Role of fluctuations in detecting QCD phase transition

- Fluctuations of the Polyakov loop and deconfinement in a pure SU(N) gauge theory and in QCD
- Fluctuations of conserved charges as probe for the chiral phase transition and deconfinement
- Probability distribution and O(4) criticality

  ⇨ theoretical expectation and STAR data

Krzysztof Redlich University of Wroclaw & EMMI

Susceptibilities of net charge and order parameters

– The generalized susceptibilities probing fluctuations of net -charge number in a system and its critical properties

pressure:

$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V, T, \mu_{B,Q,S}, m_{u,d,s})$$

generalized susceptibilities

$$\chi_q^{(i+j+k)} = \frac{\partial^{(i+j+k)} p / T^{\wedge 4}}{\partial T^i \partial \mu_x^j \partial m^i} :$$

Order parameter

$$< O_h > = \frac{1}{V} \frac{\partial \ln Z}{\partial h}$$

particle number density

$$\frac{n_q}{T^3} = \frac{1}{VT^3} \frac{\partial \ln Z}{\partial \mu_q / T}$$

quark number susceptibility

$$\chi_q^{(2)} = \frac{\partial\ n_q / T^3}{\partial \mu_q / T}$$

4^{th} order cumulant

$$\chi_q^{(4)} = \frac{1}{VT^3} \frac{\partial^4 \ln Z}{\partial (\mu_q / T)^4}$$

$$\chi_q^1 = \frac{1}{VT^3} < N >, \quad \chi_q^2 = \frac{1}{VT^3} (< N^2 > - < N >^2) \quad \chi_q^4 = \frac{1}{VT^3} (< (\delta N)^4 > -3 < (\delta N)^3 >)$$

expressed by $N = N_q - N_{\bar{q}}$ **and central moment** $\delta N = N - < N >$

Polyakov loop on the lattice needs renormalization

- Introduce Polyakov loop:

$$L \Rightarrow c_N L$$

$$c_N = e^{2\pi\, i\, k/N} \in Z(N)$$

$$L^{\rm bare}_{\vec{x}} = \frac{1}{N_c} Tr \prod_{\tau=1}^{N_\tau} U_{(\vec{x},\tau),4}$$

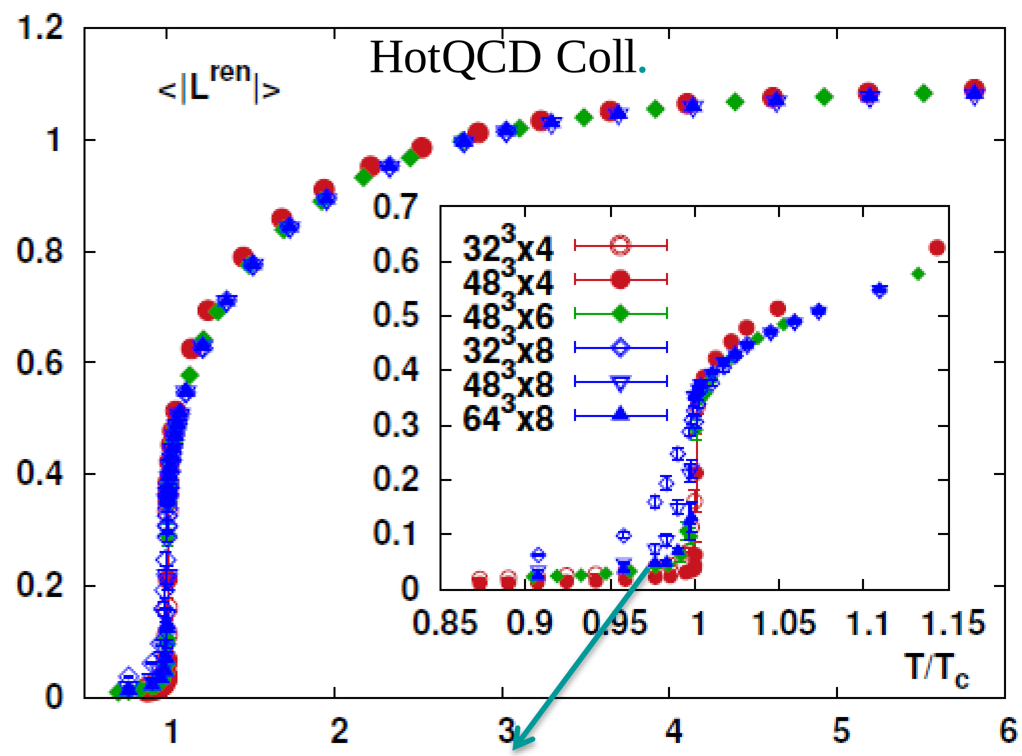
$$L^{\rm bare} = \frac{1}{N_\sigma^3} \sum_{\vec{x}} L^{\rm bare}_{\vec{x}}$$

$$<| L^{\rm ren} |> = e^{-\beta F_q^{\rm ren}} \rightarrow \begin{cases} \neq 0 \Leftrightarrow \text{deconfined} \;\; T > T_c \\ 0 \Leftrightarrow \text{deconfined} \;\; T > T_c \end{cases}$$

- Renormalized ultraviolet divergence

$$L^{\rm ren} = \left(Z(g^2)\right)^{N_\tau} L^{\rm bare}$$

- Usually one takes $\langle|L^{\rm ren}|\rangle$ as an order parameter



$1/\sqrt{V}$

To probe deconfinement : consider fluctuations

- Fluctuations of modulus of the Polyakov loop

$$T^3\chi_A = \frac{N_\sigma^3}{N_\tau^3}\left(\langle|L^{\rm ren}|^2\rangle - \langle|L^{\rm ren}|\rangle^2\right)$$

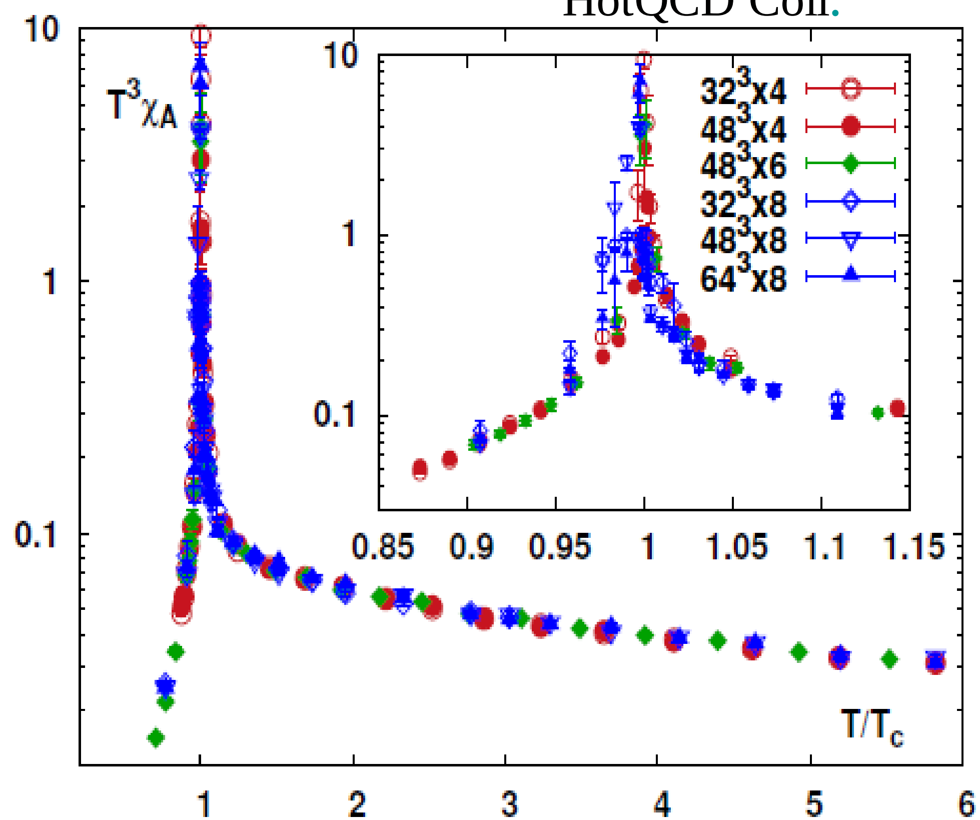
However, the Polyakov loop

$$L = L_R + iL_I$$

Thus, one can consider fluctuations of the real χ_R and the imaginary part χ_I of the Polyakov loop.

SU(3) pure gauge: LGT data

HotQCD Coll.

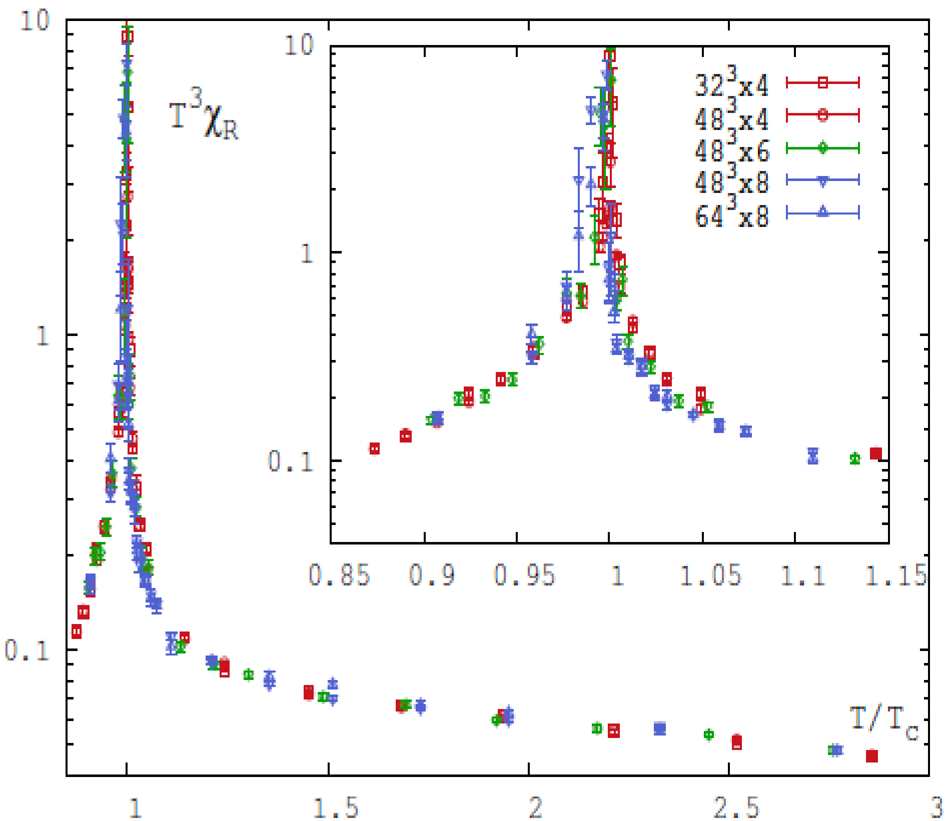


Fluctuations of the real and imaginary part of the renormalized Polyakov loop

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.

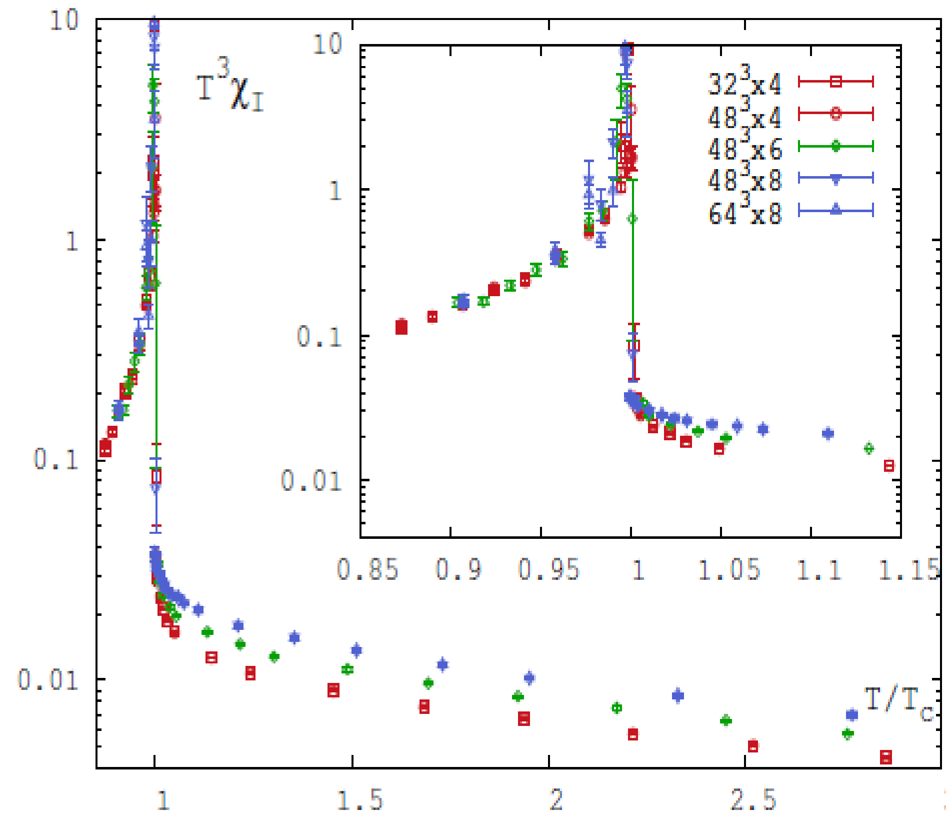
- Real part fluctuations

$$T^3\chi_R = \frac{N_\sigma^3}{N_\tau^3}\left[\langle (L_R^{\rm ren})^2\rangle - \langle L_R^{\rm ren}\rangle^2\right]$$



