The role of the input scale in parton distribution analyses

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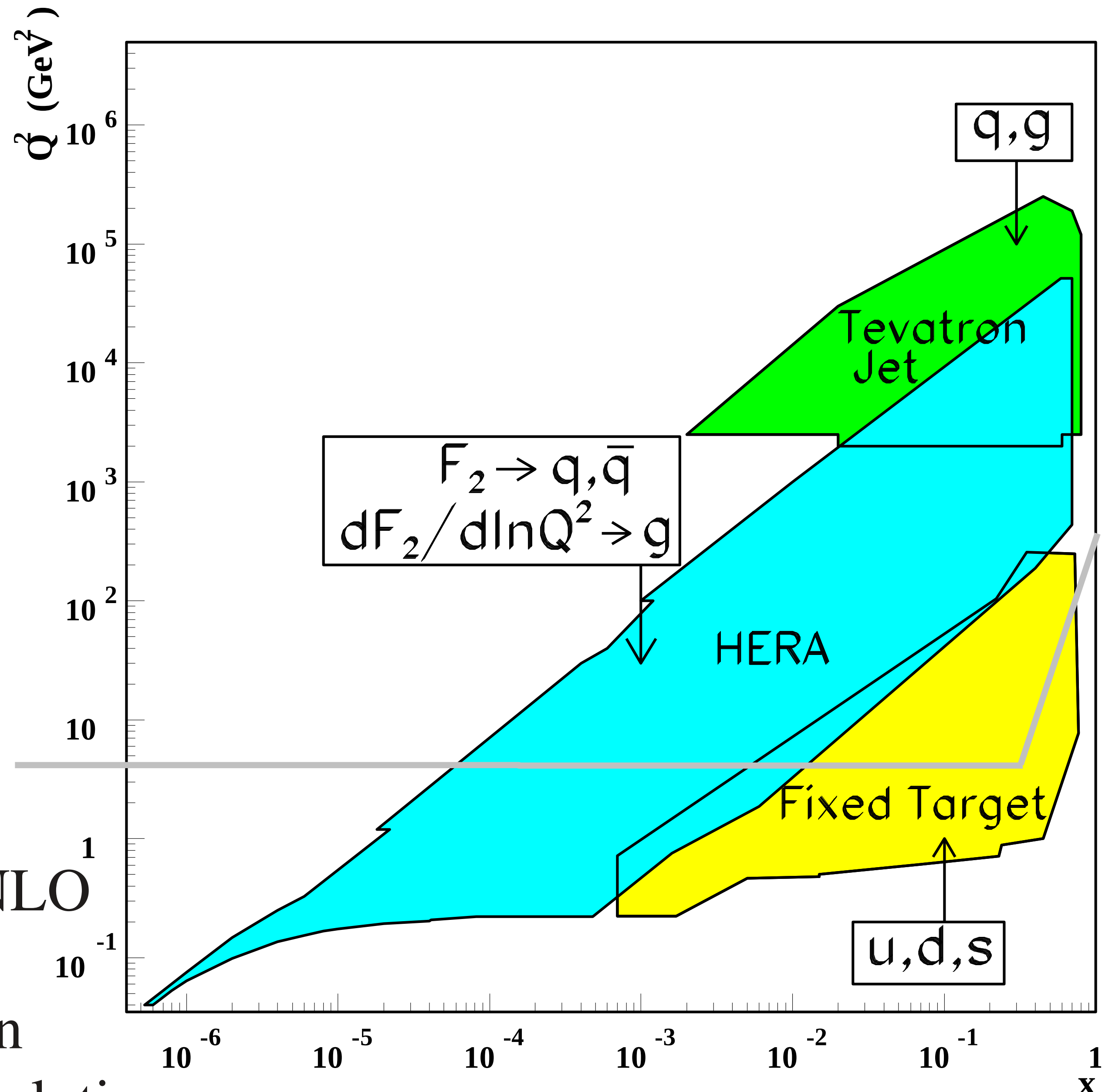
DIS data and kinematic cuts

Inclusive DIS structure functions at small (and large) x often excluded via kinematic cuts:

$$Q^2 \gtrsim 4\ \mathrm{GeV}^2,\ \ W^2 \gtrsim 10\ \mathrm{GeV}^2$$

Gluon only enters (at LO) in jet production (large-x) and semi-inclusive heavy-quark production in DIS (small-x) $\Rightarrow$

Cannot be *directly* and *consistently* determined at NNLO



[Particle Data Group]

However these regions enter in most calculations trough convolutions

Parametrizations and the dynamical approach

Parametrizations: $xf(x, Q_0^2) = Nx^a(1-x)^b\ \text{function}(x)$

"function" may be polynomial, contain exponentials, neural networks, ...

Since we are free to (and have to) select an input scale for the RGE:

At low-enough Q^2 only "valence" partons would be "resolved"

⇒ *structure at higher Q^2 appears radiatively (QCD* ***dynamics****)*

DYNAMICAL:

$Q_0^2 < 1\text{GeV}^2$ optimally determined

Valence-like structure

QCD "predictions" for small-x

More predictive, less uncertainties

"STANDARD":

