UNIVERSITÀ DEGLI STUDI DI CATANIA
INFN-LNS

THE ELLIPTIC FLOW AND THE SHEAR VISCOSITY OF THE QGP FROM A BEAM ENERGY SCAN

S. Plumari, A. Puglisi, F. Scardina,
V. Greco, L.P. Csernai

- **Transport approach at fixed η/s:**
  - **η ↔ σ(θ), M, T comparing Green-Kubo and Chapman-Enkog approach.**
- **η/s(T) and generation of v_2: from RHIC to LHC.**
- **Conclusions**

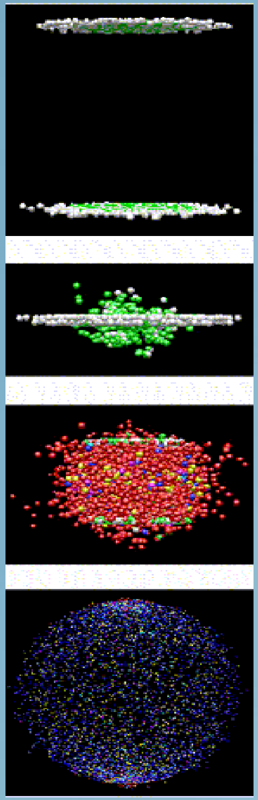
Motivation for a kinetic approach:

$$\left\{p^{\mu}\partial_{\mu}+\left[p_{\nu}F^{\mu\nu}+M\,\partial^{\mu}M\right]\partial^{p}_{\mu}\right\}f(x,p)=C_{22}+C_{23}+\dots$$

Free streaming

Field Interaction → $\varepsilon\neq 3P$

Collisions → $\eta\neq 0$



- It is not a gradient expansion in η/s.
- Valid at intermediate p_T out of equilibrium.
- Valid at high η/s (cross over region).
- Include hadronization by coalescence + fragmentation.
- Allows to study the jet-bulk talk.

Parton Cascade model

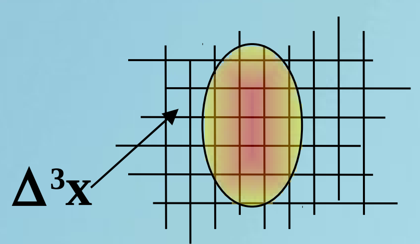
$$p^{\mu}\partial_{\mu} f(X,p) = C = C_{22} + C_{23} + \ldots$$

Collisions → $\left\{ \begin{array}{l} \varepsilon - 3p = 0, \\ \eta \neq 0 \end{array} \right.$

$$C_{22} = \frac{1}{2E_1}\int \frac{d^3 p_2}{(2\pi)^3 2E_2}\frac{1}{v}\int \frac{d^3 p'_1}{(2\pi)^3 2E'_1}\frac{d^3 p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |M_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$

For the numerical implementation of the collision integral we use the stochastic algorithm. (Z. Xu and C. Greiner, PRC 71 064901 (2005))

$$P_{22} = \frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{rel}\,\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$



$\Delta t \to 0$

$\Delta^3 x \to 0$

⇨ **right solution**

Passed several numerical test on the box.

Extraction of the Shear Viscosity: Box calculation

Green – Kubo relation

$$\eta = \frac{1}{T}\int dt \int d^3x \langle \pi^{xy}(x,t)\,\pi^{xy}(0,t)\rangle$$

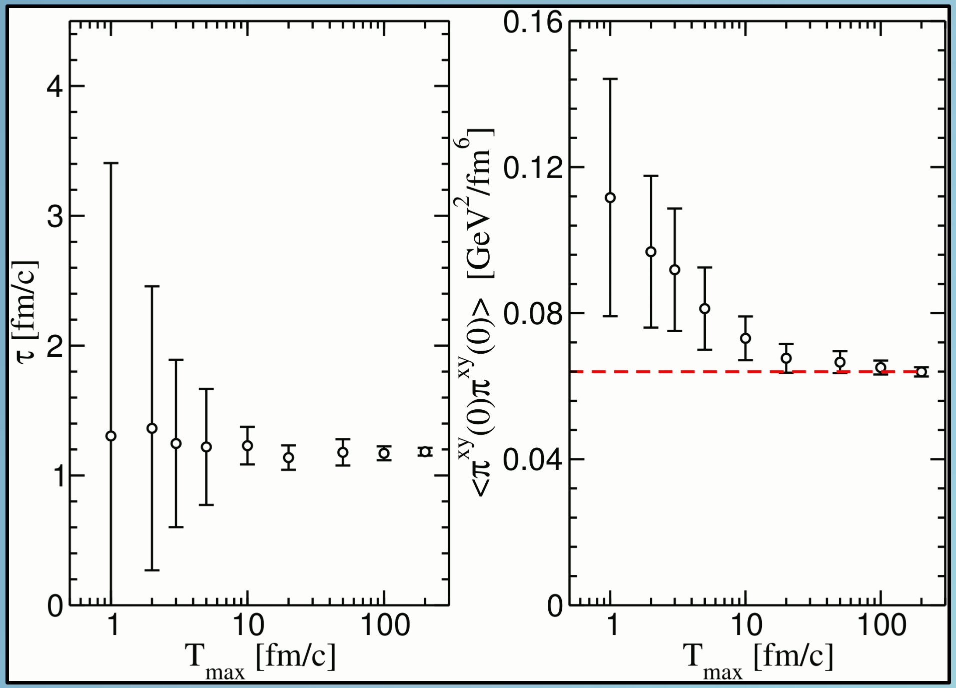
$$\langle \pi^{xy}(\vec{x},t)\,\pi^{xy}(\vec{0},t)\rangle = \langle \pi^{xy}(0)\,\pi^{xy}(0)\rangle \cdot e^{-t/\tau}$$

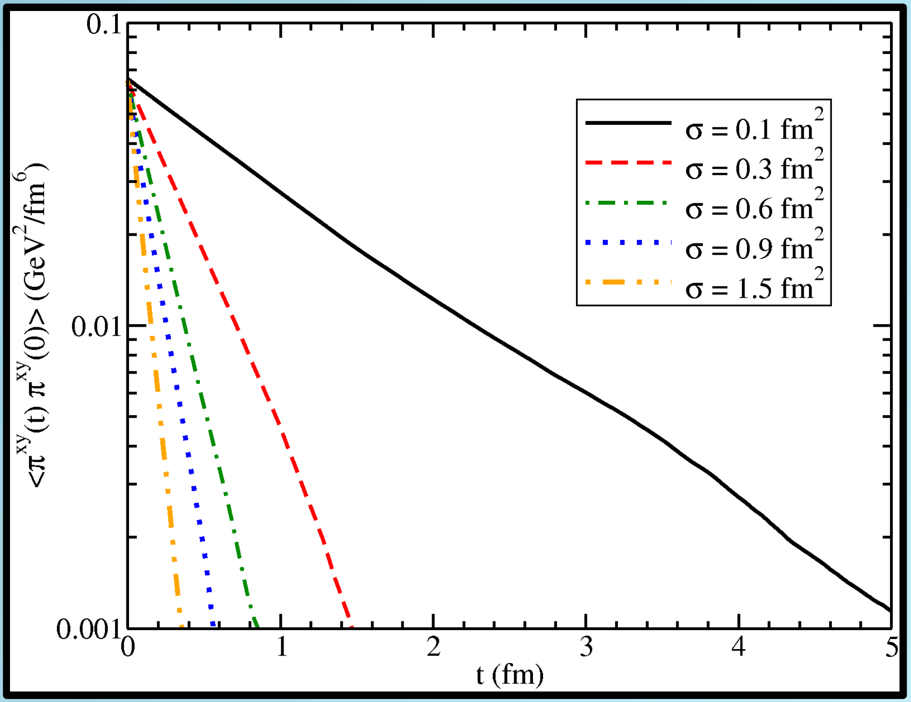
Depends on microscopical details: τ(σ)

$$\eta = \frac{V}{T}\langle \pi^{xy}(0)\,\pi^{xy}(0)\rangle \cdot \tau$$

Depends on macroscopical details:

$$= \frac{4}{15}\,\frac{eT}{V}$$





S. Plumari et al.,Phys.Rev. C86 (2012) 054902.
C. Wesp et al., Phys. Rev. C 84, 054911 (2011).
J. Fuini III et al. J. Phys. G38, 015004 (2011).

Extraction of the Shear Viscosity: Box calculation

$$\eta^{IS}_{relax}/s = \frac{1}{15}\langle p\rangle \tau_r = \frac{1}{15}\frac{\langle p\rangle}{\sigma_{tot}\langle f(a) v_{rel}\rangle \rho}$$

$$\sigma_{tr} = \int d\Omega \sin^2(\theta_{cm}) \frac{d\sigma}{d\Omega_{cm}} = \sigma_{tot} f(a) \leq \frac{2}{3}\sigma_{tot}$$

Employed also for non-isotropic cross section:

G.Ferini, PLB(2009); D. Molnar, JPG35(2008); V.Greco, PPNP(2009);

For the standard pQCD-like cross section

$$\frac{d\sigma}{d\Omega_{cm}} = \frac{9\pi\alpha_S^2}{2}\frac{1}{(q^2+m_D^2)^2}\left(1+\frac{m_D^2}{s}\right)$$

→

m_D regulates the anisotropy of collision
$m_D \to \infty$ we recover the isotropic limit

$$f(a) = 4a(1+a)\left[(2a+1)\ln(1+a^{-1}) - 2\right], \quad a = m_D^2/s$$

1st Chapman-Enskog approximation

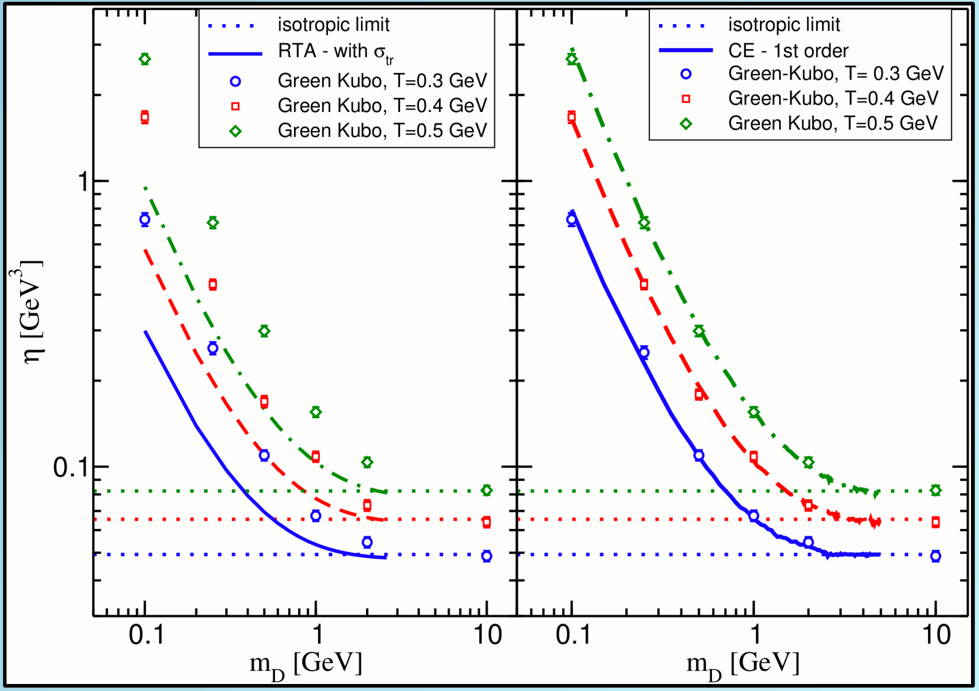
$$[\eta]_{1st}/s = \frac{1}{15}\langle p\rangle \tau_\eta = \frac{1}{15}\frac{\langle p\rangle}{\sigma_{tot}\, g(a)\rho}$$

$$g(a) = \frac{1}{50}\int dy\, y^6\left[\left(y^2+\frac{1}{3}\right)K_3(2y) - yK_2(2y)\right] f(a), \quad a = \frac{m_D}{2T}$$

- **CE and RTA can differ by a factor of 2**
- **Green-Kubo agree with CE (< 5%)**

