Predictions for Drell-Yan $\phi^*$ and $Q_T$ observables at the LHC

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Available resummations of large logarithms $\ln(Q_T/M_Z)$ in the Z-boson $Q_T$ distribution follow two distinct philosophies:

- **Catani et al:** very accurate (NNLL+NNLO) resummation and no non-perturbative (NP) effects [Catani et al’10]

- **RESBOS:** less accurate (NLL +NLO) resummation and intrinsic parton $k_t$ modelled with a NP form factor
Small-x broadening

- RESBOS NP form factor predicts a significant broadening of the Z-boson $Q_T$ spectrum at the LHC due to small-$x$ effects

[ Berge Nadolsky Olness Yuan ’04]

- The understanding of this small-$x$ broadening needs a dedicated study with precision observables probing the low-$Q_T$ domain
**New precision observables in DY**

In recent years new observables have been introduced that probe low $p_T$ physics, but have better resolution than $Z$ transverse momentum $\vec{Q}_T$

$$\vec{a}_T = \vec{Q}_T \times \frac{\vec{p}_{T1} - \vec{p}_{T2}}{|\vec{p}_{T1} - \vec{p}_{T2}|}$$

$$\phi^* = \tan(\phi_{acop}/2) \sin \theta^* \simeq \frac{a_T}{M_Z}$$

- $a_T$ performs much better than $Q_T$ in the low $Q_T$ region
- Observables like $\phi^*$ or $\tan(\phi_{acop}/2)$ are determined only by lepton directions and can be measured very precisely

[Vesterinen Wyatt '09, AB Redford Vesterinen Waller Wyatt '10]
Comparison of $\phi^*$ distribution for large $Z$ rapidity ($|y| > 2$) with RESBOS raised issues with small-$x$ broadening

However, agreement between Tevatron data and RESBOS seems to be restored in the new version of RESBOS

We have decided to perform a dedicated theoretical study of the $\phi^*$ distribution using theoretical tools from perturbative QCD only, to see to what extent one really needs NP effects
At small $\phi^*$ the perturbative series does not converge because of the appearance of large logarithms up to $\alpha_s^n [\ln^{2n-1} \phi^*/\phi^*] +$ 

$$\frac{1}{\sigma} \frac{d\sigma}{dM^2 d\phi^*} = \alpha_s \left[ \frac{\ln(1/\phi^*)}{\phi^*} \right] + \ldots$$

These logarithms can be resummed at all orders 

$$\frac{d\sigma}{dM^2 d\phi^*} = \int_0^{\infty} dbM \cos(bM \phi^*) \mathcal{L}(\vec{b}^{-1}) e^{-R(\vec{b}M)} \quad \vec{b} = \frac{b e^{\gamma_E}}{2}$$

$R(\vec{b}M)$ is the same Sudakov exponent as for boson $Q_T$

$\mathcal{L}(1/\vec{b})$ is a process-dependent term, containing the parton luminosities at the scale $1/\vec{b}$
Our predictions are NNLL accurate, i.e. $\alpha_s^n \ln^{n-1}(\bar{b}M)$, and their ingredients $R(\bar{b}M)$ and $\mathcal{L}(1/\bar{b})$ are all known from $Q_T$ resummation [AB Marzani Dasgupta ’11]

$$\frac{d\sigma}{dM^2 d\phi^*} = \int_0^{\infty} db M \cos(b M \phi^*) \mathcal{L}(b^{-1}) e^{-R(\bar{b}M)} \quad \bar{b} = \frac{b e^{\gamma_E}}{2}$$

The “parton luminosity” $\mathcal{L}(1/\bar{b})$ contains all process-dependent terms

$$\mathcal{L}(\bar{b}^{-1}) = \int_{\text{lepton cuts}} [dk_1][dk_2] \int_0^1 dx_1 dx_2 \delta(x_1 x_2 s - M^2) \sum_{i,j} \left\{ f_i(x_1, b^{-1}) f_j(x_2, b^{-1}) + \frac{\alpha_s(\bar{b}^{-1})}{2\pi} \left[ \sum_k \int_0^1 \frac{dz}{z} C_{ik}(z) f_k \left( \frac{x_1}{z}, b^{-1} \right) f_j(x_2, b^{-1}) \right] \right\} M^2(x_1 p_1, x_2 p_2, k_1, k_2)$$

The Born matrix element $M^2(x_1 p_1, x_2 p_2, k_1, k_2)$ for hadronic dilepton production is taken directly from MCFM: we are then fully differential in the lepton momenta by construction

All convolutions are perform directly in $x$ space (i.e. no Mellin transform) using the HOPPET package by Salam and Rojo
We integrate numerically over the impact parameter \( b \): this requires prescriptions both at large and at small \( b \)

\[
\frac{d\sigma}{dM^2 d\phi^*} = \int_0^\infty dbM \cos(bM\phi^*) \mathcal{L}(\bar{b}^{-1}) e^{-R(\bar{b}M)}
\]

Large \( b \): avoid Landau pole in \( R(\bar{b}M) \) as well as factorisation scale \( 1/\bar{b} \) become less than \( Q_0 \lesssim 1 \text{GeV} \) ⇒ sharp cutoff \( b_{\text{max}} \geq 1/Q_0 \) and freeze the pdfs for \( b \geq 1/Q_0 \)

Small \( b \): freeze \( R(\bar{b}M) \) and \( \mathcal{L}(\bar{b}^{-1}) \) at their value for \( \bar{b}M \leq 1 \) ⇒ the \( \phi^* \) distribution is normalised to the total cross section