Arbitrarily fixed $Q_0^2 \gtrsim 1\text{GeV}^2$

Fine tunning to particular data

Extrapolations to unmeasured regions

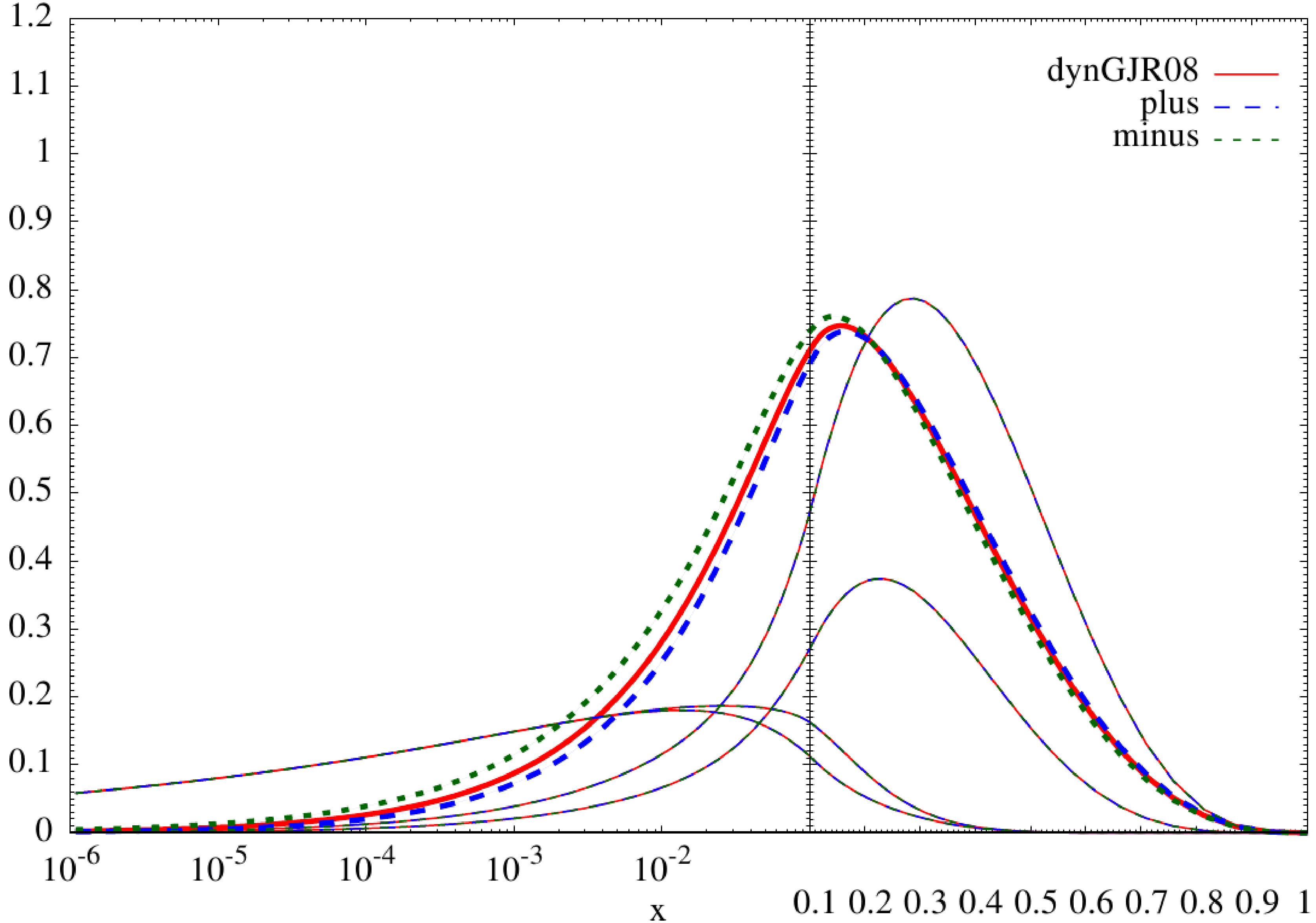
More adaptable, marginally smaller χ^2

There are no extra constraints involved in the dynamical approach

Physical motivation for contour conditions ≠ non-perturbative structure

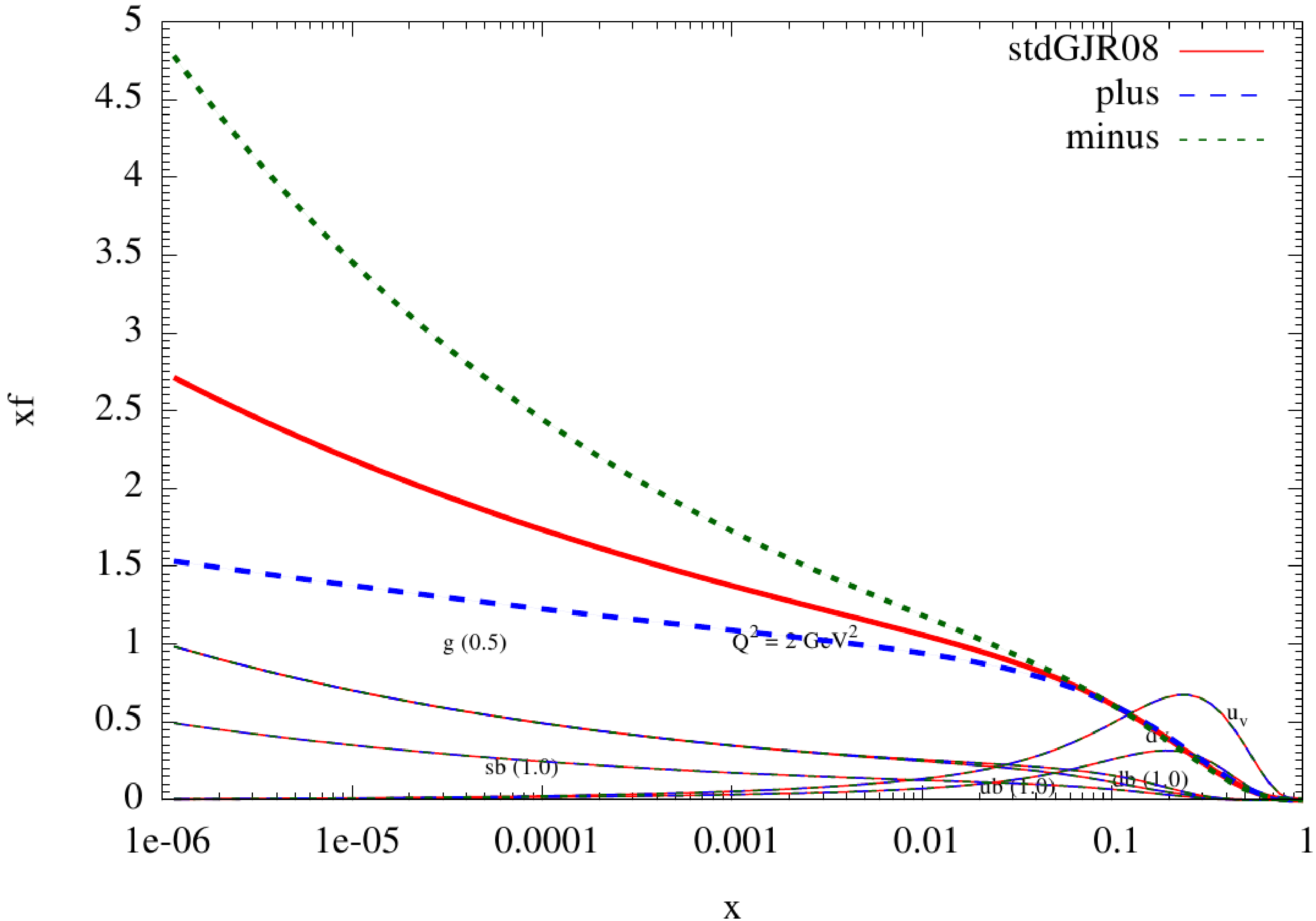
Parametrizations and the dynamical approach

