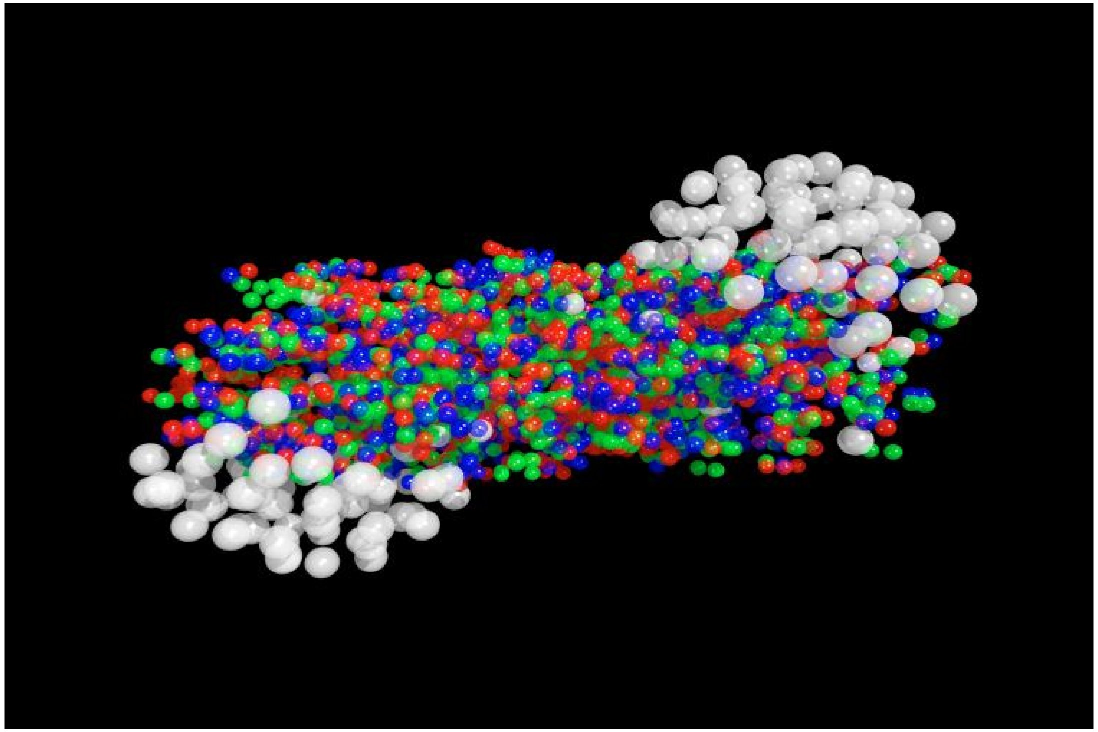
University of Catania
INFN-LNS

Heavy flavor Suppression : Langevin vs Boltzmann



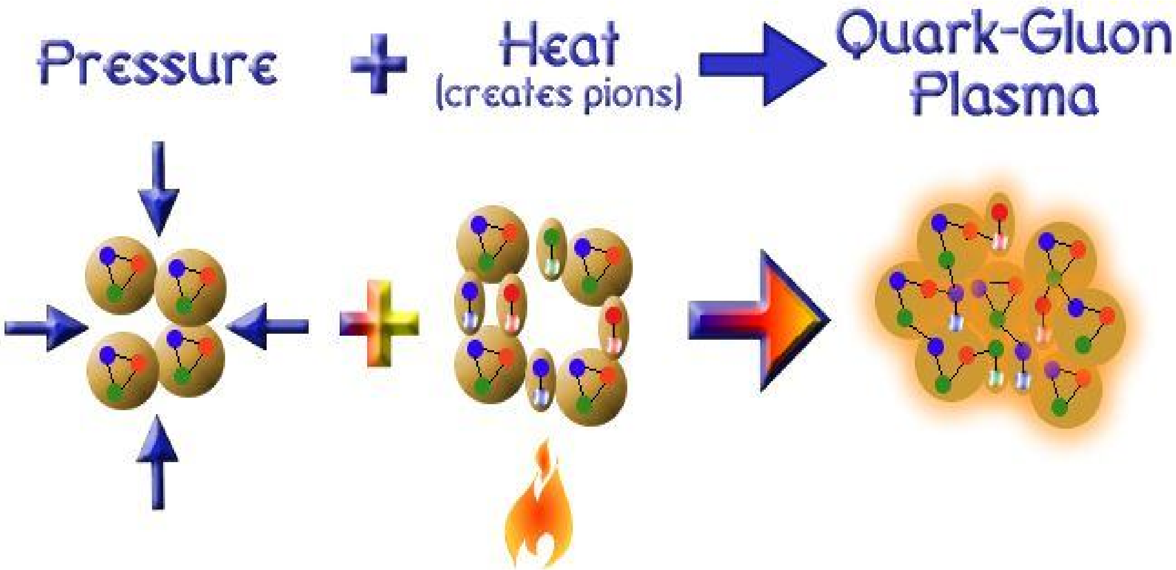
S. K. Das, F. Scardina and V. Greco

Outline Of my talk.............

- Introduction
- Langevin Equation and the Thermalization Issue
- Boltzmann Equation and the Thermalization Issue
- Nuclear Suppression: Langevin vs Boltzmann
- Summary and outlook