A. Wiranata, M. Prakash, PRC 85 (2012) 054908.
O. N. Moroz, arXiv:1112.0277 [hep-ph].

S. Plumari et al.,PRC86 (2012) 054902.



- **We know how to fix locally $\eta/s(T)$**

- **We have checked the Chapmann-Enskog:**

- CE good already at I° order ≈ 5% (≈ 3% at II° order)

- RTA even with σtr severely underestimates η

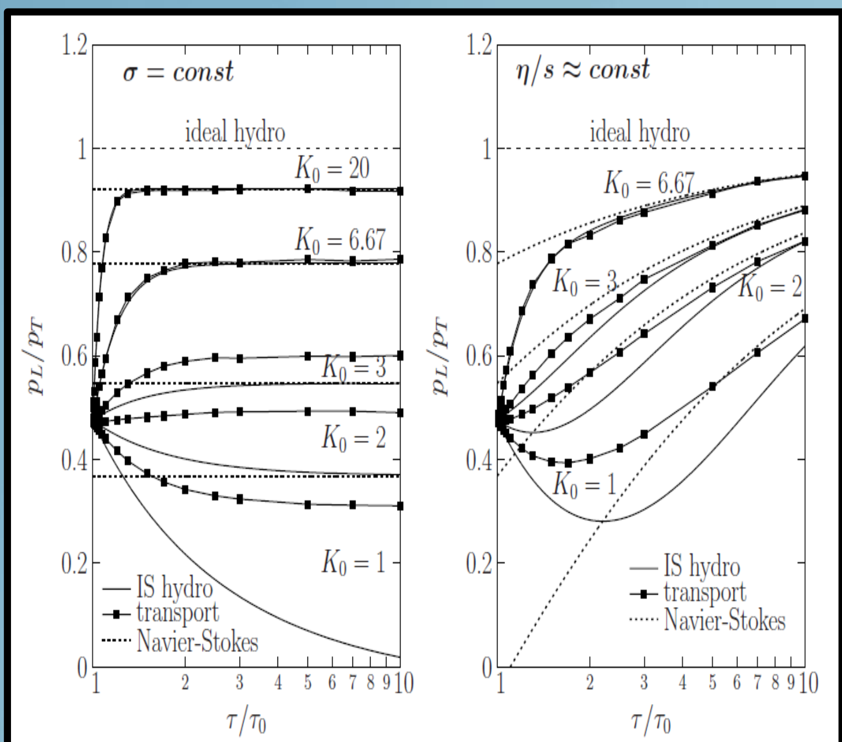
Simulating a constant η/s

For the general case of anisotropic cross section and massless particles:

$$\eta(\vec{x},t)/s = \frac{1}{15}\langle p\rangle\,\tau_\eta \quad\Longrightarrow\quad \sigma_{tot}^{\eta/s} = \frac{1}{15}\,\frac{\langle p\rangle}{g(m_D/2T)\,n}\,\frac{1}{\eta/s} = K\sigma_{tot}^{pQCD}$$

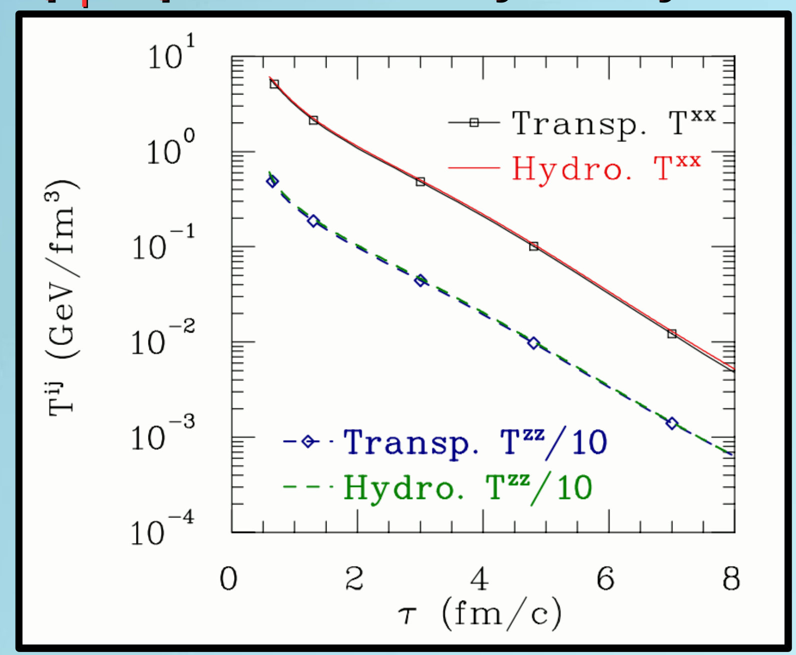
σ is evaluated in such way to keep the η/s=cost. during the dynamics.

(similar to D. Molnar, arXiv:0806.0026[nucl-th] but our approach is local.)



Huovinen and Molnar, PRC79(2009)

At low p_T equivalent to Hydro dynamics



D. Molnar et al., JPG 35, (2008). QM08

Initial condition of our simulation

- r-space: standard Glauber model
- p-space: Boltzmann-Juttner Tmax=1.7-3.5 Tc
- [pT<2 GeV]+ minijet [pT>2-3GeV]

Discarded in viscous hydro

We fix maximum initial T at RHIC 200 AGeV

T_{max0} = 340 MeV

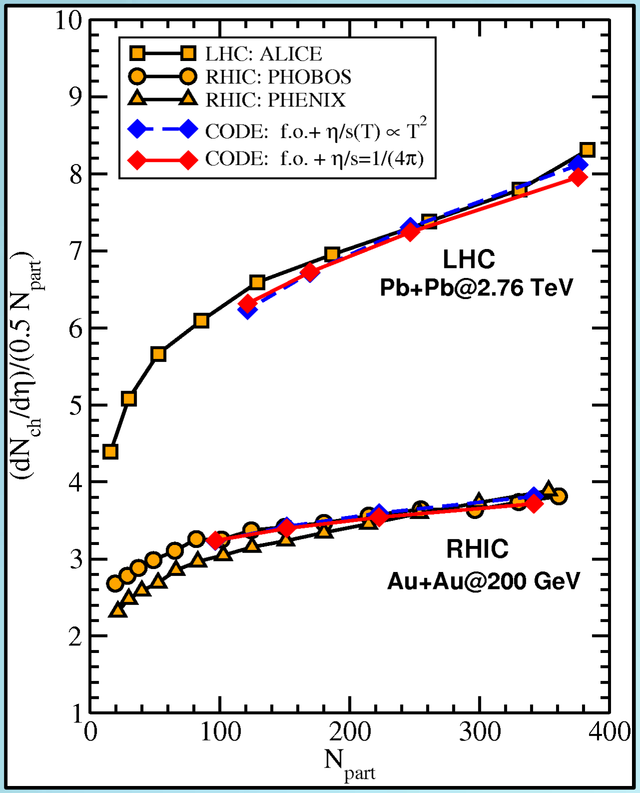
$T_0\ \tau_0$ =1 -> τ_0=0.6 fm/c

Typical hydro condition

Then we scale it according to

$$\frac{1}{\tau A_T}\frac{dN_{ch}}{d\eta} \propto T^3$$

√s	62 GeV	200 GeV	2.76 TeV
T_0	290 MeV	340 MeV	590 MeV
τ_0	0.7 fm/c	0.6 fm/c	0.3 fm/c



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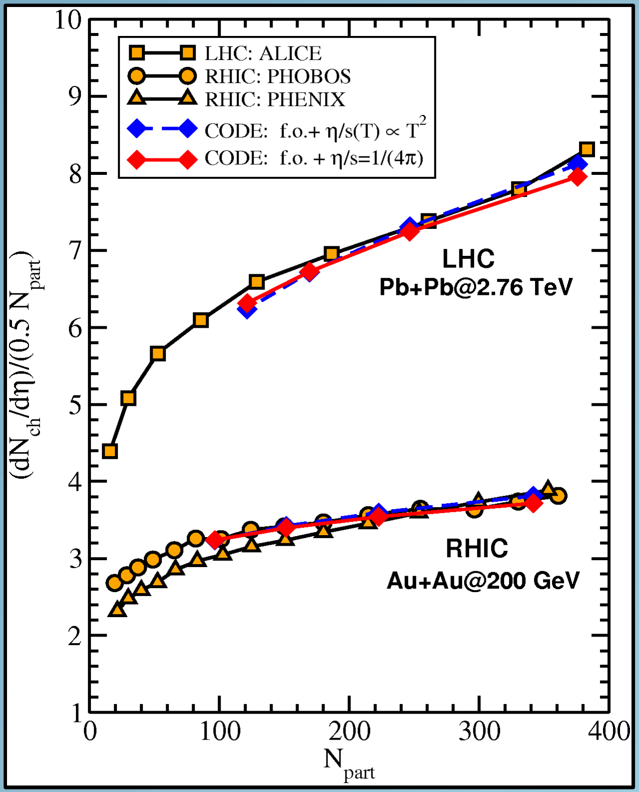
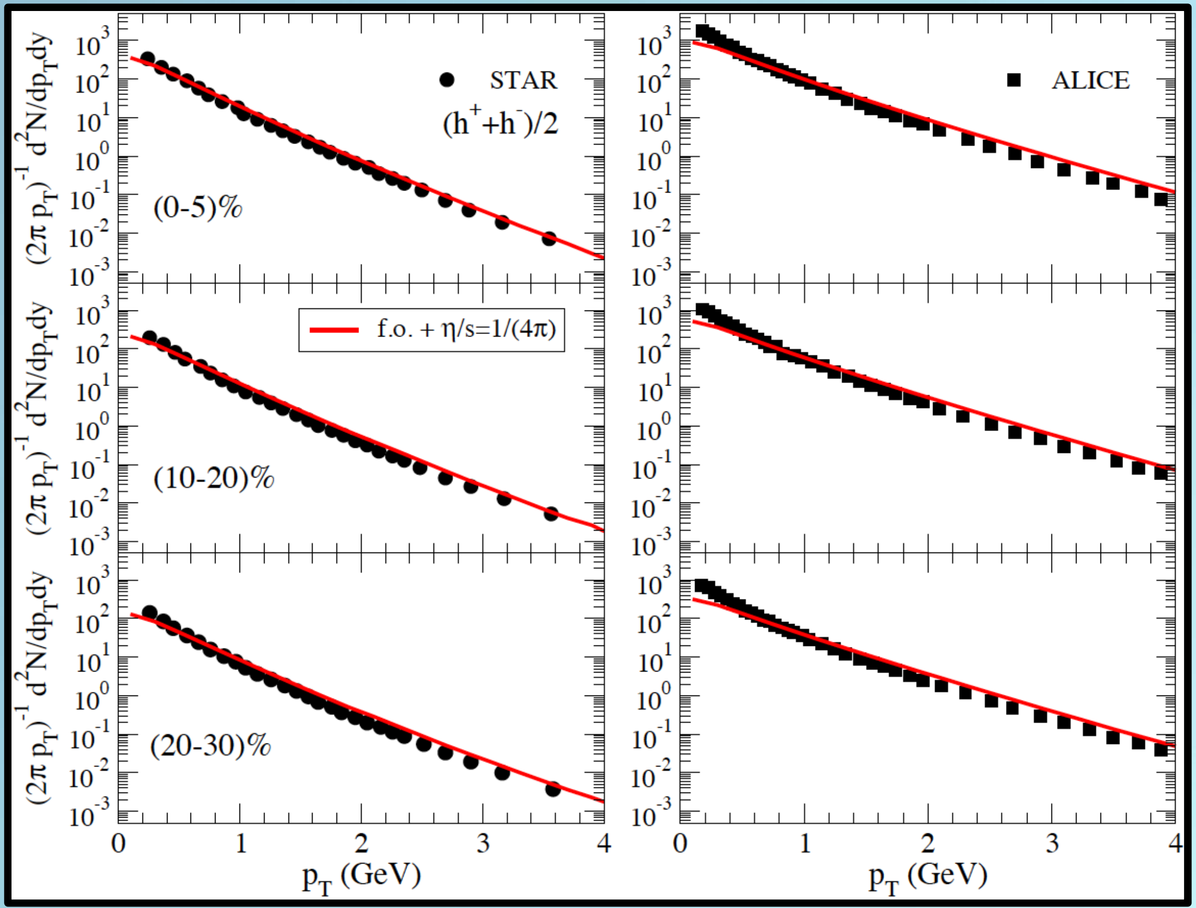
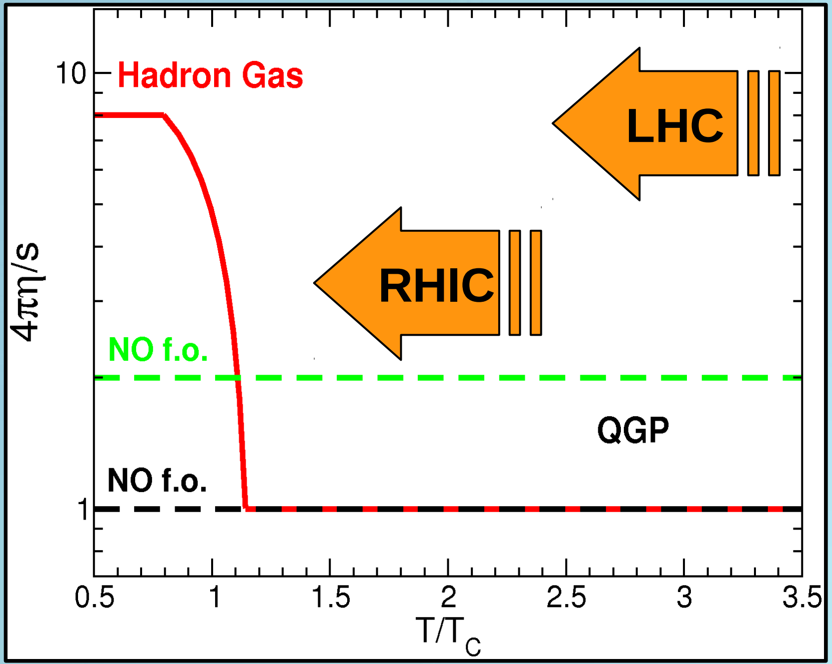




