Synchrotron Radiation in the FFS

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January 29, 2013

CLIC Workshop 2013

Table of contents

Beamsize

We are interested in the horizontal beamsize at the IP.

Horizontal plane

$$\sigma^2 = \sigma_0^2 + \sigma_i^2 + \sigma_{rad}^2$$

$$\sigma_0 \equiv \text{zero}^{\text{th}} \text{ order approx.}$$
$$\sigma_i \equiv \text{result from aberrations}$$
$$\sigma_{rad} \equiv \text{interaction with magnets}$$

Evaluated by:

- tracking of particles
- mathematical approximations

Beam Radiation Model

x describes the displacement of a particle at $(s = L$, e.g. IP), where all other effects are included in the reference orbit.

$$x = \sum_{i=1}^{N(T)} \Delta x_{i,total} - x_0 \tag{1}$$

- $\Delta x_{i,total}$: is the total deviation due to the i^{th} photon radiated
- x_0: is $\langle \sum_{i=1}^{N(T)} \Delta x_{i,total} \rangle$, in order to make $\langle x \rangle = 0$, and $\sigma_{rad}^2 = \langle x^2 \rangle$
- N: is the number of photons radiated
- T: time to cross the bending magnet

Finding Δx_i

Δx_i is the effect at $(s = L)$ due to a photon of energy u radiated at $s = s_i$, therefore it has to be propagated from s_i to L.

$$\Delta x_i = (u/E) R_{16}(s_i, L) \tag{2}$$

According to Sands [1], $N \sim Poisson$, then

$$\sigma_N^2 = \langle N \rangle, \qquad x_0 = \langle N \rangle \langle \Delta x_{i,total} \rangle \tag{3}$$

$$\Delta x_{i,total} = \frac{u}{E}\sqrt{\frac{\beta_L}{\beta}}\left[\eta \cos \Delta\phi_{s_i,L} + (\alpha\eta + \beta\eta') \sin \Delta\phi_{s_i,L}\right] \tag{4}$$
$$= \Delta x_i + \Delta x_{i,\eta} \quad {}^1$$

1

[1] betatron oscillation + displacement

Finding Δx_i (cont.)

We could do the following approximation,

$$\langle \Delta x_{i,total} \rangle = \overset{0}{\cancel{\langle \Delta x_i \rangle}} + \langle \Delta x_{i,\eta} \rangle = (u/E)\eta_L \tag{5}$$

when, $\eta_L = 0$ it returns exactly the same result as Sands [1].
when, $\eta_L \neq 0$ this is the term to be substracted from $\Delta x_{i,total}$ in order to obtain the correct contribution to radiation.

$$x = \sum_{i=1}^{N(T)} (\Delta x_{i,total} - \langle \Delta x_{i,total} \rangle) \tag{6}$$

Now we have an approximation when $\eta_L \neq 0$.

Result (from approximation)

The contribution to beamsize due to radiation now can be calculated as:

$$\sigma_{rad}^2 \approx C_2 \int \frac{E^5}{\rho^3} \left\{ \sqrt{\frac{\beta_L}{\beta_s}} \left[\eta \cos \Delta\phi(s,L) + (\alpha\eta + \beta\eta') \sin \Delta\phi(s,L) \right] - \eta_L \right\}^2 ds$$

- $C_2 = 4.13 \times 10^{-11} [\mathrm{m}^2 \cdot \mathrm{GeV}^{-5}]$
- E: is the beam energy

This expression was included in MAPCLASS2.
It is possible to obtain a mathematical expression for one sbend magnet.

Low number of photons

Average number of photons emitted is

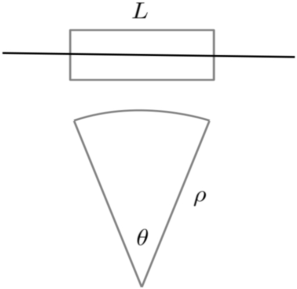
$$\langle N \rangle = \frac{1}{c} \int_0^L ds \int_0^\infty du \; n(u,s) \quad (7)$$

$$\approx C_1 E \theta \qquad ; C_1 = 20.61[\text{GeV}]^{-1} \quad (8)$$

if $\langle N \rangle < 1$, then, N could be modelled by a binomial distribution. Adding-up the statistic result from many particles, the binomial distribution converges again to Poisson, under the condition that $n(u,s)$ remains the same on both cases.

