Diffusion of Non-Gaussianity in Heavy Ion Collisions

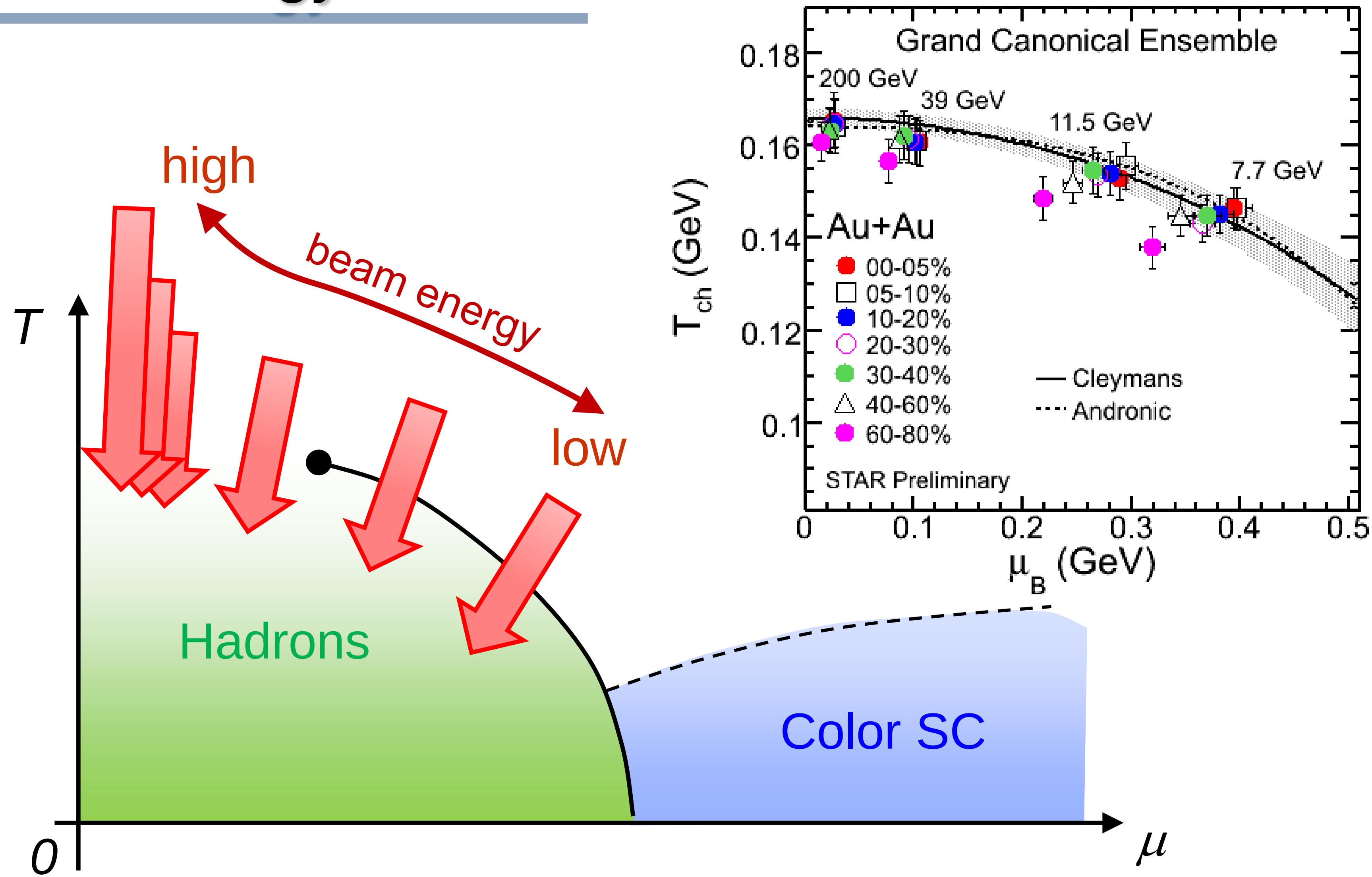
Masakiyo Kitazawa
(Osaka U.)

MK, Asakawa, Ono, arXiv:1307.2978

SQM, Birmingham, 23, July 2013

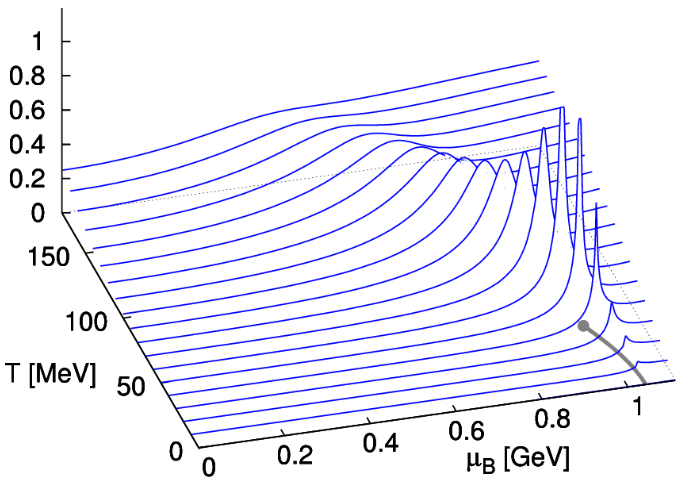
Beam-Energy Scan

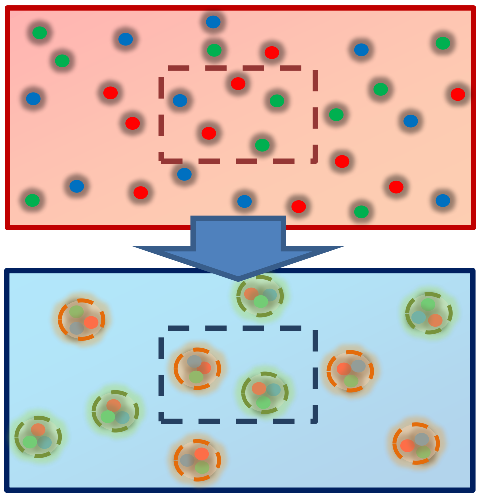
STAR 2012

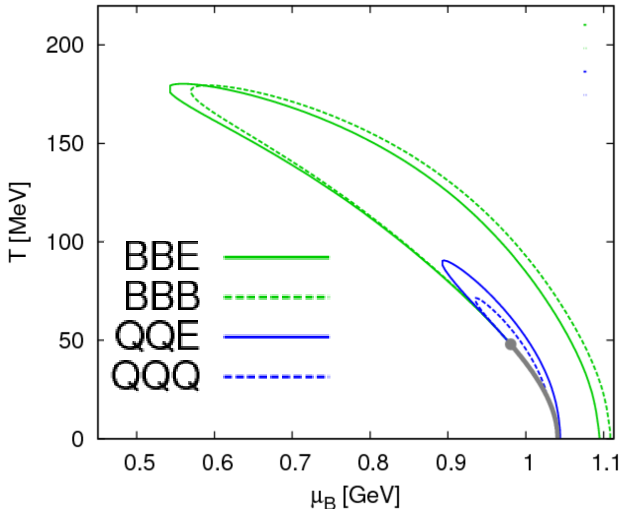


Fluctuations

- Fluctuations reflect properties of matter.
  - Enhancement near the critical point
    Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);…
  - Ratios between cumulants of conserved charges
    Asakawa,Heintz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)
  - Signs of higher order cumulants
    Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)



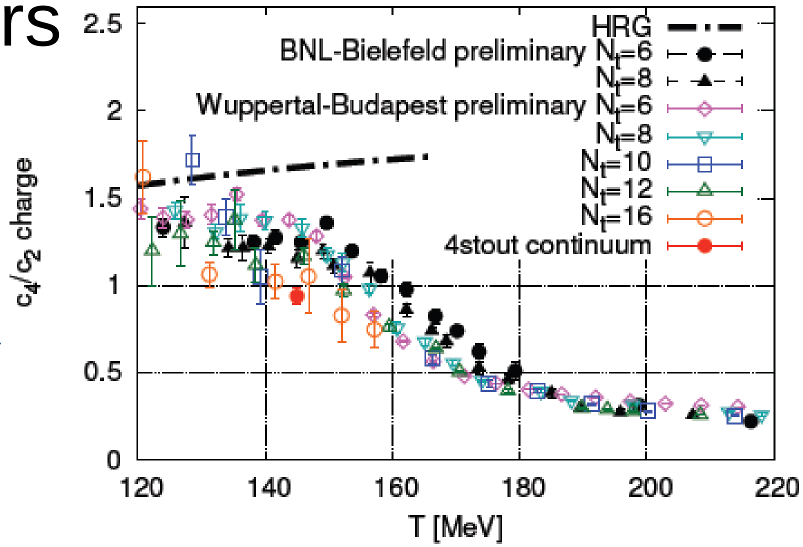




Conserved Charges : Theoretical Advantage

- Definite definition for operators
  - as a Noether current
  - calculable on any theory

ex: on the lattice



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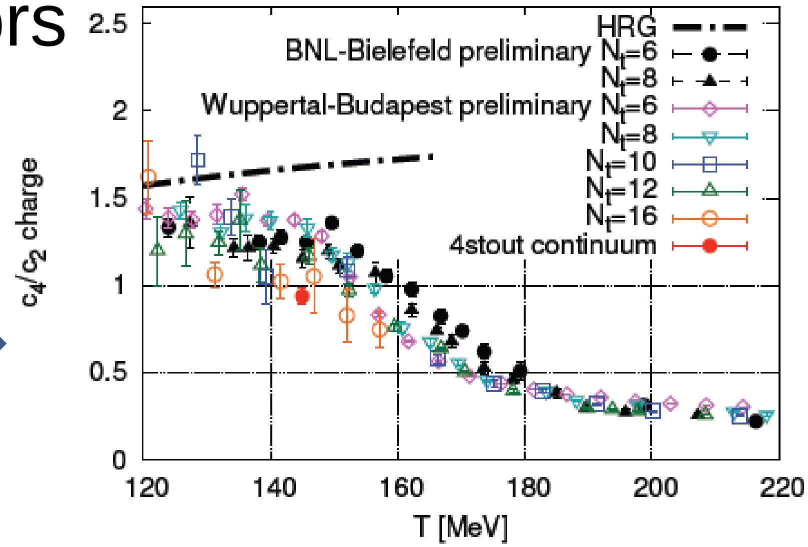


- Simple thermodynamic relations

$$\langle \delta N_c^n \rangle = \frac{1}{VT^{n-1}} \frac{\partial^n \Omega}{\partial \mu_c^n}$$

