

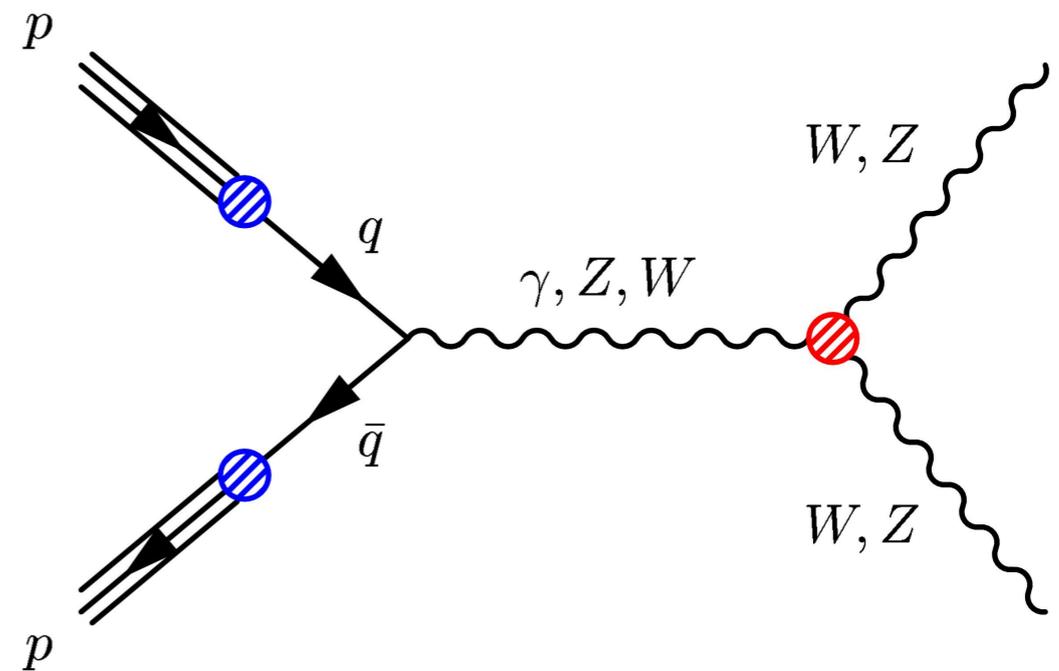
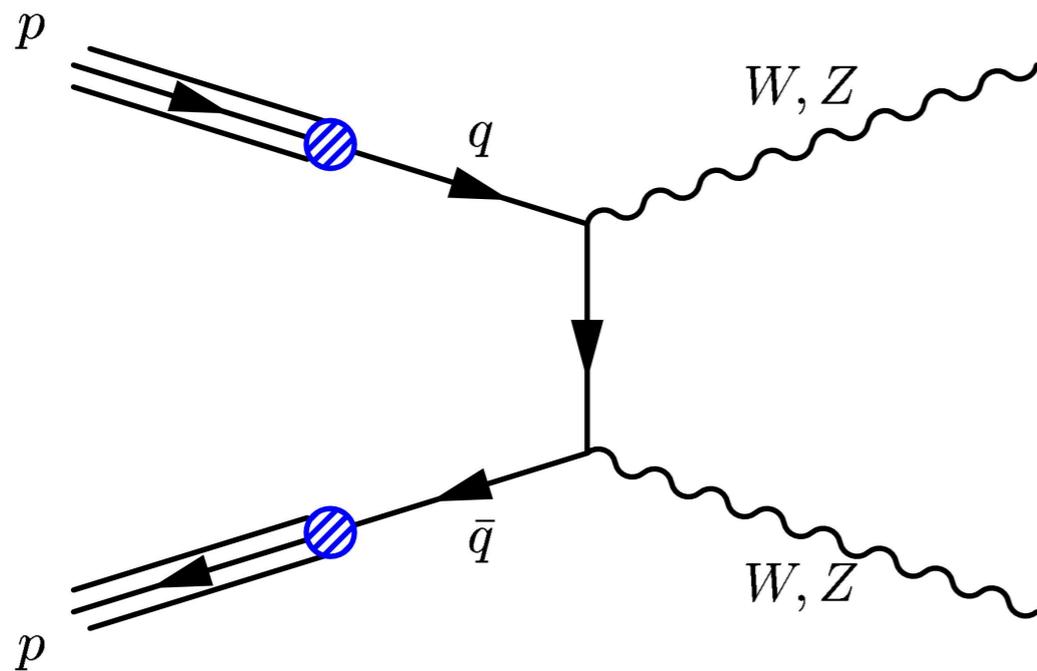
Diboson Physics in CMS

- Ian Ross, University of Wisconsin-Madison, on behalf of CMS
- LHC Electroweak Working Group Meeting
 - 10.October.2012

- Introduction
- Results
 - WW cross section (7 and 8 TeV)
 - WZ cross section (7 TeV)
 - $Z \rightarrow 4l$ branching ratio (7 TeV)
 - ZZ Cross section (7 and 8 TeV)
 - $V\gamma$ (7 TeV, 2010 only)
- Conclusions and CMS outlook

- Diboson processes at the LHC
 - Standard Model measurements
 - Cross sections
 - Direct measurement of Triple Gauge Couplings (TGCs)
 - Higgs
 - SM ZZ and WW serve as irreducible background to Higgs searches and must be well understood within the context of our new boson
 - BSM physics
 - New particle decays (e.g. $W' \rightarrow WZ$)
 - Anomalous triple gauge couplings (aTGCs)

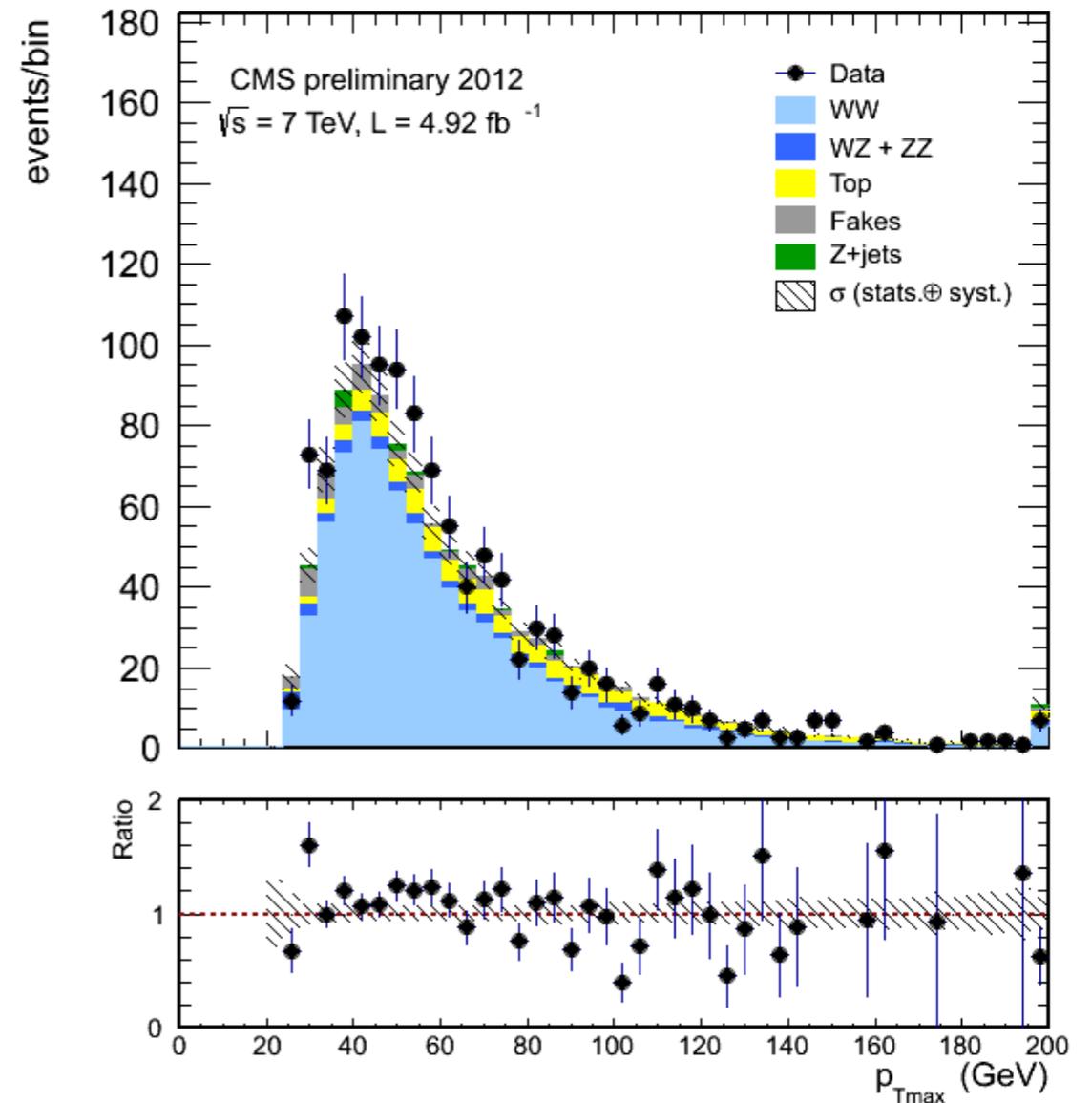
Diboson Production (Leading order)



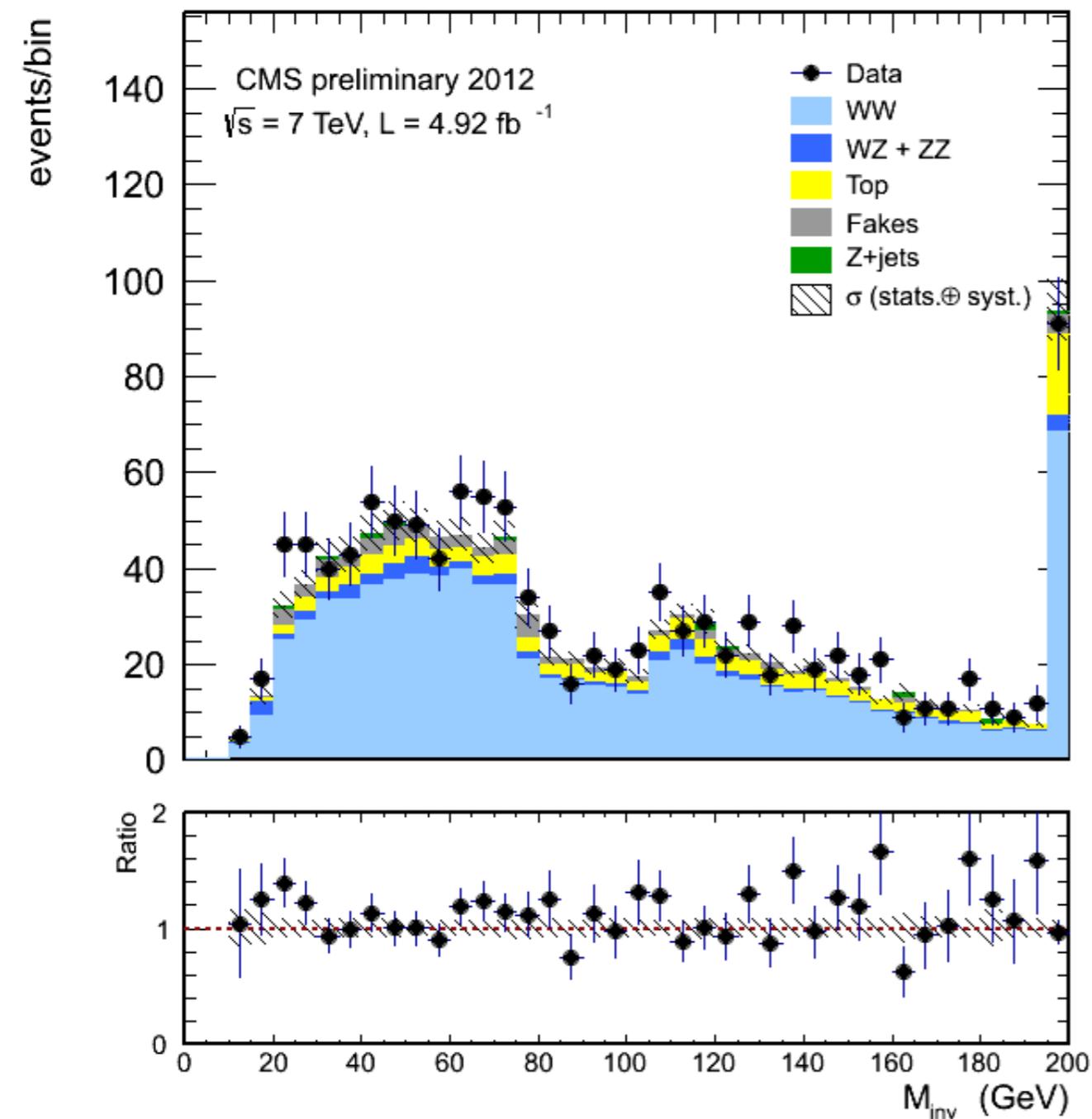
- Triple gauge couplings:
 - Charged triple gauge couplings (WWZ , $WW\gamma$) allowed
 - Neutral triple gauge couplings (ZZZ , $ZZ\gamma$) forbidden in Standard Model
- Anomalous couplings lead to enhanced cross section, larger $V p_T$

$WW \rightarrow 2l2\nu$, Analysis Overview

- Key features:
 - Non-resonant
 - Important in Higgs search
- Signature: two high p_T leptons + high ME_T
- Background treatment:
 - W+jets reduced using central- and b-jet vetoes, estimated from data via lepton fake rate extrapolation
 - Top background estimated from data via top veto fake rate
 - DY contribution (outside Z window) estimated by normalizing the simulation to the yield within the Z window



WW, 7 TeV (4.92 fb⁻¹)

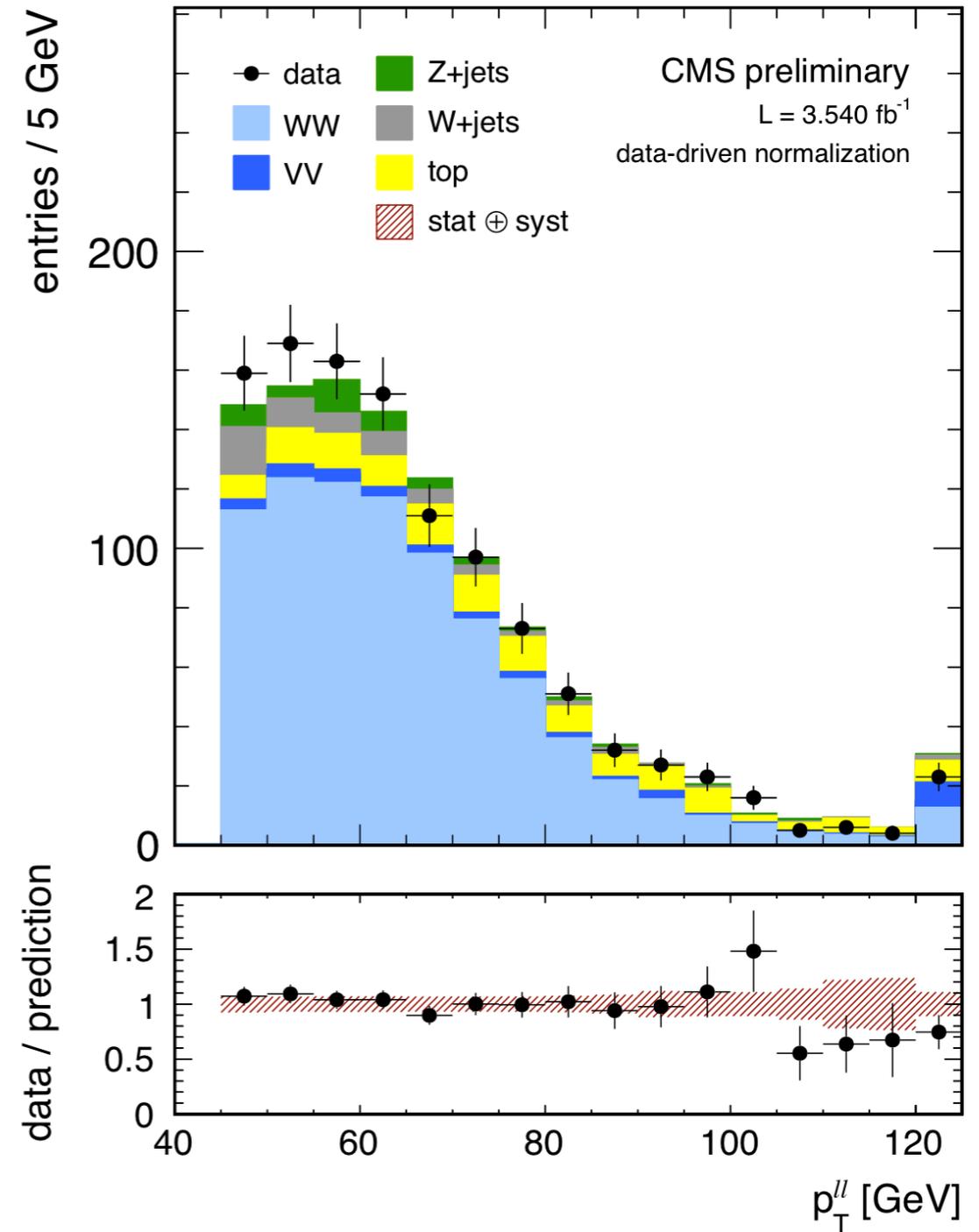


Sample	Yield \pm stat. \pm syst.
$gg \rightarrow W^+W^-$	$46.0 \pm 0.6 \pm 14.2$
$q\bar{q} \rightarrow W^+W^-$	$750.9 \pm 4.1 \pm 53.1$
$t\bar{t} + tW$	$128.5 \pm 12.8 \pm 19.6$
W+jets	$59.5 \pm 3.9 \pm 21.4$
WZ+ZZ	$29.4 \pm 0.4 \pm 2.0$
Z/ γ^*	$11.0 \pm 5.1 \pm 2.6$
W+ γ	$18.8 \pm 2.8 \pm 4.7$
Z/ $\gamma^* \rightarrow \tau\tau$	$0.0 \pm 1.0 \pm 0.1$
Total Background	$247.1 \pm 14.6 \pm 29.5$
Signal + Background	$1044.0 \pm 15.2 \pm 62.4$
Data	1134

$\sigma_{WW} = 52.4 \pm 2.0 \text{ (stat.)} \pm 4.5 \text{ (syst.)} \pm 1.2 \text{ (lumi.) pb.}$
Theory: $47.0 \pm 2.0 \text{ pb}$ (MCFM)

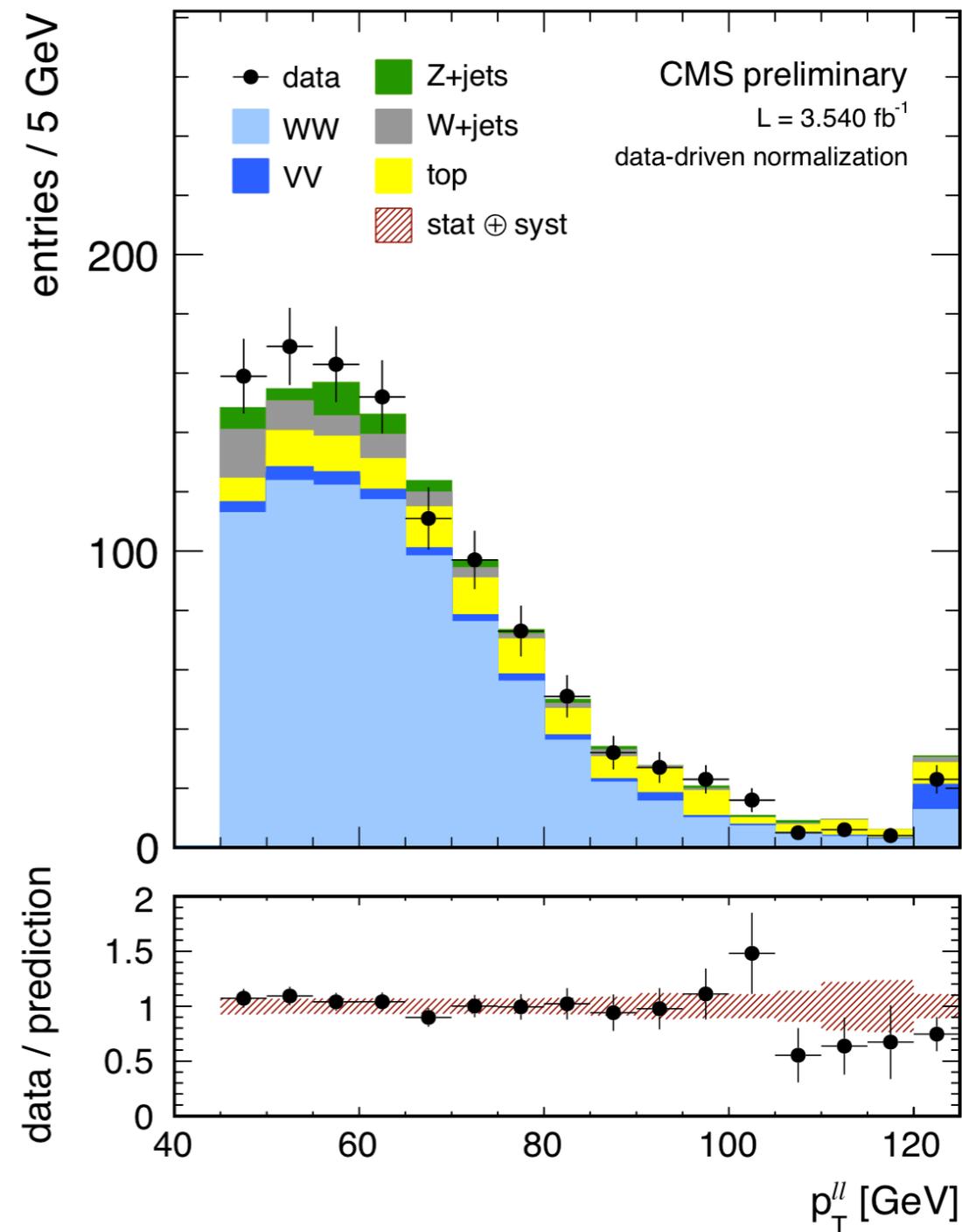
WW, 8 TeV (3.54 fb⁻¹)

sample	yield \pm stat. \pm syst.
gg \rightarrow WW	43.3 \pm 1.0 \pm 13.4
qq \rightarrow WW	640.3 \pm 4.9 \pm 47.4
t \bar{t} + tW	131.6 \pm 12.7 \pm 19.5
W + jets	60.0 \pm 4.3 \pm 21.6
WZ + ZZ	27.4 \pm 0.5 \pm 2.9
Z/ γ^*	42.5 \pm 6.0 \pm 9.9
W γ + W γ^*	13.6 \pm 2.4 \pm 4.3
total background	275.2 \pm 14.9 \pm 31.2
signal + background	958.8 \pm 15.7 \pm 58.3
data	1111 \pm 33



$$\sigma_{WW} = 69.9 \pm 2.8 \text{ (stat.)} \pm 5.6 \text{ (syst.)} \pm 3.1 \text{ (lumi.) pb.} \quad \text{Theory: } 57.3^{+2.4}_{-1.6} \text{ pb (MCFM)}$$

sample	yield \pm stat. \pm syst.
gg \rightarrow WW	43.3 \pm 1.0 \pm 13.4
qq \rightarrow WW	640.3 \pm 4.9 \pm 47.4
t \bar{t} + tW	131.6 \pm 12.7 \pm 19.5
W + jets	60.0 \pm 4.3 \pm 21.6
WZ + ZZ	27.4 \pm 0.5 \pm 2.9
Z/ γ^*	42.5 \pm 6.0 \pm 9.9
W γ + W γ^*	13.6 \pm 2.4 \pm 4.3
total background	275.2 \pm 14.9 \pm 31.2
signal + background	958.8 \pm 15.7 \pm 58.3
data	1111 \pm 33

