Transverse PDFs at NNLO

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Outline

1. q_T resummation at hadron colliders
2. Transverse parton distribution functions at next-to-next-to-leading order
3. Status and outlook

Hadron colliders

- $\Rightarrow$ extract σ_{process}, $\frac{d\sigma_{\text{process}}}{dv}$, ...

  e.g. for $p\bar{p} \to Z + X$ differential in q_T
- $\Rightarrow$ results depend on many mass scales

  s, M_Z, q_T, ...

  can be of very different size.

Perturbation theory

- perturbative expansion:

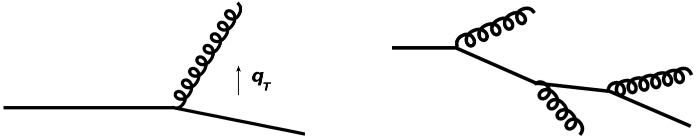
$$d\sigma = \sum_{n=0}^{\infty} \alpha_S^n d\sigma^{(n)}$$

  fixed order: stop at e.g. $n = 2$ (NNLO).

- For each power in α_S find large logarithms of scale ratios, which
  - spoil convergence,
  - need to be resummed to all orders in α_S.

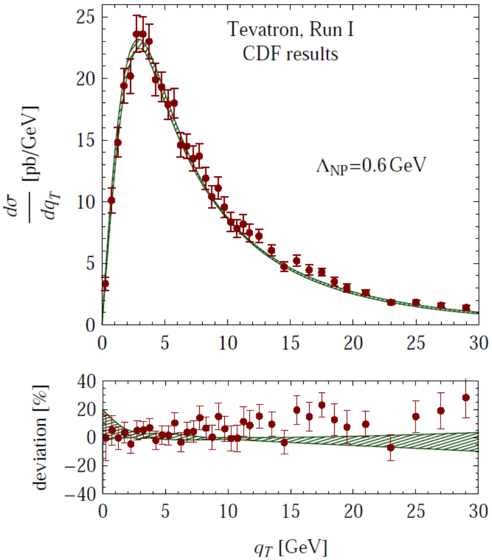
Large logarithms

Momenta of emitted particle soft, collinear
$\Rightarrow$ up to $\log^2 \frac{q_T^2}{Q^2}$ for each power α_S.



Due to specific structure, factorize, can account for (resum) them to all orders in α_S.

q_T spectrum, $p\bar{p} \longrightarrow Z + R$



resummed result

for large $q_T \longrightarrow$ fixed order

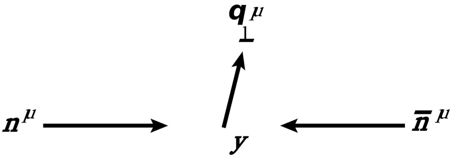
for small $q_T \longrightarrow$ resummation

phenomenologically important, e.g. analog $W \longrightarrow$ mass

[Becher, Neubert, Wilhelm, 2011]

Notation

Directions of incoming hadrons and outgoing vector boson:



- With $n^2, \bar{n}^2 = 0, \; n \cdot \bar{n} = 2.$
- Decompose $v^\mu = (\bar{n} \cdot v)\frac{n^\mu}{2} + (n \cdot v)\frac{\bar{n}^\mu}{2} + v_\perp^\mu$
- $v_\perp^2 = -v_T^2$

Factorized differential cross section

At $\Lambda \ll q_T \ll M_V$ factorization formula [Becher, Neubert, 2010]:

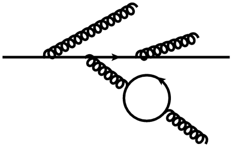
$$\frac{d^2\sigma_{N_1 N_2 \to V+X}}{dq_T^2\,dy} \sim \left|C_V(-M_V^2,\mu)\right|^2 \int d^2x_\perp\, e^{-i\,q_\perp\cdot x_\perp} \sum_q g_q^2$$

$$\times \left[\mathcal{B}_{q/N_1}(\xi_1, x_T^2, \mu)\bar{\mathcal{B}}_{\bar{q}/N_2}(\xi_2, x_T^2, \mu) + (q \leftrightarrow \bar{q})\right]_{M_V^2} + \mathcal{O}\left(\frac{q_T^2}{M_V^2}\right),$$

- Compare to $d\sigma = \hat{\sigma}_{12}\phi_1\phi_2$.
- q_T dependent; Fourier space.
- $\xi_{1,2} = \sqrt{\tau}e^{\pm y}$, $\tau = (M_V^2 + q_T^2)/s$.
- $\mathcal{B}$ are TPDFs, C_V is the hard function.