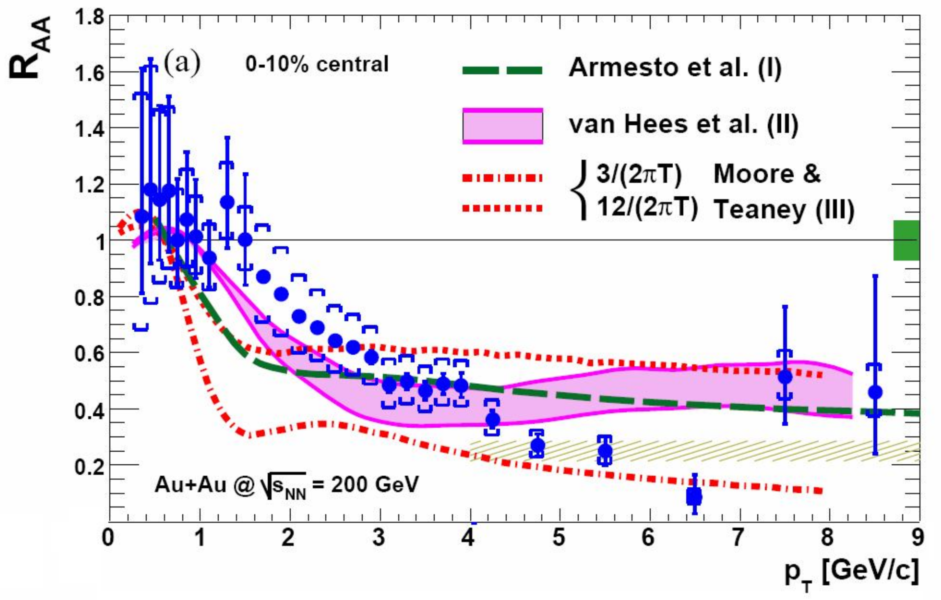
Introduction

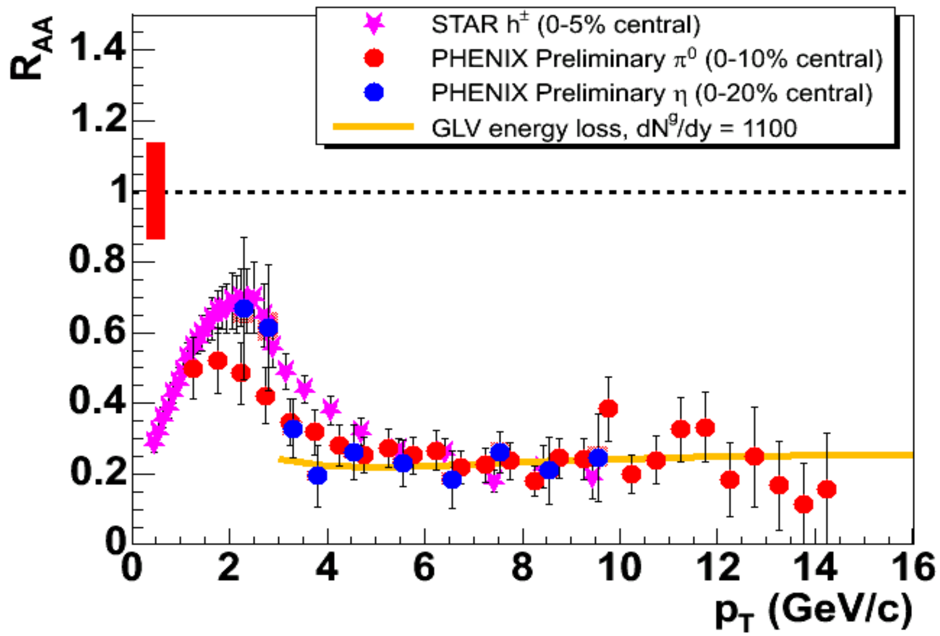
At very high temperature and density hadrons melt to a new phase of matter called Quark Gluon Plasma (QGP).



$$\tau_{HQ} > \tau_{LQ}, \quad \tau_{HQ} \sim (M/T)\,\tau_{LQ}$$

Heavy flavor at RHIC

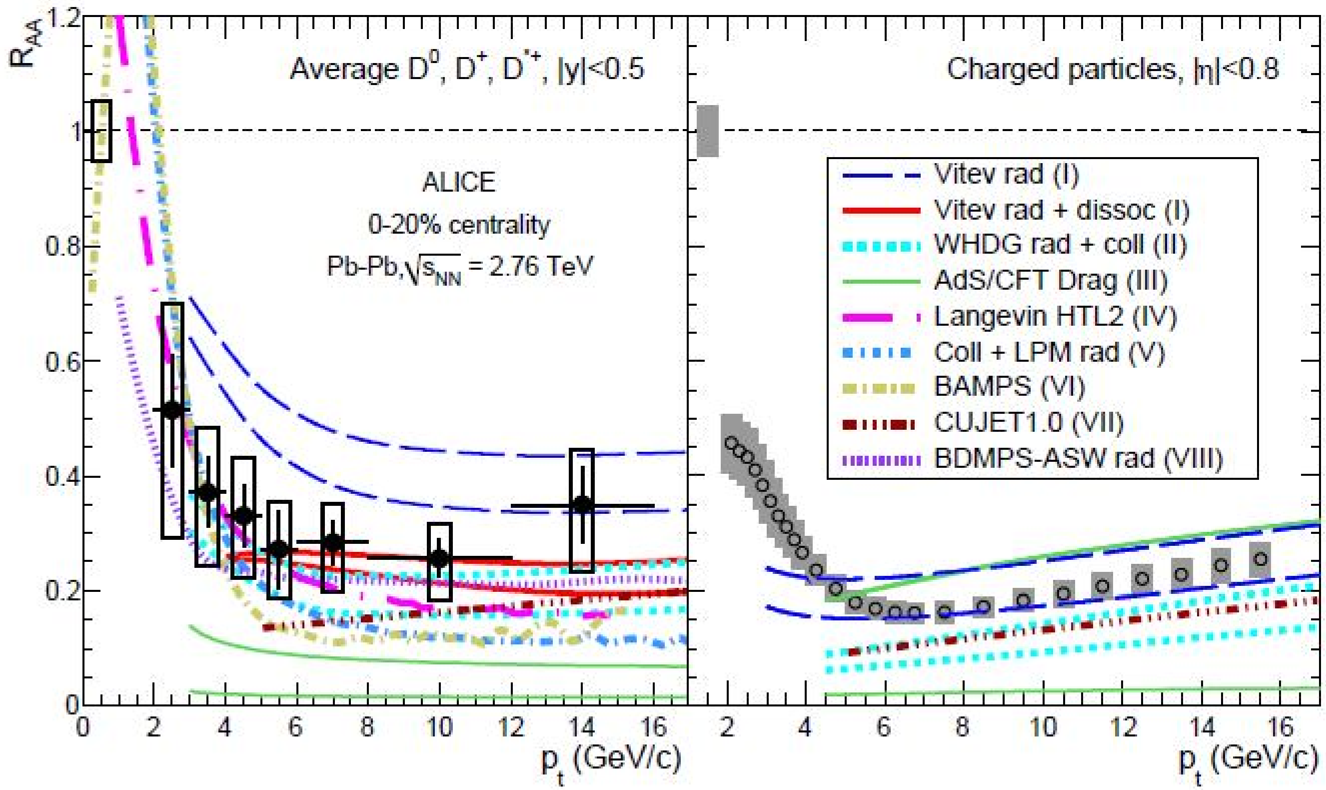




PHENIX: *PRL98(2007)172301*

At RHIC energy heavy flavor suppression is similar to light flavor

Heavy Flavors at LHC



Again at LHC energy heavy flavor suppression is similar to light flavor

Is the momentum transfer really small !