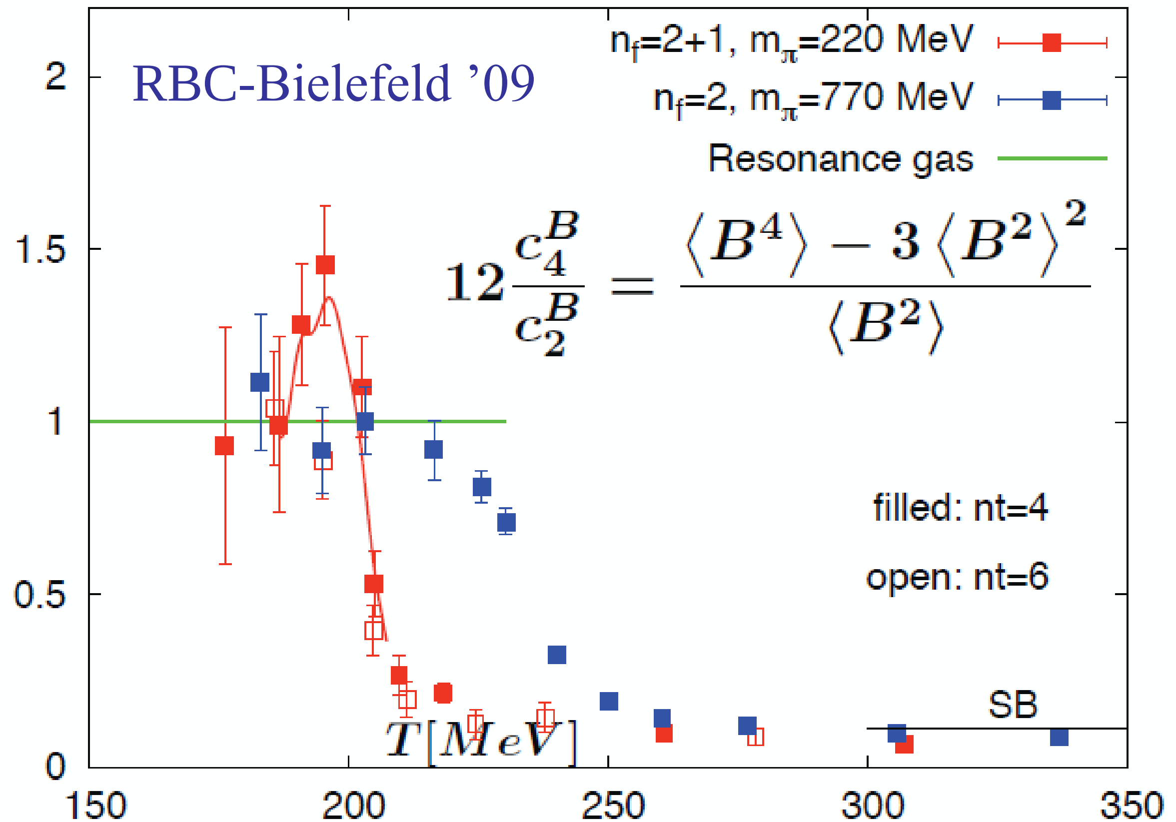
  - Intuitive interpretation for the behaviors of cumulants

ex: $\langle \delta N_B^3 \rangle = \frac{1}{VT^2} \frac{\partial \langle \delta N_B^2 \rangle}{\partial \mu_B}$

Asakawa, Ejiri, MK, 2009

Conserved Charge Fluctuations

$$\frac{\langle \delta N_B^4 \rangle_c}{\langle \delta N_B^2 \rangle}$$

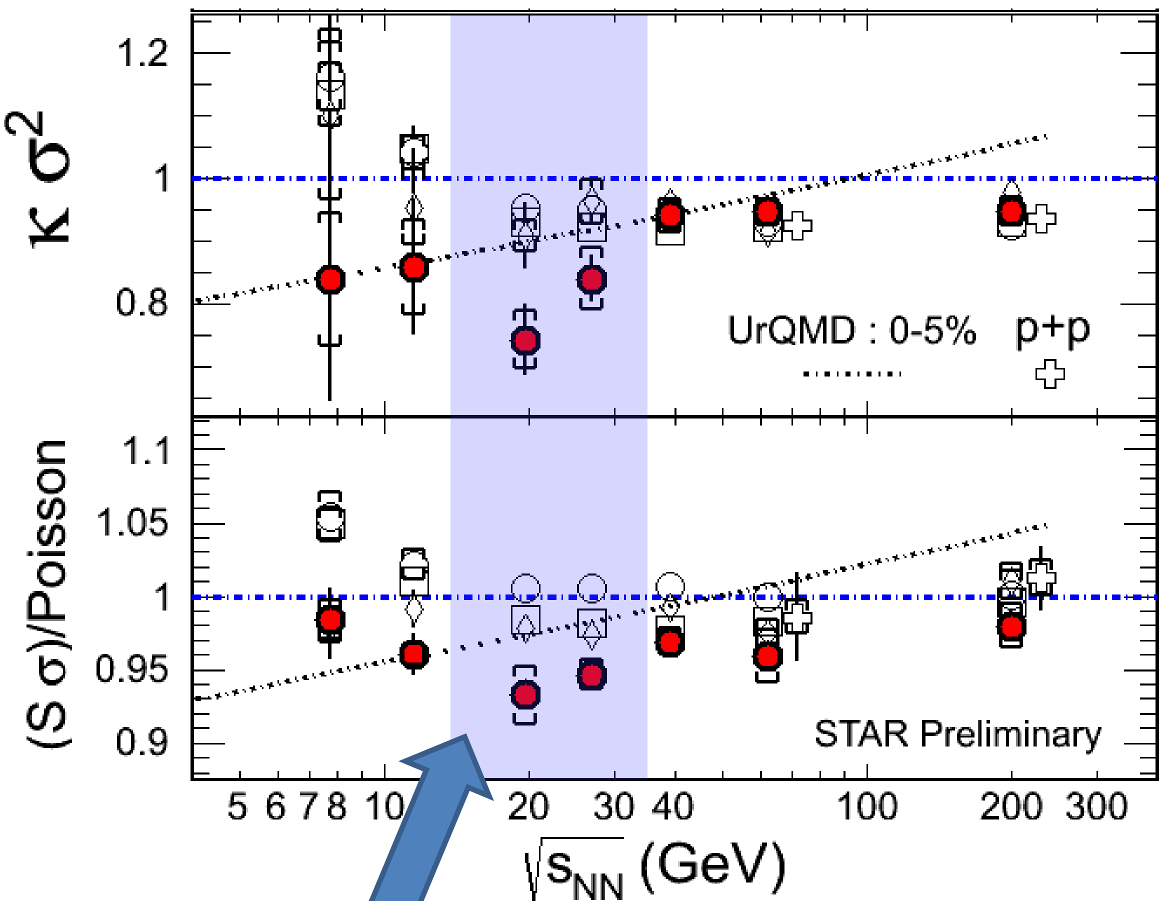


Cumulants of N_B and N_Q are **suppressed** at high T.

Asakawa, Heintz, Muller, 2000; Jeon, Koch, 2000;
Ejiri, Karsch, Redlich, 2006; Asakawa, Ejiri, MK, 2009;
Friman, et al., 2011; Stephanov, 2011

Proton # Cumulants @ STAR-BES

STAR,QM2012



$$\frac{C_4}{C_2}$$

$$\frac{C_3}{C_1} = \frac{C_3/C_2}{\text{Poissonian}}$$

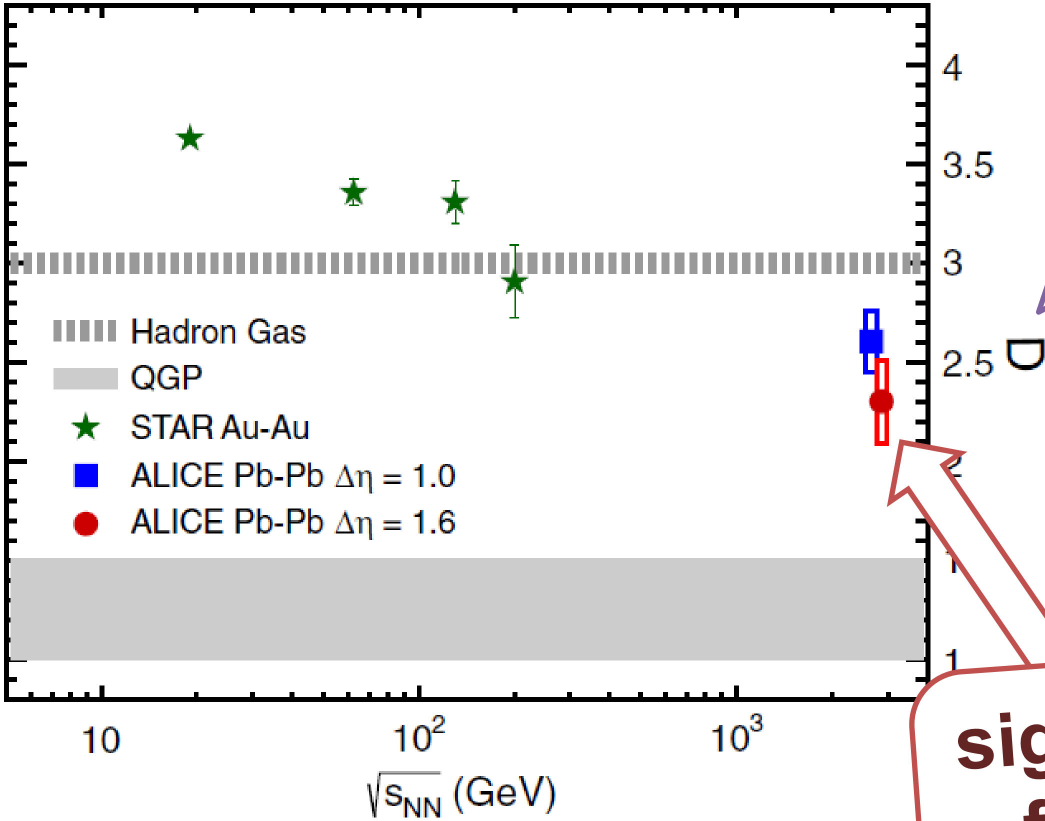
Something interesting??

CAUTION!

proton number $\neq$ baryon number

MK, Asakawa, 2011;2012

Charge Fluctuation @ LHC



ALICE, PRL110,152301(2013)

D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

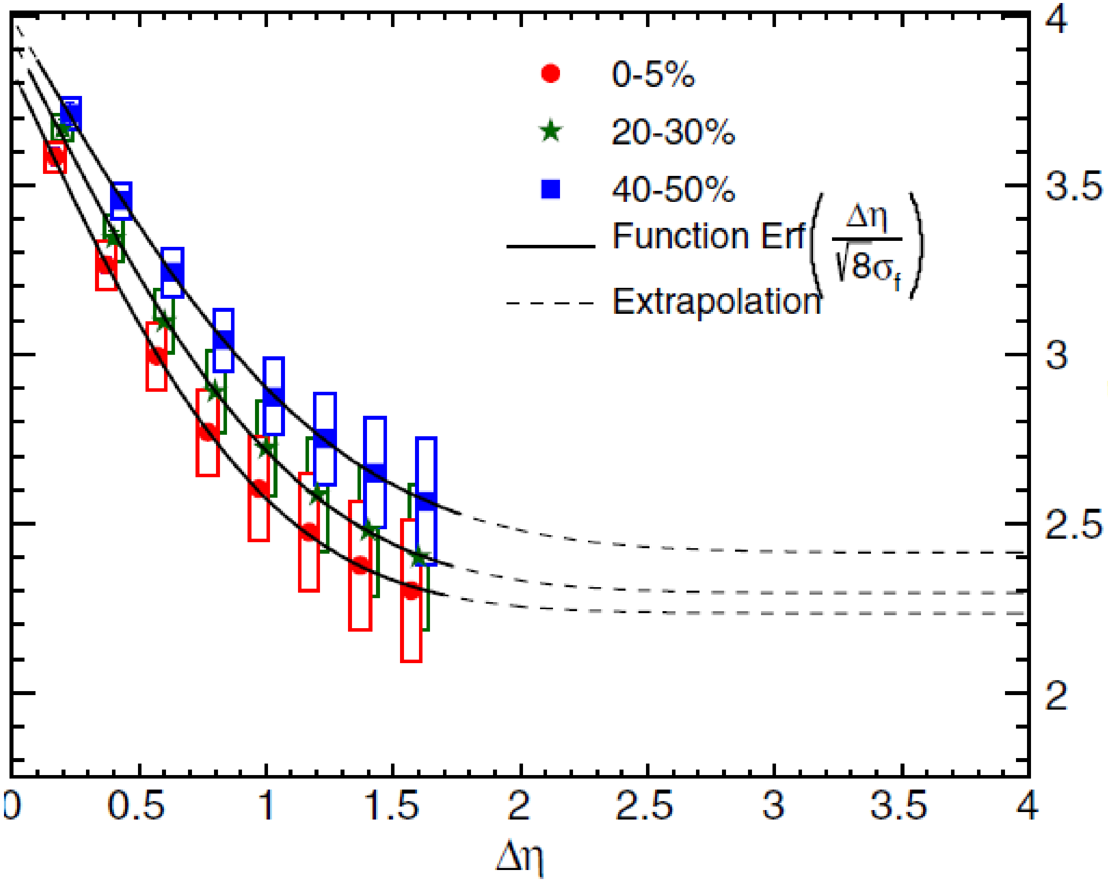
- D ~ 3-4 Hadronic
- D ~ 1 Quark

significant suppression from hadronic value at LHC energy!