kinetic freeze-out scheme

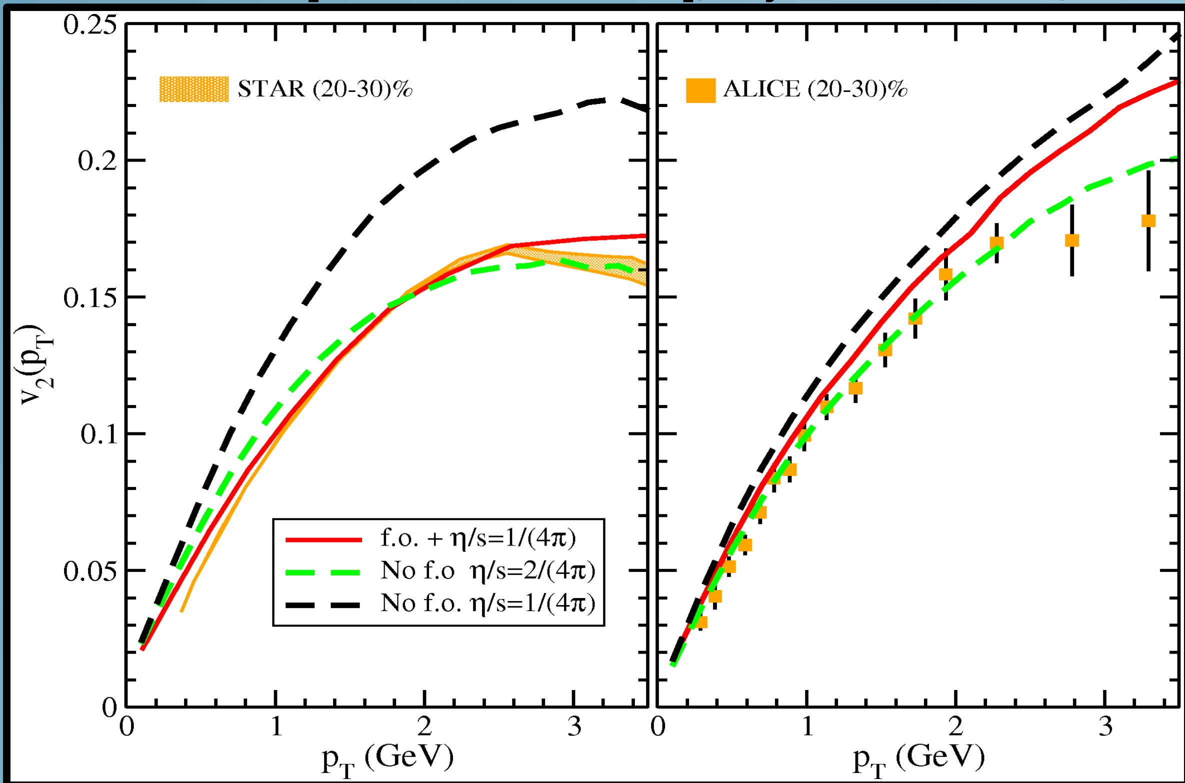
- The f.o. Is the increase of η/s in the cross-over region, with a smooth transition between the QGP and the hadronic phase, the collisions are switched off.

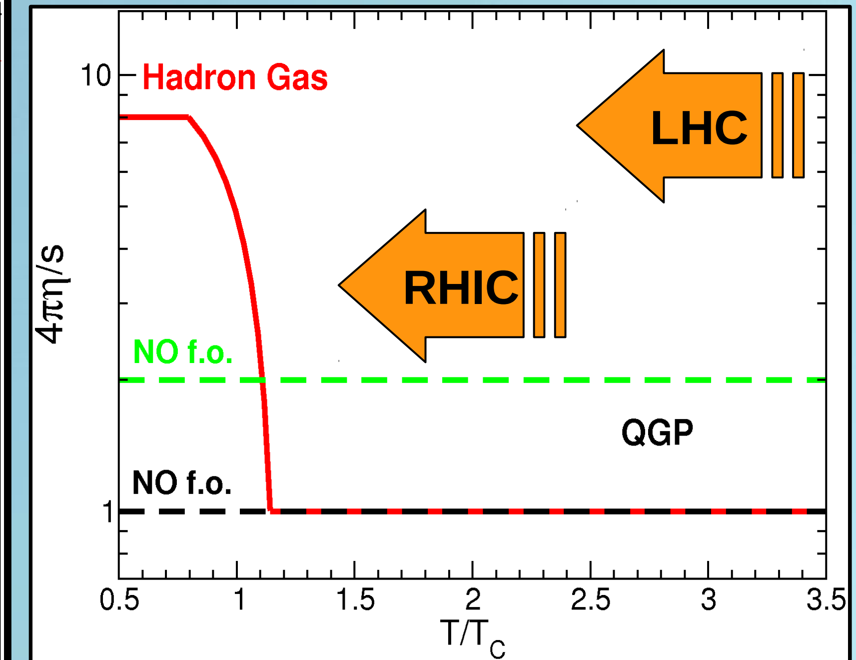


For the v_2 similar to cut-off at ε_0=0.7 GeV/fm^3

kinetic freeze-out scheme

K. Aamodt et al. [ALICE Collaboration], Phys. Rev.Lett. 105, 252302 (2010).





S. Plumari et al., J. Phys.: Conf. Ser. 420 012029 (2013). arXiv:1209.0601.

RHIC:

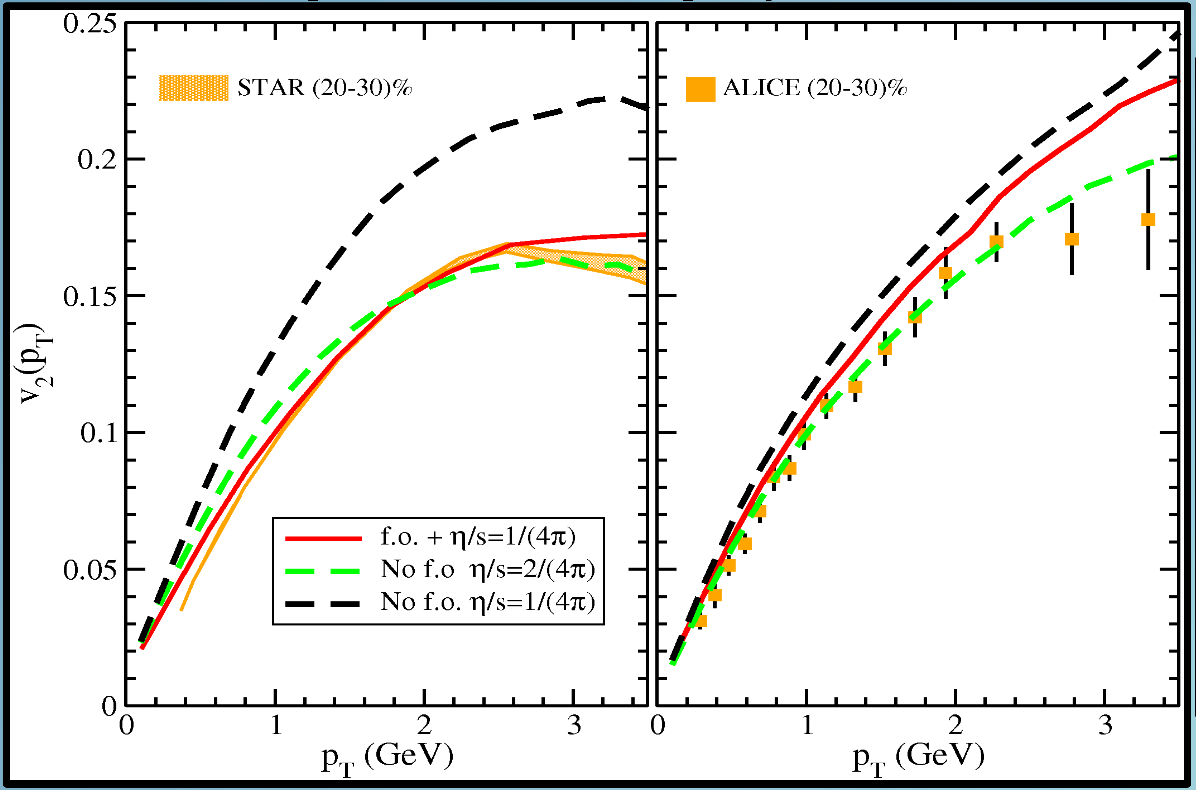
- Like viscous hydro the data are close to $\eta/s=1/(4\pi)$ + f.o.
- Sensitive reduction of the v_2 when the f.o. is included the effect is about of 20%.
- p_T < 2.5 GeV good agreement with the experimental data.

