# Measurement of the Luminosity Spectrum at CLIC

André Sailer, Stéphane Poss

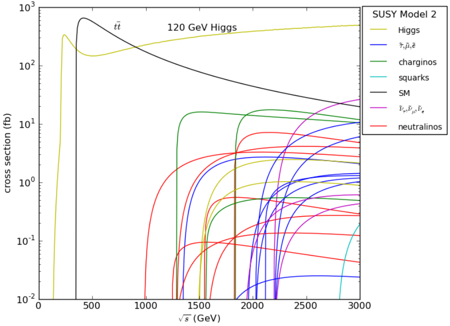
CERN-PH-LCD

CLIC Workshop, CERN
Jan.-Feb. 2013

# Table Of Contents

# Goal and Limits of our Study

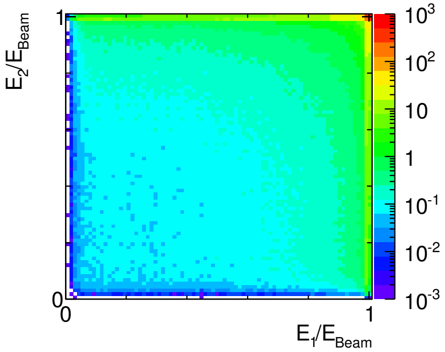
- How does the uncertainty in the luminosity spectrum affect measurements at CLIC 3 TeV?
  - Benchmark processes far above threshold, very few events with $\sqrt{s'} < 1.5$ TeV
  - Integrated Luminosity 2 $\mathrm{ab}^{-1}$
- Studied luminosity spectrum in light of these benchmarks
- Including relevant effects for reconstruction
- Can use a minimal model to describe the luminosity spectrum, do not need a complete and global description of the spectrum from $\sqrt{s'} = 0$ TeV – 3 TeV



# What is the Goal of this Measurement?

- Goal: The distribution of the pairs of particle energies prior to initial state radiation $\mathscr{L}(x_1, x_2)$
  - Only reconstructing the centre-of-mass energy ignores the longitudinal boost of the system
  - Strong correlation between the two particle energies
  - Account for Asymmetric beams
  - Initial state radiation depends on the specific process and centre-of-mass energy
- Note: We mostly show the c.m.s. luminosity spectrum $\mathscr{L}(\sqrt{s'})$ because it is easier to compare and interpret

Particle Energy Spectrum from GUINEAPIG

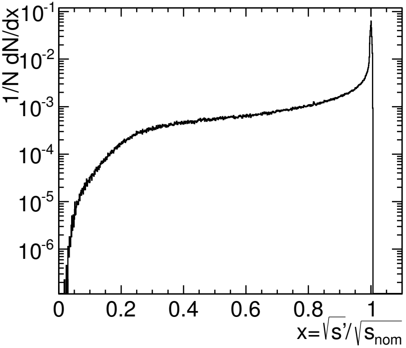



$$\mathscr{L}(\sqrt{s'}) = \int dx_1 \int dx_2 \mathscr{L}(x_1, x_2)\, \delta\left(\frac{\sqrt{s'}}{\sqrt{s_{nom}}} - \sqrt{x_1 x_2}\right)$$

# What is the Goal of this Measurement?

- Goal: The distribution of the pairs of particle energies prior to initial state radiation $\mathscr{L}(x_1, x_2)$
  - Only reconstructing the centre-of-mass energy ignores the longitudinal boost of the system
  - Strong correlation between the two particle energies
  - Account for Asymmetric beams
  - Initial state radiation depends on the specific process and centre-of-mass energy
- Note: We mostly show the c.m.s. luminosity spectrum $\mathscr{L}(\sqrt{s'})$ because it is easier to compare and interpret

Luminosity Spectrum from GUINEAPIG

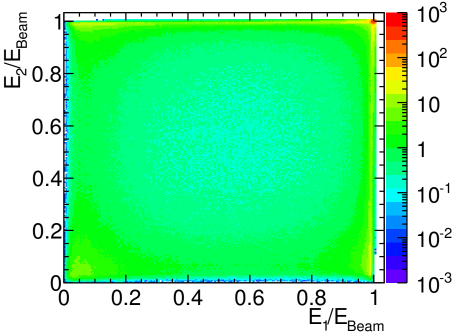



$$\mathscr{L}(\sqrt{s'}) = \int \mathrm{d}x_1 \int \mathrm{d}x_2 \mathscr{L}(x_1, x_2)\,\delta\Big(\frac{\sqrt{s'}}{\sqrt{s_{\mathrm{nom}}}} - \sqrt{x_1 x_2}\Big)$$

# What Do We Measure in the Detector?

- Need large cross-section and well known process: Bhabha scattering
- In the detector we measure the final state particles affected by the cross-section (initial state radiation, final state radiation, $\sqrt{s'}$ dependence)
- There is no way, for an individual event, to know if the energy was lost from initial state radiation or Beamstrahlung
- The measured values are also affected by the resolution of the respective subdetector

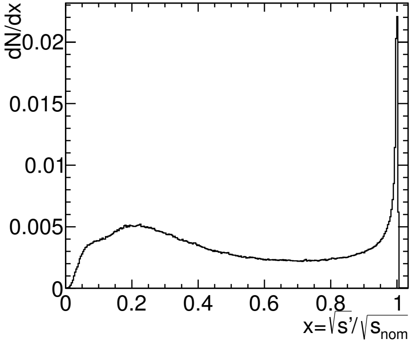
Distributions after Bhabha scattering and cross-section (without detector resolutions)



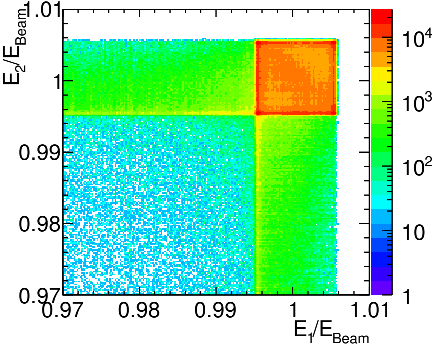
# What Do We Measure in the Detector?

- Need large cross-section and well known process: Bhabha scattering
- In the detector we measure the final state particles affected by the cross-section (initial state radiation, final state radiation, $\sqrt{s'}$ dependence)
- There is no way, for an individual event, to know if the energy was lost from initial state radiation or Beamstrahlung
- The measured values are also affected by the resolution of the respective subdetector

Distributions after Bhabha scattering and cross-section (without detector resolutions)



# The Model




GUINEAPIG

# The Model

$$\mathscr{L}(x_1, x_2) =$$

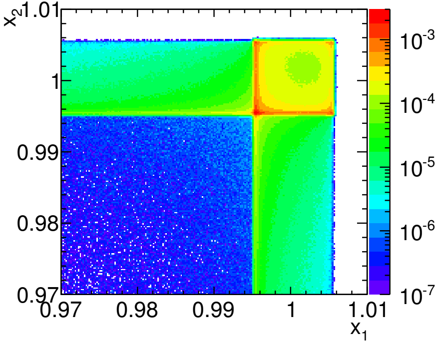
$$p_{\text{Peak}}\,\delta(1 - x_1) \otimes \text{BES}\left(x_1; [p]_1^{\text{Peak}}\right)$$
$$\delta(1 - x_2) \otimes \text{BES}\left(x_2; [p]_2^{\text{Peak}}\right)$$
$$+p_{\text{Arm1}}\,\delta(1 - x_1) \otimes \text{BES}\left(x_1; [p]_1^{\text{Arm}}\right)$$
$$\text{BB}\left(x_2; [p]_2^{\text{Arm}}, \beta_{\text{limit}}^1\right)$$
$$+p_{\text{Arm2}}\,\text{BB}\left(x_1; [p]_1^{\text{Arm}}, \beta_{\text{limit}}^1\right)$$
$$\delta(1 - x_2) \otimes \text{BES}\left(x_2; [p]_2^{\text{Arm}}\right)$$
$$+p_{\text{Body}}\,\text{BG}\left(x_1; [p]_1^{\text{Body}}, \beta_{\text{limit}}^2\right)$$
$$\text{BG}\left(x_2; [p]_2^{\text{Body}}, \beta_{\text{limit}}^2\right)$$