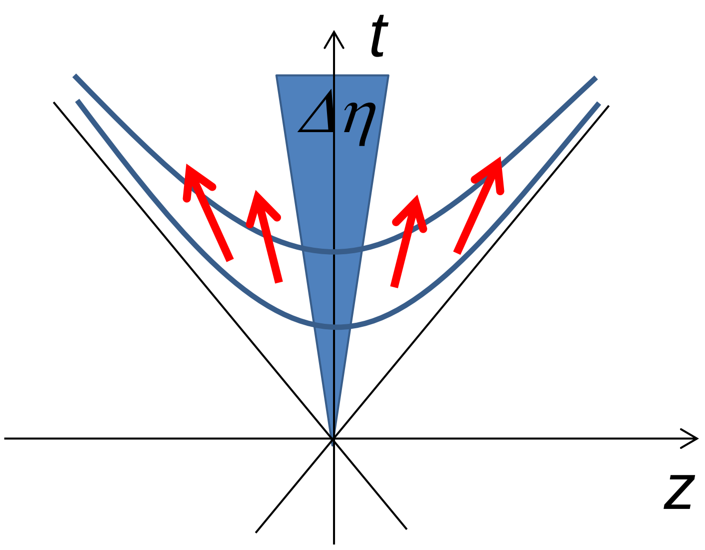
$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

Δη Dependence @ ALICE

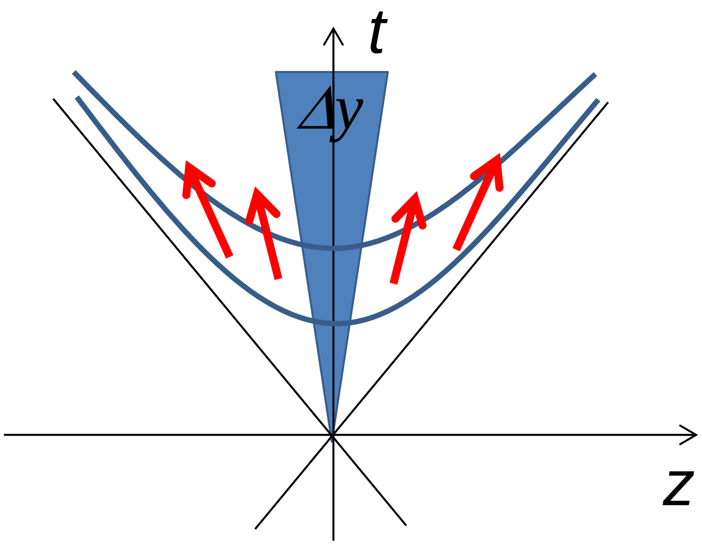
ALICE
PRL 2013

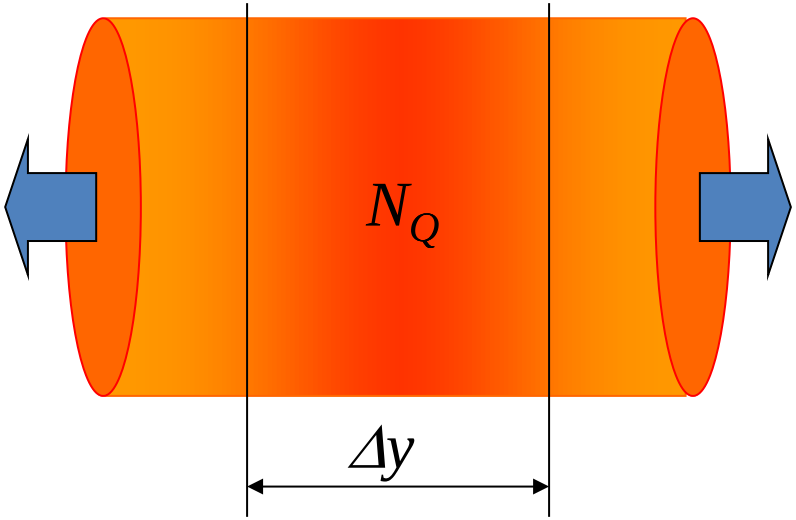


rapidity window



Time Evolution of CC



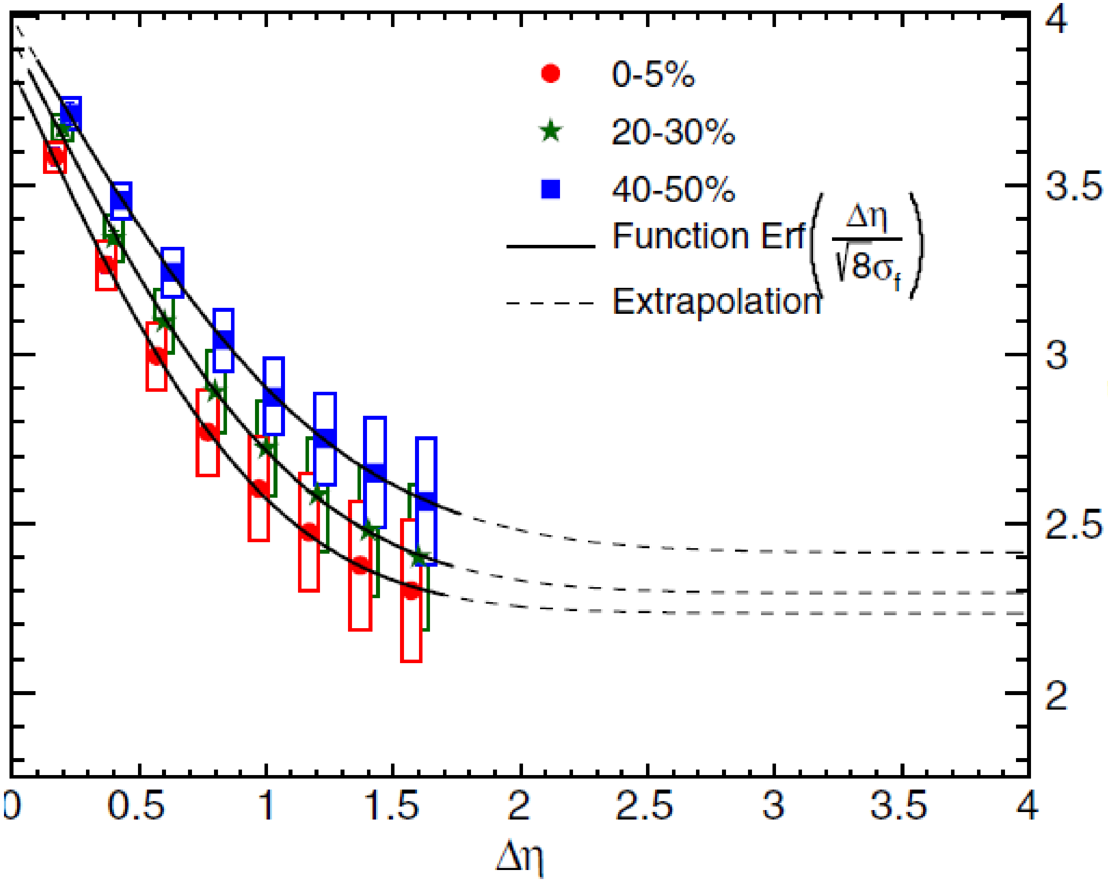


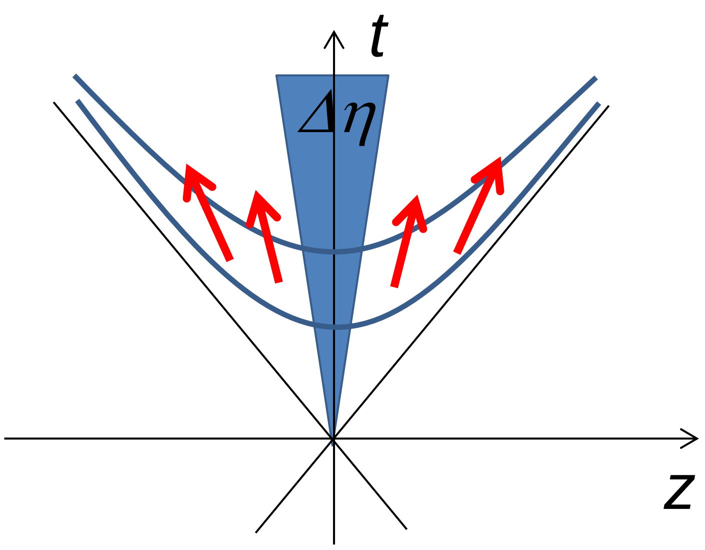
Variation of a conserved charge in Δy is achieved only through diffusion.

The larger Δy, the slower diffusion

Δη Dependence @ ALICE

ALICE
PRL 2013

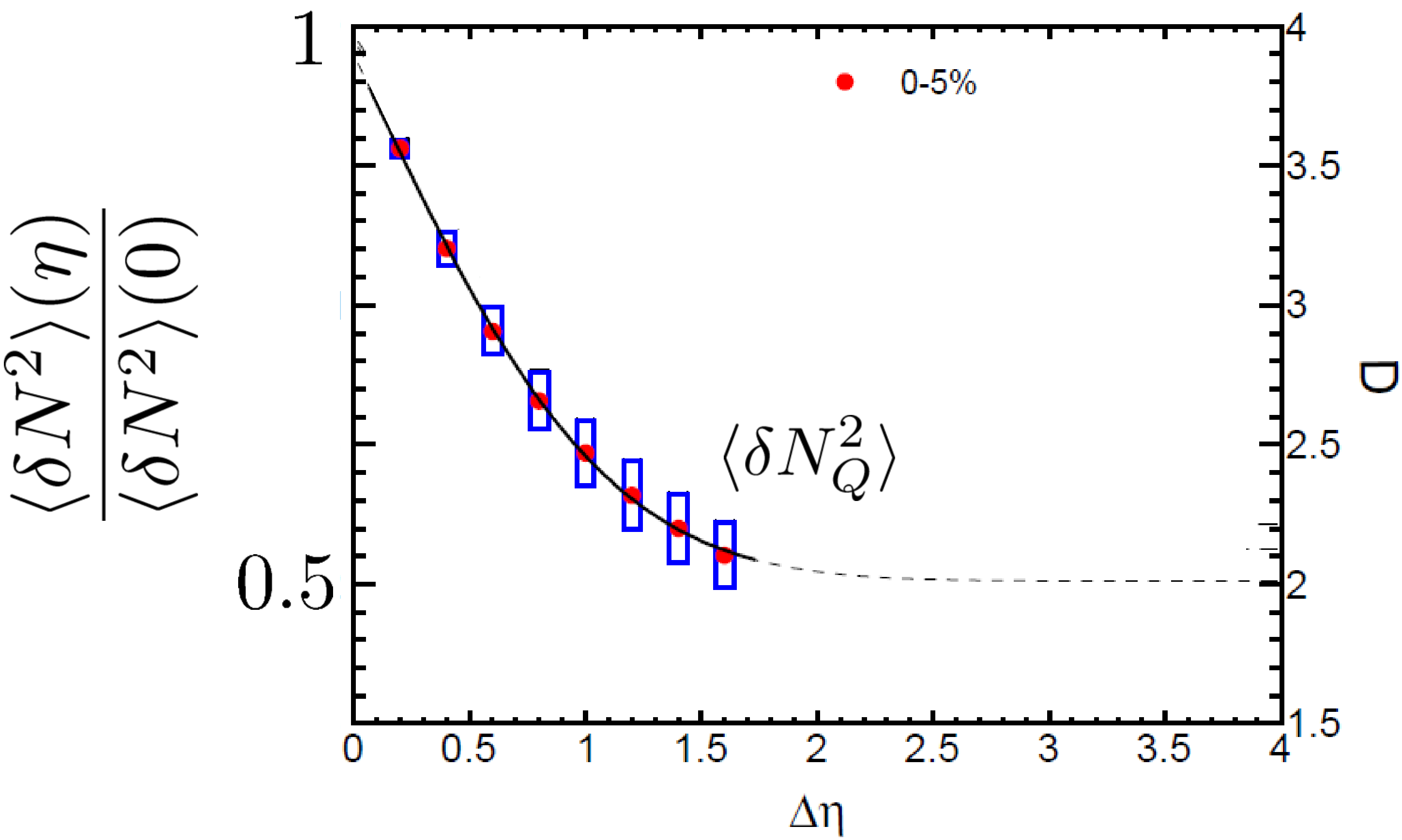




Δη dependences of fluctuation observables encode history of the hot medium!

