phi_c \sim \begin{pmatrix} v \\ 0 \end{pmatrix} \end{aligned}$$

Gauge invariant coupling to gauge fields

$\Rightarrow$  mass terms for gauge bosons and fermions

## 1.) $VV\Phi\Phi$ coupling:

$$V_{\text{wavy}} \longrightarrow \text{wavy} + \text{wavy} \begin{matrix} \times \times \\ \diagup \diagdown \end{matrix} v + \text{wavy} \begin{matrix} \times \times \times \\ \diagup \diagdown \diagdown \end{matrix} + \dots$$

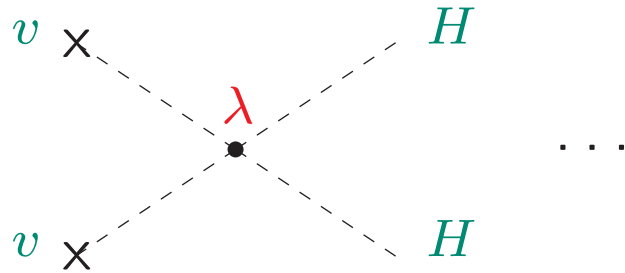
$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} + \sum_j \frac{1}{q^2} \left[ \left( \frac{gv}{\sqrt{2}} \right)^2 \frac{1}{q^2} \right]^j = \frac{1}{q^2 - M^2} : M^2 = g^2 \frac{v^2}{2} \Rightarrow M \propto g$$

## 2.) fermion mass terms: Yukawa couplings:

$$f \longrightarrow \text{fermion} + \text{fermion} \begin{matrix} \times \\ \diagup \end{matrix} v + \text{fermion} \begin{matrix} \times \times \\ \diagup \diagdown \end{matrix} + \dots$$

$$\frac{1}{\not{q}} \rightarrow \frac{1}{\not{q}} + \sum_j \frac{1}{\not{q}} \left[ \frac{g_f v}{\sqrt{2} \not{q}} \right]^j = \frac{1}{\not{q} - m_f} : m_f = g_f \frac{v}{\sqrt{2}} \Rightarrow m_f \propto g_f$$

### 3.) mass of the Higgs boson: self coupling



$$\lambda = M_H^2/v$$

$$M_H = v\sqrt{\lambda} \quad \text{free parameter}$$

→ last unknown(??) parameter  
of the SM

⇒ establish Higgs mechanism  $\equiv$  find the Higgs  $\oplus$  measure its couplings

Another effect of the Higgs field:

Scattering of longitudinal  $W$  bosons:  $W_L W_L \rightarrow W_L W_L$

$$\mathcal{M}_V = \begin{array}{c} W \\ \diagup \\ \text{---} \\ \diagdown \\ W \end{array} \begin{array}{c} \text{---} \\ \diagup \\ \gamma, Z \\ \diagdown \\ \text{---} \end{array} \begin{array}{c} W \\ \diagdown \\ \text{---} \\ \diagup \\ W \end{array} + \begin{array}{c} \text{---} \\ \diagup \\ \gamma, Z \\ \diagdown \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \diagup \\ \text{---} \\ \diagdown \\ \text{---} \end{array} = -g^2 \frac{E^2}{M_W^2} + \mathcal{O}(1) \quad \text{for } E \rightarrow \infty$$

⇒ violation of unitarity

Contribution of a scalar particle with couplings prop. to the mass:

$$\mathcal{M}_S = \begin{array}{c} W \\ \diagup \\ \text{---} \\ \diagdown \\ W \end{array} \begin{array}{c} \text{---} \\ \diagup \\ H \\ \diagdown \\ \text{---} \end{array} \begin{array}{c} W \\ \diagdown \\ \text{---} \\ \diagup \\ W \end{array} + \begin{array}{c} \text{---} \\ \diagup \\ H \\ \diagdown \\ \text{---} \end{array} = g_{WWH}^2 \frac{E^2}{M_W^4} + \mathcal{O}(1) \quad \text{for } E \rightarrow \infty$$

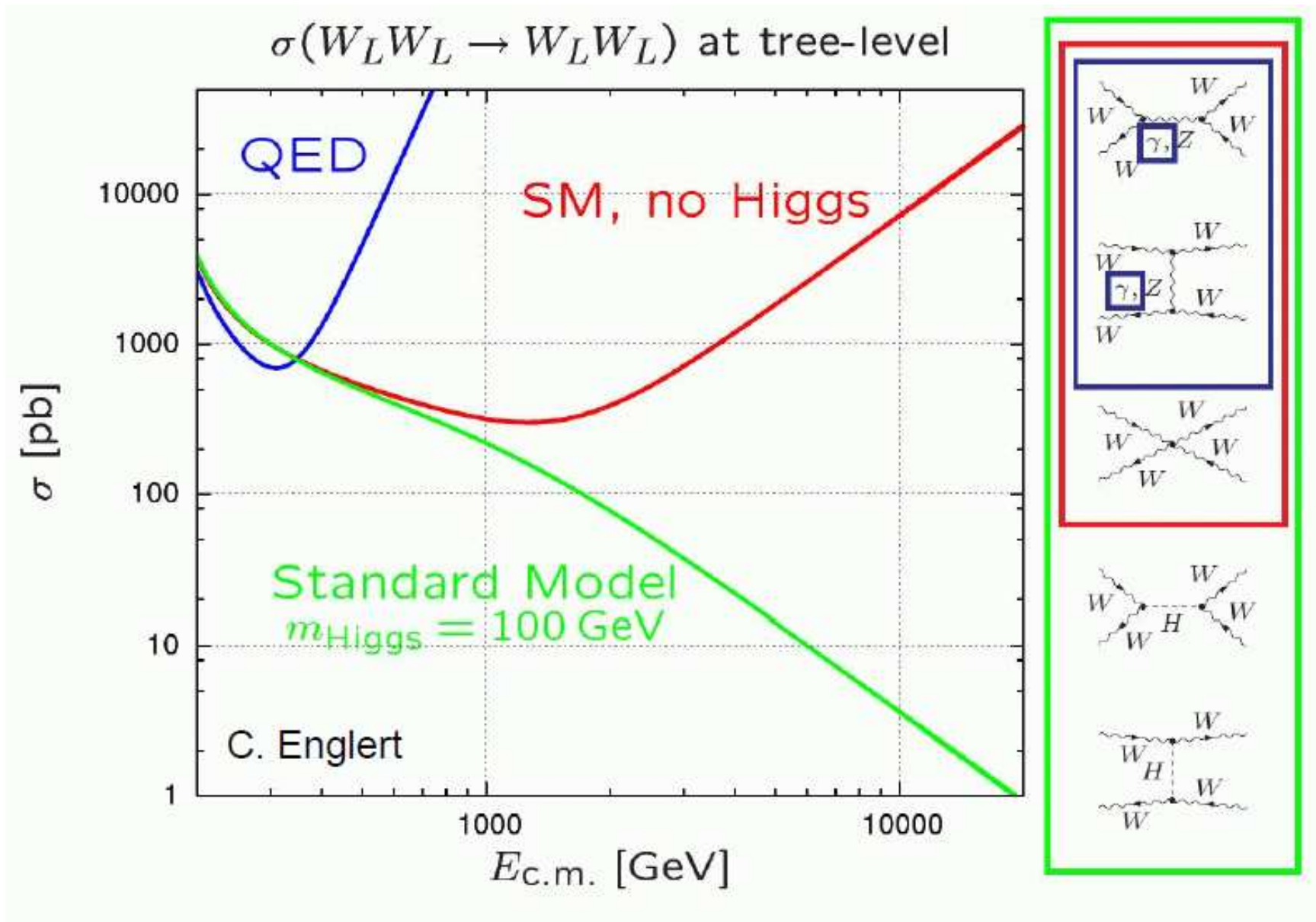
$$\mathcal{M}_{\text{tot}} = \mathcal{M}_V + \mathcal{M}_S = \frac{E^2}{M_W^4} \left( g_{WWH}^2 - g^2 M_W^2 \right) + \dots$$