With

$$\text{BES}(x) = \int_{x_{\min}}^{x_{\max}} b(\tau)\text{Gauss}(x - \tau)\mathrm{d}\tau$$

$$\text{BB}(x) = (b \otimes \text{BES})(x)$$

$$\text{BG}(x) = (b \otimes g)(x)$$




Model (arbitrary parameters)

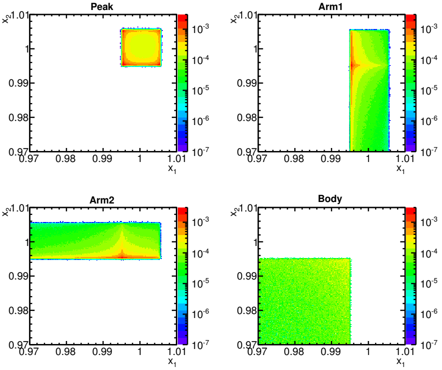
# The Model

$$\mathscr{L}(x_1, x_2) =$$

$$p_{\text{Peak}}\,\delta(1-x_1)\otimes \text{BES}\left(x_1;[p]_1^{\text{Peak}}\right)$$
$$\delta(1-x_2)\otimes \text{BES}\left(x_2;[p]_2^{\text{Peak}}\right)$$
$$+p_{\text{Arm1}}\,\delta(1-x_1)\otimes \text{BES}\left(x_1;[p]_1^{\text{Arm}}\right)$$
$$\text{BB}\left(x_2;[p]_2^{\text{Arm}},\beta^1_{\text{limit}}\right)$$
$$+p_{\text{Arm2}}\,\text{BB}\left(x_1;[p]_1^{\text{Arm}},\beta^1_{\text{limit}}\right)$$
$$\delta(1-x_2)\otimes \text{BES}\left(x_2;[p]_2^{\text{Arm}}\right)$$
$$+p_{\text{Body}}\,\text{BG}\left(x_1;[p]_1^{\text{Body}},\beta^2_{\text{limit}}\right)$$
$$\text{BG}\left(x_2;[p]_2^{\text{Body}},\beta^2_{\text{limit}}\right)$$

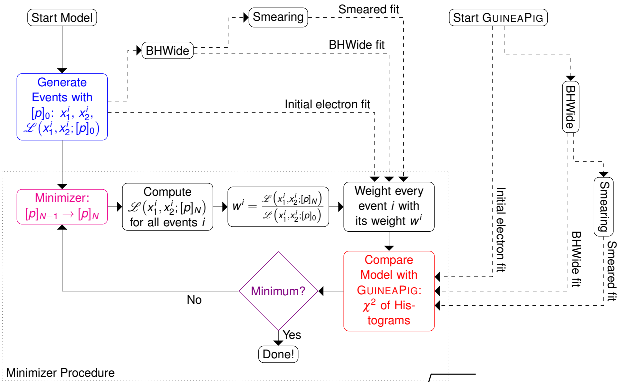
With

$$\text{BES}(x) = \int_{x_{\text{min}}}^{x_{\text{max}}} b(\tau)\text{Gauss}(x-\tau)\mathrm{d}\tau$$
$$\text{BB}(x) = (b\otimes \text{BES})(x)$$
$$\text{BG}(x) = (b\otimes g)(x)$$




Model (arbitrary parameters)

# Reweighting Fit

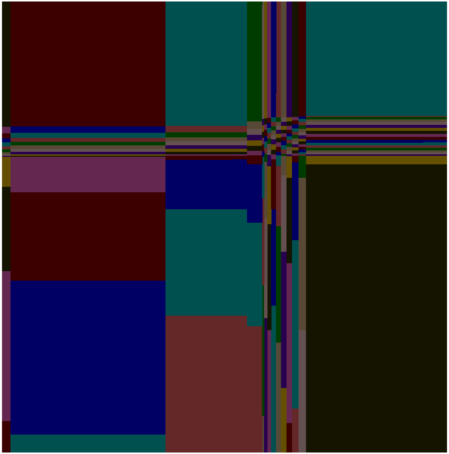


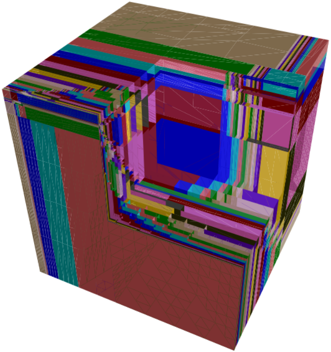

$$N^j_{\mathrm{GP}} = \sum_{\text{GP Events } i \text{ in Bin } j} 1 \qquad \sigma^j_{\mathrm{GP}} = \sqrt{N^j_{\mathrm{GP}}}$$

$$N^j_{\mathrm{Model}} = \sum_{\text{Model Events } i \text{ in Bin } j} w^i \qquad \sigma^j_{\mathrm{Model}} = \sqrt{\sum_{\text{Model Events } i \text{ in Bin } j} (w^i)^2}$$

$$w^i = \frac{\mathscr{L}(x_1^i, x_2^i; [p]_N)}{\mathscr{L}(x_1^i, x_2^i; [p]_0)} \qquad \chi^2 = \sum_{\text{Bins } j} \frac{\left(N^j_{\mathrm{GP}} - f_S \cdot N^j_{\mathrm{Model}}\right)^2}{(\sigma^j_{\mathrm{GP}})^2 + (f_S \cdot \sigma^j_{\mathrm{Model}})^2}$$

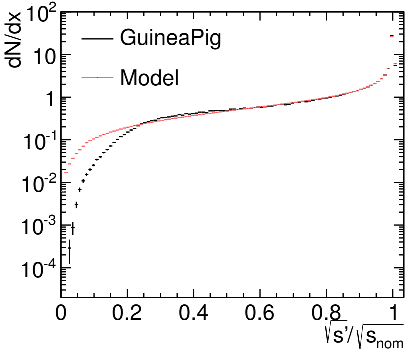
# Binning





# Fitting Spectrum Directly

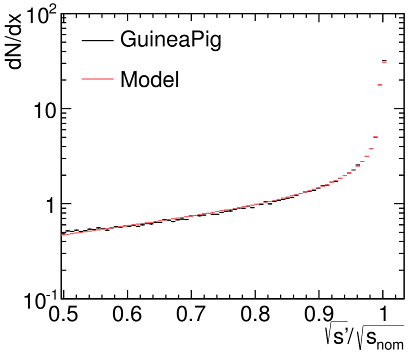
- Fit the 2D distribution of *Particle Energies*
- 3 million GP events and 10 million according to MODEL
- No cross-section, initial state radiation, or detector effects
- Difference in the width of the peak, but averages out
- Spectrum described within 5% down to $0.6\sqrt{s_{\text{nom}}}$
- Some problem with the width of the peak
  - Only statistical errors from GUINEAPIG sample
  - Error due to parameters smaller




Results for $150 \times 150$ bins and cut $\sqrt{s'} > 1.5$ TeV

# Fitting Spectrum Directly

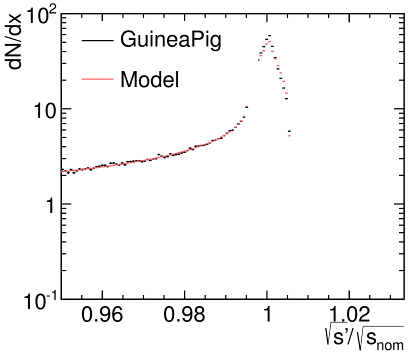
- Fit the 2D distribution of *Particle Energies*
- 3 million GP events and 10 million according to MODEL
- No cross-section, initial state radiation, or detector effects
- Difference in the width of the peak, but averages out
- Spectrum described within 5% down to $0.6\sqrt{s_{\text{nom}}}$
- Some problem with the width of the peak
  - Only statistical errors from GUINEAPIG sample
  - Error due to parameters smaller