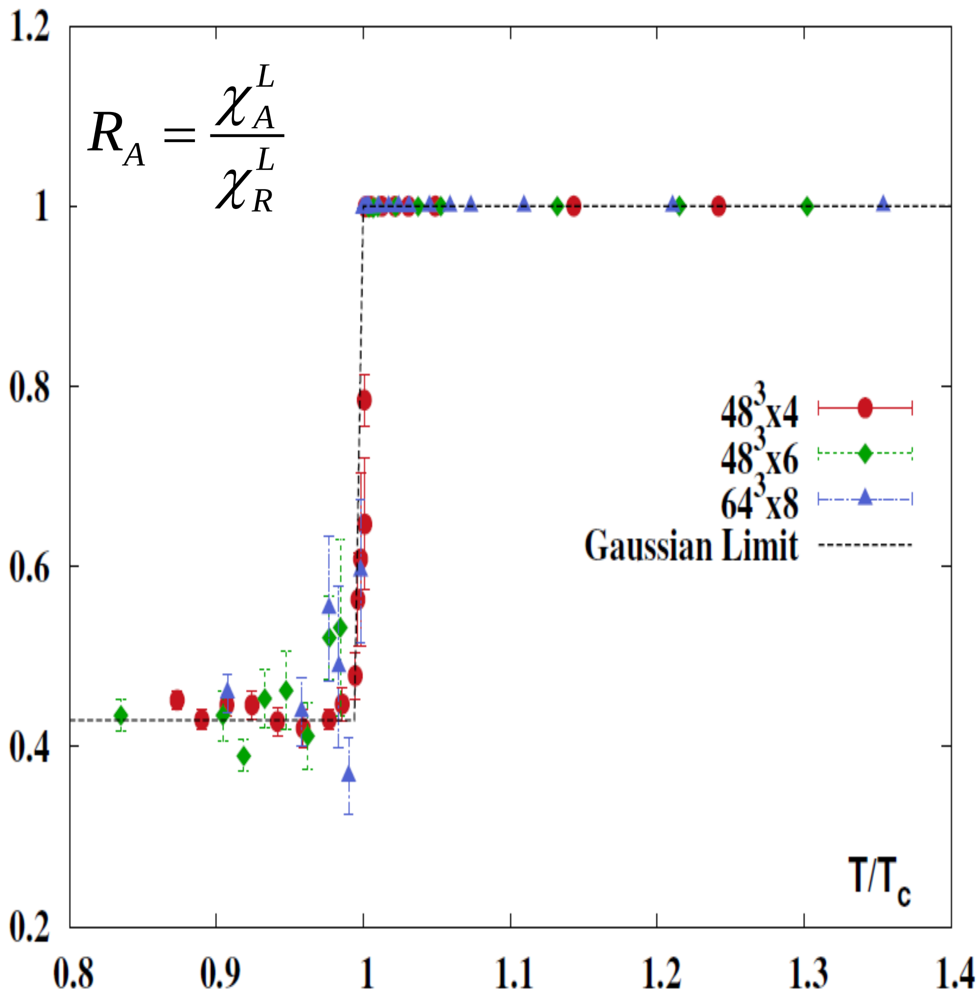
- Imaginary part fluctuations

$$T^3\chi_I = \frac{N_\sigma^3}{N_\tau^3}\left[\langle (L_I^{\rm ren})^2\rangle - \langle L_I^{\rm ren}\rangle^2\right]$$



Ratios of the Polyakov loop fluctuations as an excellent probe for deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the deconfined phase $R_A \approx 1$

Indeed, in the real sector of Z(3)

$$L_R \approx L_0 + \delta L_R \quad \text{with} \quad L_0 =< L_R >$$

$$L_I \approx L_0^I + \delta L_I \quad \text{with} \quad L_0^I = 0, \ \text{thus}$$

$$\chi_R^L = V < (\delta L_R)^2 >, \quad \chi_I^L = V < (\delta L_I)^2 >$$

Expand the modulus,

$$| L |= \sqrt{L_R^{\ 2} + L_I^{\ 2}} \approx L_0 (1 + \frac{\delta L_R}{L_0} + \frac{(\delta L_I)^2}{2 L_0^{\ 2}})$$

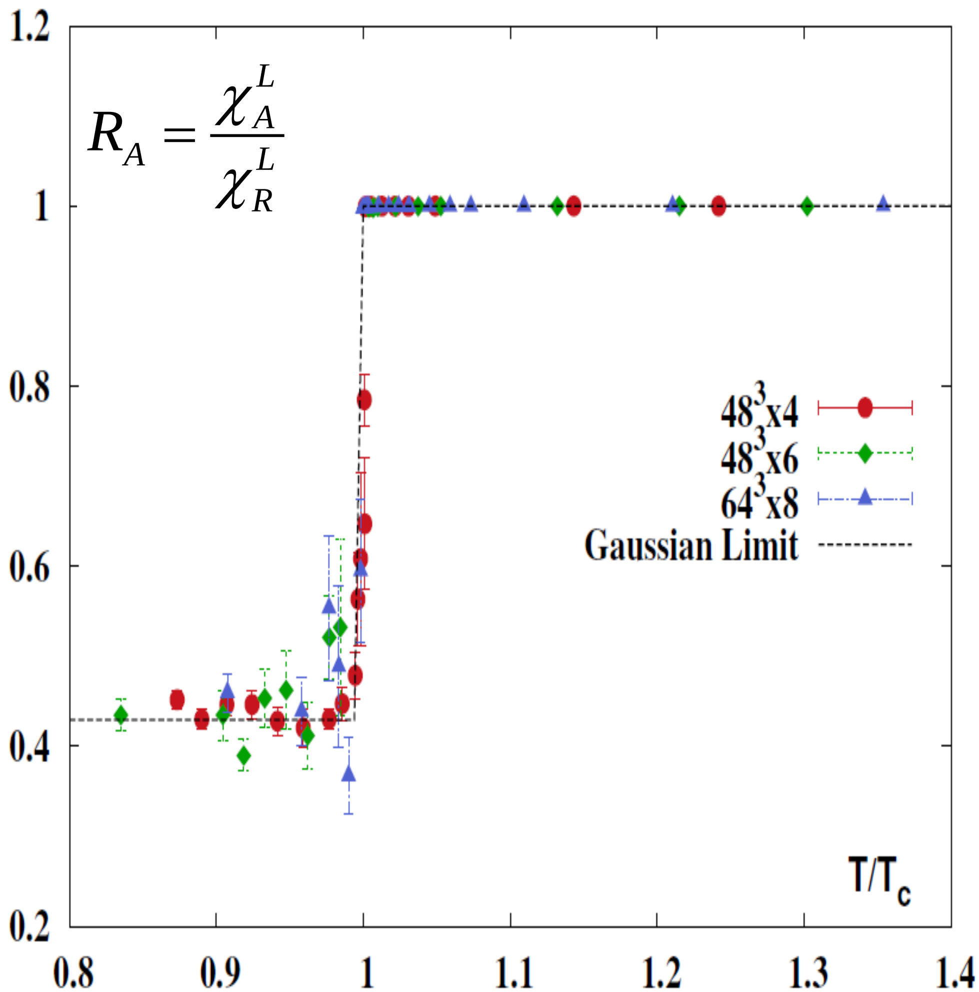
get in the leading order

$$<| L |^2> - <| L |>^2 \approx \ < (\delta L_R)^2 >$$

thus $\chi_A \approx \chi_R$

Ratios of the Polyakov loop fluctuations as an excellent probe for deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the confined phase $R_A \approx 0.43$

Indeed, in the Z(3) symmetric phase, the probability distribution is to a first approximation Gaussian with the partition function

$$Z = \int dL_R dL_I e^{VT^3[\alpha(T)(L_R{}^2 + L_I{}^2)]}$$

Thus $\chi_R = \frac{1}{2\alpha T^3}$, $\chi_I = \frac{1}{2\alpha T^3}$ and

$\chi_A = \frac{1}{2\alpha T^3}(2 - \frac{\pi}{2})$, consequently

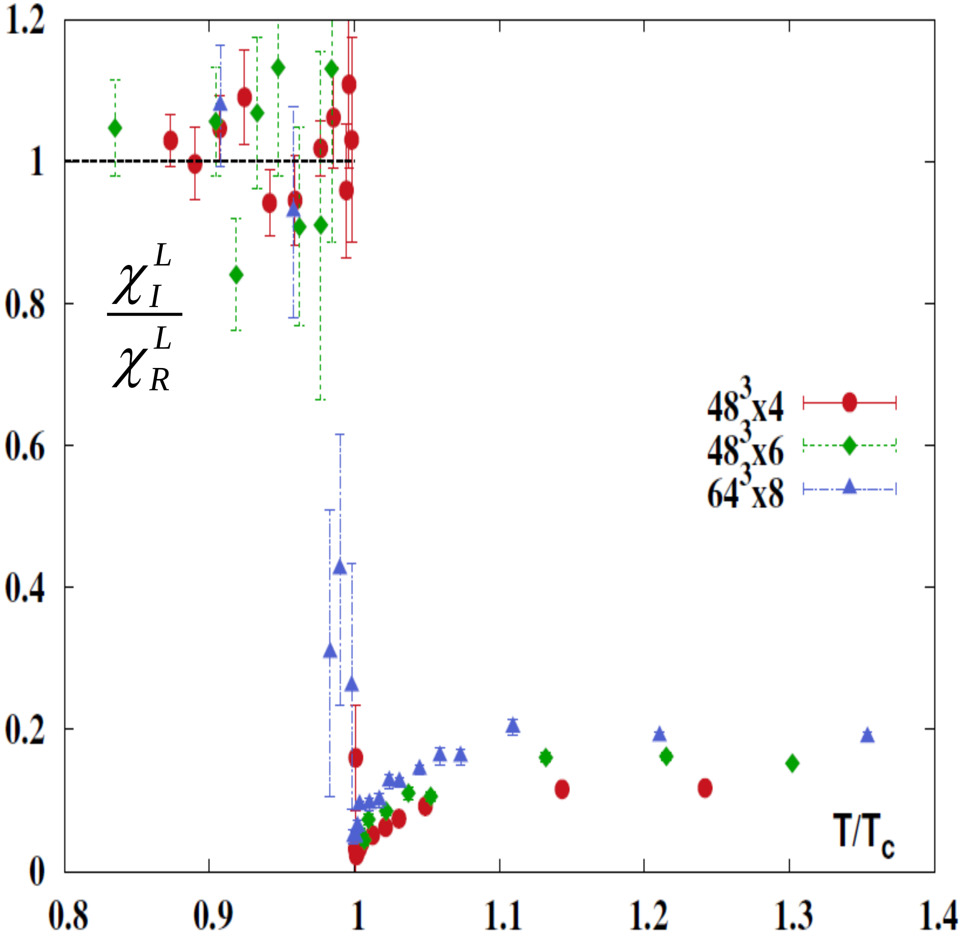
$$R_A^{SU(3)} = (2 - \frac{\pi}{2}) = 0.429$$

In the SU(2) case $R_A^{SU(2)} = (2 - \frac{2}{\pi}) = 0.363$

is in agreement with MC results

Ratio Imaginary/Real of Polyakov loop fluctuations

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the confined phase for any symmetry breaking operator its average vanishes, thus

$$\chi_{LL} = \langle L^2 \rangle - \langle L \rangle^2 = 0$$ and

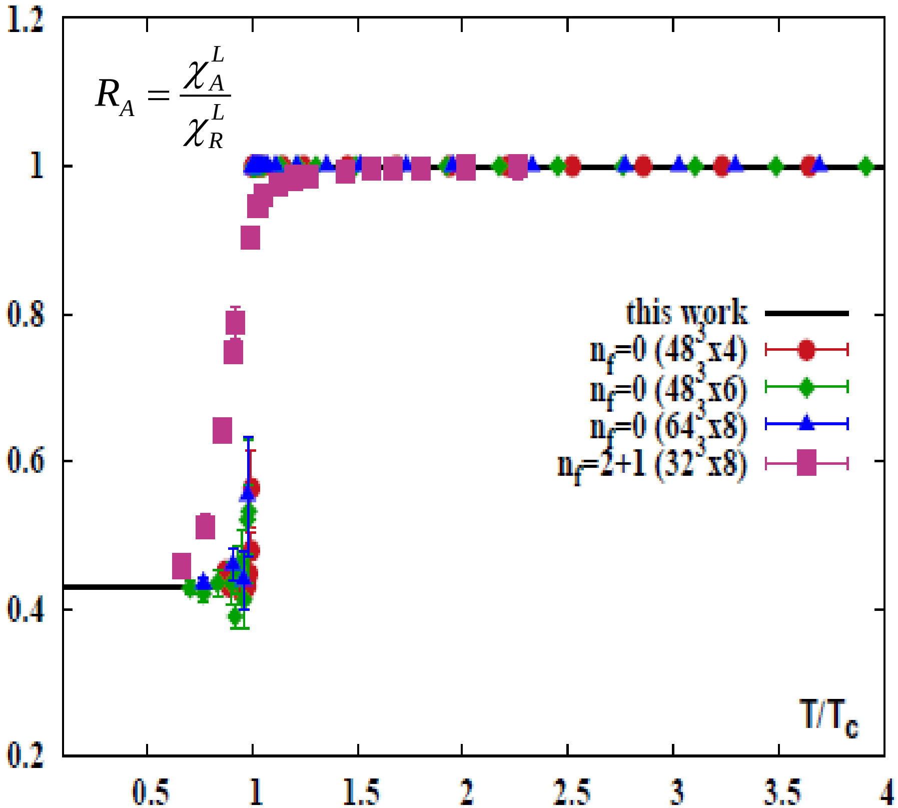
$$\chi_{LL} = \chi_R^L + \chi_I^L$$ thus $$\chi_R = \chi_I$$

