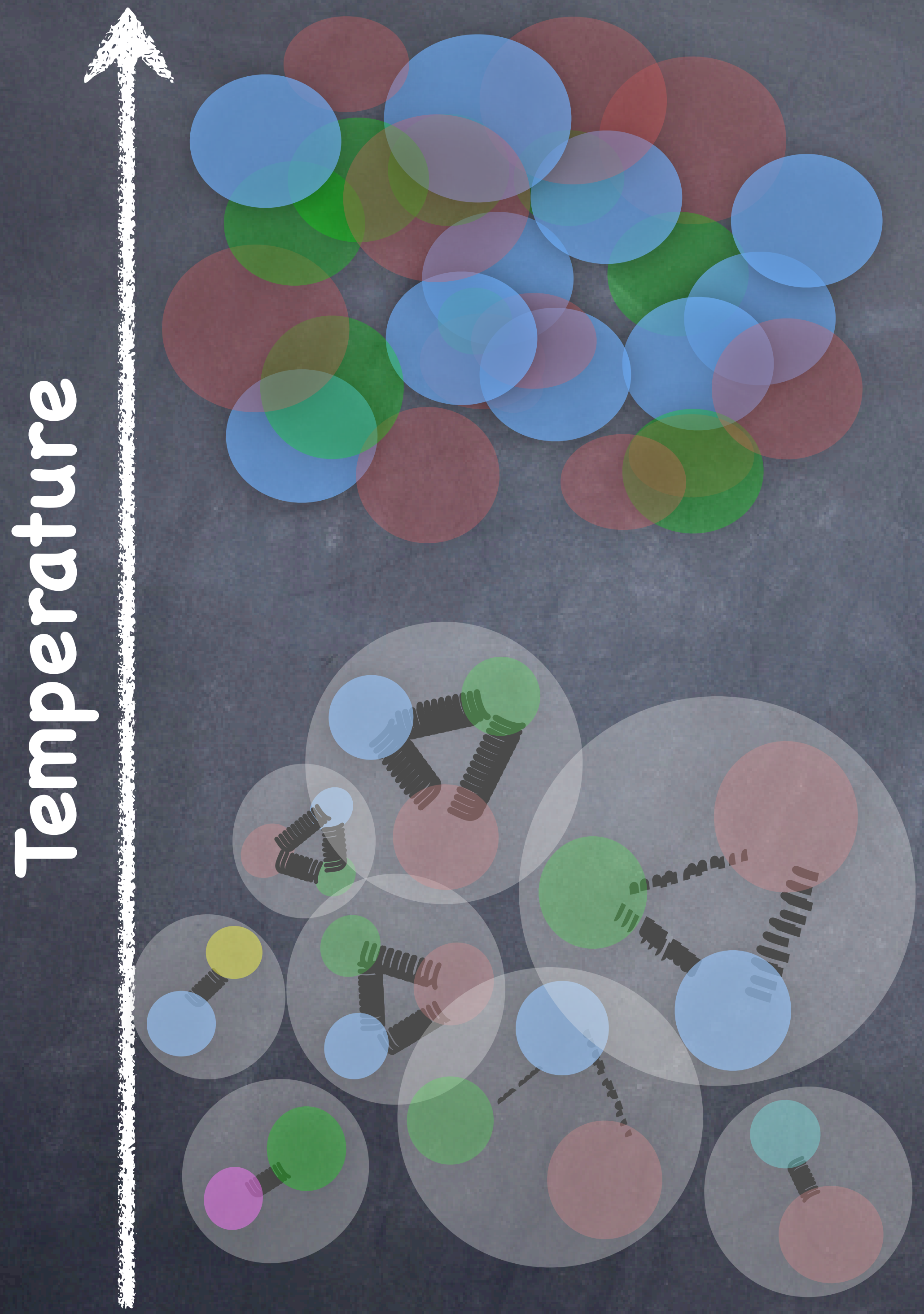
Quark-hadron phase transition in a three flavor PNJL model

Kanako Yamazaki
University of Tokyo

1. **Motivations : quark-hadron transition**
2. **3-flavor Nambu-Jona-Lasinio Model with Polyakov Loop**
3. **Pseudo-Scalar Mesons and Scalar Mesons**
4. **Equation of state**

in collaboration with **Tetsuo Matsui**

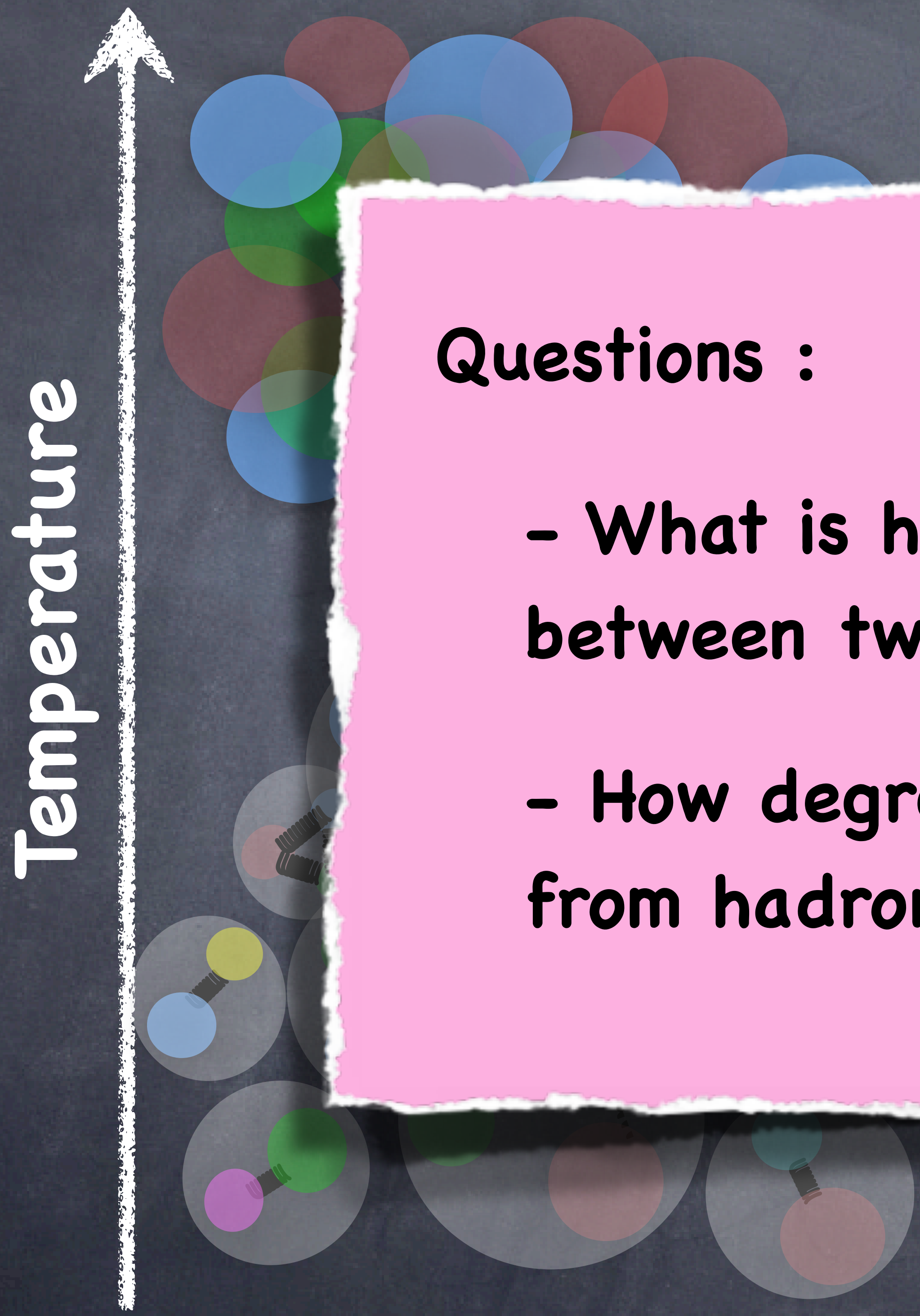
Quark-Hadron Phase Transition



- Chiral symmetry **restoration**
- Color **de-confinement**

- Chiral symmetry **breaking**
- Color **confinement**

Quark-Hadron Phase Transition



- Chiral symmetry restoration

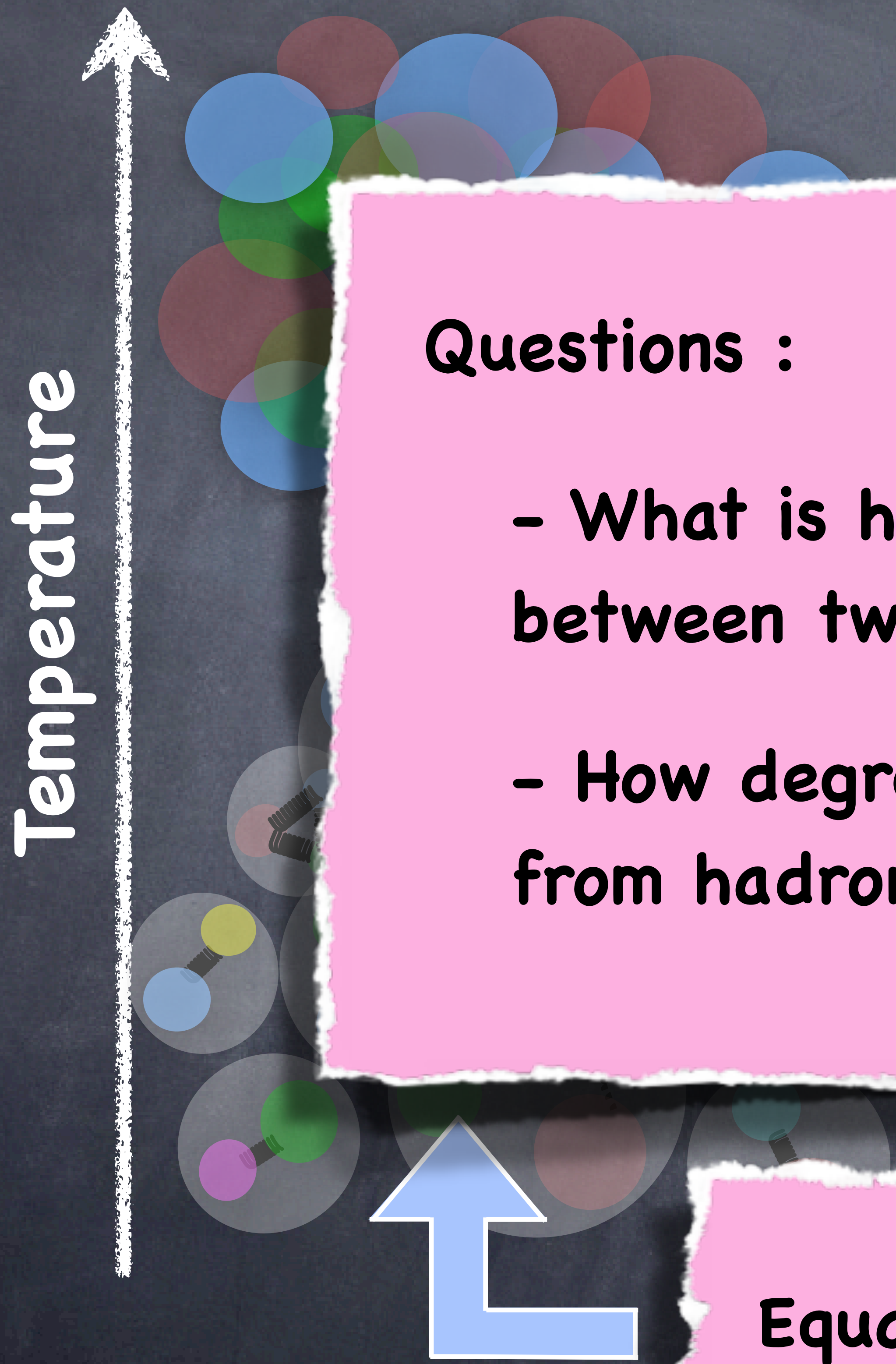
king

- Color confinement

Questions :

- What is happening in middle region between two phases ?
- How degrees of freedom change from hadrons to quarks ?

Quark-Hadron Phase Transition



- Chiral symmetry restoration

king

- Color confinement

Questions :

- What is happening in middle region between two phases ?
- How degrees of freedom change from hadrons to quarks ?