Results for $150 \times 150$ bins and cut $\sqrt{s'} > 1.5$ TeV

# Fitting Spectrum Directly

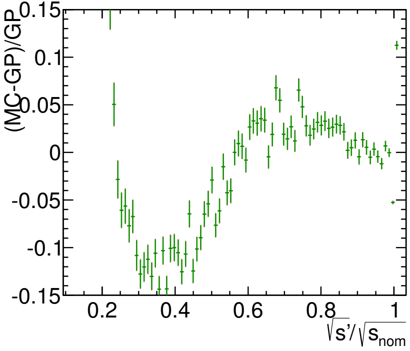
- Fit the 2D distribution of *Particle Energies*
- 3 million GP events and 10 million according to MODEL
- No cross-section, initial state radiation, or detector effects
- Difference in the width of the peak, but averages out
- Spectrum described within 5% down to $0.6\sqrt{s_{\mathrm{nom}}}$
- Some problem with the width of the peak
  - Only statistical errors from GUINEAPIG sample
  - Error due to parameters smaller




Results for 150 × 150 bins and cut $\sqrt{s'} > 1.5$ TeV

# Fitting Spectrum Directly

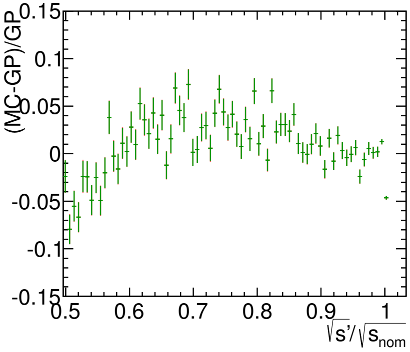
- Fit the 2D distribution of *Particle Energies*
- 3 million GP events and 10 million according to MODEL
- No cross-section, initial state radiation, or detector effects
- Difference in the width of the peak, but averages out
- Spectrum described within 5% down to $0.6\sqrt{s_{\text{nom}}}$
- Some problem with the width of the peak
  - Only statistical errors from GUINEAPIG sample
  - Error due to parameters smaller




Results for 150 $\times$ 150 bins and cut $\sqrt{s'} > 1.5$ TeV

# Fitting Spectrum Directly

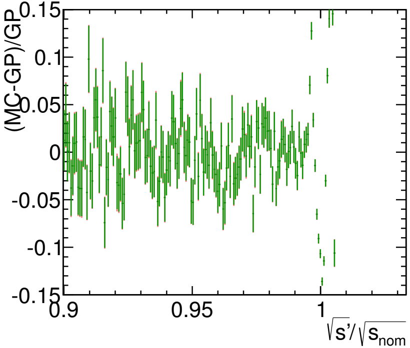
- Fit the 2D distribution of *Particle Energies*
- 3 million GP events and 10 million according to MODEL
- No cross-section, initial state radiation, or detector effects
- Difference in the width of the peak, but averages out
- Spectrum described within 5% down to $0.6\sqrt{s_{\mathrm{nom}}}$
- Some problem with the width of the peak
  - Only statistical errors from GUINEAPIG sample
  - Error due to parameters smaller




Results for 150 × 150 bins and cut $\sqrt{s'} > 1.5$ TeV

# Fitting Spectrum Directly

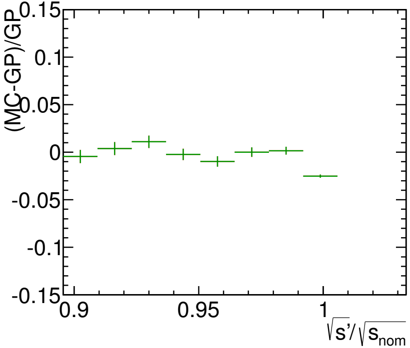
- Fit the 2D distribution of *Particle Energies*
- 3 million GP events and 10 million according to MODEL
- No cross-section, initial state radiation, or detector effects
- Difference in the width of the peak, but averages out
- Spectrum described within 5% down to $0.6\sqrt{s_{\mathrm{nom}}}$
- Some problem with the width of the peak
  - Only statistical errors from GUINEAPIG sample
  - Error due to parameters smaller




Results for $150 \times 150$ bins and cut $\sqrt{s'} > 1.5$ TeV

# Fitting Spectrum Directly

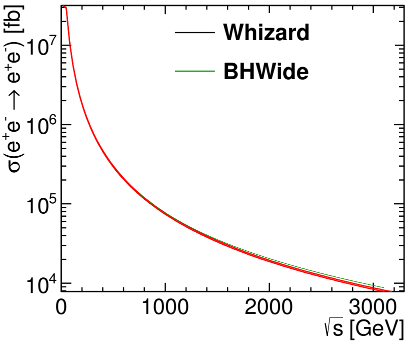
- Fit the 2D distribution of *Particle Energies*
- 3 million GP events and 10 million according to MODEL
- No cross-section, initial state radiation, or detector effects
- Difference in the width of the peak, but averages out
- Spectrum described within 5% down to $0.6\sqrt{s_{\mathrm{nom}}}$
- Some problem with the width of the peak
  - Only statistical errors from GUINEAPIG sample
  - Error due to parameters smaller




Results for 150 $\times$ 150 bins and cut $\sqrt{s'} > 1.5$ TeV

# Luminosity Spectrum with Cross-Section

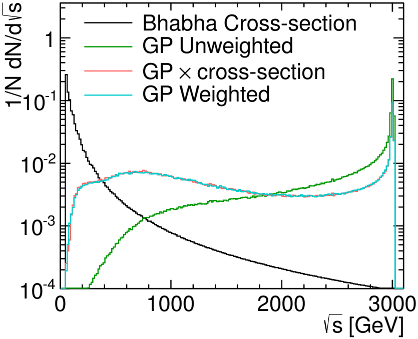
- Bhabha cross-section proportional to $1/s$
- Need Luminosity Spectrum scaled according to cross-section
- Feed these energy pairs to BHWIDE for ISR/FSR and Bhabha-scattering




Cross-section calculated by WHIZARD and BHWIDE $7^\circ < \theta_{e^\pm} < 173^\circ$, without luminosity spectrum

# Luminosity Spectrum with Cross-Section

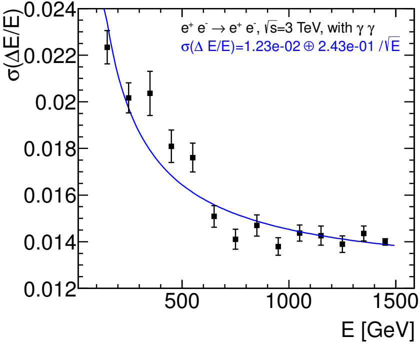
- Bhabha cross-section proportional to $1/s$
- Need Luminosity Spectrum scaled according to cross-section
- Feed these energy pairs to BHWIDE for ISR/FSR and Bhabha-scattering



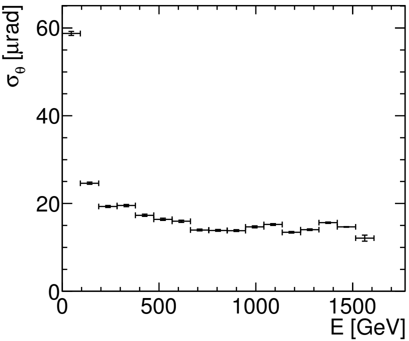

Cross-section calculated by WHIZARD and BHWIDE $7^\circ < \theta_{e^\pm} < 173^\circ$, without luminosity spectrum