⇒ compensation of terms with bad high-energy behavior for

$$g_{WWH} = g M_W$$

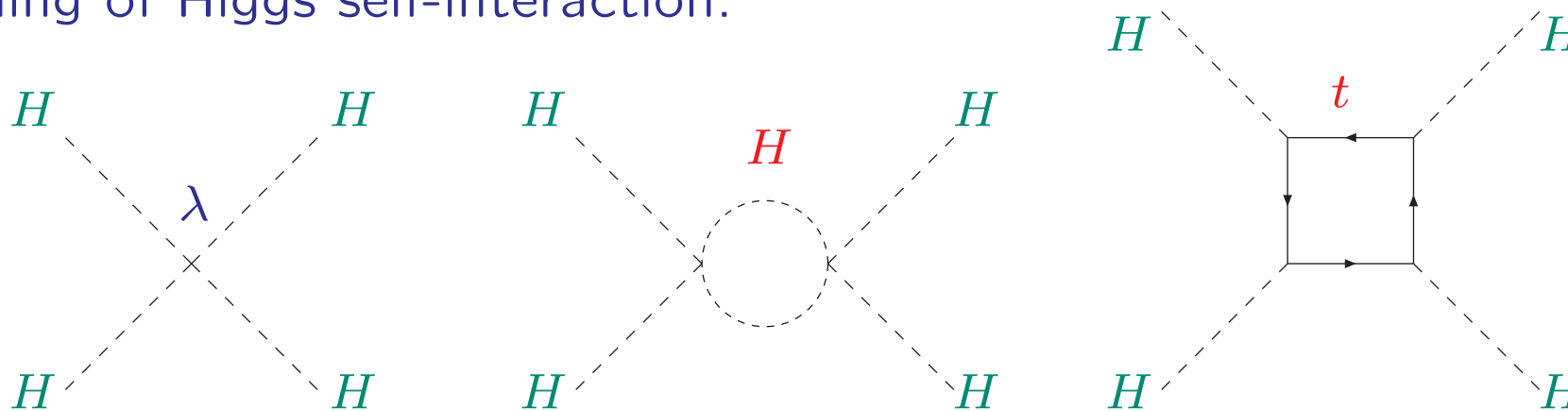
# Cross section with/without the Higgs:

[taken from M. Schumacher '12 / C. Englert]



## What else do we know about the Higgs boson?

Running of Higgs self-interaction:



Renormalization group equation:

$$\frac{d\lambda}{dt} = \frac{3}{8\pi^2} \left[ \lambda^2 + \lambda g_t^2 - g_t^4 + \frac{1}{16} (2g_2^4 + (g_2^2 + g_1^2)^2) \right], \quad t = \log \left( \frac{Q^2}{v^2} \right)$$

Two conditions:

- 1.) avoid Landau pole (for large  $\lambda \sim M_H^2$ )
- 2.) avoid vacuum instability (for small/negative  $\lambda$ )

1.) avoid Landau pole (for large  $\lambda \sim M_H^2$ )

$$\frac{d\lambda}{dt} = \frac{3}{8\pi^2} [\lambda^2]$$
$$\Rightarrow \lambda(Q^2) = \frac{\lambda(v^2)}{1 - \frac{3\lambda(v^2)}{8\pi^2} \log\left(\frac{Q^2}{v^2}\right)}$$

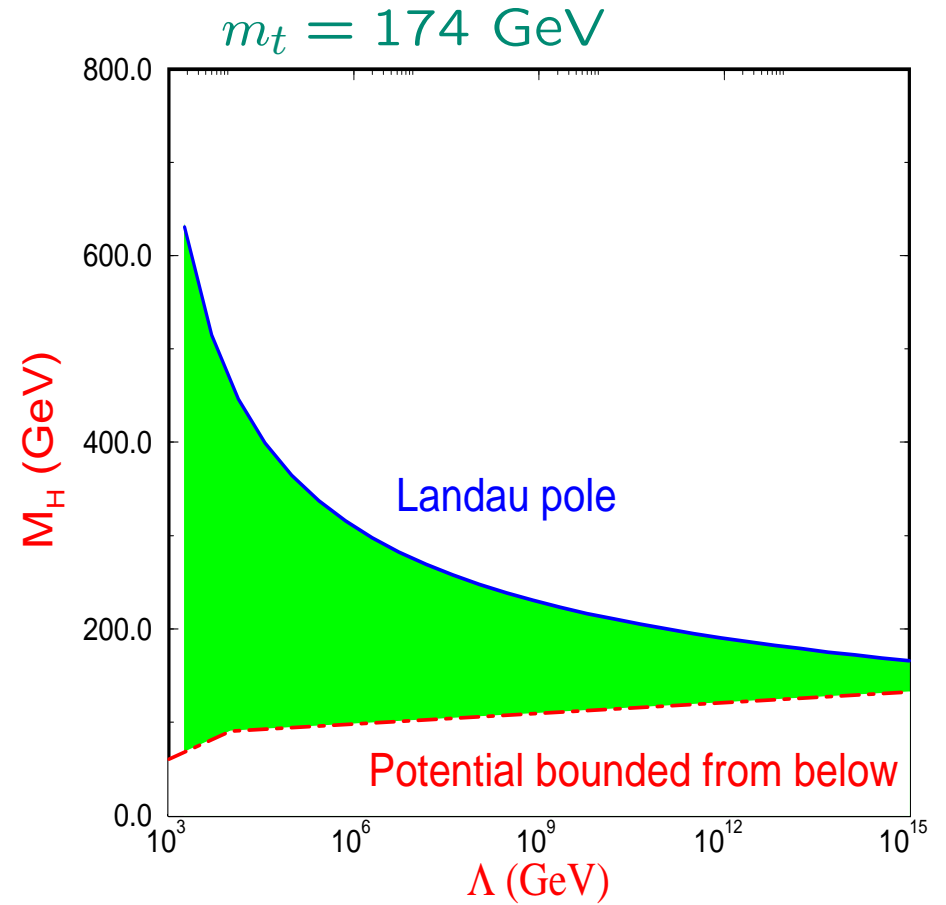
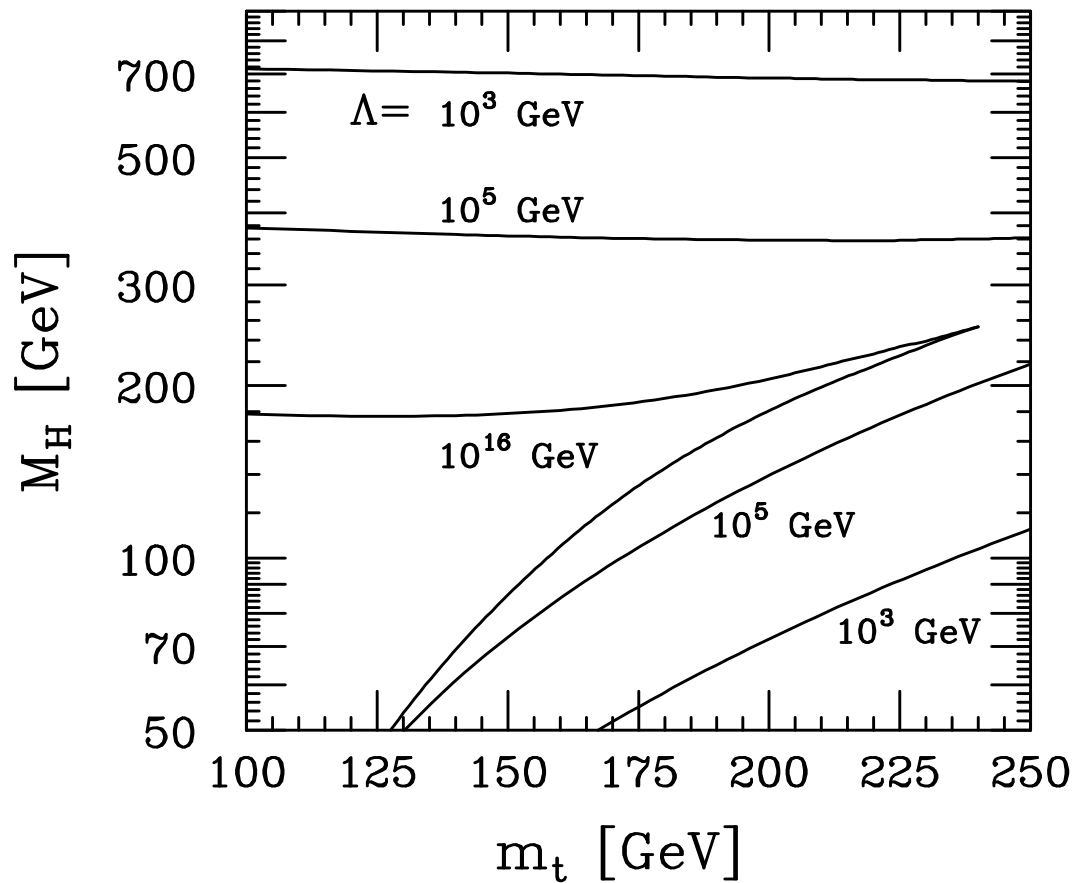
$$\lambda(\Lambda) < \infty \Rightarrow M_H^2 \leq \frac{8\pi^2 v^2}{3 \log\left(\frac{\Lambda^2}{v^2}\right)} : \text{upper bound on } M_H$$