Equation of state

Method

- Calculating partition function in path integral method

- Model choosing
  - Chiral phase transition
    - → **Nambu-Jona-Lasinio** (NJL)
  - De-confining phase transition
    - → **Polyakov loop**

PNJL model

K. Fukushima, 2004

- Bosonization
  - inserting **dummy integrals**
  - 4- and 6-point interactions --> **bosonic fields**

- Mean field approximation + **Mesonic correlations**

3 flavor PNJL model

Partition function

$$Z(T, A_4) = \int [dq][d\bar{q}] \exp\left[\int_0^\beta d\tau \int d^3x \mathcal{L}_{\mathrm{NJL}}(q, \bar{q}, A_4)\right]$$

$$\mathcal{L}_{NJL} = \sum_{i,j=1}^{3} \bar{q}_i (i\not{D} - \hat{m})_{i,j} q_j + \boxed{\mathcal{L}_4} + \boxed{\mathcal{L}_6}$$

$$D_\mu = \partial_\mu + g \boxed{A_0} \delta_{\mu,0}$$

3 flavor PNJL model

Partition function

$$Z(T, A_4) = \int [dq][d\bar{q}] \exp\left[\int_0^\beta d\tau \int d^3x \mathcal{L}_{\mathrm{NJL}}(q, \bar{q}, A_4)\right]$$

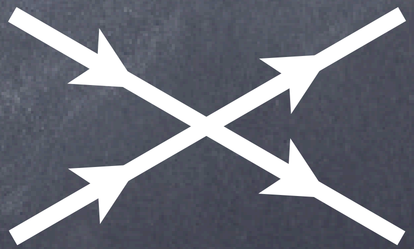
$$\mathcal{L}_{NJL} = \sum_{i,j=1}^{3} \bar{q}_i (i\not{D} - \hat{m})_{i,j} q_j + \boxed{\mathcal{L}_4} + \boxed{\mathcal{L}_6}$$

$$D_\mu = \partial_\mu + g \boxed{A_0} \delta_{\mu,0}$$

- 4 point interaction

$$\boxed{\mathcal{L}_4 = G \sum_{a=0}^{8} \left[(\bar{q}\lambda^a q)^2 + (\bar{q} i \gamma_5 \lambda^a q)^2 \right]}$$

$a = 0 \sim 8$



3 flavor PNJL model

Partition function

$$Z(T,A_4) = \int [dq][d\bar{q}]\exp\left[\int_0^\beta d\tau \int d^3x \mathcal{L}_{\rm NJL}(q,\bar{q},A_4)\right]$$

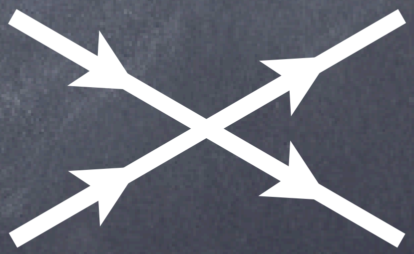
$$\mathcal{L}_{NJL} = \sum_{i,j=1}^{3} \bar{q}_i (i\not{D} - \hat{m})_{i,j} q_j + \boxed{\mathcal{L}_4} + \boxed{\mathcal{L}_6}$$

$$D_\mu = \partial_\mu + g\boxed{A_0}\delta_{\mu,0}$$

- 4 point interaction

$$\boxed{\mathcal{L}_4 = G\sum_{a=0}^{8}\left[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2\right]}$$

$$a = 0 \sim 8$$



- 6 point interaction

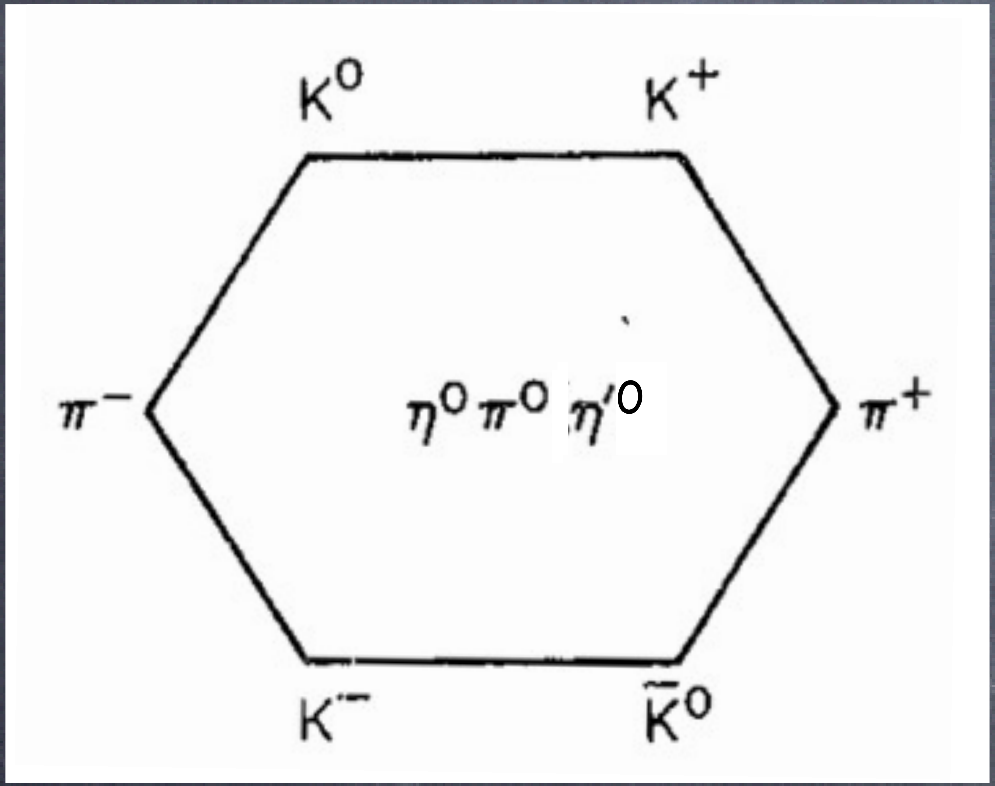
U(1)A breaking

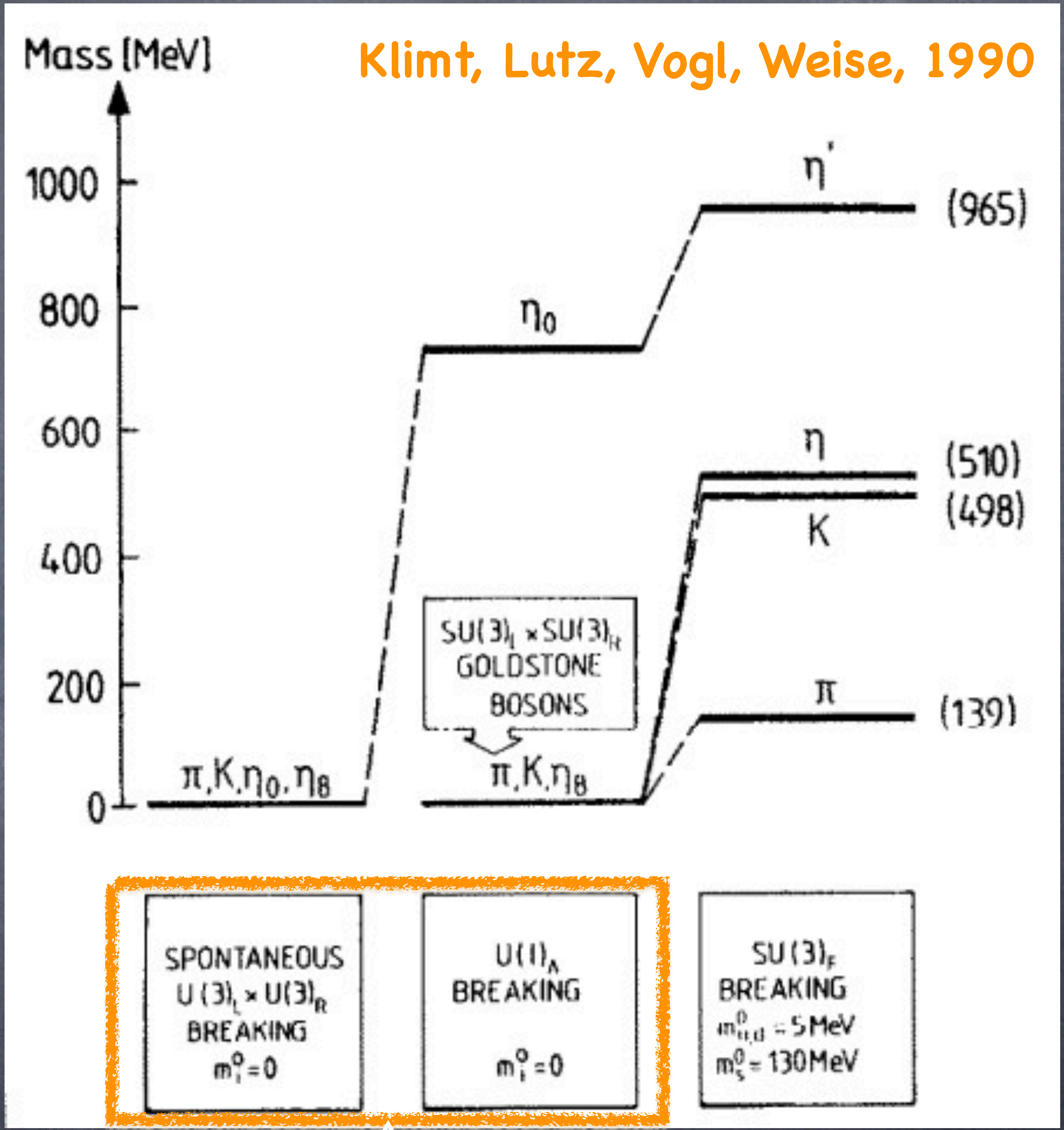
$$\boxed{\mathcal{L}_6 = -K\left[\det \bar{q}(1+\gamma_5)q + \det \bar{q}(1-\gamma_5)q\right]}$$

Meson nonets

Pseudo scalar mesons

π, K, η, η′




Klimt, Lutz, Vogl, Weise, 1990

$\mathcal{L}_6$ plays a role in the mass splitting

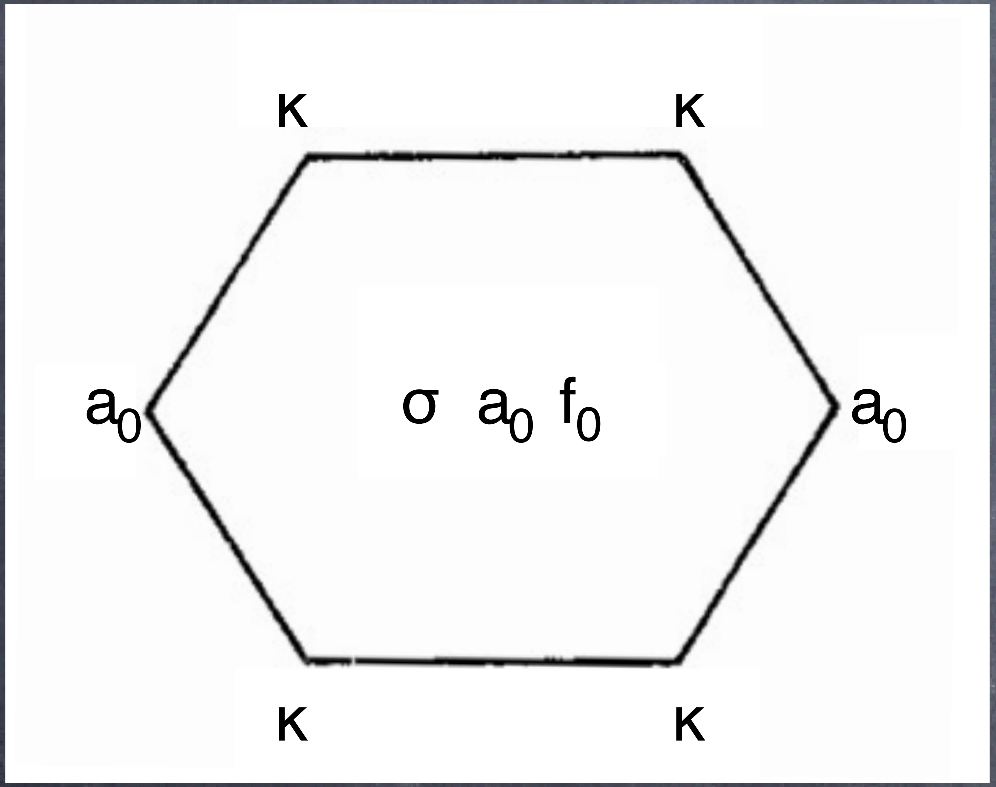
Meson nonets

Scalar mesons

σ, κ, f0, a0

Ishida, 1998

Fariborz, Jora, Schechter, 2009



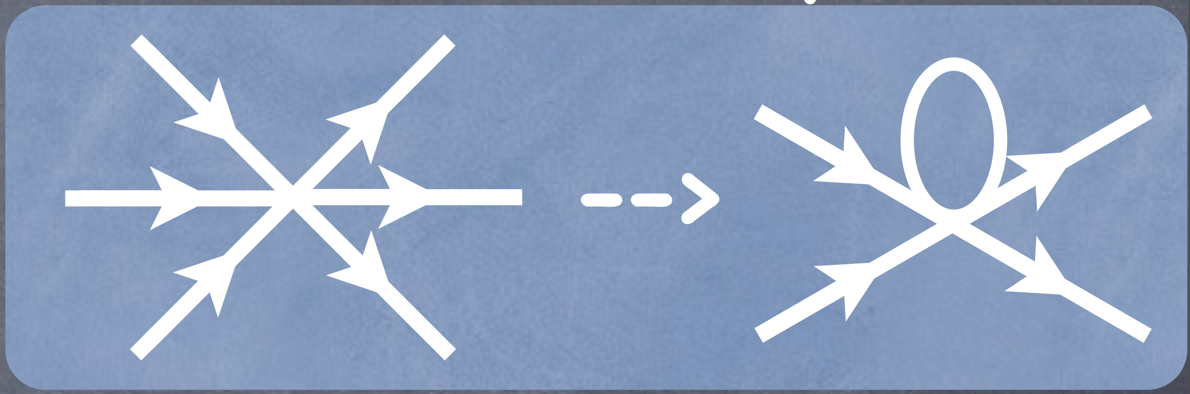
?