One dipole (theoretical expression)

Using $R_{16} = \rho(1 - \cos\theta)$

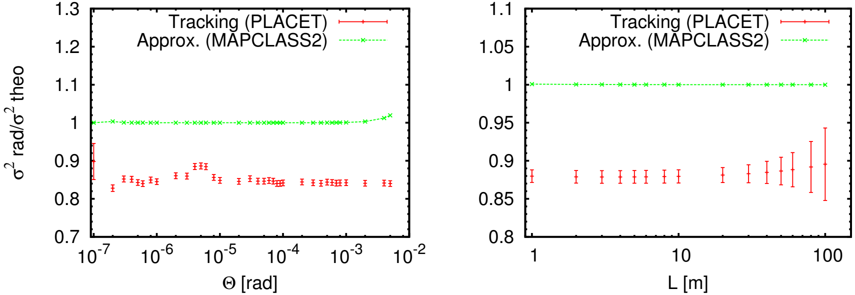


$$
\begin{aligned}
\sigma^2_{rad} &= C_2 \int_0^L \frac{E^5}{\rho^3} R^2_{16}(s, L) ds \\
&= C_2 \int_0^\theta \frac{E^5}{\rho^3} [\rho(1 - \cos(\theta - \chi))]^2 \rho d\chi \\
&= C_2 E^5 \left[\frac{1}{4}(6\theta - 8\sin\theta + \sin(2\theta))\right] \quad ^2 \\
&= C_2 E^5 \left(\frac{\theta^5}{20} - \frac{\theta^7}{168} + \frac{\theta^9}{2880} - \frac{17\theta^{11}}{1330560} + O(\theta^{13})\right)
\end{aligned}
$$

Now, the theoretical expression, the approximated result and tracking with PLACET could be compared.
Some care should be taken when using the expression above due to numerical precision.

[2] If E is considered constant.

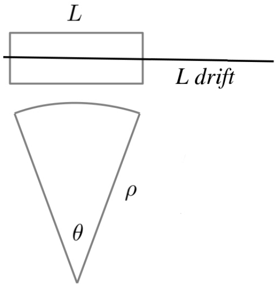
Results

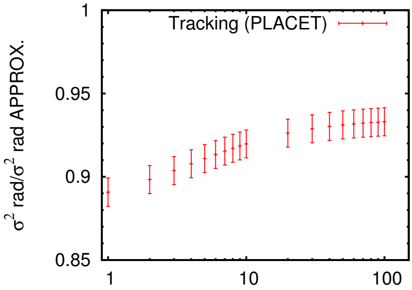


Radiation beamsize has been normalized to the theoretical value.

Results (cont.)

Including a drift in front of the bending magnet.



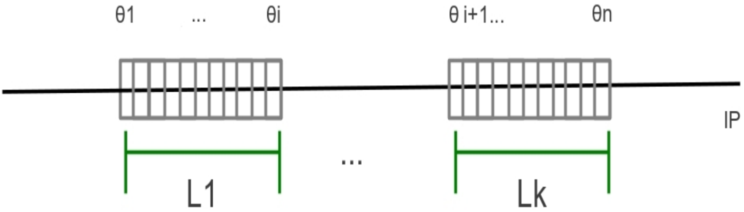


In this case the radiation beamsize has been normalized to the approximated result

Agreement between 80% and 90%

Optimization

The total length is fixed.

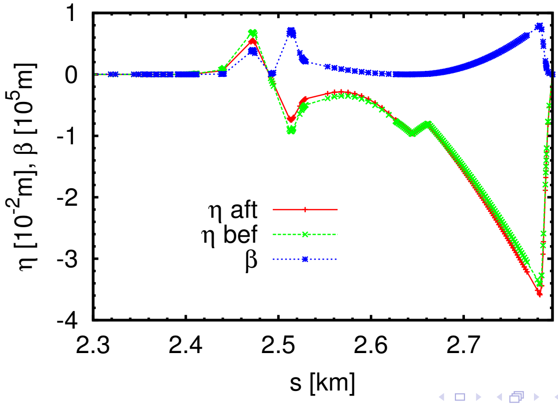


Total angle distribution will be changed to minimize σ_{rad}, under the following constraints:

- $\eta_x(IP) = 0$
- $\eta'_x(IP) =$constant value

Result

	σ_{bef}[nm]	σ_{aft}[nm]
CLIC 3TeV	46.28	46.3



Conclusions

- Lattice optimization for radiation is restricted by the required corrections of aberration.
- Radiation model seems not to be limited by low number of photon emission per electron.
- Faster method to calculate radiation beamsize has been tested. Differences between 80%∼90% with tracking code PLACET have been found
- Calculation valid for linear lattices.

References

- Sands, Matthew. Emittance growth from radiation fluctuations. SLAC/AP – 47. December, 1985.
- Renier, Ives. Implementation and validation of the linear collider final focus prototype : ATF2 at KEK (Japan). Doctoral Thesis, LAL10-91. June 2010.

Additional slide

Dispersion function

$$\begin{pmatrix} \eta(s_2) \\ \eta'(s_2) \\ 1 \end{pmatrix} = \begin{pmatrix} C(s_1,s_2) & S(s_1,s_2) & R_{16}(s_1,s_2) \\ C'(s_1,s_2) & S'(s_1,s_2) & R_{26}(s_1,s_2) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta(s_1) \\ \eta'(s_1) \\ 1 \end{pmatrix}$$

PLACET error bars

$$f = \frac{x_{rad}^2 - x_{norad}^2}{x_0^2}$$

$$\delta f = \frac{2}{\sqrt{N}} \frac{(x_{rad}^2 + x_{norad}^2)}{x_0^2}$$