$<\delta N_B^2>$ and $< \delta N_p^2 >$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
should have different $\Delta\eta$ dependence.



Baryon # cumulants are experimentally observable! MK, Asakawa, 2011;2012

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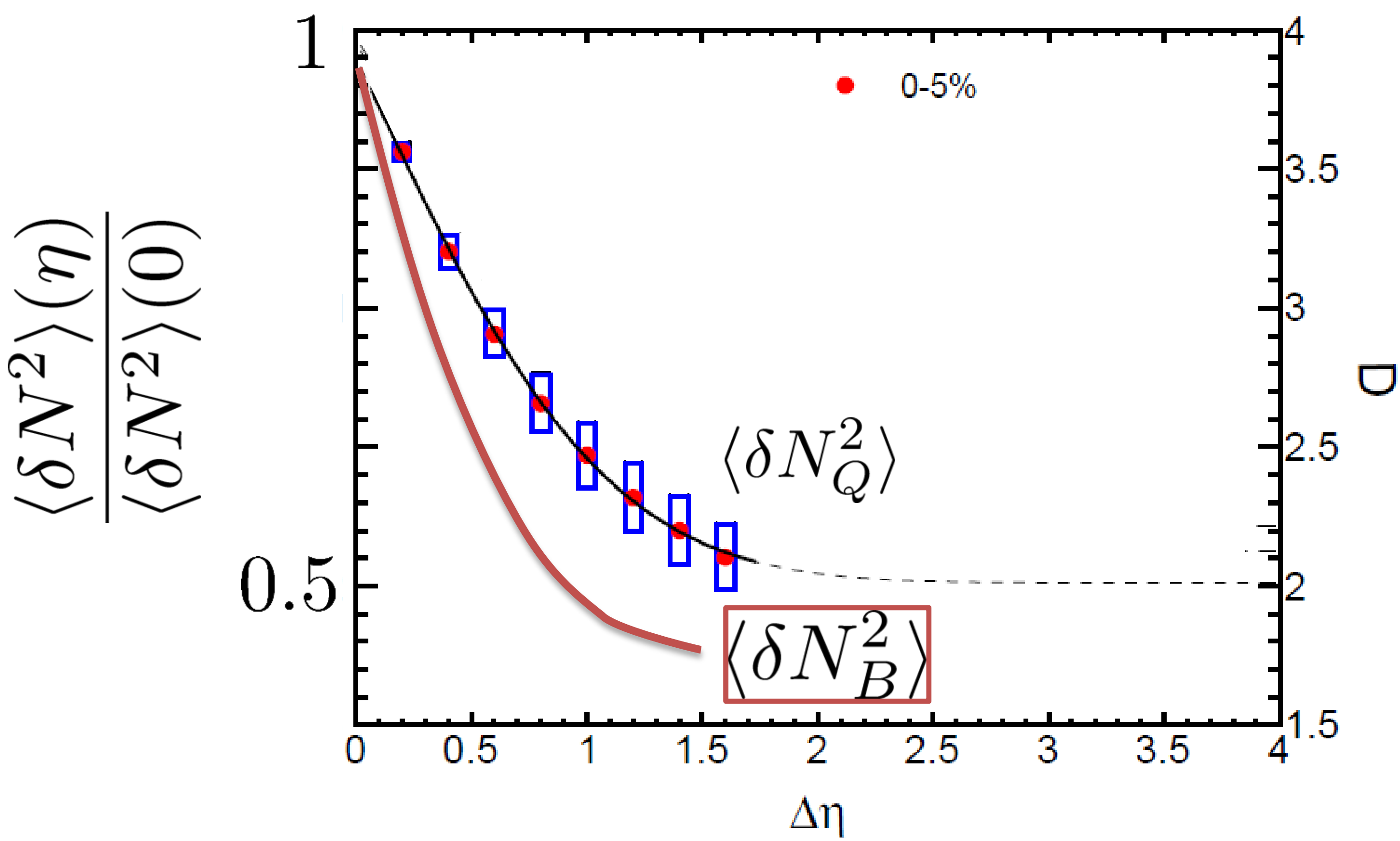

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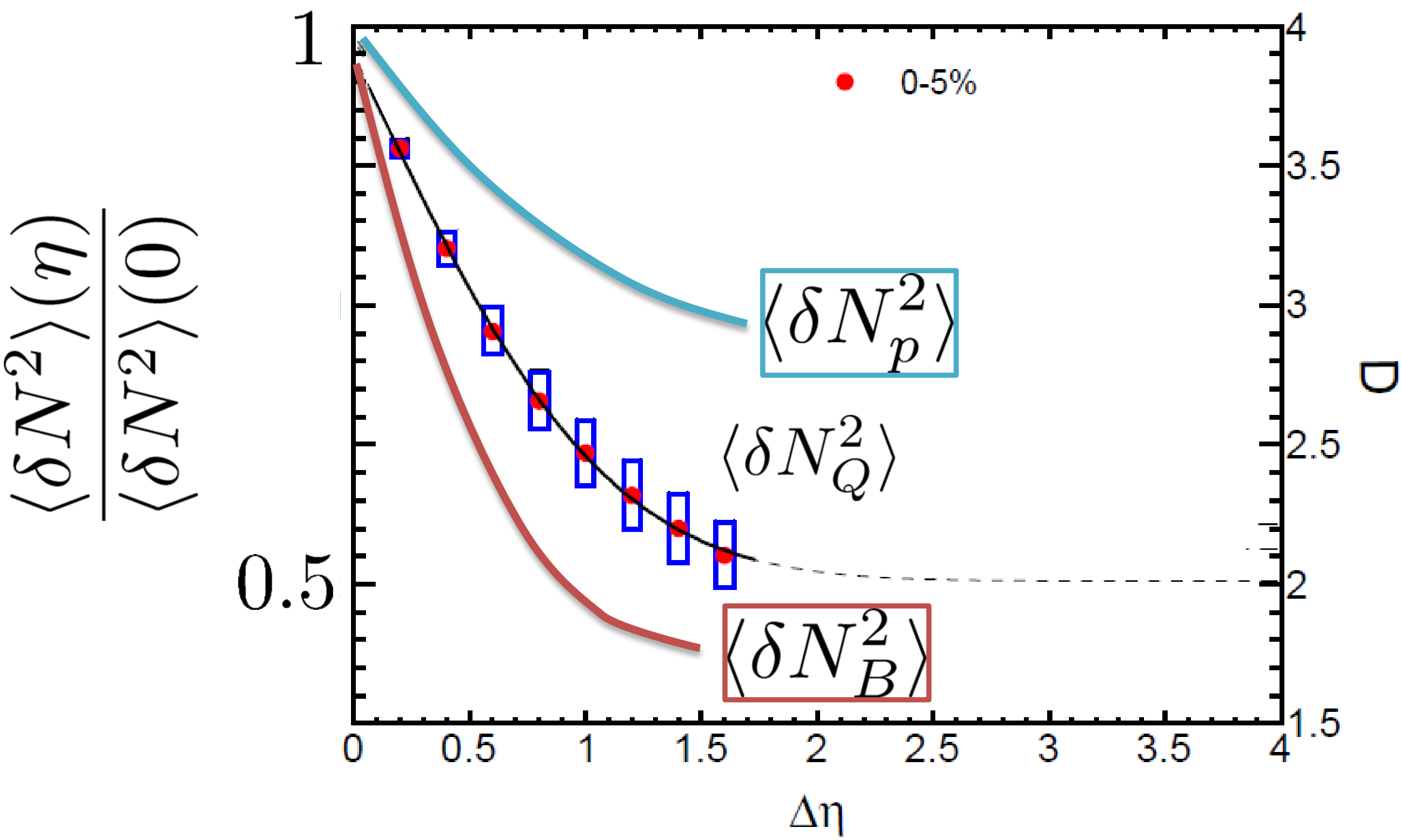
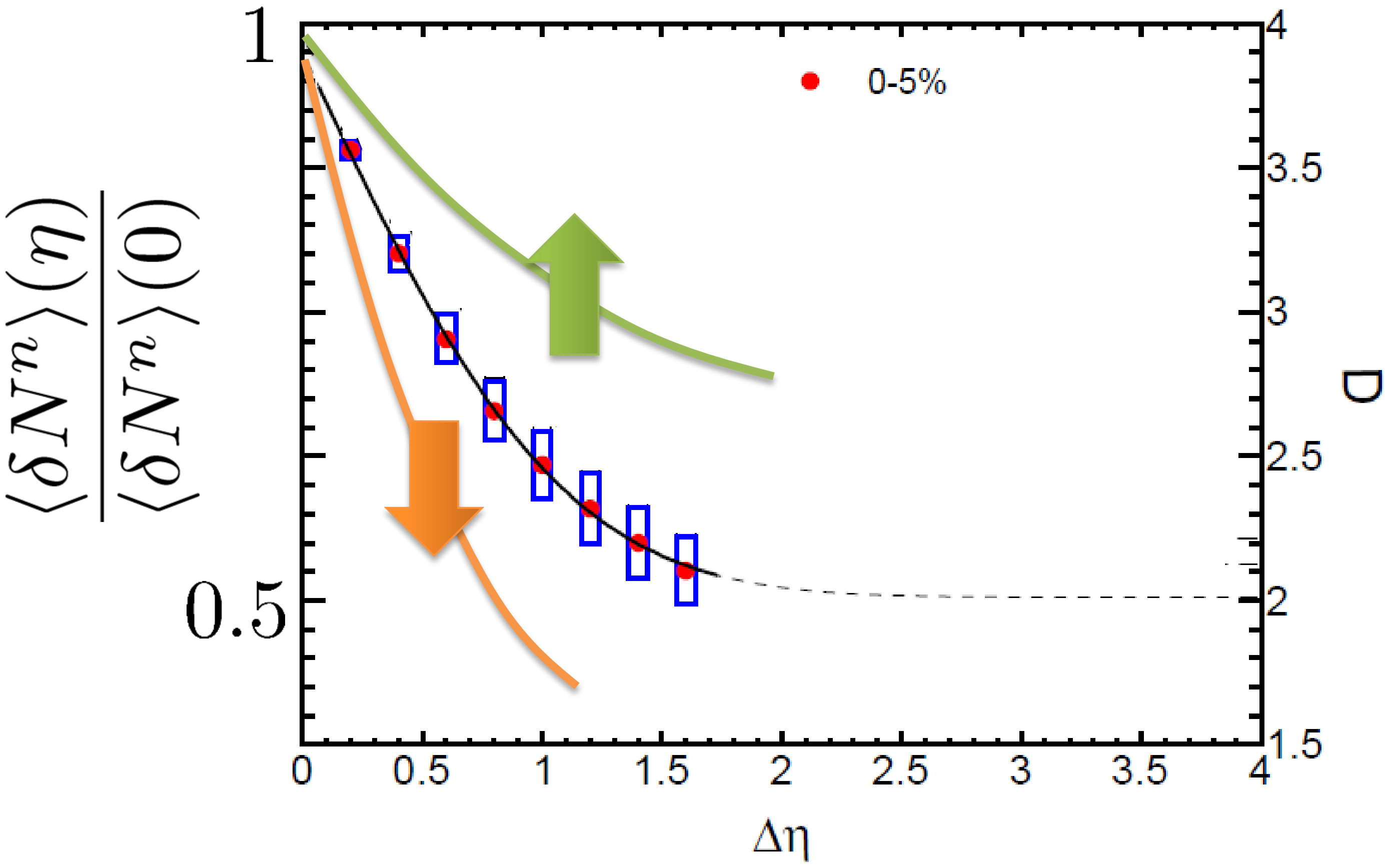


Baryon # cumulants are experimentally observable! MK, Asakawa, 2011;2012

$<\delta N_Q{}^4>$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

suppression or enhancement



Three "NON"s

Physics of non-Gaussianity in heavy-ion is a **particular** problem!