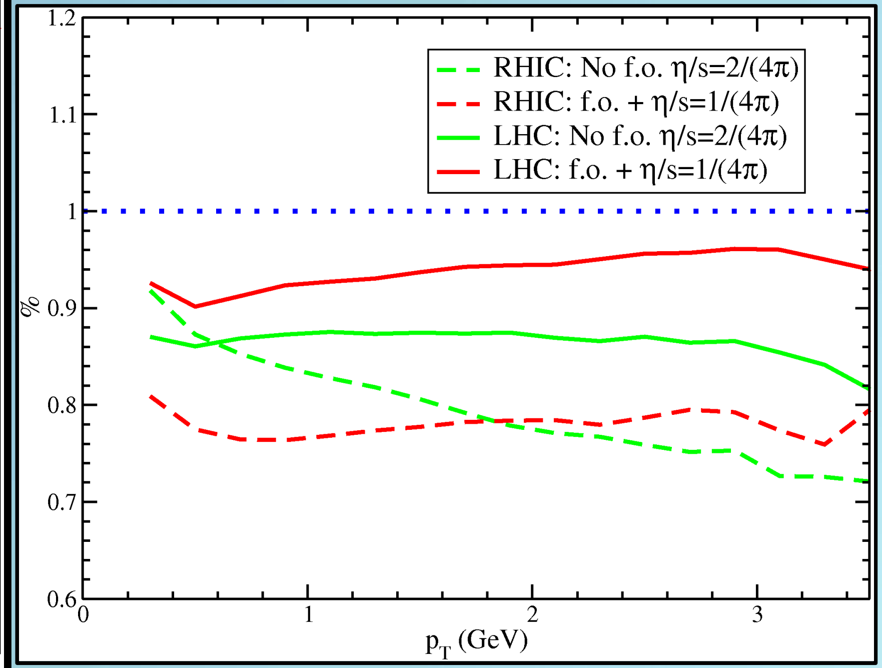
LHC:

- p_T< 2 GeV like hydro data described with $\eta/s=1/(4\pi)$ + f.o.
- Smaller effect on the reduction of the v_2 when the f.o. is included an effect of about 5%.
- Without the kinetic freezout the effect of a constant $\eta/s=2(4\pi)^{-1}$ is to reduce the v_2 of 15%.

kinetic freeze-out scheme

K. Aamodt et al. [ALICE Collaboration], Phys. Rev.Lett. 105, 252302 (2010).



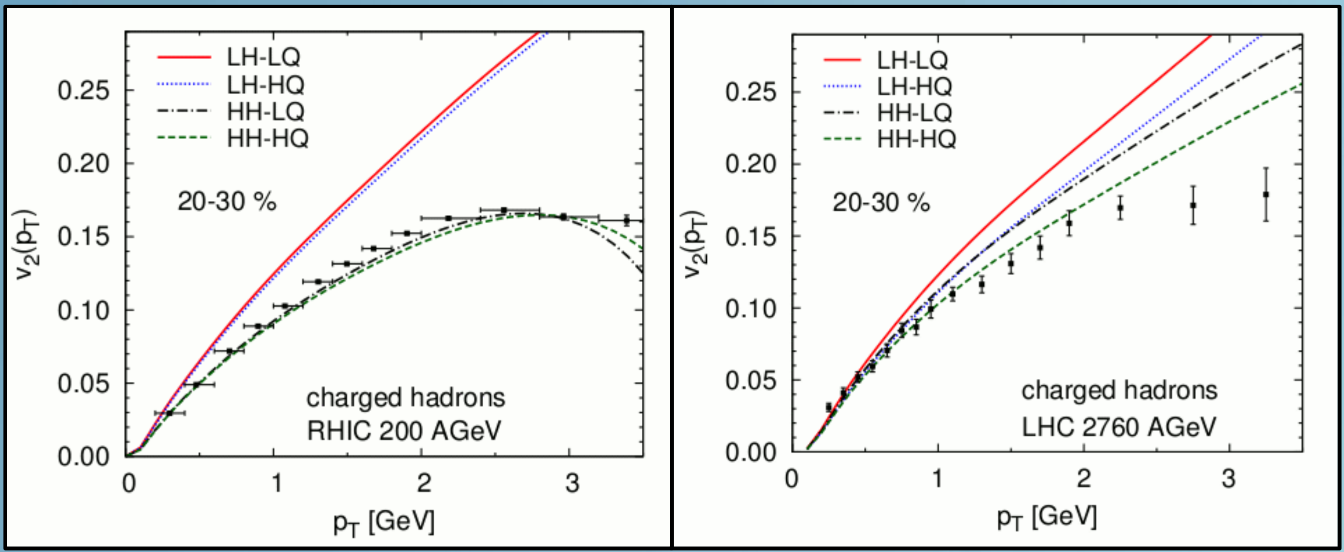


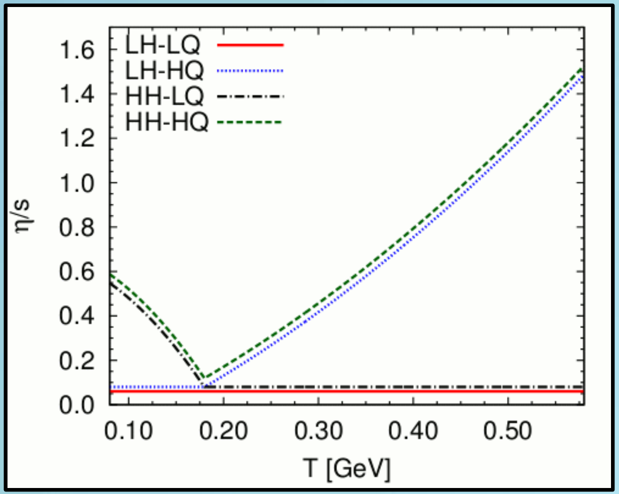
S. Plumari et al., J. Phys.: Conf. Ser. 420 012029 (2013). arXiv:1209.0601.

At LHC the contamination of mixed and hadronic phase becomes negligible

Longer life time of QGP → v_2 completely developed in the QGP phase (at least up to 3 GeV)

Effect of η/s(T) in Hydro: Niemi et al.





Niemi et al., PRL 106 (2011).

$$T^{\mu\nu} = T^{\mu\nu}_{eq} + \delta T^{\mu\nu} \leftarrow f_{eq} + \delta f$$

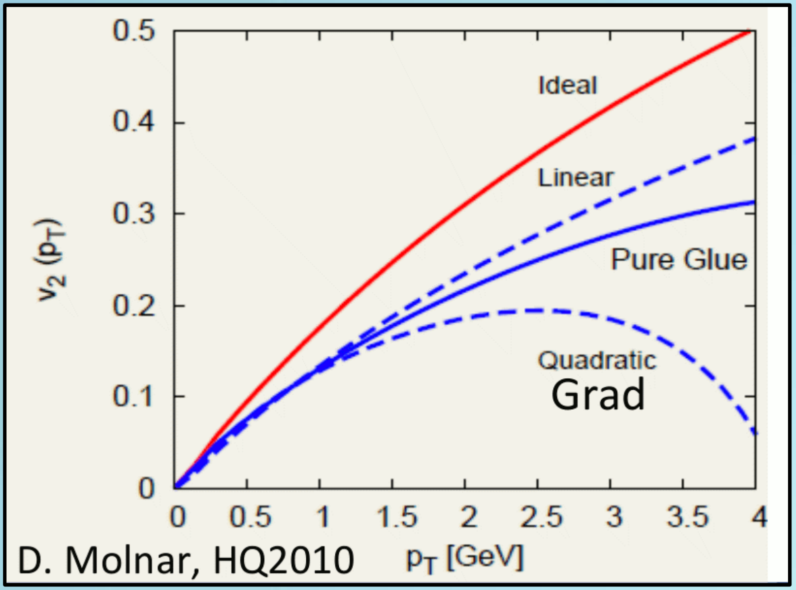
Grad ansantz

R. Lacey et al., PRC82

$$\delta f = \frac{\pi^{\mu\nu} p_\mu p_\nu}{(\varepsilon + p) T^2} f_{eq} \approx \frac{\eta}{3s} \frac{p_T^2}{\tau T^2} f_{eq}$$

- This inplies that the η is in Relaxation Time Approximation
  D. Teaney,Phys.Rev. C68 (2003) 034913

- Hydro is valid up to p_T~3 GeV



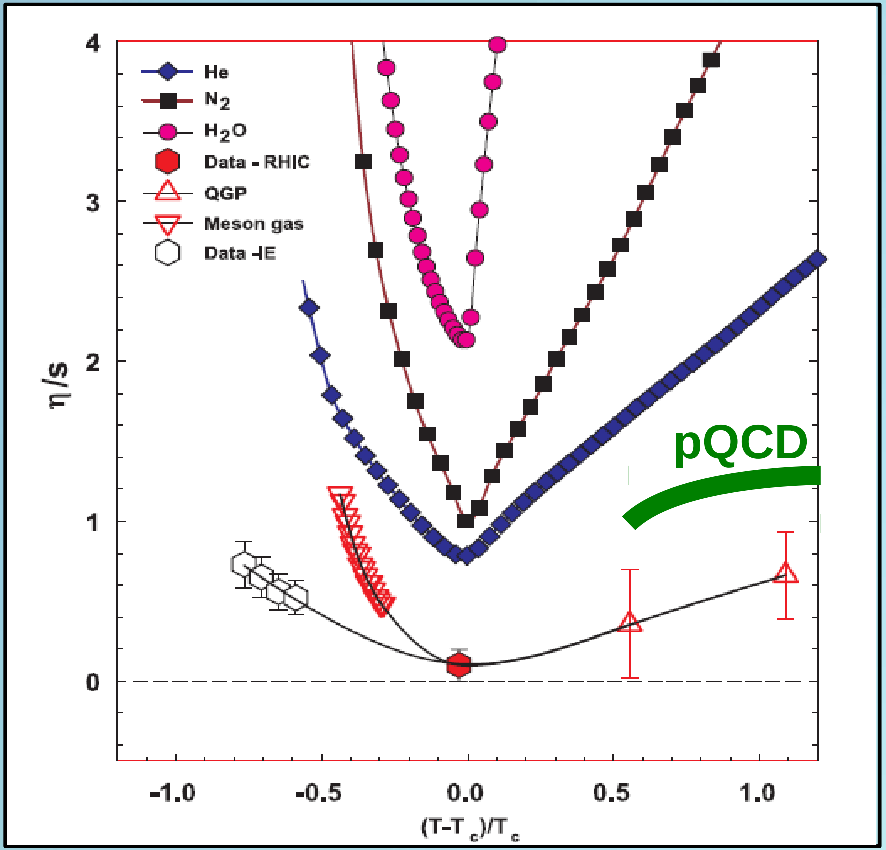
η/s(T) around to a phase transition

P. Kovtun et al.,Phys.Rev.Lett. 94 (2005) 111601.
L. P. Csernai et al., Phys.Rev.Lett. 97 (2006) 152303.
R. A. Lacey et al., Phys.Rev.Lett. 98 (2007) 092301.

- **Quantum mechanism**

$$\Delta E \cdot \Delta t \geq 1 \;\rightarrow\; \eta/s \approx \frac{1}{15}\langle p \rangle \cdot \tau > \frac{1}{15}$$

- **AdS/CFT suggest a lower bound η/s = 1/(4 π)~0.08**



The QGP viscosity is close to this bound!

Do we have signature of a 'U' shape of η/s(T) for the QCD matter ?

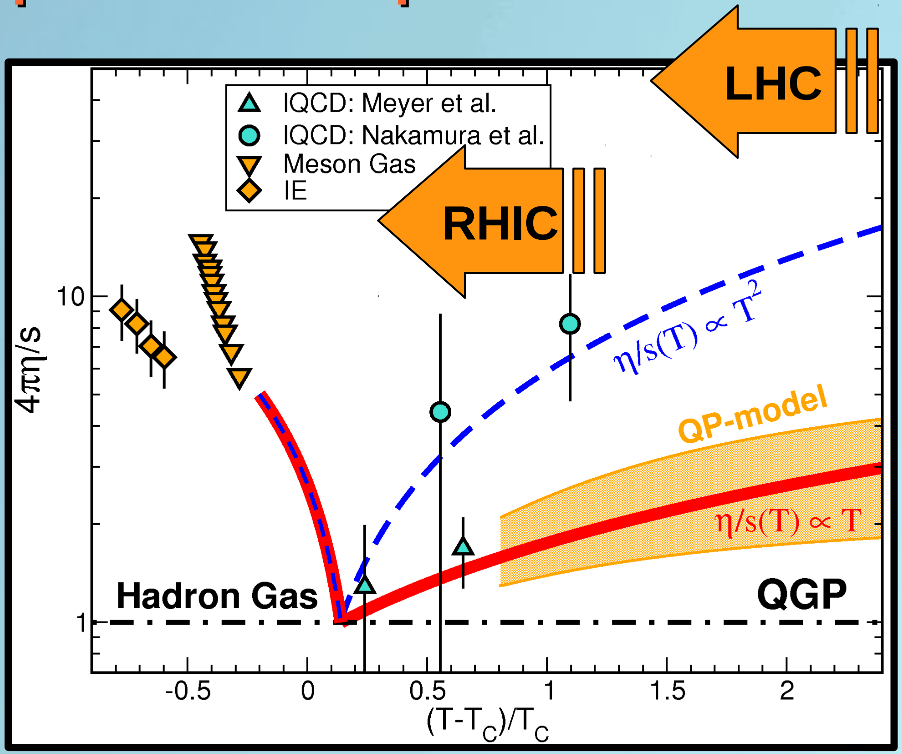
From pQCD: $\eta/s \sim \frac{1}{g^4 \ln(1/g)} \;\Rightarrow\; \eta/s \sim 1$

P.Arnold et al., JHEP 0305 (2003) 051.