An illustration: GJR08 input ± 0.05 at $Q_0^2 = 0.5\ \mathrm{GeV}^2$



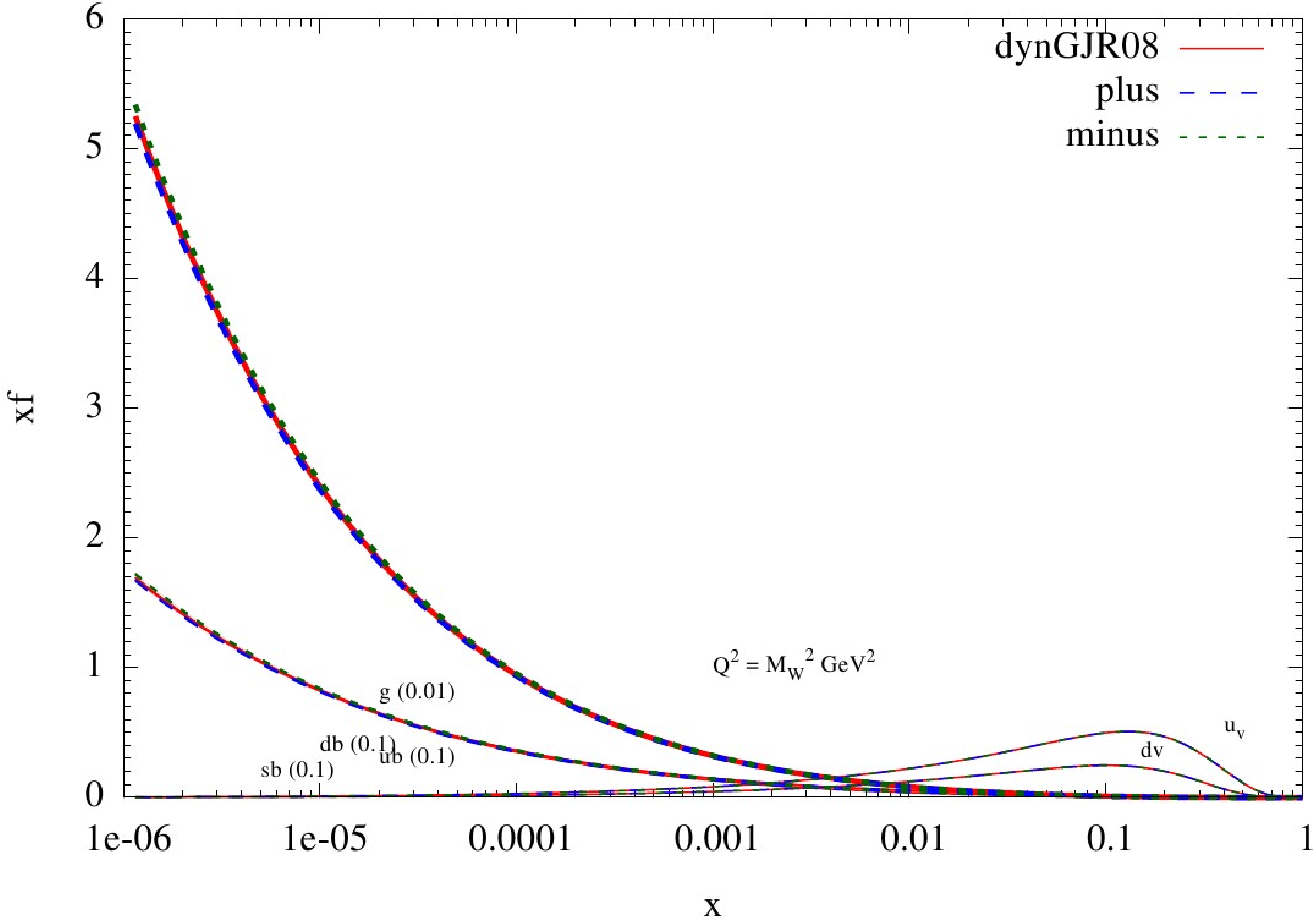
Parametrizations and the dynamical approach

