A Perturbative Realization of Miransky Scaling

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Near-Conformal Gauge Theory

We study a general gauge theory model, and in particular how it changes with the number of flavors [1]. We have calculated the beta functions to two-loop order and conducted a thorough fixed point analysis. We find that the model exhibits Miransky scaling and has distinct IR-free, conformal, walking and asymptotically free phases.

The AMS Model

The model under consideration is given by the Lagrangian [2]

$$\begin{aligned}\mathcal{L}_{AMS} = &-\frac{1}{4}\mathrm{Tr}F^2 + i\lambda^a\sigma^\mu D_\mu\bar{\lambda}^a + i\bar{q}_i\bar{\sigma}^\mu D_\mu q_i + i\bar{\widetilde{q}}_i\bar{\sigma}^\mu D_\mu \widetilde{q}_i + \mathrm{Tr}|\partial_\mu H|^2 \\ &+ y_H\widetilde{q}_i H_{ij} q_j + y_H \bar{q}_i H^\dagger_{ij}\bar{\widetilde{q}}_j - u_1 \mathrm{Tr}H^\dagger H \mathrm{Tr}H^\dagger H - u_2\mathrm{Tr}H^\dagger H H^\dagger H, \end{aligned} \tag{1}$$

and its field content is neatly expressed in Table 1.

Table 1: The first three fields are Weyl spinors in the $\left(\frac{1}{2},0\right)$ representation of the Lorentz group. H is a complex scalar and G_μ are the gauge fields.

Fields	$[SU(N_c)]$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_V$	$U(1)_{AF}$
λ	Adj	1	1	0	1
q	$\square$	$\overline{\square}$	1	$\frac{N_f-N_c}{N_c}$	$-\frac{N_c}{N_f}$
$\widetilde{q}$	$\overline{\square}$	1	$\square$	$-\frac{N_f-N_c}{N_c}$	$-\frac{N_c}{N_f}$
H	1	$\square$	$\overline{\square}$	0	$\frac{2N_c}{N_f}$
G_μ	Adj	1	1	0	0

Two Loops Beta Functions

In terms of the rescaled couplings (6), the beta functions are

$$\beta_{a_g} = -\frac{2}{3}a_g^2\left[9-2x+(18-13x)\,a_g+3x^2a_H\right] \tag{2}$$

$$\begin{aligned}\beta_{a_H} = a_H\Big[&2(x+1)a_H - 6a_g + (8x+5)a_g a_H + \frac{20x-183}{6}a_g^2 \\ &- 8xz_2a_H - \frac{x(x+12)}{2}a_H^2 + 4z_2^2\Big]\end{aligned} \tag{3}$$

$$\begin{aligned}\beta_{z_1} = &4(z_1^2+4z_1z_2+3z_2^2+z_1a_H) + 2\big[z_1a_ga_H + 2x^2a_H^3 + x(4z_2-3z_1)a_H^2 \\ &- 4z_1^2a_H - 12z_2^2a_H - 16z_1z_2a_H - 48z_2^3 - 20z_1z_2^2\big]\end{aligned} \tag{4}$$

$$\begin{aligned}\beta_{z_2} = &2\left(2z_2a_H + 4z_2^2 - xa_H^2\right) + 2\big[5z_2a_ga_H - 2xa_ga_H^2 + 2x^2a_H^3 \\ &- 3xz_2a_H^2 - 8z_2^2a_H - 12z_2^3\big]\end{aligned} \tag{5}$$

The Veneziano Limit

Though this model contains N_c colors and N_f flavors (see Table 1), we only consider the Veneziano limit where $N_c \to \infty$ and $N_f \to \infty$ with the ratio $x = N_f/N_c$ constant. In the this limit, we must use the rescaled couplings

$$a_g = \frac{g^2N_c}{(4\pi)^2}\,,\quad a_H = \frac{y_H^2N_c}{(4\pi)^2}\,,\quad z_1 = \frac{u_1N_f^2}{(4\pi)^2}\,,\quad z_2 = \frac{u_2N_f}{(4\pi)^2}\,. \tag{6}$$