# Detector Effects

Particle Energy



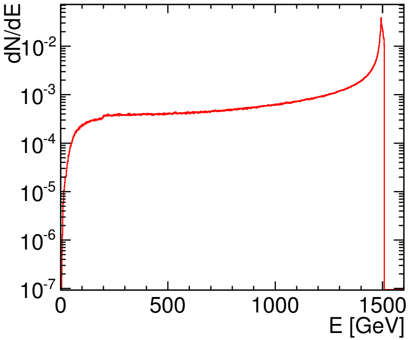

Angular Resolution ($e^{\pm}, \theta \geq 7^{\circ}$)



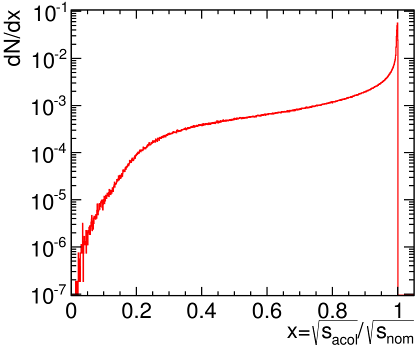

- Full simulation of millions of Bhabha events not feasible, use 4vector smearing
- Detector resolutions obtained with full simulation/reconstruction with background overlay thanks to J.J. Blaising

# Smeared Observables

Energy of the leptons:




Relative CME:

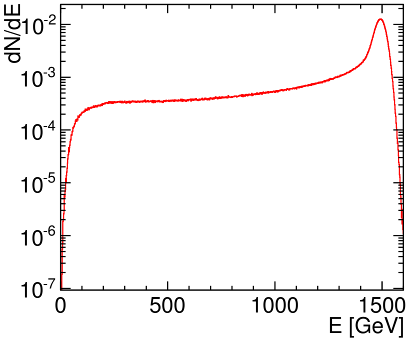



Very large effect on energy, small on relative CME: lower energy precision (higher background, different calorimeter), high angular precision (low magnetic field effect, straight tracks)

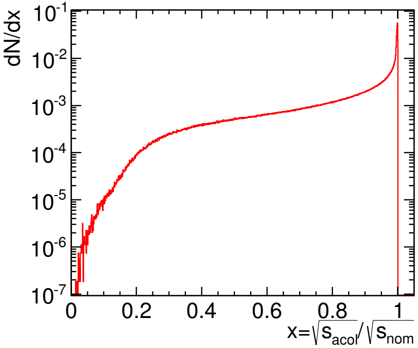
$$\frac{\sqrt{s_{\mathrm{acol}}}}{\sqrt{s_{\mathrm{nom}}}} = \sqrt{\frac{\sin(\theta_1)+\sin(\theta_2)+\sin(\theta_1+\theta_2)}{\sin(\theta_1)+\sin(\theta_2)-\sin(\theta_1+\theta_2)}},$$

# Smeared Observables

Energy of the leptons:




Relative CME:

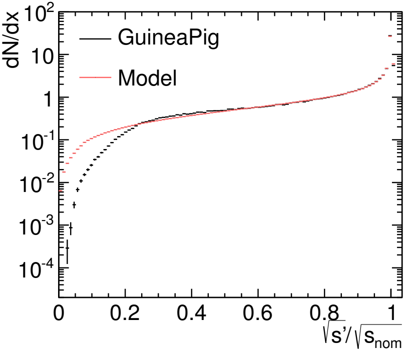



Very large effect on energy, small on relative CME: lower energy precision (higher background, different calorimeter), high angular precision (low magnetic field effect, straight tracks)

$$\frac{\sqrt{s_{\mathrm{acol}}}}{\sqrt{s_{\mathrm{nom}}}} = \sqrt{\frac{\sin(\theta_1)+\sin(\theta_2)+\sin(\theta_1+\theta_2)}{\sin(\theta_1)+\sin(\theta_2)-\sin(\theta_1+\theta_2)}},$$

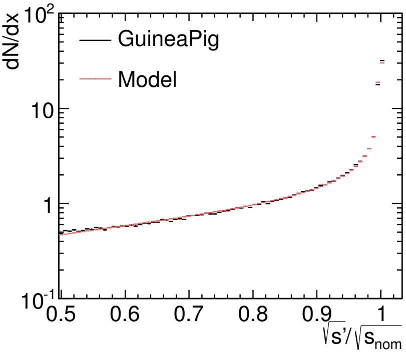
# Final Fit: All Effects

- Luminosity spectrum, cross-section weighting, ISR, FSR, detector resolutions
- Binning $60 \times 30 \times 30$
- 2 Million GP (current number of available events, approx. $400\text{fb}^{-1}$) , 10 Million MODEL
- Cut on: $\sqrt{s'} > 1.5$ TeV, $E_1 > 150$ GeV, $E_2 > 150$ GeV



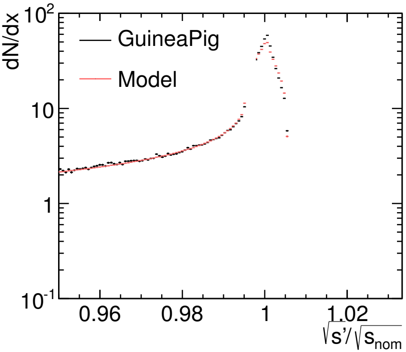
# Final Fit: All Effects

- Luminosity spectrum, cross-section weighting, ISR, FSR, detector resolutions
- Binning $60 \times 30 \times 30$
- 2 Million GP (current number of available events, approx. 400fb$^{-1}$) , 10 Million MODEL
- Cut on: $\sqrt{s'} > 1.5$ TeV, $E_1 > 150$ GeV, $E_2 > 150$ GeV



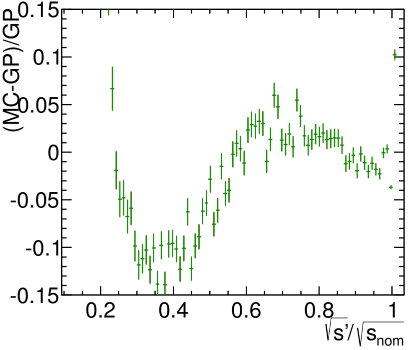
# Final Fit: All Effects

- Luminosity spectrum, cross-section weighting, ISR, FSR, detector resolutions
- Binning $60 \times 30 \times 30$
- 2 Million GP (current number of available events, approx. $400\text{fb}^{-1}$) , 10 Million MODEL
- Cut on: $\sqrt{s'} > 1.5$ TeV, $E_1 > 150$ GeV, $E_2 > 150$ GeV



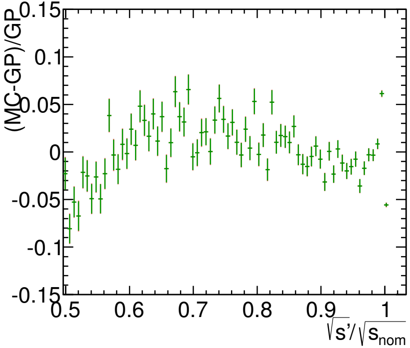
# Final Fit: All Effects

- Luminosity spectrum, cross-section weighting, ISR, FSR, detector resolutions
- Binning $60 \times 30 \times 30$
- 2 Million GP (current number of available events, approx. $400\text{fb}^{-1}$) , 10 Million MODEL
- Cut on: $\sqrt{s'} > 1.5$ TeV, $E_1 > 150$ GeV, $E_2 > 150$ GeV