- **Non-Gaussian** — Non-Gaussianitiy is irrelevant in large systems
- **Non-critical** — critical enhancement is not observed in HIC so far
- **Non-equilibrium** — Fluctuations are not equilibrated in HIC

Three "NON"s

Physics of non-Gaussianity in heavy-ion is a **particular** problem!

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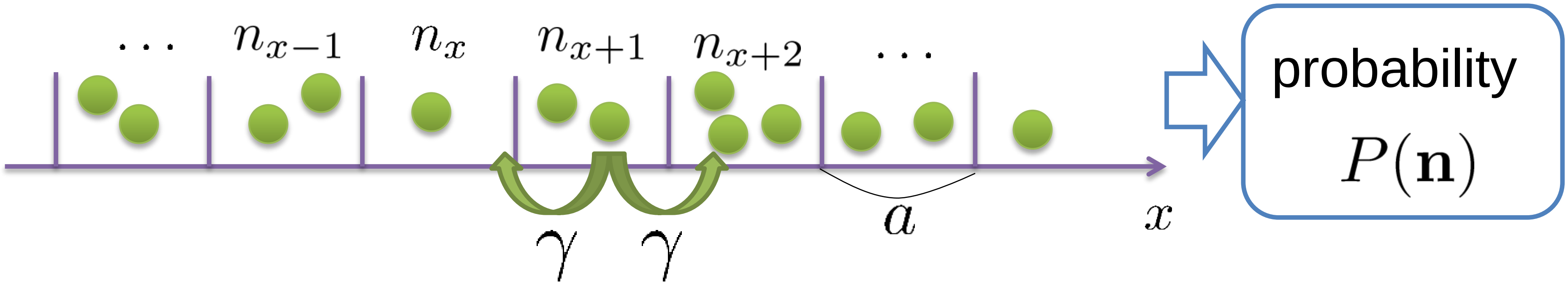
It is **impossible** to directly extend the **theory of hydro fluctuations** to treat higher orders.

Diffusion Master Equation

Divide spatial coordinate into discrete cells



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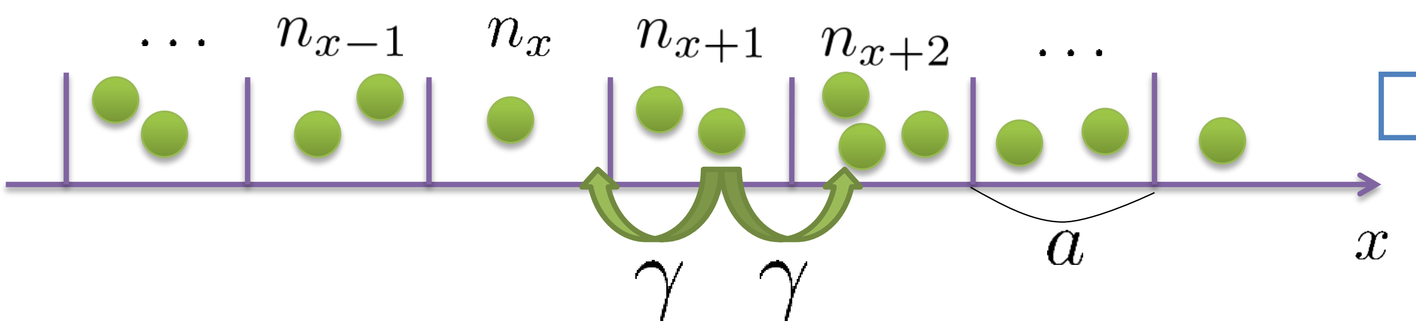

probability

$P(\mathbf{n})$

Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x \Big[(n_x+1)\Big\{P(\mathbf{n}+\hat{x}-\widehat{x+1}) + P(\mathbf{n}+\hat{x}-\widehat{x-1})\Big\} - 2n_x P(\mathbf{n})\Big]$$

x-hat: lattice-QCD notation

Solve the DME **exactly**, and take *a*→0 limit

No approx., ex. van Kampen's system size expansion

Solution of DME

1st $\langle \tilde{n}_k \rangle(t) = e^{-\omega_k t} \langle \tilde{n}_k \rangle_0$ $\qquad \omega_k \simeq \gamma a^2 k^2$

initial

- Deterministic part ←→ diffusion equation at long wave length (1/a<<k)

$$\partial_t \langle n_x(t) \rangle = \gamma a^2 \partial_x^2 \langle n_x(t) \rangle$$

- Appropriate continuum limit with $\gamma a^2 = D$

Solution of DME

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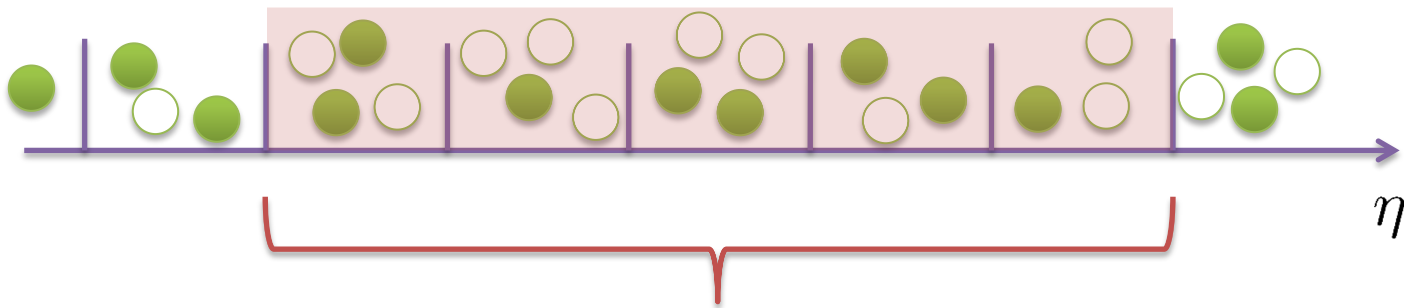
2nd $$\langle \delta\tilde{n}_{k_1} \delta\tilde{n}_{k_2} \rangle(t) = \langle \tilde{n}_{k_1+k_2} \rangle_0 (e^{-\omega_{k_1+k_2} t} - e^{-(\omega_{k_1}+\omega_{k_2})t}) + \langle \delta\tilde{n}_{k_1} \delta\tilde{n}_{k_2} \rangle_0 e^{-(\omega_{k_1}+\omega_{k_2})t}$$

- Consistent with stochastic diffusion eq. (for smooth initial condition)

Shuryak, Stephanov, 2001

Net Charge Number

Prepare 2 species of (non-interacting) particles



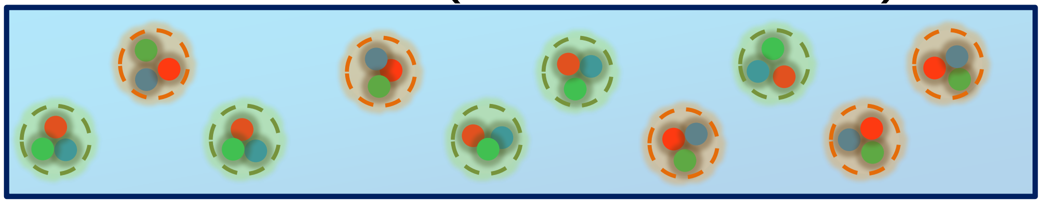
$$\bar{Q}(\tau) = \int_0^{\Delta\eta} d\eta \, (n_1(\eta,\tau) - n_2(\eta,\tau))$$

Let us investigate

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ at freezeout time t

Time Evolution in Hadronic Phase

Hadronization (initial condition)



- Boost invariance / infinitely long system
- Local equilibration / local correlation

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ $\qquad \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ $\qquad \langle Q_{(\mathrm{tot})}^2 \rangle_c$

suppression owing to local charge conservation ($\langle \bar{Q}^2 \rangle_c$, $\langle \bar{Q}^4 \rangle_c$, $\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$)

strongly dependent on hadronization mechanism ($\langle Q_{(\mathrm{tot})}^2 \rangle_c$)

Time Evolution in Hadronic Phase

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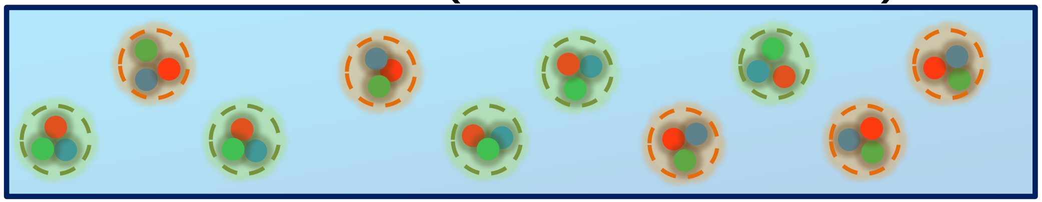


Time evolution via DME

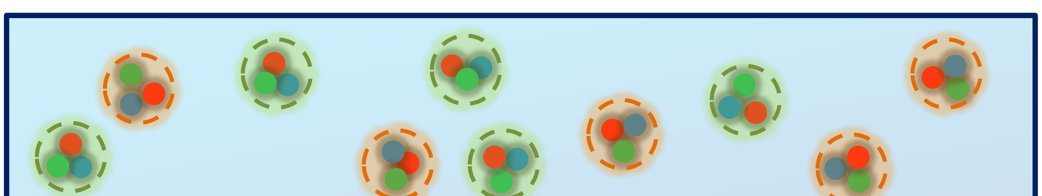
- Boost invariance / infinitely long system
- Local equilibration / local correlation

$\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c$ — suppression owing to local charge conservation

$\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$

$\langle Q^2_{(\mathrm{tot})} \rangle_c$ — strongly dependent on hadronization mechanism