Boltzmann Kinetic equation

$$\left(\frac{\partial}{\partial t} + \frac{P}{E}\frac{\partial}{\partial x} + \mathrm{F}.\frac{\partial}{\partial p}\right) f\left(x, p, t\right) = \left(\frac{\partial f}{\partial t}\right)_{col}$$

$$R(p,t) = \left(\frac{\partial f}{\partial t}\right)_{col} = \int d^3k\left[\omega(p+k,k) f(p+k) - \omega(p,k) f(p)\right]$$

$$\omega(p,k) = g\int \frac{d^3q}{(2\pi)^3} f'(q) v_{q,p}\sigma_{p,q\rightarrow p-k,q+k}$$ → is rate of collisions which change the momentum of the charmed quark from p to p-k

$$\omega(p+k,k) f(p+k) \approx \omega(p,k) f(p) + k.\frac{\partial}{\partial p}(\omega f) + \frac{1}{2}k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f)$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \frac{\partial}{\partial \mathbf{p}_i}\left[\mathbf{A}_i(\mathbf{p})\mathbf{f} + \frac{\partial}{\partial \mathbf{p}_j}\left[\mathbf{B}_{ij}(\mathbf{p})\mathbf{f}\right]\right]$$

B. Svetitsky PRD 37(1987)2484

where we have defined the kernels

$$\mathbf{A}_i = \int \mathbf{d}^3\mathbf{k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k}_i$$ → **Drag Coefficient**

$$\mathbf{B}_{ij} = \int \mathbf{d}^3\mathbf{k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k}_i\mathbf{k}_j$$ → **Diffusion Coefficient**

$$\omega(p+k,k)\,f(p+k) \approx \omega(p,k)\,f(p) + k.\frac{\partial}{\partial p}(\omega f) + \frac{1}{2}k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f)$$

Boltzmann Equation

Fokker Planck

It will interesting to study both the equation in a identical environment to ensure the validity of this assumption.

Langevin Equation

$$dx_j = \frac{p_j}{E} dt$$

$$dp_j = -\Gamma p_j dt + \sqrt{dt}\, C_{jk}(t, p + \xi dp)\rho_k$$

where Γ **is the deterministic friction (drag) force**

C_{ij} **is stochastic force in terms of independent**

Gaussian-normal distributed random variable

$$\rho = (\rho_x, \rho_y, \rho_z) \quad , \quad P(\rho) = \left(\frac{1}{2\pi}\right)^3 \exp\left(-\frac{\rho^2}{2}\right)$$

With $<\rho_i(t)\rho_k(t')>=\delta(t-t')\delta_{jk}$

H. v. Hees and R. Rapp
arXiv:0903.1096

$\xi = 0$ **the pre-point Ito**

interpretation of the momentum argument of the covariance matrix.

Langevin process defined like this is equivalent to the Fokker-Planck equation:

$$\frac{\partial f}{\partial t} + \frac{p_j}{E}\frac{\partial f}{\partial x_j} = \frac{\partial}{\partial p_j}\left[\left(p_j \Gamma - \xi C_{lk}\frac{\partial C_{jk}}{\partial p_l}\right) f\right] + \frac{1}{2}\frac{\partial^2}{\partial p_j \partial p_k}(C_{jl}C_{kl}f)$$

the covariance matrix is related to the diffusion matrix by

$$C_{jk} = \sqrt{2B_0}P^{\perp}_{jk} + \sqrt{2B_1}P^{\parallel}_{jk}$$

and $$A_i = p_j \Gamma - \xi C_{lk}\frac{\partial C_{ij}}{\partial p_l}$$

With $$B_0 = B_1 = D \qquad C_{jk} = \sqrt{2D(E)}\delta_{jk}$$

Relativistic dissipation-fluctuation relation

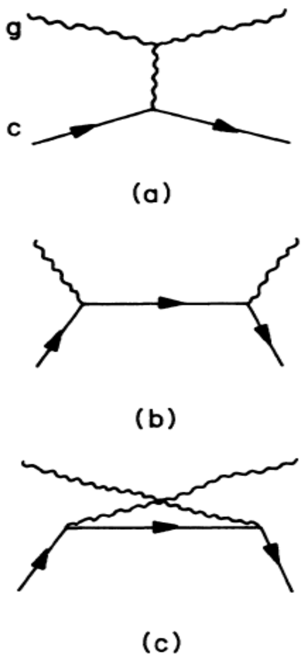
$$A(E)ET - D(E) + T(1-\xi)D'(E) = 0$$

For Collision Process the A_i and B_{ij} can be calculated as following :

$$A_i = \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}}\int\frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma_c}\sum|M|^2(2\pi)^4\delta^4(p+q-p'-q')f(q)[(p-p')_i] = \langle\langle(p-p')_i\rangle\rangle$$

$$B_{ij} = \frac{1}{2}\langle\langle(p-p')_i(p'-p)_j\rangle\rangle$$

Elastic processes



$$gc \to gc$$

✓ **We have introduce a mass into the internal gluon propagator in the t and u-channel-exchange diagrams, to shield the infrared divergence.**

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Thermalization in Langevin approach in a static medium

1) Diffusion D=Constant
   Drag A= D/ET from FDT

2) Diffusion D(p) and Drag A(p) both from pQCD

$$A(E)ET - D(E) + T(1-\xi)D'(E) = 0$$

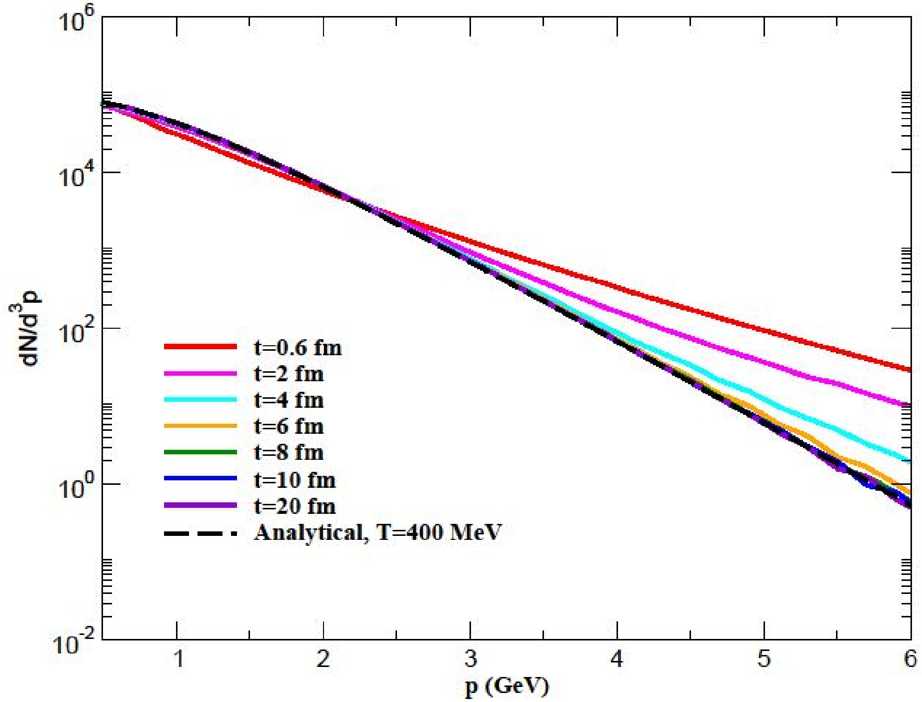
We are solving Langevin Equation in a box.