Fixed Points

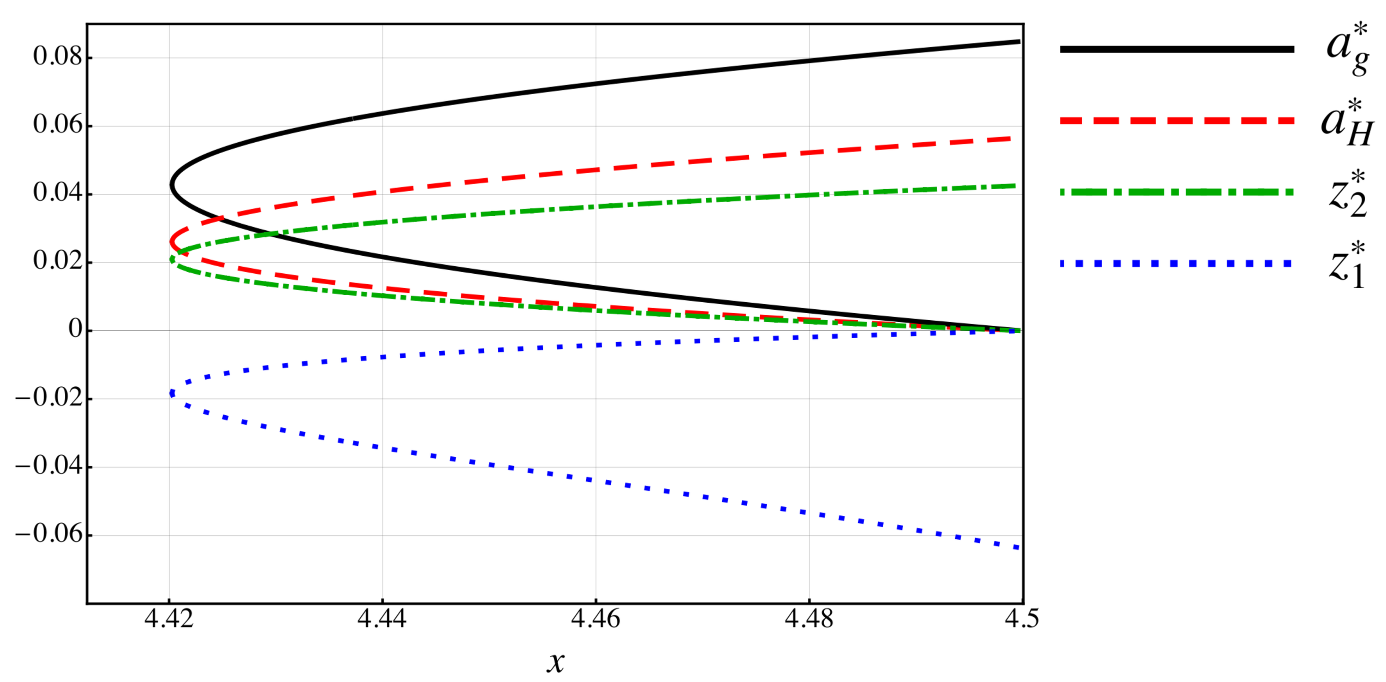

Figure 1: Demanding that the beta functions all vanish yield these values of the couplings and a *conformal window* from $x \approx 4.42$ to $x = 4.5$.