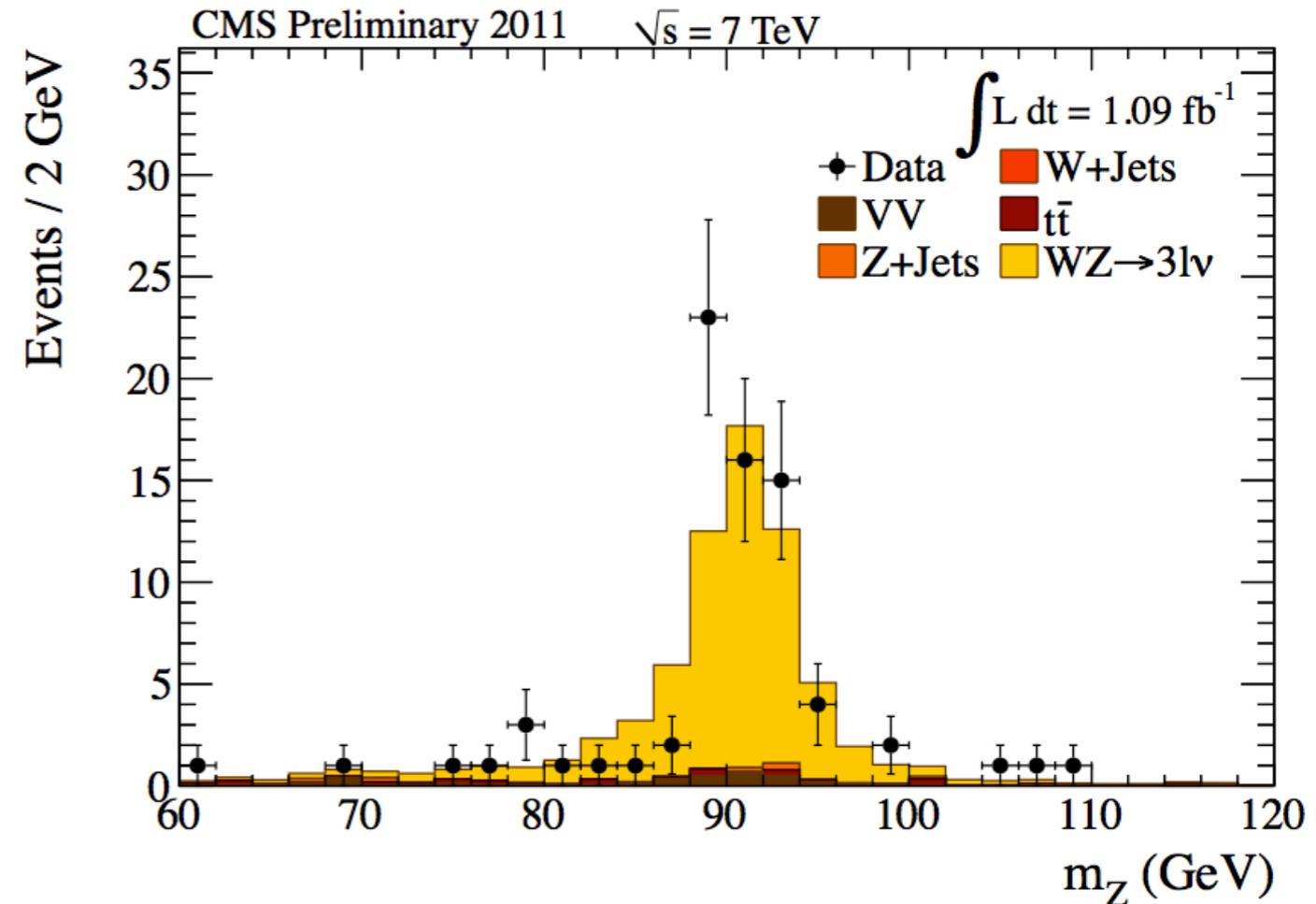


Key systematics: jet veto efficiency,
PDF uncertainty

$$\sigma_{WW} = 69.9 \pm 2.8 \text{ (stat.)} \pm 5.6 \text{ (syst.)} \pm 3.1 \text{ (lumi.) pb.} \quad \text{Theory: } 57.3^{+2.4}_{-1.6} \text{ pb (MCFM)}$$

$WZ \rightarrow 3lv$ (1.09 fb^{-1} @ 7 TeV)

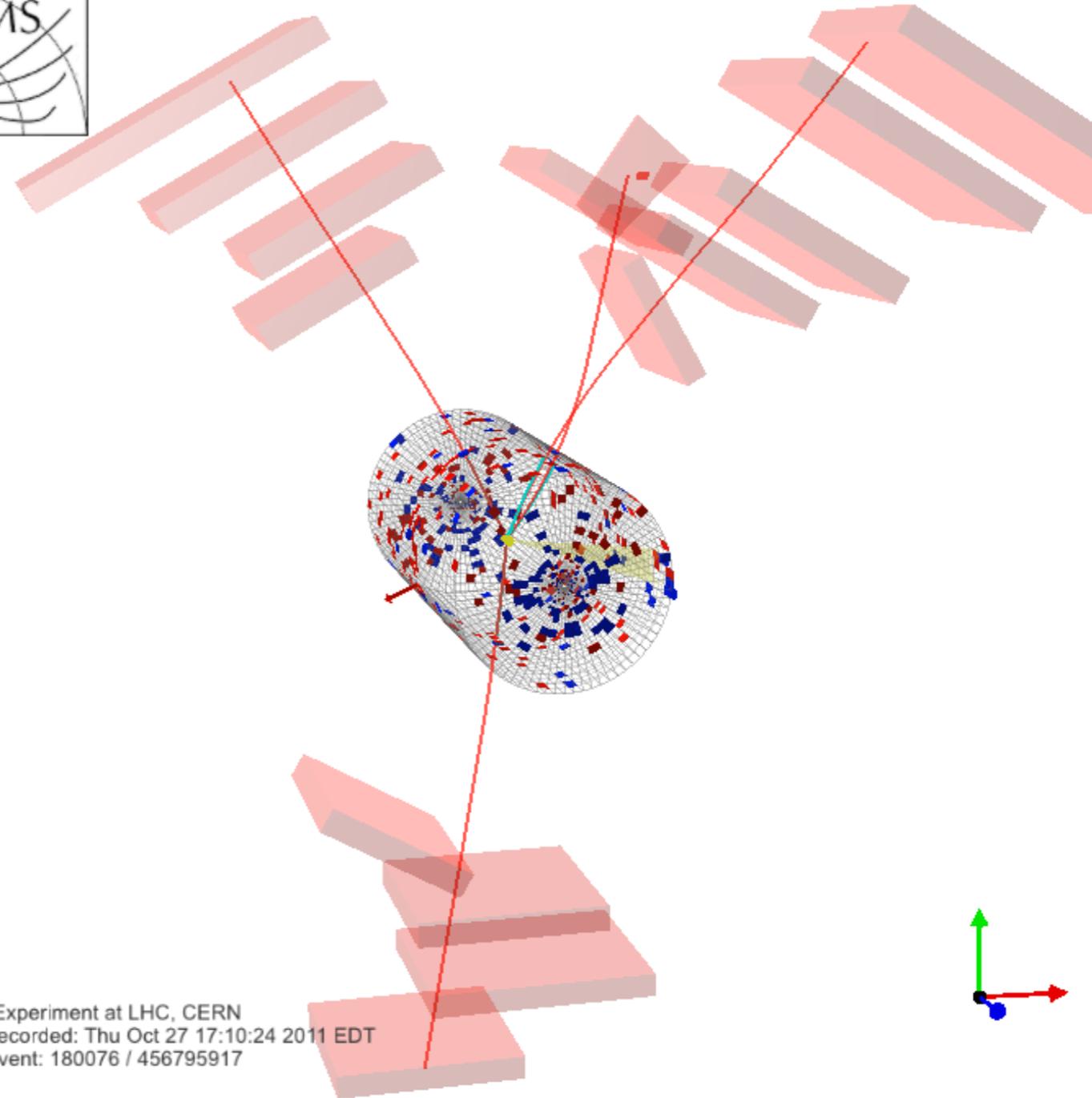
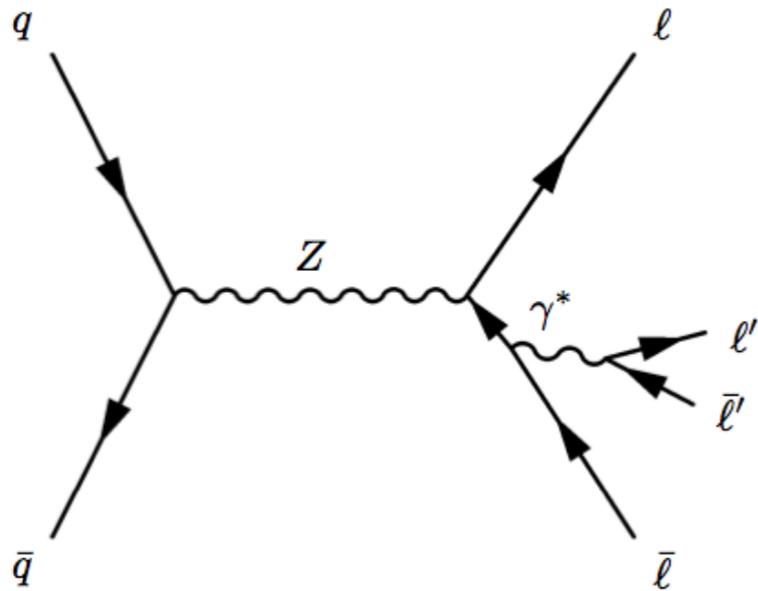
- Signature:
 - Two leptons consistent with $60 \leq M_{ll} \leq 120$, third lepton + ME_T
- Backgrounds:
 - Data-driven: Z+Jets, $t\bar{t}$
 - From MC:
 - $ZZ \rightarrow 4l$, $Z\gamma$, WZ to τ decays
- Key systematics: Efficiency, theory, background estimations



channel	$N_{observed}$	cross section (pb)
$\sigma_{WZ \rightarrow eee\nu}$	22	$0.086 \pm 0.022(stat) \pm 0.007(syst) \pm 0.005(lumi)$
$\sigma_{WZ \rightarrow ee\mu\nu}$	20	$0.060 \pm 0.017(stat) \pm 0.005(syst) \pm 0.004(lumi)$
$\sigma_{WZ \rightarrow \mu\mu e\nu}$	13	$0.053 \pm 0.018(stat) \pm 0.004(syst) \pm 0.003(lumi)$
$\sigma_{WZ \rightarrow \mu\mu\mu\nu}$	20	$0.060 \pm 0.016(stat) \pm 0.004(syst) \pm 0.004(lumi)$

$$\sigma(pp \rightarrow WZ + X) = 17.0 \pm 2.4 \text{ (stat.)} \pm 1.1 \text{ (syst.)} \pm 1.0 \text{ (lumi.) pb.}$$

Theory: 17.5 ± 0.6 pb (MCFM)



CMS Experiment at LHC, CERN
 Data recorded: Thu Oct 27 17:10:24 2011 EDT
 Run/Event: 180076 / 456795917

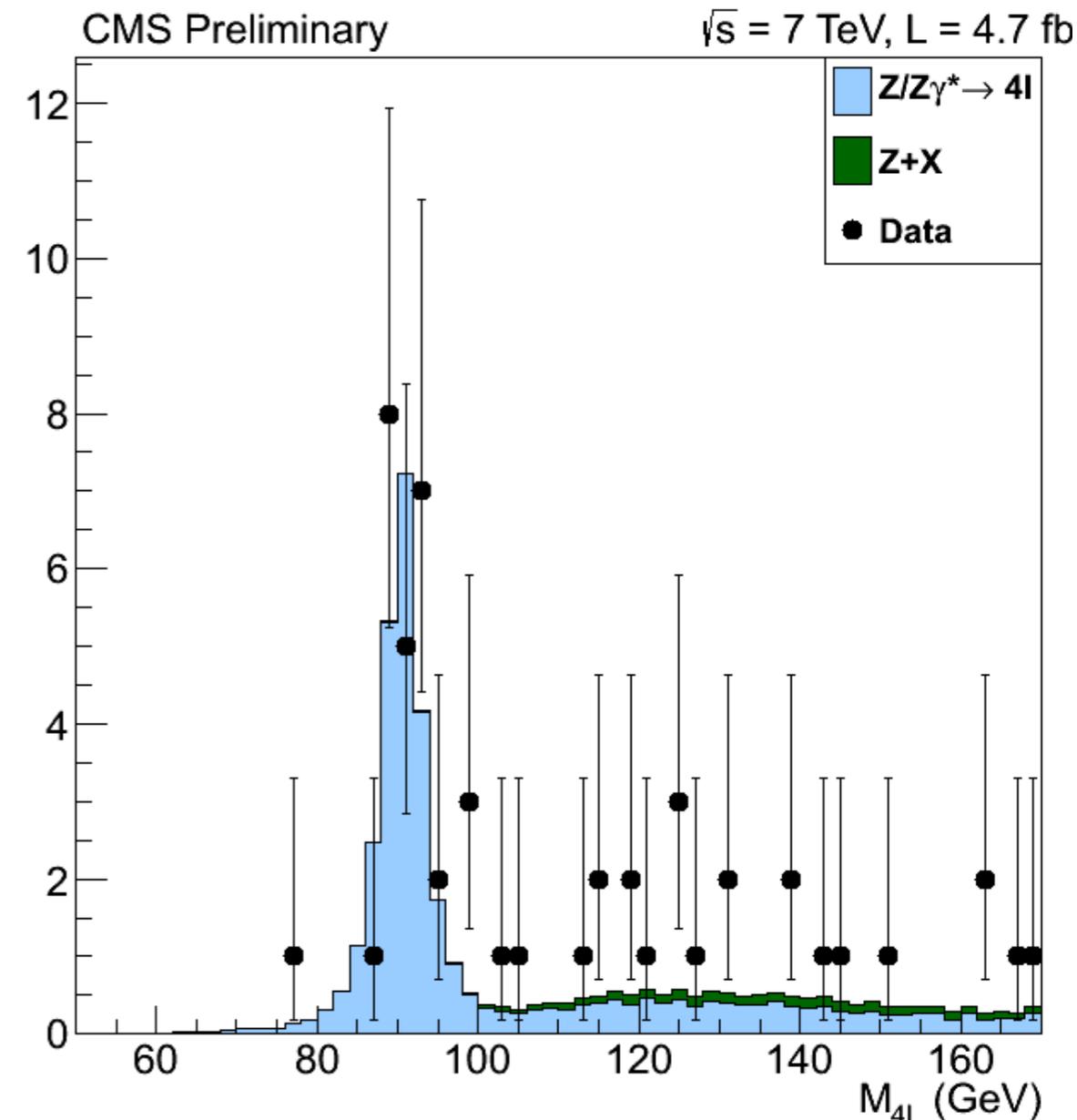


- First observation of $Z \rightarrow 4l$ peak at a hadron collider
- Key backgrounds:
 - $pp \rightarrow Z\gamma^* \rightarrow 4l$ (from MC)
 - $Z+X$ (from Data)
- Define signal region as $80 < M_{4L} < 100 \text{ GeV}$

Final state channels	4e	4μ	2e2μ	4ℓ
Irreducible background ($pp \rightarrow Z\gamma^* \rightarrow 4\ell$)	0.04	0.16	0.08	0.3 ± 0.03
Other reducible backgrounds	0.01	0.01	0.05	0.1 ± 0.13
Expected signal ($pp \rightarrow Z \rightarrow 4\ell$)	3.1	12.3	9.2	24.6 ± 2.2
Total expected (MC)	3.2	12.5	9.3	25.0 ± 2.2
Observed events	2	14	10	26
Rate from the fit of the observed mass distribution		13.6	9.7	25.4

- 26 events observed, 24.6 expected signal, 0.4 expected background

- $\sigma \times \text{BR}(Z \rightarrow 4l) = 125_{-23}^{+26}$ (stat.) $_{-6}^{+9}$ (syst.) $_{-5}^{+7}$ (lumi) fb



- Measured $\text{BR}(Z \rightarrow 4l) = 4.4_{-0.8}^{+1.0}$ (stat.) ± 0.2 (syst.) $\times 10^{-6}$
 - Theory: 4.45×10^{-6} (CalcHEP)

ZZ → 4l Analysis Overview

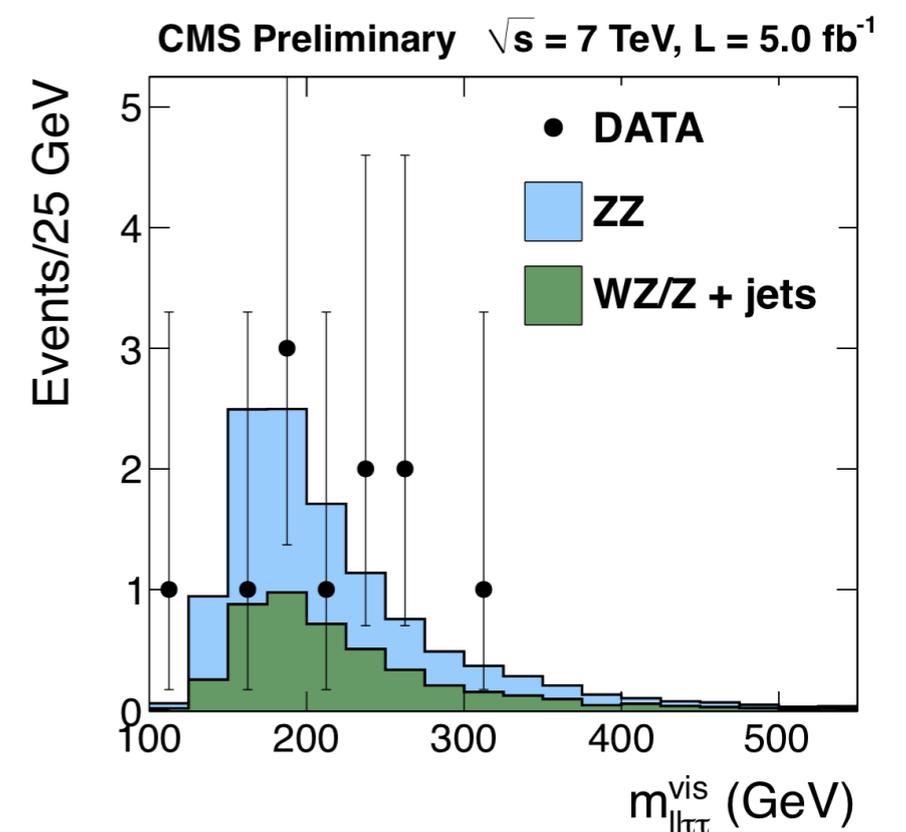
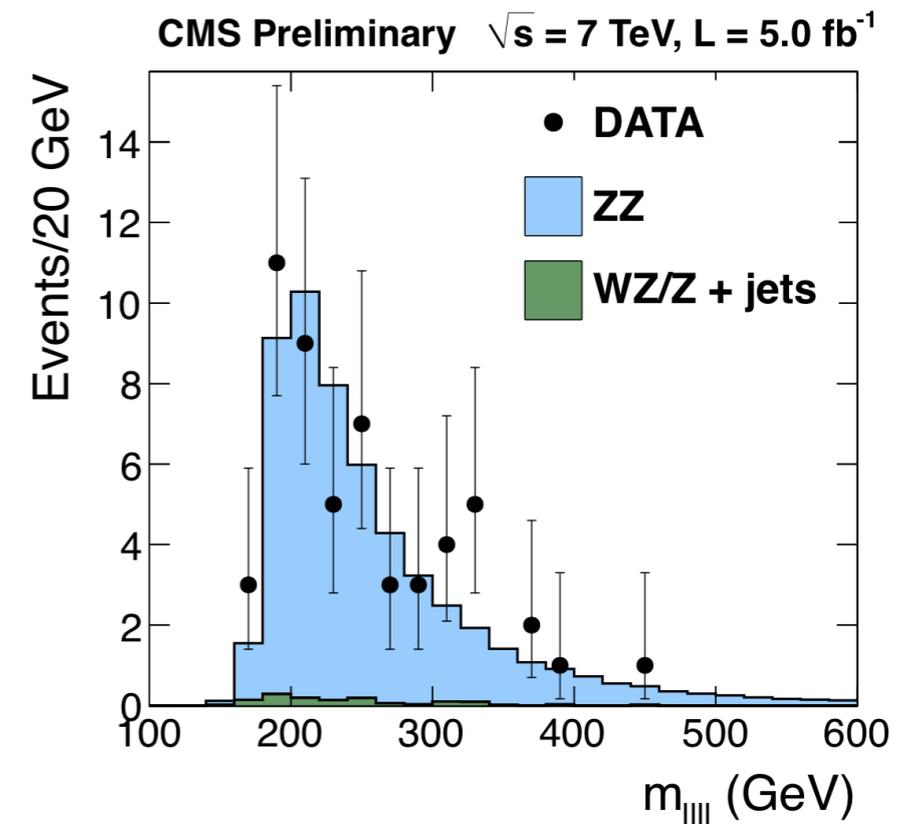
Key features:

- ZZ → 4l (l=e, μ) provides clean signature
 - Low background, high resolution ideal for Higgs physics
 - Beyond the Standard Model (ZZ decays of new particles, aTGCs)
- 2l2τ states included in cross-section measurements (~10-15% additional yield)

■ Signature: four leptons with $60 \leq M_{Z1}, M_{Z2} \leq 120$ ($30 \leq M_{\text{vis}} \leq 80$ for ττ legs)

Background treatment:

- Z+jets/WZ/ttbar, extracted from data via applying lepton fake rates to regions where one or both leptons fail selection criteria