Case:1

1) D=Constant
   A= D/ET from FDT

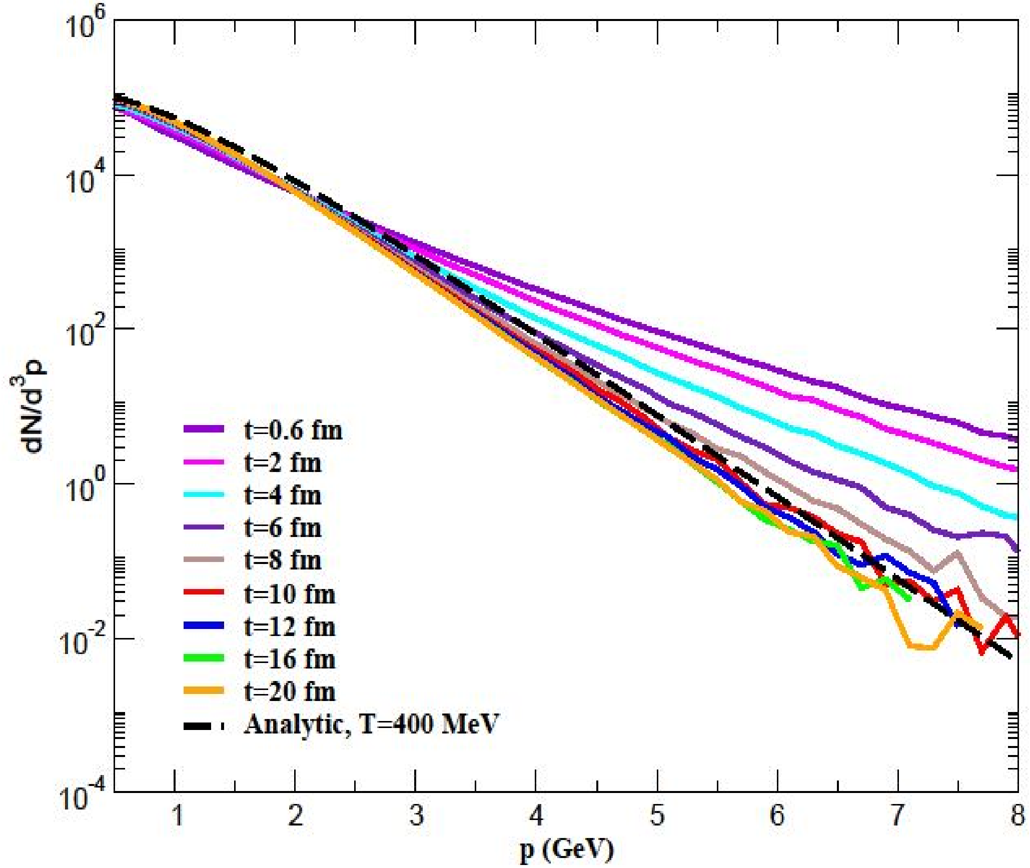
Due to the collision charm approaches to thermal equilibrium with the bulk

Bulk composed only by gluon in Thermal equilibrium at T= 400 MeV.



Case: 2

Diffusion coefficient: D(p) pQCD
Drag coefficient: A(p) pQCD



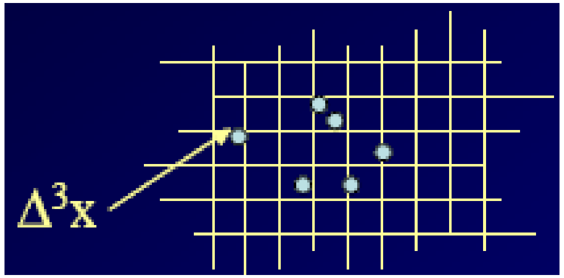
In this case we are away from thermalization.

Transport theory

$$p^{\mu}\partial_{\mu} f(x,p) = C_{22}$$

We consider two body collisions

$$C_{22} = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$
$$- \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f_1 f_2 |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p'_1 - p'_2)$$



$$\Delta t \to 0$$

$$\Delta^3 x \to 0$$

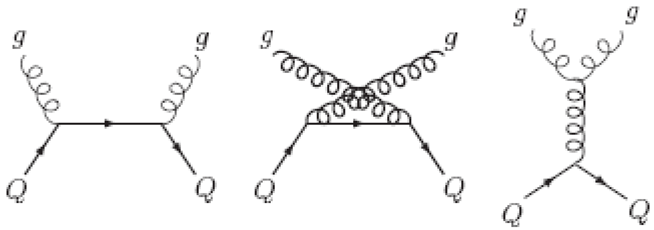
Exact solution

Collision integral is solved with a local stochastic sampling

[Z. Xhu, et al. PRC71(04)]
Greco et al PLB670, 325 (08)]

$$P_{22} = \frac{\Delta N^{2\to 2}_{\rm coll}}{\Delta N_1 \Delta N_2} = v_{\rm rel}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

