Caesium Vector Magnetometry for nEDM

Data analysis and feasibility of the idea

S. Afach

Paul Scherrer Institut,
ETH Zürich,
Universitätsklinikum Jena

Outline

1. Experimental setup
   - Measurement procedure
   - How does a Cs magnetometer work?

2. Our Results/Contribution
   - Work and progress - Simulation results
   - Future tasks

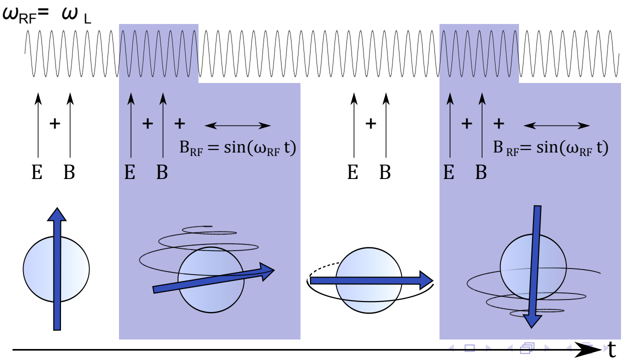
Outline

Ramsey precession

- How is the nEDM measured? The relevant Hamiltonian is:

$$H_{\text{ext}} = 2\mu_B B + 2d_n E = \hbar\omega_L$$

- Ramsey's method is used to detect the frequency in the last equation



Estimating the nEDM

- From the Hamiltonian, we use two opposite directions of E:

$$\hbar\omega_1 = 2\mu_B B_1 + 2d_n E$$

$$\hbar\omega_2 = 2\mu_B B_2 - 2d_n E$$

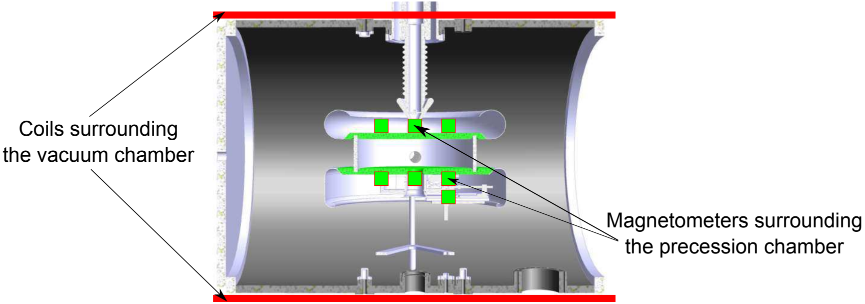
$$\hbar(\omega_1 - \omega_2) = 2\mu_B (B_1 - B_2) + 4d_n E$$

- It is easy to get the nEDM if $B_1 = B_2$. Can this be achieved?

Magnetometers in the experiment

- Vacuum chamber of the nEDM (neutron electric dipole moment) measurement experiment

Cross-section of the vacuum chamber



Required magnetic field stability

- Our goal is to detect an nEDM **of the order** 10^{-28}e $\cdot$ cm.
- Since the magnetic field's leading term in the energy formula:

$$\hbar\omega = 2\mu_B B + 2d_n E,$$

$$d_n \approx 10^{-28}\mathrm{e}\cdot\mathrm{cm},$$

$$\mu_B B \approx d_n E \quad \Rightarrow \quad \Delta B \approx 10^{-16}\,T$$

- Over a year, with **10000 measurements**:

$$\Delta B_{\text{stat}} = \sqrt{10000} \times 10^{-16=}10^{-14}\,T$$

- What about systematics?

False nEDM and geometric phase

- **Linearly correlated systematic** effects include:

$$\vec{B} = \frac{\vec{E} \times \vec{v}}{c^2}$$

- This could produce a **frequency shift** in ω_L that is **linearly dependent** on the electric field
- With a **gradient in the magnetic field** due to $\nabla \cdot \vec{B} = 0$, a **false EDM** shows up

$$\vec{B}_{xy} = -\frac{\partial B_{0z}}{\partial z}\frac{\vec{r}}{2} + \frac{\vec{E} \times \vec{v}}{c^2}$$

$$\Delta\omega \propto B_{xy}^2 = \left(\frac{\partial B_{0z}}{\partial z}\frac{\vec{r}}{2}\right)^2 + \left(\frac{\vec{E} \times \vec{v}}{c^2}\right)^2 - 2\frac{\partial B_{0z}}{\partial z}\frac{\vec{r}}{2}\frac{\vec{E} \times \vec{v}}{c^2}$$