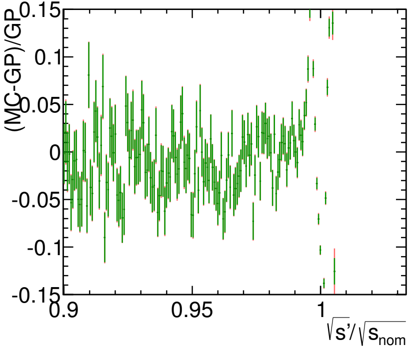
# Final Fit: All Effects

- Luminosity spectrum, cross-section weighting, ISR, FSR, detector resolutions
- Binning $60 \times 30 \times 30$
- 2 Million GP (current number of available events, approx. 400fb$^{-1}$) , 10 Million MODEL
- Cut on: $\sqrt{s'} > 1.5$ TeV, $E_1 > 150$ GeV, $E_2 > 150$ GeV



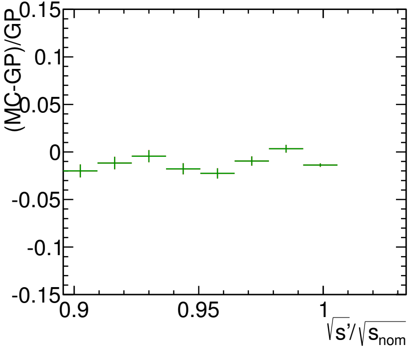
# Final Fit: All Effects

- Luminosity spectrum, cross-section weighting, ISR, FSR, detector resolutions
- Binning $60 \times 30 \times 30$
- 2 Million GP (current number of available events, approx. $400\text{fb}^{-1}$) , 10 Million MODEL
- Cut on: $\sqrt{s'} > 1.5$ TeV, $E_1 > 150$ GeV, $E_2 > 150$ GeV

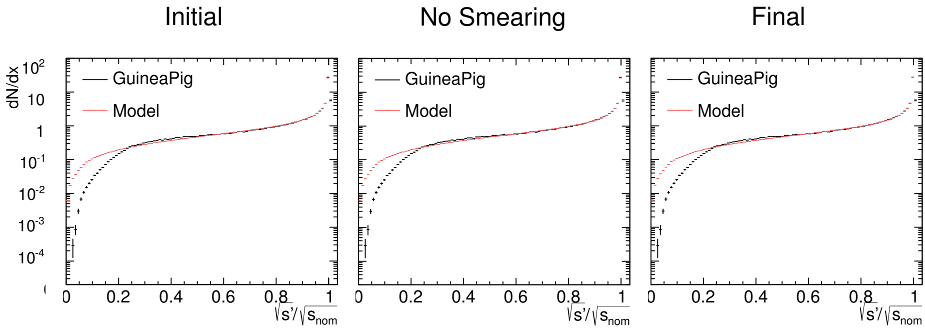


# Final Fit: All Effects

- Luminosity spectrum, cross-section weighting, ISR, FSR, detector resolutions
- Binning $60 \times 30 \times 30$
- 2 Million GP (current number of available events, approx. $400 \text{fb}^{-1}$) , 10 Million MODEL
- Cut on: $\sqrt{s'} > 1.5$ TeV, $E_1 > 150$ GeV, $E_2 > 150$ GeV

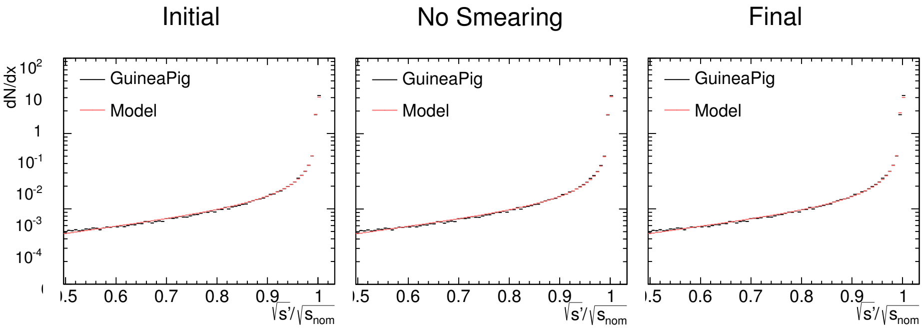


# Comparison: Initial vs. Final



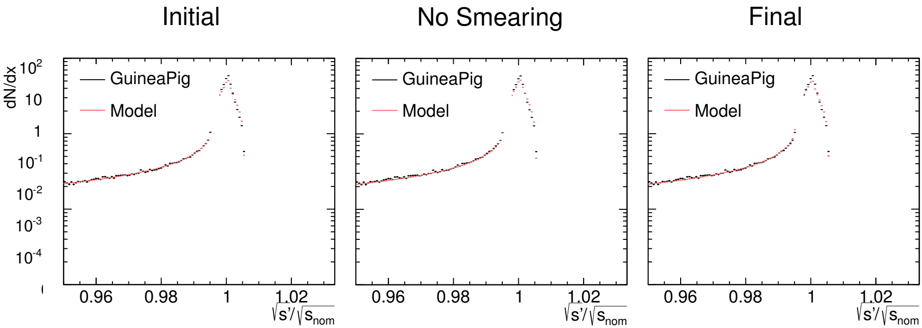

N.B.:The GUINEAPIG sample for all these plots is the same.

# Comparison: Initial vs. Final



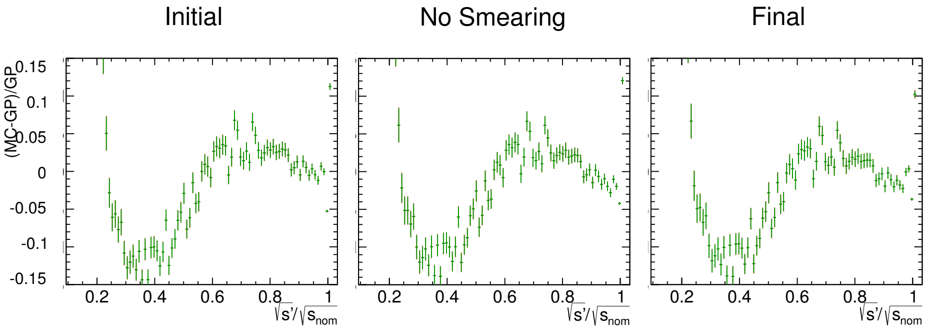

N.B.:The GUINEAPIG sample for all these plots is the same.

# Comparison: Initial vs. Final



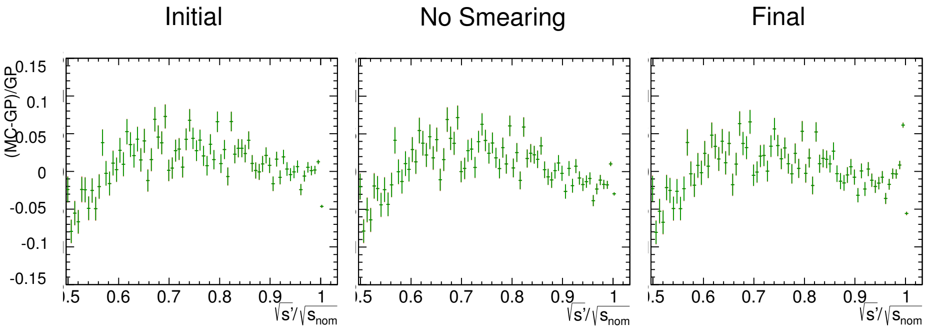

N.B.:The GUINEAPIG sample for all these plots is the same.

# Comparison: Initial vs. Final



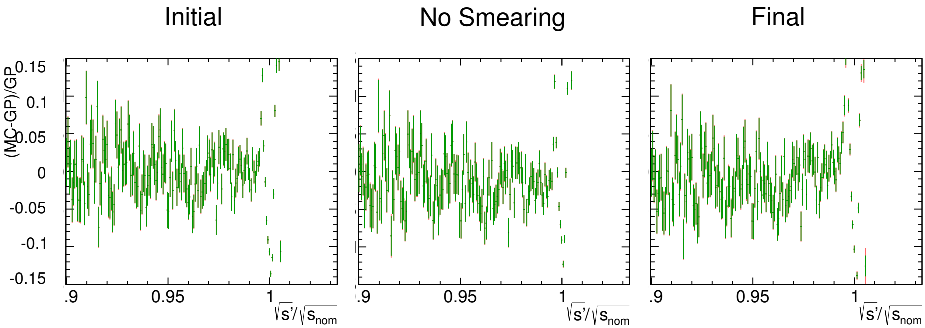

N.B.:The GUINEAPIG sample for all these plots is the same.