	Mass[MeV]	Width[MeV]
σ	∼550	400 - 700
κ	∼800	
$f_0(980)$	∼980	40 - 100
$a_0(980)$	∼980	50 - 100

Rewrite P.F. by auxiliary fields

- Lagrangian contains **4th power** and **6th power** of fermion fields.

- 6th power can be effectively rewritten to 4th power by **replacing with condensate**.



- eliminating 4point interactions to bosonic fields : ϕ^a, π^a

- We get partition function **as a function of auxiliary bosonic fields** :

$$Z(T, A_4) = \int \underline{[d\phi][d\pi]} \exp\Big[-I_{eff}(\underline{\phi^a}, \underline{\pi^a}, A_4)\Big]$$

Thermodynamic potential

- Expanding effective action up to **second order of fluctuation** around **stationary point**

- Stationary point is determined by **stationary condition** :

$$\left.\frac{\delta I}{\delta \phi_a}\right|_{\phi=\phi_0} = 0.$$

- Performing **Gaussian integrals** over bosonic fields

- Thermodynamic potential

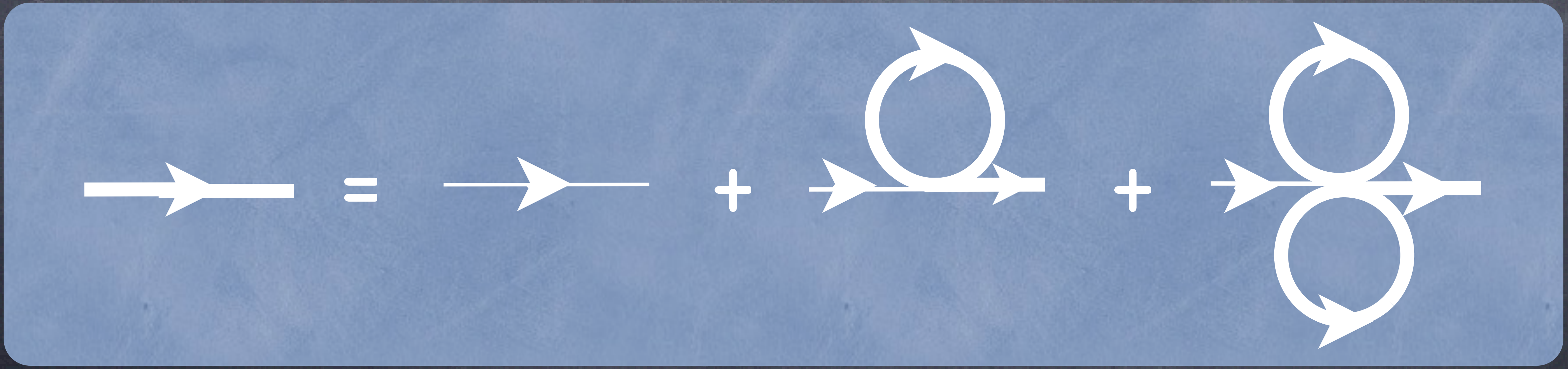
$$\Omega(T, A_4) = T\Big(\underbrace{I_0}_{\text{mean field}} + \underbrace{\frac{1}{2}\mathrm{Tr}_M \ln \frac{\delta^2 I}{\delta\phi_a \delta\phi_b} + \frac{1}{2}\mathrm{Tr}_M \ln \frac{\delta^2 I}{\delta\pi_a \delta\pi_b}}_{\text{mesonic excitations}} \Big)$$

Constituent quark mass

- Pressure depends on **constituent quark masses**.

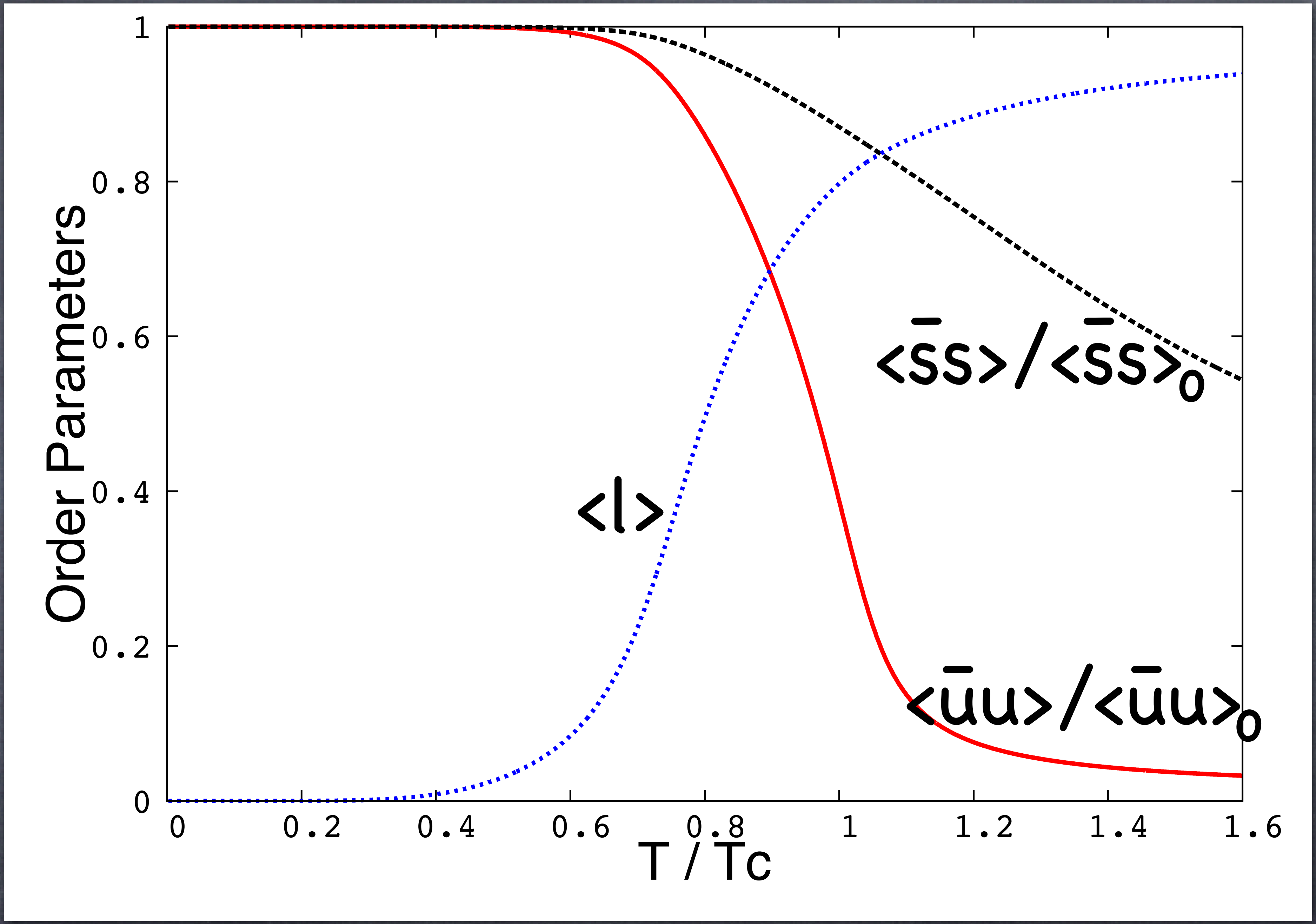
- Constituent quark masses are determined by solving **gap equations** :

$$\begin{cases} \boxed{M_u = m_u - 4G\langle\bar{u}u\rangle + 2K\langle\bar{d}d\rangle\langle\bar{s}s\rangle} \\ \boxed{M_s = m_s - 4G\langle\bar{s}s\rangle + 2K\langle\bar{u}u\rangle\langle\bar{d}d\rangle} \end{cases}$$



- Chiral condensates : $\boxed{\langle\bar{u}u\rangle(=\langle\bar{d}d\rangle),\langle\bar{s}s\rangle}$

Order Parameters



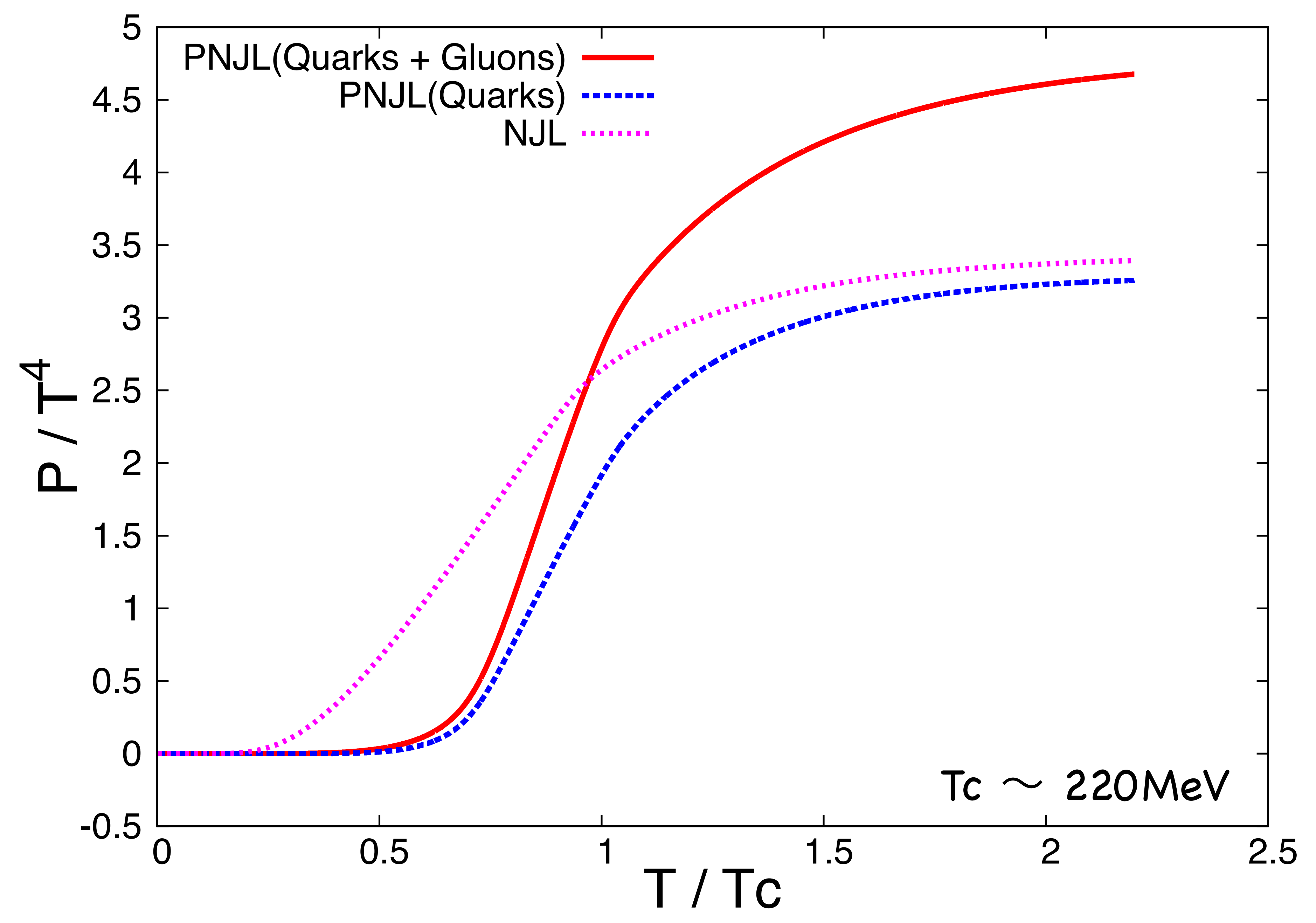
$\langle l\rangle$: Expectation value of Polyakov loop