Beta Function Below the Merger

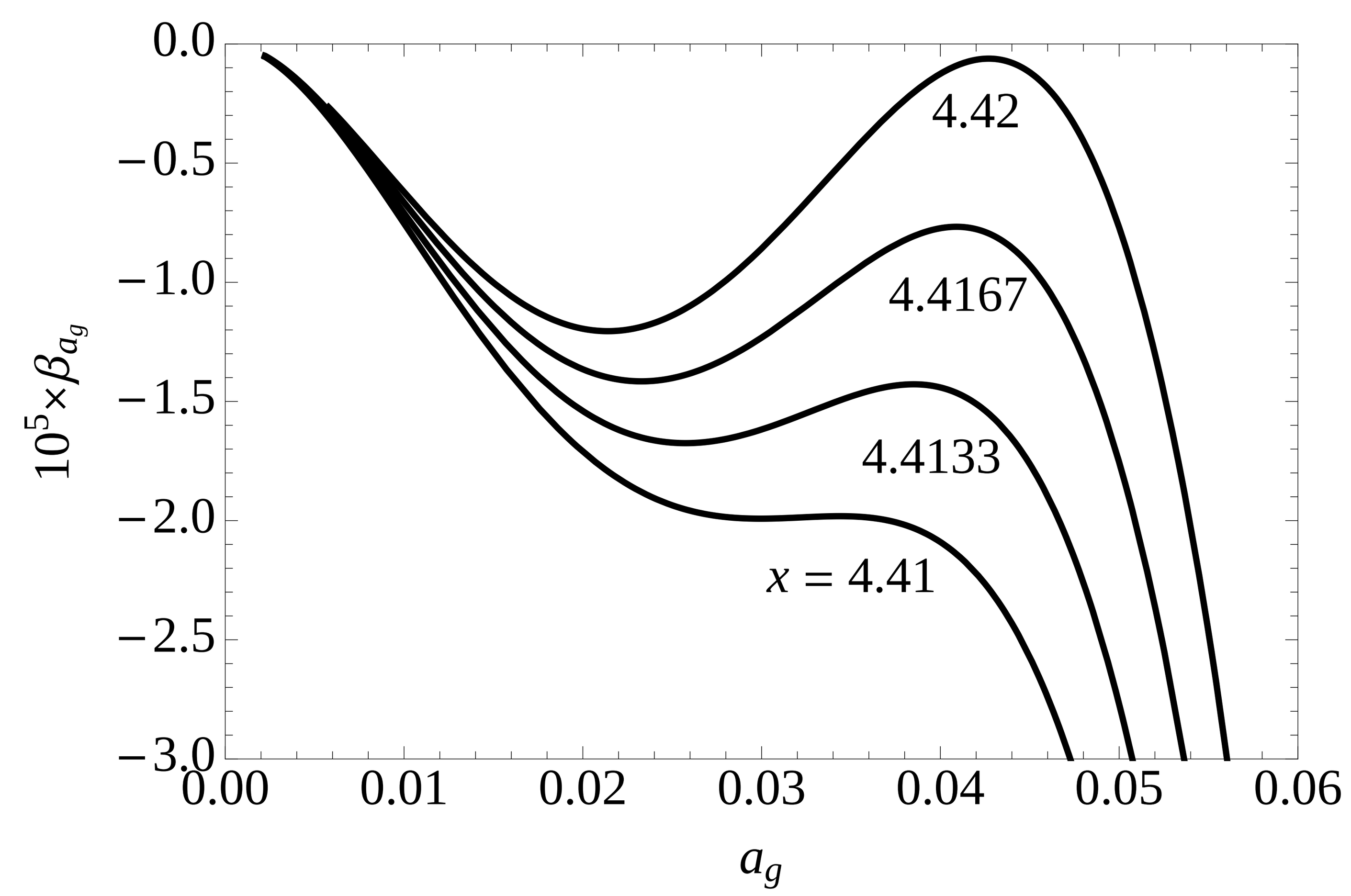

Figure 2: β_{a_g} for values of x just below the merger.

Miransky Scaling

When the beta function has a local maximum, but no fixed points (see Figure 2), the theory acquires a physical scale. As x moves away from the critical value x^*, this scale increases and the rate at which it does this can be computed by integrating the inverse of the beta function:

$$\log\frac{\Lambda}{\mu_0} = \int_{a_g(\mu_0)}^{a_g(\Lambda)}\frac{\mathrm{d}a_g}{\beta_{a_g}} \approx \int_{a_g^*}^{\infty}\frac{1}{\beta_0 + \frac{\beta_2(a_g-a_g^*)^2}{2!}}\mathrm{d}a_g \tag{7}$$

$$\approx -\frac{\pi}{c_1\sqrt{x^*-x}}\,,\qquad c_1 = 2.99\times 10^{-2} \tag{8}$$