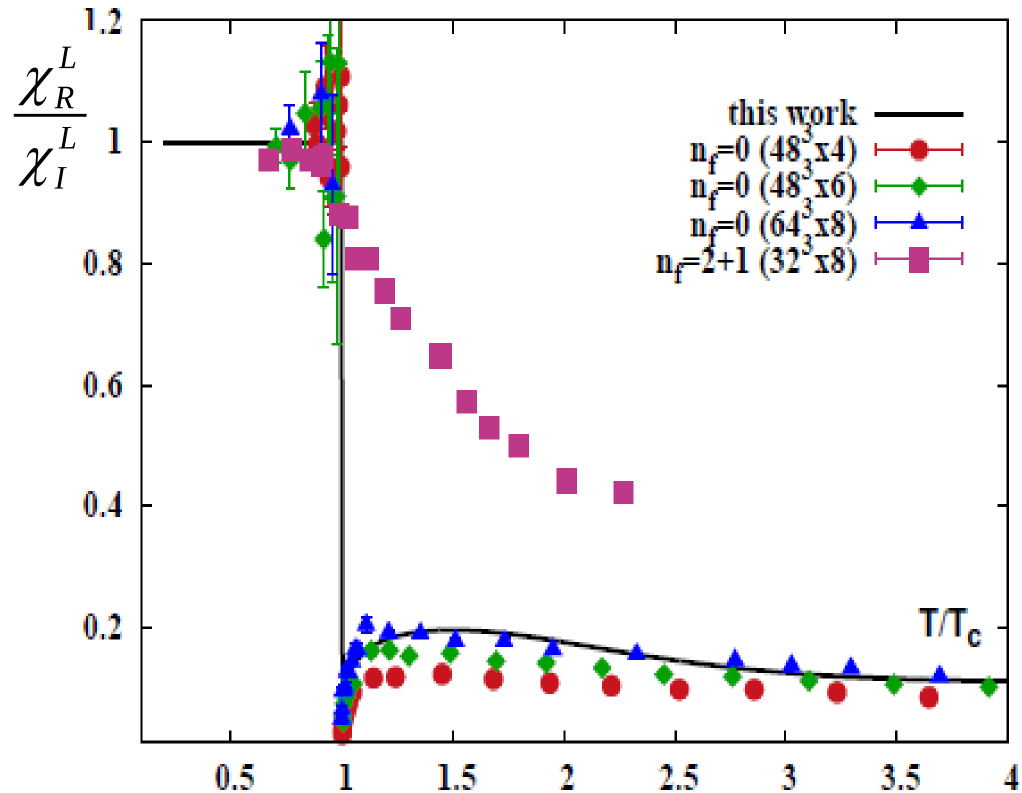
- In the deconfined phase the ratio of $\chi_I^L / \chi_R^L \neq 0$ and its value is model dependent

The influence of fermions on ratios of the Polyakov loop susceptibilities

- Z(3) symmetry broken, however ratios still showing the transition
- Change of the slopes at fixed T

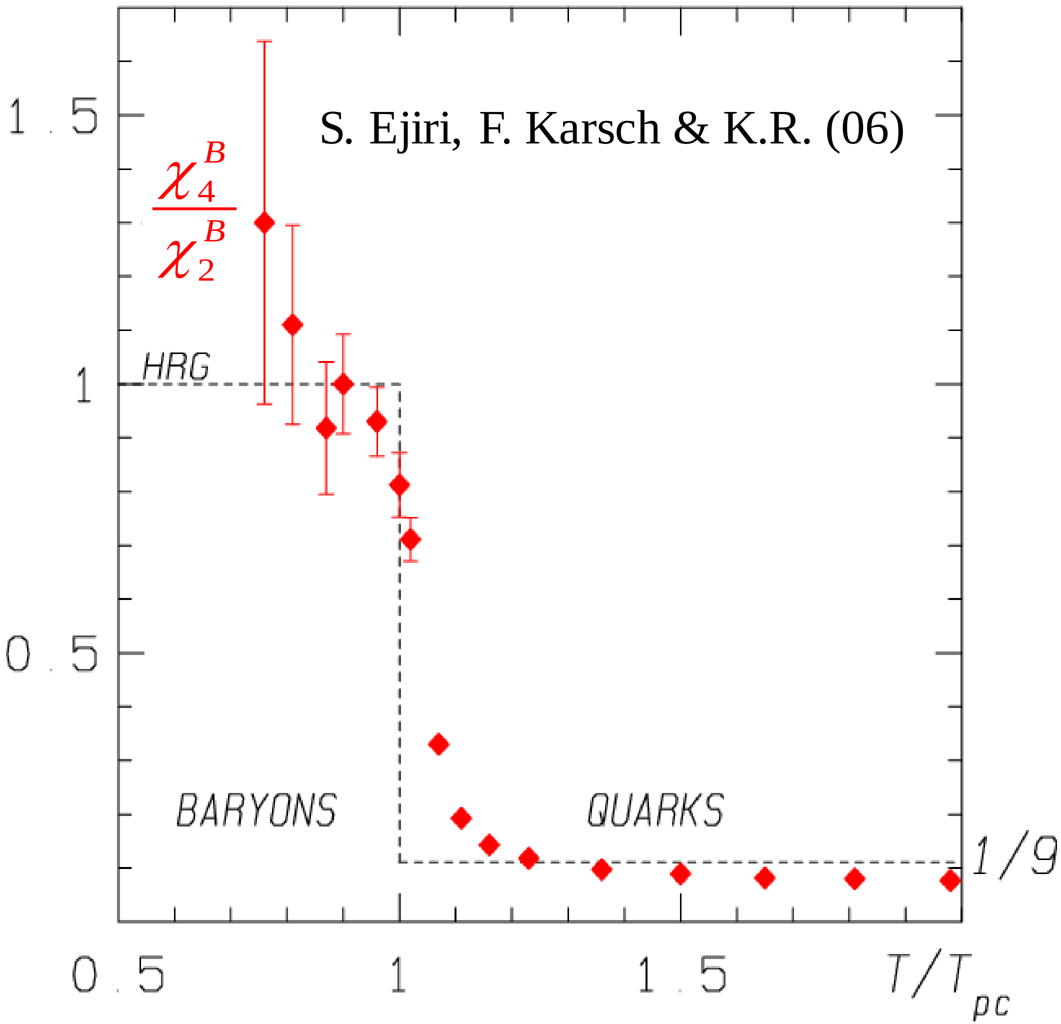
Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.





Probing deconfinement in QCD

$16^3 \times 4$ lattice with p4 fermion action



- HRG factorization of pressure:

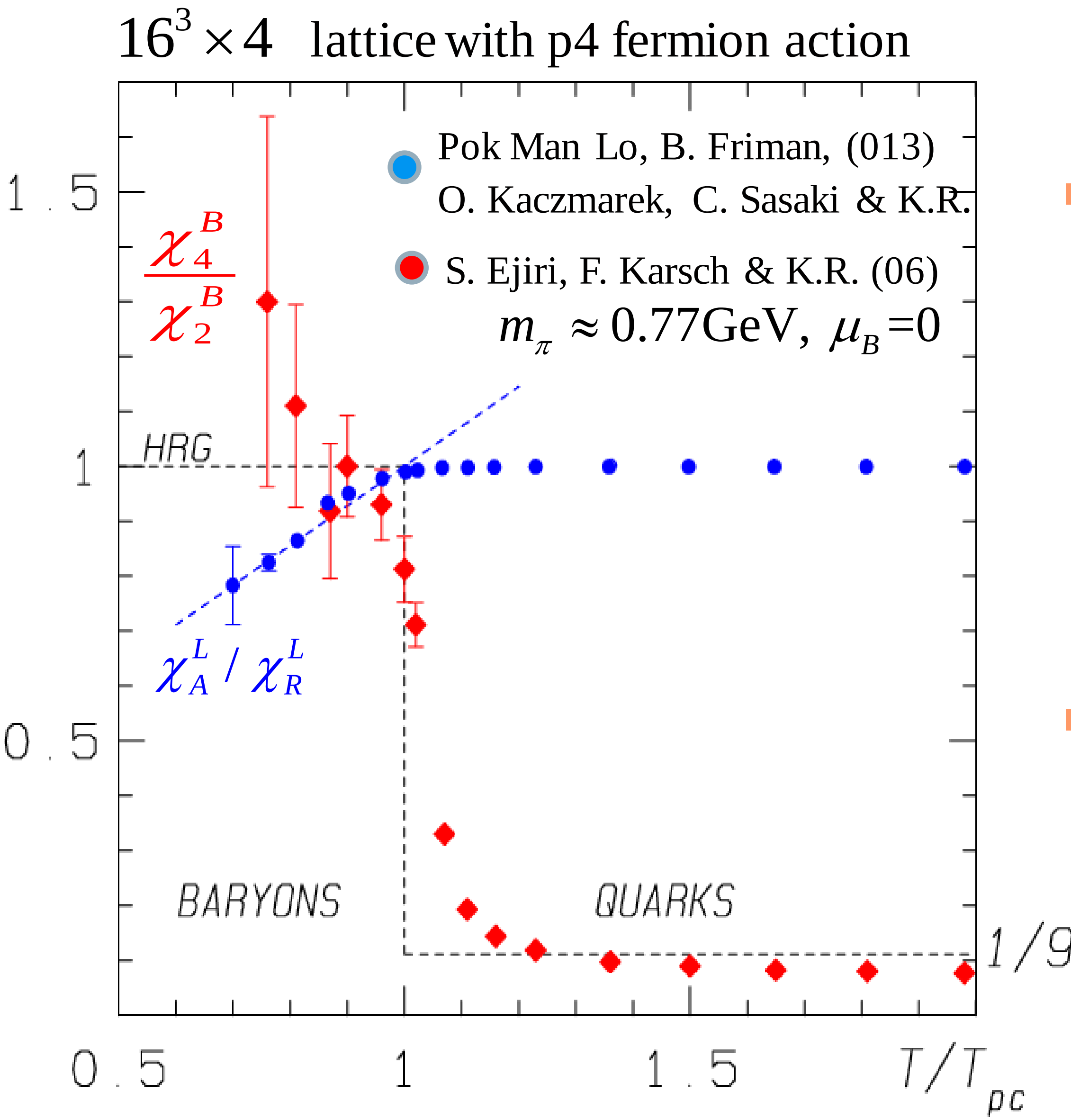
$$P^B(T,\mu_q) = F(T)\cosh(3\mu_q/T)$$

- Kurtosis measures the squared of the baryon number carried by leading particles in a medium

S. Ejiri, F. Karsch & K.R. (06)

$$\kappa\,\sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{pmatrix}$$

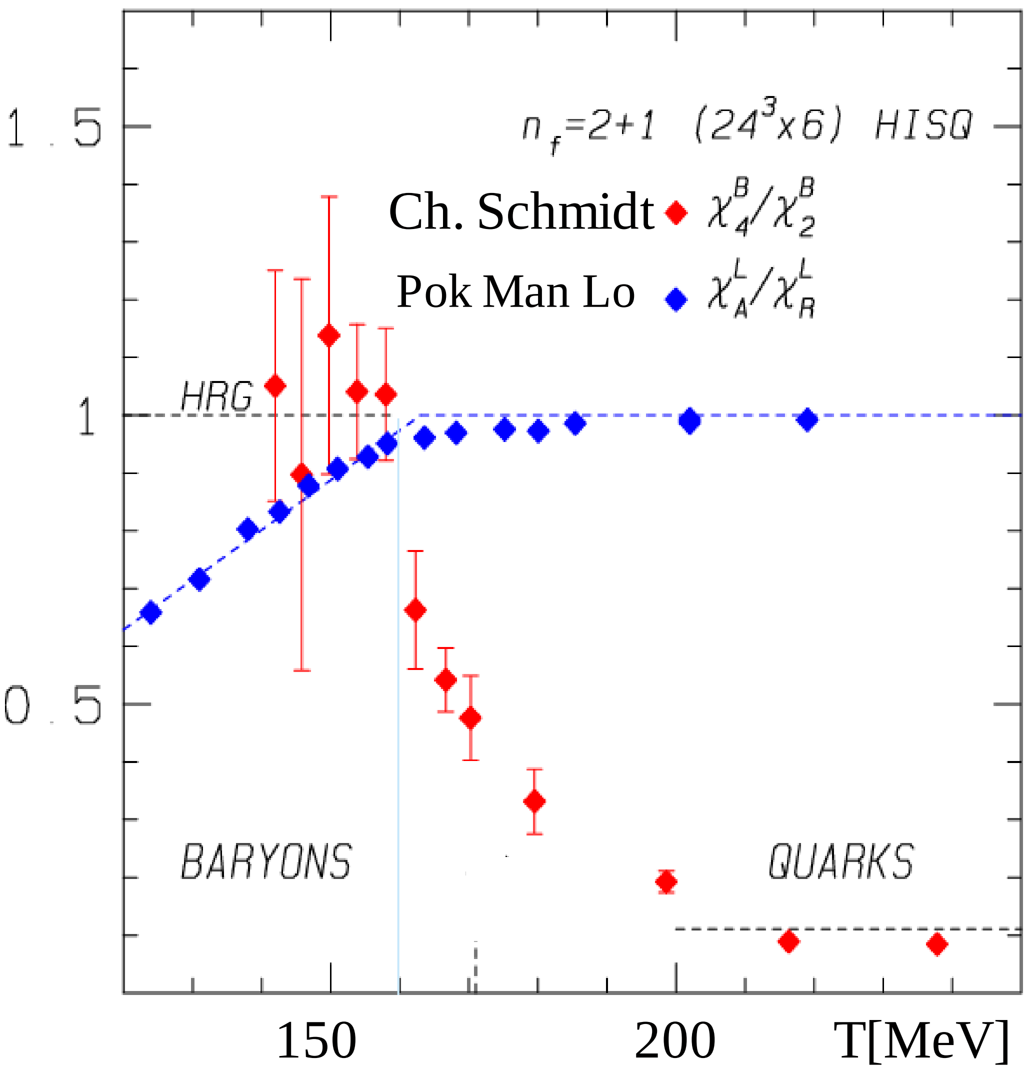
Probing deconfinement in QCD



- The change of the slope of the ratio of the Polyakov loop susceptibilities χ_A^L / χ_R^L appears at the same T where the kurtosis drops from its HRG asymptotic value
- In the presence of quarks there is “remnant” of Z(N) symmetry in the χ_A^L / χ_R^L ratio, indicating deconfinement of quarks

Probing deconfinement in QCD

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.