Cross Section gc -> gc

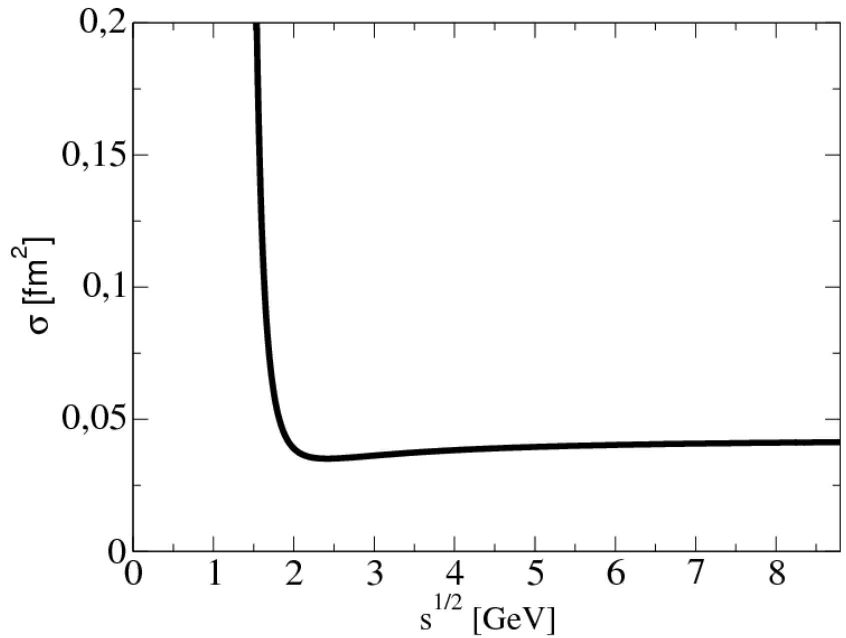


The infrared singularity is regularized introducing a Debye-screaning-mass μ_D

$$\frac{1}{t} \rightarrow \frac{1}{t - m_D} \qquad m_D = \sqrt{4\pi\alpha_s}T$$

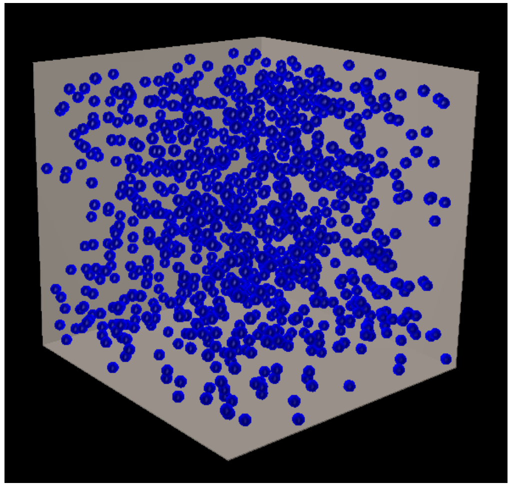
$$\sum |\mathcal{M}|^2 = \pi^2\alpha^2(Q^2)\left[\frac{32(s-M^2)(M^2-u)}{t^2} + \frac{64}{9}\frac{(s-M^2)(M^2-u)+2M^2(s+M^2)}{(s-M^2)^2} + \frac{64}{9}\frac{(s-M^2)(M^2-u)+2M^2(M^2+u)}{(M^2-u)^2} + \frac{16}{9}\frac{M^2(4M^2-t)}{(s-M^2)(M^2-u)} + 16\frac{(s-M^2)(M^2-u)+M^2(s-u)}{t(s-M^2)} - 16\frac{(s-M^2)(M^2-u)-M^2(s-u)}{t(M^2-u)}\right]$$

$$\hat{\sigma} = \frac{1}{16\pi(s-M^2)^2}\int_{-(s-M^2)^2/s}^{0} dt \sum |\mathcal{M}|^2 \longrightarrow$$