Temperature dependent η/s(T)

Phase transition physic suggest a T dependence of η/s also in the QGP phase

- LQCD some results for quenched approx. large error bars
- Quasi-Particle models seem to suggest a $\eta/s \sim T^{\alpha}$ $\alpha \sim 1 - 1.5$.
- Chiral perturbation theory → Meson Gas
- Intermediate Energies – IE ($\mu_B > T$)



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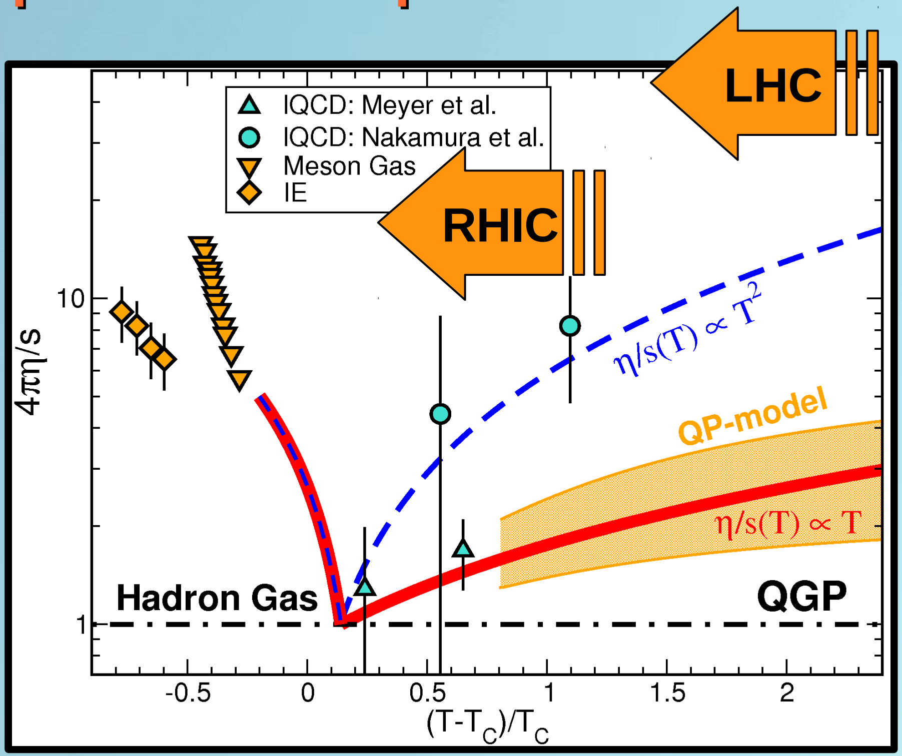

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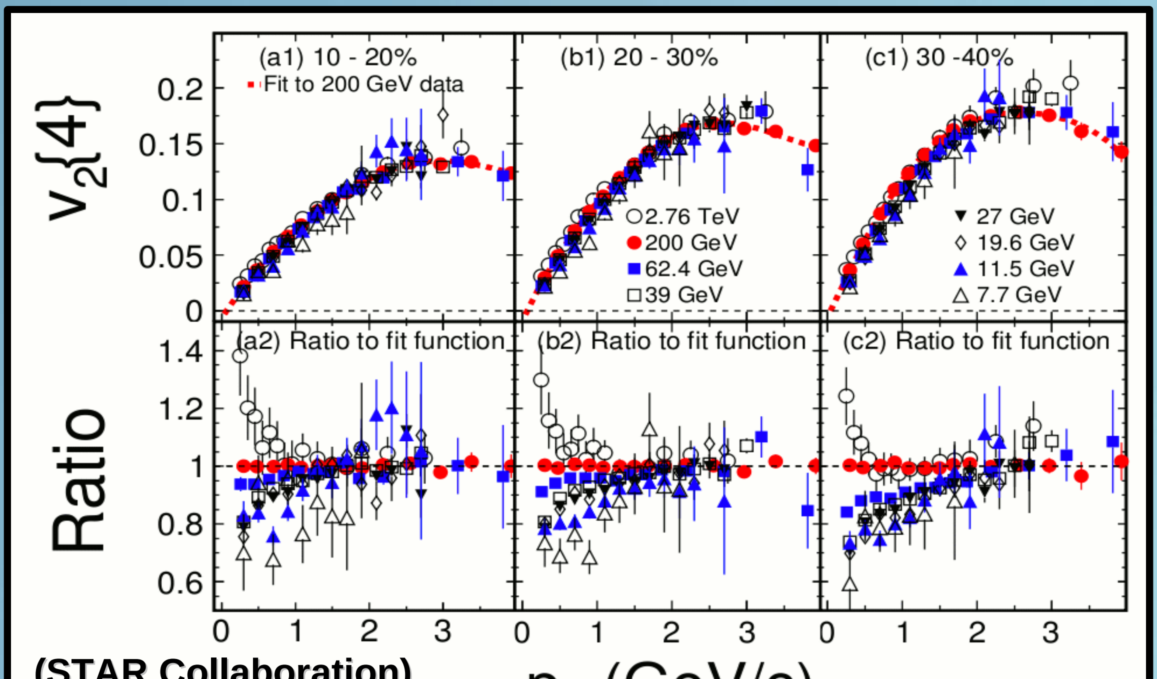
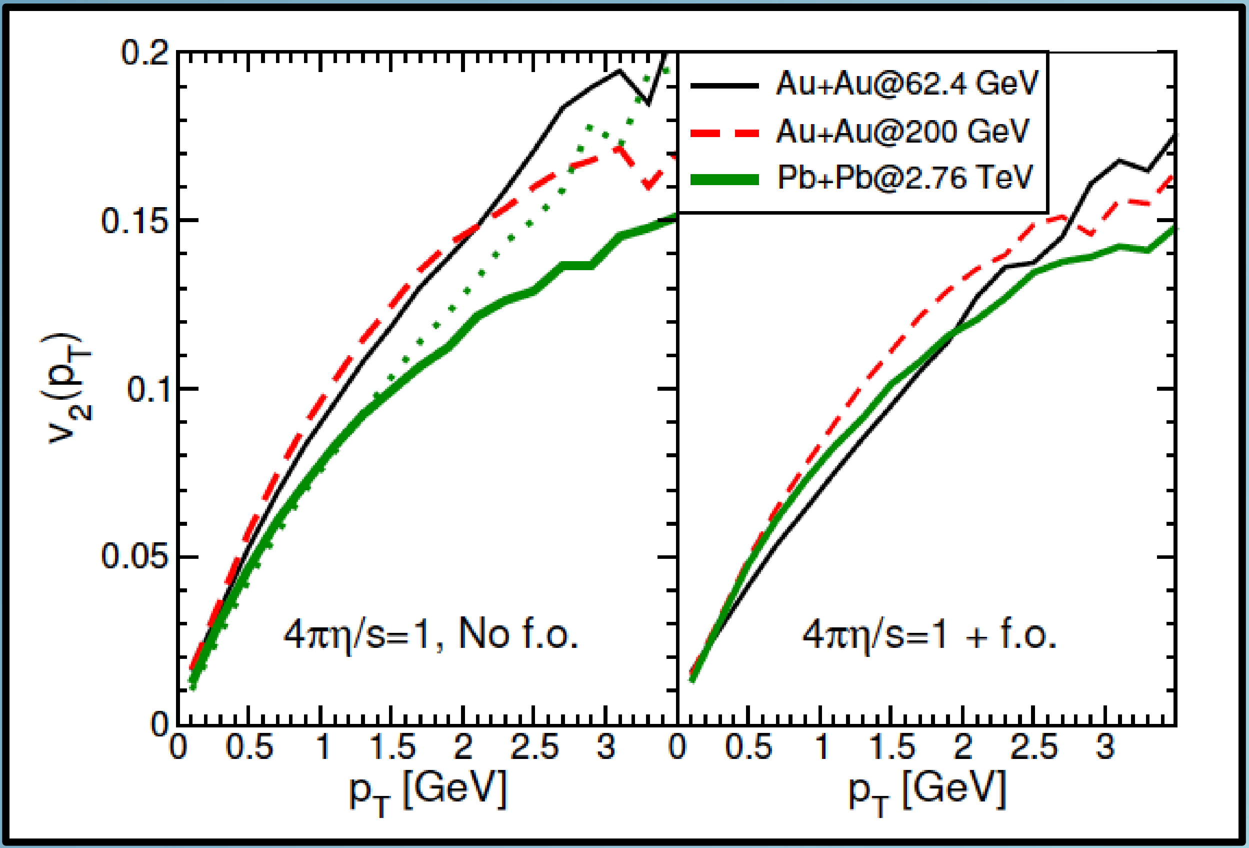
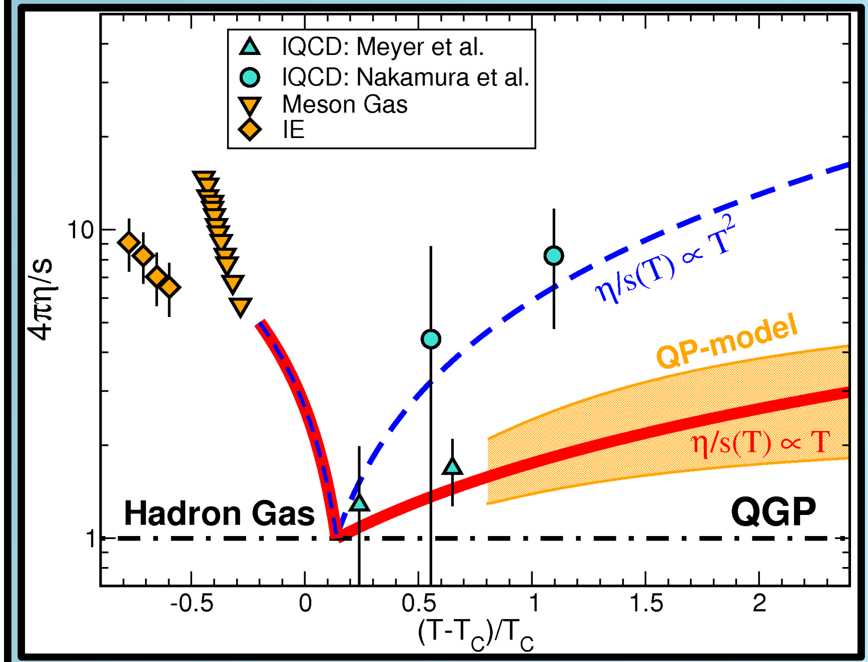


(STAR Collaboration), arXiv:1206.5528 [nucl-ex].