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Remnant of the O(4) chiral phase transtion in QCD

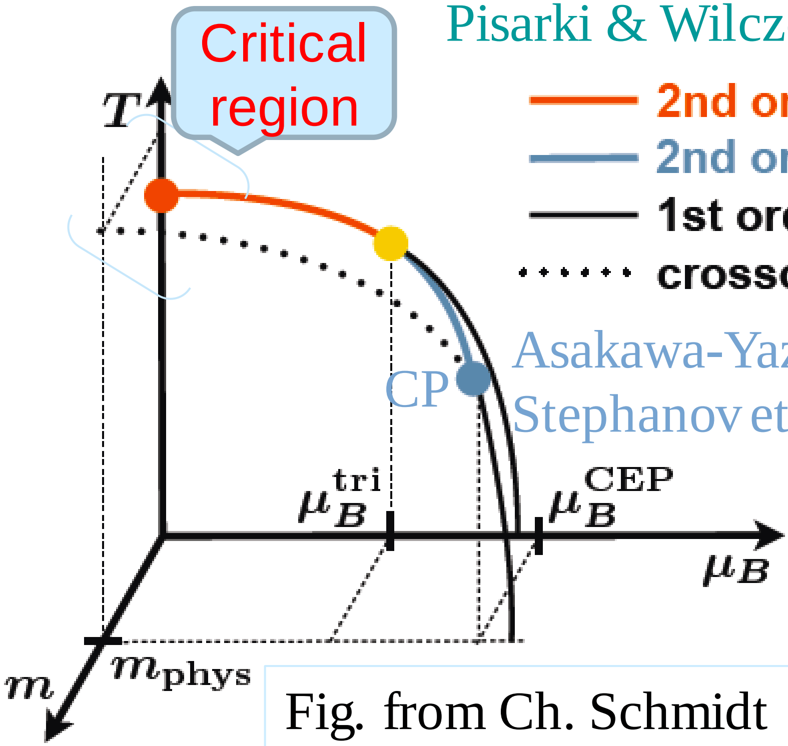


Fig. from Ch. Schmidt

At the CP:
Divergence of Fluctuations, Correlation length and specific heat

- The QCD crossover line can appear in the O(4) critical region!

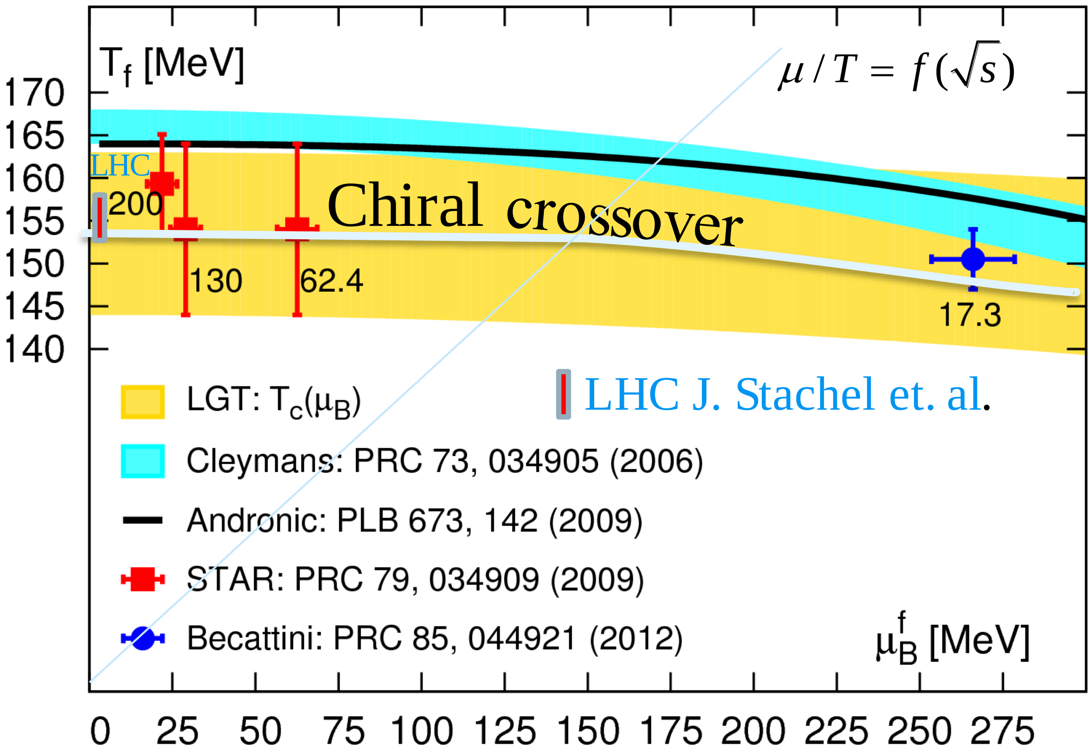
This has been indeed shown in LQCD calculations by:

BNL-Bielefeld group

Phys. Rev. D83, 014504 (2011)

Phys. Rev. D80, 094505 (2009)

Chemical freezeout and the QCD chiral crossover



Chiral crossover Temperature from LGT

$T_c = 154 \pm 9$ MeV

HotQCD Coll. (QM'12)

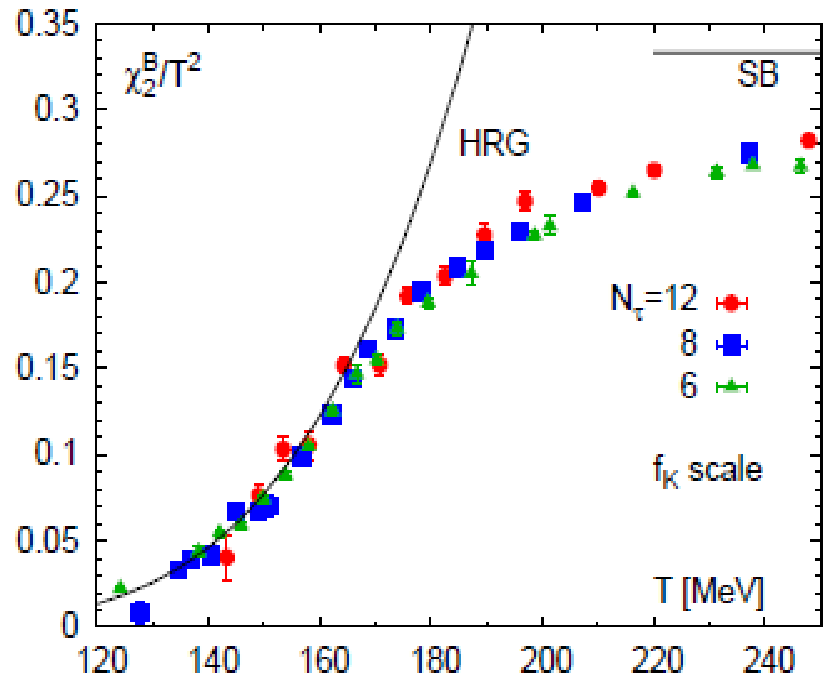
Chemical Freezeout LHC (ALICE)

$T_f = 156 \pm 2$ MeV

see talk of J. Stachel

- Is there a memory that the system has passed through a region of QCD O(4)-chiral crossover transition?

HotQCD Collaboration



Quark fluctuations and O(4) universality class

- Due to the expected O(4) scaling in QCD the free energy:

$$F = F_R(T, \mu_q, \mu_I) + b^{-1} F_S(b^{(2-\alpha)^{-1}} t, b^{\beta\delta/\nu} h)$$

- Consider generalized susceptibilities of net-quark number

$$c_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} = c_R^{(n)} + c_S^{(n)} \quad \text{with} \quad c_s^{(n)} = d\ h^{(2-\alpha-n)/\beta\delta}\ f_{\pm}^{(n)}(z)$$

- Since for $T < T_{pc}$, $c_R^{(n)}$ are well described by the HRG

  search for deviations (in particular for larger n) from HRG

  ⇨ to quantify the contributions of $c_S^{(n)}$, i.e. the O(4) criticality

S. Ejiri, F. Karsch & K.R. Phys. Lett. B633, (2006) 275
M. Asakawa, S. Ejiri and M. Kitazawa, Phys. Rev. Lett. 103 (2009) 262301
V. Skokov, B. Stokic, B. Friman &K.R. Phys. Rev. C82 (2010) 015206
F. Karsch & K. R. Phys.Lett. B695 (2011) 136
B. Friman, et al. . Phys.Lett. B708 (2012) 179, Nucl.Phys. A880 (2012) 48

Effective chiral models
Renormalisation Group Approach

$$S=\int_0^{\beta=1/T} d\tau \int_V d^3x [i\bar{q}(\gamma_\mu \partial_\mu - A_\mu \delta_{\mu 4})q - V^{int}(q,\bar{q}) + \mu_q q^+ q - U(L,L^*)]$$

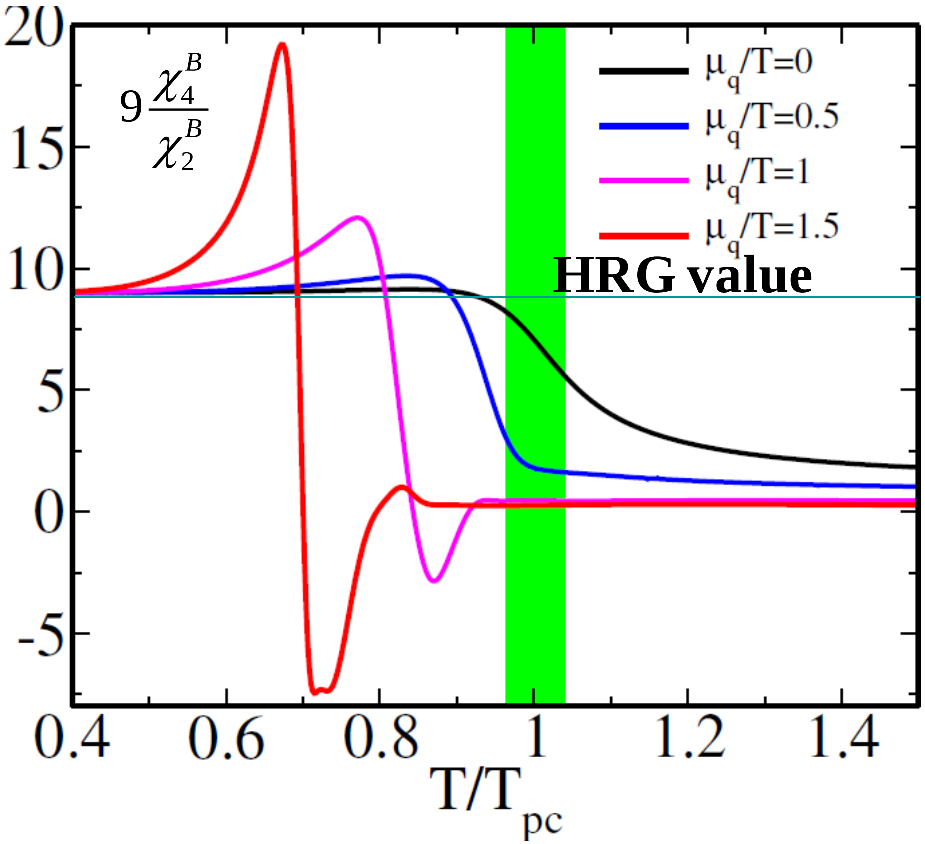
$U(L,L^*)$ – the $Z(3)$ - invariant Polyakov loop potential
(Get potential from YM theory, C. Sasaki &K.R. Phys.Rev. D86, (2012);
Parametrized LGT data: Pok Man Lo, B. Friman, O. Kaczmarek &K.R.)