# Comparison: Initial vs. Final



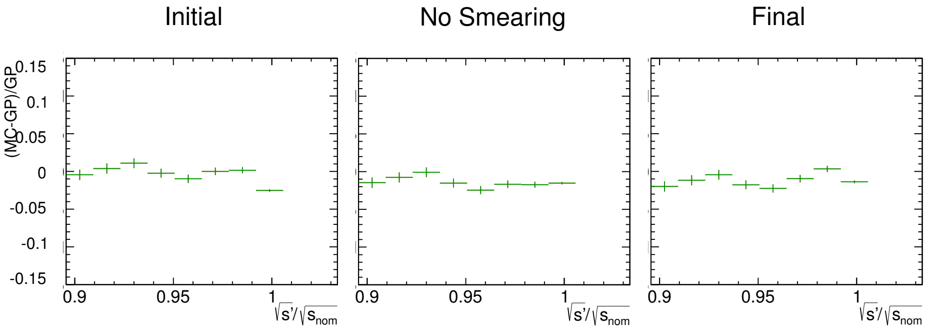

N.B.:The GUINEAPIG sample for all these plots is the same.

# Comparison: Initial vs. Final




N.B.:The GUINEAPIG sample for all these plots is the same.

# Comparison: Initial vs. Final




N.B.:The GUINEAPIG sample for all these plots is the same.

# Effect on the $\widetilde{\mu}$ mass measurement (I) (LCD-Note-2011-018)

- $e^+e^- \to \widetilde{\mu}^+\widetilde{\mu}^- \to \mu^+\mu^-\widetilde{\chi}_1^0\widetilde{\chi}_1^0$
- Fit background subtracted muon energy distribution to extract smuon and neutralino mass with

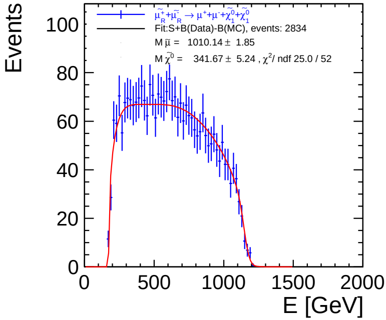
  $f(E_\mu; m_{\widetilde{\mu}}, m_\chi) = \text{Box} \times \sigma(\sqrt{s'}) \otimes \mathscr{L}(\vec{p}) \otimes \text{ISR} \otimes \text{DetRes}$
- Fit with all parameters of luminosity spectrum varied by $\pm\sigma_p^i/2$ individually

$$\sigma^2_{m_{\widetilde{\mu}}} = \sum_{i,j} \delta_i C_{ij} \delta_j$$

$$\delta_i = m_{+_i} - m_{-_i}$$

$$m_{+_i} = f\left(\vec{p} + \vec{e}_i \frac{\sigma_{p_i}}{2}\right)$$

$$m_{-_i} = f\left(\vec{p} - \vec{e}_i \frac{\sigma_{p_i}}{2}\right)$$




with the correlation matrix

$$C = \begin{pmatrix} 1 & -0.6 & \dots & -0.02 \\ -0.6 & 1 & \dots & 0.04 \\ \dots & \dots & \dots & \dots \\ -0.02 & 0.04 & \dots & 1 \end{pmatrix}$$

# Effect on the $\widetilde{\mu}$ mass measurement (II)(LCD-Note-2011-018)

Results:

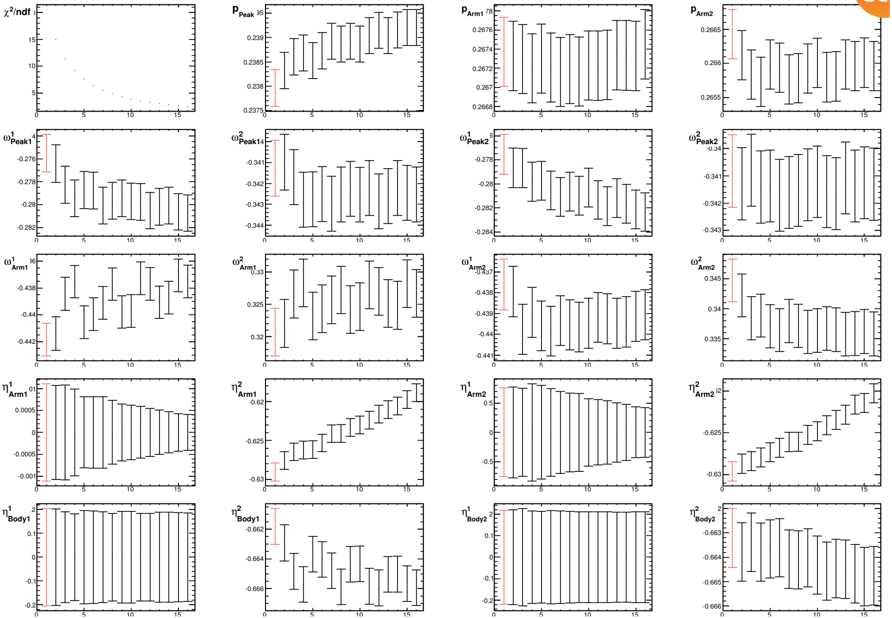
- Using our measured spectrum:

$$m_{\widetilde{\mu}} = (1010.1 \pm 1.8(\text{stat}) \pm 0.2(\text{par}))\ \text{GeV},$$
$$m_{\chi} = (\ \ 341.7 \pm 5.2(\text{stat}) \pm 0.2(\text{par}))\ \text{GeV}$$

Conclusion:

- With this result the luminosity spectrum measurement has no significant effect on $\widetilde{\mu}/\chi$ mass measurements
- Have code to do the fit in hand, will study this measurement with respect to the spread of our results