L. Combridge, Nucl. Phys. B151, 429 (1979)]
[B. Svetitsky, Phys. Rev. D 37, 2484 (1988)]

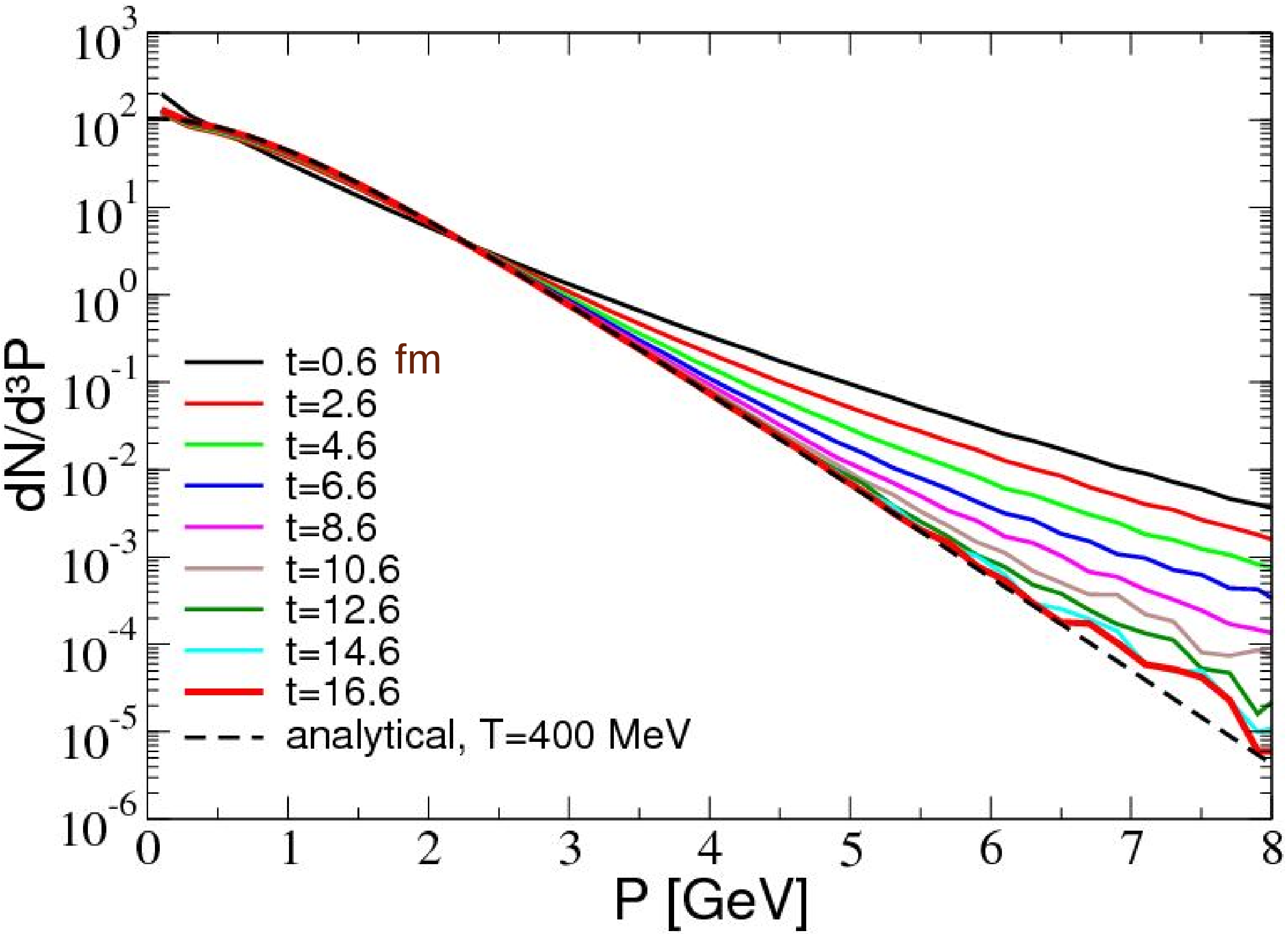
Charm evolution in a static medium



Simulations in which a particle ensemble in a **box** evolves dynamically

Bulk composed only by **gluons in thermal equilibrium** at **T=400 MeV**

C and Cbar are initially distributed: uniformily in r-space, while in p-space



Due to collisions charm approaches to thermal equilibrium with the bulk

Langevin vs Boltzmann

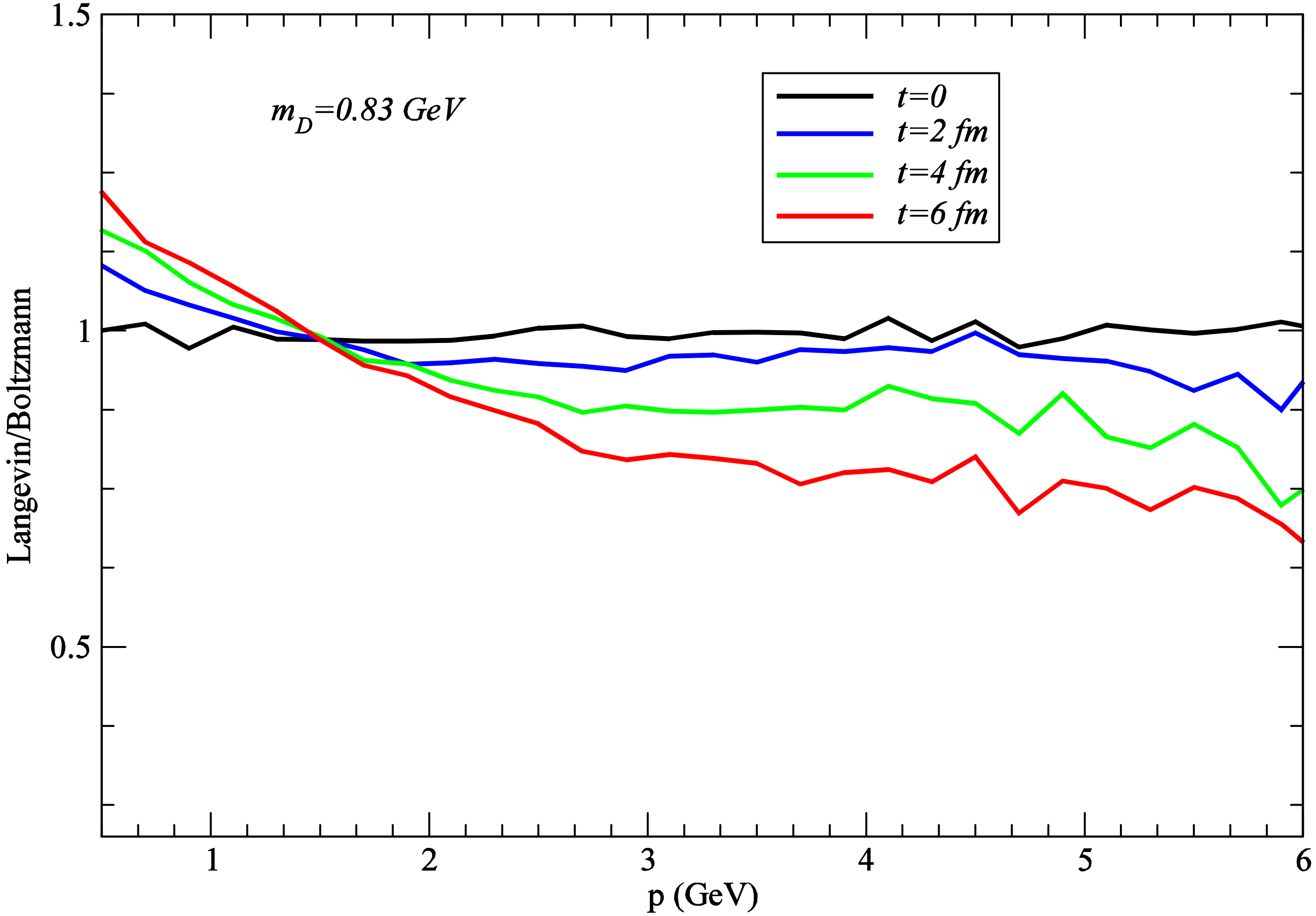
Case:1 m_D = 0.83 GeV (~gT, pQCD)

Case:2 m_D = 3 GeV (Isotropic)

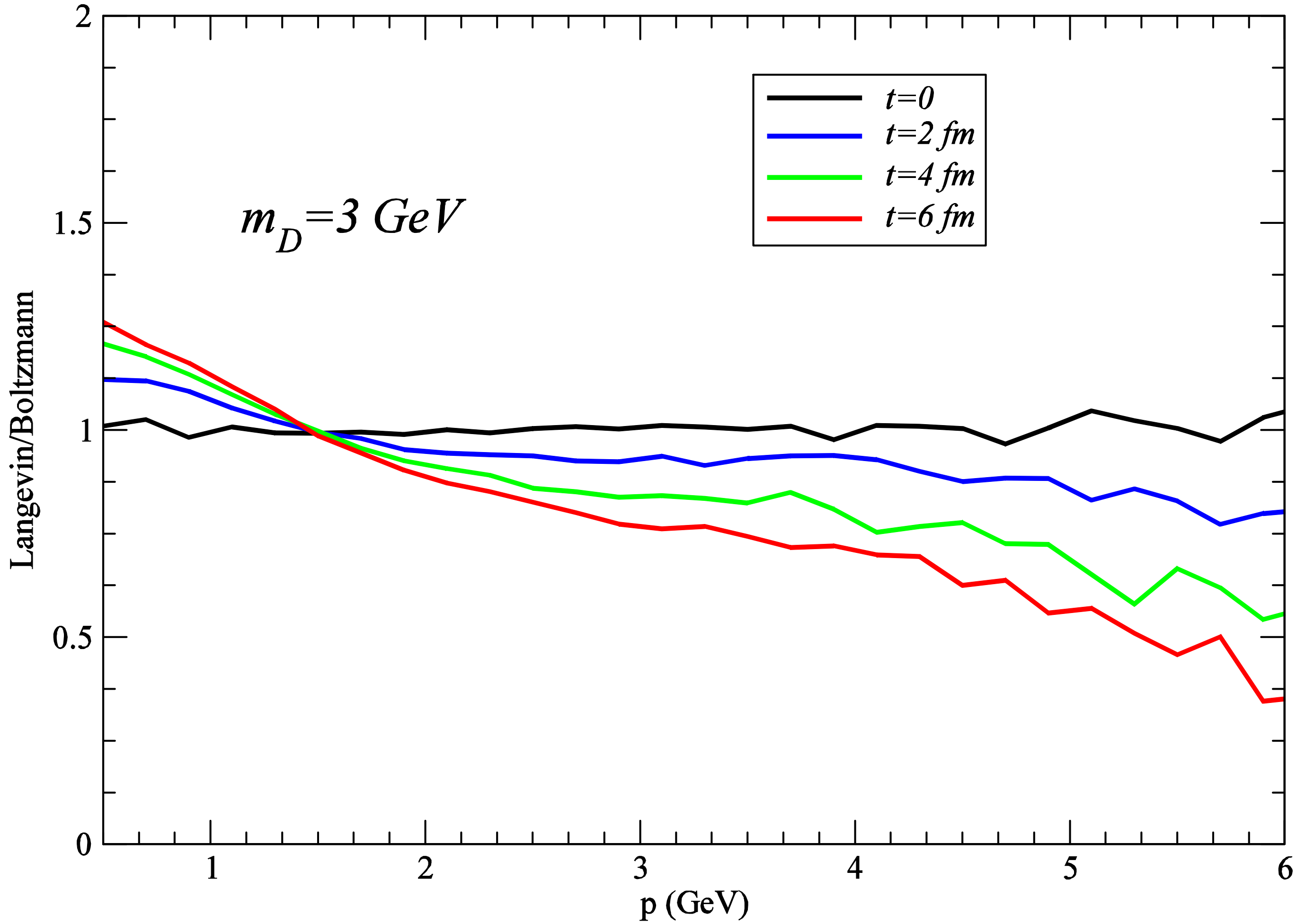
Case:3 m_D = 0.4 GeV (Forward-backword peak)