ZZ → 4l 7 TeV Results

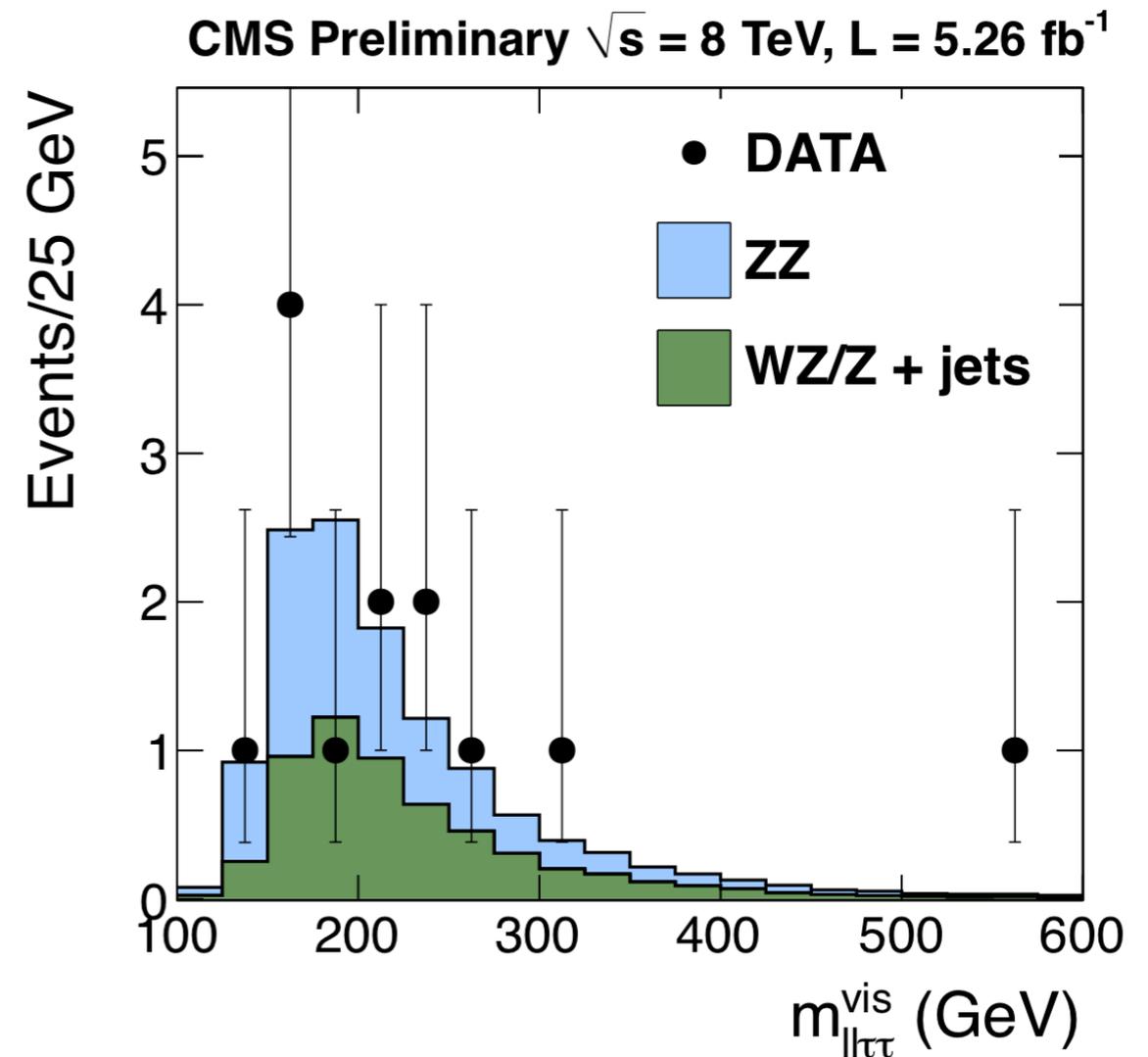
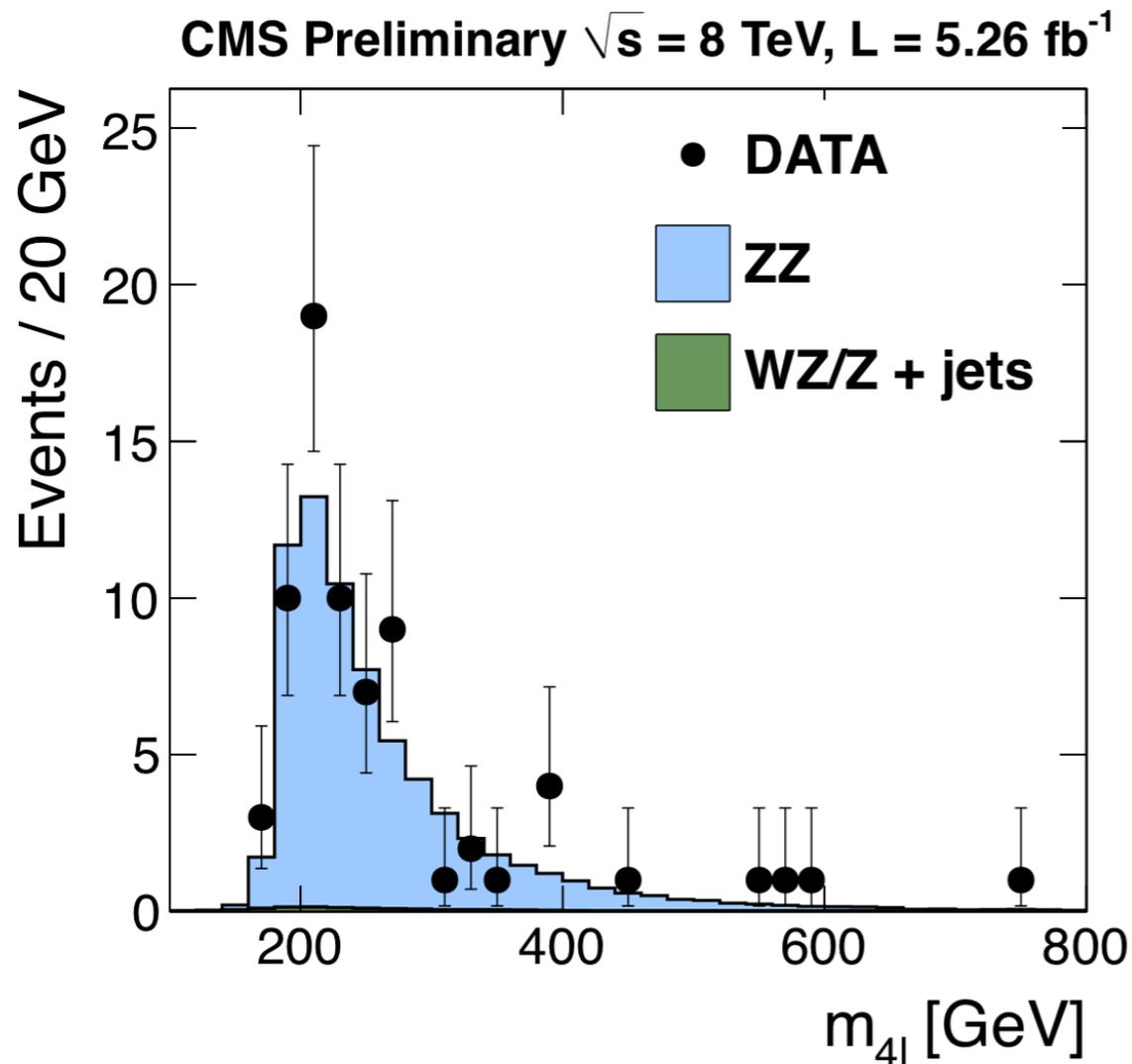
Decay channel	N_{ZZ}^{exp}	Background	Total expected	Observed
$\mu\mu\mu\mu$	$15.91 \pm 0.05 \pm 1.43$	$0.52 \pm 0.26 \pm 0.25$	$16.43 \pm 0.26 \pm 1.45$	14
$eeee$	$10.50 \pm 0.04 \pm 0.95$	$0.25 \pm 0.14 \pm 0.07$	$10.75 \pm 0.14 \pm 0.95$	9
$\mu\mu ee$	$26.74 \pm 0.10 \pm 2.41$	$0.58 \pm 0.18 \pm 0.23$	$27.32 \pm 0.17 \pm 2.41$	31
$\mu\mu\tau_h\tau_h$	$0.82 \pm 0.02 \pm 0.07$	$0.75 \pm 0.16 \pm 0.08$	$1.57 \pm 0.16 \pm 0.11$	0
$ee\tau_h\tau_h$	$0.75 \pm 0.01 \pm 0.07$	$0.76 \pm 0.16 \pm 0.05$	$1.51 \pm 0.16 \pm 0.09$	1
$ee\tau_e\tau_h$	$1.17 \pm 0.02 \pm 0.11$	$0.96 \pm 0.34 \pm 0.12$	$2.29 \pm 0.34 \pm 0.16$	3
$\mu\mu\tau_e\tau_h$	$1.15 \pm 0.02 \pm 0.10$	$0.35 \pm 0.34 \pm 0.11$	$1.60 \pm 0.34 \pm 0.15$	3
$\mu\mu\tau_\mu\tau_h$	$1.08 \pm 0.02 \pm 0.10$	$0.55 \pm 0.24 \pm 0.11$	$1.64 \pm 0.24 \pm 0.15$	2
$ee\tau_\mu\tau_h$	$0.94 \pm 0.02 \pm 0.08$	$0.22 \pm 0.14 \pm 0.04$	$1.17 \pm 0.14 \pm 0.06$	0
$ee\tau_e\tau_\mu$	$0.54 \pm 0.01 \pm 0.05$	$0.64 \pm 0.44 \pm 0.16$	$1.22 \pm 0.44 \pm 0.17$	0
$\mu\mu\tau_e\tau_\mu$	$0.60 \pm 0.01 \pm 0.05$	$0.14 \pm 0.30 \pm 0.10$	$0.74 \pm 0.30 \pm 0.11$	2

4l Total:
54 observed,
54.6 expected
(53.2+1.4)

2l2τ Total:
11 observed,
11.5 expected
(7.1+4.4)

$$\sigma(pp \rightarrow ZZ) = 6.24_{-0.80}^{+0.86} (stat.)_{-0.32}^{+0.41} (sys.) \pm 0.14 (lumi.) \text{ pb}$$

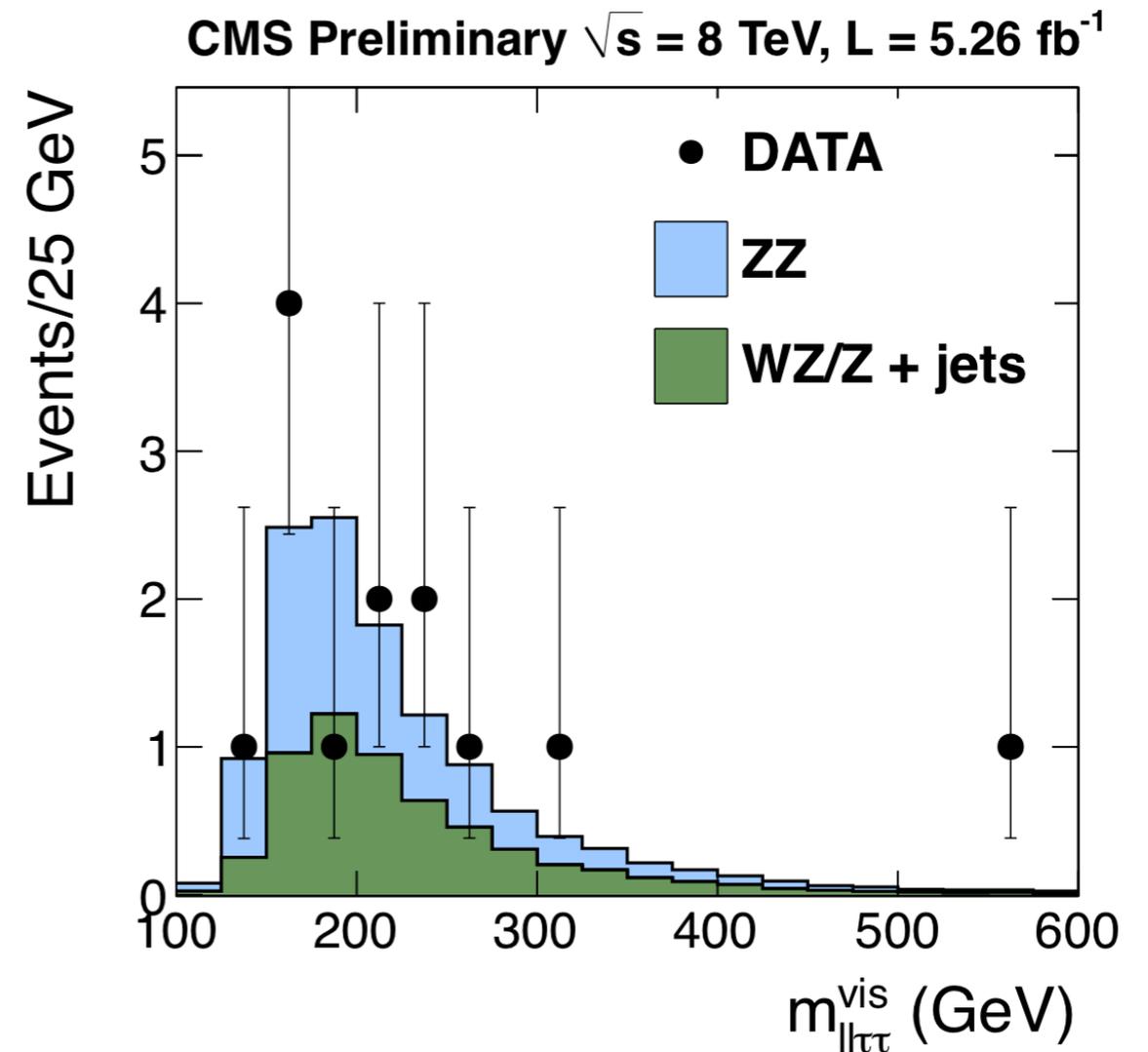
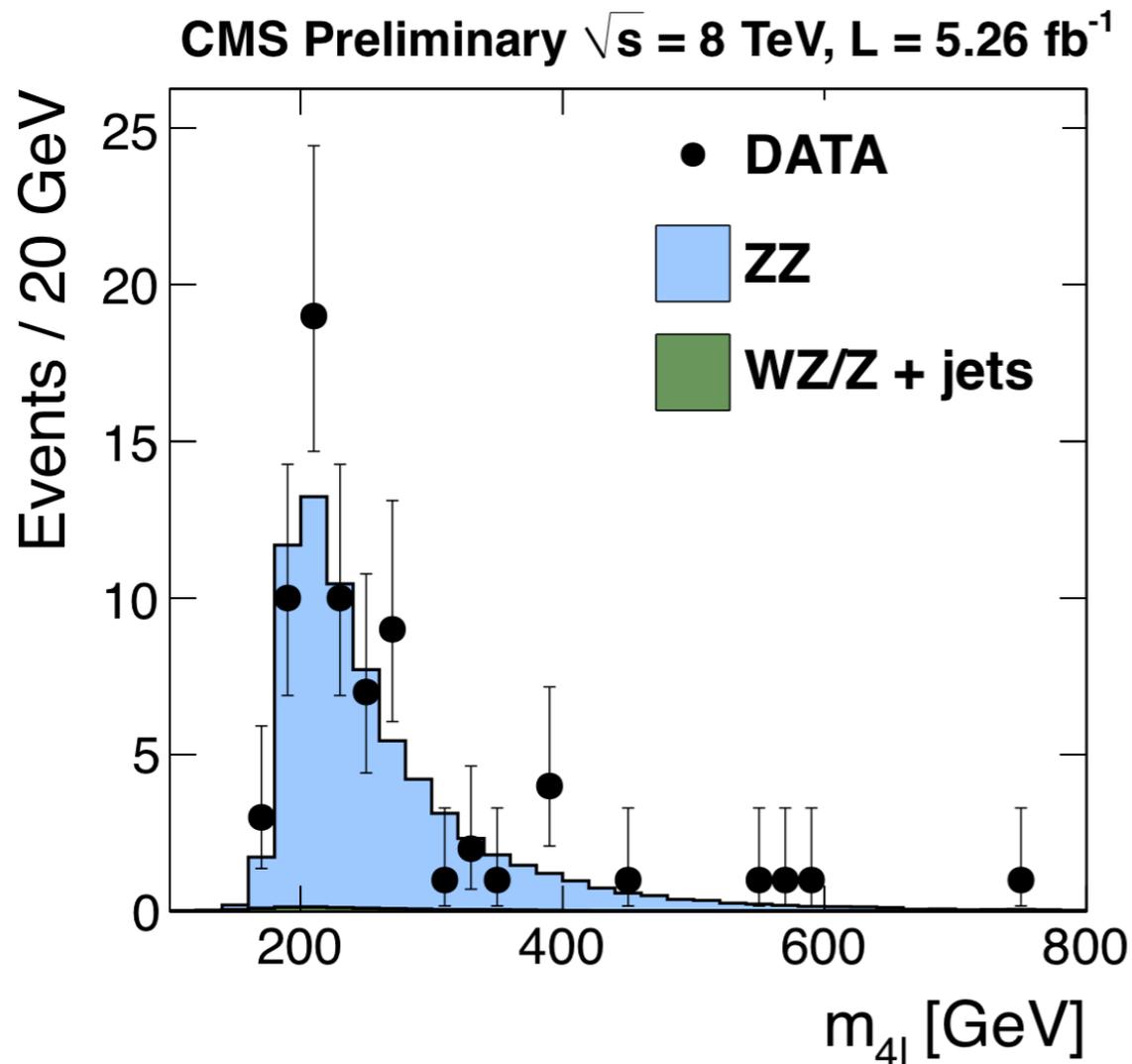
Theory: $6.3 \pm 0.4 \text{ pb}$ (MCFM)



Channel	4e	4 μ	2e2 μ	2 ℓ 2 τ
ZZ	11.6 ± 1.4	20.3 ± 2.2	32.4 ± 3.5	6.5 ± 0.8
Background	0.4 ± 0.2	0.4 ± 0.3	0.5 ± 0.4	5.6 ± 1.4
Total	12.0 ± 1.4	20.7 ± 2.2	32.9 ± 3.5	12.1 ± 1.6
Observed	14	19	38	13

■ $\sigma(pp \rightarrow ZZ) = 8.4 \pm 1.0(\text{stat.}) \pm 0.7(\text{sys.}) \pm 0.4(\text{lumi.}) \text{ pb}$

Theory: $7.7 \pm 0.4 \text{ pb}$ (MCFM)



Channel	4e	4 μ	2e2 μ	2 ℓ 2 τ
ZZ	11.6 ± 1.4	20.3 ± 2.2	32.4 ± 3.5	6.5 ± 0.8
Background	0.4 ± 0.2	0.4 ± 0.3	0.5 ± 0.4	5.6 ± 1.4
Total	12.0 ± 1.4	20.7 ± 2.2	32.9 ± 3.5	12.1 ± 1.6
Observed	14	19	38	13

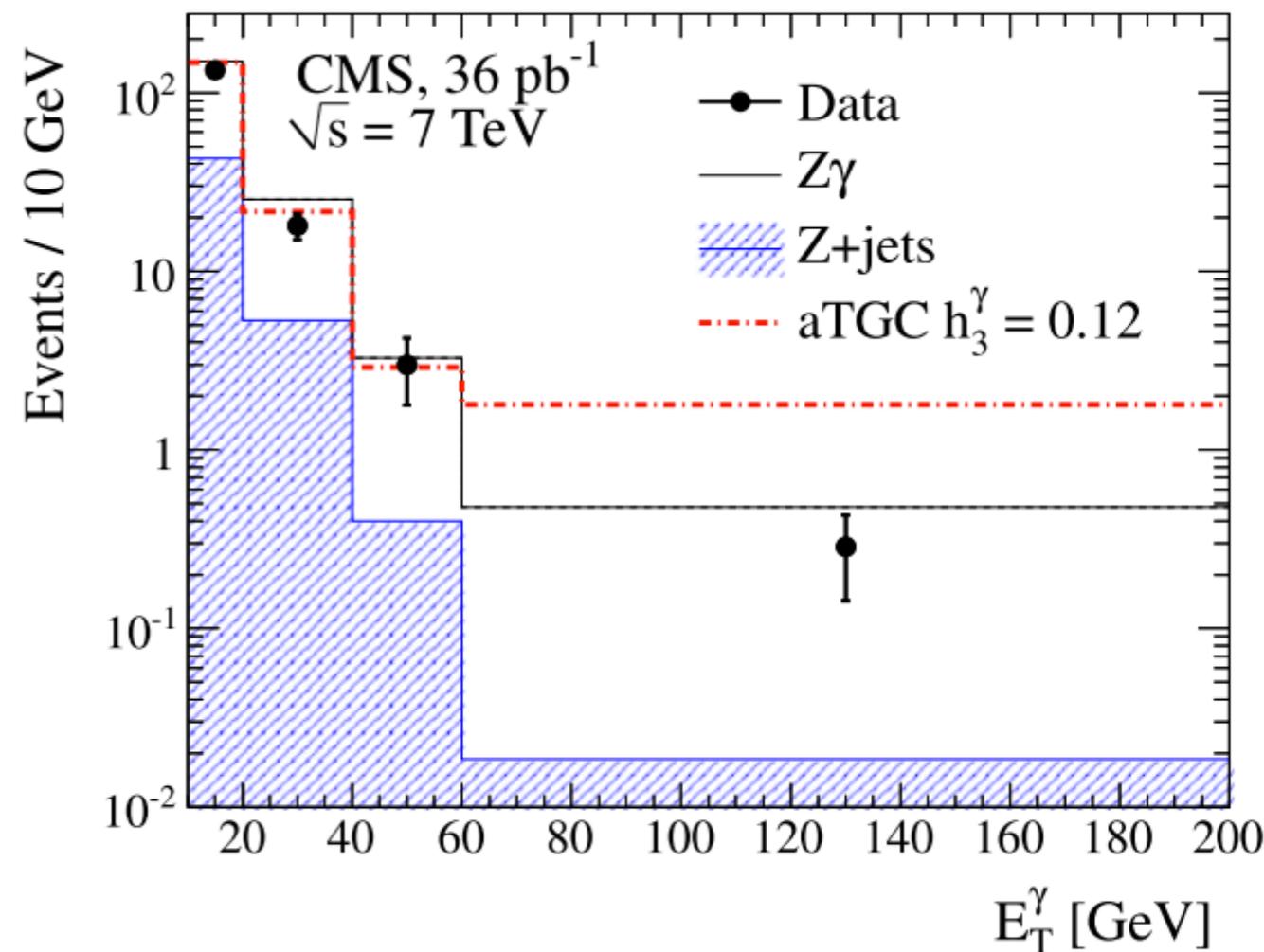
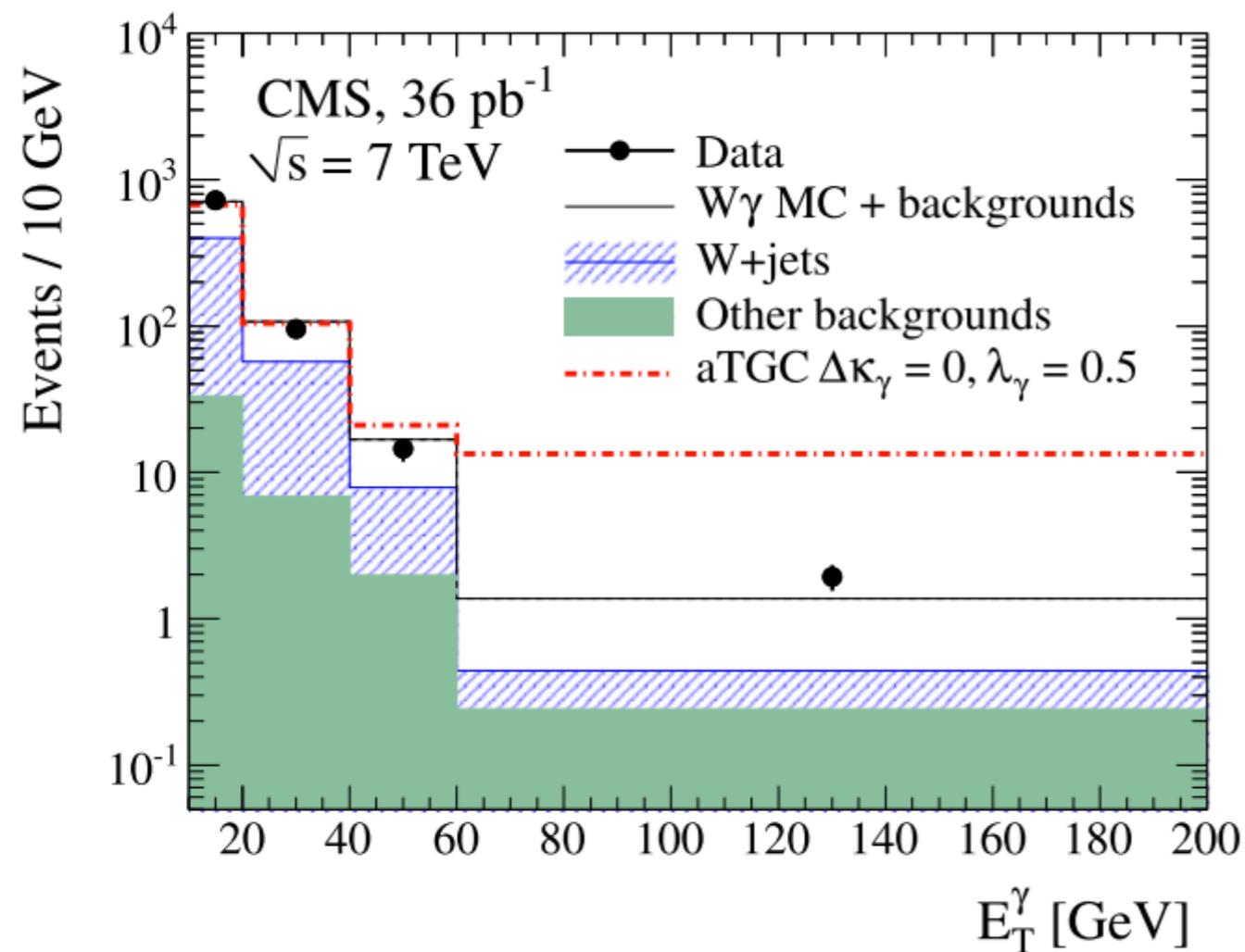
■ $\sigma(\text{pp} \rightarrow \text{ZZ}) = 8.4 \pm 1.0(\text{stat.}) \pm 0.7(\text{sys.}) \pm 0.4(\text{lumi.}) \text{ pb}$