# Initial Fit: Parameter Dependence on Binning

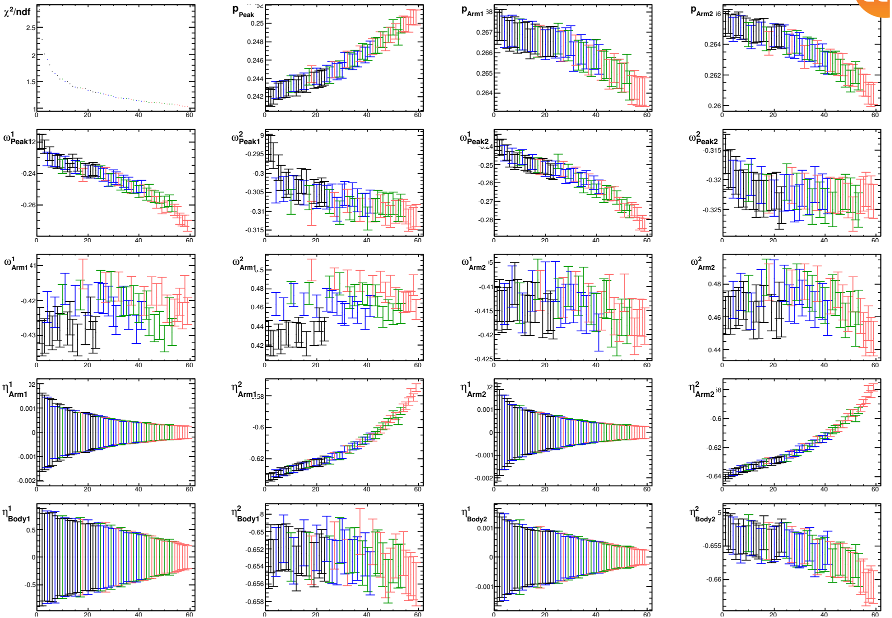


# Initial Fit:Parameter Dependence on Binning

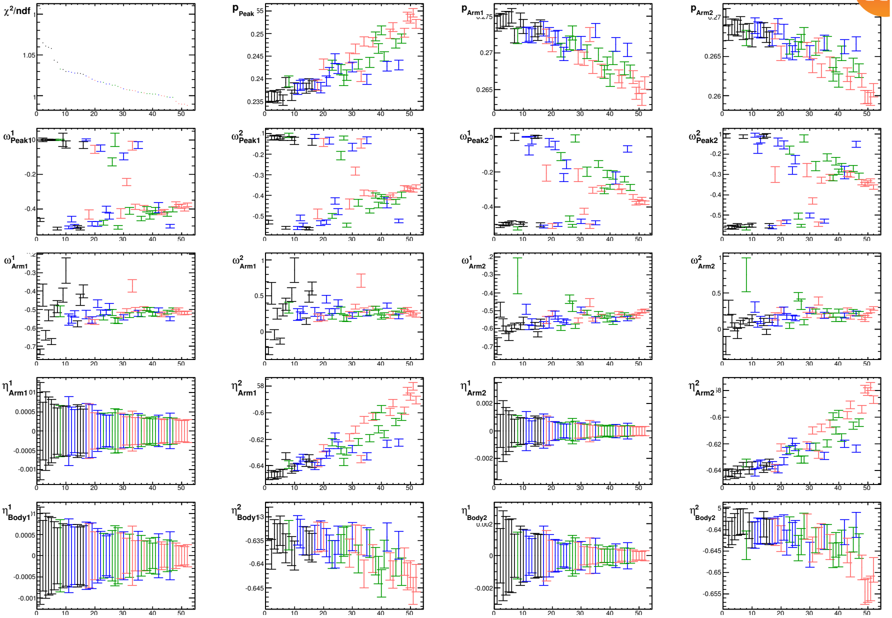
- Vary number of bins from $50 \times 50$ to $200 \times 200$
- Sorted by $\chi^2/\mathrm{ndf}$
- 3 Million GP, 10 Million MODEL
- Constant number of events, i.e., lower number of events per bin
- Some parameters show strong dependence on binning, some show (anti)correlation also seen in correlation matrix
- There is a bias in the reconstruction of the parameters, but we do not know the 'real' parameters of the spectrum
- Currently 'running'* 15000 Fits to find least biasing binning based on MODEL to MODEL fits, where we know the real parameters

*or waiting for them to run

# Parameter Dependence on Binning (No Smearing)



# Parameter Dependence on Binning (W/ Smearing)



# Parameter Dependence on Binning

- Vary number of bins from $10 \times 10 \times 10$ to $80 \times 50 \times 50$
- Sorted by $\chi^2/\text{ndf}$
- Some of the binnings fail to result in converging fit
- Constant number of events, i.e., lower number of events per bin
- Some parameters show strong dependence on binning, some show (anti)correlation also seen in correlation matrix
- A minimum number of bins is necessary for proper reconstruction
- There is a bias in the reconstruction of the parameters, but we do not know the 'real' parameters of the spectrum
- With current CPU power we cannot systematically evaluate a least biasing result, because it would take 150 times longer to do the same study as is currently running for the initial state fit

# Summary/Conclusion/Outlook

- Have modelled the CLIC luminosity spectrum
- Implemented sophisticated reconstruction procedure
- Systematically studied impact of reconstruction
  - Still some issue with description of the width of peak region
  - There is some dependence on the binning
  - When choosing the right binning, adding effects does not worsen reconstruction
  - Reconstruction within 5% down to $0.5\sqrt{s_{\mathrm{nom}}}$
- Two more quick studies for completion (submitting $< 200$ batchjobs):
  - Have larger GUINEAPIG sample, will study dependence on number of events ($1.0\ \mathrm{ab}^{-1}$, $1.5\ \mathrm{ab}^{-1}$, $2.0\ \mathrm{ab}^{-1}$)
  - Study impact of Luminosity Spectrum on Smuon pair-production measurements
- Other applications of reconstructed spectrum are welcome! (3 TeV only for now)

# Acknowledgement

Thanks to Barbara Dalena for the GUINEAPIG beam profiles

# Thank you for your attention!

# Backup Slides

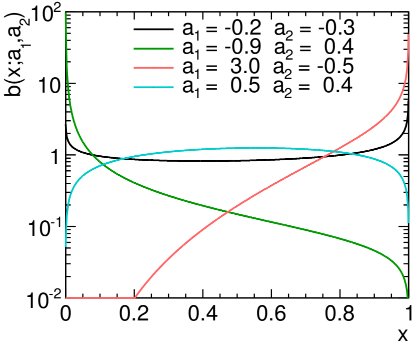
# Beta-Distributions

- Mostly using Beta-Distributions for the description of the luminosity spectrum

$$b(x) = \frac{1}{N} x^{a_1} (1-x)^{a_2}$$

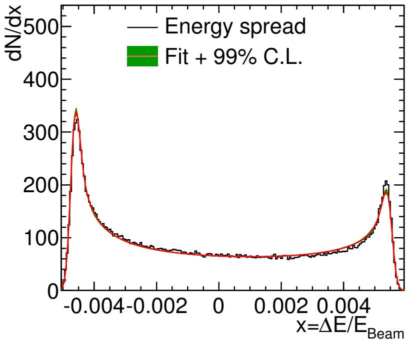
with different parameter bounds

- Range: $0 < x < 1$



# Beam-Energy Spread Function

Particle energy distribution from accelerator simulation




- Beam-Energy Spread:
  Beta-distribution convoluted with Gauss

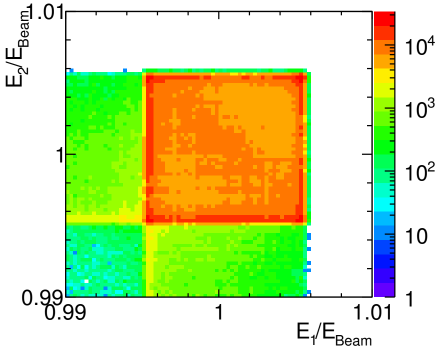
$$\mathrm{BES}(x) = \int_{x_{\mathrm{min}}}^{x_{\mathrm{max}}} b(\tau)\mathrm{Gauss}(x-\tau)\mathrm{d}\tau$$

- 5 parameters, including min. and max. of beta-distribution range
- $\chi^2/\mathrm{ndf}$ = 764/195

# Luminosity-weighted Beam-Energy Spread

- Due to the correlation, Beamstrahlung, and beam-beam effects two vastly different beam-energy spread distributions emerge for the luminosity spectrum
- *Peak* Region: Both particles with $E > 0.995 E_{\text{Beam}}$
- *Arms* Region: Only one of the particles with $E > 0.995 E_{\text{Beam}}$
- Both can be fit with a beta-distribution convoluted with a Gauss (keeping $x_{\text{min}}$, $x_{\text{max}}$, and $\sigma$ fixed)

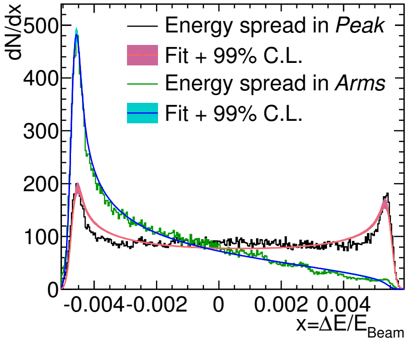
Peak of the luminosity spectrum



# Luminosity-weighted Beam-Energy Spread

- Due to the correlation, Beamstrahlung, and beam-beam effects two vastly different beam-energy spread distributions emerge for the luminosity spectrum
- *Peak* Region: Both particles with $E > 0.995 E_{\text{Beam}}$
- *Arms* Region: Only one of the particles with $E > 0.995 E_{\text{Beam}}$
- Both can be fit with a beta-distribution convoluted with a Gauss (keeping $x_{\text{min}}$, $x_{\text{max}}$, and $\sigma$ fixed)