We have scaled our interaction in such a way that our thermalization time is always same for all the three case.

Ratio between Langevin and Boltzmann
At fixed time (pQCD)



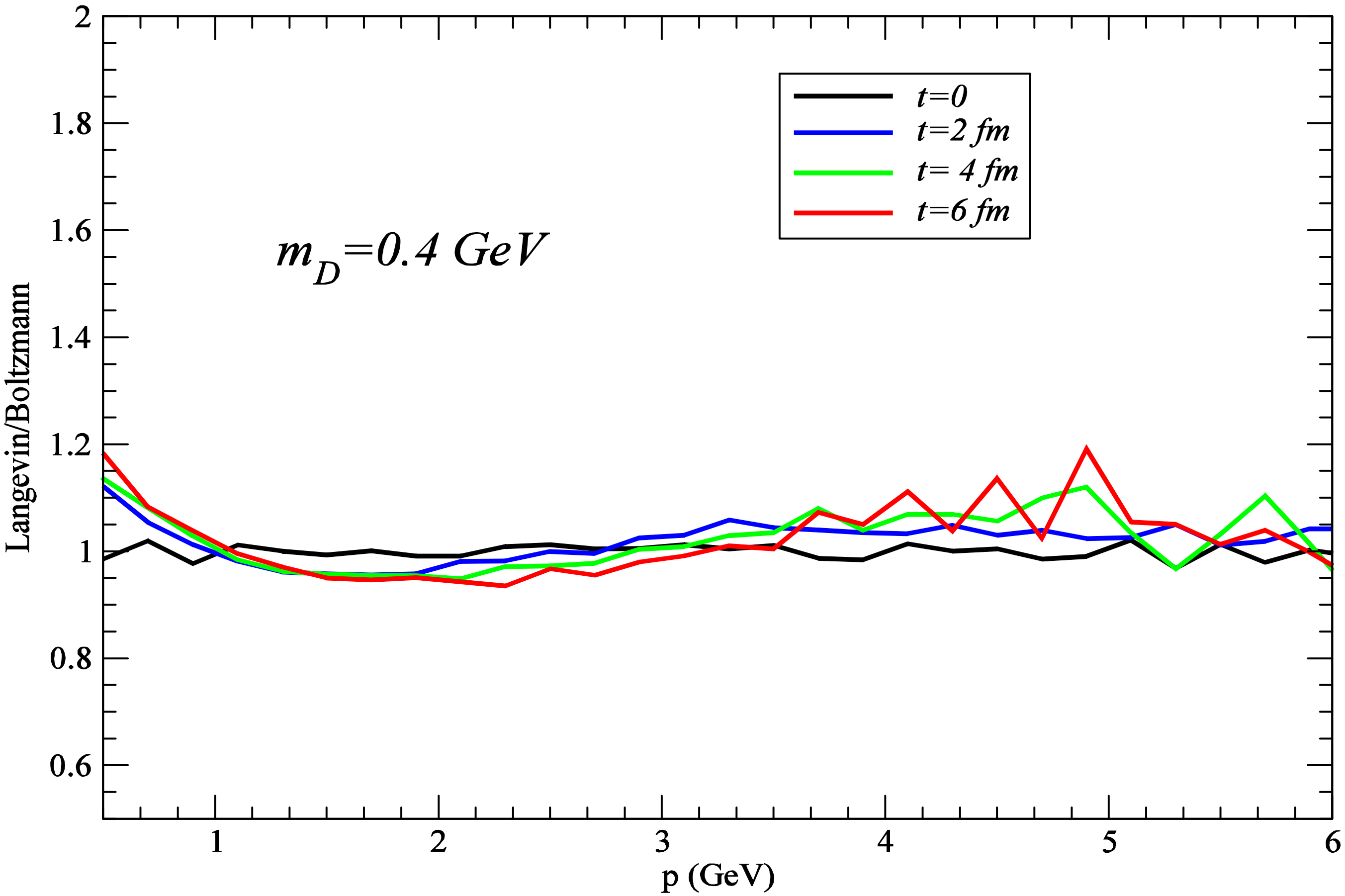
Ratio between Langevin and Boltzmann
At fixed time (Isotropic case)