Theory: $7.7 \pm 0.4 \text{ pb}$ (MCFM)

Key systematics: QCD scale, efficiencies

2010 Results, $\gamma\gamma$ and $V\gamma$

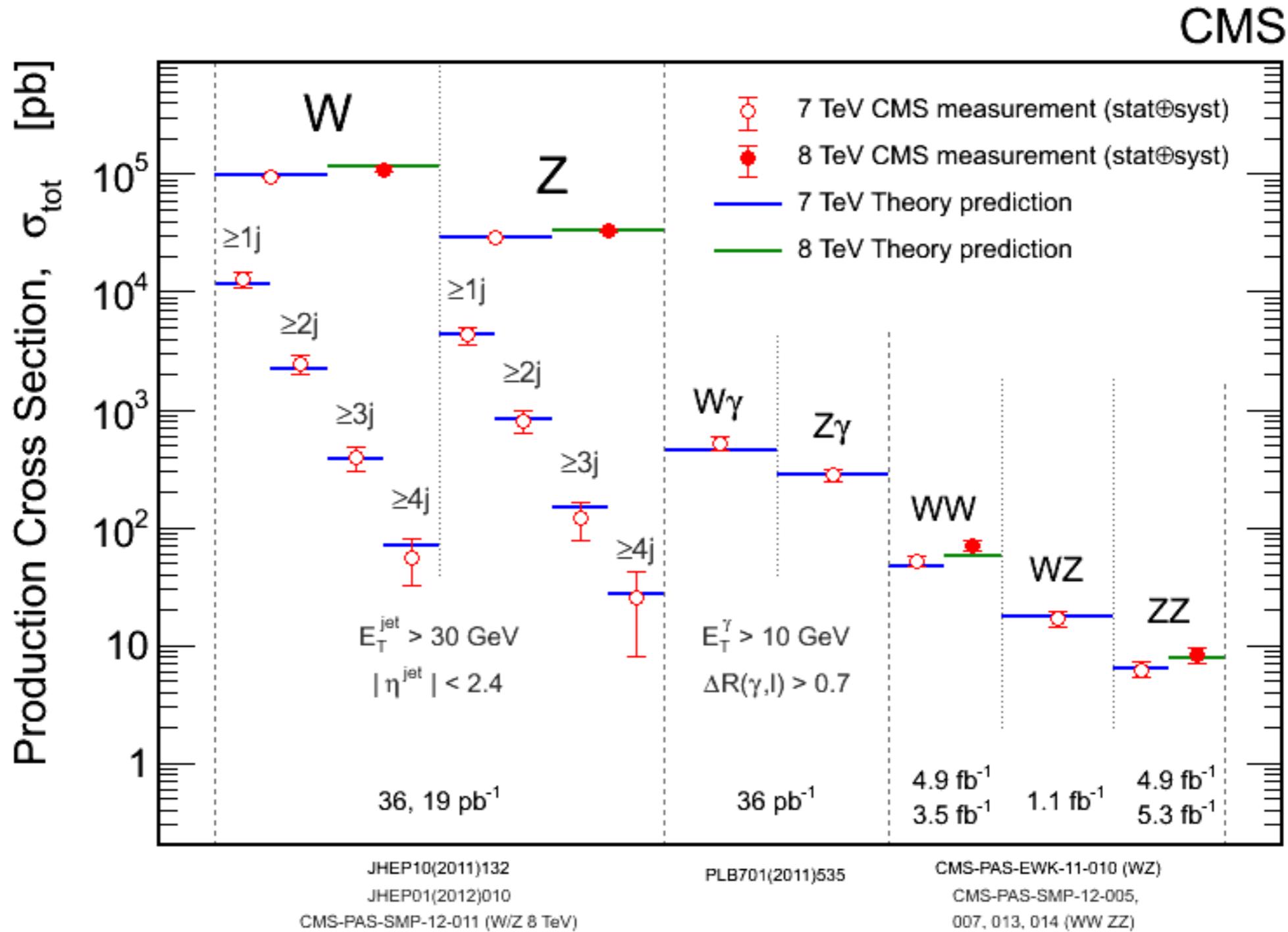
- $\sigma(pp \rightarrow W\gamma + X) \times B(W \rightarrow l\nu) = 56.3 \pm 7.4 \text{ pb}$
Theory: $49.4 \pm 3.8 \text{ pb}$
- $\sigma(pp \rightarrow Z\gamma + X) \times B(Z \rightarrow ll) = 9.4 \pm 1.2 \text{ pb}$
■ Theory: $9.6 \pm 0.4 \text{ pb}$



- 2011 update coming to HCP!

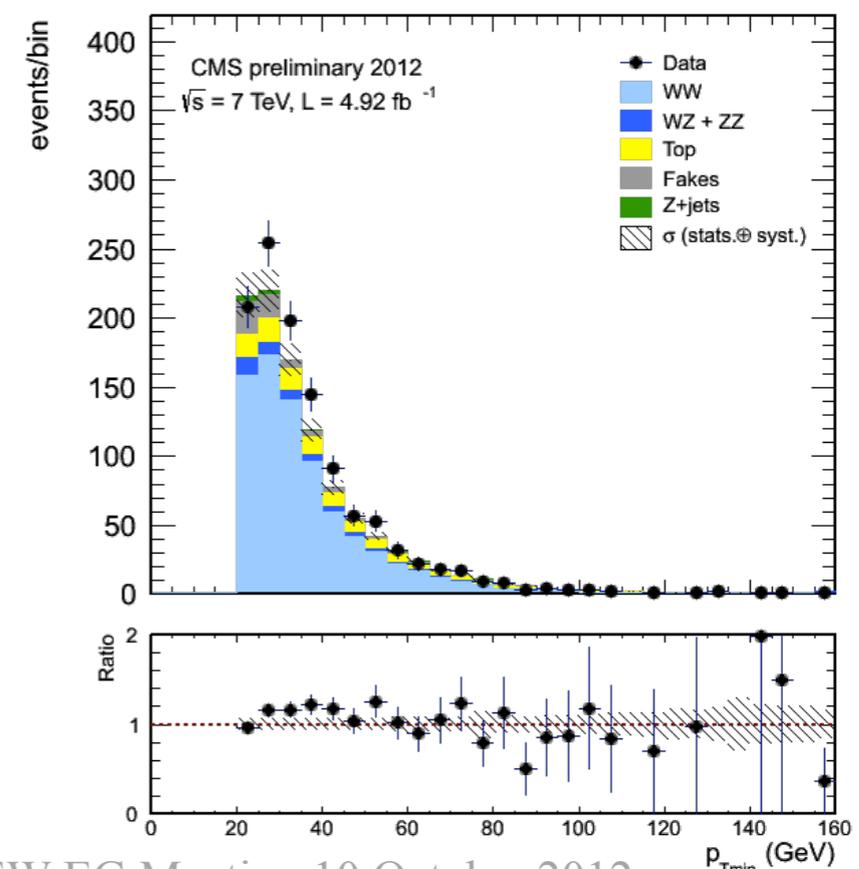
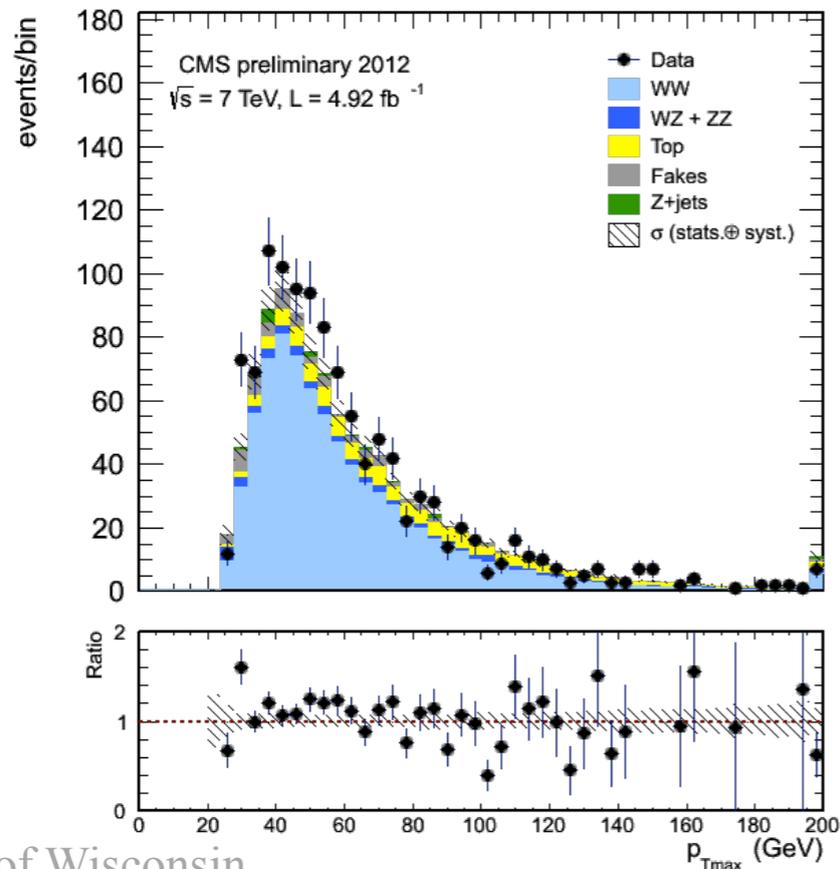
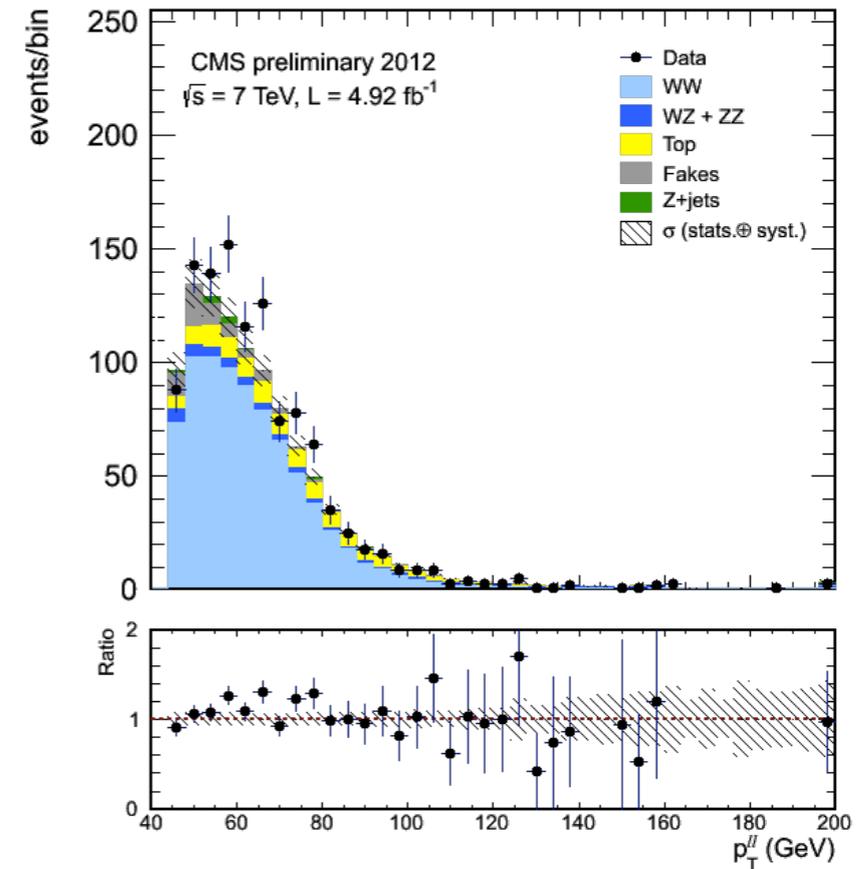
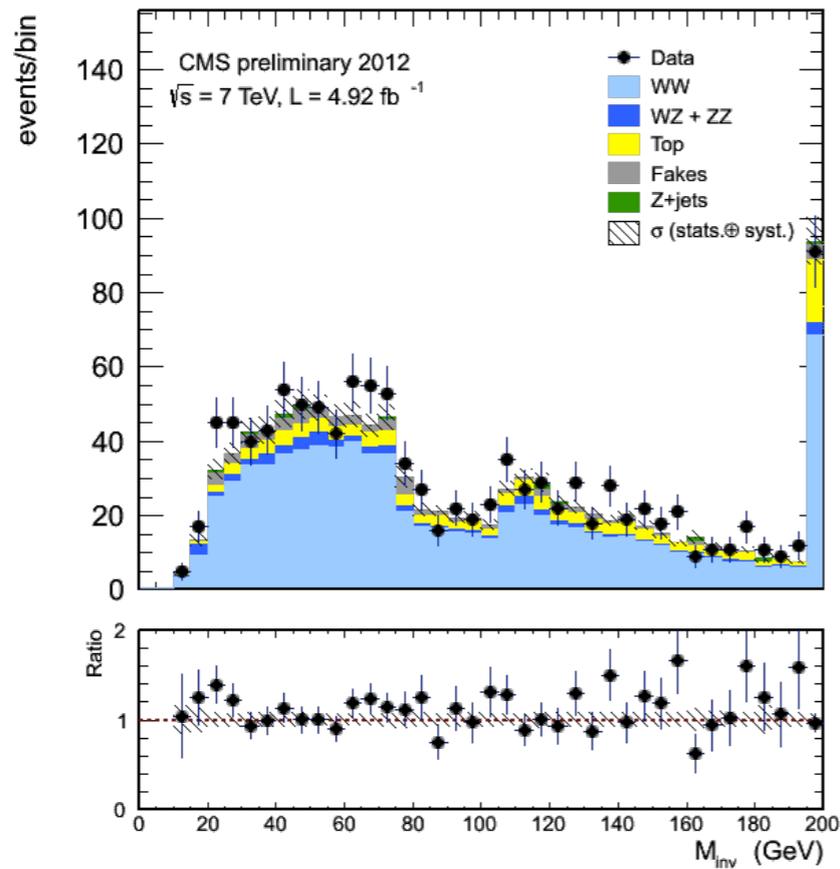
- Diboson production measurements at CMS indicate good agreement with Standard Model predictions
- aTGC limits have been produced in a handful of couplings, with sensitivity comparable to (or better than) current LEP and Tevatron limits (See M. Herndon's talk this afternoon)
- New measurements and updates coming soon!

CMS Boson Measurement outlook



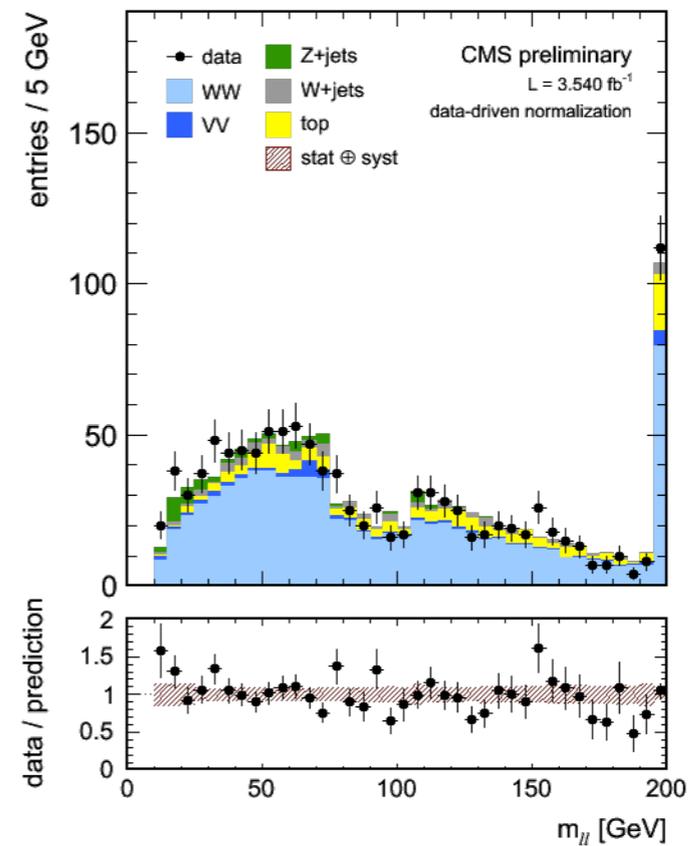
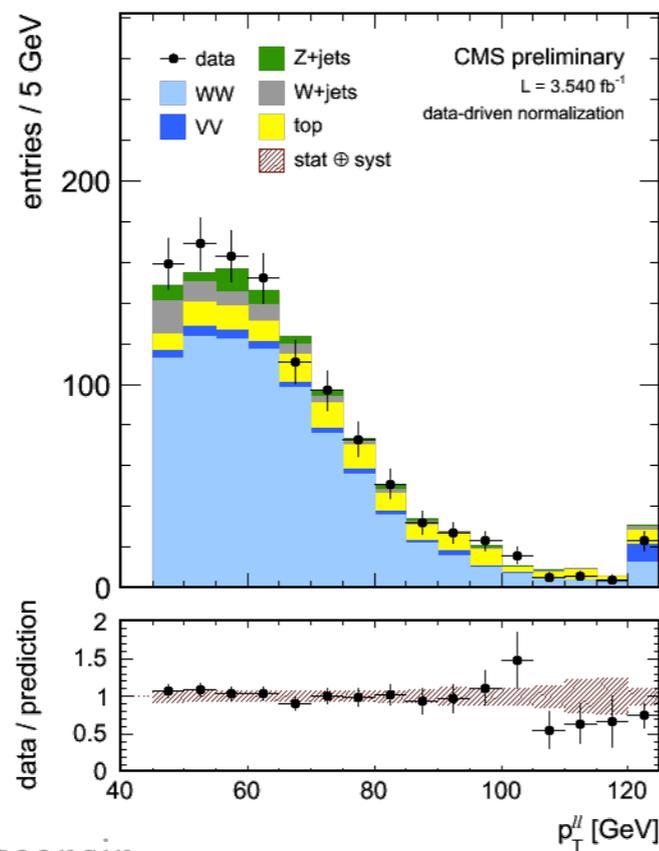
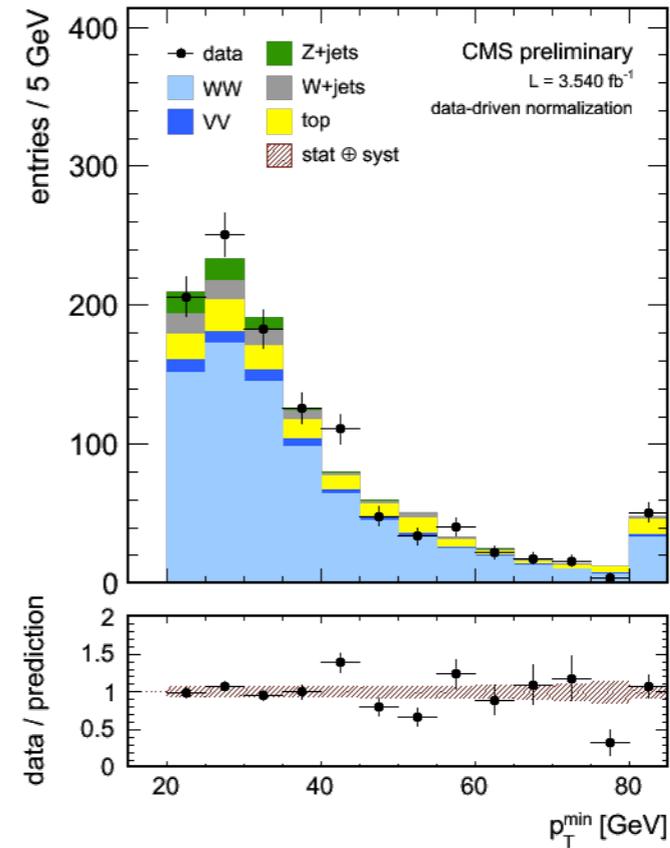
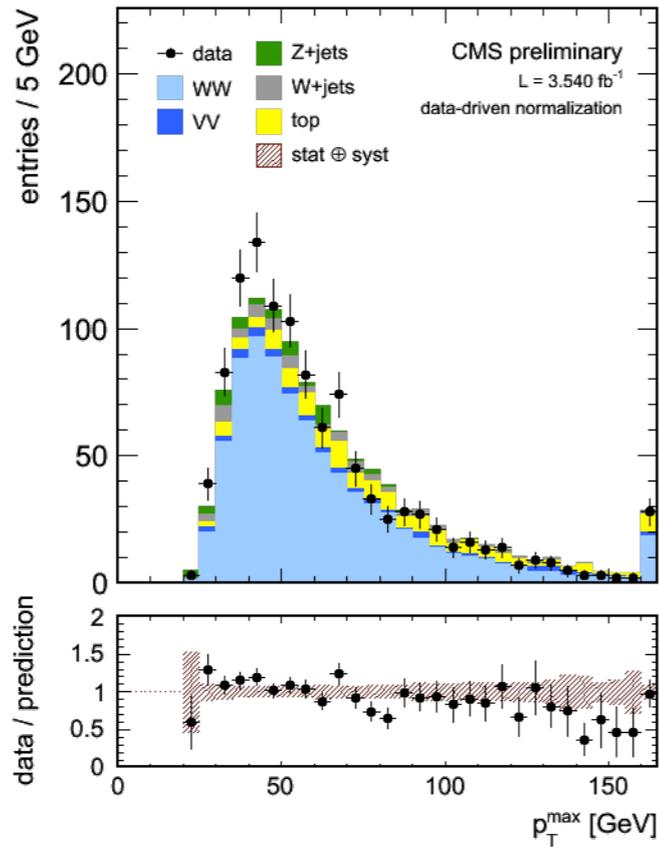
WW, 7 TeV Systematics