2.) avoid vacuum instability (for small/negative  $\lambda$ ):  $V(v) < V(0) \Rightarrow \lambda(\Lambda) > 0$

$$\frac{d\lambda}{dt} = \frac{3}{8\pi^2} \left[ -g_t^4 + \frac{1}{16} (2g_2^4 + (g_2^2 + g_1^2)^2) \right]$$
$$\Rightarrow \lambda(Q^2) = \lambda(v^2) \frac{3}{8\pi^2} \left[ -g_t^4 + \frac{1}{16} (2g_2^4 + (g_2^2 + g_1^2)^2) \right] \log\left(\frac{Q^2}{v^2}\right)$$

$$\lambda(\Lambda) > 0 \Rightarrow M_H^2 > \frac{v^2}{4\pi^2} \left[ -g_t^4 + \frac{1}{16} (2g_2^4 + (g_2^2 + g_1^2)^2) \right] \log\left(\frac{\Lambda^2}{v^2}\right) : \text{lower bound}$$

Both limits combined:



$\Lambda$ : scale up to which the SM is valid

$$\Lambda = M_{\text{GUT}} \Rightarrow 130 \text{ GeV} \lesssim M_H \lesssim 180 \text{ GeV}$$



## Top quark physics in (B)SM

Top-quark mass is a fundamental parameter of the electroweak theory

By far the largest quark mass,  
largest mass of all known fundamental particles

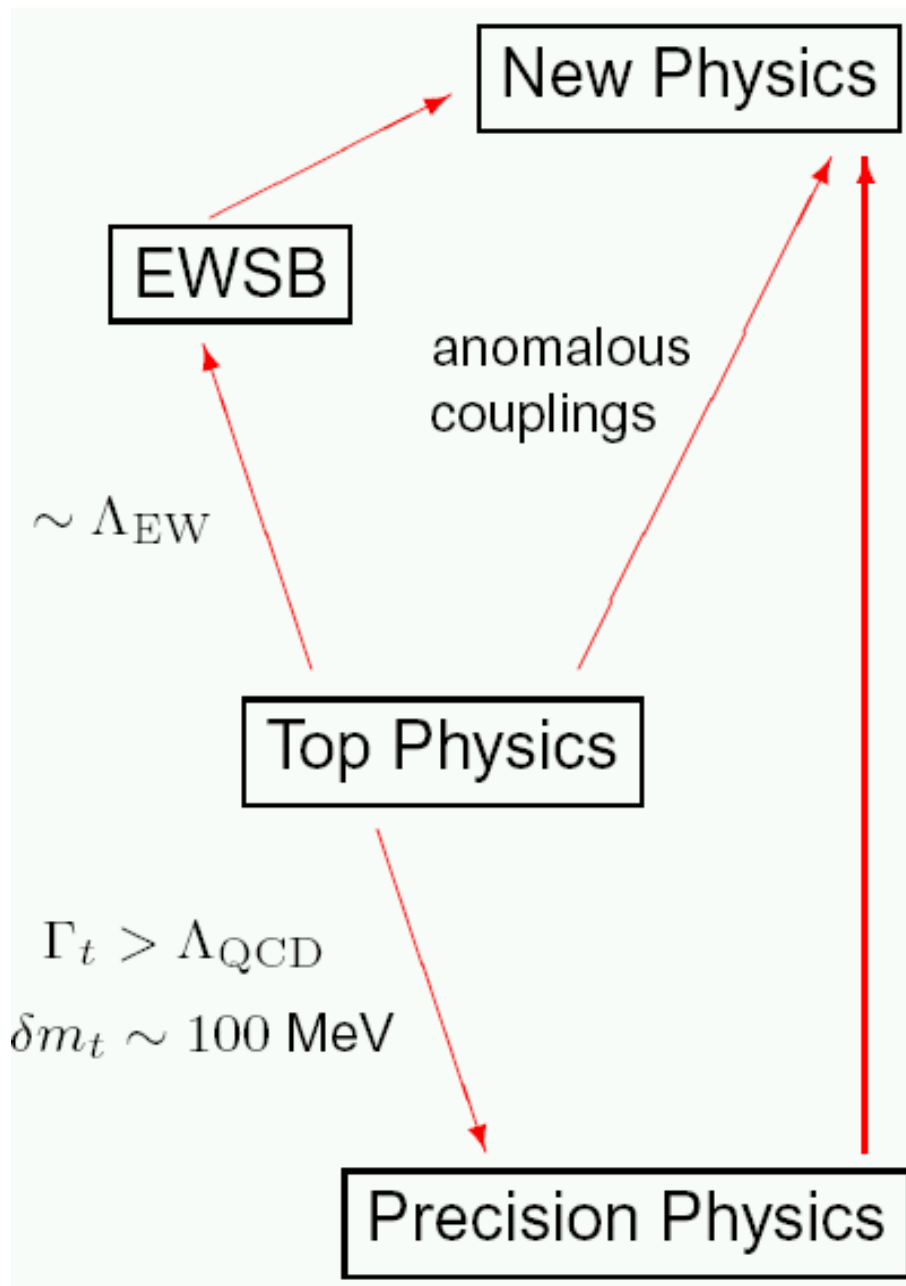
### Window to new physics?

Large coupling to the Higgs boson; physics of flavor;  
prediction of  $m_t$  from underlying theory?

Radiative corrections

⇒ non-decoupling effects proportional to powers of  $m_t$

⇒ Need to know  $m_t$  very precisely in order to have  
sensitivity to effects of new physics



EWSB: just a heavy quark?  
 special role for  $t$  in EWSB?  
 strong constraint on any model

Precision physics:  
 $\delta m_t^{\text{exp}}$  leading parametric uncertainty  
 $\rightarrow$  could obscure new physics

SUSY:  $m_t$  crucial input parameter  
 drives SSB/unification

Little Higgs: heavier top

What can be done at the LHC?

## What is the top mass?

Particle masses are **not** observables  
one can only measure cross sections, decay rates, ...

Additional problem for the top mass:

**what is the mass of a colored object?**

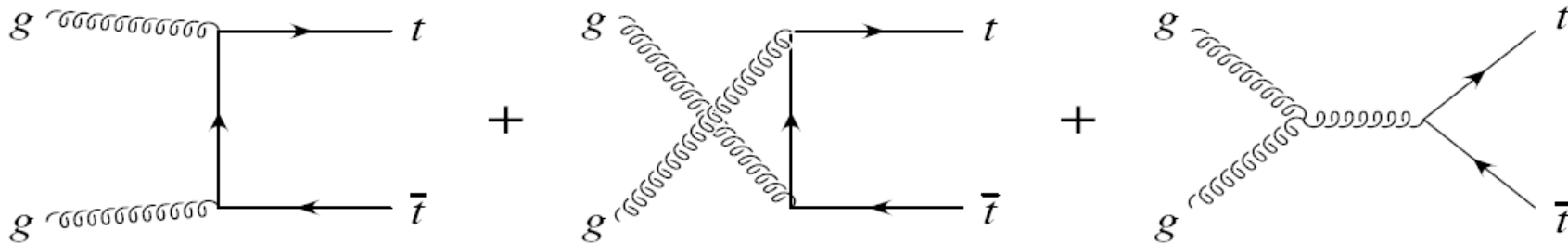
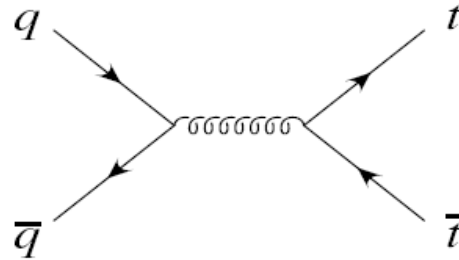
Top pole mass is not IR safe (affected by large long-distance contributions), cannot be determined to better than  $\mathcal{O}(\Lambda_{\text{QCD}})$