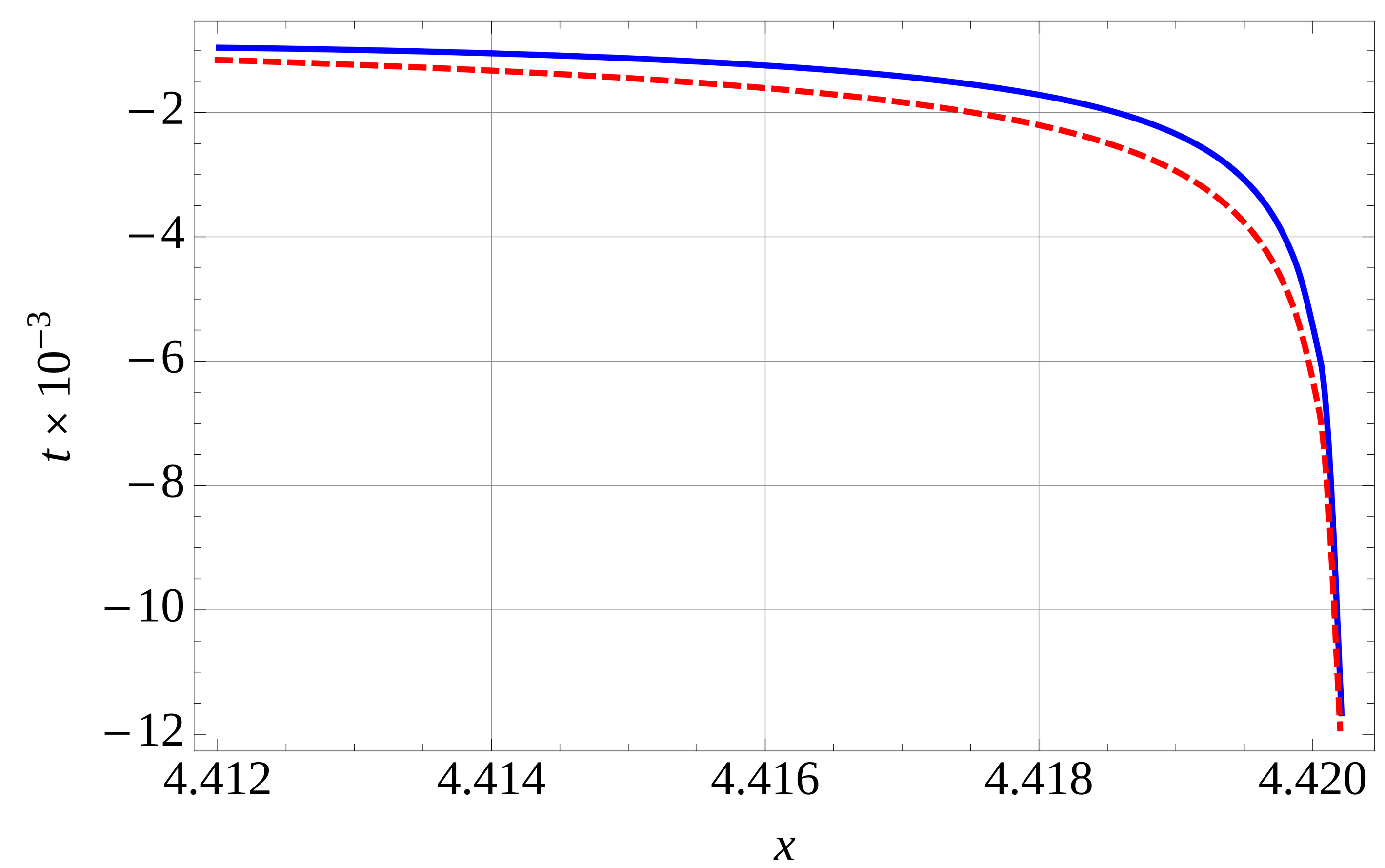

Figure 3: The blue line is a numerical integration of the beta function, the red line is equation (8).

Distinct Regions

Table 2: We have labeled the values of x where Miransky scaling holds the *walking window*, and see four distinct regions.

$x \lesssim 4.41$	$4.41 \lesssim x \lesssim 4.42$	$4.42 \lesssim x < 4.5$	$4.5 < x$
Asymptotically free	Walking window	Conformal window	Infrared free

References

[1] O. Antipin, S. Di Chiara, M. Mojaza, E. Mølgaard and F. Sannino, arXiv:1205.6157 [hep-ph].

[2] O. Antipin, M. Mojaza and F. Sannino, Phys. Lett. B **712**, 119 (2012) [arXiv:1107.2932 [hep-ph]].