An illustration: GJR08 input ± 0.05 at $Q_0^2 = 2\ \mathrm{GeV}^2$



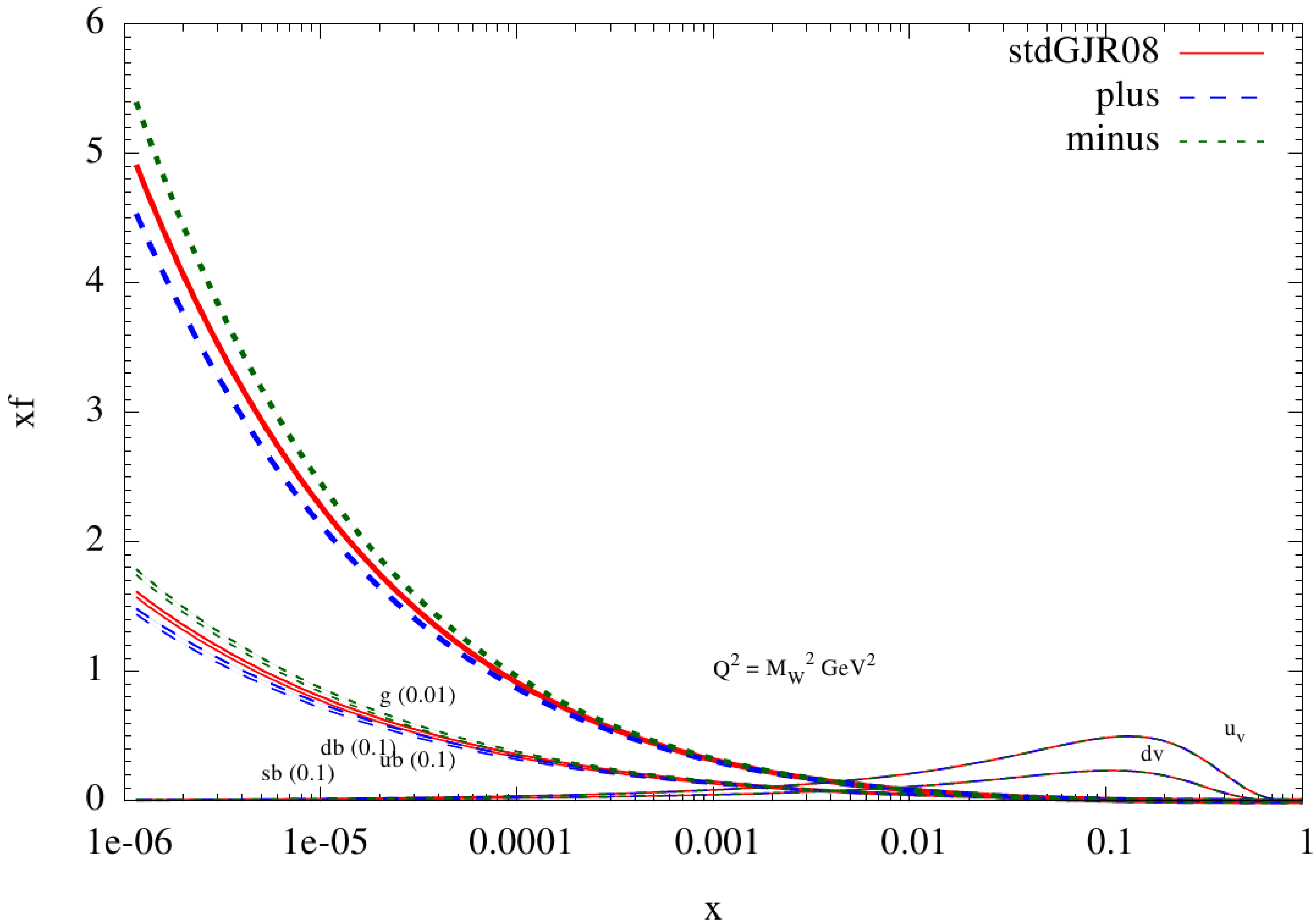
Parametrizations and the dynamical approach