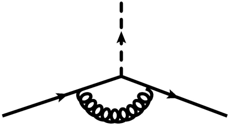
Factorized differential cross section, pictorial

$$\mathcal{B}_{q/N_1}(\xi_1, x_T^2, \mu)$$



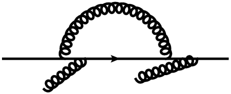
'transverse PDF'

$$C_V(-M_V^2, \mu)$$



'hard function'

$$\bar{\mathcal{B}}_{\bar{q}/N_2}(\xi_2, x_T^2, \mu)$$



'transverse PDF'

$n^\mu \longrightarrow$

$\longleftarrow \bar{n}^\mu$

Operator definition of (T)PDF

Operator definition of TPDF (quark case) in SCET

$$\mathcal{B}_{q/N}(z, x_T^2, \mu) = \frac{1}{2\pi} \int \mathrm{d}t \; e^{-izt\bar{n}\cdot k} \langle N(k) | \, \bar{\chi}(t\bar{n} + x_\perp) \frac{\bar{\not{n}}}{2} \chi(0) \, | N(k) \rangle \; .$$

Compare to usual collinear PDF

$$\phi_{q/N}(z, \mu) = \frac{1}{2\pi} \int \mathrm{d}t \; e^{-izt\bar{n}\cdot k} \langle N(k) | \, \bar{\chi}(t\bar{n}) \frac{\bar{\not{n}}}{2} \chi(0) \, | N(k) \rangle \; .$$

Above $\bar{\chi}(x) = \bar{\xi}(x) W(x)$, with the hard-collinear quark field $\xi(x)$ and the hard-collinear Wilson line $W(x)$.

Replace above hadron N by parton p $\Rightarrow$ perturbatively calculable $\mathcal{B}_{ip}$ and ϕ_{jp}.

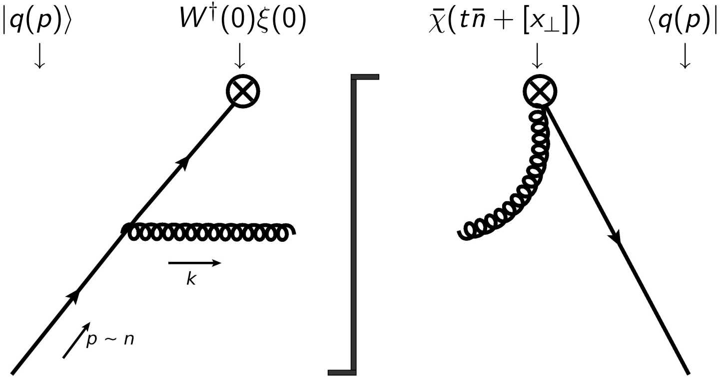
Matching kernel

Matching kernel $\mathcal{I}_{i\leftarrow j}$ relates $\mathcal{B}$ and ϕ

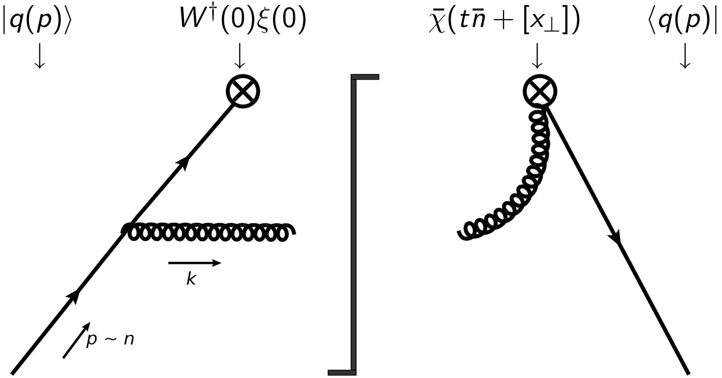
$$\mathcal{B}_{i/N}(\rho, x_T^2, \mu) = \sum_j \int_\rho^1 \frac{dz}{z} \mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu) \phi_{j/N}(\rho/z, \mu) + \mathcal{O}(\Lambda^2 x_T^2)$$

- Compare to $\frac{d\phi}{d\log\mu^2} = P \otimes \phi$ (DGLAP).
- ϕ usual PDFs. Non-perturbative, but extractable from data.
- For $x_T^{-2} \gg \Lambda^2_{\mathrm{QCD}}$, $\mathcal{I}_{i\leftarrow j}$ can be determined from $\mathcal{B}_{ip}$ and ϕ_{pj} (perturbatively).