Tc : pseudo critical temperature

Tc ~ 220MeV

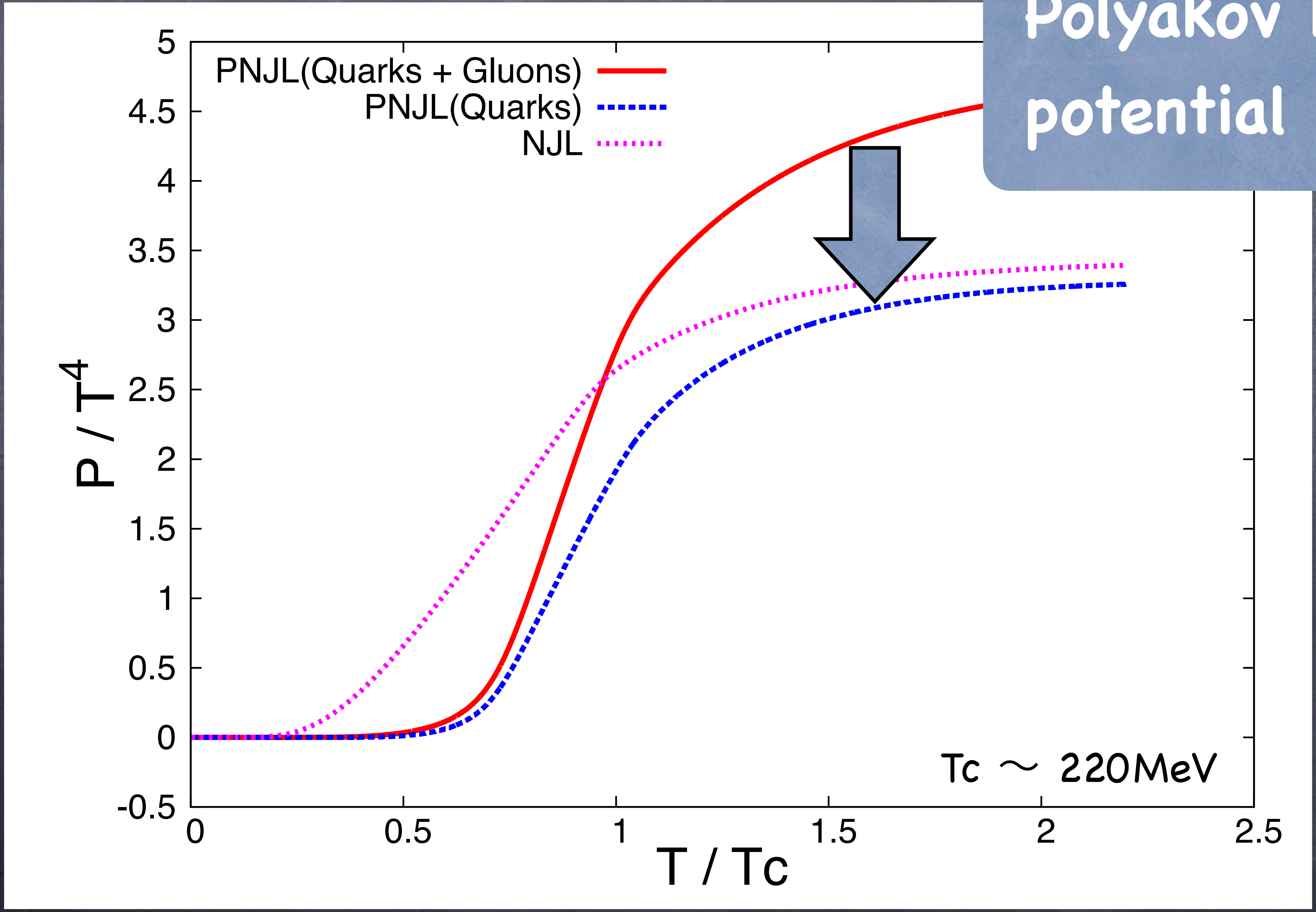
Pressure under the MFA

$$p_{MF}(T) = \sum_f p^0_{M_f} + 4N_c \sum_f \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_f} f_{\langle l \rangle}(E_f) - \mathcal{U}(T)$$



Pressure under the MFA

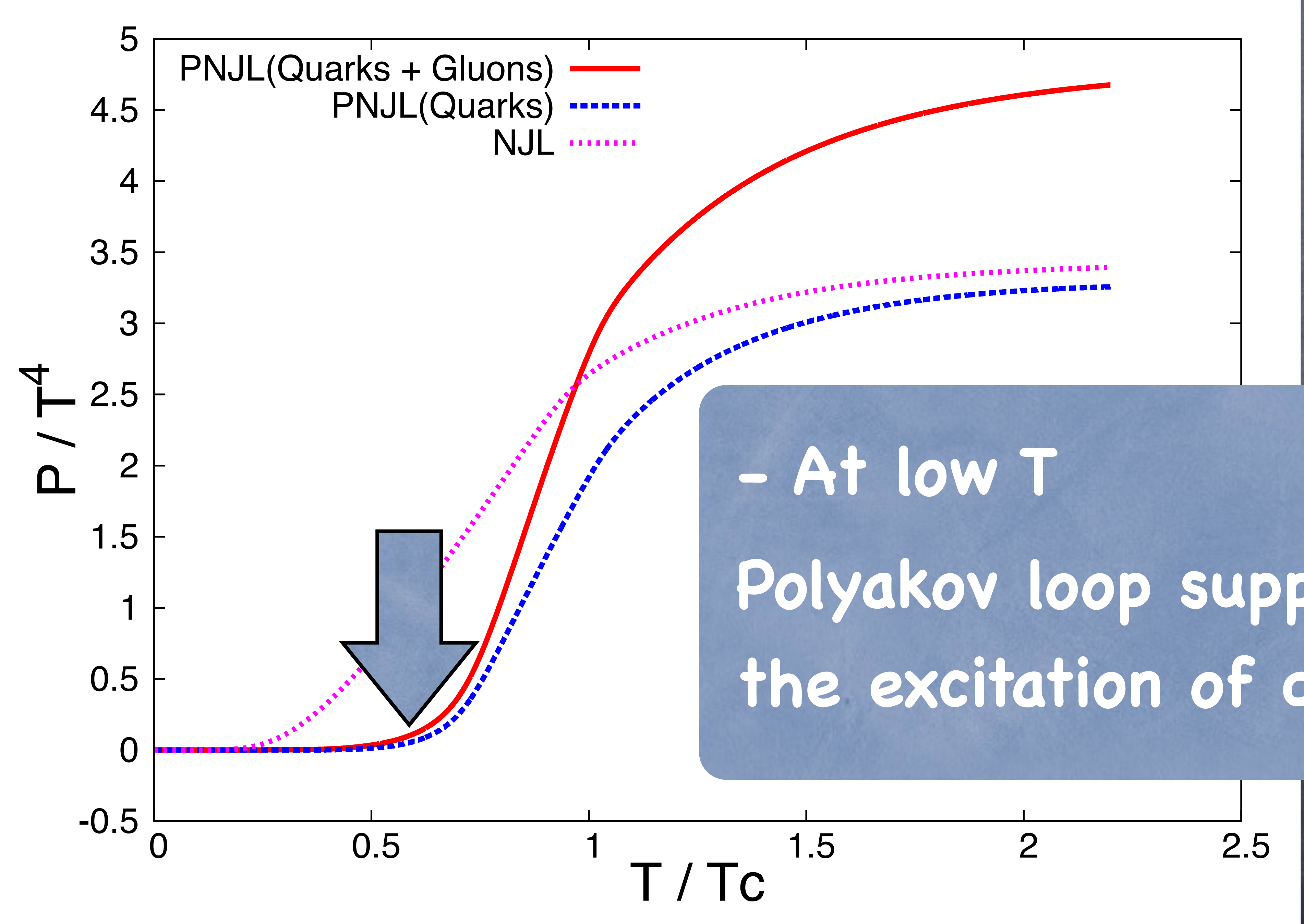
$$p_{MF}(T) = \sum_f p^0_{M_f} + 4N_c \sum_f \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_f} f_{\langle l \rangle}(E_f) - \mathcal{U}(T)$$



Polyakov loop potential : $\mathcal{U}(T)$

Pressure under the MFA

$$p_{MF}(T) = \sum_f p^0_{M_f} + 4N_c \sum_f \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_f} f_{\langle l \rangle}(E_f) - \mathcal{U}(T)$$

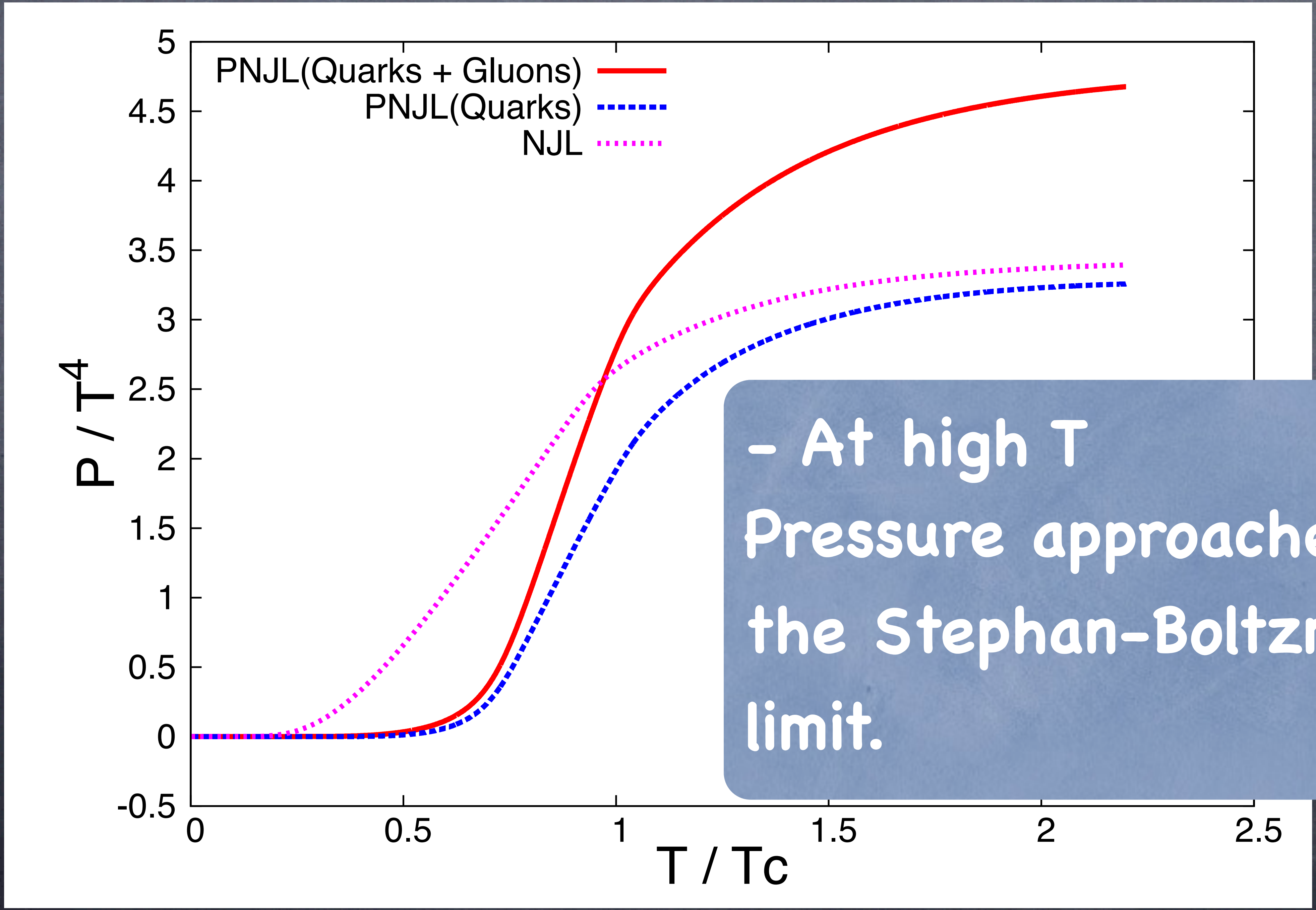


- At low T

Polyakov loop suppresses the excitation of quarks.