Temperature dependent η/s(T)

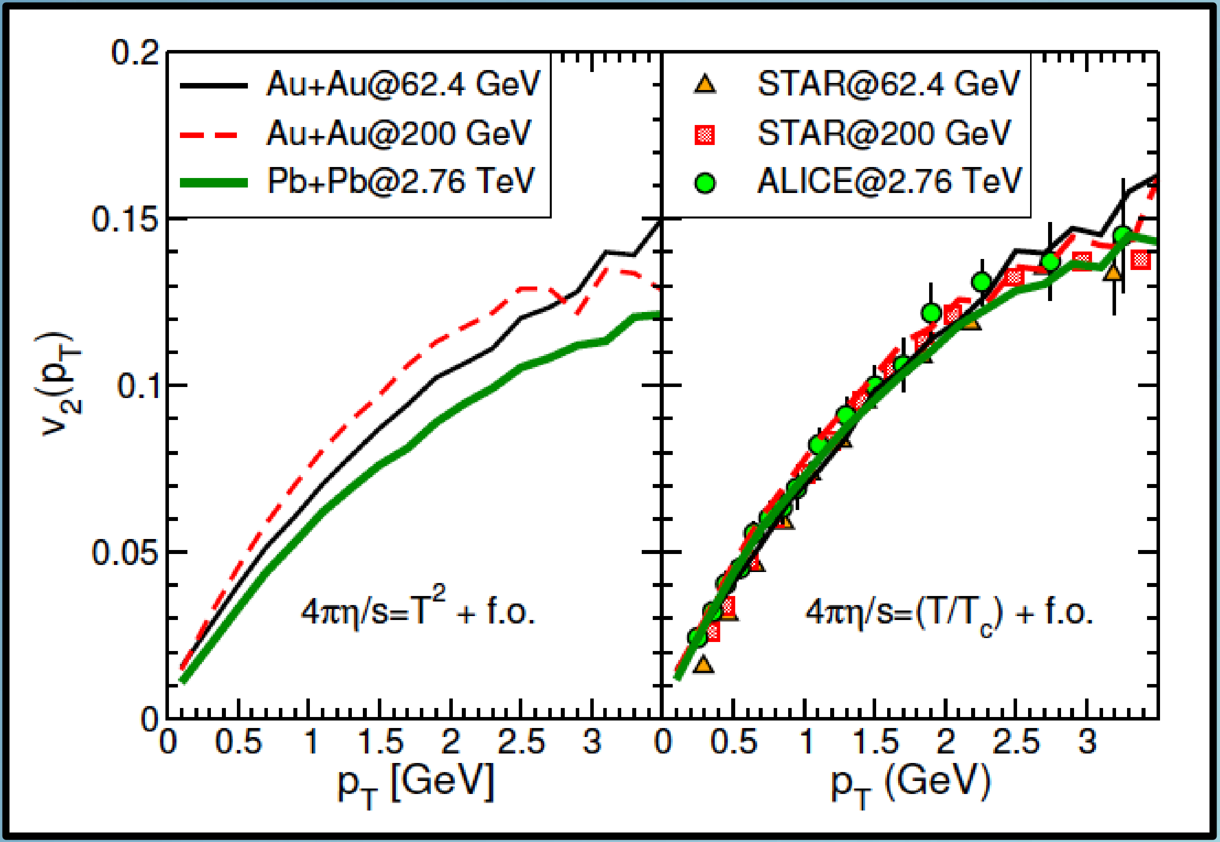


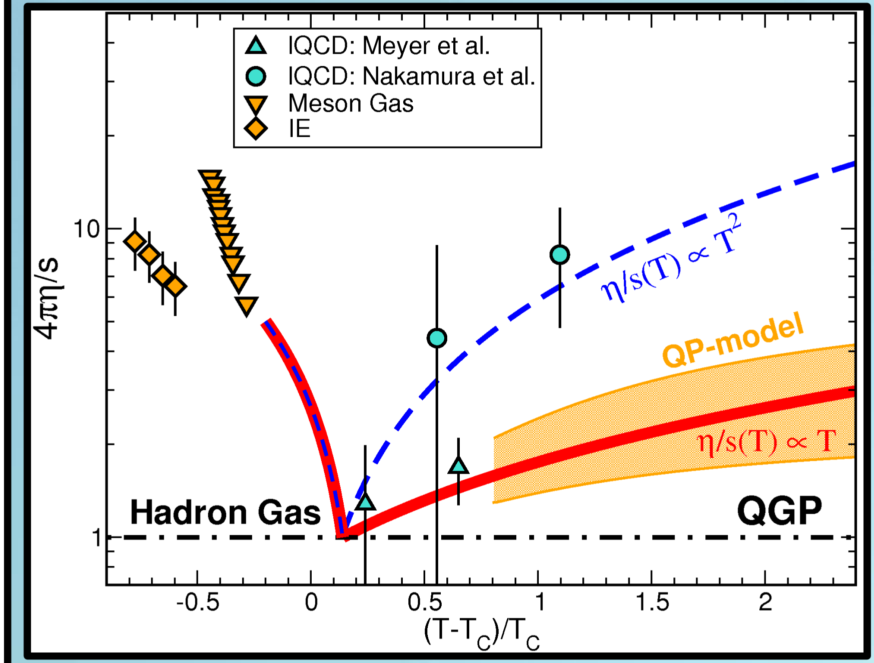


Plumari, Greco, Csernai, arXiv:1304.6566

- **For 4πη/s=1 during all the evolution of the fireball we get a discrepancy for the $v_2(p_T)$, in particular we observe a smaller $v_2(p_T)$ at LHC.**
- **Similar results for η/s∝ T² → a discrepancy about 20%.**
- **Notice only with RHIC → scaling for 4πη/s=1 LHC data play a key role**

Temperature dependent η/s(T)





Plumari, Greco, Csernai, arXiv:1304.6566

- **Invariance of $v_2(p_T)$ in BES suggest that the system goes through a phase transition.**
- **Hope: v_n, n>3 with an event-by-event analysis will put even stronger consstraint**
- **Implementation of local fluctuation under development**

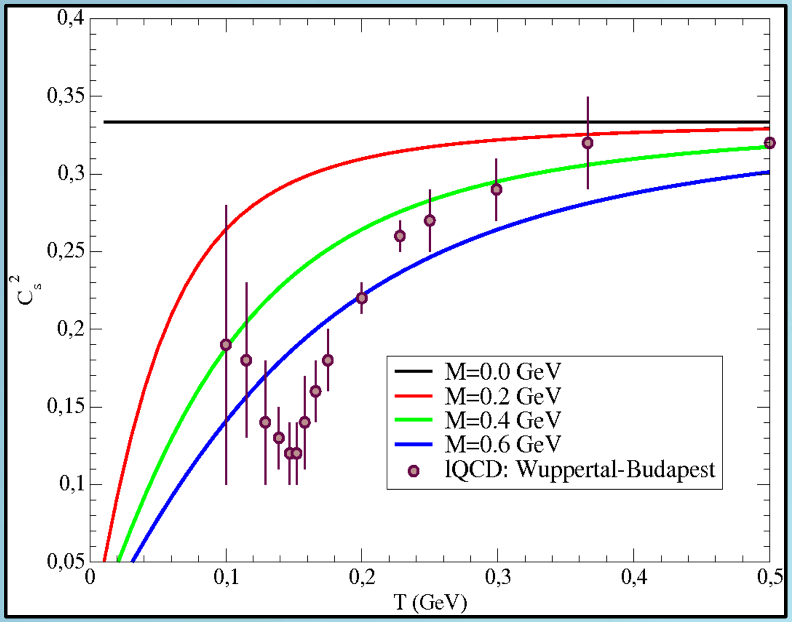
- **Similar results: R. A. Lacey et al., arXiv:1305.3341 [nucl-ex].**

Conclusions and Outlook

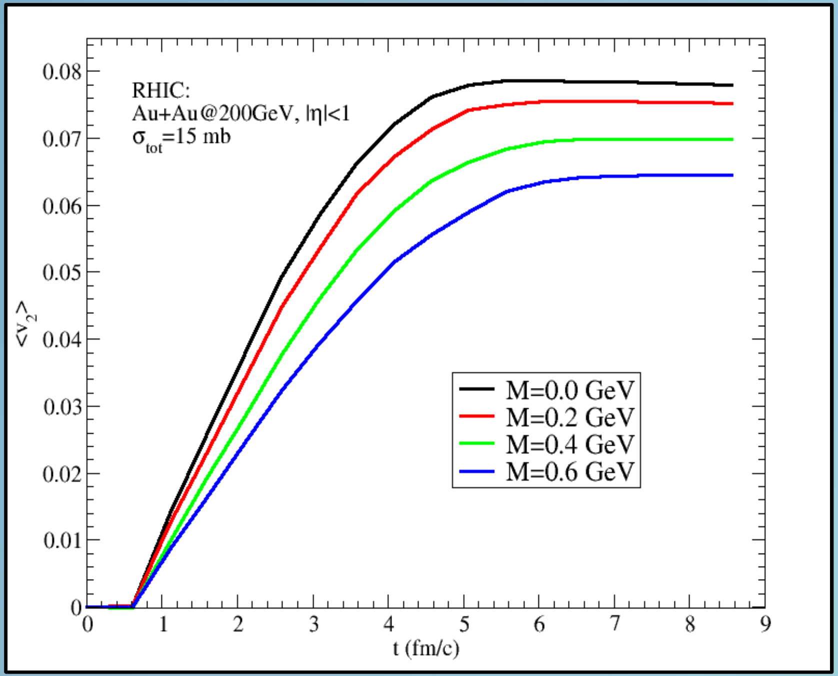
- Enhancement of $\eta/s(T)$ in the cross-over region affect differently the expanding QGP from RHIC to LHC.
- At LHC nearly all the v_2 from the QGP phase.
- The scaling of $v_2(p_T)$ from Beam Energy Scan indicate a 'U' shape of $\eta/s(T)$ this would be a first signature of $\eta/s(T)$ behavior typical of a phase transition.

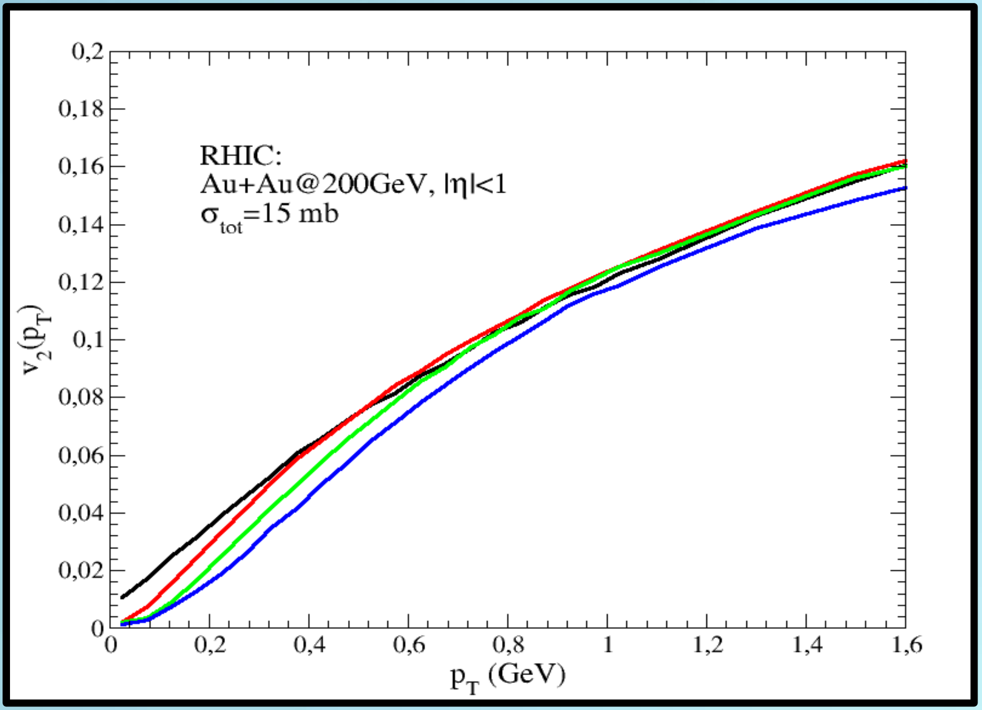
Finite masses and EoS

$$p^{\mu}\partial_{\mu}f(x,p)=C_{22}$$



$$M \neq 0 \rightarrow \begin{cases} \varepsilon - 3p \neq 0 \\ C_s^2 \leq \frac{1}{3} \end{cases}$$





Extraction of the Shear Viscosity: Box calculation

Relaxation Time Approximation

Kapusta, PRC(2010); Gavin NPA(1985);

$$\eta = \frac{1}{15T}\int \frac{d^3p}{(2\pi)^3}\frac{p^4}{E^2}\tau(E) f^{eq}(E) \qquad \tau^{-1}(E) = \rho\langle \sigma_{tot} v_{rel}\rangle$$

$$\eta_{relax} = 0.8\frac{T}{\sigma_{tot}} \;\longrightarrow\; \eta \sim \frac{1}{\sigma_{tot}}$$

Usual as Relax. Time Approx. - Israel Stewart $\sigma_{tot} \to \sigma_{tr} = (2/3)\sigma_{tot}$

$$\eta^{IS}_{relax} = 0.8\frac{T}{\sigma_{tr}} = 1.2\frac{T}{\sigma_{tot}}$$

Molnar-Huovenin PRC(2009),
G. Ferini PLB(2009),
Khvorostukhin PRC (2010)
....