A PT resummation with such prescriptions to perform the \( b \) integral is what we mean by PT prediction

By NP effects we mean either a gaussian smearing \( \exp[-g_{\text{NP}}b^2] \) or even introducing \( k_t \) dependent parton densities
Perturbative uncertainties

- We vary renormalisation, factorisation and resummation scales $\mu_R, \mu_F$ and $\mu_Q$ in the range $[M/2, 2M]$ with $1/2 \leq \mu_i/\mu_j \leq 2$

\[ \frac{d\sigma}{dM^2d\phi^*} = \int_0^\infty dbM \cos(bM\phi^*) \left[ C \left( \frac{\mu_F}{\mu_Q} \right) \otimes \mathcal{L} \left( \alpha_s(\mu_R), \frac{\mu_R}{M}, \frac{\mu_F}{b\mu_Q} \right) \right] e^{-R[\alpha_s(\mu_R), \frac{\mu_R}{M}, \frac{\mu_Q}{M}, \delta\mu_Q]} \]

- We match our results to Z+1jet@NLO, obtained with MCFM

\[ \left( \frac{d\sigma}{d\phi^*} \right)_{\text{matched}} = \left( \frac{d\sigma}{d\phi^*} \right)_{\text{resummed}} + \left( \frac{d\sigma}{d\phi^*} \right)_{\text{fixed order}} - \left( \frac{d\sigma}{d\phi^*} \right)_{\text{expanded}} \]

- We compute $1/\sigma d\sigma/d\phi^*$ by dividing $d\sigma/d\phi^*$ by its area: dividing by the NLO total cross section gives 1% difference

- We validate our predictions by comparing $1/\sigma d\sigma/d\phi^*$ to Tevatron data for electrons and muons, in different bins of Z-boson rapidity

[AB Marzani Tomlison Dasgupta '11]
Phistar at the Tevatron

- Going from NLL to NNLL resummation reduces the theoretical uncertainty from 10% to 5-6%

- Different prescriptions to evaluate the $b$ integral (e.g. changing the freezing point of the pdfs $Q_0$ by a factor of two) give curves within our uncertainty band
**Validation against Tevatron data (I)**

- $Z \rightarrow \mu^+ \mu^-$ with two different bins in $Z$-boson rapidity

Good agreement with Tevatron data even at very low values of $\phi^*$ without any NP effects
Validation against Tevatron data (II)

- $Z \rightarrow e^+e^-$ with three different bins in $Z$ rapidity

![Graphs showing comparison between data and theory for different rapidity bins.]

- Agreement of our predictions with data persists even in the small-$x$ region $|y| > 2$ ⇒ No need for small-$x$ broadening

- Slight disagreement in the large $\phi^*$ region where multi-jet configurations become important ⇒ case for $Z+1\text{jet@NNLO}$
**Impact of non-perturbative effects**

- We take the curve with $\mu_R = \mu_F = \mu_Q = M_Z$ and add to the resummation a gaussian smearing $\exp[-g_{NP}b^2]$

- Inclusion of NP corrections gives an effect that is comparable to the variation of perturbative scales $\Rightarrow N^3LL$ resummation needed?
**Issues with Qt resummation at LHC**

- Our resummation can be applied to the $Q_T$ distribution \(\Rightarrow\) validation of our predictions against LHC data!

\[
\frac{d\sigma}{dM^2 dQ_T} = Q_T \int_0^\infty db \, b J_0(bQ_T) \mathcal{L}(1/\bar{b}) e^{-R(\bar{b}M)}
\]

- Freezing of the pdfs below $Q_0 = 1\text{GeV}$ leads to an unphysical oscillatory behaviour at large $Q_T \Rightarrow$ extrapolate the pdfs for $b \geq 1/Q_0$

- All curves with $\mu_F/\mu_Q = 1/2$ are very sensitive to variation of $Q_0$ by a factor 2 around $1\text{GeV} \Rightarrow$ sensitivity to Physics beyond collinear factorisation?

- We have decided therefore to evaluate our perturbative uncertainties by varying all scales in the range $\mu_F/\mu_Q \geq 1$ only
Predictions for $Q_T$ at the LHC

We can provide predictions for the $Q_T$ distribution with the fiducial cuts employed by ATLAS and CMS.

Also at the LHC data lie within our PT uncertainty band.

The inclusion of a NP term $\exp[-0.5 GeV^2 b^2]$ gives a distribution compatible with our theoretical uncertainty.
Having validated our resummation with the $Q_T$ distribution, we can confidently provide predictions for $\phi^*$ at the LHC.
Conclusions and open issues

- The novel observable $\phi^*$ can be measured very precisely and provides an accurate probe of $Q_T$ physics over a wide range of scales.

- We have a code that can produce perturbative resummed predictions for $\phi^*$ and $Q_T$ distributions in the Drell-Yan process with arbitrary cuts on lepton momenta.

- Our code is easily generalisable to other $\bar{Q}_T$-type resummations in other processes (e.g. $\vec{p}_{T,t\bar{t}}$ in top production), since it is just a reweighting of MCFM.

- Tevatron data call for more precise theoretical predictions both at low ($N^3LL$ resummation) and at high $\phi^*$ ($Z+1\text{jet@NNLO}$).

- Our theoretical predictions at the LHC are sensitive to behaviour of pdfs below 1GeV: this calls for a better theoretical understanding of the breaking of collinear factorisation (need for $k_t$ dependent pdfs?) ⇒ comparisons with other resummation codes would be valuable.