Example diagram at NLO



Example diagram at NLO

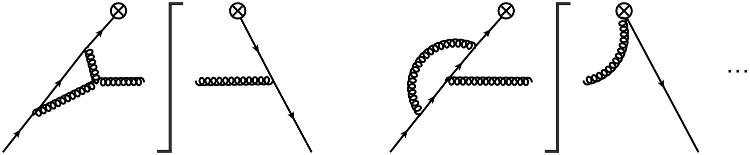


$$\frac{\alpha_S}{4\pi}\left(\mathcal{B}^{(1)}_{qq},\phi^{(1)}_{qq}\right)=\int\frac{\mathrm{d}^d k}{(2\pi)^d}(2\pi)\delta^+(k^2)\delta(\bar{n}\cdot[k-(1-z)p])\left(e^{ik_T\cdot x_T},1\right)\mathcal{M}$$

Contribution from purely virtual diagrams cancel for $\mathcal{I}$.

Example diagrams at NNLO

Virtual-Real contribution:



Example diagrams at NNLO

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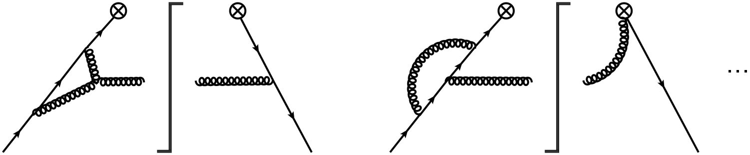
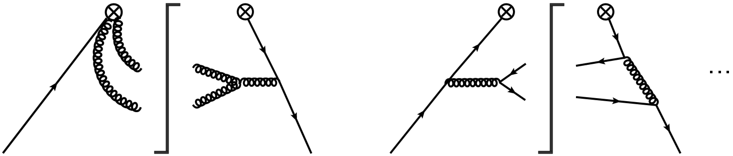


Real-Real contribution:



Analytic regularization

- Integrals contain poles, which are not regularized in dimensional regularization. Definition of $\mathcal{B}$ problematic.

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- Chose 'analytic regularization' [Becher, Bell, 2011]:
  - $\Rightarrow$ For each emitted particle (with momentum k) multiply integrand by $\left(\frac{\nu}{n \cdot k}\right)^{\alpha}$.
- Advantage: Do not introduce new denominators. Can expand in α (for NNLO only need up to α^0).

Anomaly

- $\mathcal{B}$ has poles in new regulator α.
- Have to consider $\mathcal{B}\bar{\mathcal{B}}$, which should not depend way of regularization. In especially all poles in α should cancel. Remaining poles in $d-4$ removed by $\overline{MS}$ renormalization.
- However, resulting expression depends on hard scale M_V^2 $\longrightarrow$ anomaly.
- Can be refactorized [Becher, Neubert, 2010].

Refactorization

Refactorize $\mathcal{B}\bar{\mathcal{B}}$:

$$[\mathcal{B}_{ip_1}(z_1, x_T^2, \mu)\bar{\mathcal{B}}_{\bar{i}p_2}(z_2, x_T^2, \mu)]_{M_V^2} = \left(\frac{x_T^2 M_V^2}{4e^{-2\gamma_E}}\right)^{-F_{i\bar{i}}(x_T^2,\mu)} \times B_{ip_1}(z_1, x_T^2, \mu) B_{\bar{i}p_2}(z_2, x_T^2, \mu) \,.$$

- Defines TPDFs B, which are independent of hard scale M_V^2. F parameterizes M_V^2 dependence, which naively was not expected.