	$q\bar{q} \rightarrow W^+W^-$	$gg \rightarrow W^+W^-$	top	W + jets	WZ + ZZ	$Z/\gamma^* \rightarrow \ell\ell$	W + γ	W + γ^*	$Z/\gamma^* \rightarrow \tau\tau$
Luminosity	2.2	2.2	-	-	2.2	-	2.2	-	-
Trigger efficiency	1.5	1.5	-	-	1.5	-	1.5	-	-
Lepton id efficiency	2.0	2.0	-	-	2.0	-	2.0	-	-
Muon momentum scale	1.5	1.5	-	-	1.5	-	1.5	-	-
Electron energy scale	2.5	2.5	-	-	1.9	-	2.0	-	-
E_T^{miss} resolution	2.0	2.0	-	-	2.0	-	2.0	-	-
Jet veto efficiency	4.7	4.7	-	-	4.7	-	4.7	-	-
pile-up	2.3	2.3	-	-	2.3	-	2.3	-	-
top normalisation	-	-	18	-	-	-	-	-	-
W + jets normalisation	-	-	-	36.0	-	-	-	-	-
$Z/\gamma^* \rightarrow \ell^+\ell^-$ normalisation	-	-	-	-	-	50.0	-	-	-
W + γ normalisation	-	-	-	-	-	-	30.0	-	-
W + γ^* normalisation	-	-	-	-	-	-	-	30.0	-
$Z/\gamma^* \rightarrow \tau^+\tau^-$ normalisation	-	-	-	-	-	-	-	-	10.0
PDFs	2.3	0.8	-	-	5.9	-	-	-	-
Higher order corrections	1.5	30.0	-	-	3.3	-	-	-	-
Sample statistics	0.8	1.3	-	6.6	1.5	-	48.9	10.3	15.9



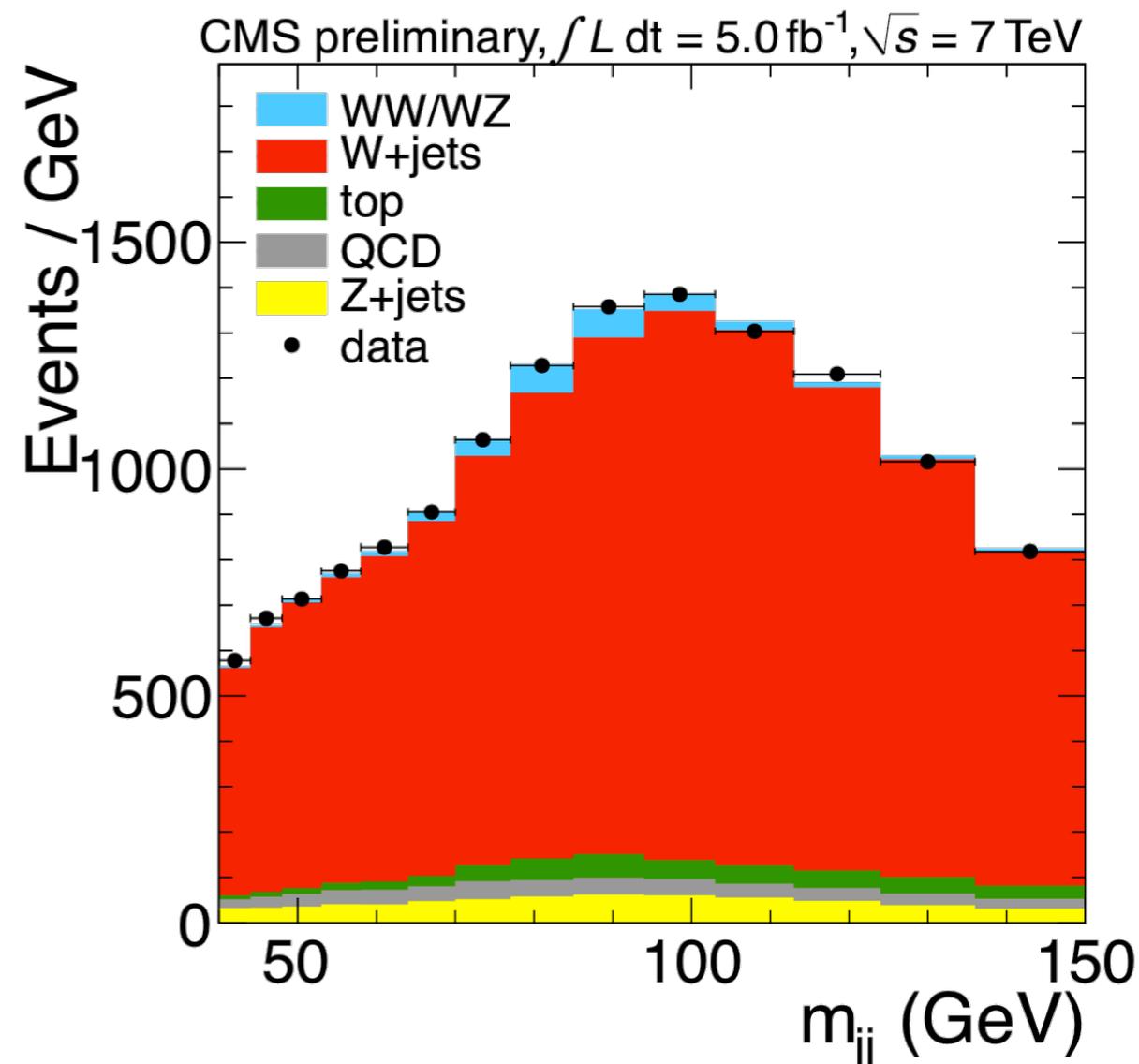
WW, 8 TeV Systematics

	$q\bar{q} \rightarrow WW$	$gg \rightarrow WW$	top	W+jets	WZ + ZZ	Z/ γ^*	W + γ	W + γ^*
Luminosity	5.0	5.0	-	-	5.0	-	5.0	-
Trigger efficiency	1.5	1.5	-	-	1.5	-	1.5	-
Lepton id efficiency	2.0	2.0	-	-	2.0	-	2.0	-
Muon momentum scale	1.5	1.5	-	-	1.5	-	1.5	-
Electron energy scale	2.5	2.5	-	-	1.9	-	2.0	-
E_T^{miss} resolution	2.0	2.0	-	-	2.0	-	2.0	-
Jet veto efficiency	4.7	4.7	-	-	4.7	-	4.7	-
pile-up	2.3	2.3	-	-	2.3	-	2.3	-
top normalisation	-	-	$9 \oplus 13$	-	-	-	-	-
Wjets normalisation	-	-	-	$7 \oplus 36$	-	-	-	-
Z normalisation	-	-	-	-	-	$36 \oplus 20$	-	-
W + γ normalisation	-	-	-	-	-	-	30	-
W + γ^* normalisation	-	-	-	-	-	-	-	30
PDFs	2.3	0.8	-	-	5.9	-	-	-
Higher order corrections	1.5	30.0	-	-	3.3	-	-	-
Sample statistics	1.1	3.1	-	-	4.1	-	8.4	8.4



WW/WZ in dijets

- Key features:
 - Large branching ratio
 - Full handle on boson p_T
- Signature: Leptonically decaying W with exactly two non-b jets
- Background treatment:
 - Yields extracted via unbinned maximum likelihood fit to dijet invariant mass
- $\sigma(WW+WZ) = 68.89 \pm 8.71$
(stat) ± 9.70 (syst) ± 1.52 (lumi) pb
 - Theory: 65.6 ± 2.2 pb

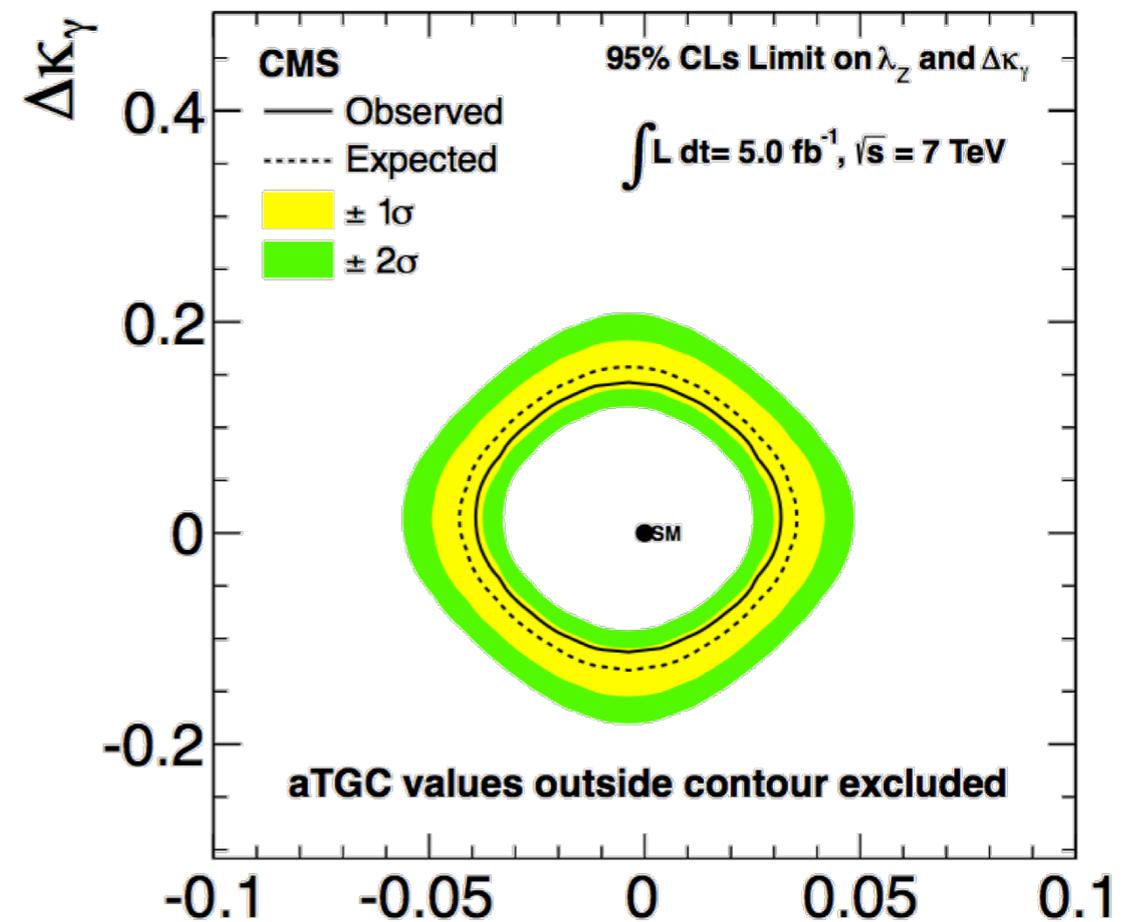
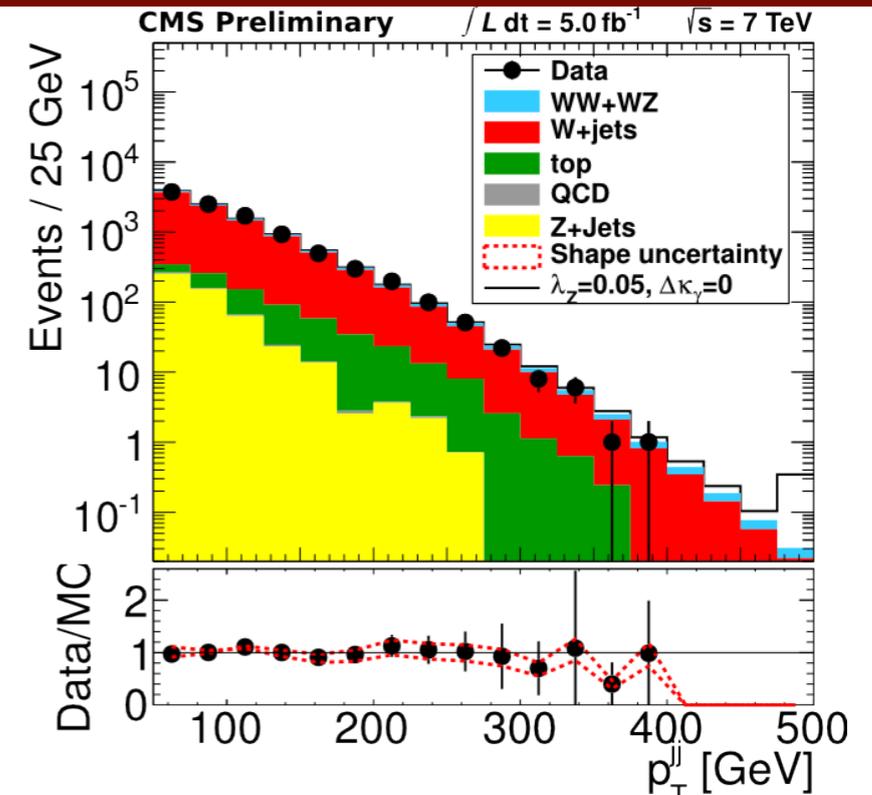


Process	Muon channel	Electron channel
Diboson (WW+WZ)	1899 ± 373	783 ± 306
W+jets	67384 ± 586	31644 ± 850
$t\bar{t}$	1662 ± 117	946 ± 67
Single top	650 ± 33	308 ± 17
Drell-Yan+jets	3609 ± 155	1408 ± 64
Multijet (QCD)	296 ± 317	4195 ± 867
Fit χ^2/dof (probability)	9.73/12 (0.64)	5.30/12 (0.95)
Total from fit	75420	39371
Data	75419	39365
Acceptance \times efficiency ($\mathcal{A}\epsilon$)	5.153×10^{-3}	2.633×10^{-3}
Expected WW+WZ yield from simulation	1697 ± 57	867 ± 29

- Anomalous couplings parameterized via Δg_Z^1 , λ_Z , and $\Delta\kappa_\gamma$
- Assume the SM value $\Delta g_Z^1 = 0$, set limits on λ_Z and $\Delta\kappa_\gamma$
- Dijet p_T chosen as observable
- Require:
 - $75 < m_{jj} < 90$ GeV
- Set 95% upper limit in $(\lambda_Z, \Delta\kappa_\gamma)$ space using CLs methodology

$$-0.038 < \lambda_Z < 0.030$$

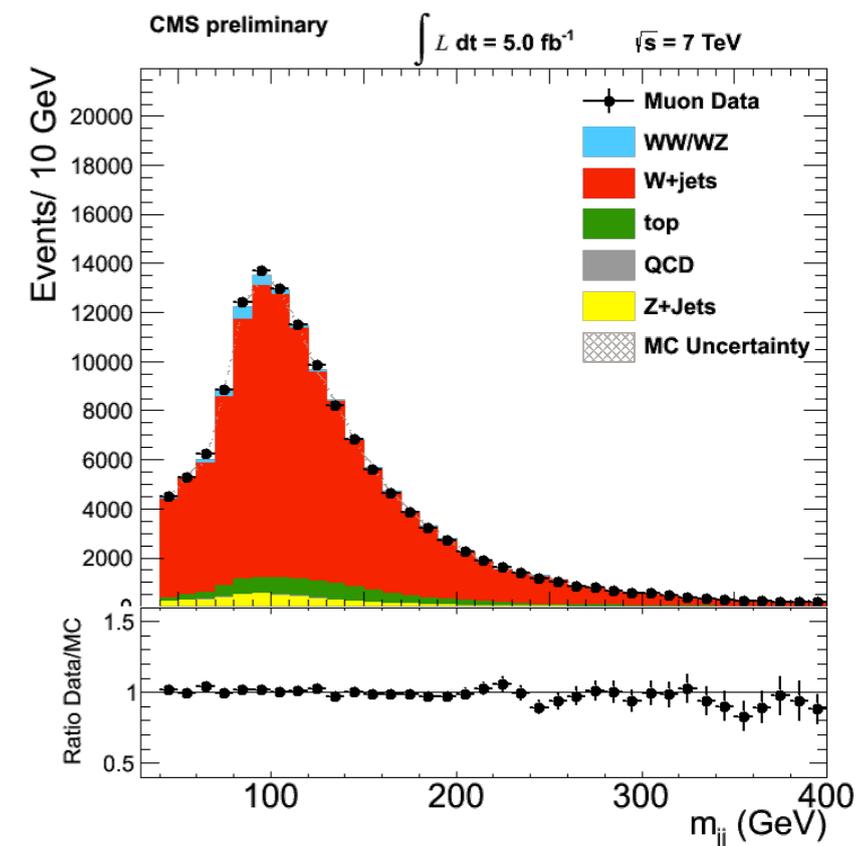
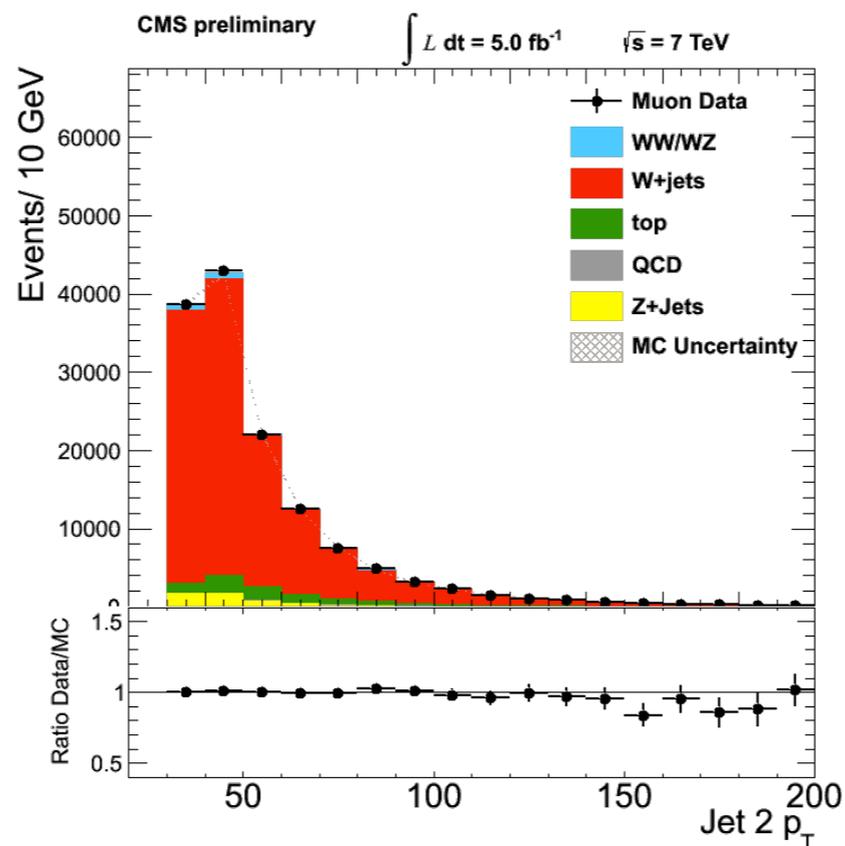
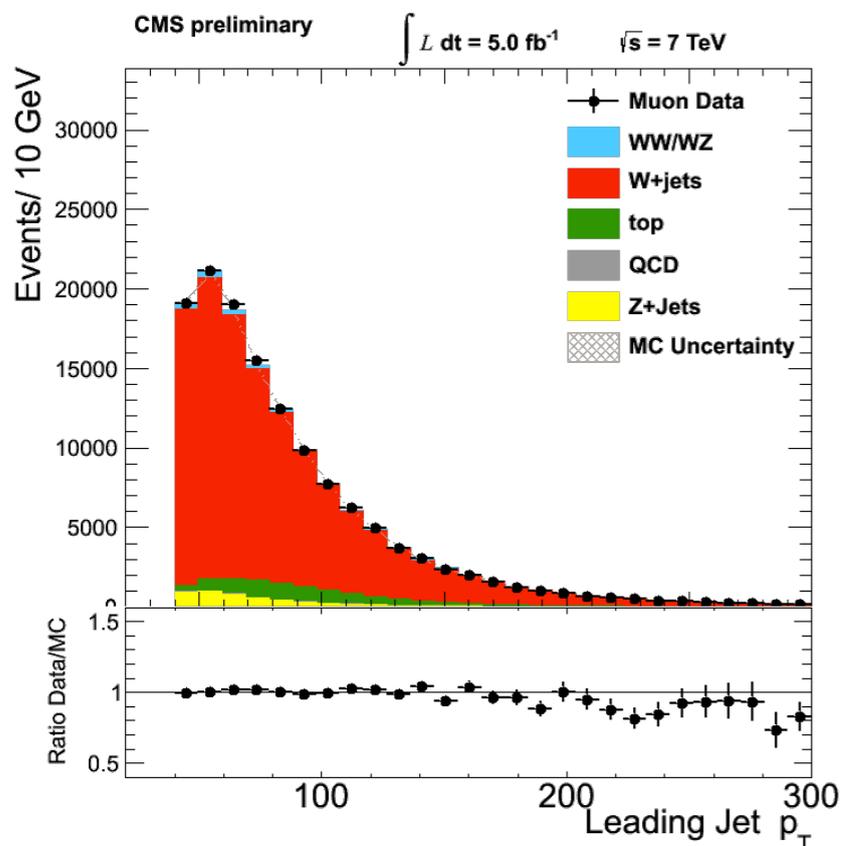
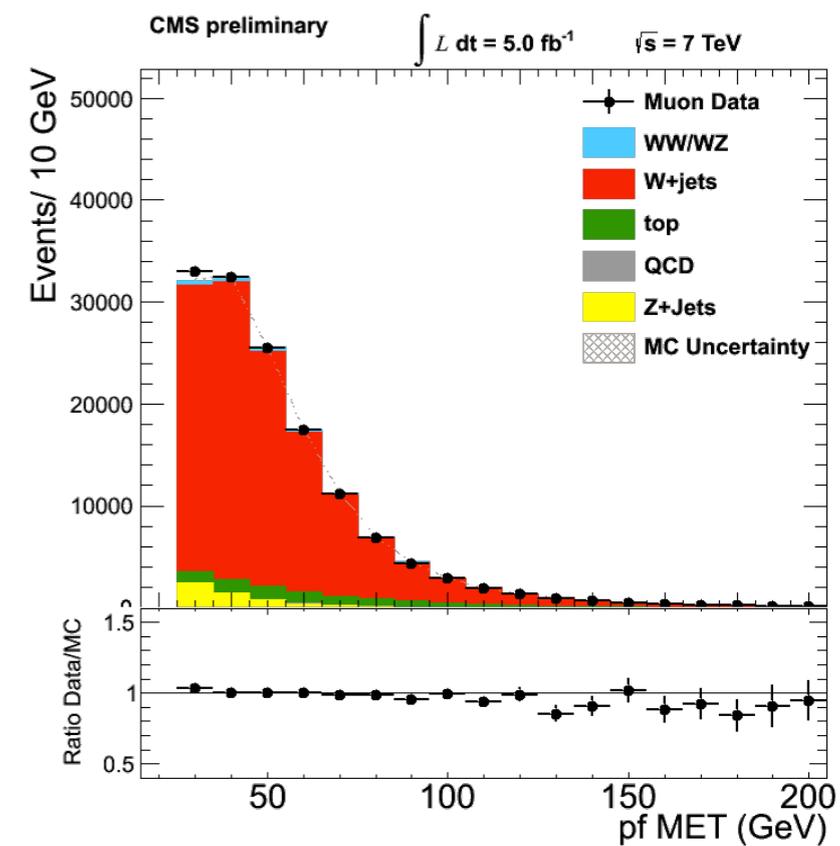
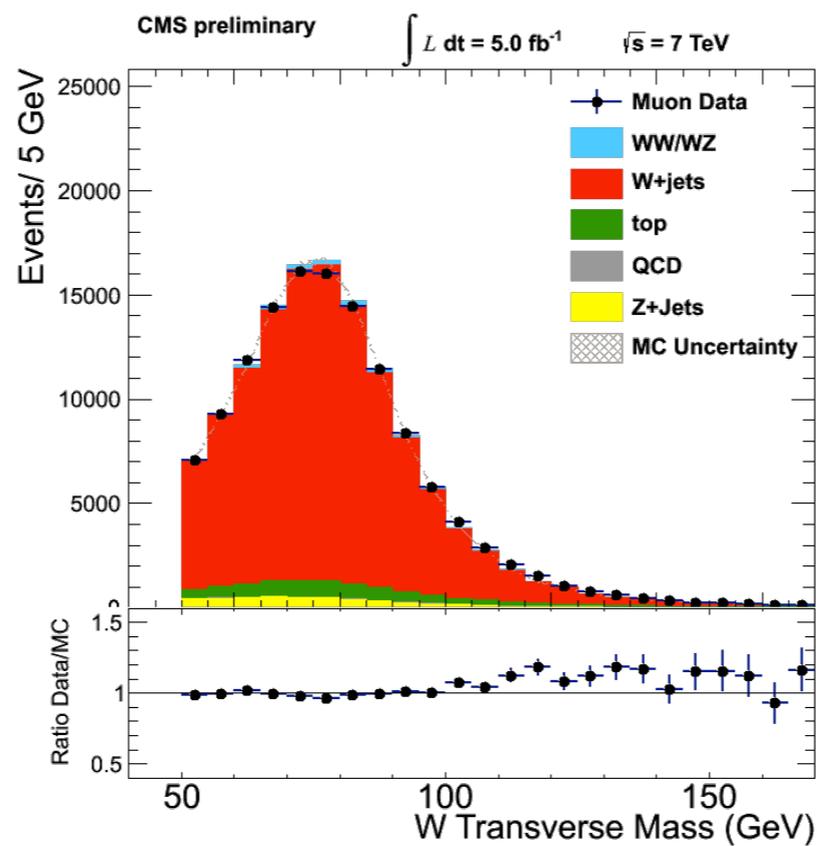
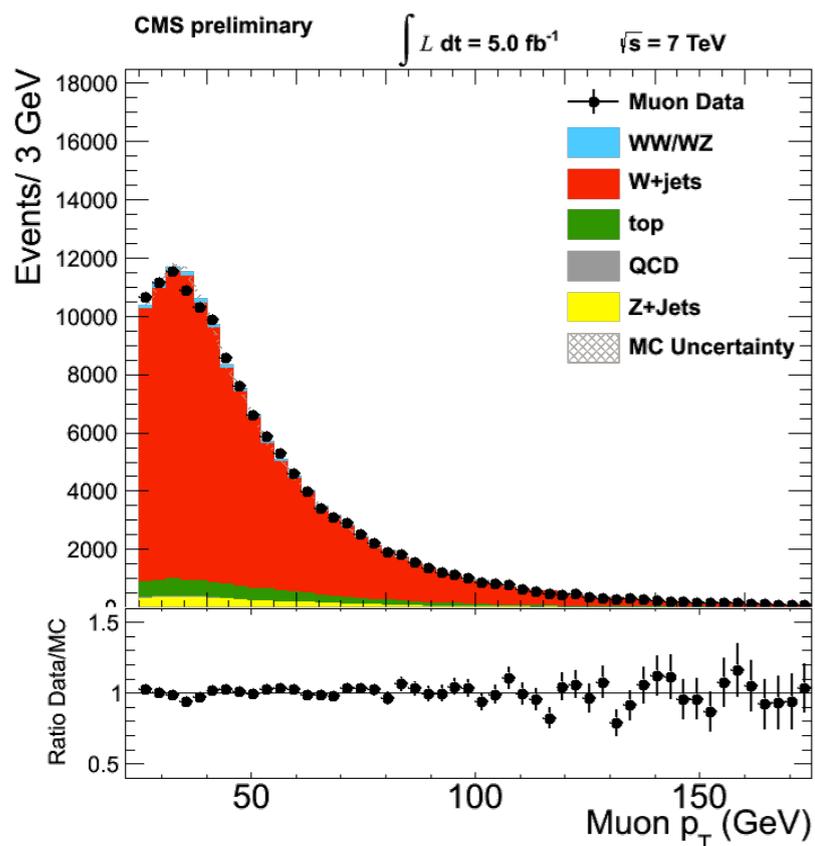
$$-0.111 < \Delta\kappa_\gamma < 0.142$$