An illustration: GJR08 input ± 0.05 at $Q^2 = M_W^2$

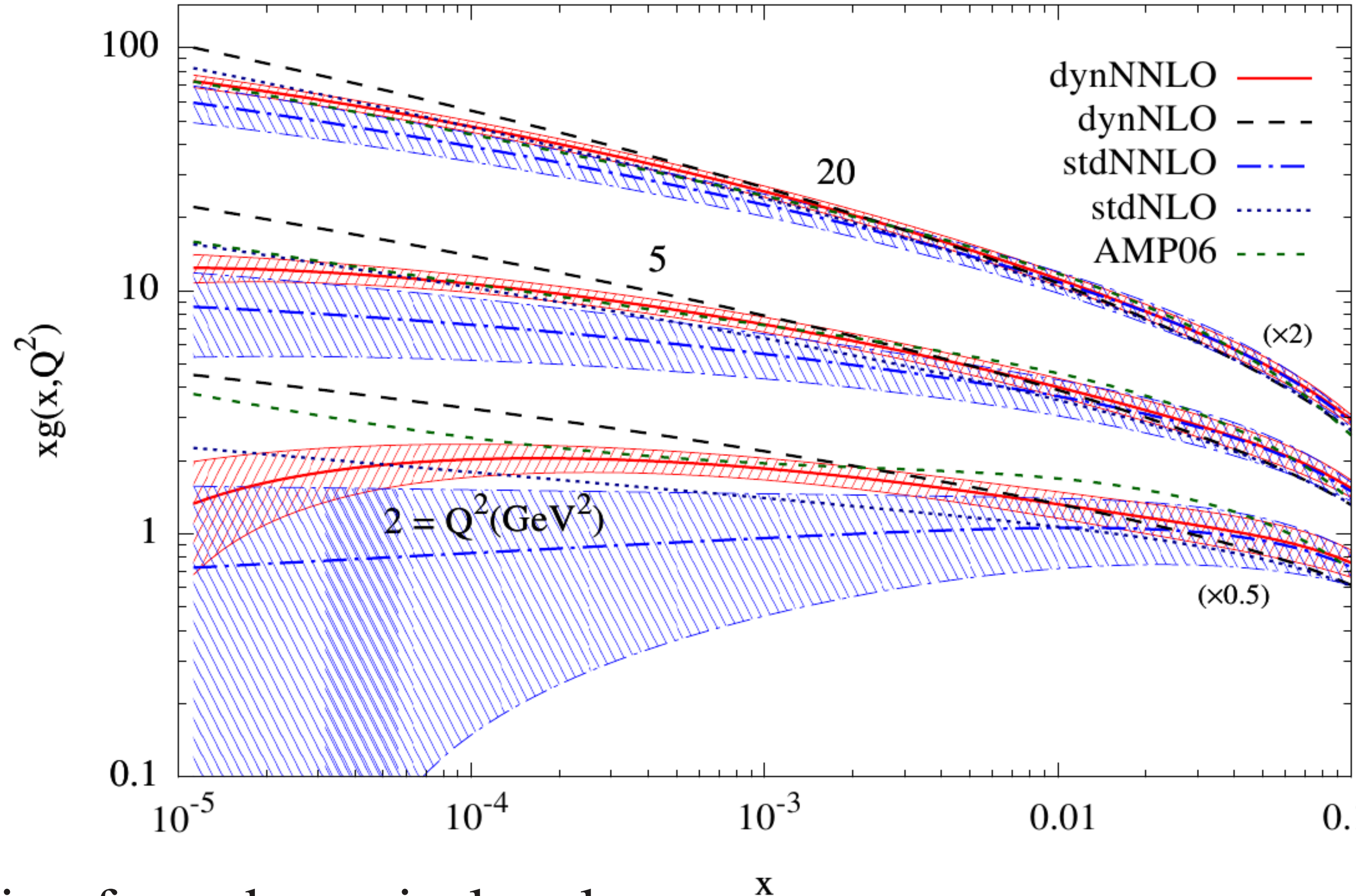


Parametrizations and the dynamical approach

An illustration: GJR08 input ± 0.05 at $Q^2 = M_W^2$



Parametrizations and the dynamical approach



Evolution from dynamical scales:

larger “evolution distance”+ valence-like structure (of the input distributions)

⇒ less uncertainties and steeper gluons (correspondingly smaller χ^2)

Fine tunning marginal (e.g. for DIS in JR09, $\chi^2_{dyn} = 0.90$, $\chi^2_{std} = 0.87$)

The role of the input scale

Once an optimal solution is found using an input scale, equally good solutions do exist at different scales:

*Any dependence is due to shortcomings of the estimation: **procedural bias***

For example (but not exclusively) parametization bias

Note, e.g., that backwards evolution to low scales leads to oscillating gluons (imposible to cast with finite precision)

Excersise: systematic study with progresively more flexible parametrizations

$$xf(x, Q_0^2) = N_f \; x^{a_f}(1-x)^{b_f}(1 + A_f\sqrt{x} + B_f x + C_f x^2)$$

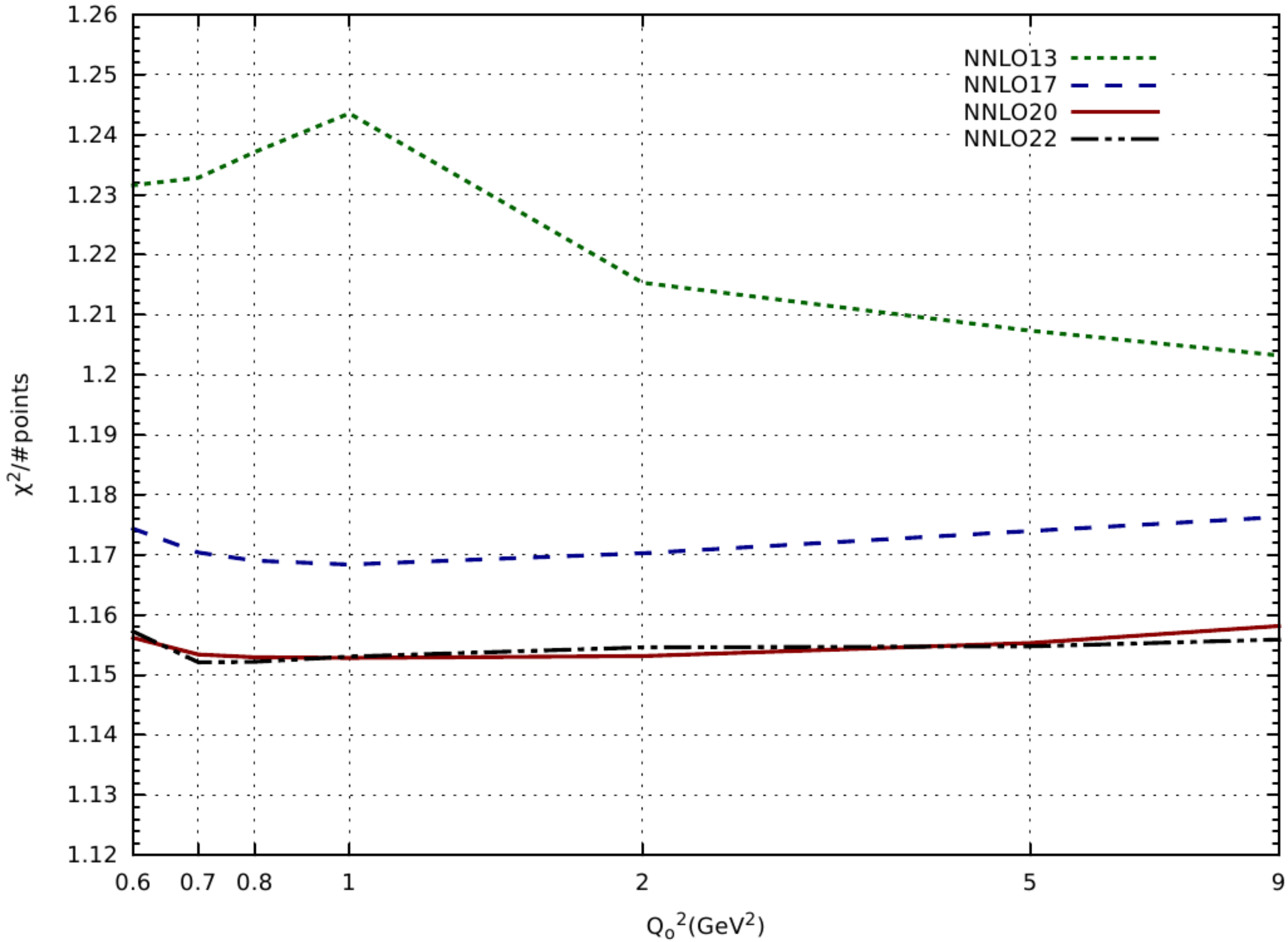
Allow also for negative input gluons:

$$\mathrm{xg(x,Q_0^2)} = N_g \; x^{a_g}(1-x)^{b_g}\left(1 + N_g' \; x^{a_g'}(1-x)^{25}\right)$$

	NNLO13	NNLO17	NNLO20	NNLO22
a_{u_v}	$\surd$	$\surd$	$\surd$	$\surd$
b_{u_v}	$\surd$	$\surd$	$\surd$	$\surd$
A_{u_v}	–	–	$\surd$	$\surd$
B_{u_v}	–	$\surd$	$\surd$	$\surd$
C_{u_v}	$\surd$	$\surd$	$\surd$	$\surd$
a_{d_v}	$\surd$	$\surd$	$\surd$	$\surd$
b_{d_v}	$\surd$	$\surd$	$\surd$	$\surd$
A_{d_v}	–	$\surd$	$\surd$	$\surd$
B_{d_v}	–	–	$\surd$	$\surd$
C_{d_v}	–	–	–	–
N_Δ	$\surd$	$\surd$	$\surd$	$\surd$
a_Δ	$\surd$	$\surd$	$\surd$	$\surd$
b_Δ	$\surd$	$\surd$	$\surd$	$\surd$
A_Δ	–	$\surd$	$\surd$	$\surd$
B_Δ	–	–	–	–
C_Δ	–	–	–	–
N_Σ	$\surd$	$\surd$	$\surd$	$\surd$
a_Σ	$\surd$	$\surd$	$\surd$	$\surd$
b_Σ	$\surd$	$\surd$	$\surd$	$\surd$
A_Σ	–	$\surd$	$\surd$	$\surd$
B_Σ	–	–	$\surd$	$\surd$
C_Σ	–	–	–	–
a_g	$\surd$	$\surd$	$\surd$	$\surd$
b_g	$\surd$	$\surd$	$\surd$	$\surd$
N_g'	–	–	–	$\surd$
a_g'	–	–	–	$\surd$

The role of the input scale

In this particular exercise χ^2 decreases at first and stabilizes at NNLO20: allowing for negative gluons does not improve the description