Particle energy distribution from the GUINEAPIG simulation

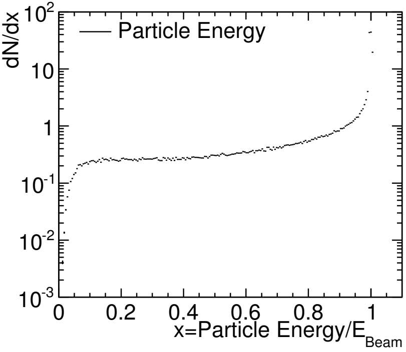


# Beamstrahlung

- Second contribution to luminosity spectrum is energy loss due to Beamstrahlung
- Potentially large loss of energy for some particles

Fitting the particle Energy Spectrum

- Upper bound of $0.995\sqrt{s_{\text{nom}}}$, because of impact of beam-energy spread (Particle energy is convolution of Beamstrahlung and beam-energy spread effect)
- Single Beta-Distribution not enough to describe full range of particle energies
- Keep small number of parameters: Limit to $0.5\sqrt{s_{\text{nom}}}$ and a single beta-distribution (limited $a_1 \geq 0.0$), but can extend

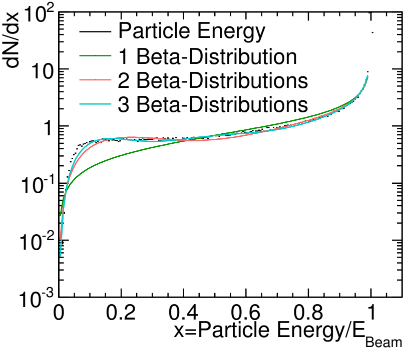


# Beamstrahlung

- Second contribution to luminosity spectrum is energy loss due to Beamstrahlung
- Potentially large loss of energy for some particles

Fitting the particle Energy Spectrum

- Upper bound of $0.995\sqrt{s_{\text{nom}}}$, because of impact of beam-energy spread (Particle energy is convolution of Beamstrahlung and beam-energy spread effect)
- Single Beta-Distribution not enough to describe full range of particle energies
- Keep small number of parameters: Limit to $0.5\sqrt{s_{\text{nom}}}$ and a single beta-distribution (limited $a_1 \geq 0.0$), but can extend




NEW:
Using proper normalisation

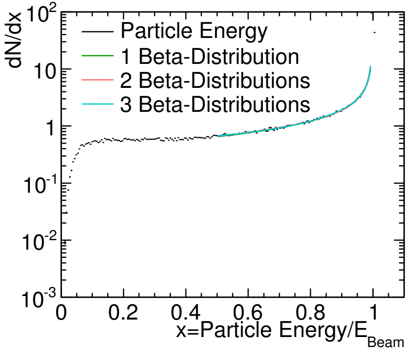
$$\int_0^{0.995\sqrt{s_{\text{nom}}}} = 1$$

# Beamstrahlung

- Second contribution to luminosity spectrum is energy loss due to Beamstrahlung
- Potentially large loss of energy for some particles

Fitting the particle Energy Spectrum

- Upper bound of $0.995\sqrt{s_{\mathrm{nom}}}$, because of impact of beam-energy spread (Particle energy is convolution of Beamstrahlung and beam-energy spread effect)
- Single Beta-Distribution not enough to describe full range of particle energies
- Keep small number of parameters: Limit to $0.5\sqrt{s_{\mathrm{nom}}}$ and a single beta-distribution (limited $a_1 \geq 0.0$), but can extend




NEW:

Using proper normalisation

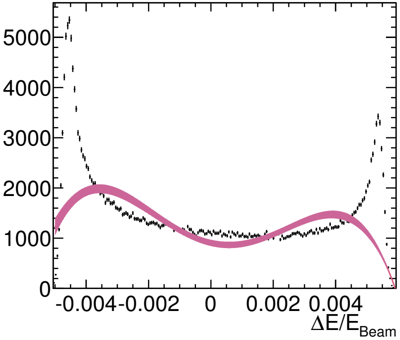
$$\int_0^{0.995\sqrt{s_{\mathrm{nom}}}} = 1$$

# Fitting with Chebyshev Polynomials

- Fitting with Chebyshev polynomials would avoid trouble of MODEL description
- $f(x) = \sum_i p_i \mathrm{Cheb}_i(x)$

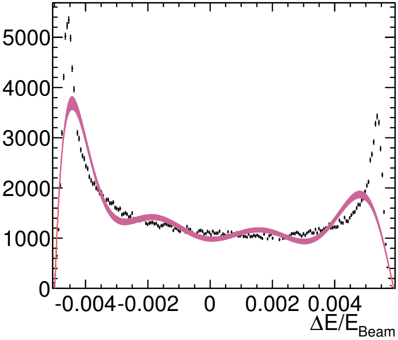
# Fitting with Chebyshev Polynomials

- Fitting with Chebyshev polynomials would avoid trouble of MODEL description
- $f(x) = \sum_i p_i \text{Cheb}_i(x)$
- But
  - 5 Parameters



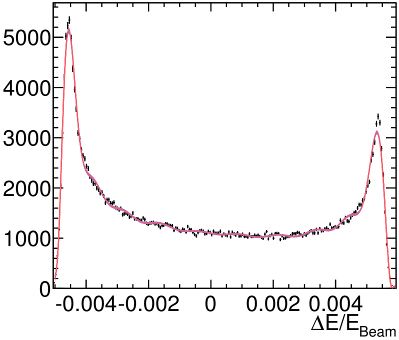
# Fitting with Chebyshev Polynomials

- Fitting with Chebyshev polynomials would avoid trouble of MODEL description
- $f(x) = \sum_i p_i \mathrm{Cheb}_i(x)$
- But
  - 5 Parameters
  - 10 Parameters



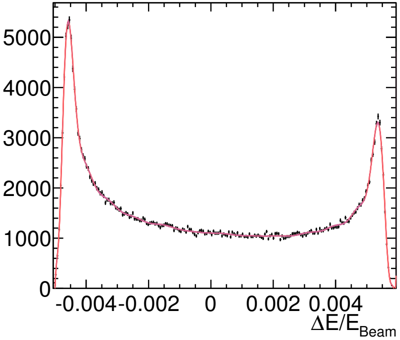
# Fitting with Chebyshev Polynomials

- Fitting with Chebyshev polynomials would avoid trouble of MODEL description
- $f(x) = \sum_i p_i \mathrm{Cheb}_i(x)$
- But
  - 5 Parameters
  - 10 Parameters
  - 26 Parameters: $\chi^2/\mathrm{ndf}$ = 668/173



# Fitting with Chebyshev Polynomials

- Fitting with Chebyshev polynomials would avoid trouble of MODEL description
- $f(x) = \sum_i p_i \mathrm{Cheb}_i(x)$
- But
  - 5 Parameters
  - 10 Parameters
  - 26 Parameters: $\chi^2/\mathrm{ndf}$ = 668/173
  - 35 Parameters: $\chi^2/\mathrm{ndf}$ = 226/164



# Fitting with Chebyshev Polynomials

- Fitting with Chebyshev polynomials would avoid trouble of MODEL description
- $f(x) = \sum_i p_i \text{Cheb}_i(x)$
- But
  - 5 Parameters
  - 10 Parameters
  - 26 Parameters: $\chi^2/\text{ndf} = 668/173$
  - 35 Parameters: $\chi^2/\text{ndf} = 226/164$
- Saves trouble of convolution, but at the cost of many parameters
- Could also fit centre only and do convolution with Gauss, but still need larger number of parameters