Pressure under the MFA

$$p_{MF}(T) = \sum_f p^0_{M_f} + 4N_c \sum_f \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_f} f_{\langle l \rangle}(E_f) - \mathcal{U}(T)$$



- At high T
Pressure approaches the Stephan-Boltzmann limit.

Mesonic Correlations

- Pressure of mesonic correlation is given as sum of contributions of each meson.

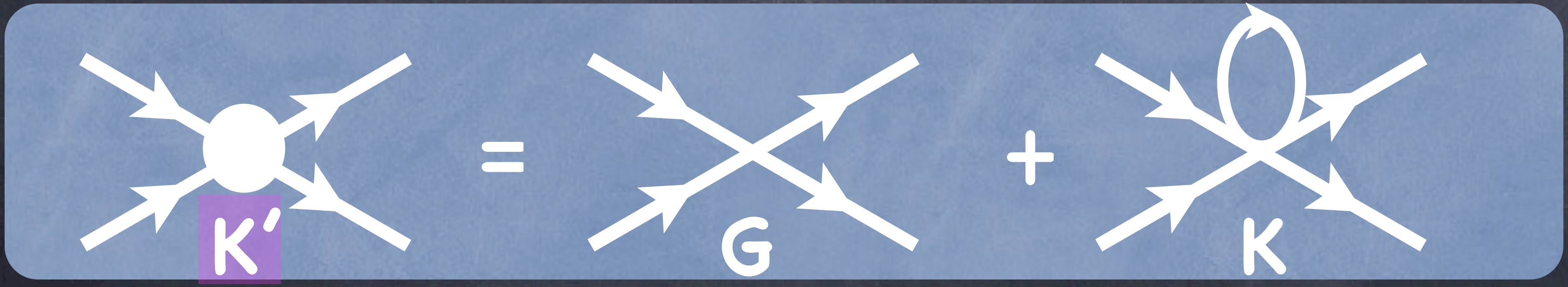
$$p_M = -\sum_n \int \frac{d^3q}{(2\pi)^3} \Big\{ 3\ln\mathcal{M}_\pi(\omega_n, q) + 4\ln\mathcal{M}_K(\omega_n, q) + \ln\mathcal{M}_\eta(\omega_n, q) + \ln\mathcal{M}_{\eta'}(\omega_n, q)$$

$$+\ln\mathcal{M}_\sigma(\omega_n, q) + 4\ln\mathcal{M}_\kappa(\omega_n, q) + 3\ln\mathcal{M}_{a_0}(\omega_n, q) + \ln\mathcal{M}_{f_0}(\omega_n, q) \Big\}$$

$$\mathcal{M}(\omega_n, q) = \frac{1}{2K'} - \Pi(\omega_n, q)$$

$\Pi(\omega_n, q)$ =

- K' is effective coupling, combination of G and K :



Mesonic Correlations

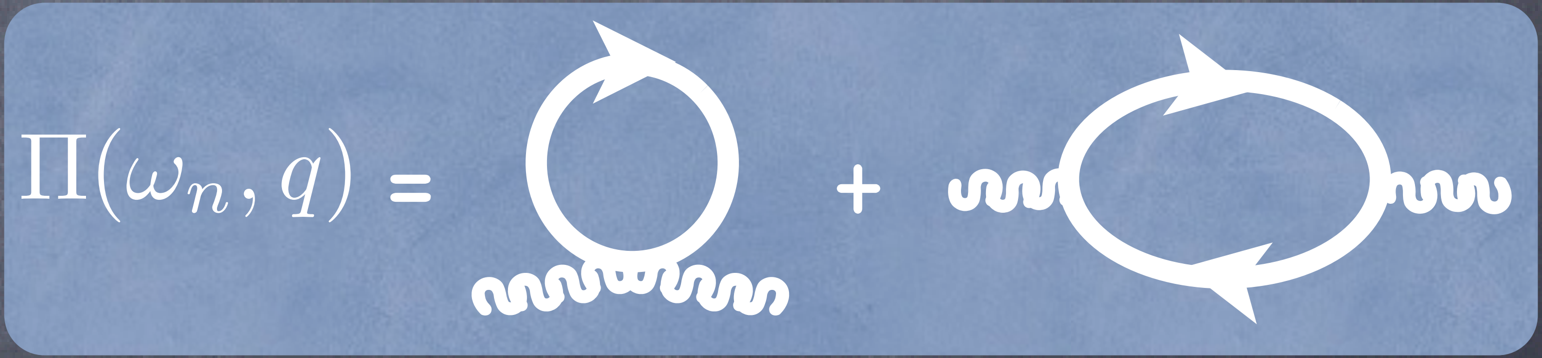
- Pressure of mesonic correlation is given as sum of contributions of each meson.

$$p_M = -\sum_n \int \frac{d^3q}{(2\pi)^3}\Big\{3\ln\mathcal{M}_\pi(\omega_n,q) + 4\ln\mathcal{M}_K(\omega_n,q) + \ln\mathcal{M}_\eta(\omega_n,q) + \ln\mathcal{M}_{\eta'}(\omega_n,q)$$

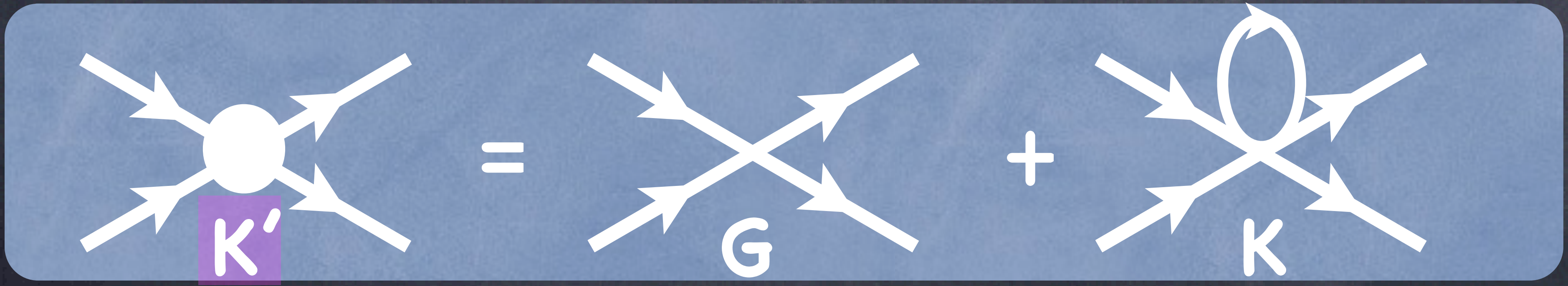
$$+\ln\mathcal{M}_\sigma(\omega_n,q) + 4\ln\mathcal{M}_\kappa(\omega_n,q) + 3\ln\mathcal{M}_{a_0}(\omega_n,q) + \ln\mathcal{M}_{f_0}(\omega_n,q)\Big\}$$

Pseudo scalar

$$\mathcal{M}(\omega_n,q) = \frac{1}{2K'} - \Pi(\omega_n,q)$$



- **K'** is effective coupling, combination of G and K :



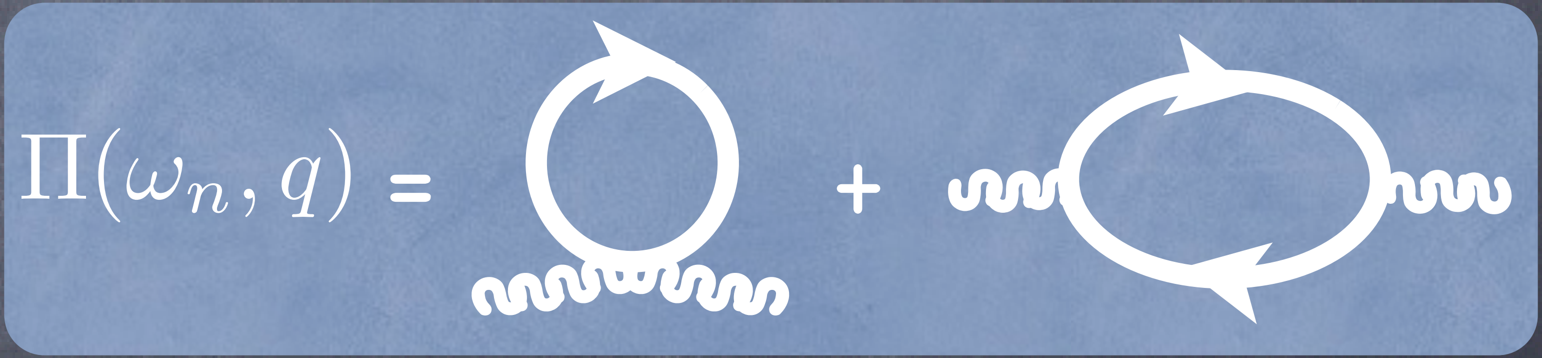
Mesonic Correlations

- Pressure of mesonic correlation is given as sum of contributions of each meson.

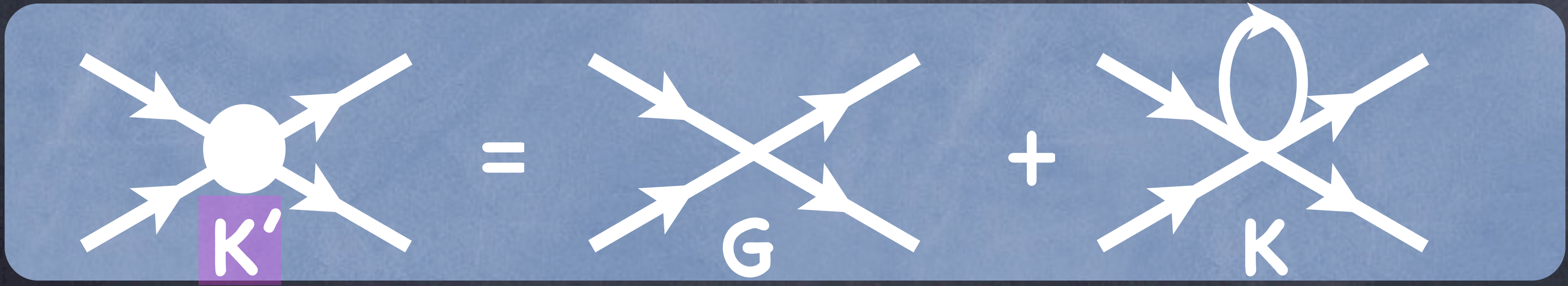
Scalar

$$p_M = -\sum_n \int \frac{d^3q}{(2\pi)^3}\Big\{3\ln\mathcal{M}_\pi(\omega_n,q) + 4\ln\mathcal{M}_K(\omega_n,q) + \ln\mathcal{M}_\eta(\omega_n,q) + \ln\mathcal{M}_{\eta'}(\omega_n,q)$$
$$+\ln\mathcal{M}_\sigma(\omega_n,q) + 4\ln\mathcal{M}_\kappa(\omega_n,q) + 3\ln\mathcal{M}_{a_0}(\omega_n,q) + \ln\mathcal{M}_{f_0}(\omega_n,q)\Big\}$$

$$\mathcal{M}(\omega_n,q) = \frac{1}{2K'} - \Pi(\omega_n,q)$$



- K' is effective coupling, combination of G and K :



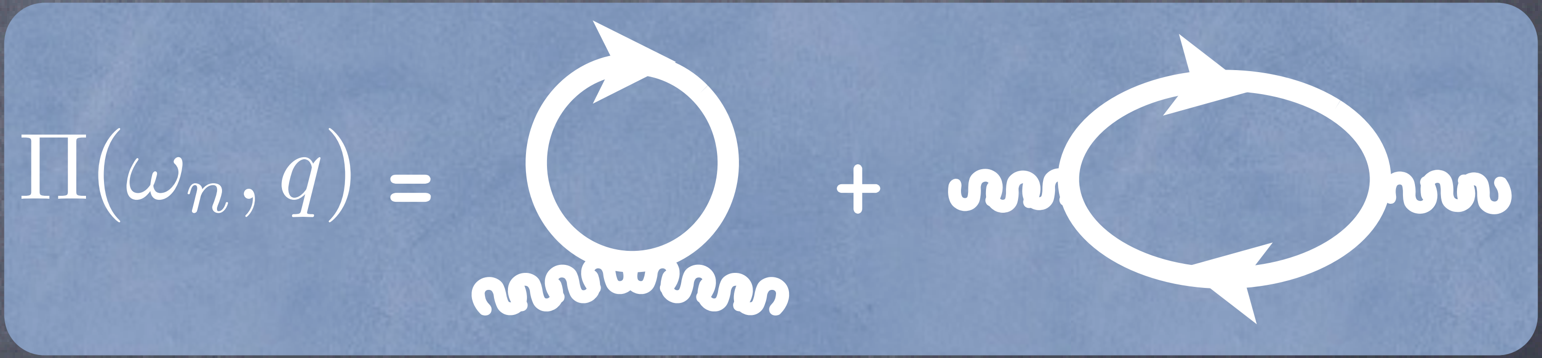
Mesonic Correlations

- Pressure of mesonic correlation is given as sum of contributions of each meson.