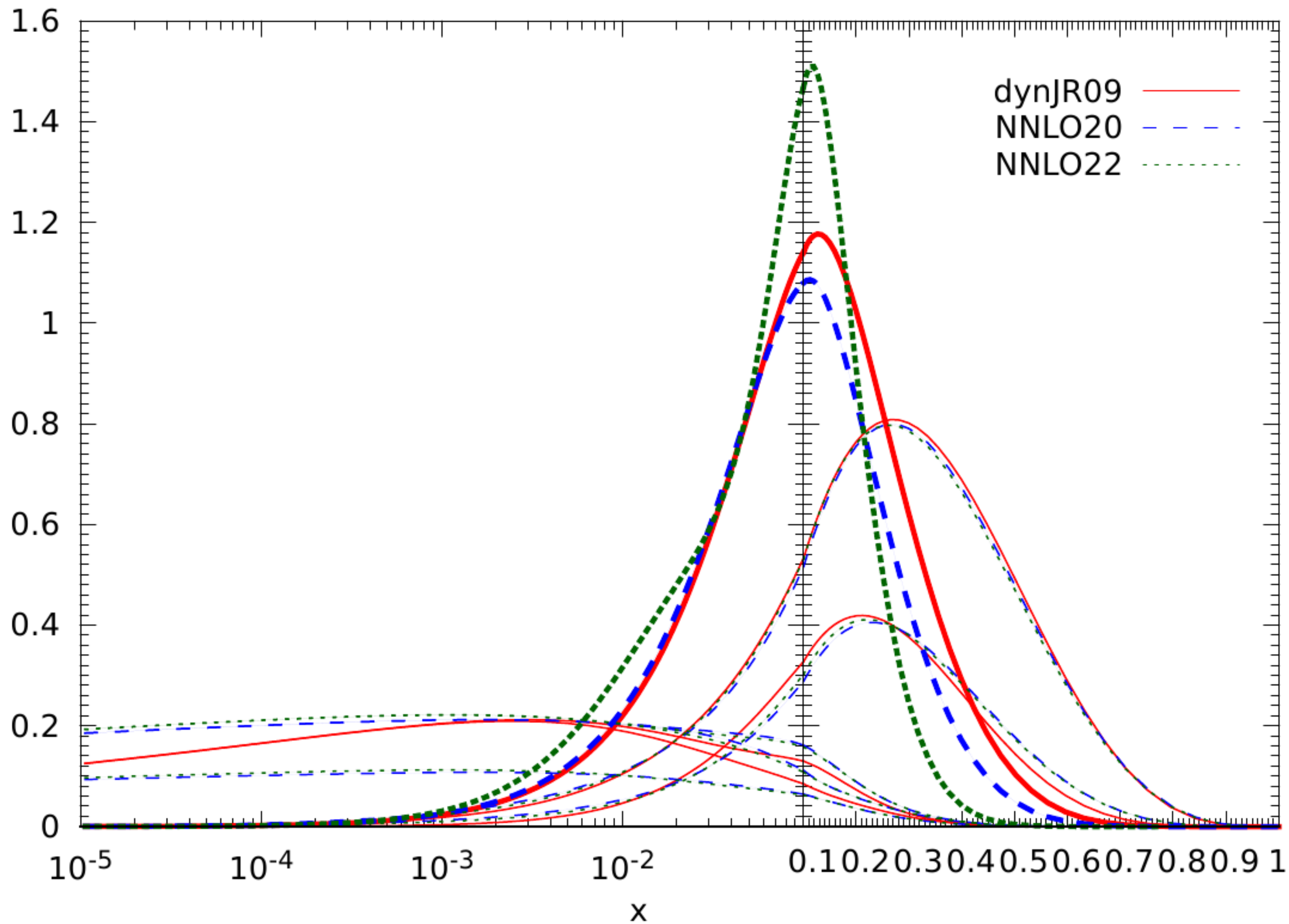


These variations can be used to estimate the (remaining) procedural bias!

(devise a measure: e.g. in (G)JR half the difference between dynamical and standard)

Positive input gluon distributions

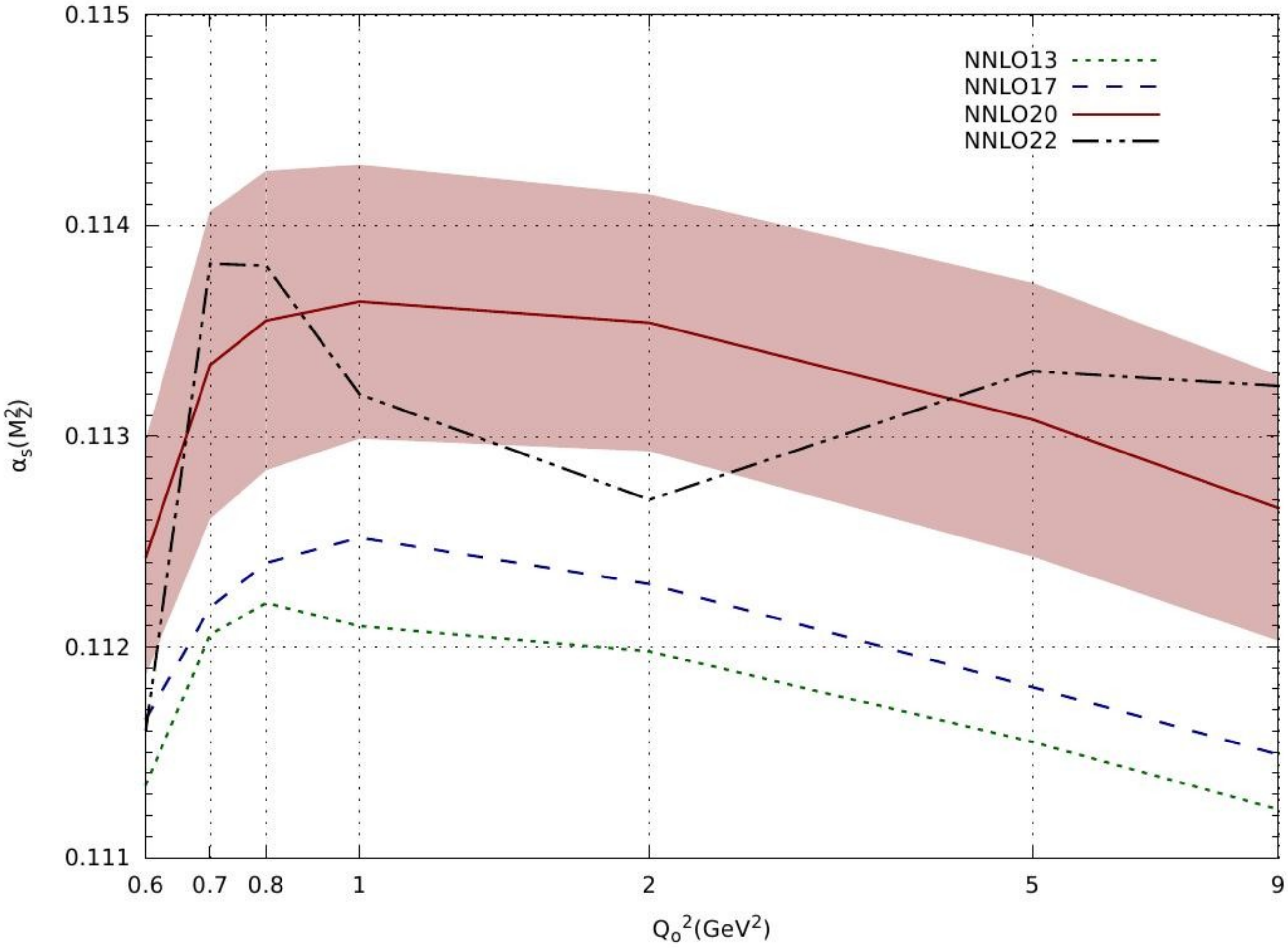
In this particular exercise χ^2 decreases at first and stabilizes at NNLO20: allowing for negative gluons does not improve the description



NNLO22 with $Q^2 = 0.6\ \mathrm{GeV}^2$ does not turn negative, remains *valencelike* ($a_g \simeq 1, a'_g \simeq 1.2$) ⇒ natural tendency of the input gluons at low scales

The role of the input scale

By considering variations with Q_0^2 one can estimate (a lower limit to) the procedural error $\Rightarrow$ additional uncertainty for each quantity



Results stabilize at NNLO20 but variations do not (substantially) decrease

Following our "recipe" we would estimate $\Delta_{\rm bias} \simeq 0.0006$; about the same size than the error from experimental uncertainties!