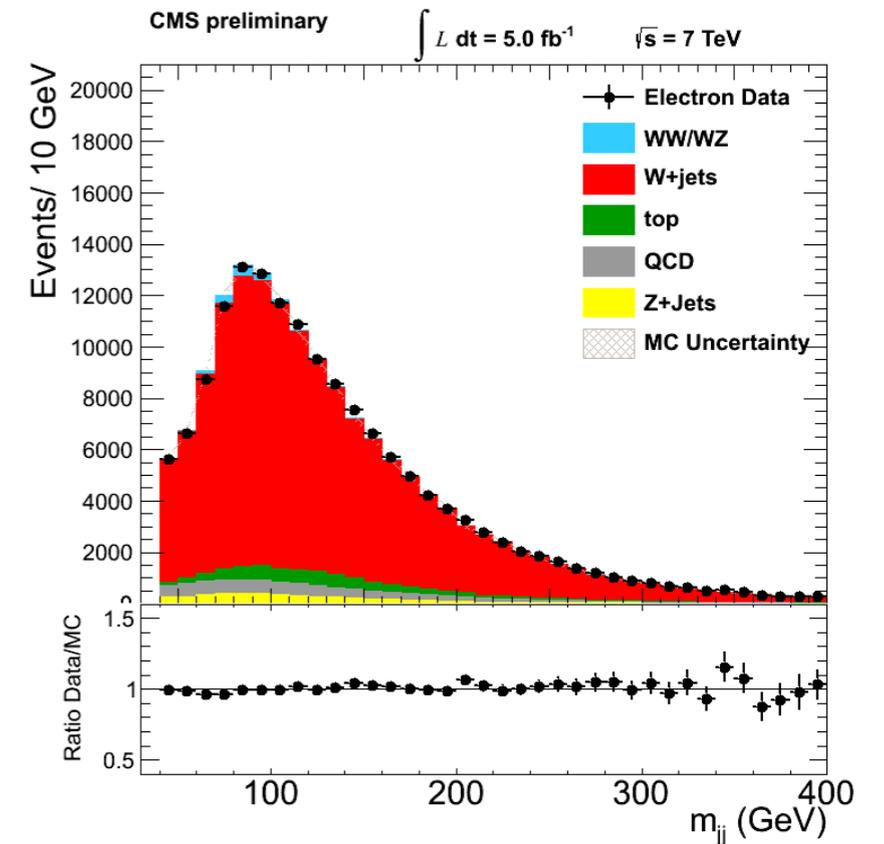
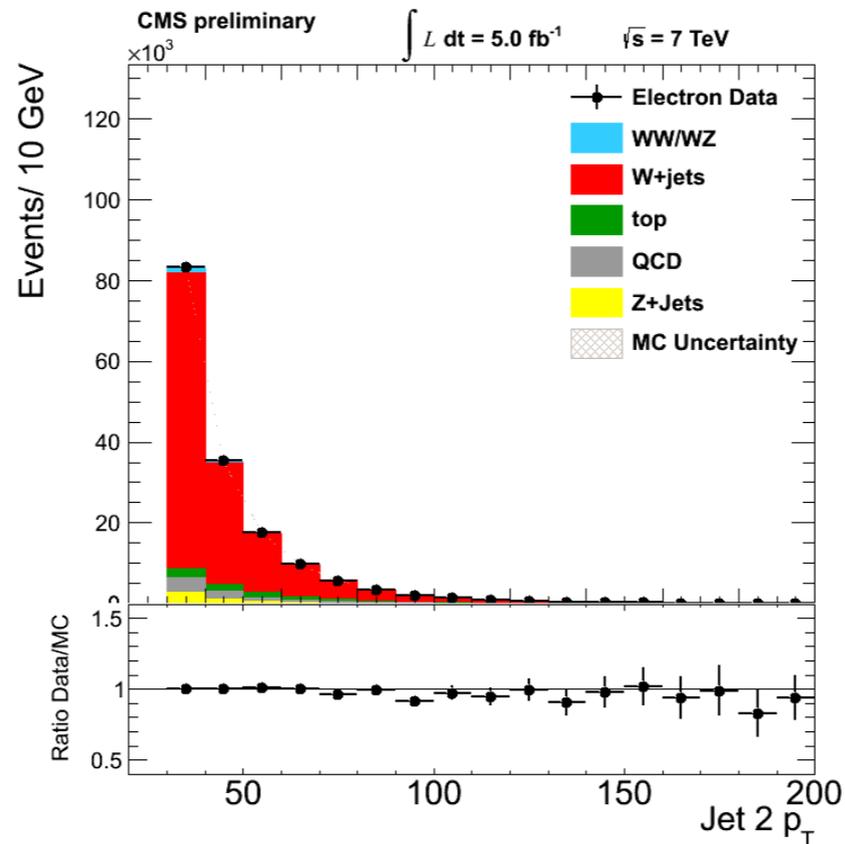
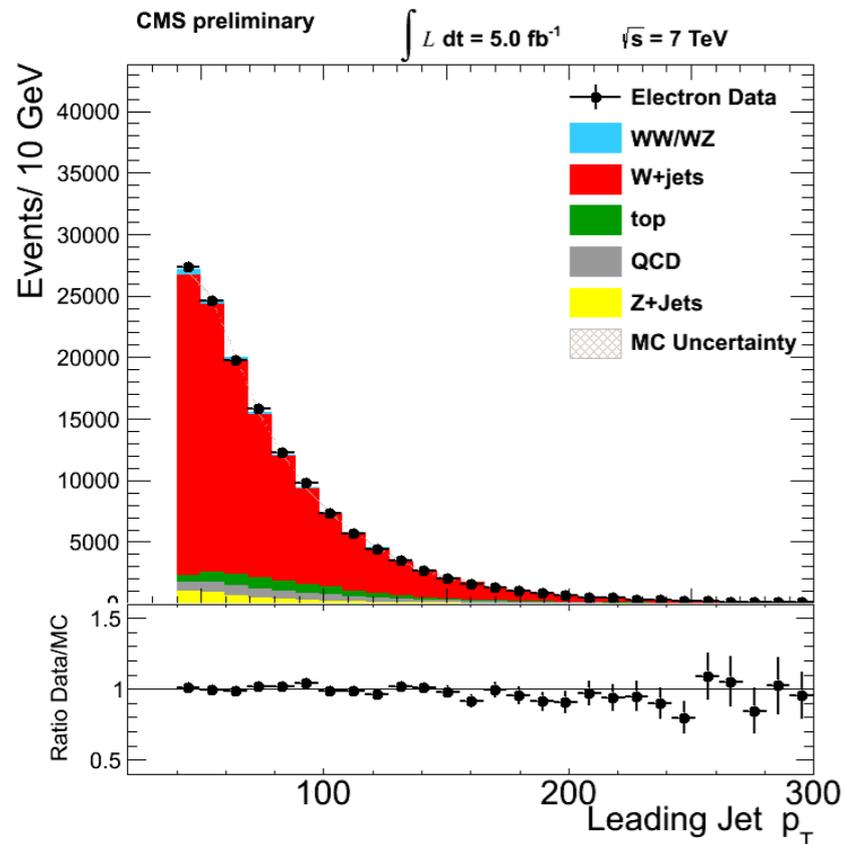
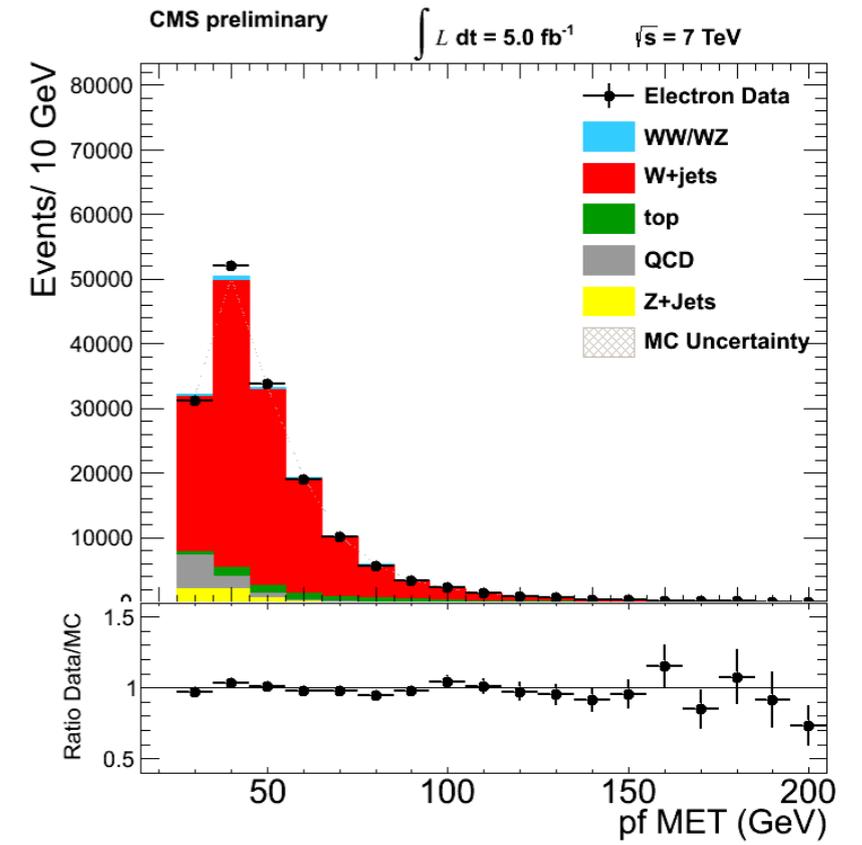
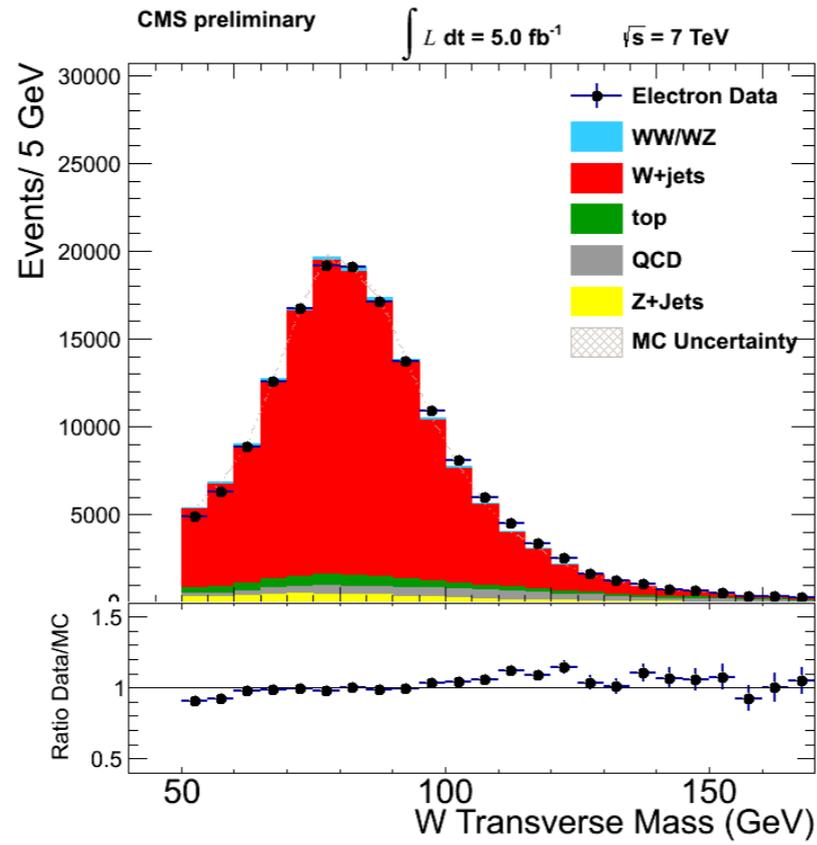
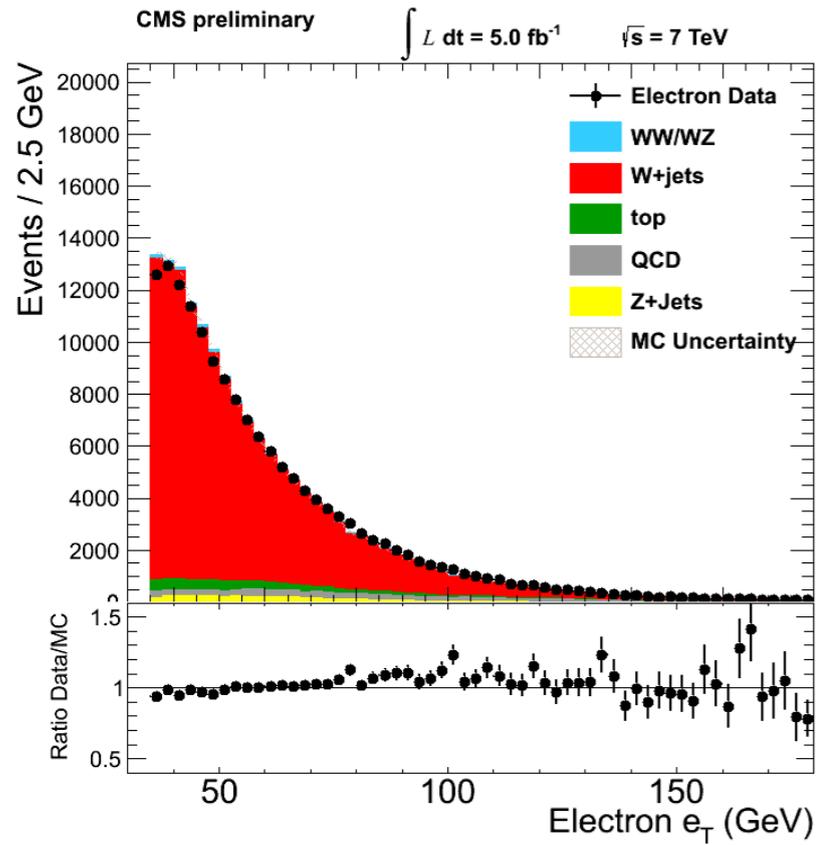
WW/WZ dijet, Systematics

- Systematic uncertainties:
 - Trigger Efficiency - 1%
 - Lepton Reconstruction and selection efficiency - 2%
 - Jet Energy scale - 0.6%
 - Missing Transverse Energy Resolution - 0.5%
 - Fit uncertainty - 0.2%
 - Luminosity Determination - 2.2%
 - Theory uncertainty on acceptance - 4%