### Measurement of $m_t$ :

- At Tevatron, LHC:  
kinematic reconstruction, fit to invariant mass distribution  
 $\Rightarrow$  “pole” mass
- At the ILC:  
mainly from threshold behavior  $\Rightarrow$  threshold mass

## Top quark production at the LHC:

Top production through **quark antiquark** annihilation and **gluon gluon** fusion

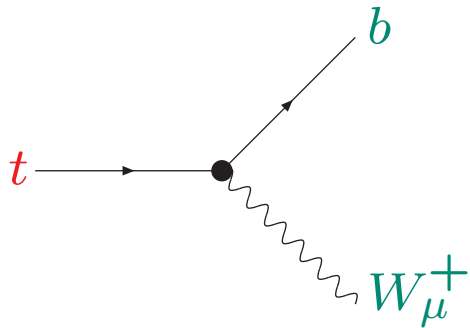


$$q\bar{q} \rightarrow t\bar{t} : 10\% \quad gg \rightarrow t\bar{t} : 90\%$$

$$\sigma_{\text{NLO}}^{\text{LHC}} = 830 \text{ pb} \pm 15\%$$

## Top quark decays (I):

The dominant decay is  $t \rightarrow W^+ b$ :



$$= -i \frac{g}{2\sqrt{2}} |V_{tb}|^2 \gamma_\mu (1 - \gamma_5)$$

$$\Gamma(t \rightarrow W^+ b) = \frac{G_\mu m_t^2}{8\pi\sqrt{2}} |V_{tb}|^2 \left(1 - \frac{M_W^2}{m_t^2}\right) \left(1 - \frac{2M_W^2}{m_t^2}\right) \approx |V_{tb}|^2 \times 1.42 \text{ GeV}$$

Unitarity of CKM matrix:  $|V_{tb}|^2 + |V_{cb}|^2 + |V_{ub}|^2 = 1 \Rightarrow |V_{tb}|^2 \approx 1$

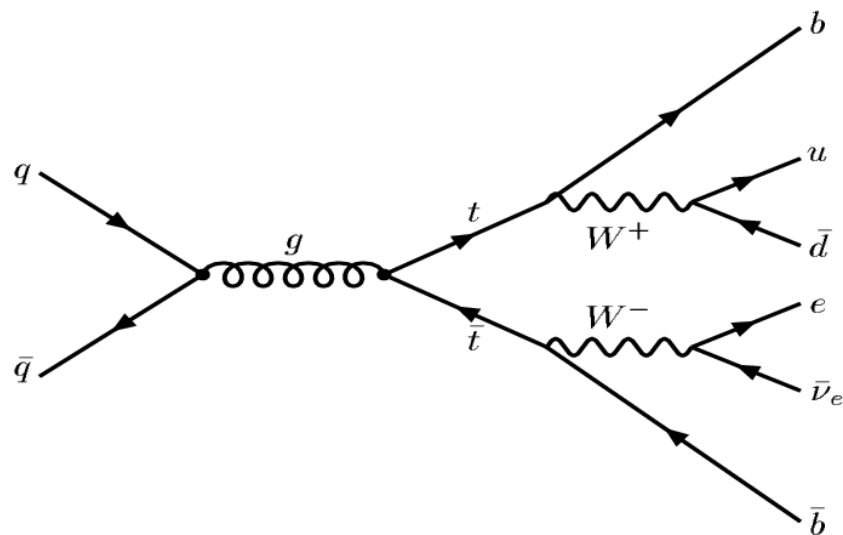
$\Rightarrow$  top quark life time  $\tau_t \approx 5 \times 10^{-25}$  sec

Typical QCD time scale for hadron formation:  $\tau_{\text{QCD}} \approx 3 \times 10^{-24}$  sec

$\Rightarrow$  the top quark decays before it can form bound states

## Top quark decays (II):

Signature depends on the  $WW$  decay modes



$W \rightarrow$	$jj$	$e\nu$	$\mu\nu$	$\tau\nu$
$W \downarrow$				
$jj$				
$e\nu$				
$\mu\nu$				
$\tau\nu$				

$\Rightarrow$  often semi-leptonic channels easiest

## Measurement of $V_{tb}$ (I):

Measure the ratio

$$\frac{\text{BR}(t \rightarrow W^+ b)}{\text{BR}(t \rightarrow W^+ q)} = \frac{|V_{tb}|^2}{|V_{tb}|^2 + |V_{cb}|^2 + |V_{ub}|^2}$$

If one assumes 3 generations then  $|V_{tb}|^2 + |V_{cb}|^2 + |V_{ub}|^2 = 1$

Current Tevatron measurements:

$$|V_{tb}| = \begin{cases} 1.05^{+0.10}_{-0.09} & ([\text{CDF '10}]) \\ 0.95^{+0.02}_{-0.02} & ([\text{D}\emptyset \text{ '11}]) \end{cases}$$

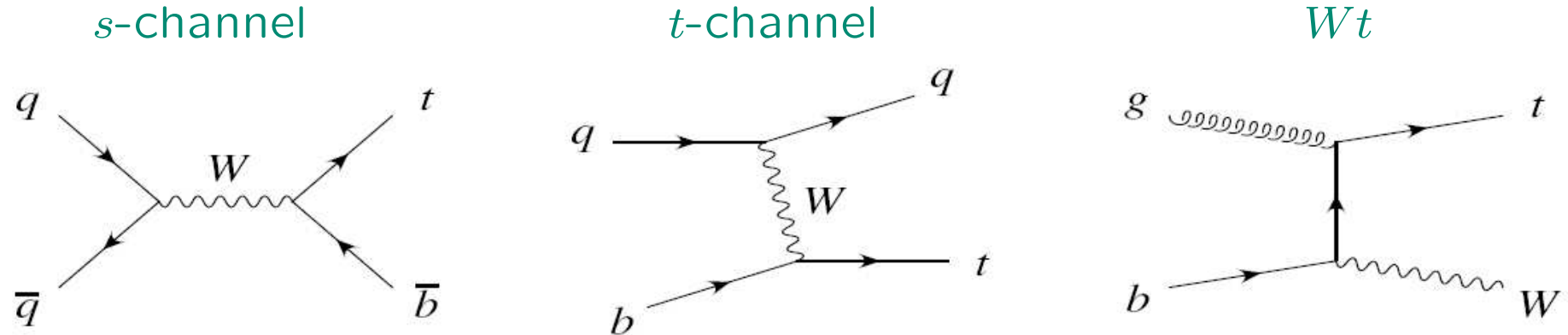
⇒ ATLAS and CMS are taking over now ...

However:

assuming three generations we know  $0.9990 < |V_{tb}| < 0.9993$  anyway from unitarity of the CKM matrix

## Measurement of $V_{tb}$ (II):

Cleaner: single top production



	s-channel	t-channel	$Wt$
$\sigma_t^{\text{NLO}}$ [pb]	$\sim 7$	$\sim 153$	$\sim 31$
$\sigma_{\bar{t}}^{\text{NLO}}$ [pb]	$\sim 4$	$\sim 90$	$\sim 31$

$\Rightarrow$  better prospects, no assumption on unitarity needed



## Top quark physics at the LHC:

The top cross section is  $\sigma_{tt} \approx 830 \text{ pb}$

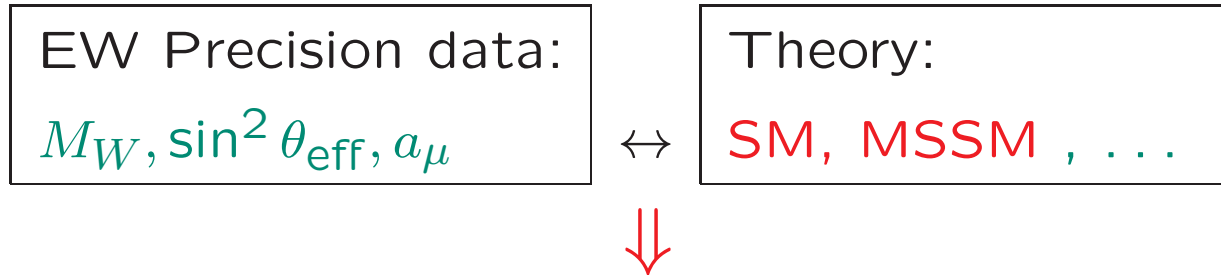
One year LHC running at low luminosity,  $\sim 10 \text{ fb}^{-1} \Rightarrow \mathcal{O}(10^7)$  top events