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- Defines TPDFs B, which are independent of hard scale M_V^2. F parameterizes M_V^2 dependence, which naively was not expected.
- In analogy to $\mathcal{I}$ define I by

$$B_{i/N}(\rho, x_T^2, \mu) = \sum_j \int_\rho^1 \frac{dz}{z} I_{i\leftarrow j}(z, x_T^2, \mu)\phi_{j/N}(\rho/z, \mu) + \mathcal{O}(\Lambda_{\rm QCD}^2 x_T^2) \,.$$

Status

- Above discussed relations and calculation allows extraction of the functions I and F as well as $B = I \otimes \phi$.
- We performed the NNLO calculation of $I_{q \leftarrow q}(z_1, x_T^2, \mu)$ and $F_{q\bar{q}}(x_T^2, \mu)$ using 'analytic' regularization.

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- Solved all integrals: many parameters, non-integer denominators, not Lorentz invariant $\rightarrow$ many standard methods inapplicable.
- All poles in α canceled $\Rightarrow$ analytic regularization applicable at NNLO (first time explicitly found).
- $F_{q\bar{q}}$ agrees with literature [Becher, Neubert 2010].
- $I_{q \leftarrow q}$: correct differential dependence on $\log(\mu)$ [Curci, Furmanski, Petronzio, 1980].

RG evolution & application

§ Once have factorized result, s.t. each function depends only on its typical mass M_{typical} and μ,

$\longrightarrow$ can consistently determine each function in fixed order pertrubation theory at $\mu \sim M_{\text{typical}}$,

$\longrightarrow$ subsequently use RG equation to evolve to common scale μ_m.

$\Rightarrow$ Resumms logarithms.

$\longrightarrow$ All ingredients at NNLO $\Rightarrow$ NNLO+NNNLL precision.

Outlook and conclusions

- For phenomenology at small q_T need resummation and TPDFs.
- Definition & calculation suffer from additional singularities.
- NNLO calculation completed for $q \to q$ using analytic regularization.
- Remaining parton combinations will follow.
- In combination with hard functions, will improve theory accuracy of $d\sigma$ at small q_T to NNLO+NNNLL for processes with color neutral final state: single and multiple vector/Higgs boson production, new color neutral particles...

THE END

Factroized cross section

Use results for phenomenology
⇒Back to our starting expression (rephrased):

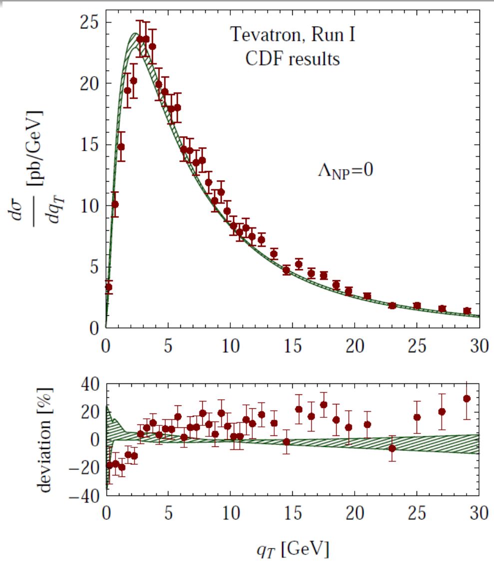
$$\frac{d^2\sigma}{dM^2 dq_T^2 dy} = \frac{\pi\alpha^2}{3N_c M^2 s} \left|C_V(-M^2,\mu)\right|^2 \sum_{i=q,\bar{q},g} \int_{\xi_1}^1 \frac{dz_1}{z_1} \int_{\xi_2}^1 \frac{dz_2}{z_2} \int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}$$

$$\times \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T,\mu)} \sum_q e_q^2 \left[I_{q\leftarrow i}(z_1,x_T^2,\mu) I_{\bar{q}\leftarrow j}(z_2,x_T^2,\mu)\right.$$

$$\left.\times\ \phi_{i/N_1}(\xi_1/z_1,x_T^2,\mu)\phi_{j/N_2}(\xi_2/z_2,x_T^2,\mu) + (q \leftrightarrow \bar{q})\right]$$

- Separated *all* physical scales into different functions, consistently use fixed order expressions choosing μ at their typical scale, such that no large logs appear.

q_T spectrum, $p\bar{p} \longrightarrow Z + R$



resummed result

Apart from PDFs no non-perturbative effects included.

[Becher, Neubert, Wilhelm, 2011]