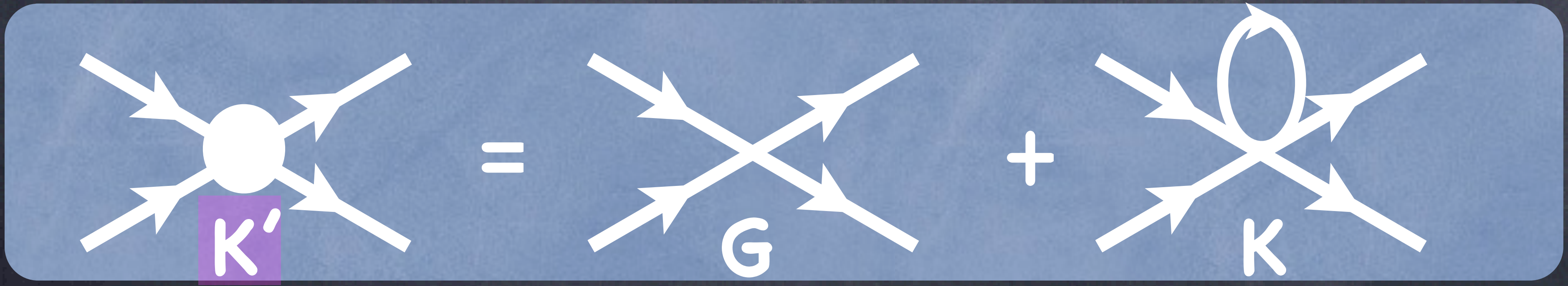
$$p_M = -\sum_n \int \frac{d^3q}{(2\pi)^3} \Big\{ 3\ln\mathcal{M}_\pi(\omega_n, q) + 4\ln\mathcal{M}_K(\omega_n, q) + \ln\mathcal{M}_\eta(\omega_n, q) + \ln\mathcal{M}_{\eta'}(\omega_n, q)$$

$$+\ln\mathcal{M}_\sigma(\omega_n, q) + 4\ln\mathcal{M}_\kappa(\omega_n, q) + 3\ln\mathcal{M}_{a_0}(\omega_n, q) + \ln\mathcal{M}_{f_0}(\omega_n, q) \Big\}$$

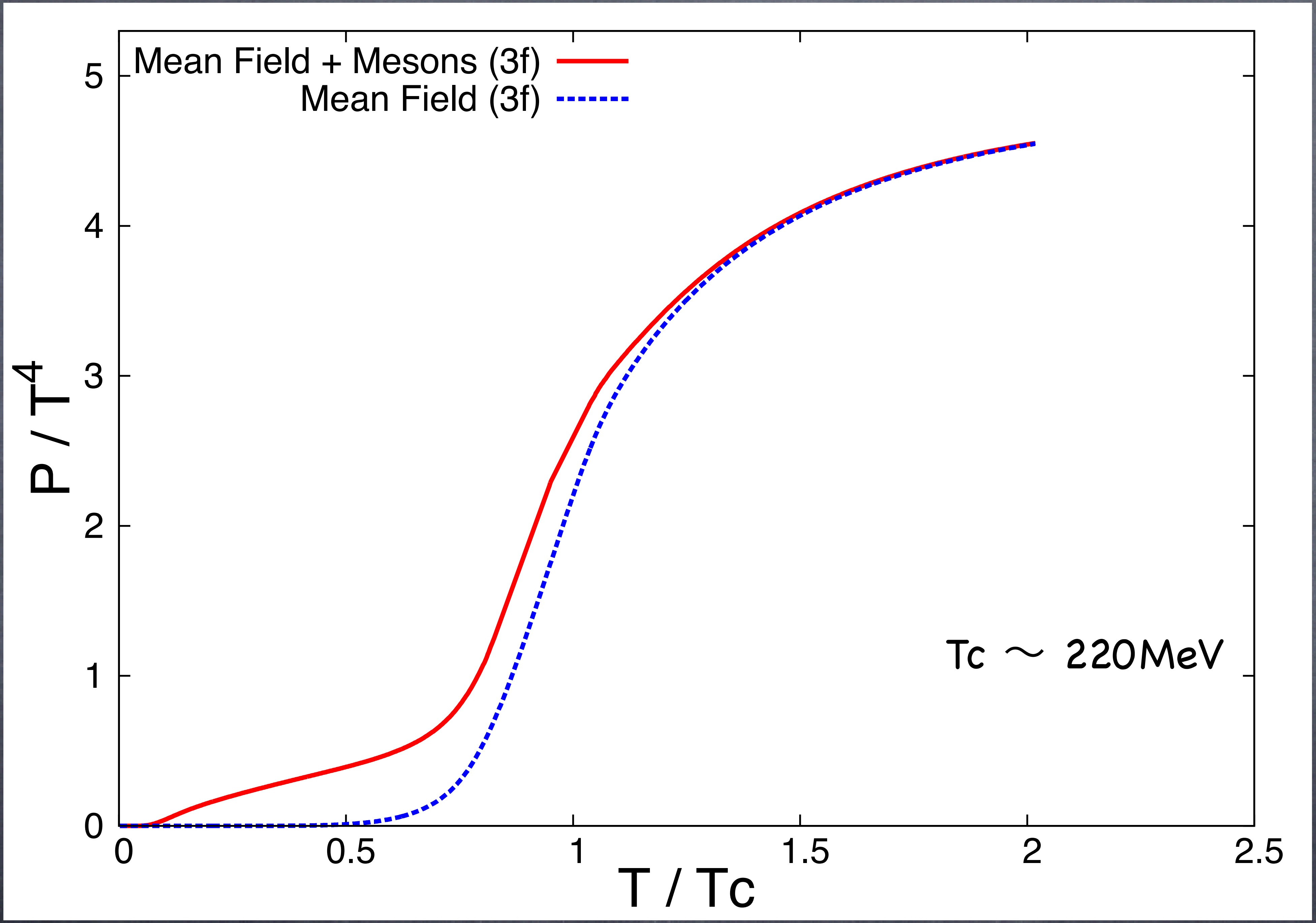
$$\mathcal{M}(\omega_n, q) = \frac{1}{2K'} - \Pi(\omega_n, q)$$



- K' is effective coupling, combination of G and K :

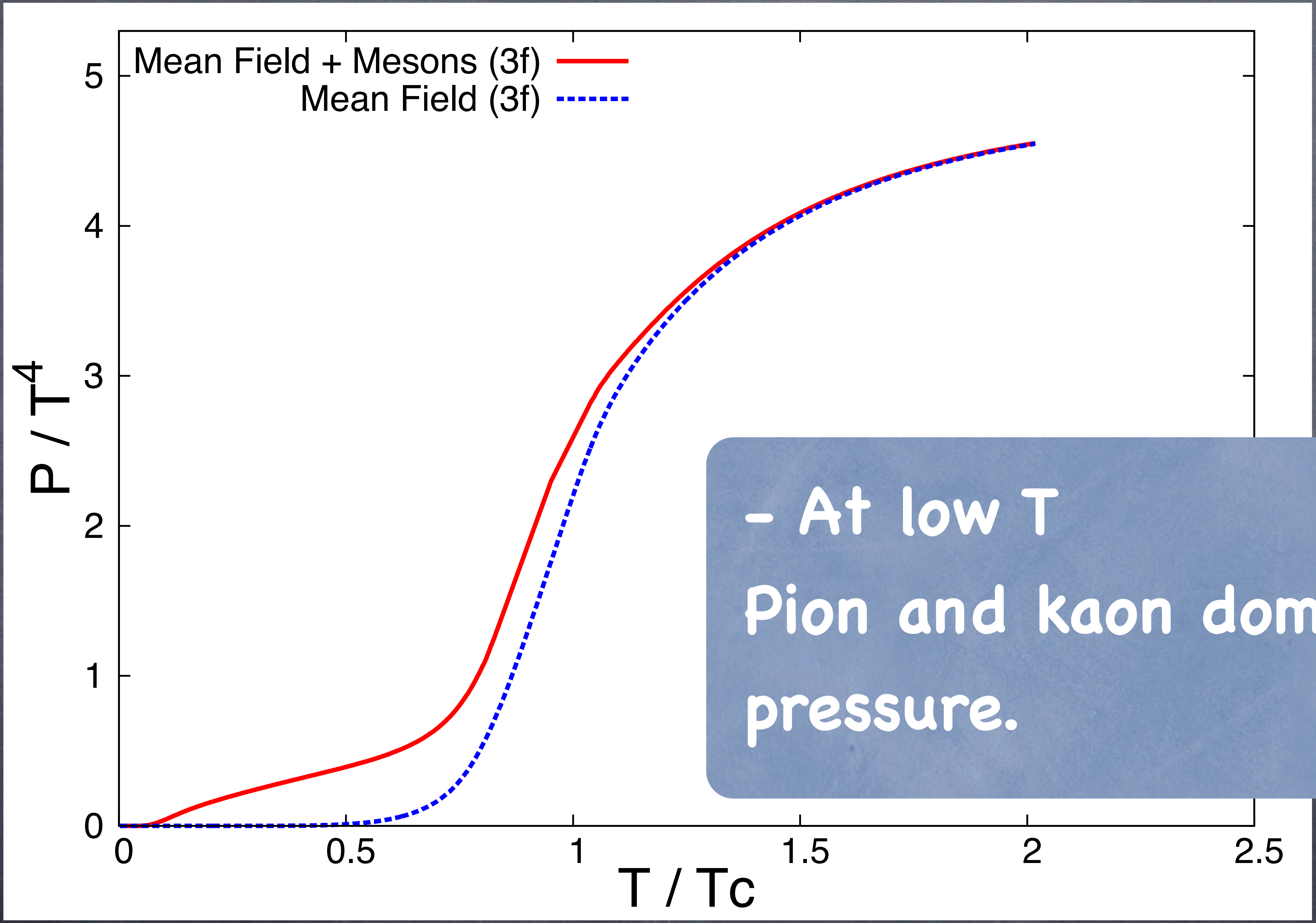


Pressure



π, K and σ are taken into this calculation.