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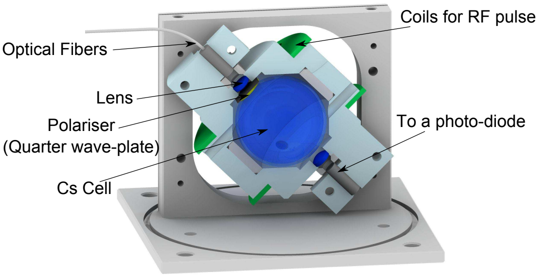
Main design units

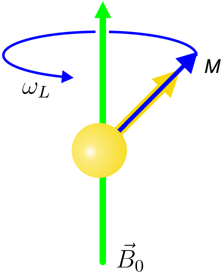


Sensor components:

- Caesium cell (magnetic field moments)
- Laser beam (that pumps the moments)
- Photo-diode (that receives the modulated laser beam)
- Radio frequency magnetic field (that drives the moments)

Response of Cs atoms to a magnetic field

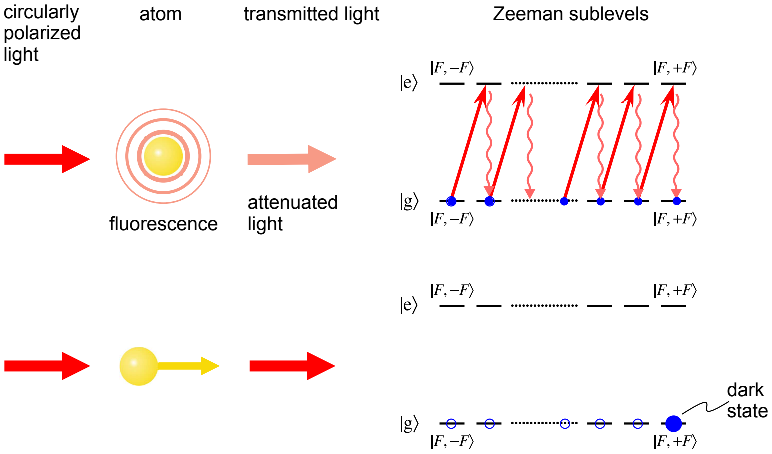
- A magnetic moment M is associated with an ensemble of atomic spins



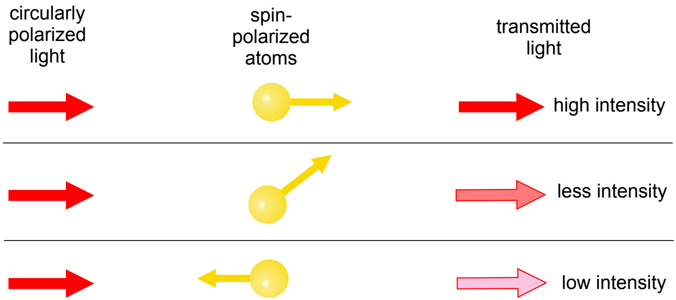
- This moment precesses in a magnetic field according to Bloch's equation

$$\omega_L = 2\pi\nu_L = \gamma\left|\vec{B}\right|$$

Dark states - laser pumping



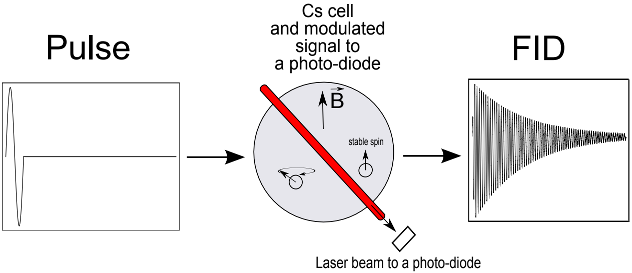
Laser modulation



- Laser **intensity depends on the orientation** of the spin
- In other words, the transmitted intensity is proportional to the **projection of the laser on the atoms**