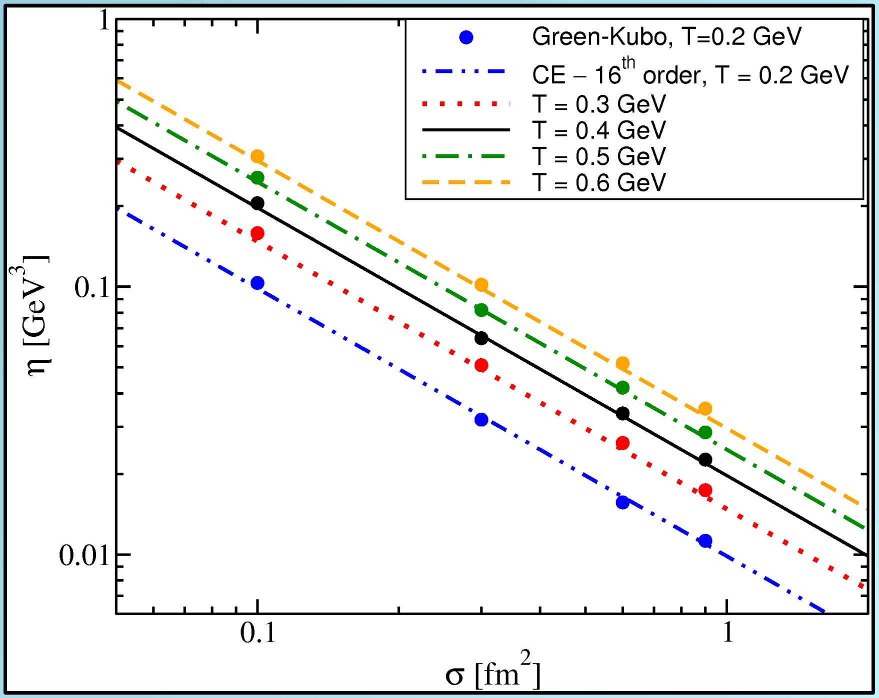
S. Plumari et al.,PRC86 (2012) 054902.

Isotropic cross section: massless case

- At 1st order of approx. in the Chapman-Enskog: $[\eta]^{CE}_{1st} = 1.2\frac{T}{\sigma_{tot}}$

- successive approx. up to 16 order: $[\eta]^{16th}_{CE} = 1.267\frac{T}{\sigma_{tot}}$

A. Wiranata, M. Prakash, arXiv:1203.0281 [nucl-th].
O. N. Moroz, arXiv:1112.0277 [hep-ph].



Extraction of the Shear Viscosity: Box calculation

Isotropic cross section: massive case

Massive case is relevant in quasi-particle models where M(T).
Good agreement with CE 1st order for isotropic cross section and massive particles.

1st Chapman-Enskog approximation

$$[\eta]_{1st} = 10\,T \left[\frac{K_3(z)}{K_2(z)}\right]^2 \frac{1}{c_{00}} = g(m_D, T)\frac{T}{\sigma_{tot}}$$

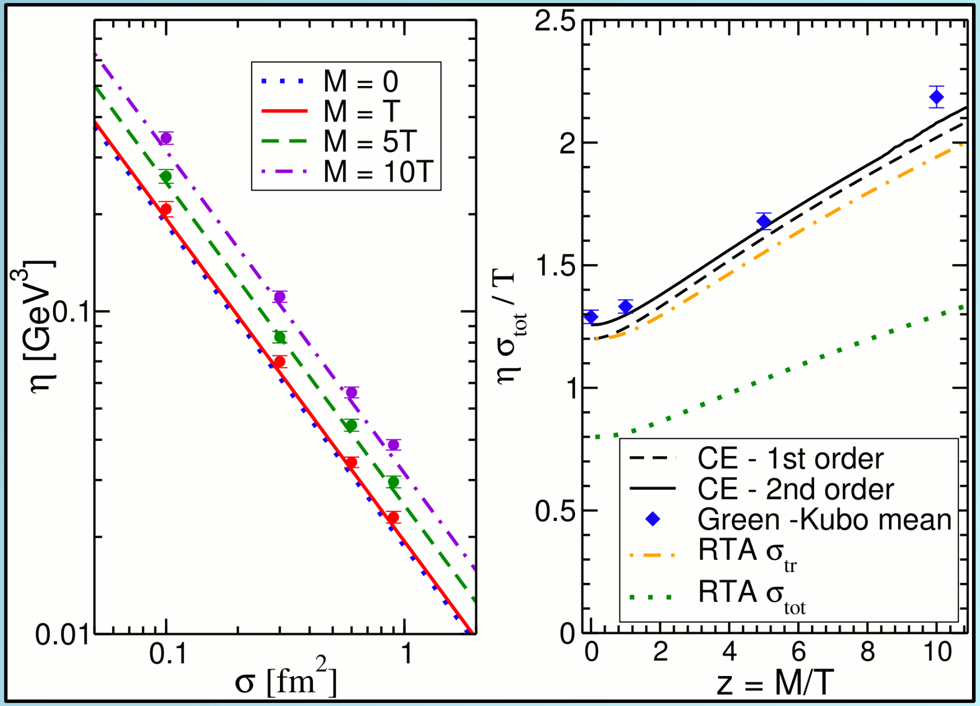
$$c_{00} = 16\left[\omega_2^{(2)} - z^{-1}\omega_1^{(2)} + (3z^2)^{-1}\omega_0^{(2)}\right] \qquad for \quad s=2 \propto \sigma_{tr}$$

$$\omega_i^{(s)} = \frac{2\pi z^3}{[K_2(z)]^2}\int dy\,(y^2-1)^3 y^i K_j(2zy) \int d\Theta \sin\Theta \frac{d\sigma}{d\Omega}(1-\cos^s\Theta)$$

$$[\eta]_{1st}^{CE} = f(z)\frac{T}{\sigma_{tot}}$$

$$f(z) = \frac{15}{16}\frac{z^4 K_3^2(z)}{(15z^2+2)K_2(2z) + (3z^3+49z)K_3(2z)}$$

S. Plumari et al., arXiv:1208.0481 [nucl-th].



A. Wiranata, M. Prakash, arXiv:1203.0281 [nucl-th].
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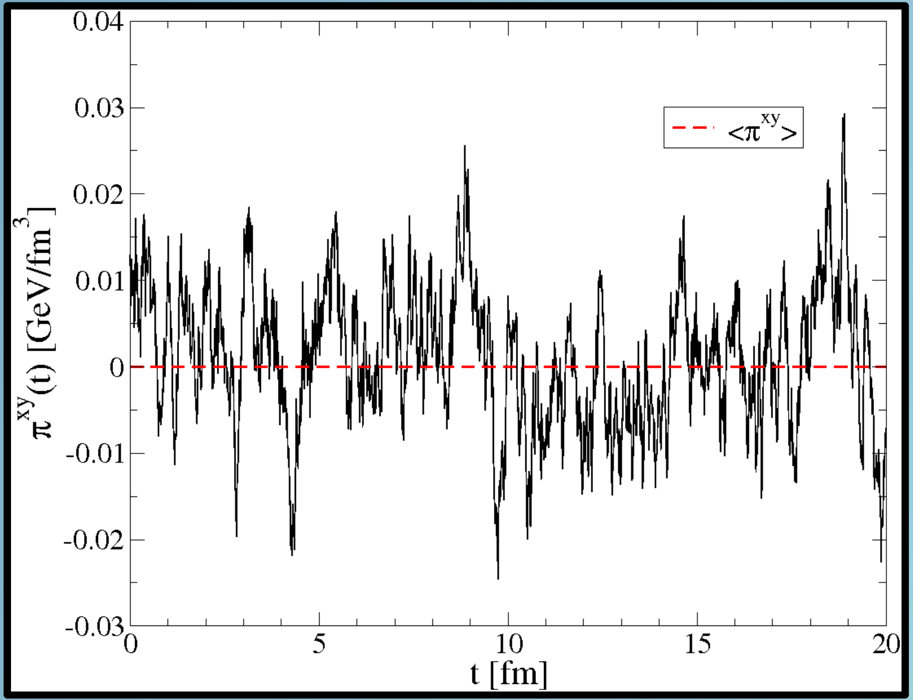
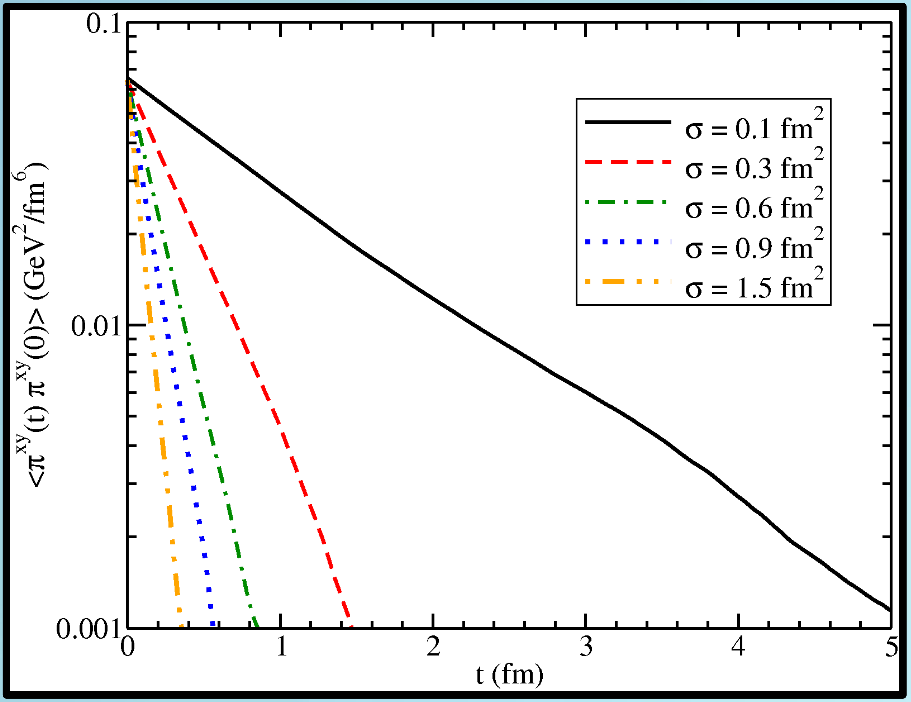
$$\eta = \frac{1}{T}\int dt \int d^3x \, \langle \pi^{xy}(x,t)\, \pi^{xy}(0,t) \rangle$$

$$\langle \pi^{xy}(\vec{x},t)\, \pi^{xy}(\vec{0},t) \rangle = \langle \pi^{xy}(0)\, \pi^{xy}(0) \rangle \cdot e^{-t/\tau}$$

$$\Rightarrow \quad \eta = \frac{V}{T} \langle \pi^{xy}(0)\, \pi^{xy}(0) \rangle \cdot \tau$$