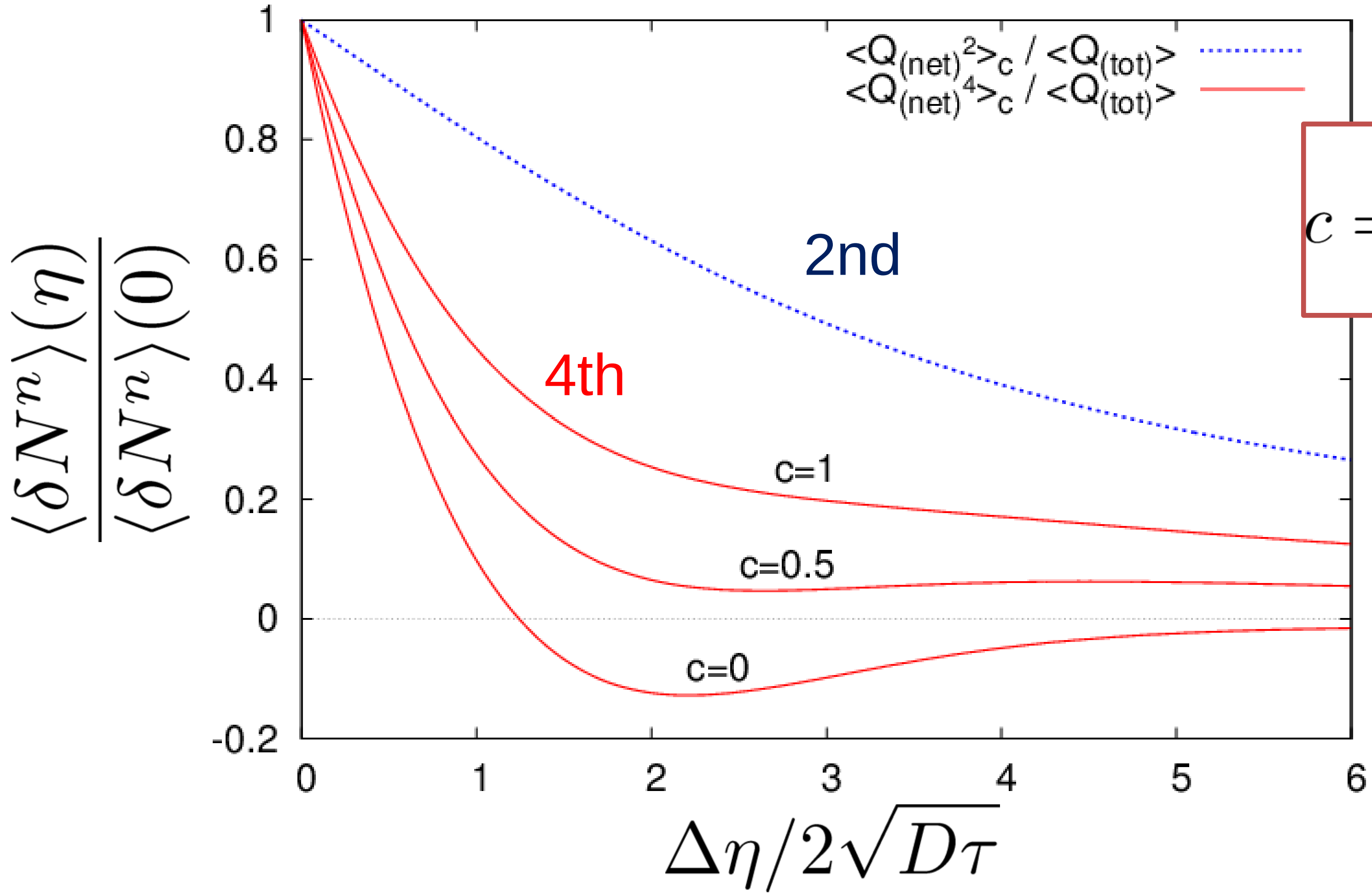
Freezeout



$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$



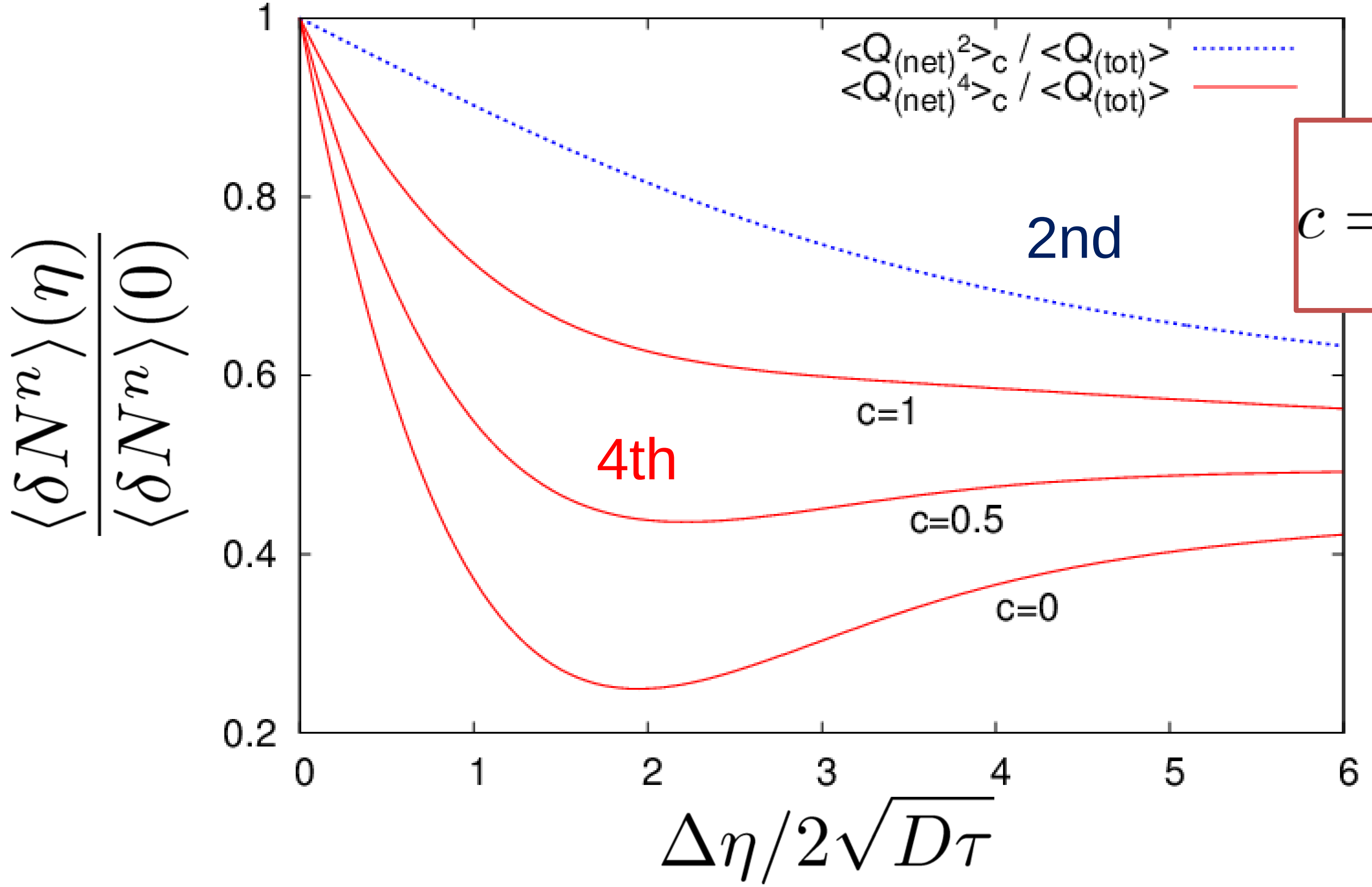
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0.5 \langle Q_{(\mathrm{tot})} \rangle$$

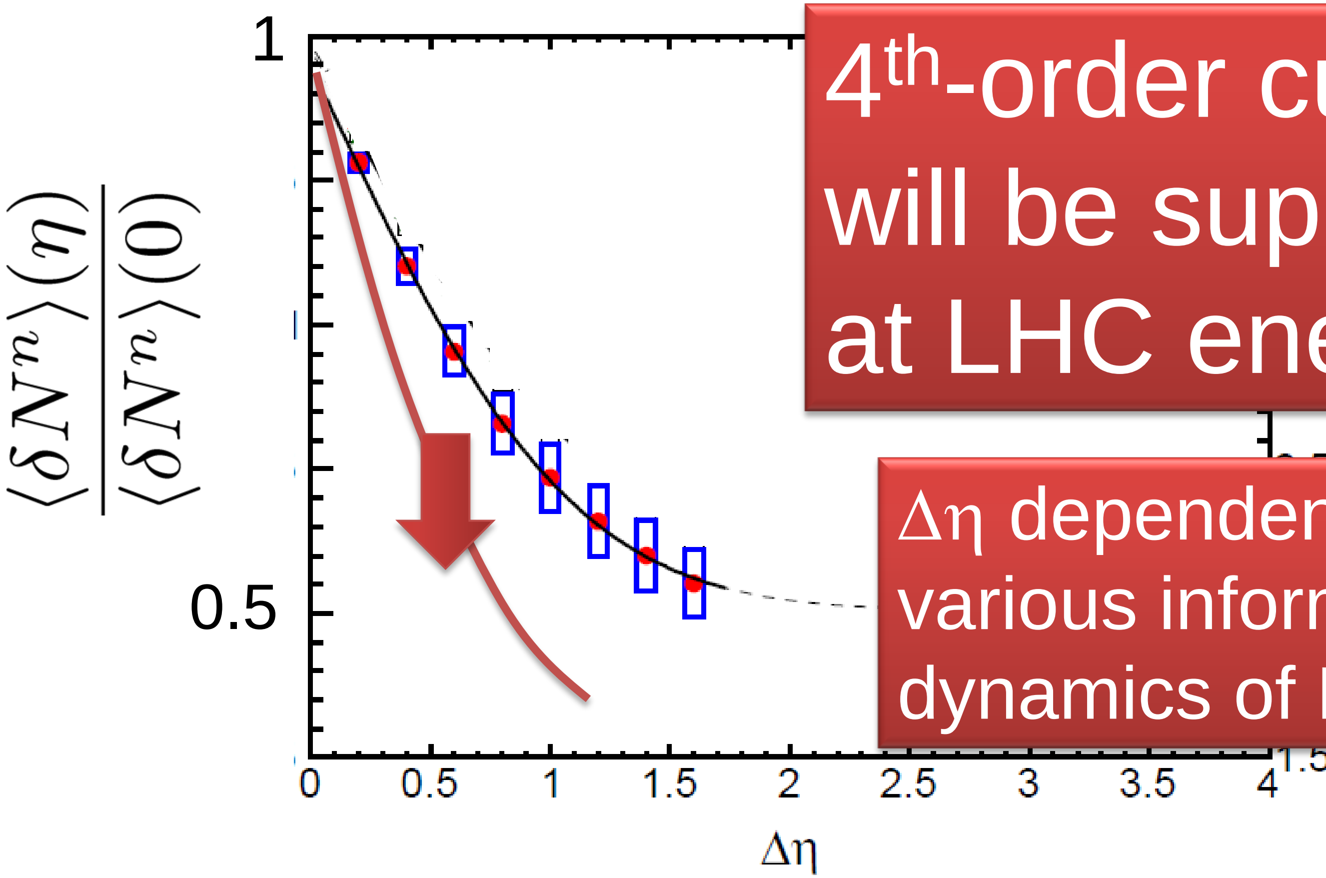


$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

$\langle \delta N_Q^4 \rangle$ @ LHC

Assumptions
- boost invariant system
- small fluctuations of CC at hadronization
- short correlation in hadronic stage



4th-order cumulant will be suppressed at LHC energy!

Δη dependences encode various information on the dynamics of HIC!

Summary

Plenty of physics in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2 \rangle_c, \ \langle N_B^2 \rangle_c, \ \langle N_Q^4 \rangle_c, \ \langle N_B^4 \rangle_c, \ \langle N_{ch}^2 \rangle_c, \cdots$$

Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC

- history of hot medium
- mechanism of hadronization
- diffusion constant

Summary

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Physical meanings of fluctuation obs. in experiments.