$V^{int}(q,\bar{q})$ – the SU(2)xSU(2) χ –invariant quark interactions described through:

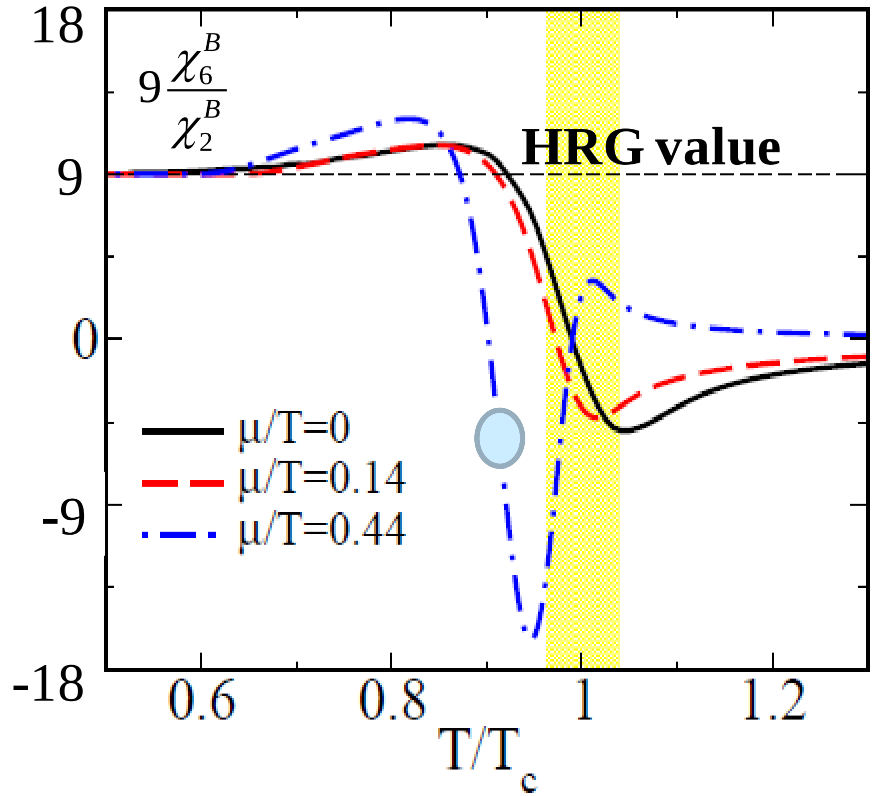
- Nambu-Jona-Lasinio model ⇨ PNJL chiral model
  K. Fukushima; C. Ratti & W. Weise; B. Friman , C. Sasaki ., ….
- coupling with meson fileds fields ⇨ PQM chiral model
  B.-J. Schaefer, J.M. Pawlowski & J. Wambach; B. Friman, V. Skokov, ...
- FRG thermodynamics of PQM model:
  B. Friman, V. Skokov, B. Stokic & K.R.

Ratios of cumulants at finite density in PQM model with FRG

B. Friman, F. Karsch, V. Skokov &K.R.
Eur.Phys.J. C71 (2011) 1694

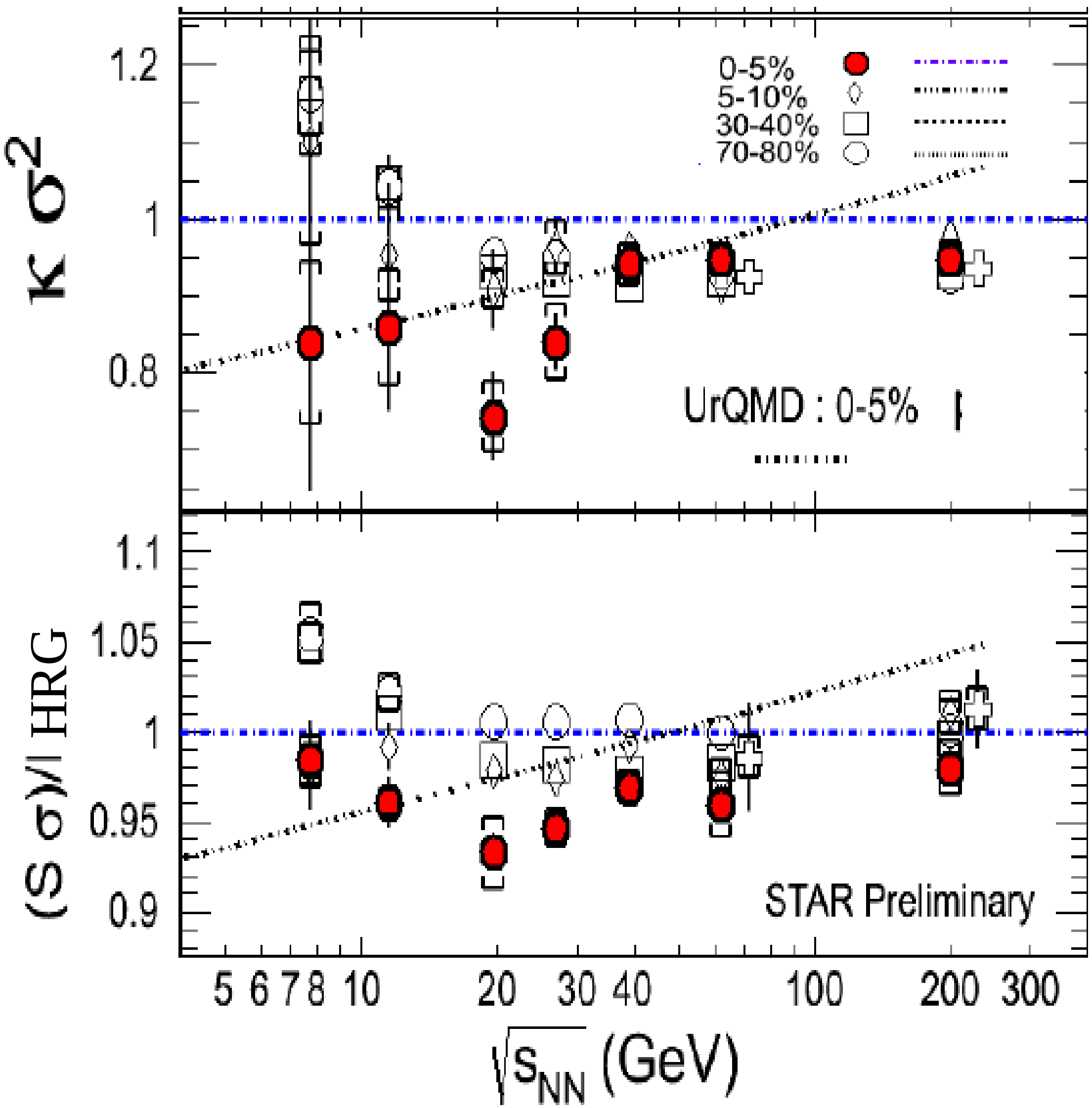


B. Friman, V. Skokov &K.R.
Phys.Rev. C83 (2011) 054904



Deviations from low -T HRG values are increasing with μ/T and the cumulant order . Negative fluctuations near the chiral crossover.

STAR data on the first four moments of net baryon number



Deviations from the HRG

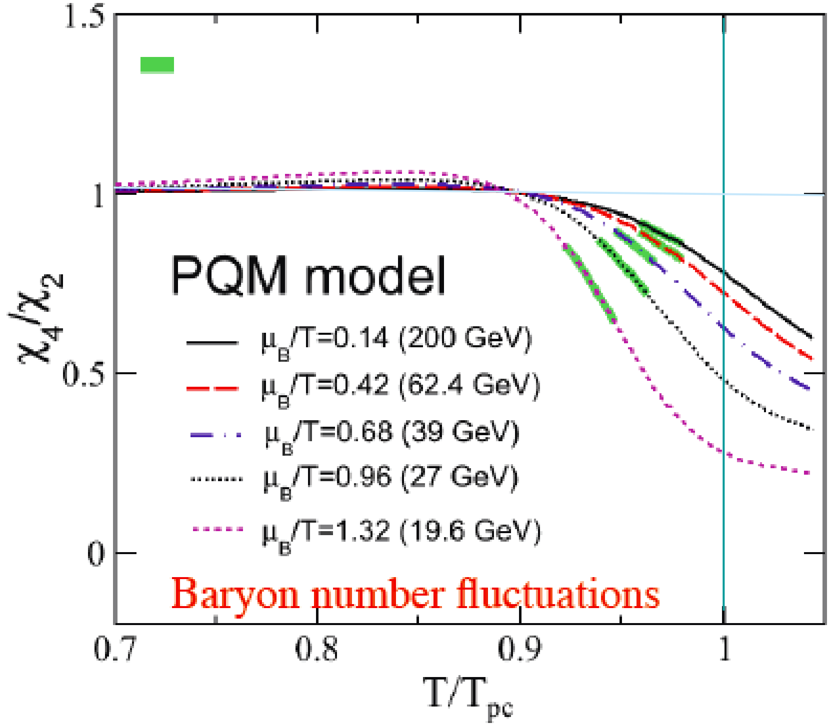
$$S\,\sigma = \frac{\chi_B^{(3)}}{\chi_B^{(2)}} \quad , \quad \kappa\,\sigma^2 = \frac{\chi_B^{(4)}}{\chi_B^{(2)}}$$

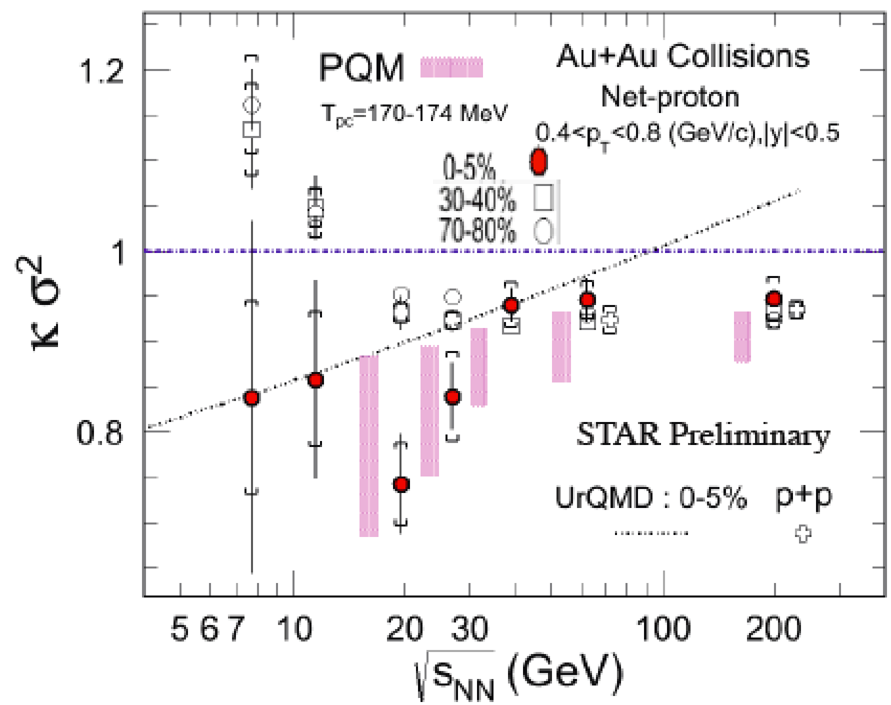
$$S\,\sigma\,|_{HRG} = \frac{N_p - N_{\bar{p}}}{N_p + N_{\bar{p}}} \quad , \kappa\sigma\,|_{HRG} = 1$$

Data qualitatively consistent with the change of these ratios due to the contribution of the O(4) singular part to the free energy

Kurtosis saturates near the O(4) phase boundary

B. Friman, et al. EPJC 71, (2011)





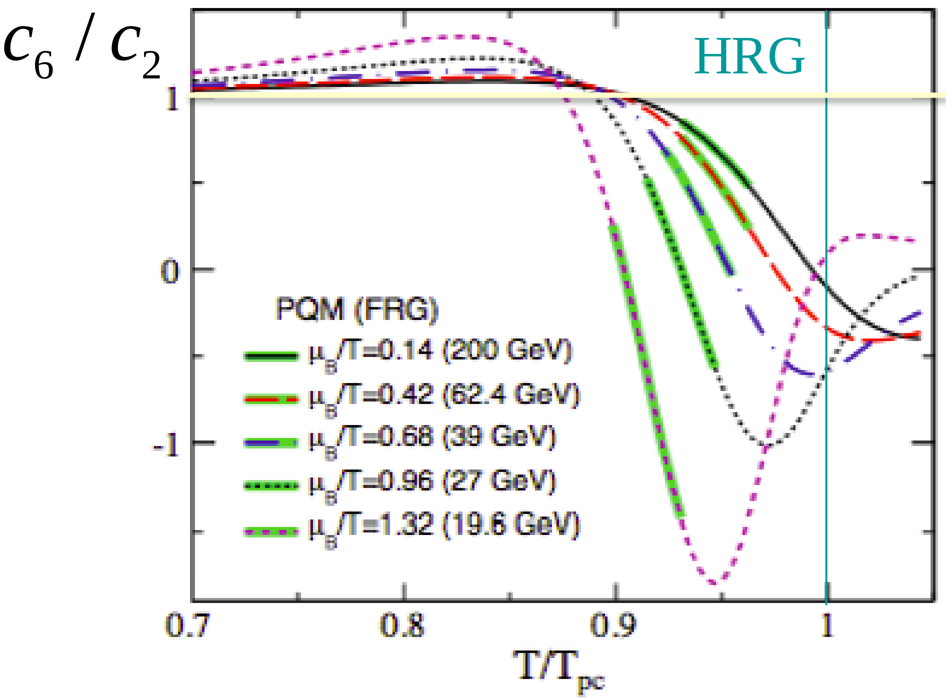
- The energy dependence of measured kurtosis consistent with expectations due to contribution of the O(4) criticality.