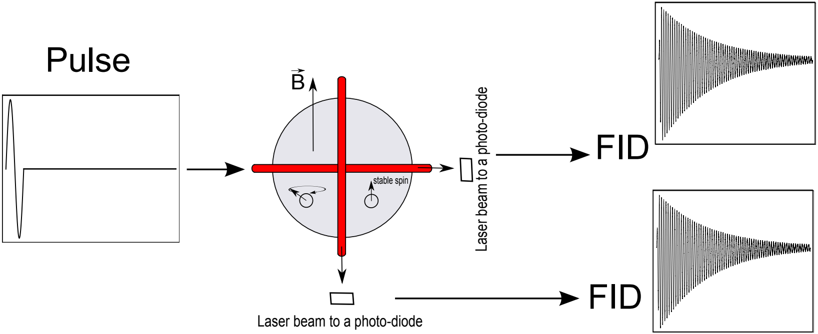
Single laser magnetometer's operation

- $\frac{\pi}{2}$ pulse stimulates precession around the applied magnetic field
- Photo-diode reads a typical FID (Free Induction Decay), whose frequency is linearly proportional to the magnetic field



Multi-laser magnetometer's operation

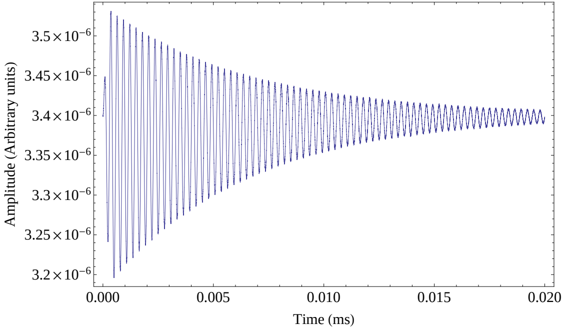
- The laser is modulated according to the projection of the magnetic moment of the spins
- Multiple laser beams allow us to follow the motion of the spins in 3D



Outline

Analytical model for FID time dependence

Simulated with Bloch equation, with 1 MHz sampling rate



2-Level spin FID (Free Induction Decay) general equation

$$f(t) = c + a \cdot \exp\left(-\frac{t}{\tau_1}\right) + A \cdot \exp\left(-\frac{t}{\tau_2}\right)\cos(\omega t + \phi)$$

$c, a, \tau_1, A, \tau_2, \omega, \phi$ are parameters to be determined

Vector magnetometer's FIDs

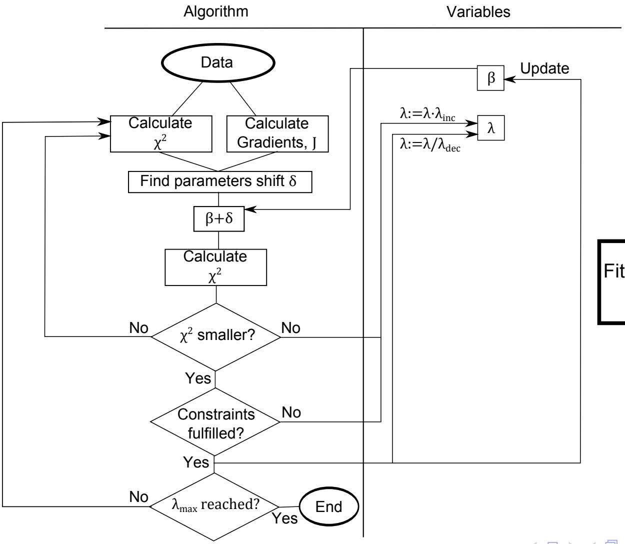
Fitting the equations simultaneosly with some shared parameters:

$$f_x(t) = c_x + a_x \cdot \exp\left(-\frac{t}{\tau_1}\right) + A_x \cdot \exp\left(-\frac{t}{\tau_2}\right)\cos(\omega t + \phi_x)$$
$$f_y(t) = c_y + a_y \cdot \exp\left(-\frac{t}{\tau_1}\right) + A_y \cdot \exp\left(-\frac{t}{\tau_2}\right)\cos(\omega t + \phi_y)$$
$$f_z(t) = c_z + a_z \cdot \exp\left(-\frac{t}{\tau_1}\right) + A_z \cdot \exp\left(-\frac{t}{\tau_2}\right)\cos(\omega t + \phi_z)$$