Least squares estimation and correlations

The need for an appropriate treatment of experimental correlations have been recognized in the last years (accuracy)

A convenient method for doing it (equivalent to the standard correlation matrix approach) is by shifts between theory and data [CTEQ]:

$$\chi^2 = \sum_{i=1}^{N} \frac{1}{\Delta_i^2} \left(D_i + \sum_{j=1}^{M} r_j \Delta_{ji} - T_i \right)^2 + \sum_{j=1}^{M} r_j^2$$

The optimal shifts for a given theory can be determined analytically:

$$r_j = - \sum_{k=1}^{M} A_{jk}^{-1} B_k \, , \quad B_j = \sum_{i=1}^{N} \Delta_{ji} \frac{D_i - T_i}{\Delta_i^2} \, , \quad A_{jk} = \delta_{jk}^{K} + \sum_{i=1}^{N} \frac{\Delta_{ji} \Delta_{ki}}{\Delta_i^2}$$

Least squares estimation and correlations

Care needed for multiplicative errors

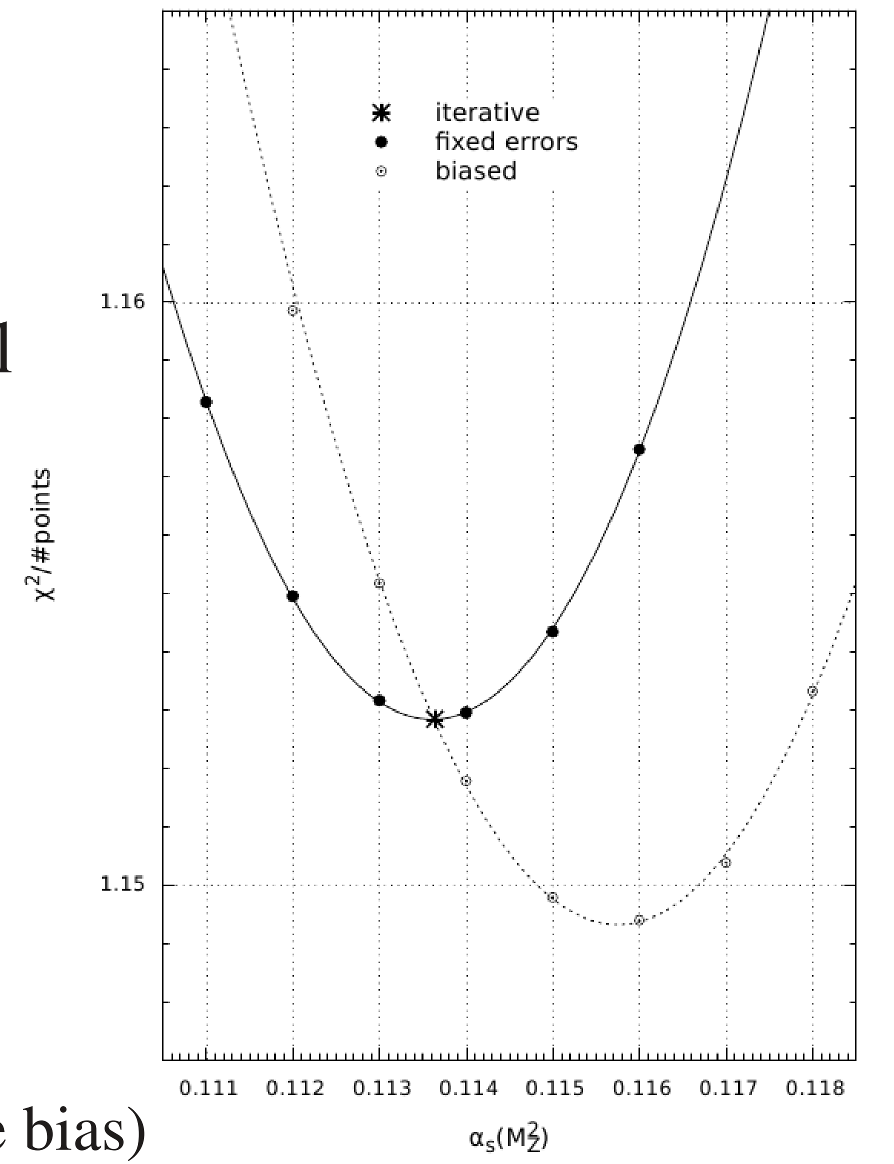
$$\Delta_{ji} = \delta_{ji} D_i,\ \Delta_i^2 = \delta_{stat,i}^2 D_i^2 + \delta_{unc,i} D_i^2$$

leads to bias towards smaller theoretical predictions (smaller errors and shifts for lower central values) [d'Agostini]

A solution is to take:

$$\Delta_{ji} = \delta_{ji} T_i,\quad \Delta_i^2 = \delta_{stat,i}^2 D_i^2 + \delta_{unc,i} T_i^2$$

but iteratively! (otherwise bias towards larger predictions). For parameter scans a fixed theory must be used (not to reintroduce the bias)



This might be relevant for NNPDF and/or HERAPDF

Outlook

Dynamical approach has greater predictive power in the small-x region: More constrained without additional constraints

At low input scales the natural tendency of gluons (even if allowed to go negative) is a valence-like structure

There is a (remaining) uncertainty (**procedural bias**) which it is usually disregarded, i.e. total errors underestimated

It can be estimated (lower limit) from (substantial) input-scale variations