### Physics goals:

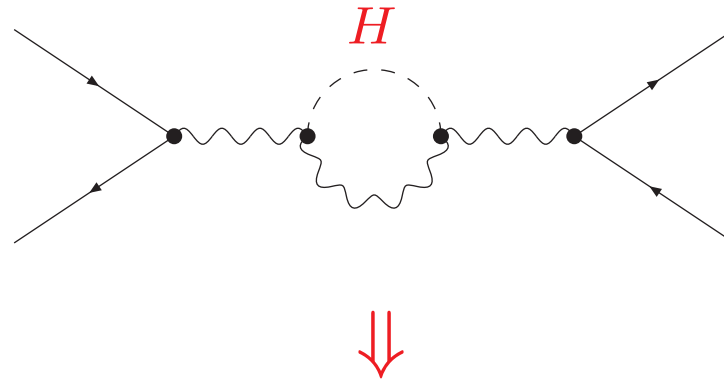
- $\delta m_t = 1 \text{ GeV}$  with  $\int \mathcal{L} = 100 \text{ fb}^{-1}$
- Observation of **single top** production, measurement of  $V_{tb}$  with  $\int \mathcal{L} = 30 \text{ fb}^{-1}$
- test of **quantum numbers**
- measurement of **rare (BSM) decay** modes

# Electroweak Precision Observables (EWPO):

Comparison of electro-weak precision observables with theory:



Test of theory at quantum level: Sensitivity to loop corrections, e.g.  $H$



SM: limits on  $M_H$

Very high accuracy of measurements and theoretical predictions needed

## Example: prediction of $M_W$ , $\sin^2 \theta_{\text{eff}}$

A) Theoretical prediction for  $M_W$  in terms

of  $M_Z, \alpha, G_\mu, \Delta r$ :

$$M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r)$$



loop corrections

Evaluate  $\Delta r$  from  $\mu$  decay  $\Rightarrow M_W$

One-loop result for  $M_W$  in the SM:

[A. Sirlin '80] , [W. Marciano, A. Sirlin '80]

$$\begin{aligned} \Delta r_{1\text{-loop}} = & \quad \Delta\alpha & - & \quad \frac{c_W^2}{s_W^2} \Delta\rho & + & \quad \Delta r_{\text{rem}}(M_H) \\ & \sim \log \frac{M_Z}{m_f} & & \sim m_t^2 & & \log(M_H/M_W) \\ & \sim 6\% & & \sim 3.3\% & & \sim 1\% \end{aligned}$$

## Example: prediction of $M_W$ , $\sin^2 \theta_{\text{eff}}$

A) Theoretical prediction for  $M_W$  in terms

of  $M_Z$ ,  $\alpha$ ,  $G_\mu$ ,  $\Delta r$ :

$$M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r)$$



loop corrections

B) Effective mixing angle:

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4 |Q_f|} \left( 1 - \frac{\text{Re } g_V^f}{\text{Re } g_A^f} \right)$$

Higher order contributions:

$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

# Comparison of SM prediction of $M_W$ with direct measurements:

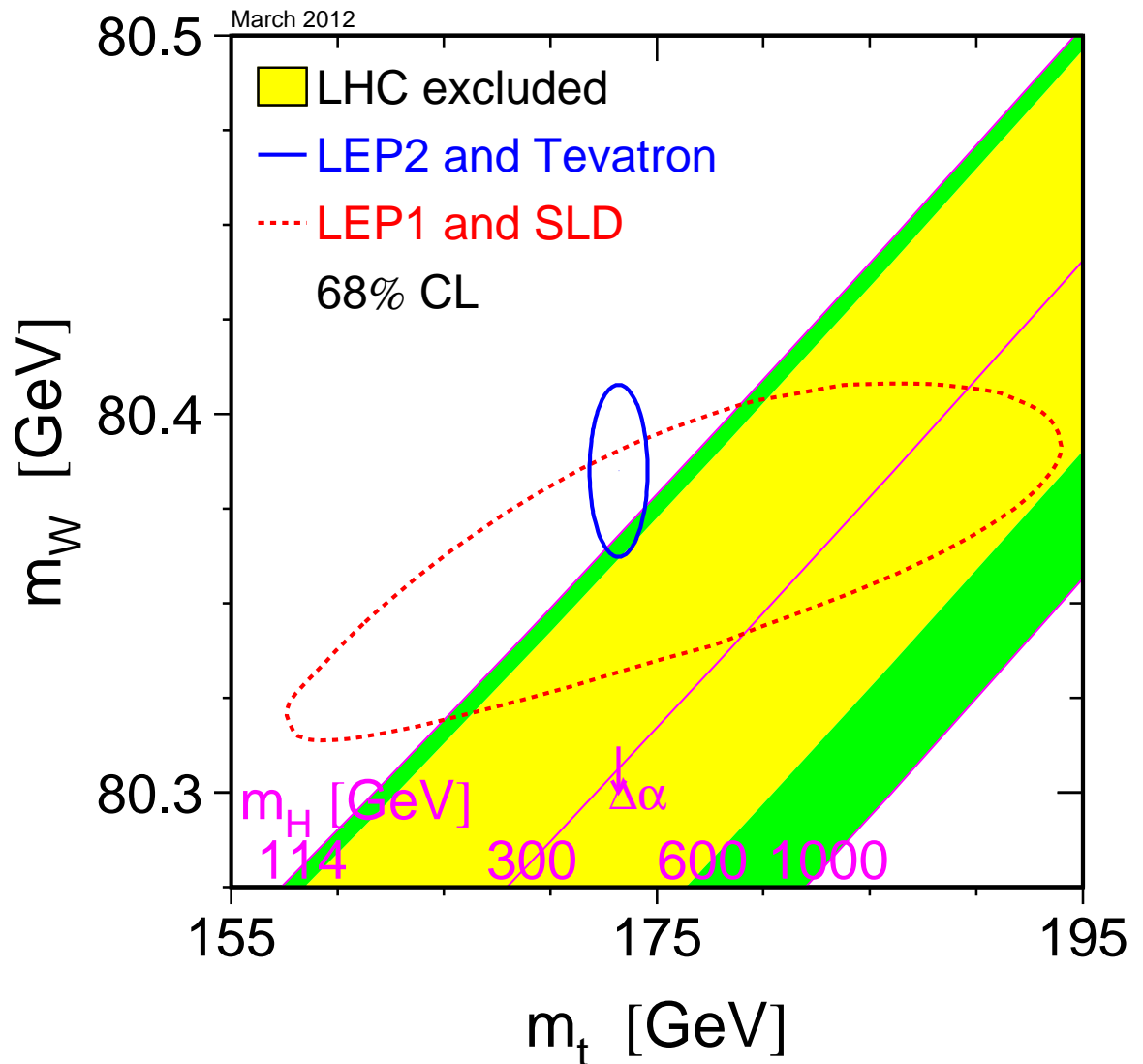
$$\Delta r = -\frac{11g_2^2 s_W^2}{96\pi^2 c_W^2} \log\left(\frac{M_H}{M_W}\right)$$

general for EWPO:

$$\Delta \sim g_2^2 \left[ \log\left(\frac{M_H}{M_W}\right) + g_2^2 \frac{M_H^2}{M_W^2} \right]$$

leading term:  $\log(M_H)$

first term  $\sim M_H^2$  with  $g_2^4$



$\Rightarrow$  light Higgs boson preferred

[LEPEWWG '12]

# Results for $M_H$ from other EWPO:

light Higgs preferred by:

$M_W, A_l^{LR}$  (SLD)