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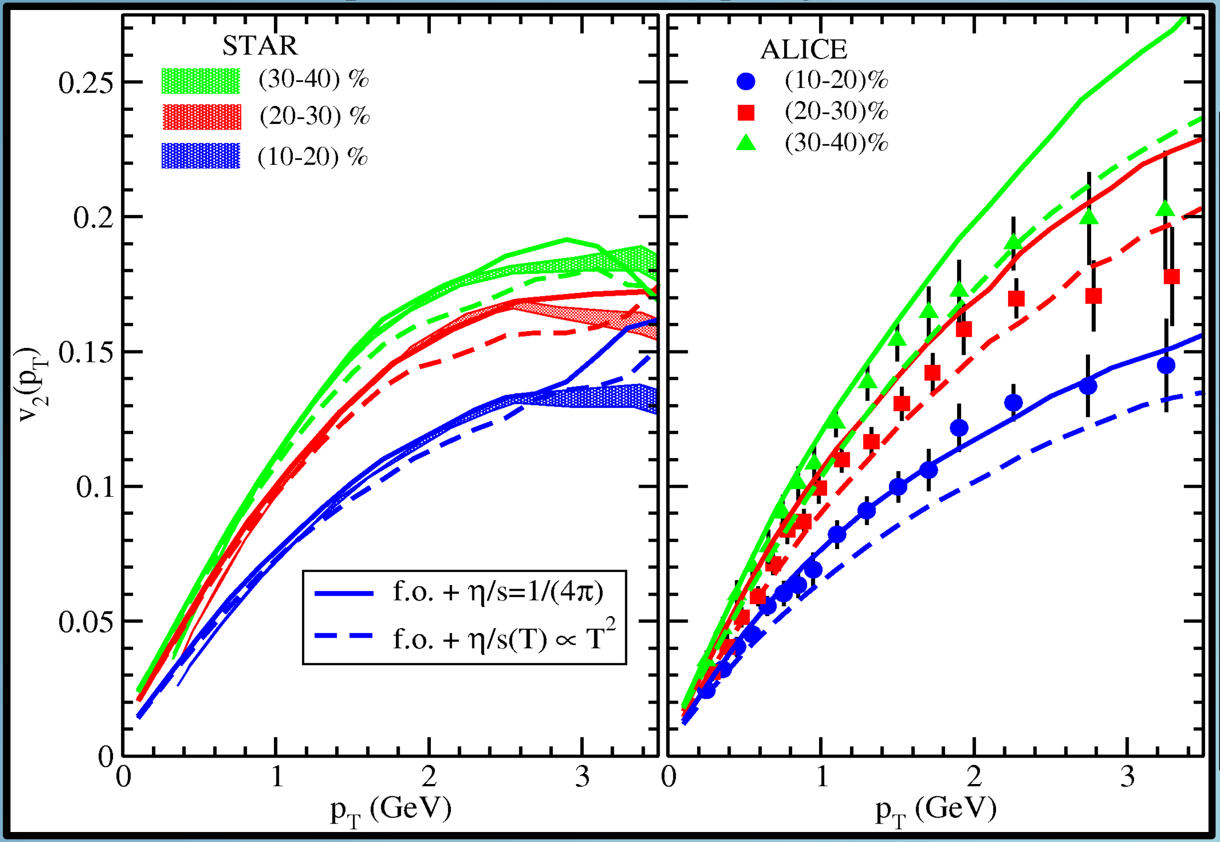
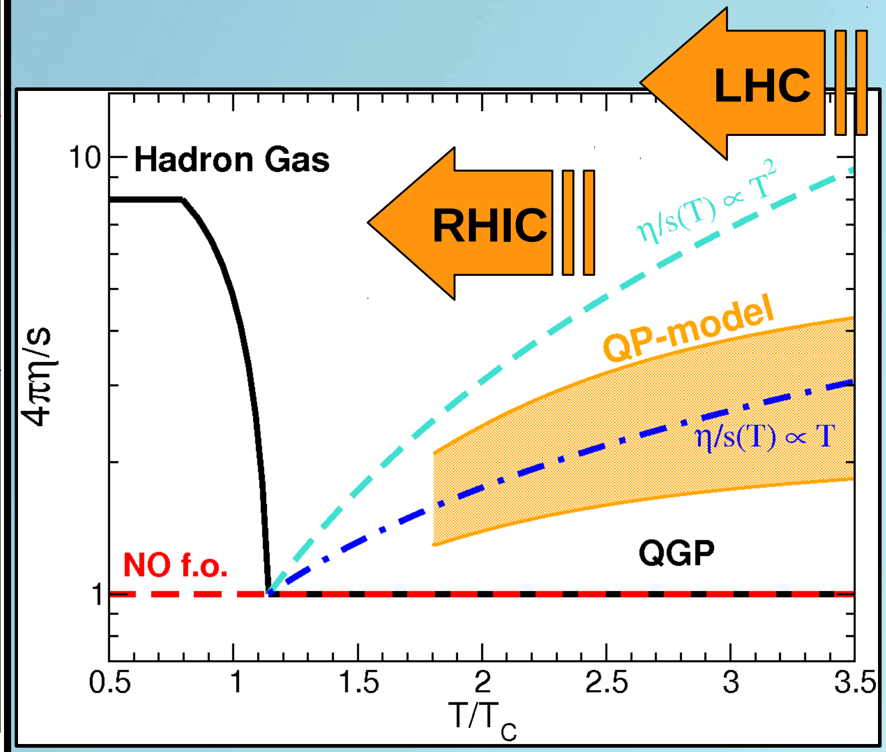
Temperature dependent $\eta/s(T)$

K. Aamodt et al. [ALICE Collaboration], Phys. Rev.Lett. 105, 252302 (2010).





S. Plumari et al., arXiv:1209.0601 [hep-ph].

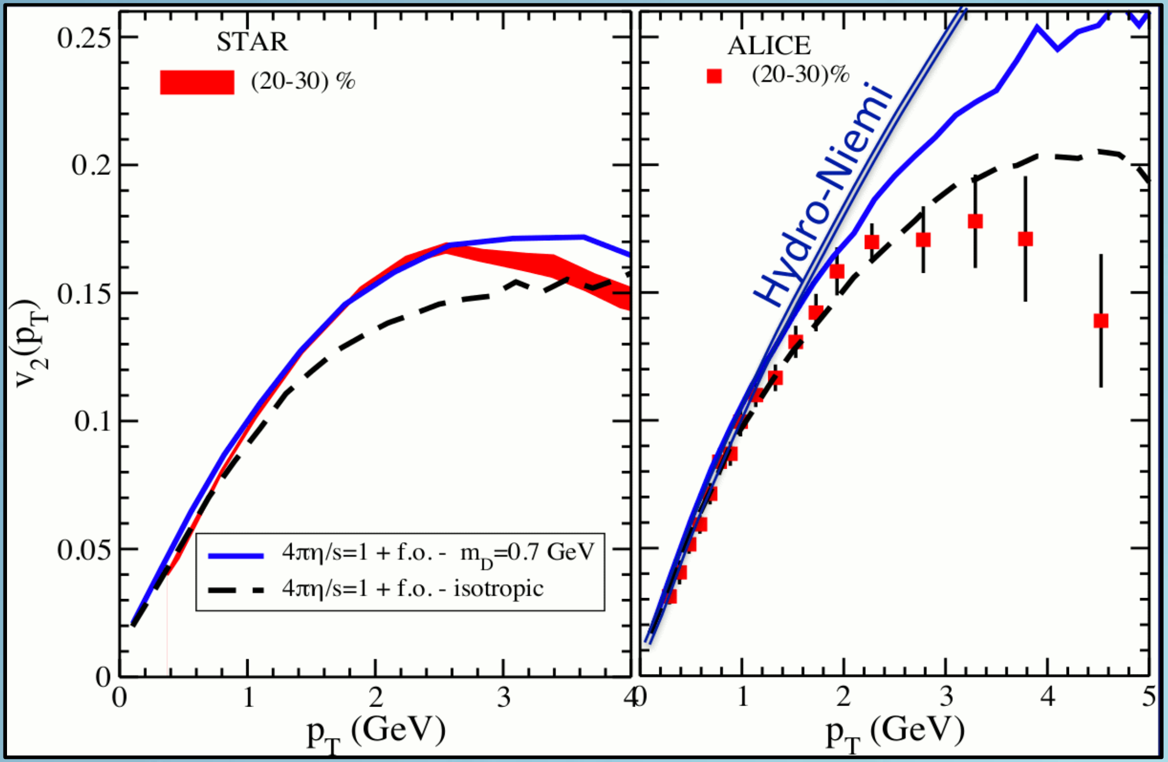
RHIC:

- The v_2 is insensitive to the value of η/s in the QGP phase
- $\eta/s \sim T^2$ cannot account for the v_2 decrease for $p_T > 2.5$ GeV.

LHC:

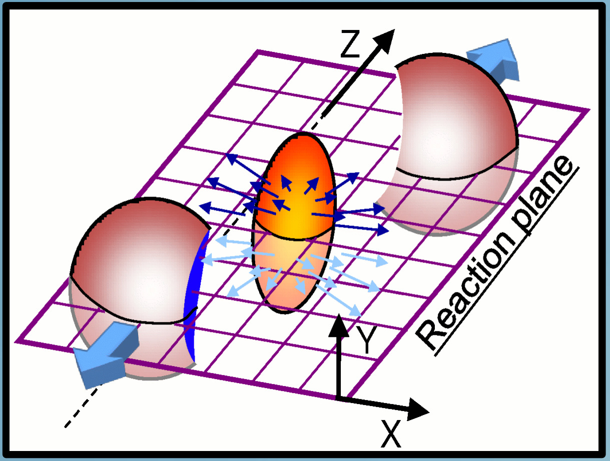
- The v_2 more sensitive to the QGP phase but still a strong temperature dependence in η/s has a small effect in the $v_2(p_T)$.

Relevance of the microscopic scale



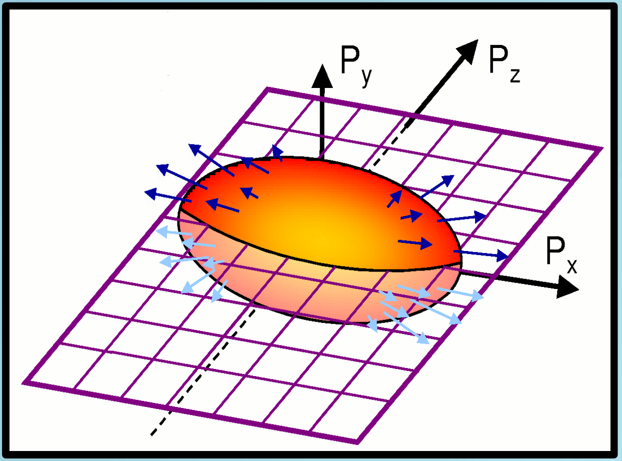
- Microscopic details of the cross section have an effect on v_2 for $p_T>1.5$ GeV
- Fixed the ratio η/s of the fluid the isotropic cross section leads to a smaller v_2
- An appropriate $m_D(T) = g(T)\,T$ gives a different behaviour between RHIC and LHC.

Information from non-equilibrium: elliptic flow



$\lambda=(\sigma\rho)^{-1}$ or η/s viscosity

$c_s^2=dP/d\varepsilon$, EoS-lQCD



$$\varepsilon_x = \left\langle \frac{y^2 - x^2}{y^2 + x^2} \right\rangle$$

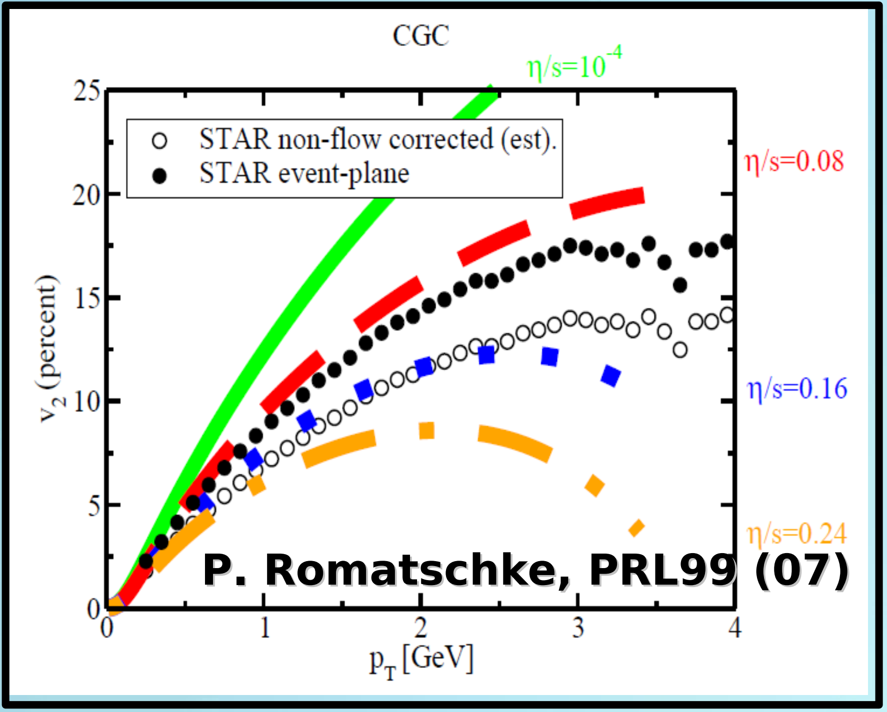
The v_2/ε measures efficiency in converting the eccentricity from Coordinate to Momentum space
J.Y. Ollitrault, PRD 46 (1992).

$$v_2 = \langle \cos 2\phi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

Can be seen also as Fourier expansion

$$\frac{dN}{dp_T d\phi} = \frac{dN}{dp_T}\left[1 + 2v_2\cos(2\phi) + 2v_4\cos(4\phi) + \dots\right]$$

by symmetry v_n with n odd expected to be zero … (but event by event fluctuations)



Viscosity η / s