Pressure

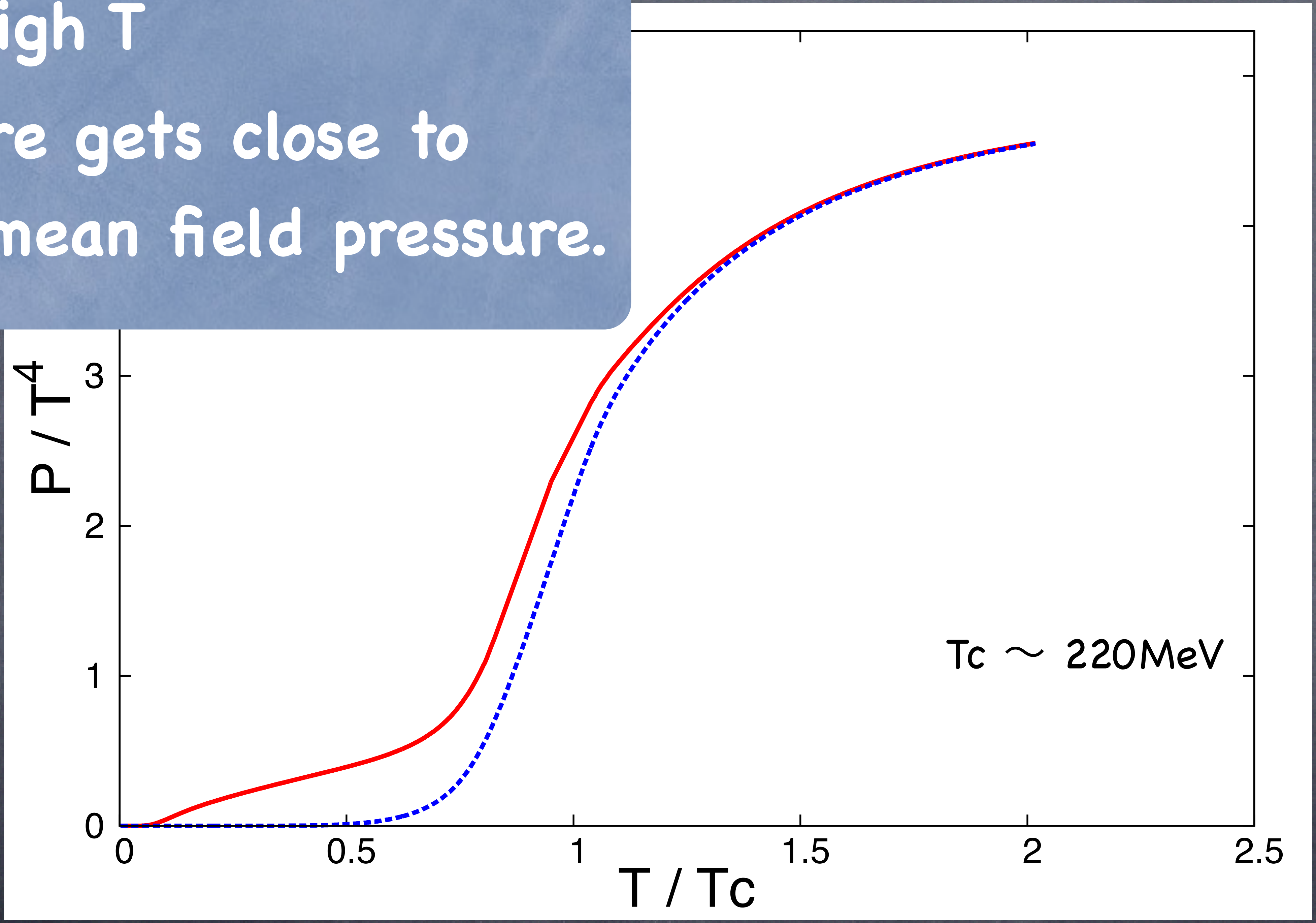


- At low T

Pion and kaon dominate pressure.

π, K and σ are taken into this calculation.

Pressure

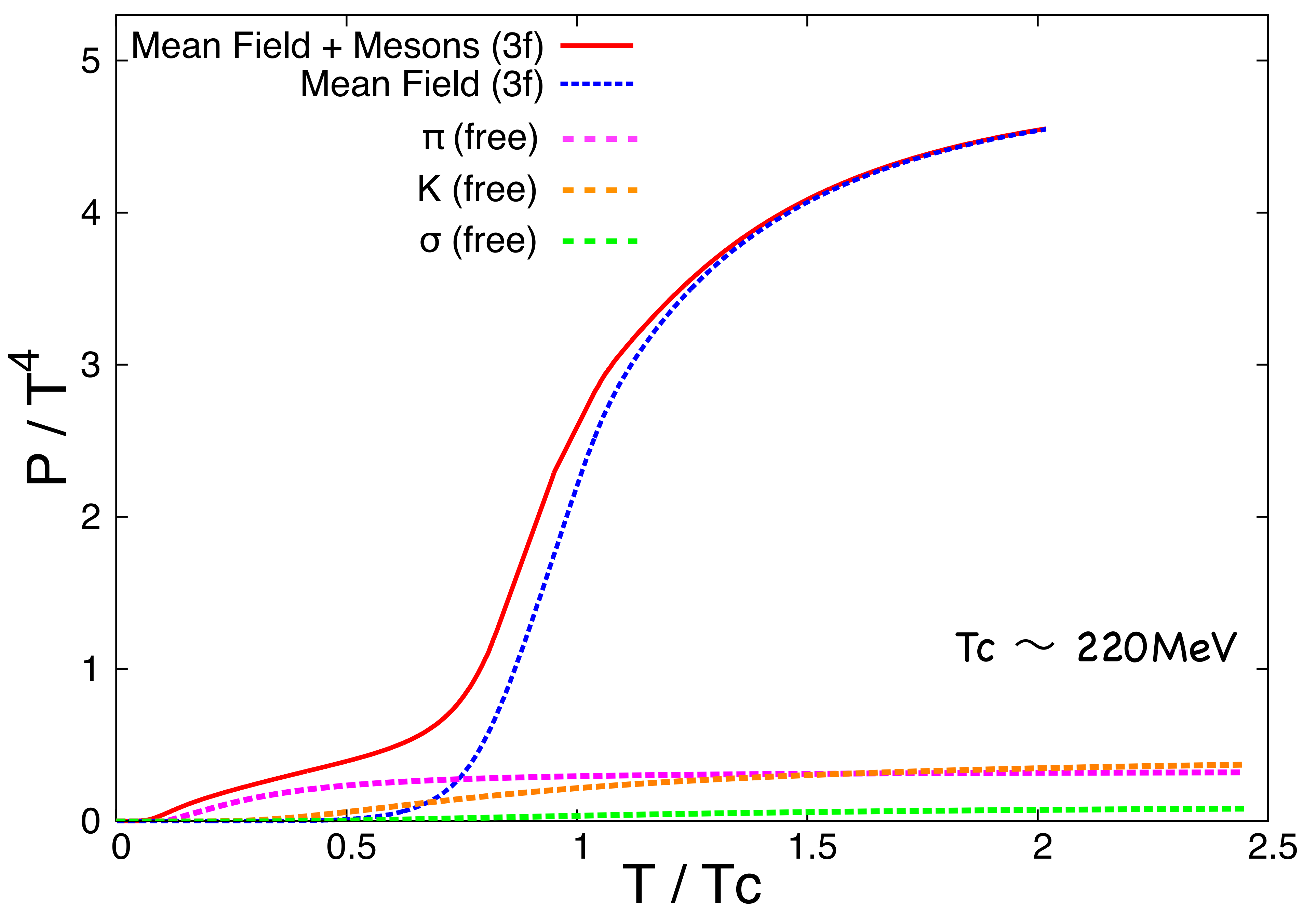


- At high T

Pressure gets close to quark mean field pressure.

π, K and σ are taken into this calculation.

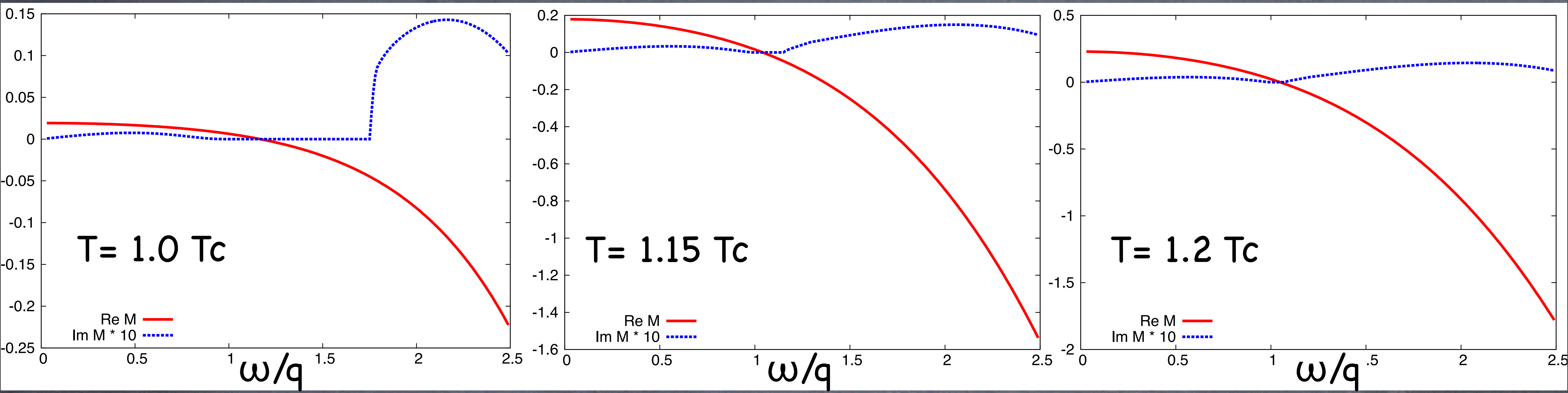
Comparison with free mesons



π, K and σ are taken into this calculation.