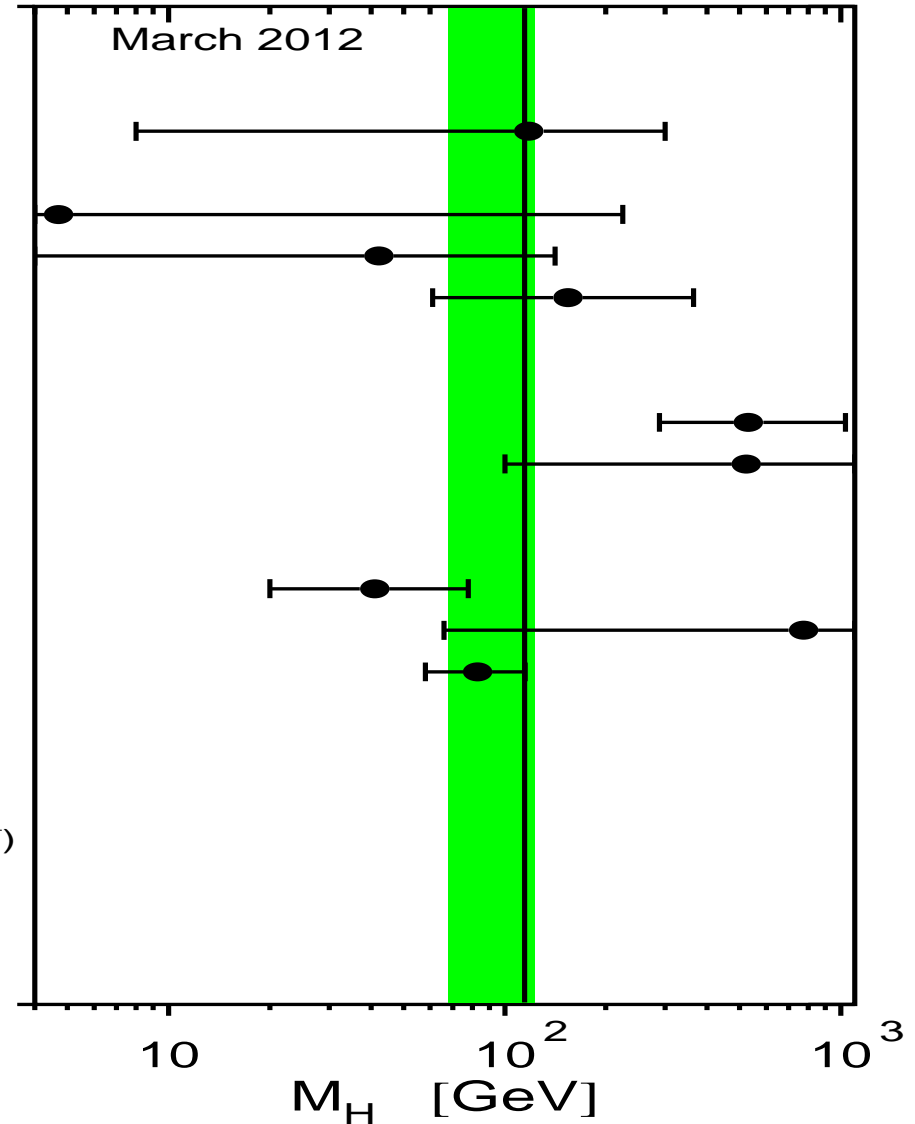
heavier Higgs preferred by:

$A_b^{FB}$  (LEP)

⇒ keeps SM alive

⇒ light Higgs boson preferred

- $\Gamma_Z^0$
- $\sigma_{had}^0$
- $R_l^0$
- $A_{fb}^{0,l}$
- $A_l(P_\tau)$
- $R_b^0$
- $R_c^0$
- $A_{fb}^{0,b}$
- $A_{fb}^{0,c}$
- $A_b$
- $A_c$
- $A_l(SLD)$
- $\sin^2\theta_{eff}^{lept}(Q_{fb})$
- $m_W$
- $\Gamma_W$
- $Q_W(Cs)$
- $\sin^2\theta_{MS}(e^-e^-)$
- $\sin^2\theta_W(\nu N)$
- $g_L^2(\nu N)$
- $g_R^2(\nu N)$



[LEPEWWG '12]

# Global fit to all SM data:

[LEPEWWG '12]

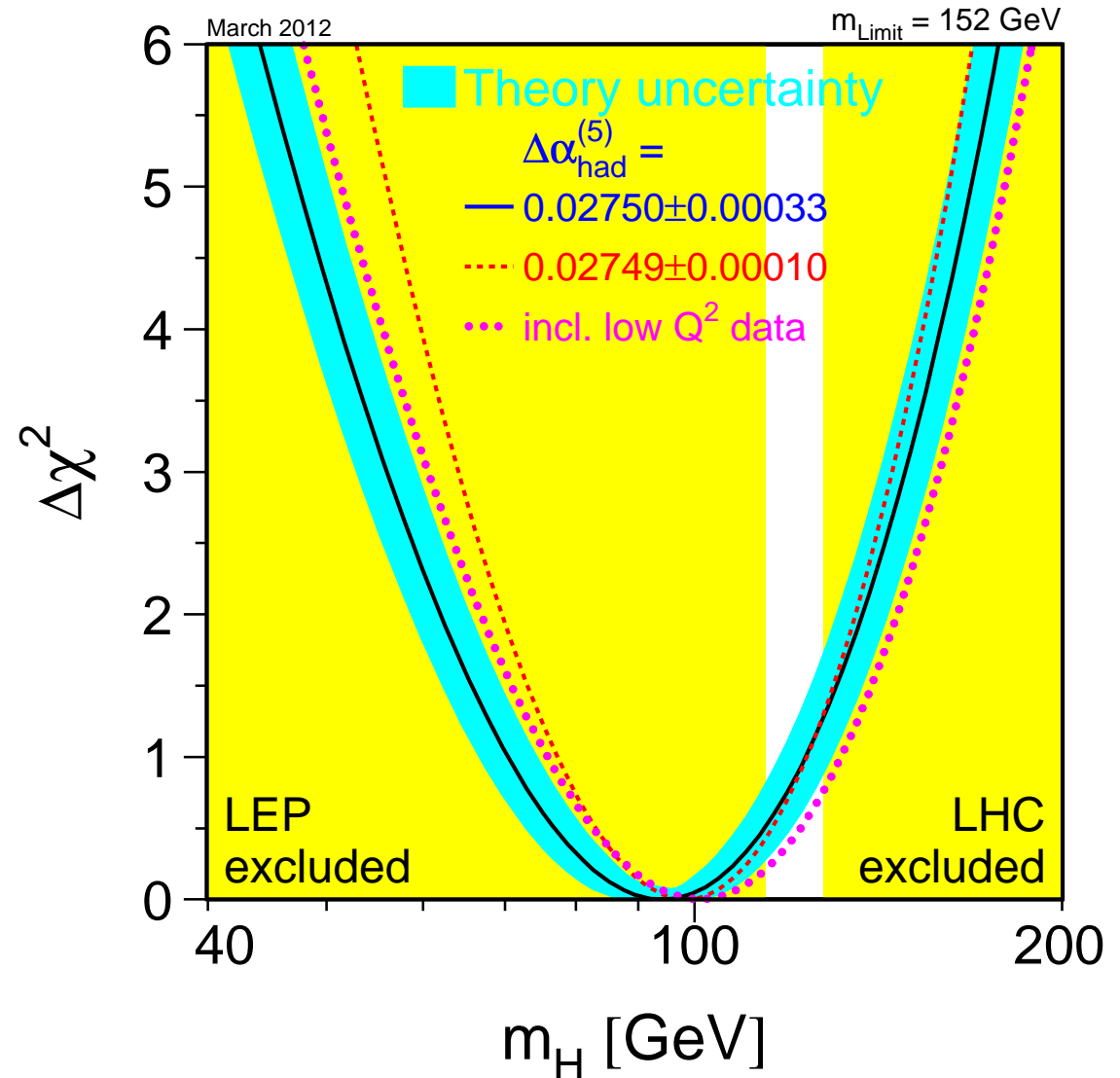
$$\Rightarrow M_H = 94^{+29}_{-24} \text{ GeV}$$

$$M_H < 152 \text{ GeV, 95\% C.L.}$$

Assumption for the fit:

SM incl. Higgs boson

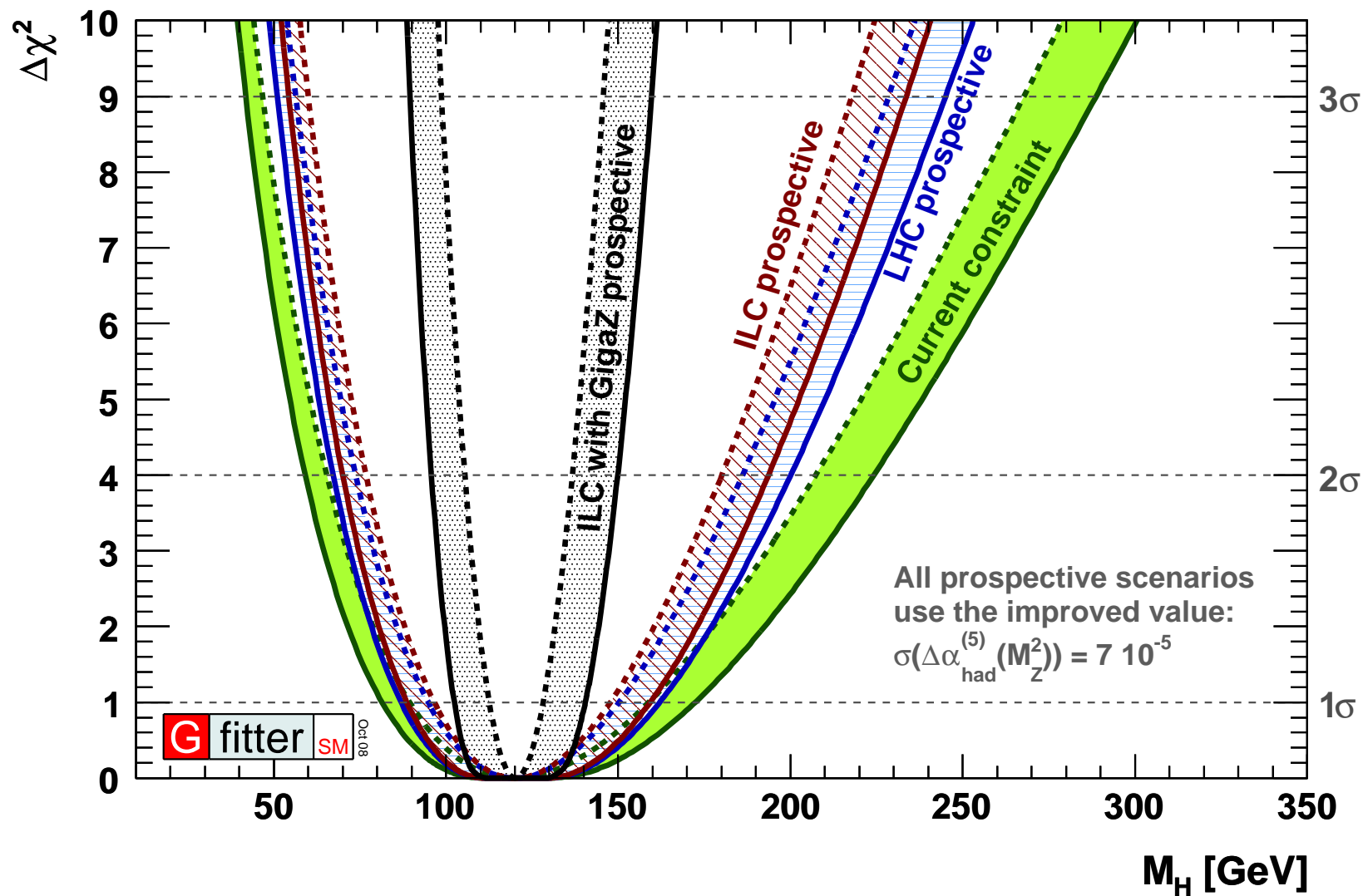
$\Rightarrow$  no confirmation of Higgs mechanism



$\Rightarrow$  Higgs boson in the SM must be light,  $M_H \lesssim 160 \text{ GeV}$

# Improvement in the Blue Band plot:

[GFitter '09]



(note: artificially  $M_H^{\text{SM}} = 120$  GeV)



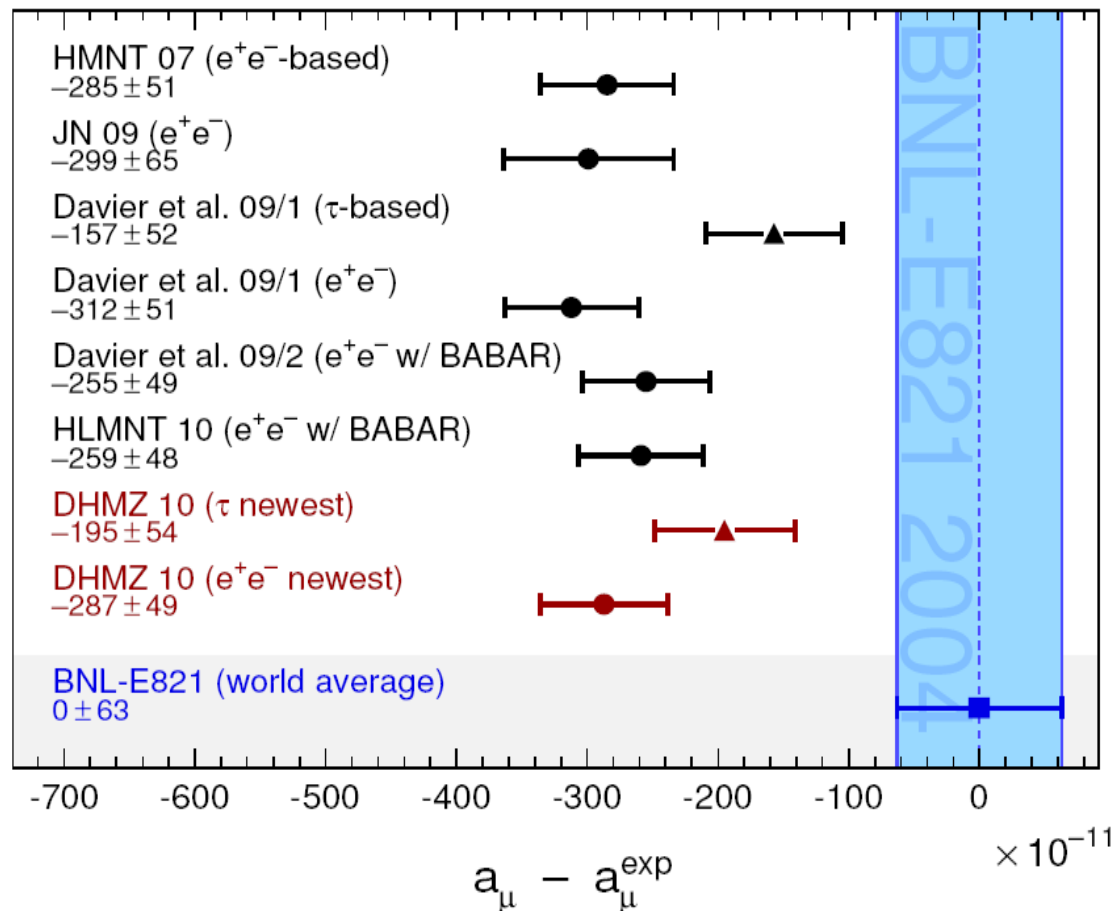
# Another EWPO: the anomalous magnetic moment of the muon

$$a_\mu \equiv (g - 2)_\mu / 2$$

Overview about the current **experimental** and SM (theory) result:

[M. Davier, A. Hoecker, B. Malaescu, Z. Zhang '10]