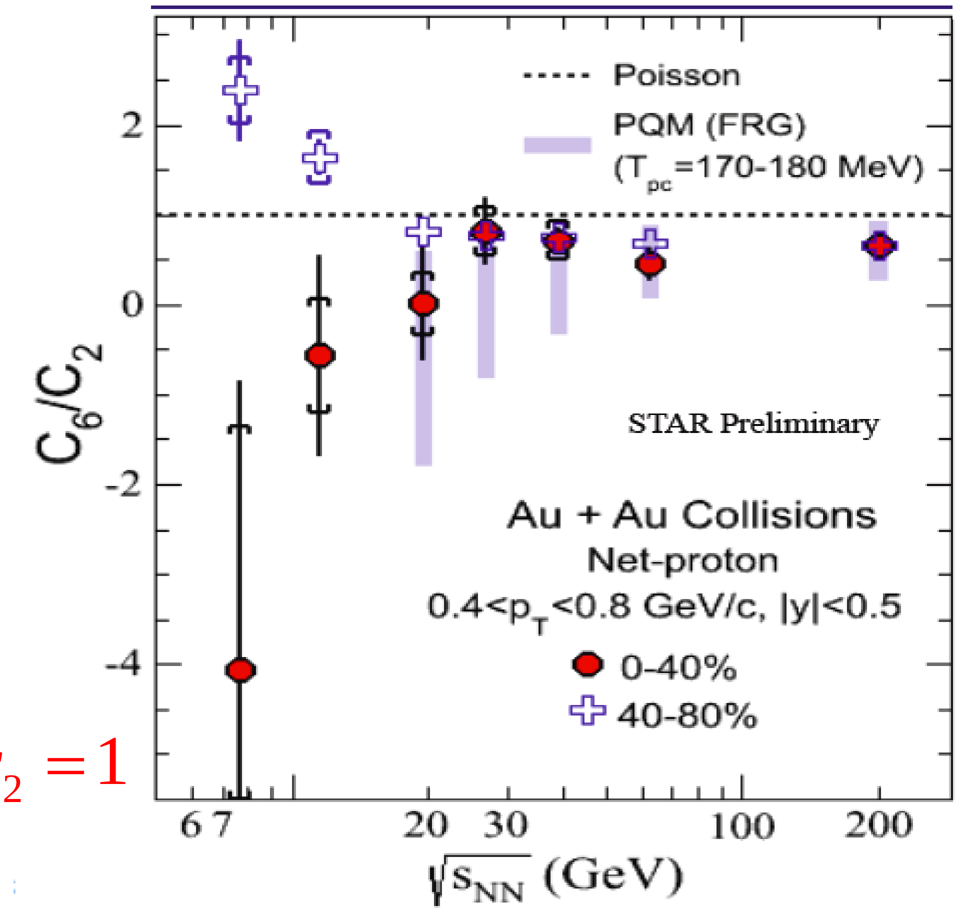
  Can that be also seen in the higher moments?

STAR DATA Presented at QM'12

Lizhu Chen for STAR Coll.

V. Skokov, B. Friman & K.R., F. Karsch et al.





The HRG reference predicts: $c_6 / c_2 = 1$

O(4) singular part contribution:

strong deviations from HRG: negative structure already at vanishing baryon density

Moments obtained from probability distributions

- Moments obtained from probability distribution

$$< N^k >= \sum_N N^k P(N)$$

- Probability quantified by all cumulants

$$P(N) = \frac{1}{2\pi}\int_0^{2\pi} dy \exp[iyN - \chi(iy)]$$

Cumulants generating function: $\chi(y) = \beta V[p(T, y+\mu) - p(T,\mu)] = \sum_k \chi_k y^k$

- In statistical physics

$$P(N) = \frac{Z_C(N)}{Z_{GC}} e^{\frac{\mu N}{T}}$$

What is the influence of O(4) criticality on P(N)?

- For the net baryon number use the Skellam distribution (HRG baseline)

$$P(N) = \left(\frac{B}{\bar{B}}\right)^{N/2} I_N(2\sqrt{B\bar{B}}) \exp[-(B+\bar{B})]$$

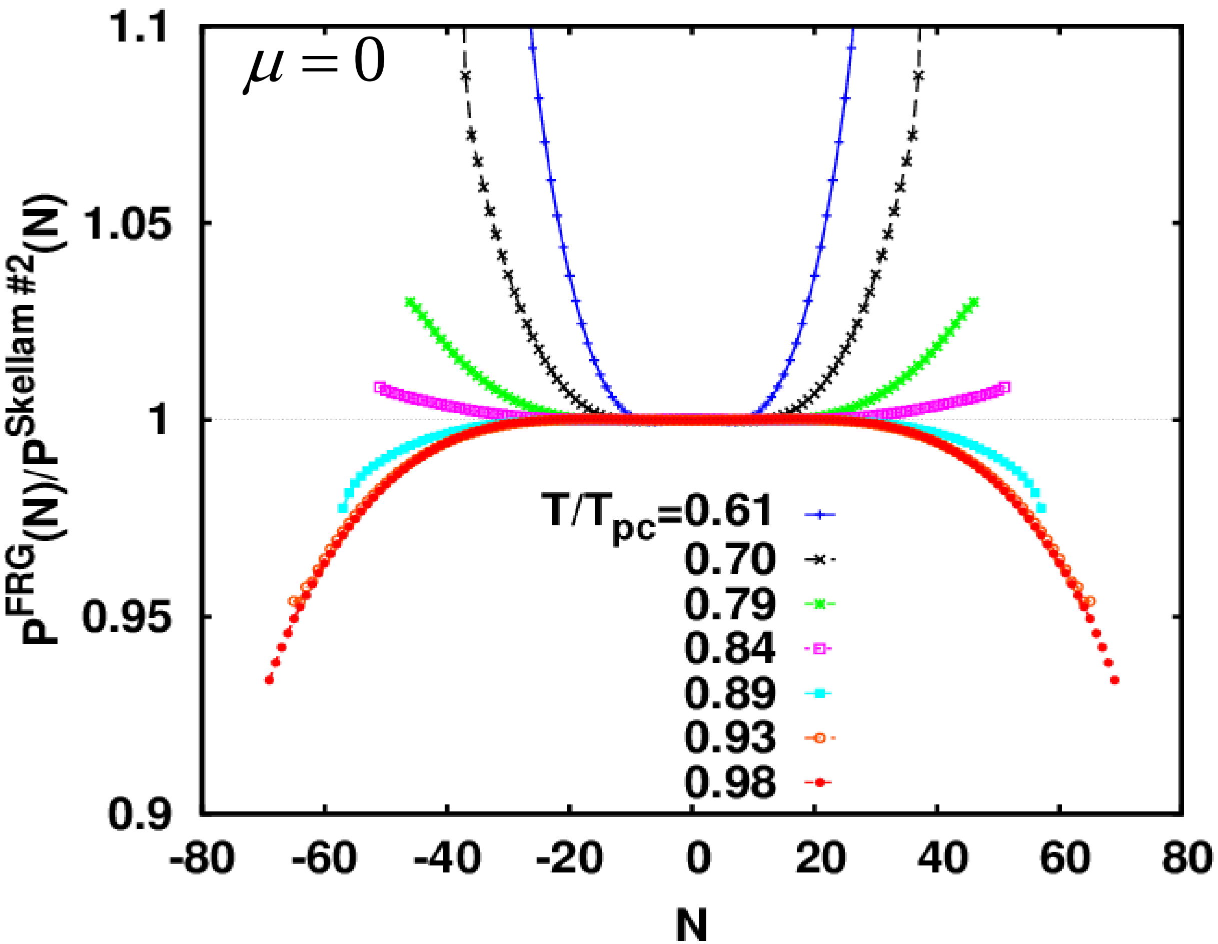
as the reference for the non-critical behavior

- Calculate P(N) in an effective chiral model which exhibits O(4) scaling and compare to Skellam distribtuion

The influence of O(4) criticality on P(N) for $\mu = 0$

- Take the ratio of $P^{FRG}(N)$ which contains O(4) dynamics to Skellam distribution with the same Mean and Variance at different T/T_{pc}

K. Morita, B. Friman &K.R. (PQM model)

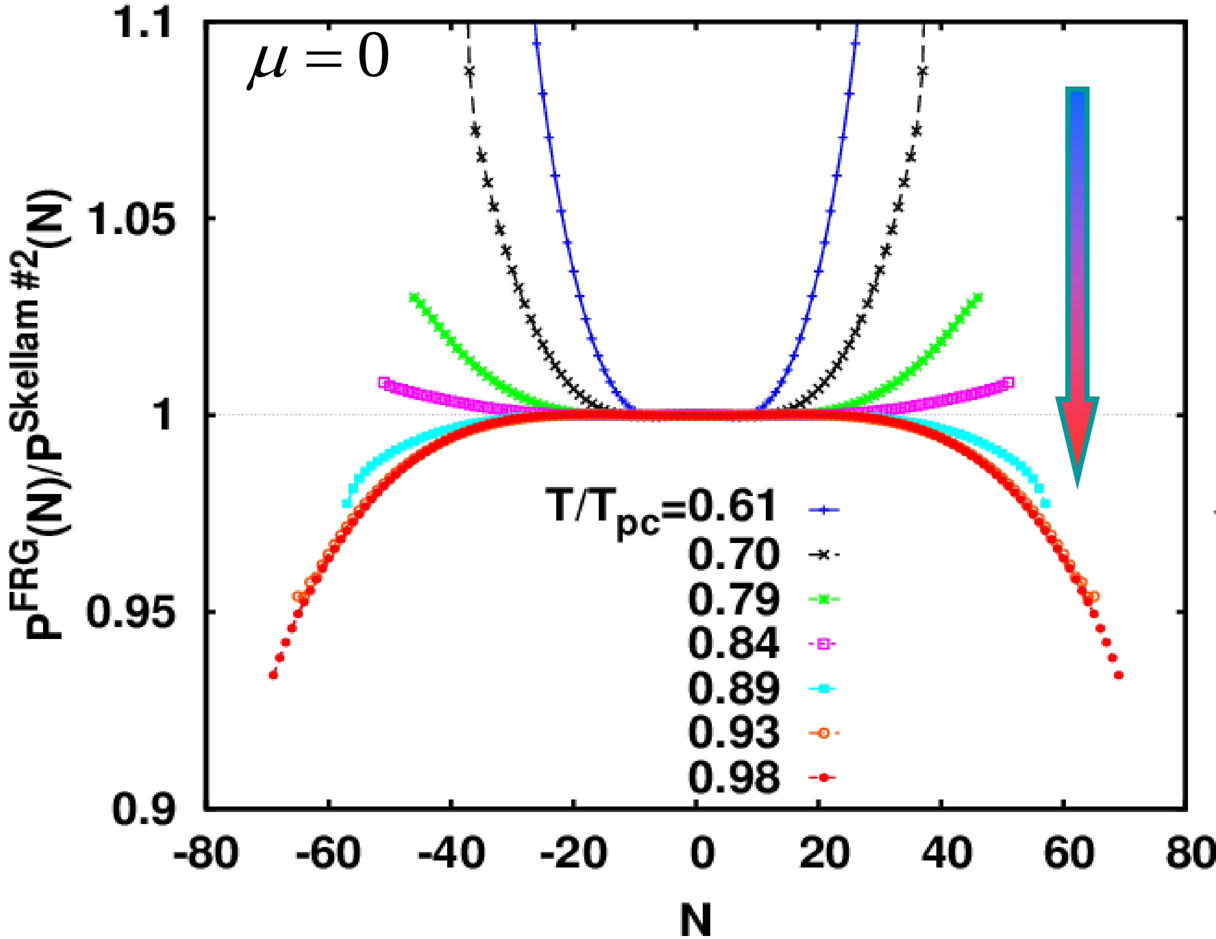


- Ratios less than unity near the chiral crossover, indicating the contribution of the O(4) criticality to the thermodynamic pressure

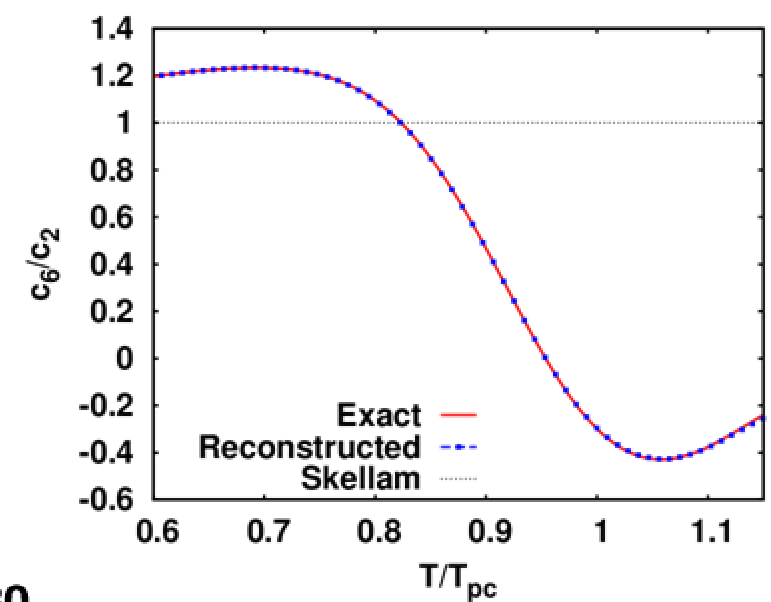
The influence of O(4) criticality on P(N) for $\mu = 0$