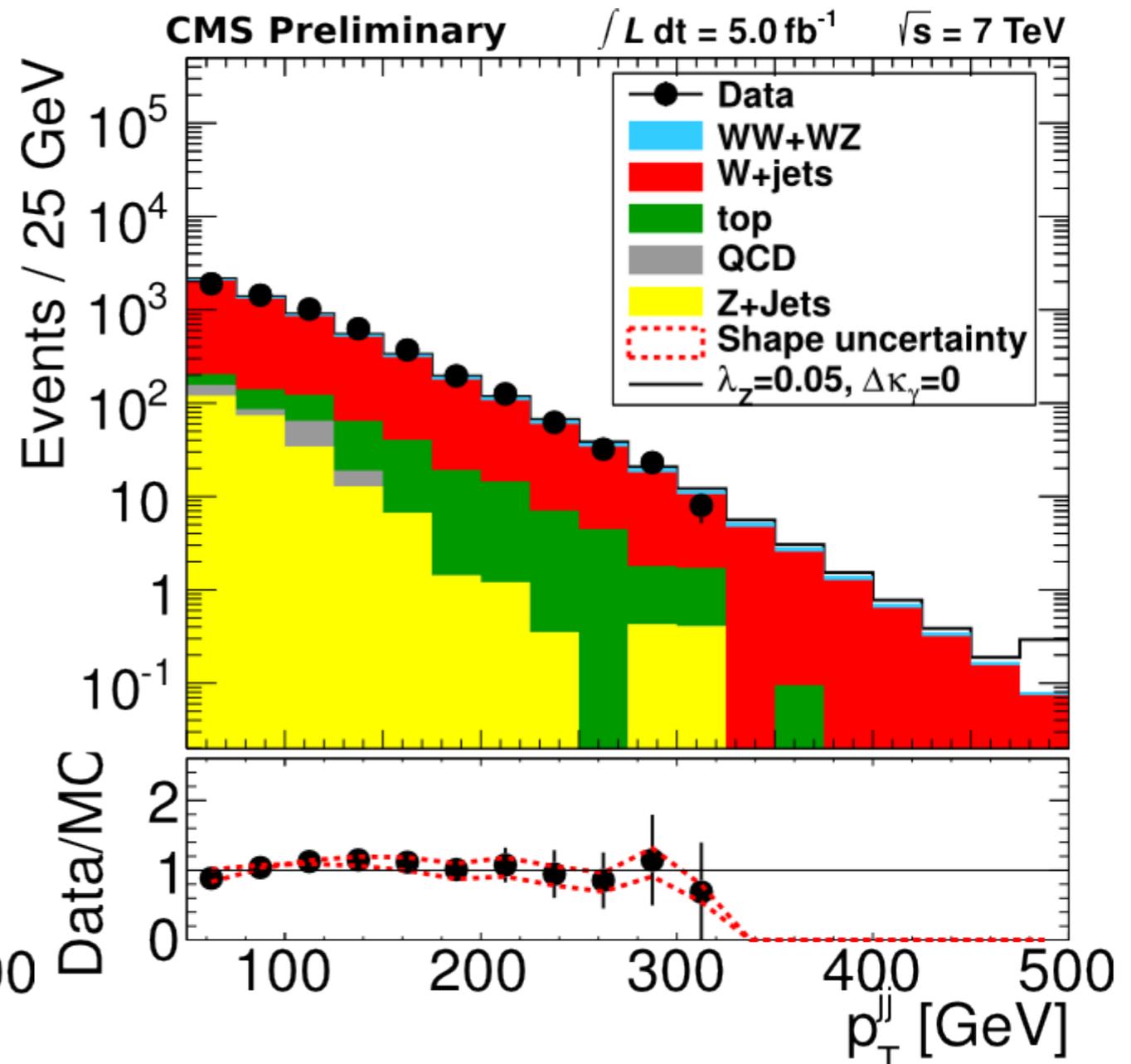
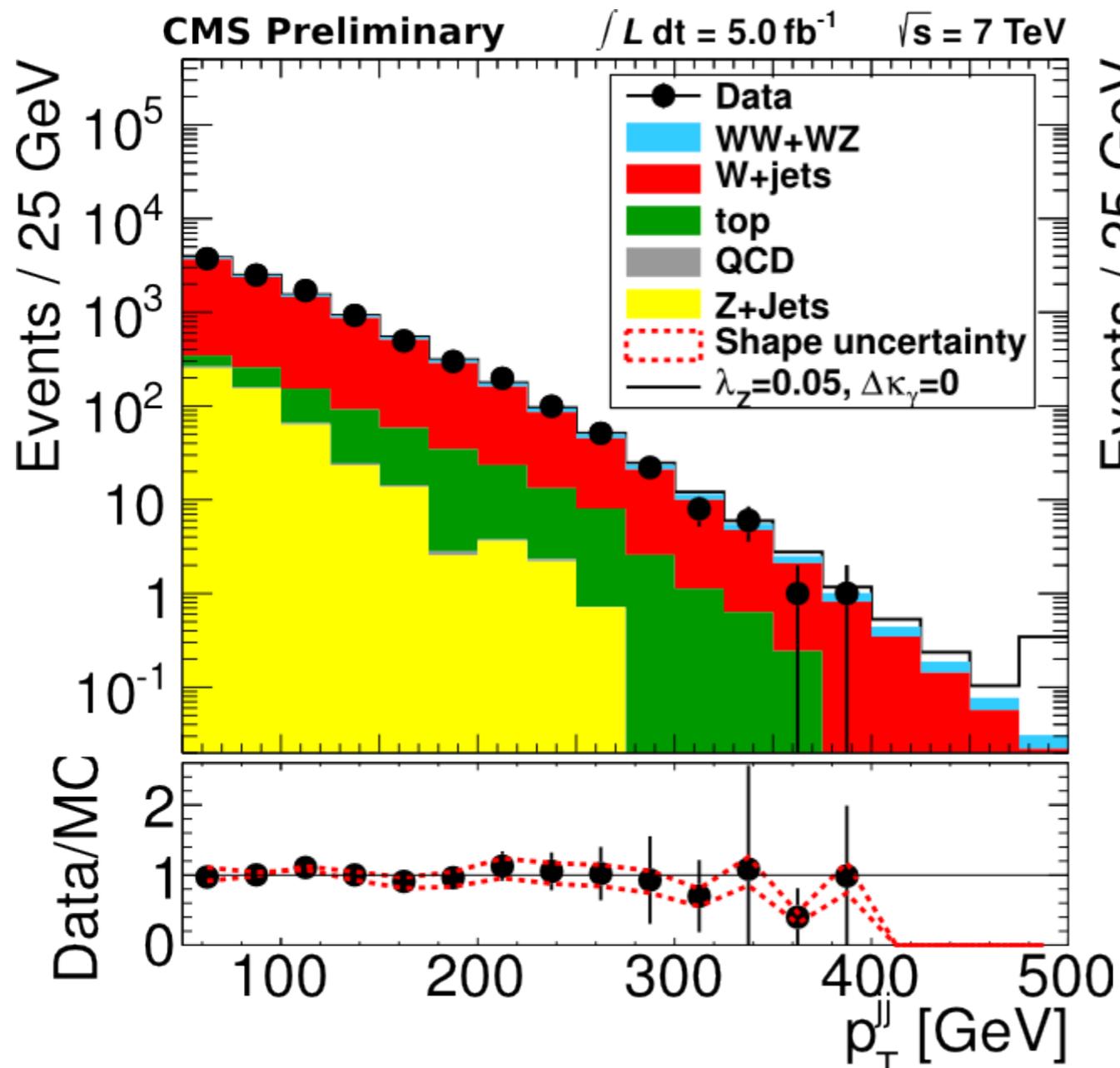
WW/WZ to Dijets, μ channel



WW/WZ to Dijets, e channel

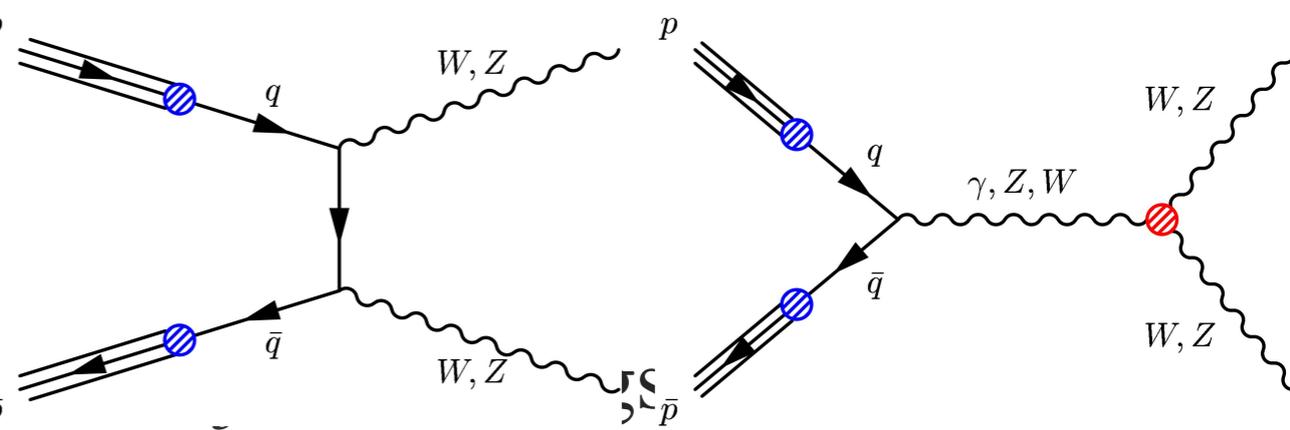


Manifestation of an aTGC signal, WW/WZ dijets



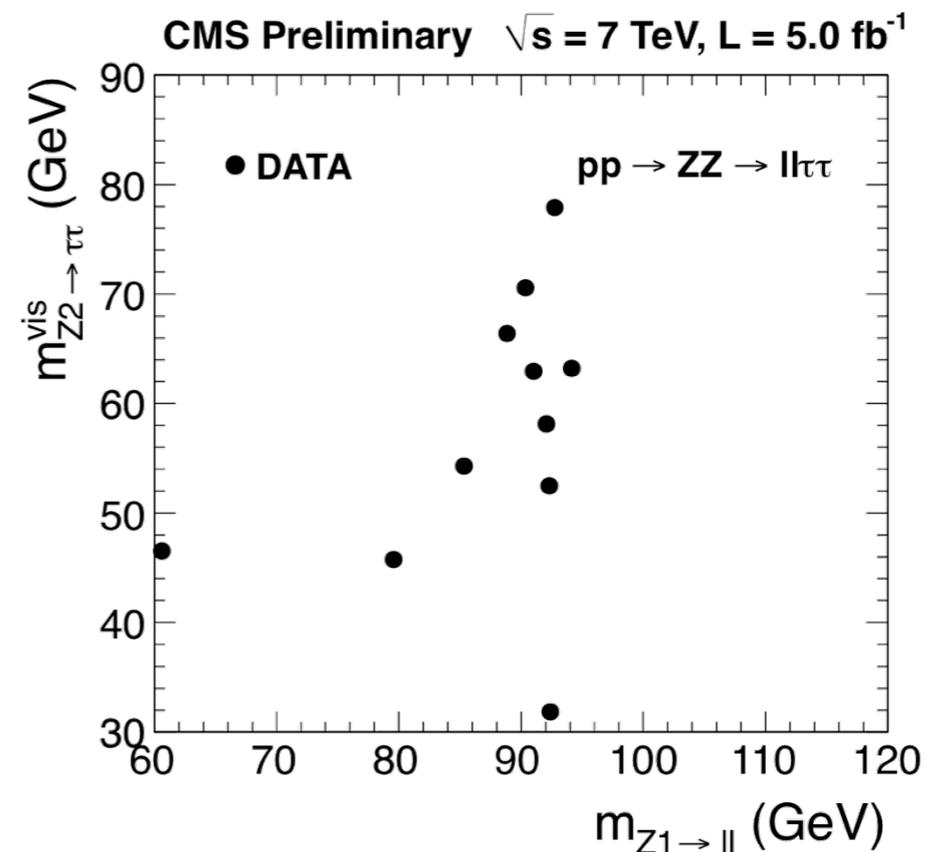
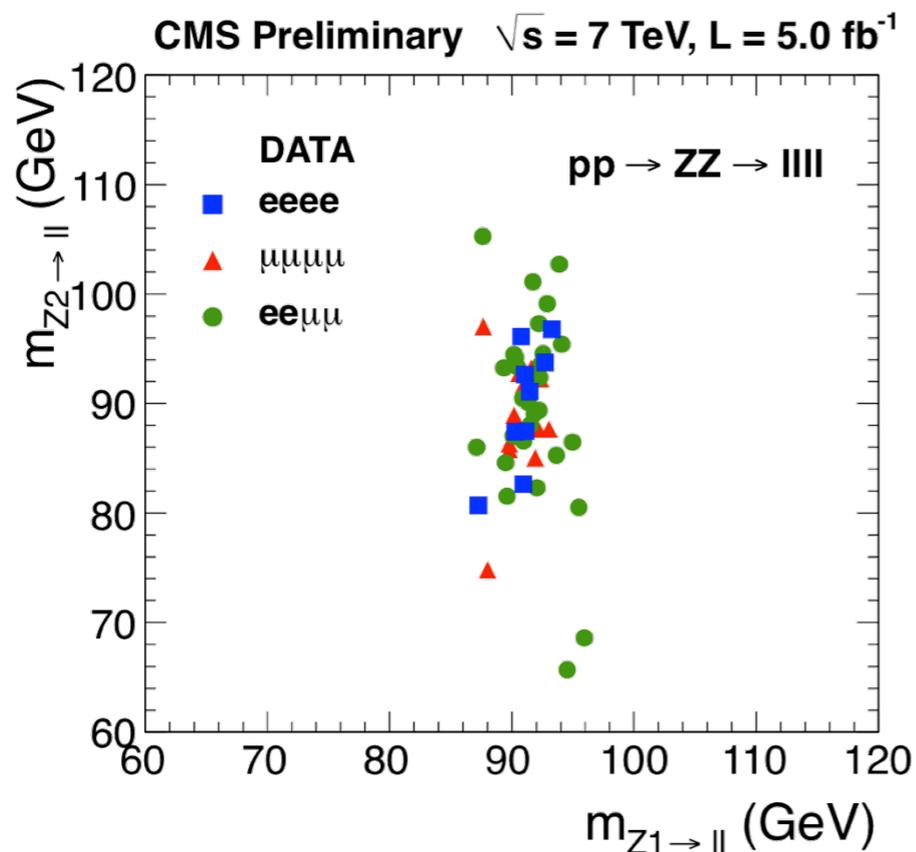
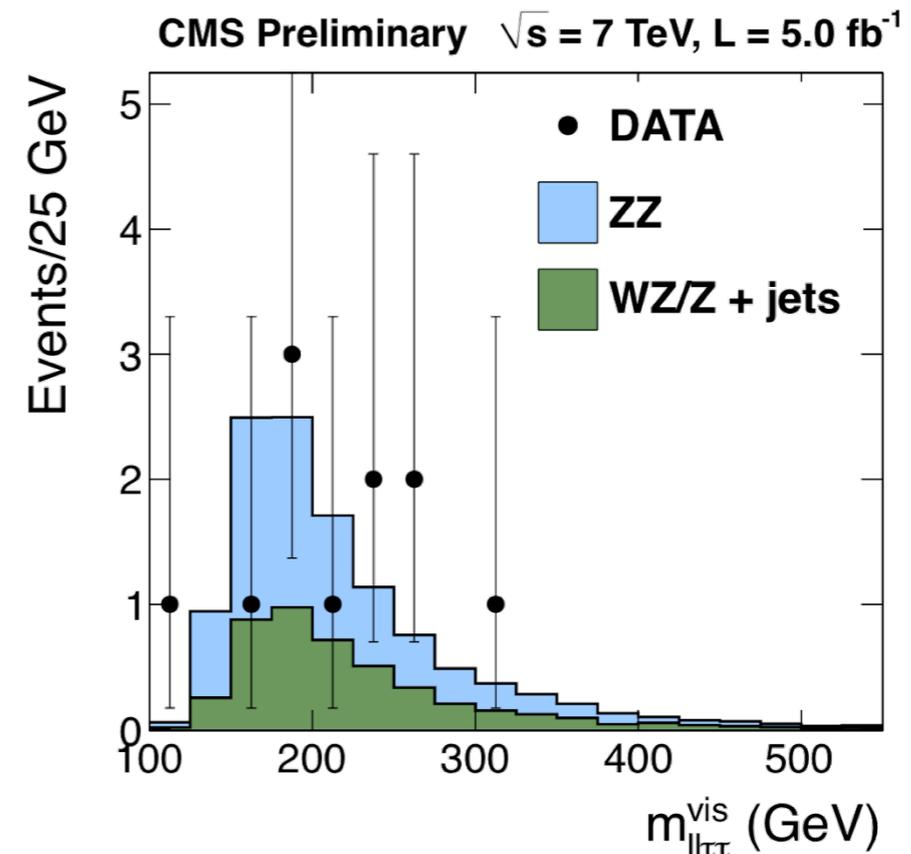
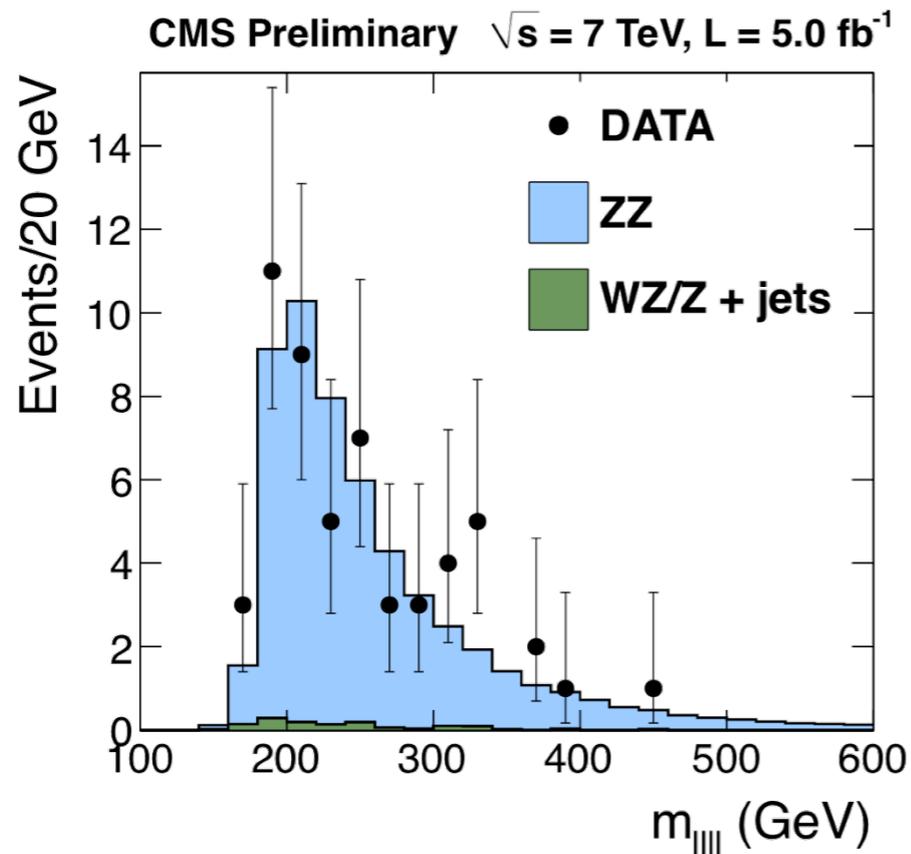
■ aTGCs motivation :

- Neutral TGC (ZZZ/ZZ γ) forbidden in SM (no s-channel at tree level)
- A way to prospect new physics
- ZZZ/ZZ γ vertex described with $f_4^Z, f_4^\gamma, f_5^Z, f_5^\gamma$
- Previous results:

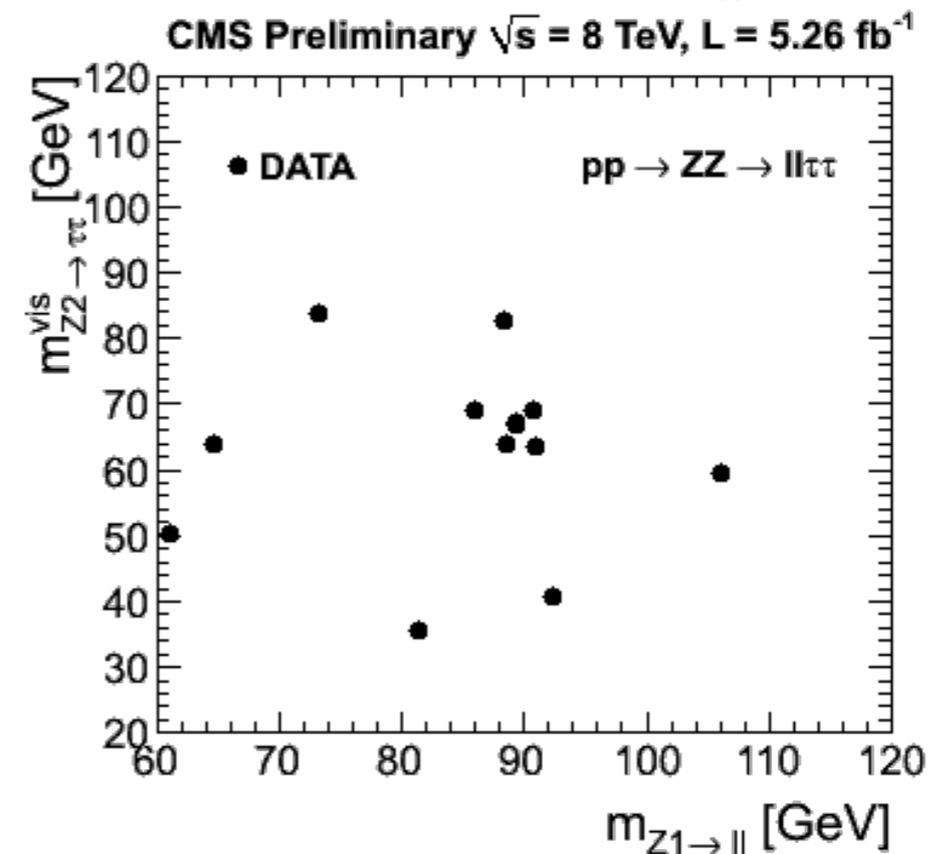
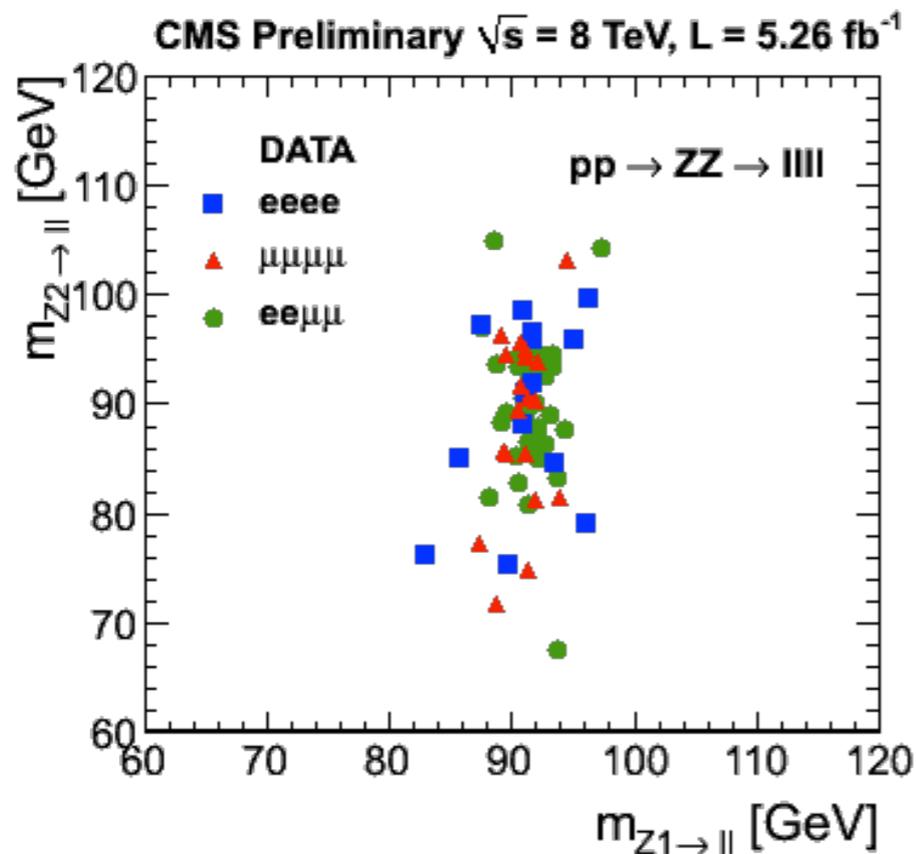
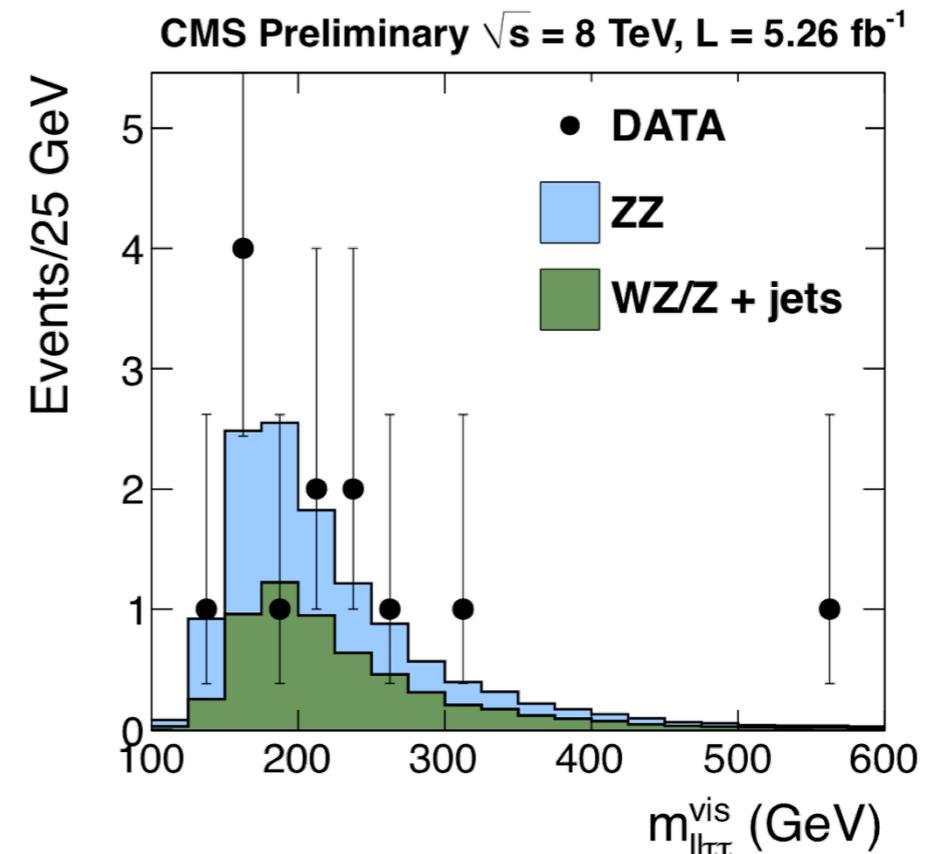
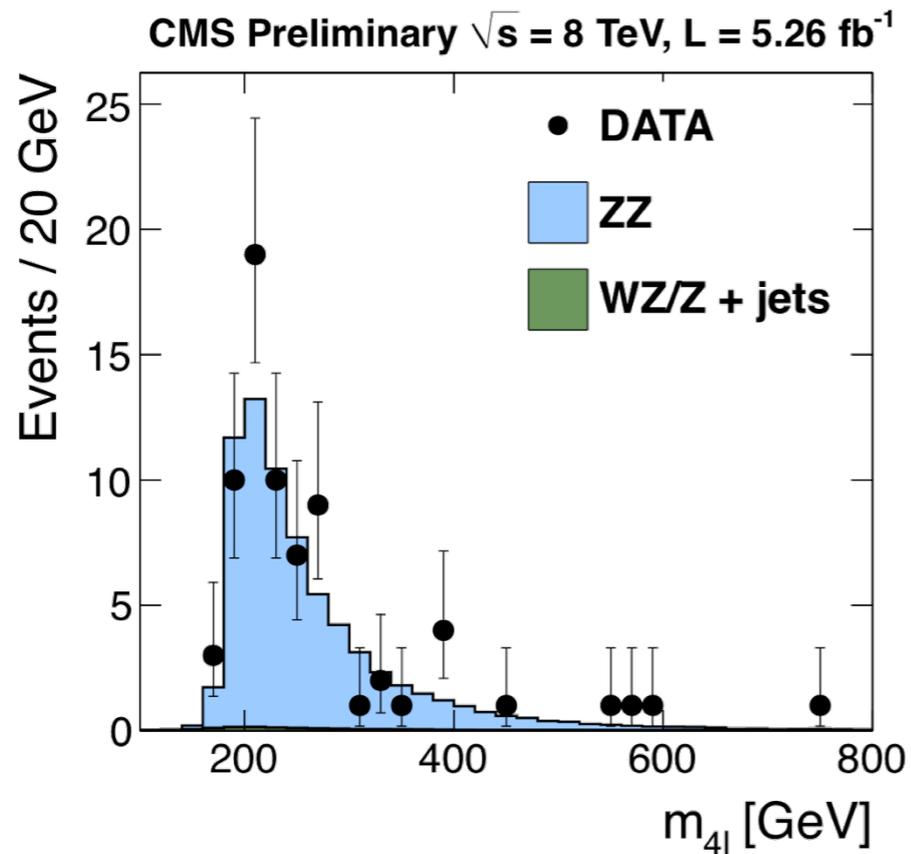