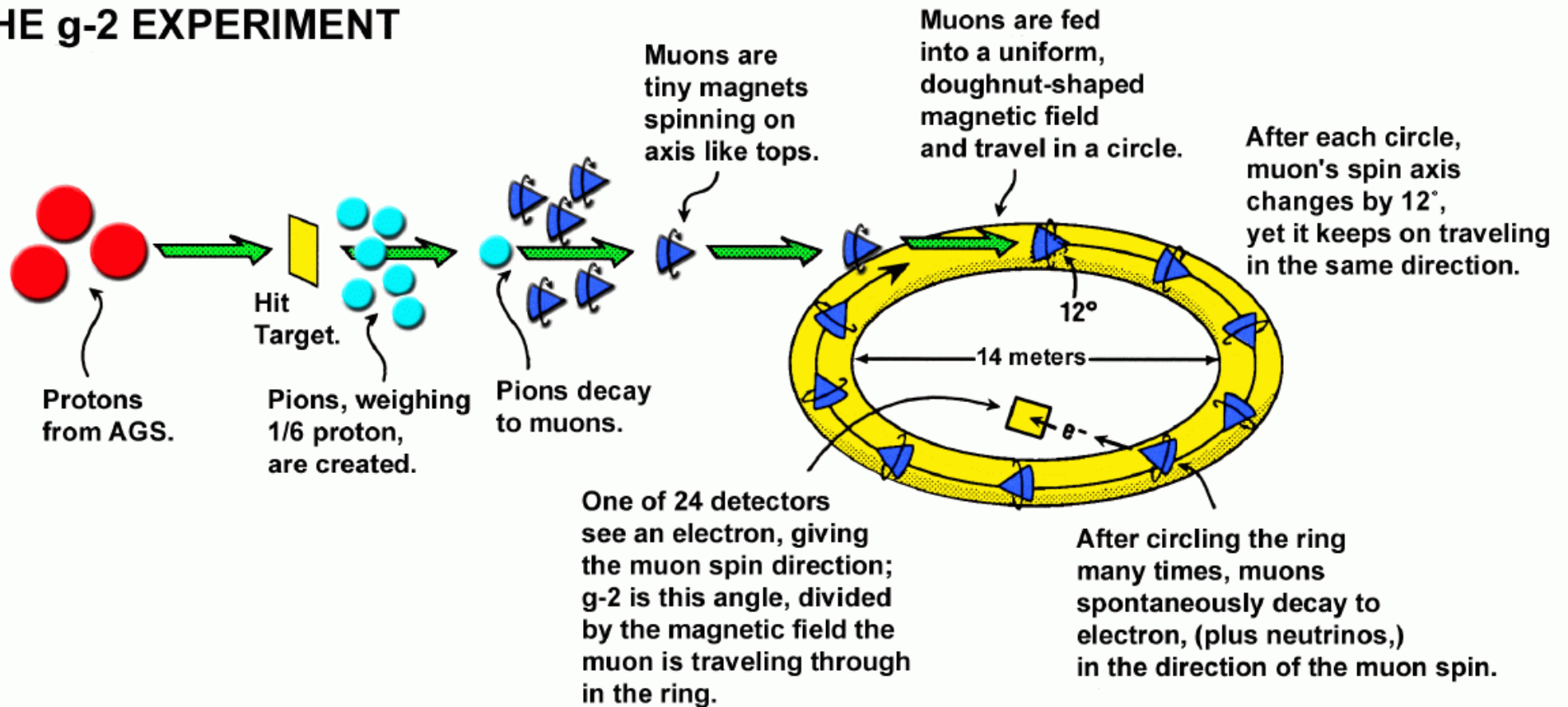
→ T



$$a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} \approx (28.7 \pm 8) \times 10^{-10} : 3.6 \sigma$$

# The $(g - 2)_\mu$ experiment:

## LIFE OF A MUON: THE g-2 EXPERIMENT



Coupling of muon to magnetic field :  $\mu - \mu - \gamma$  coupling

$$\bar{u}(p') \left[ \gamma^\mu F_1(q^2) + \frac{i}{2m_\mu} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p) A_\mu \quad F_2(0) = a_\mu$$

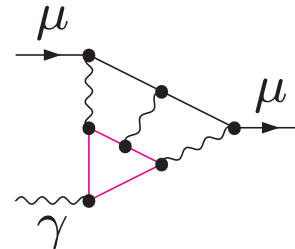
## Current status of $(g - 2)_\mu$ :

### Experiment:

- 2001 - 2006: very stable development
- final error:  $6 \times 10^{-10}$ , still statistically dominated

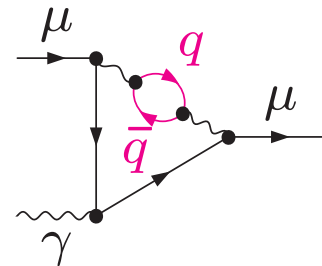
### Theory:

- the **light-by-light** contribution:



2002: sign error discovered; since then stabilized

- the **hadronic vacuum** contribution:



problems with the  $\tau$  data  $\Rightarrow$  hardly used anymore

'direct'  $e^+e^-$  data:

from **CMD-II**, **SND**, **KLOE** (radiative return)

$\Rightarrow$  agree quite well (also with old  $e^+e^-$  data)

new SM evaluations, based on new exp  $e^+e^-$  data for  $a_\mu^{\text{had}}$  :

$$a_\mu(\text{Exp-SM}) = \left\{ \begin{array}{ll} [\text{HMNT '06}] & 28(8) \\ [\text{DEHZ '06}] & 28(8) \\ [\text{FJ '07}] & 29(9) \\ [\text{MRR '07}] & 29(9) \\ [\text{DH '10}] & 28.7(8.0) \end{array} \right\} \times 10^{-10}$$

better agreement between evaluations, more precise,  
larger deviation from exp than ever before



$3\sigma$  deviation has now been definitely established

(based on  $e^+e^-$  data)

## New development for $\tau$ data:

[F. Jegerlehner, R. Szafron '11]

Re-evaluation of  $\tau$  data: improved evaluation of  $\rho$ - $\gamma$  mixing

$\Rightarrow$  shift in  $\tau$  data:

Now: agreement with  $e^+e^-$  data!  $\Rightarrow$  still tbc!

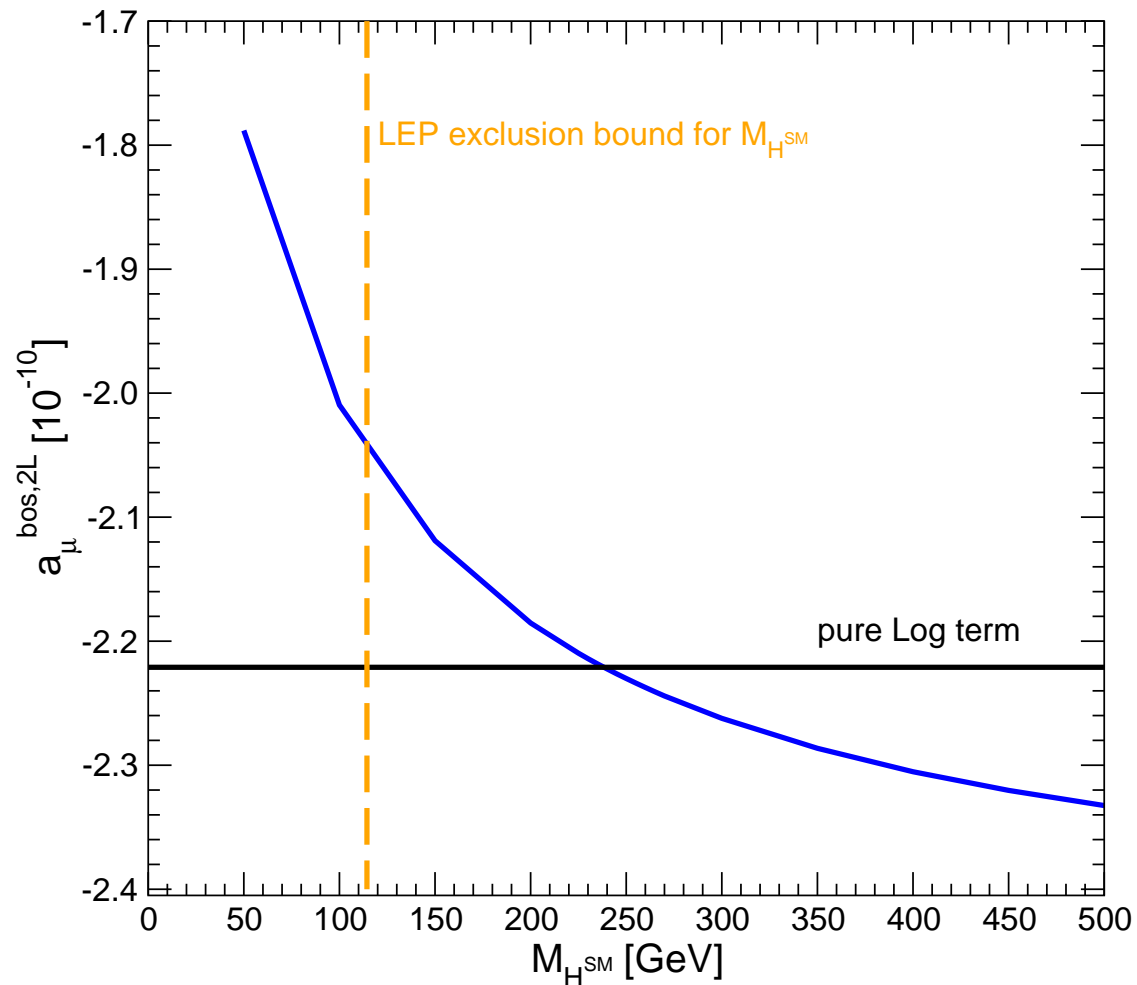
If correct:  $\Rightarrow$  new average of all data possible . . .

## Restrictions on $M_H$ from $a_\mu$ ?

⇒ Higgs enters only at the two-loop level

Example for  $M_H$  dependence:

[S.H., D. Stöckinger, G. Weiglein '04]



⇒ no restrictions on  $M_H$  (but just wait a bit ... :-)