- Take the ratio of $P^{FRG}(N)$ which contains O(4) dynamics to Skellam distribution with the same Mean and Variance at different T/T_{pc}

K. Morita, B. Friman et al.



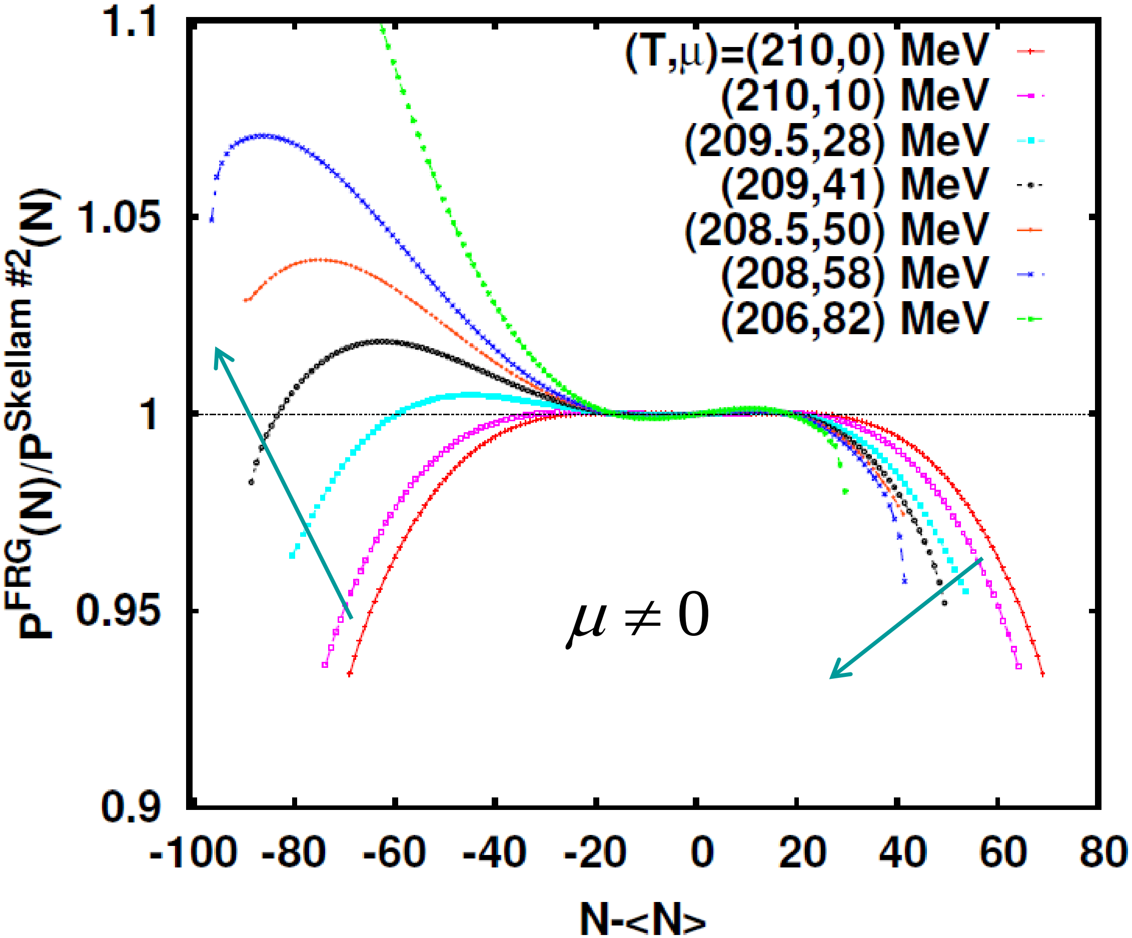
Ratio < 1 at larger |N| if $c_6/c_2 < 1$



The influence of O(4) criticality on P(N) for $\mu \neq 0$

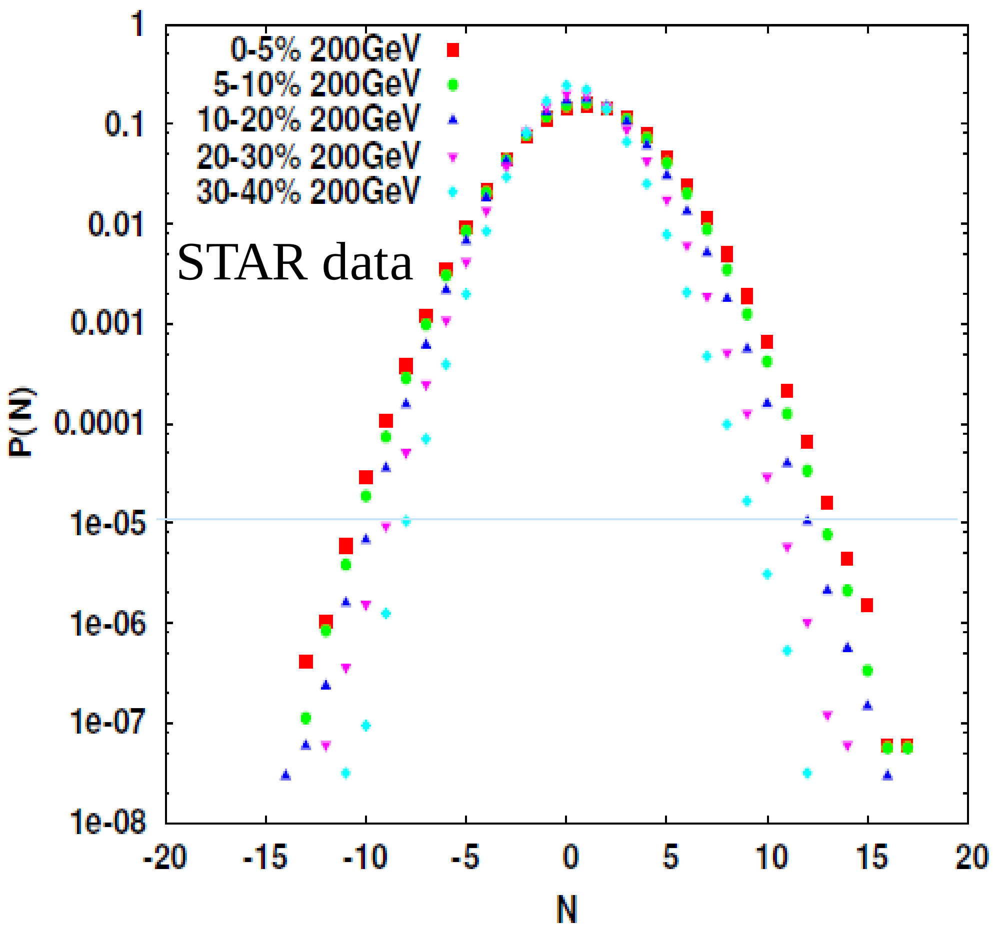
- Take the ratio of $P^{FRG}(N)$ which contains O(4) dynamics to Skellam distribution with the same Mean and Variance near $T_{pc}(\mu)$

K. Morita, B. Friman et al.

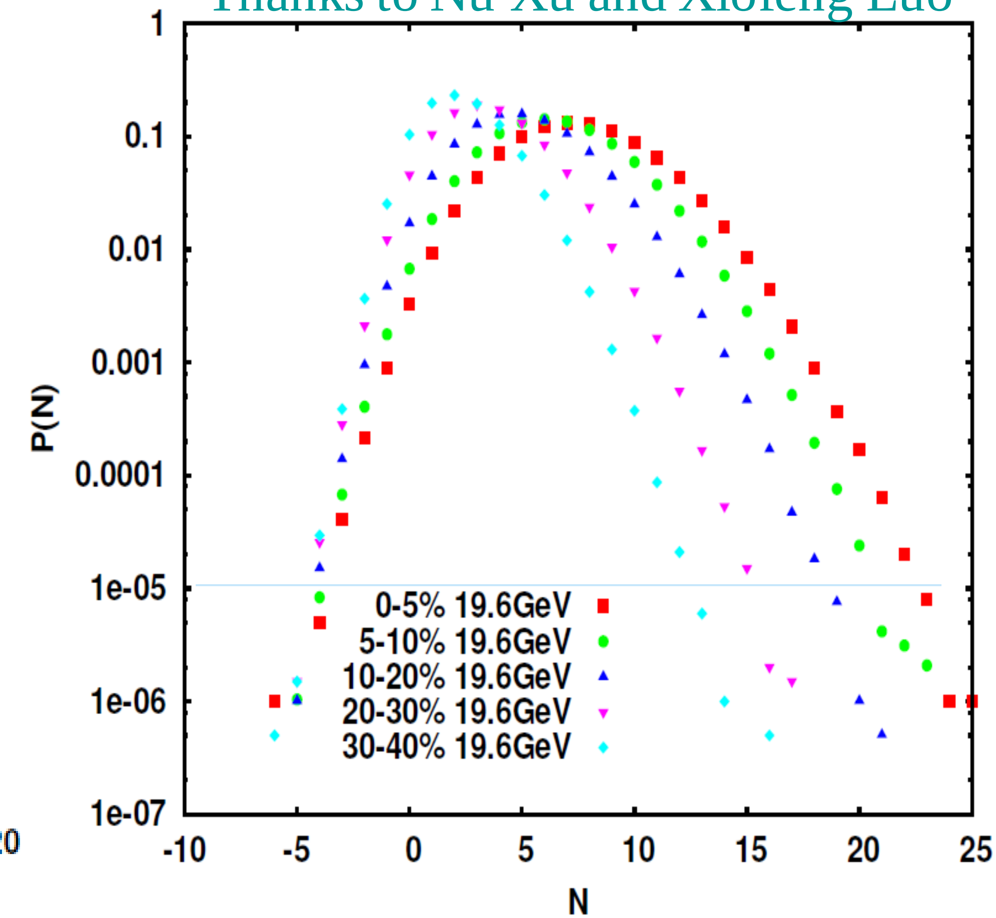


- Asymmetric P(N)
- Near $T_{pc}(\mu)$ the ratios less than unity for $N > \langle N \rangle$
- For sufficiently large μ the $P^{FRG}(N) / P^{Skellam(N)} > 1$ for $N < \langle N \rangle$