Diagnosing dynamics of HIC

- history of hot medium
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- diffusion constant

Search of QCD Phase Structure

Open Questions & Future Work

- Why the primordial fluctuations are observed only at LHC, and not RHIC ?
- Extract more information on each stage of fireballs using fluctuations

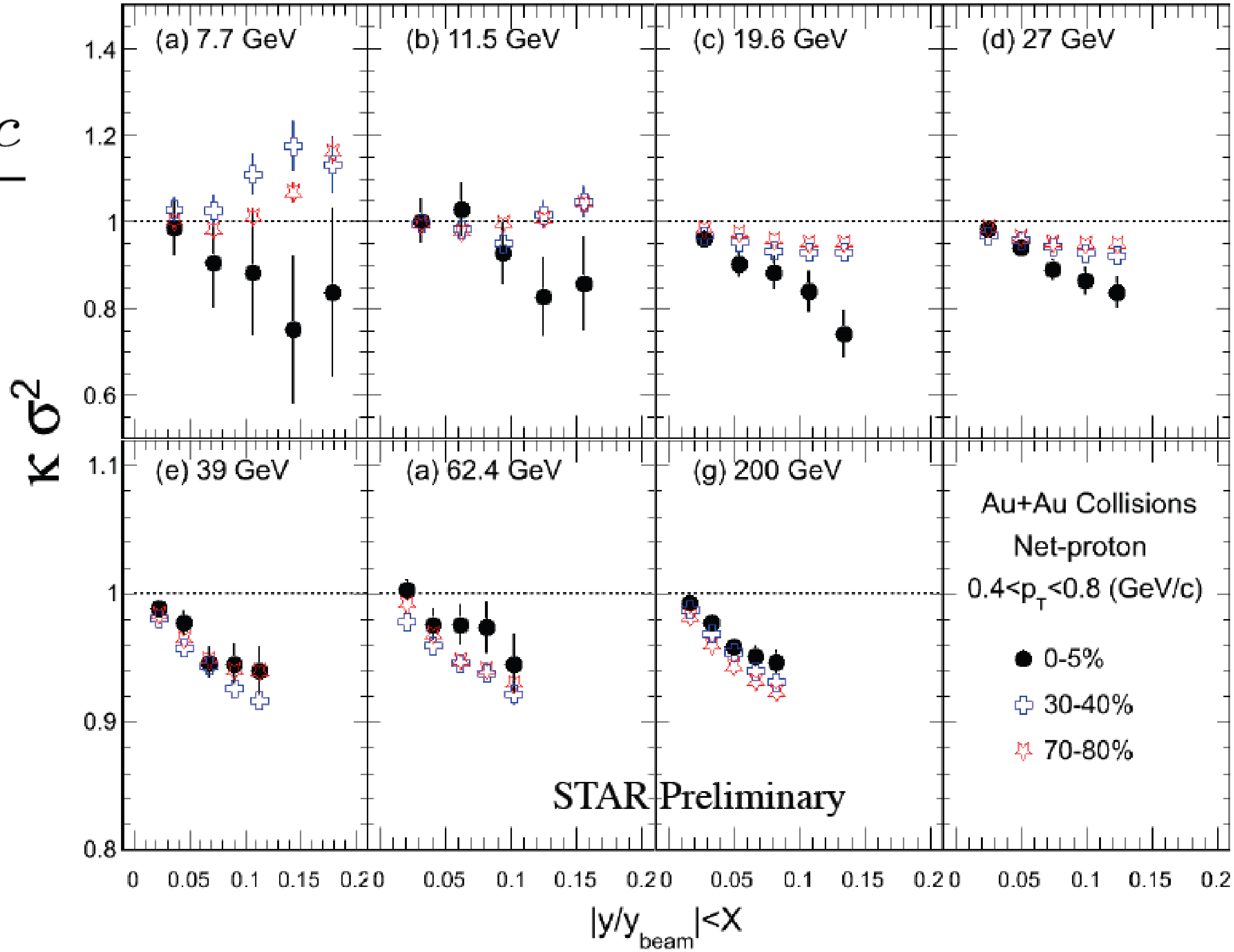
- Model refinement
  - Including the effects of
    nonzero correlation length / relaxation time
    global charge conservation

  - Non Poissonian system ← interaction of particles

$\Delta\eta$ Dependence at STAR

STAR, QM2012

$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$



$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$ decreases as $\Delta\eta$ becomes larger at RHIC energy.

Chemical Reaction 1

$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

x: # of X
a: # of A (**fixed**)

Master eq.:
$$\frac{\partial}{\partial t}P(x,t) = k_2 a P(x-1,t) + k_1(x+1)P(x+1,t) - (k_1 x + k_2 a)P(x,t)$$

Cumulants with fixed initial condition $P(x,0) = \delta_{x,N_0}$

$$\langle x(t)\rangle = N_0 e^{-k_1 t} + N_{eq}(1-e^{-k_1 t})$$

$$\langle \delta x(t)^2\rangle = N_0(e^{-k_1 t} - e^{-2k_1 t}) + N_{eq}(1-e^{-k_1 t})$$

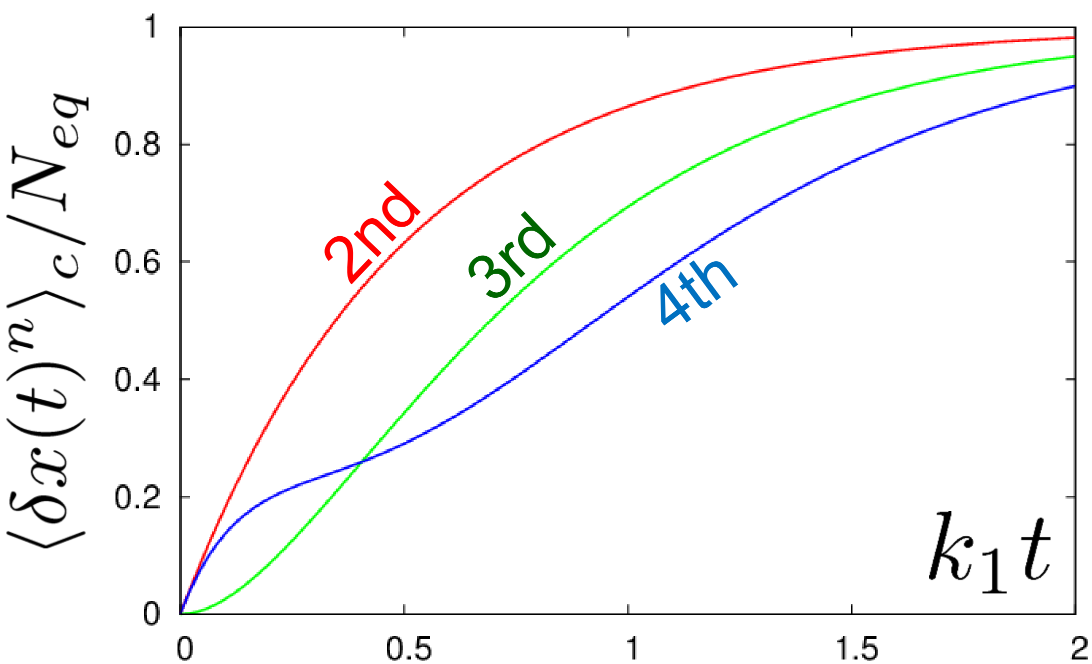
$$\langle \delta x(t)^3\rangle = N_0(e^{-k_1 t} - 3e^{-2k_1 t} + 2e^{-3k_1 t}) + N_{eq}(1-e^{-k_1 t})$$

initial　　　equilibrium

Chemical Reaction 2

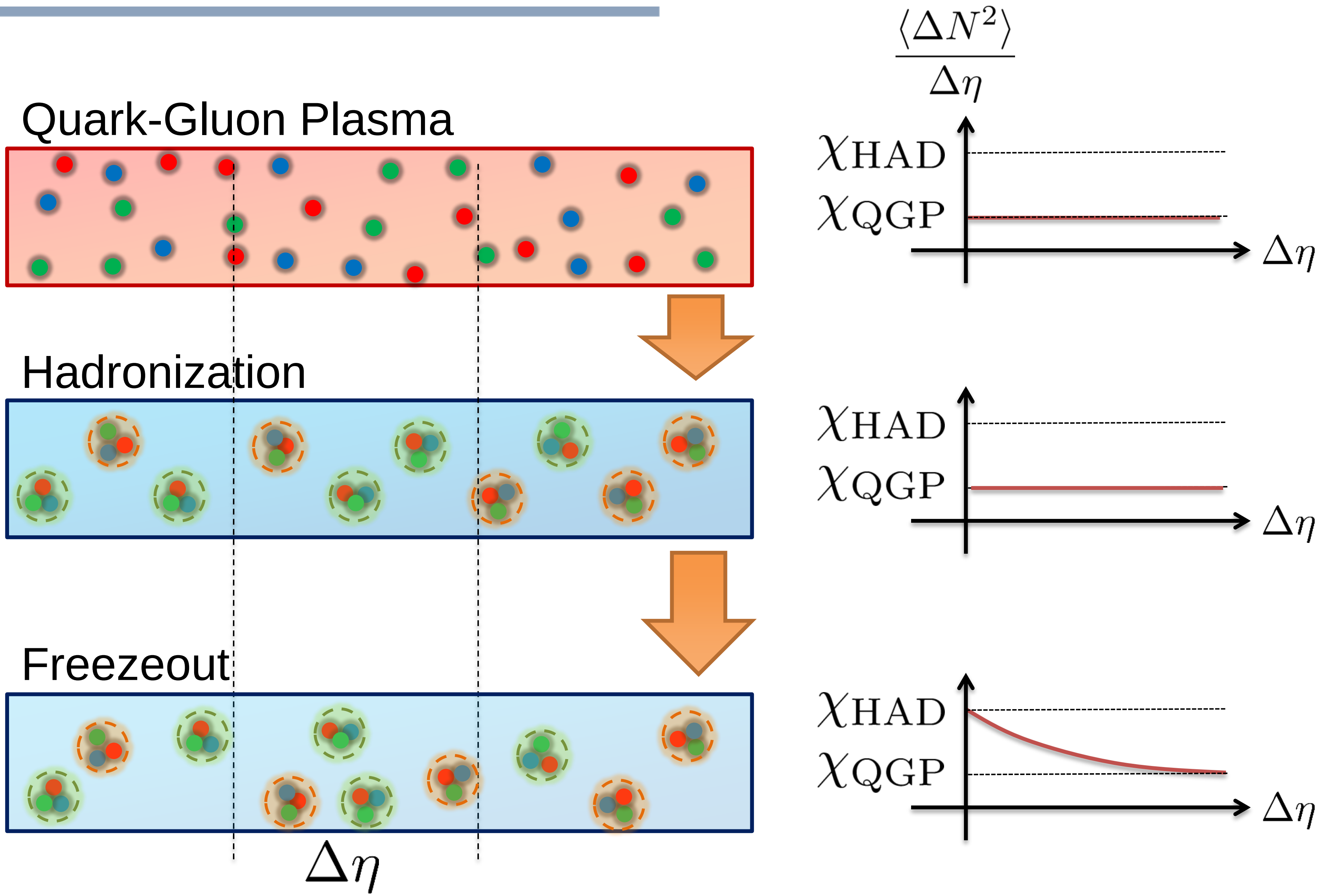
$$X \underset{k_2}{\overset{k_1}{\rightleftharpoons}} A$$

$$N_0 = N_{eq} \Rightarrow \begin{cases} \langle x(t) \rangle = N_{eq} \\ \langle \delta x(t)^2 \rangle = N_{eq}(1 - e^{-2k_1 t}) \\ \langle \delta x(t)^3 \rangle = N_{eq}(1 - 3e^{-2k_1 t} + 2e^{-3k_1 t}) \end{cases}$$



Higher-order cumulants grow slower.