Parameters to estimate:

$c_x, c_y, c_z, a_x, a_y, a_z, \tau_1, A_x, A_y, A_z, \tau_2, \omega, \phi_x, \phi_y, \phi_z$

Modified Levenberg-Marquardt fitting algorithm



Fitting time achieved: 10-15 ms

Extracting the magnetic field's direction

,

$$A_i \cdot \exp\left(-\frac{t}{\tau_2}\right)\cos\left(\omega t + \phi_i\right)$$

$$i \equiv x, y, z.$$

$$B_\theta = \arccos\left(\frac{\sqrt{A_x^2\cos^2\phi_x + A_y^2\cos^2\phi_y + A_z^2\cos^2\phi_z - A_z^2}}{\sqrt{A_x^2\cos^2\phi_x + A_y^2\cos^2\phi_y + A_z^2\cos^2\phi_z}}\right);$$

$$B_\phi = \arctan\left(\frac{\sqrt{A_x^2\cos^2\phi_x + A_y^2\cos^2\phi_y + A_z^2\cos^2\phi_z - A_y^2}}{\sqrt{A_x^2\cos^2\phi_x + A_y^2\cos^2\phi_y + A_z^2\cos^2\phi_z - A_x^2}}\right).$$

$$\vec{B} \propto \left(A_x\cos\phi_x, A_y\cos\phi_y, A_z\cos\phi_z\right) \times \left(A_x\sin\phi_x, A_y\sin\phi_y, A_z\sin\phi_z\right)$$

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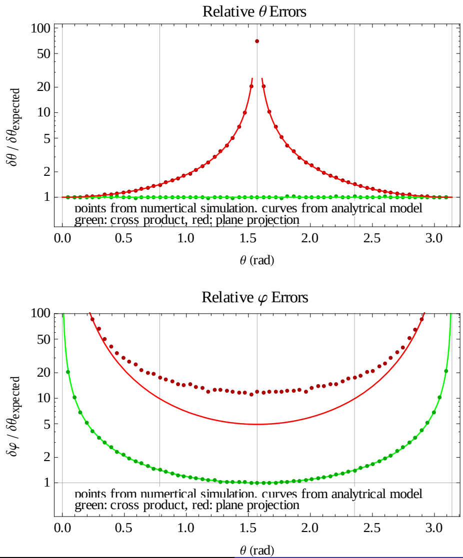
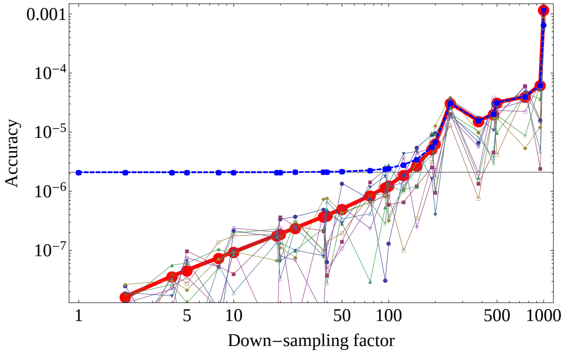


Outline

Find the lowest allowed sampling rate

Frequency estimation deviation as a function of the down-sampling factor

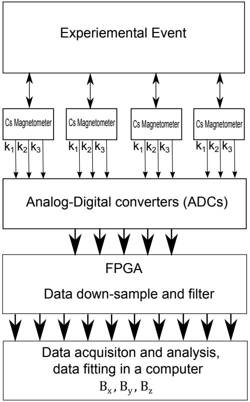


Systematic errors due to down-sampling (red) Vs statistical and systematic errors (blue)

Summary

- **Accurate magnetometers** necessary to find an nEDM
- **Cs vector magnetometry is an idea** that is based on having more than one laser on a Cs magnetometer
- **The idea** of Cs vector magnetometery **looks feasible** from our initial results and error estimation

Final experimental setup



- Outlook
    - Apply the analysis on some real data
    - Develop the devices for the measurement