Collective modes

Pion



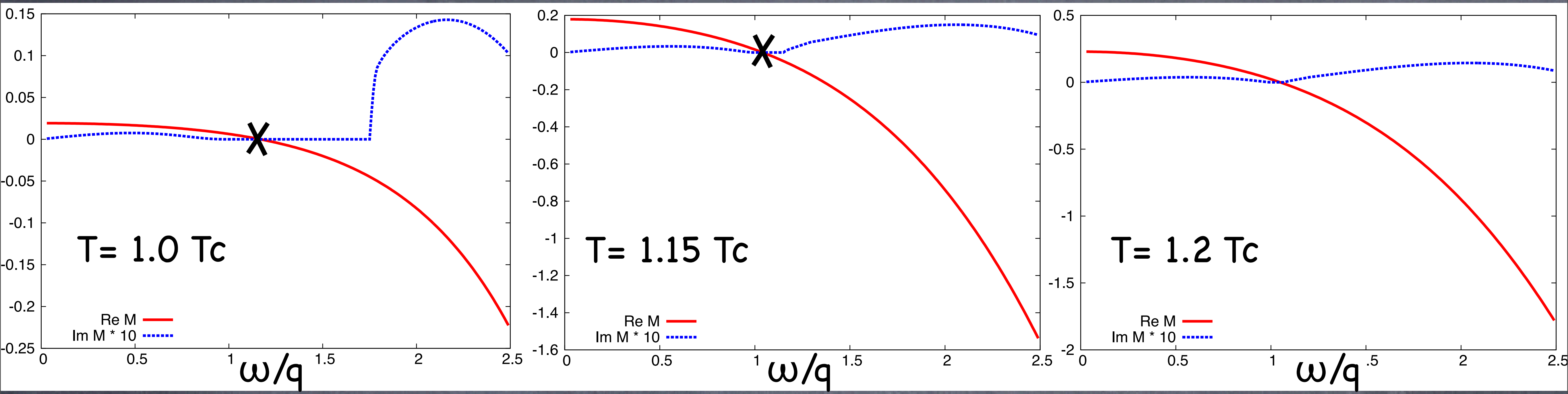
Real part of M

Imaginary part of M

$$\mathcal{M}(\omega\ \ ,q) = \frac{1}{2K'} - \Pi(\omega\ \ ,q)$$

Collective modes

Pion



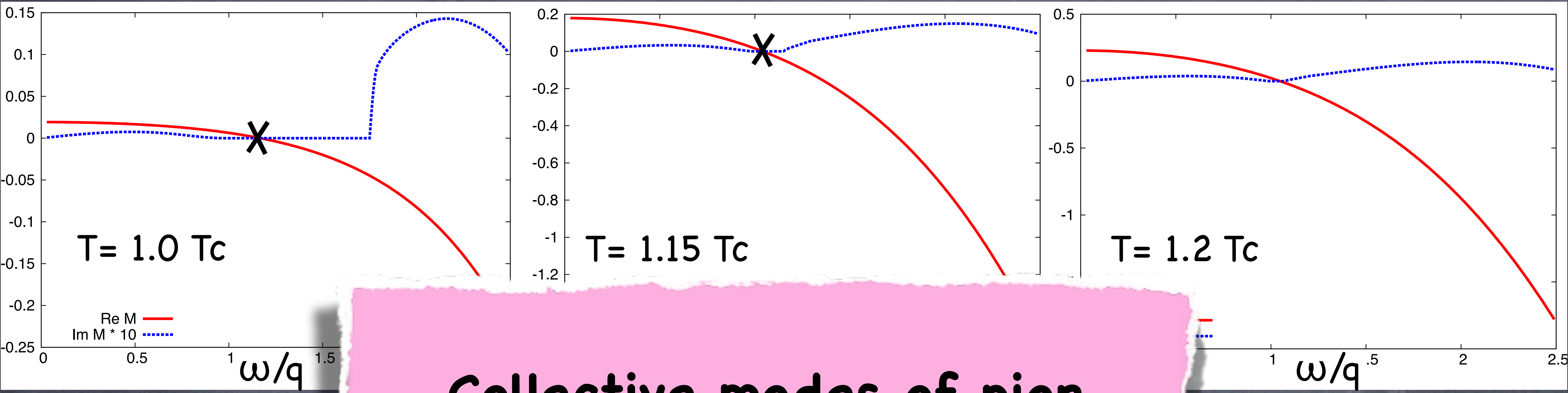
🟥 Real part of M

🟦 Imaginary part of M

$$\mathcal{M}(\omega_n, q) = \frac{1}{2K'} - \Pi(\omega_n, q)$$

Collective modes

Pion



- Collective modes of pion disappear at T=1.2Tc

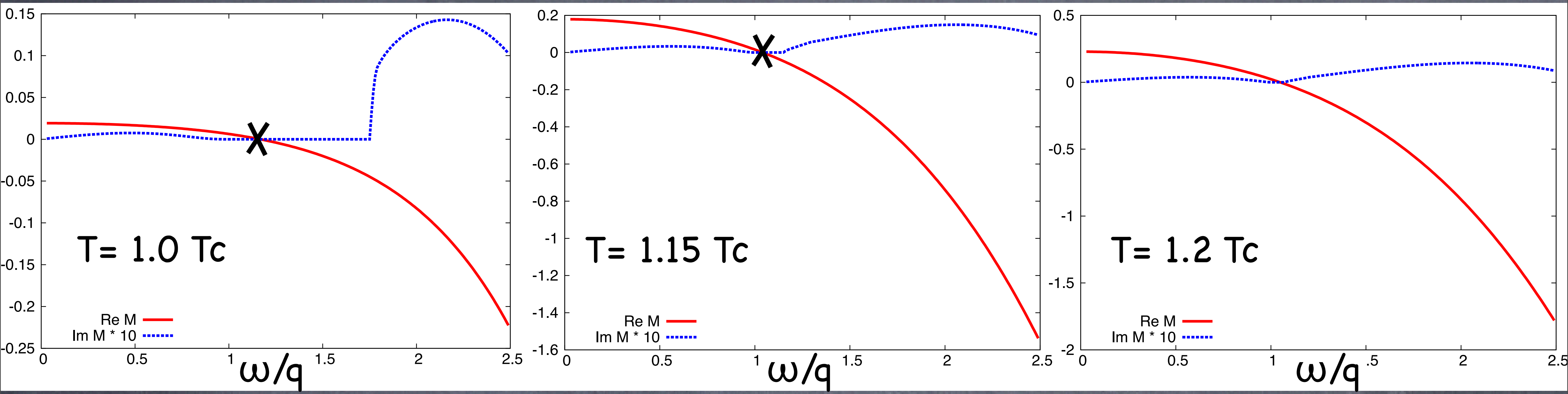
🟥 Real

🟦 Imag

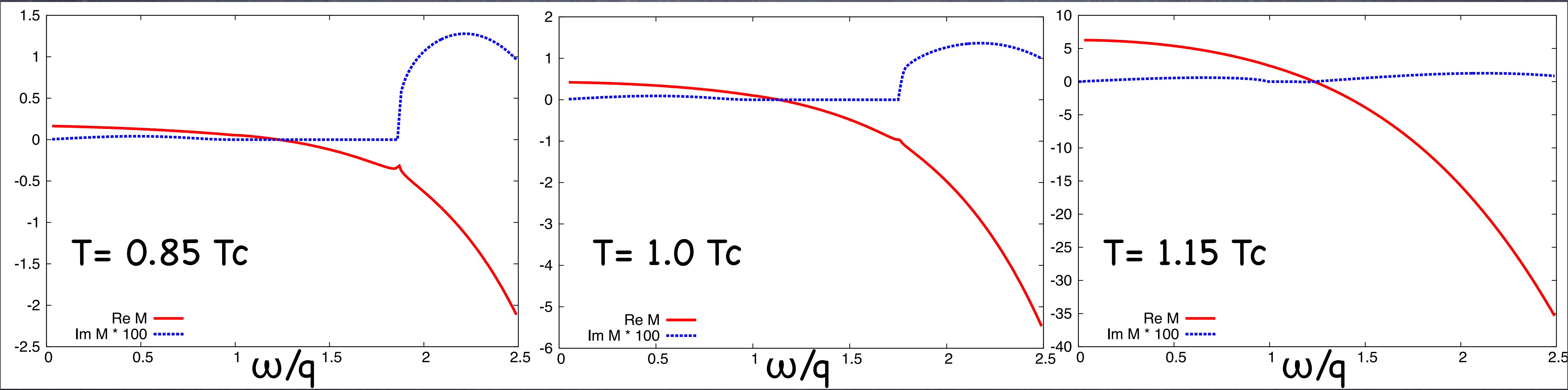
$- \Pi(\omega_n, q)$

Collective modes

Pion

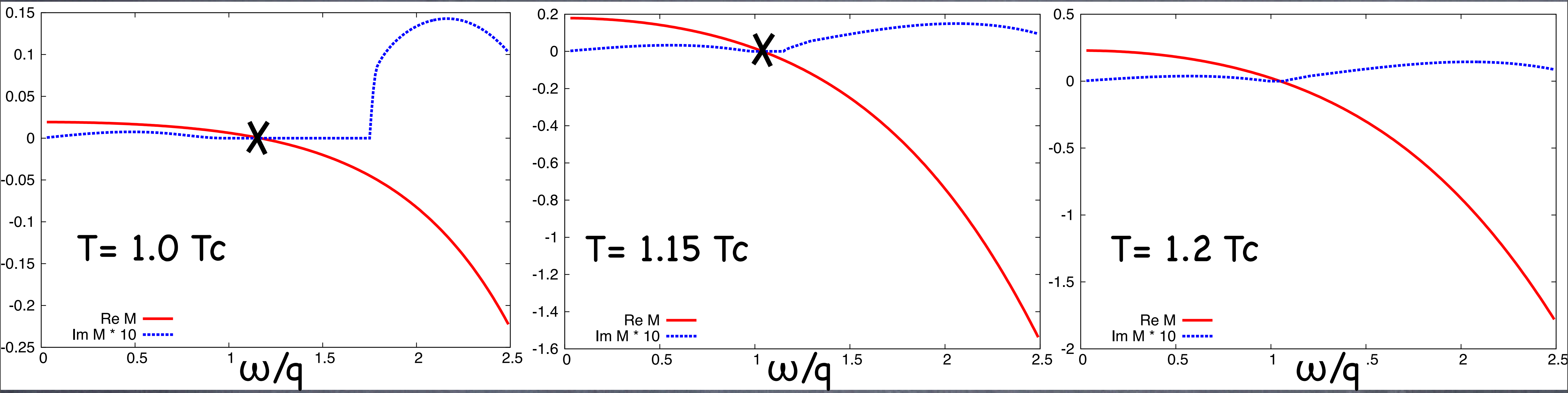


Kaon

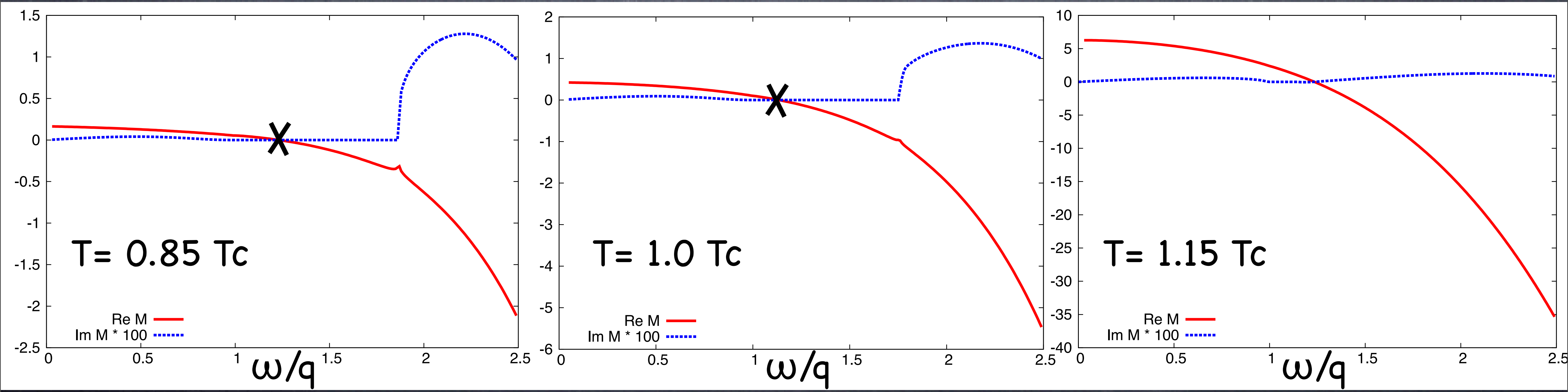


Collective modes

Pion

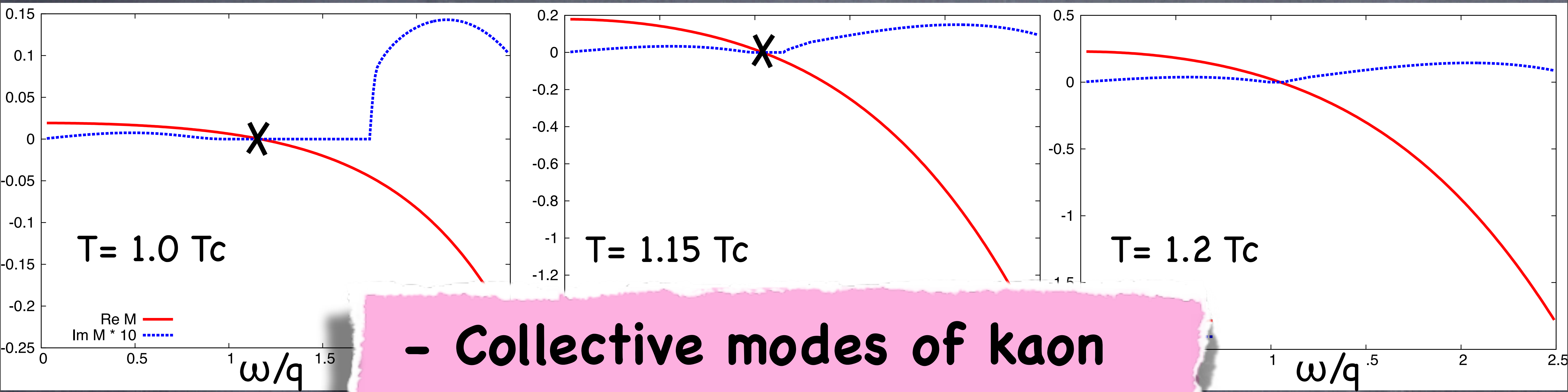


Kaon

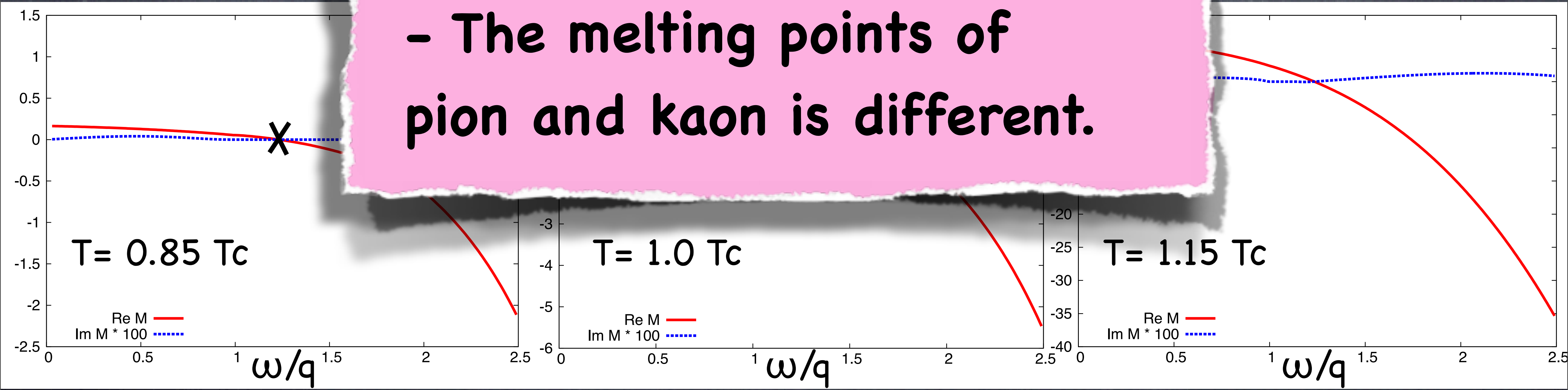


Collective modes

Pion



Kaon



- Collective modes of kaon disappear at T=1.15 Tc

- The melting points of pion and kaon is different.

Summary and Outlook

- We have described a quark-hadron phase transition from interacting quarks.

- Collective modes of mesons at low T melt with increasing temperature, and resolve to quarks.

- Melting temperature of pion is higher than that of kaon.

- How to put baryonic correlation at finite chemical potential.