Experiment	f_4^Z	f_4^γ	f_5^Z	f_5^γ	Ref.	Comments
ALEPH	[-0.60;0.61]	[-0.40;0.36]	[-1.22;1.10]	[-0.81;0.79]	[23]	2D fit results
DELPHI	[-0.40;0.42]	[-0.23;0.25]	[-0.38;0.62]	[-0.52;0.58]	[24]	
L3	[-1.9;1.9]	[-1.1;1.2]	[-5.0;4.5]	[-3.0;2.9]	[27]	$\sqrt{s} = 189$ GeV only
OPAL	[-0.45;0.58]	[-0.32;0.33]	[-0.94;0.25]	[-0.71;0.59]	[26]	
LEP WG	[-0.30;0.30]	[-0.17;0.19]	[-0.34;0.38]	[-0.32;0.36]	[25]	LEP combination
CDF	[-0.12;0.12]	[-0.10;0.10]	[-0.13;0.12]	[-0.11;0.11]	[29]	$\Lambda=1.2$ TeV
D0	[-0.28;0.28]	[-0.26;0.26]	[-0.31;0.29]	[-0.20;0.28]	[28]	$\sim 1 \text{ fb}^{-1}$, $\Lambda=1.2$ TeV
ATLAS	[-0.12;0.12]	[-0.15;0.15]	[-0.13;0.13]	[-0.13;0.13]	[30]	$\sim 1 \text{ fb}^{-1}$, $\Lambda=2$ TeV
ATLAS	[-0.07;0.07]	[-0.08;0.08]	[-0.07;0.07]	[-0.08;0.08]	[30]	$\sim 1 \text{ fb}^{-1}$, $\Lambda=\text{inf}$

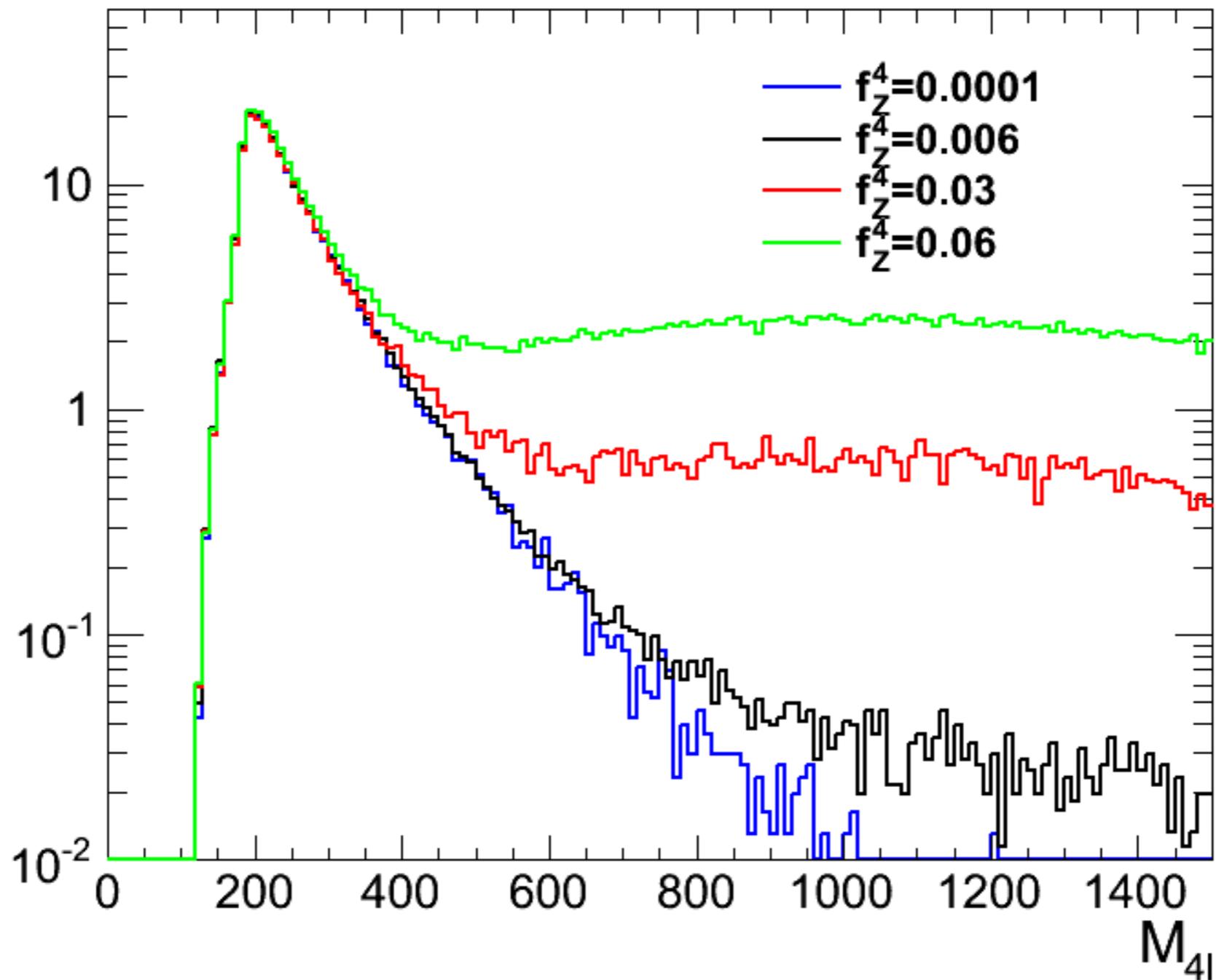
Invariant Mass Spectra, 7 TeV



Invariant Mass Spectra, 8 TeV



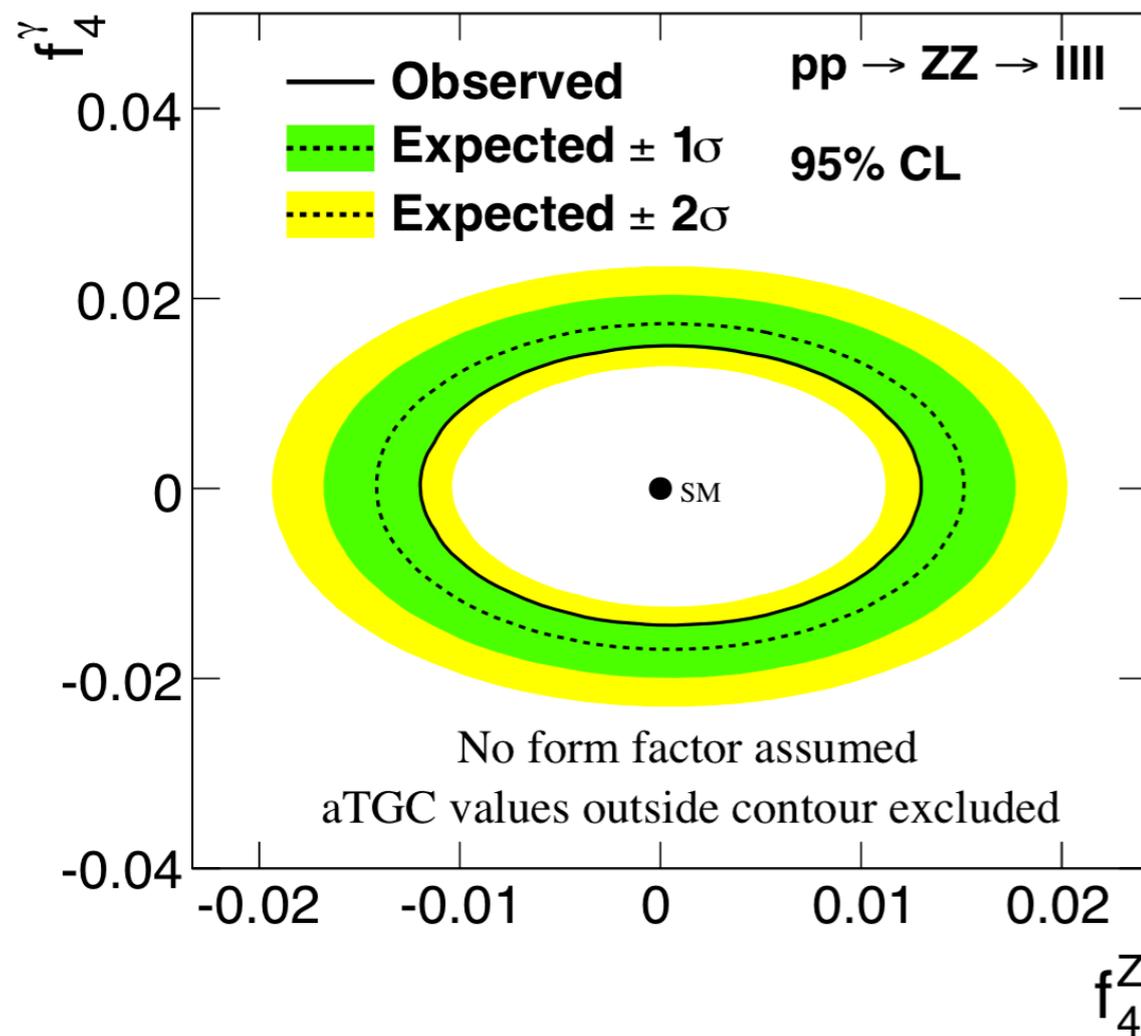
Manifestation of an aTGC signal, $ZZ \rightarrow 4l$



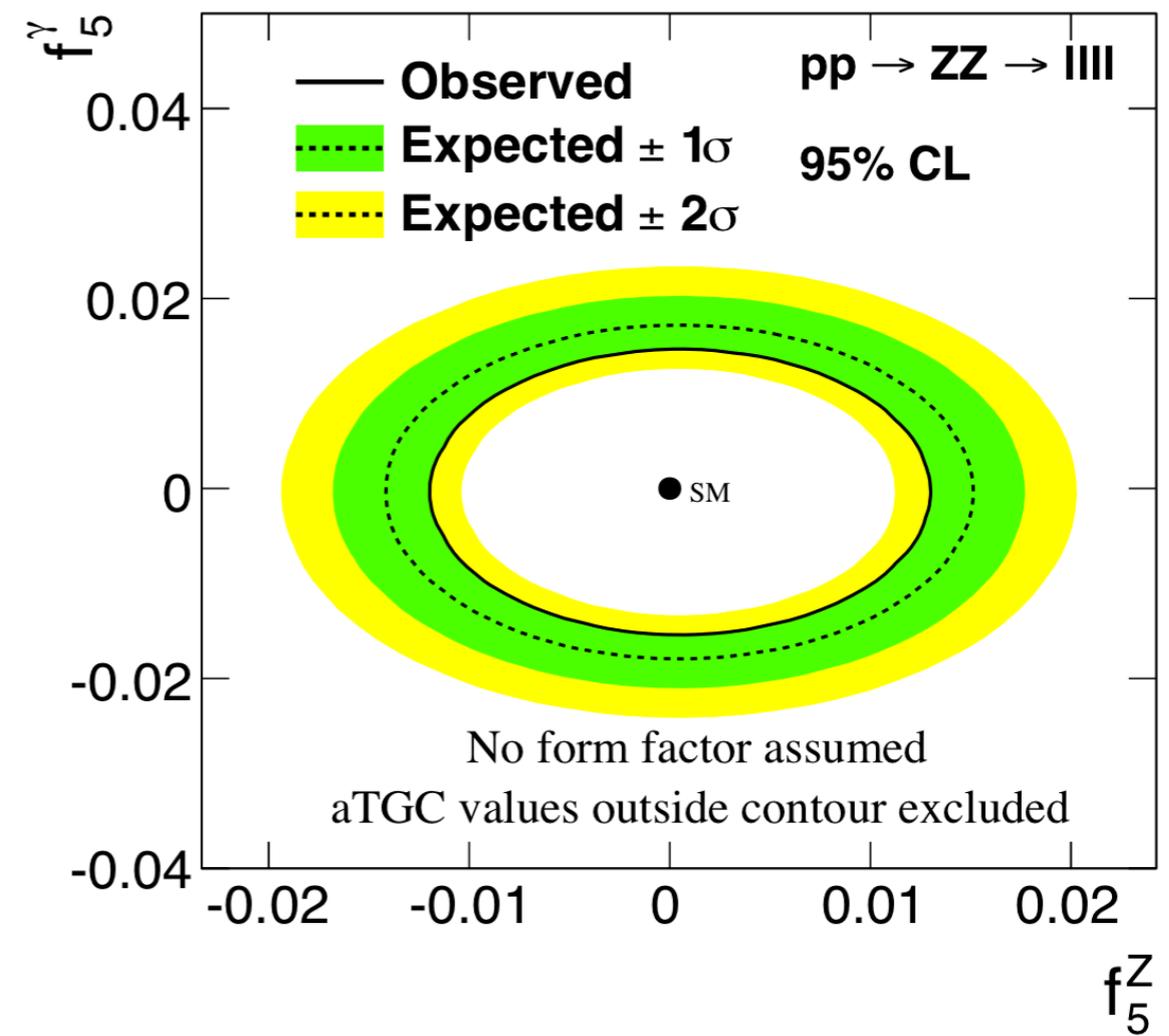
- Sensitivity to aTGCs is driven by long m_{4l} tails

- ZZZ/ZZ γ vertex described with $f_4^{Z/\gamma}$ and $f_5^{Z/\gamma}$ couplings
- 41 invariant mass used as discriminating variable

CMS Preliminary $\sqrt{s} = 7 \text{ TeV}, L = 5.0 \text{ fb}^{-1}$



CMS Preliminary $\sqrt{s} = 7 \text{ TeV}, L = 5.0 \text{ fb}^{-1}$

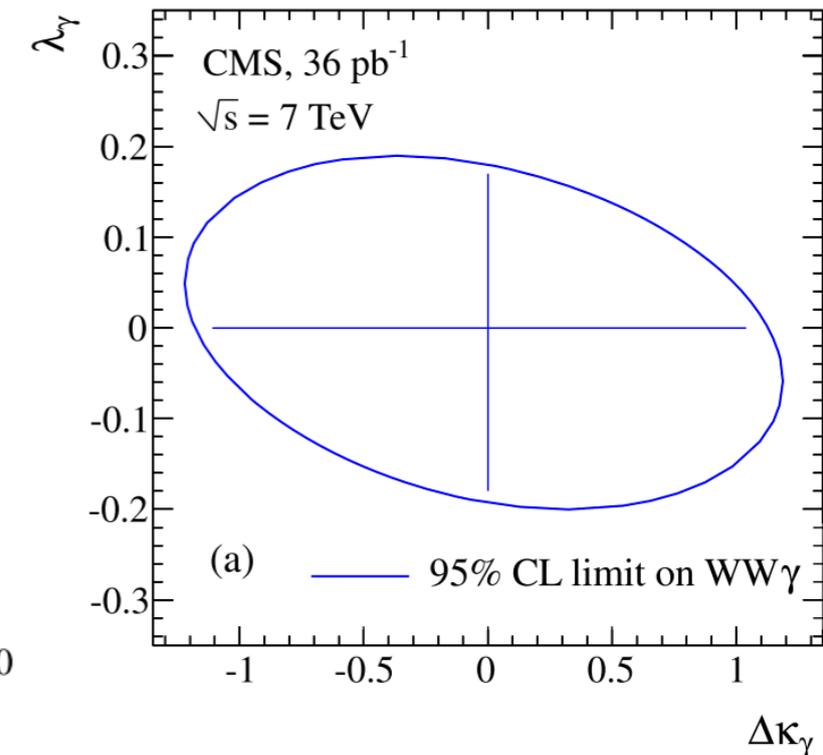
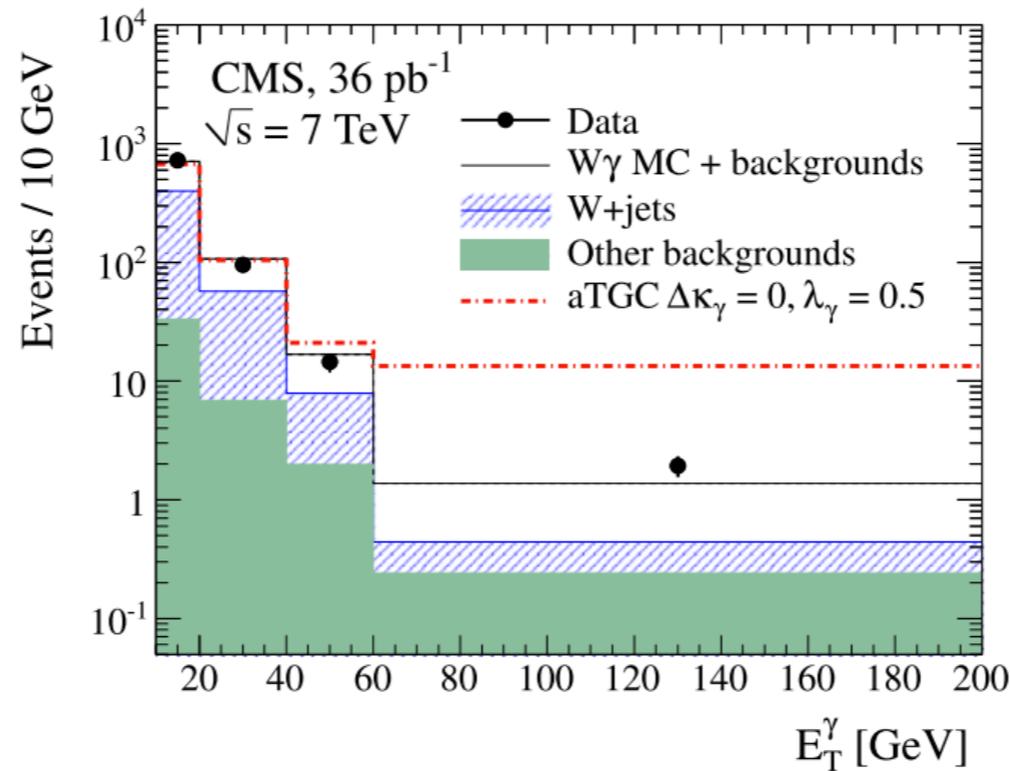


$$\begin{aligned}
 -0.012 < f_4^Z < 0.013 \\
 -0.012 < f_5^Z < 0.013
 \end{aligned}$$

$$\begin{aligned}
 -0.014 < f_4^\gamma < 0.014 \\
 -0.015 < f_5^\gamma < 0.015
 \end{aligned}$$

■ $W\gamma$:

- $\sigma(pp \rightarrow W\gamma + X) \times B(W \rightarrow l\nu) = 56.3 \pm 5.0(\text{stat.}) \pm 5.0(\text{syst.}) \pm 2.3(\text{lumi.})$
- Theory: $49.4 \pm 3.8 \text{ pb}$



■ $Z\gamma$:

- $\sigma(pp \rightarrow Z\gamma + X) \times B(Z \rightarrow ll) = 9.4 \pm 1.0 (\text{stat.}) \pm 0.6(\text{syst.}) \pm 0.4 (\text{lumi.}) \text{ pb}$
- Theory: $9.6 \pm 0.4 \text{ pb}$