Time Evolution in HIC



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n$$

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

Stochastic Force

determined by fluctuation-dissipation relation

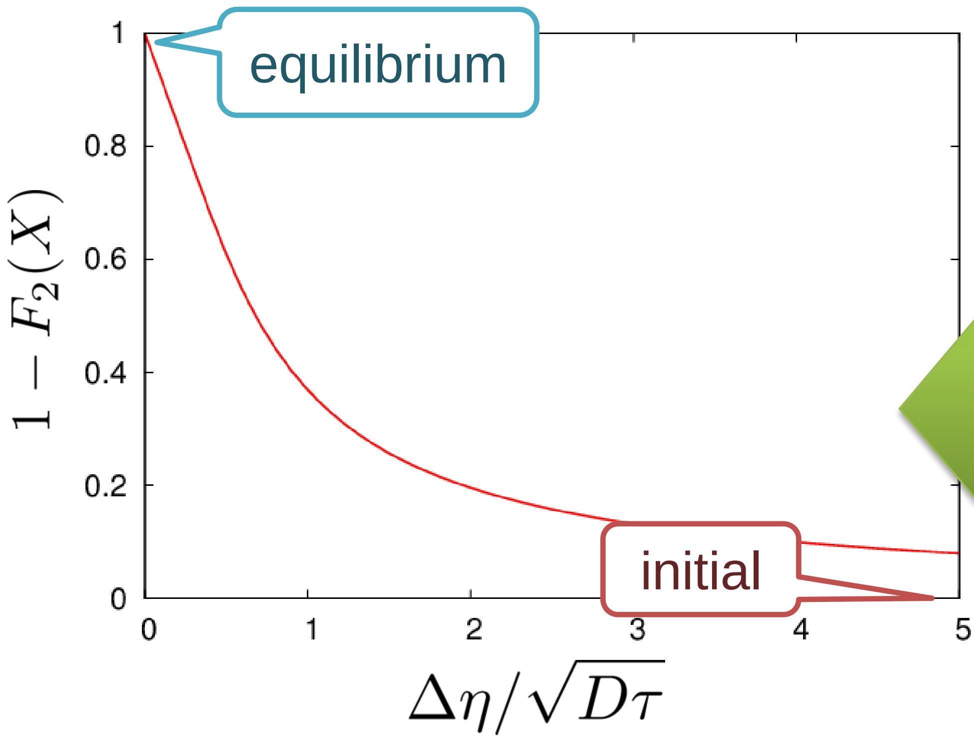
$\Delta\eta$ Dependence

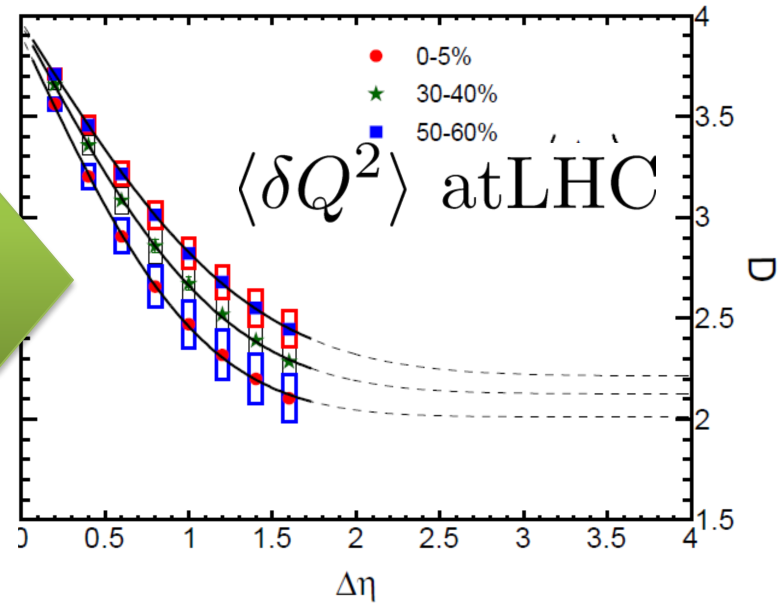
Shuryak, Stephanov, 2001

- Initial condition: $\langle \delta n(\eta_1, 0)\delta n(\eta_2, 0)\rangle = \sigma_2 \delta(\eta_1 - \eta_2)$
- Translational invariance

$$Q(\tau) = \int_0^{\Delta\eta} d\eta\, n(\eta, \tau)$$

$$\langle \delta Q(\tau)^2 \rangle = \underbrace{\sigma_2 F_2(X)}_{\text{initial}} + \underbrace{\chi_2(1 - F_2(X))}_{\text{equilibrium}}$$





Non-Gaussianity in Fluctuating Hydro?

It is **impossible** to directly extend the theory of hydro fluctuations to treat higher orders.

- No a priori extension of FD relations to higher orders
- **Theorem**

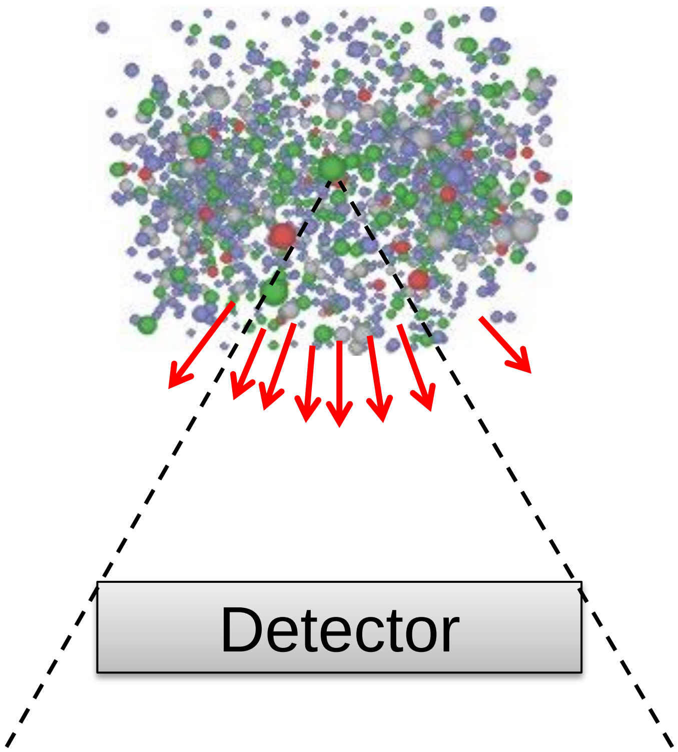
Markov process + continuous variable

➔ Gaussian random force

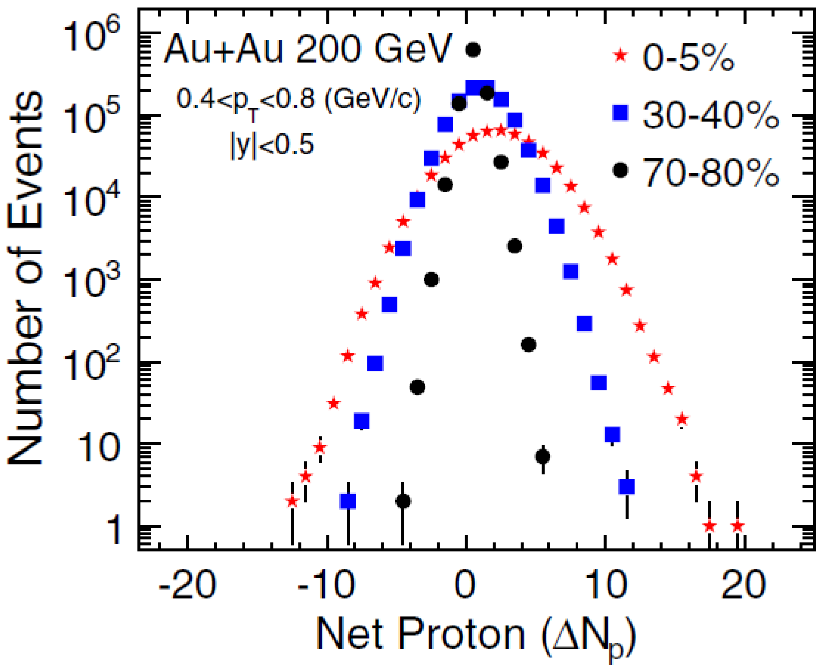
cf) Gardiner, "Stochastic Methods"

Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in HIC.



STAR, PRL**105** (2010)



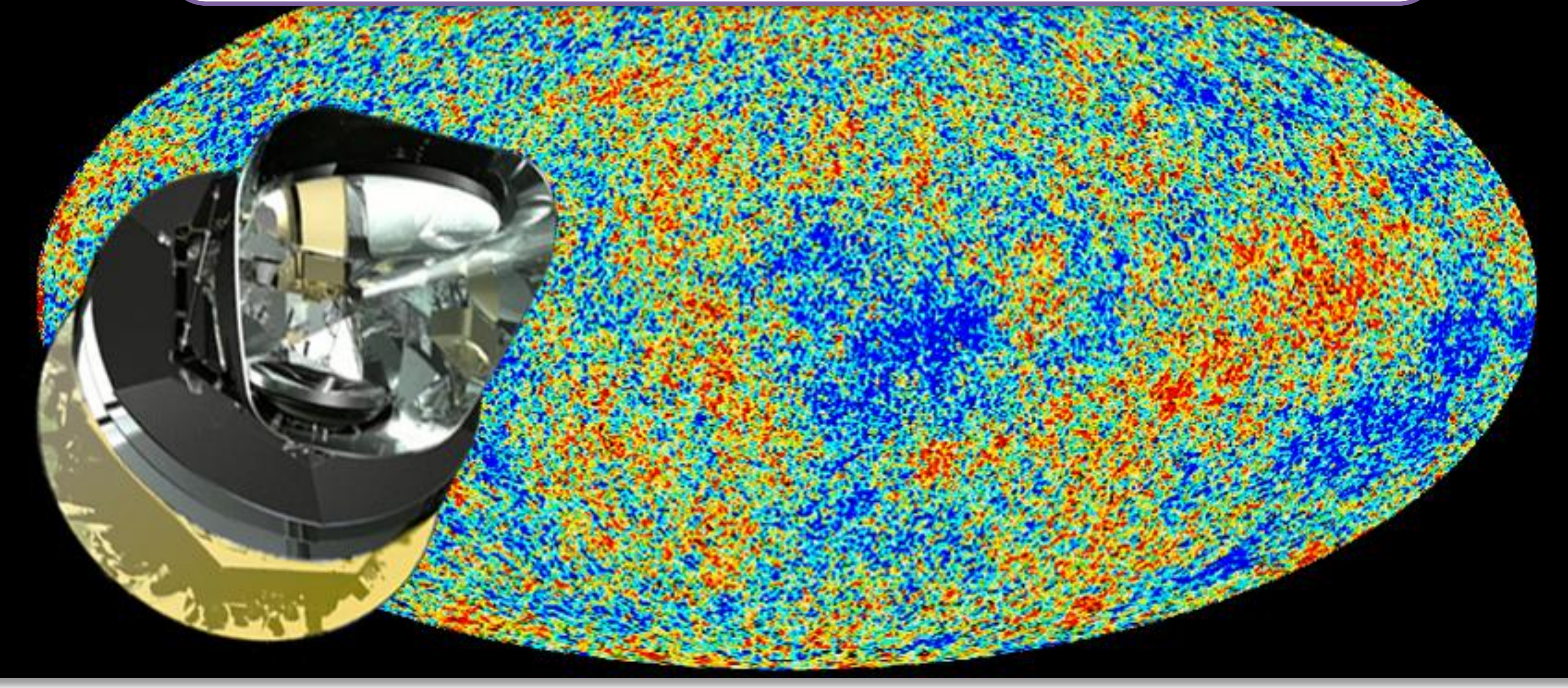
$$\langle \delta N^2 \rangle, \langle \delta N^3 \rangle, \langle \delta N^4 \rangle_c, \cdots$$

Non-Gaussianity

fluctuations (correlations)

$$\langle \delta n_1 \delta n_2 \rangle, \langle \delta n_1 \delta n_2 \delta n_3 \rangle, \langle \delta n_1 \delta n_2 \delta n_3 \delta n_4 \rangle_c, \cdots$$

→ Non-Gaussianity



PLANCK : statistics insufficient to see non-Gaussianity…(2013)