Probability distribution of net proton number STAR Coll. data at RHIC



Thanks to Nu Xu and Xiofeng Luo

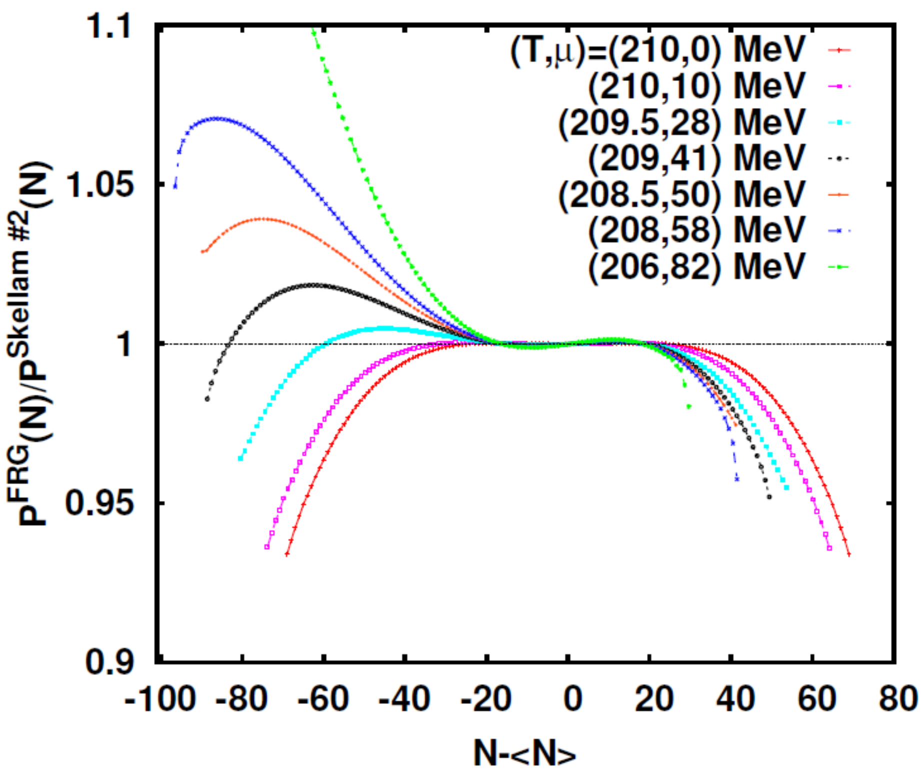


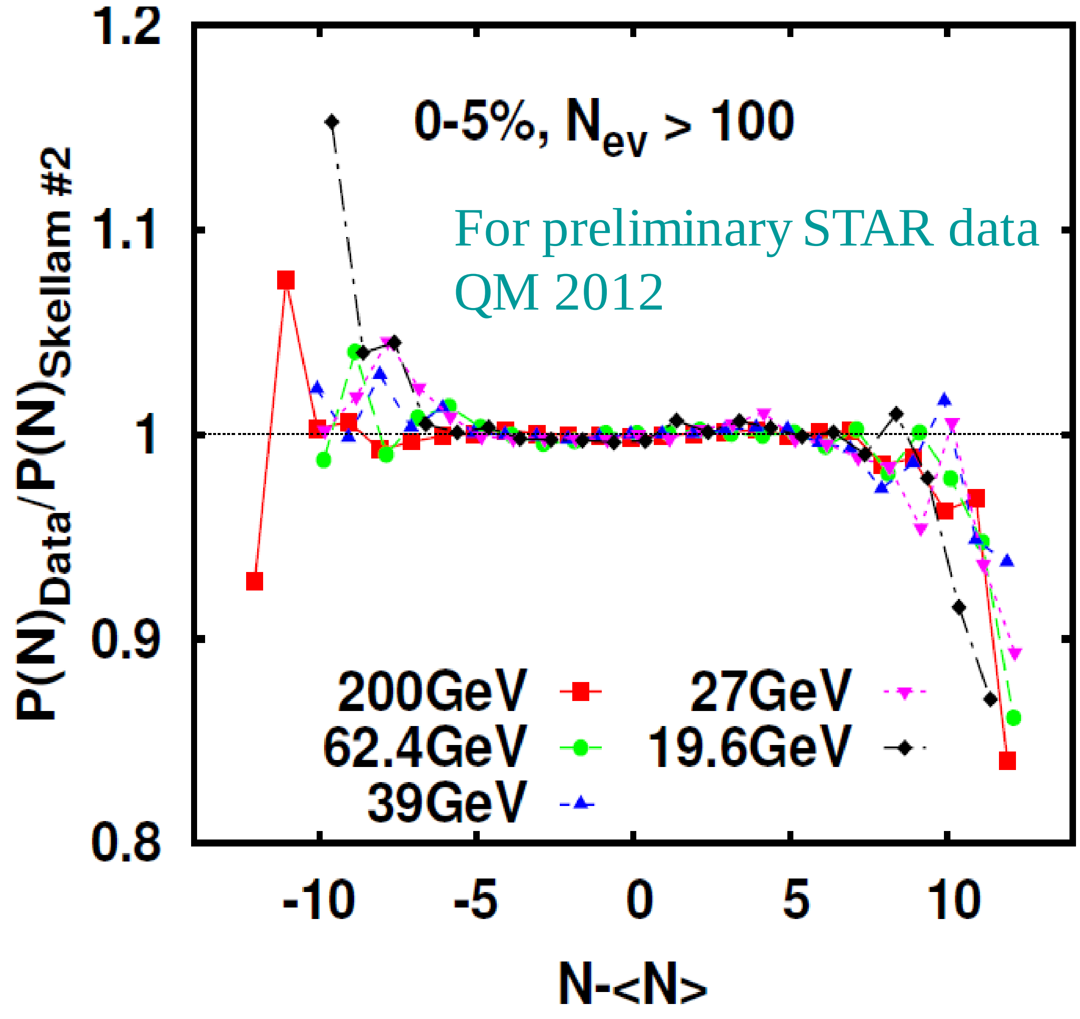
Do we also see the O(4) critical structure in these probability distributions ?

The influence of O(4) criticality on P(N) for $\mu \neq 0$

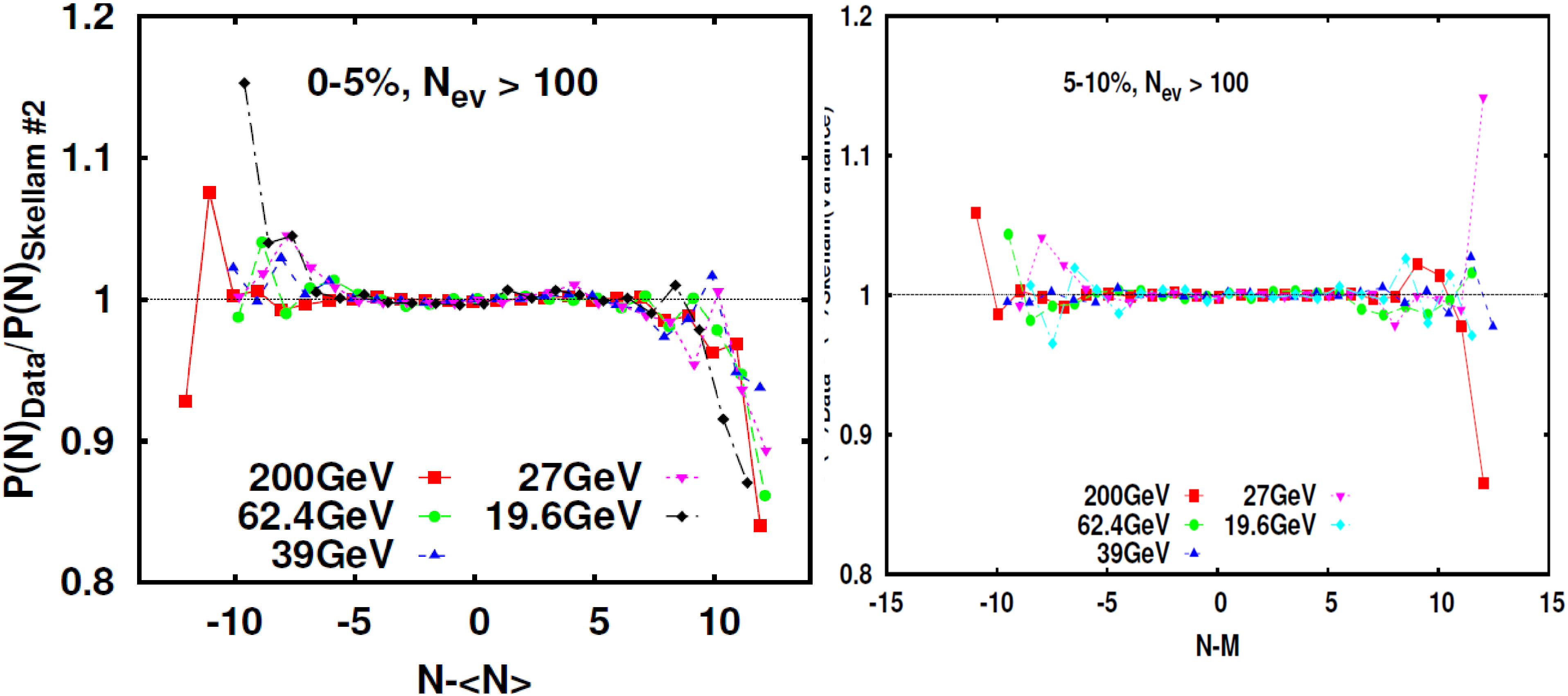
- In central collisions the probability behaves as being influenced by the chiral transition

K. Morita, B. Friman & K.R.





Energy dependence for different centralities



- Ratios at central collisions show properties expected near the O(4) chiral pseudocritical line
- For less central collisions the critical structure is lost

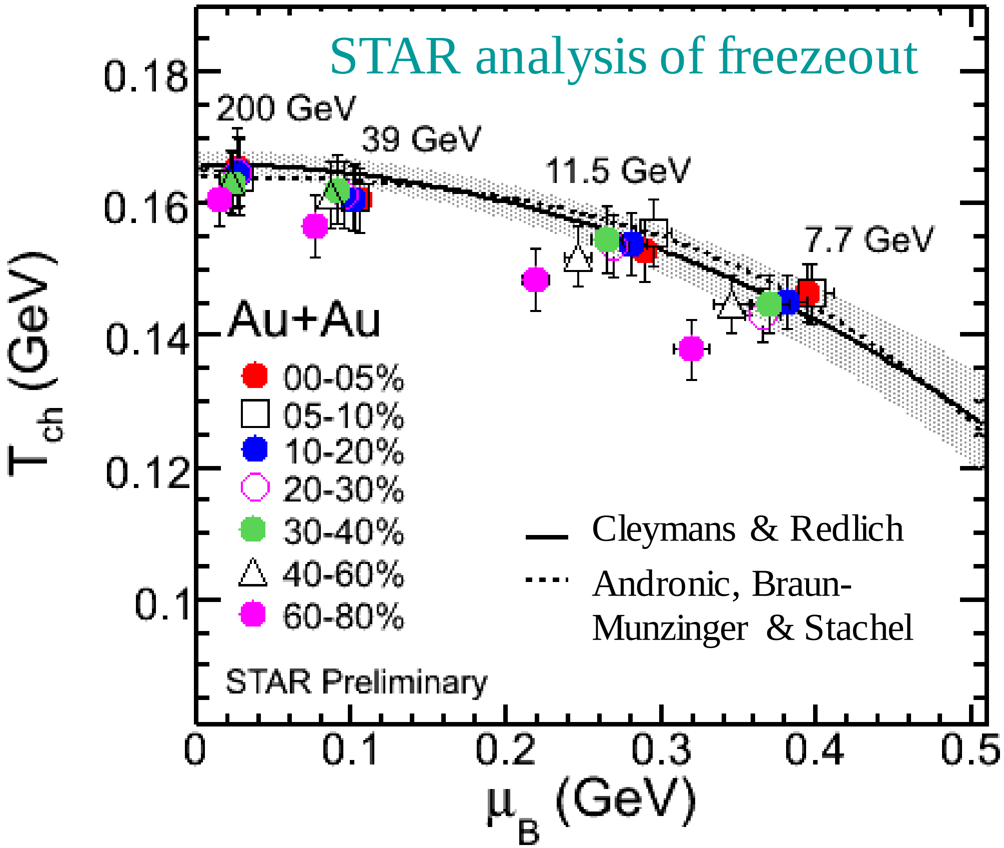
Conclusions:

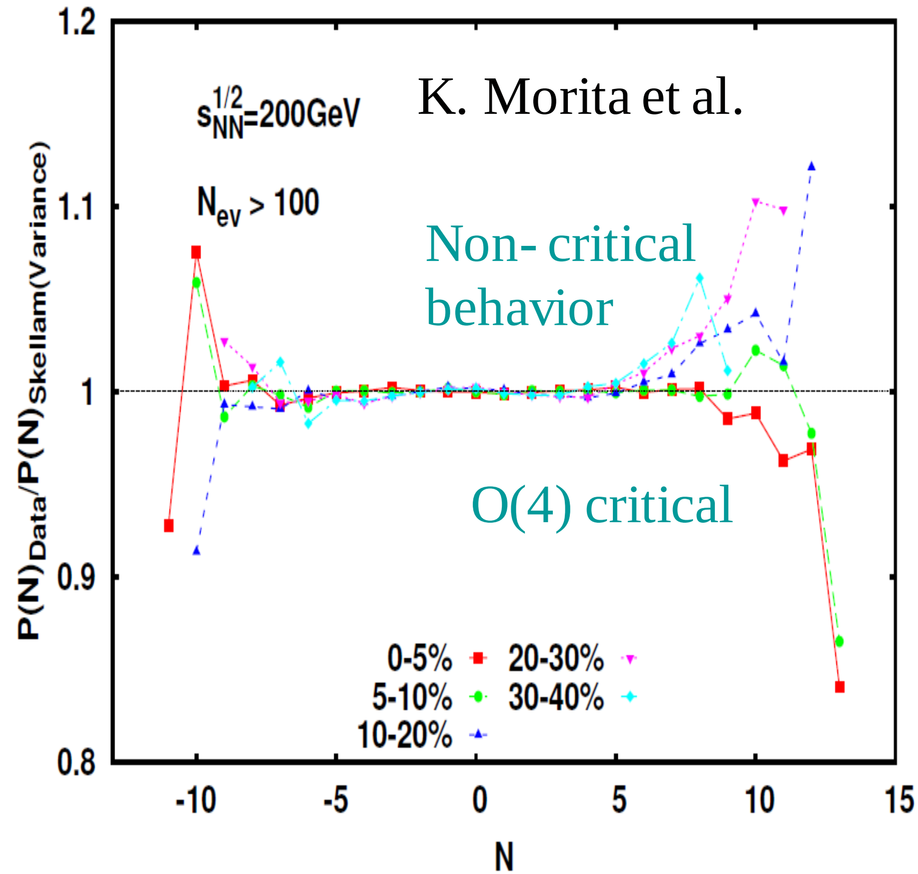
- Ratios of the Polyakov loop and the net charge susceptibilities are excellent probes of deconfinement and/or the O(4) chiral crossover transition in QCD
- Systematics of the net-proton fluctuations and their probability distributions measured by STAR are qualitatively consistent with the expectations that they are influenced by the O(4) criticality.

However, other effects could possibly also influence data:

- Exact charge conservation (Koch, Bzdak, Skokov)
- Acceptance corrections (Bzdak & Koch)
- Effects of final state interactions (Ono, Asakawa & Kitazawa)
- Non-equilibrium effects (Kitazawa, Asakawa & Ono)
- Volume fluctuations (Friman, Skokov & K.R.)
- Etc.

Centrality dependence of probability ratio





- For less central collisions, the freezeout appears away the pseudocritical line, resulting in an absence of the O(4) critical structure in the probability ratio.