T-matrix cross section are usually isotropic

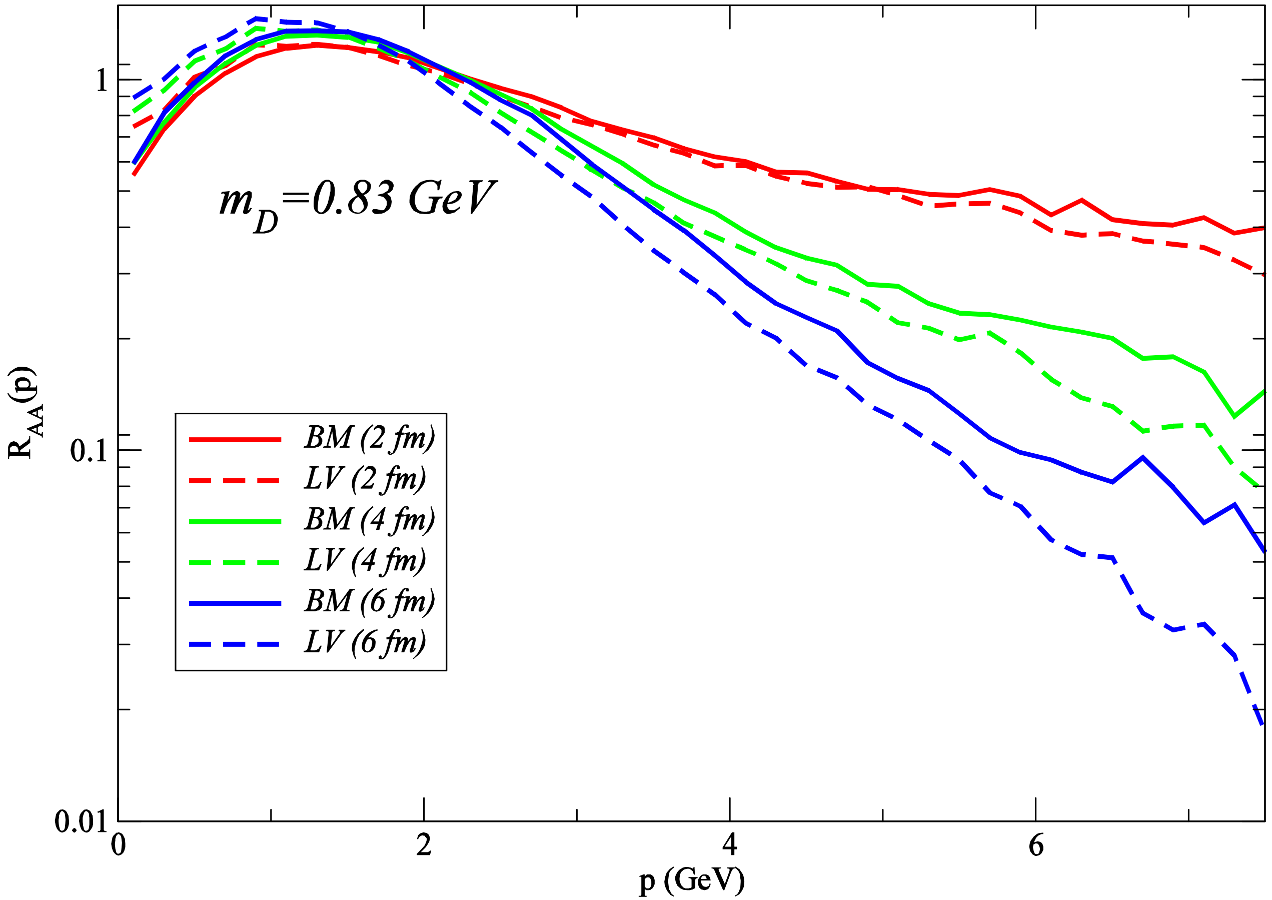
Hees, Mannarelli, Greco, and R. Rapp
Phys. Rev. Lett. 100, 192301 (2008)

Ratio between Langevin and Boltzmann
At fixed time (FBP)



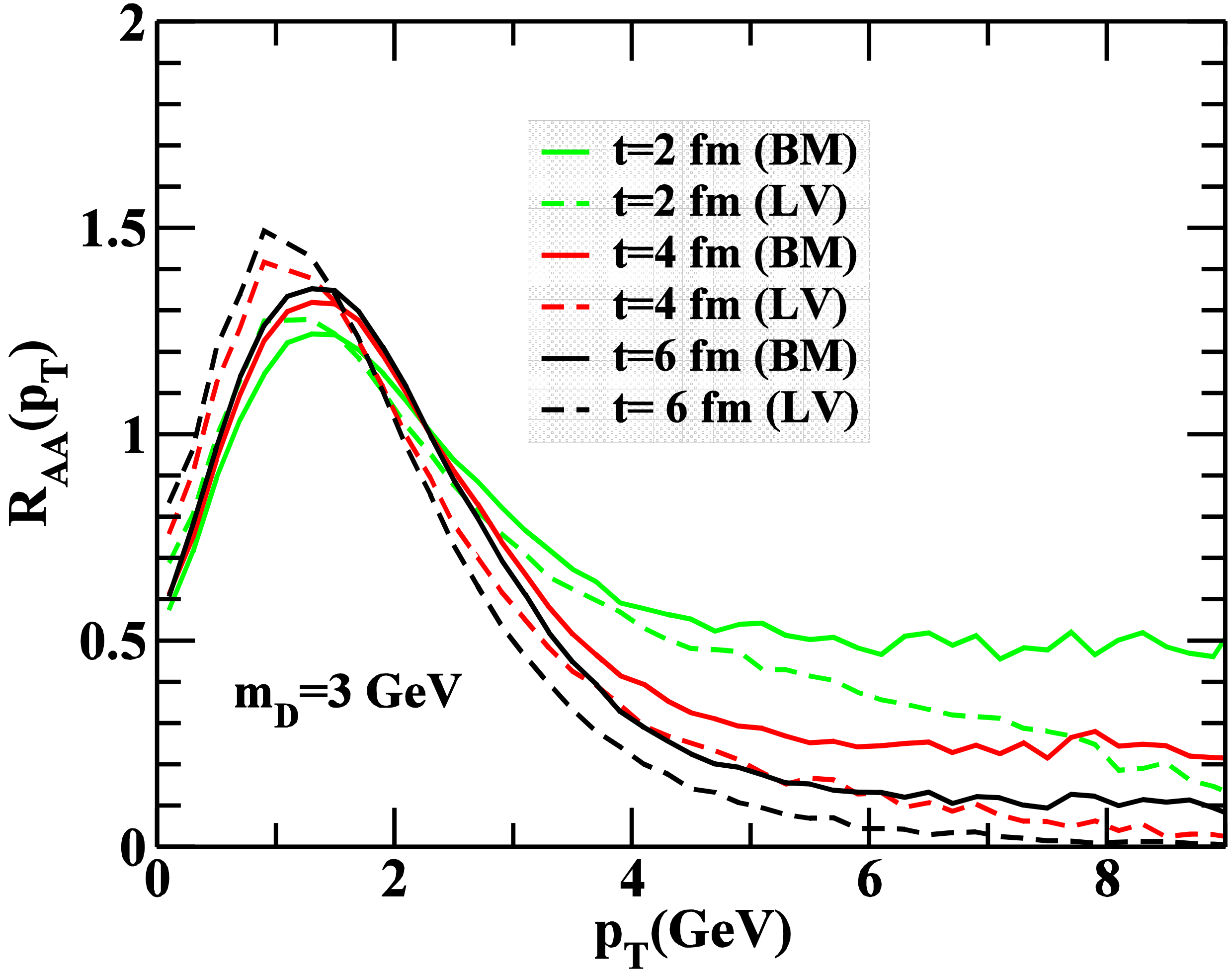
Nuclear Suppression: Langevin vs Boltzmann (pQCD)

$$R_{AA} = \frac{\left(\frac{dN}{d^3p}\right)_{output}}{\left(\frac{dN}{d^3p}\right)_{input}}$$



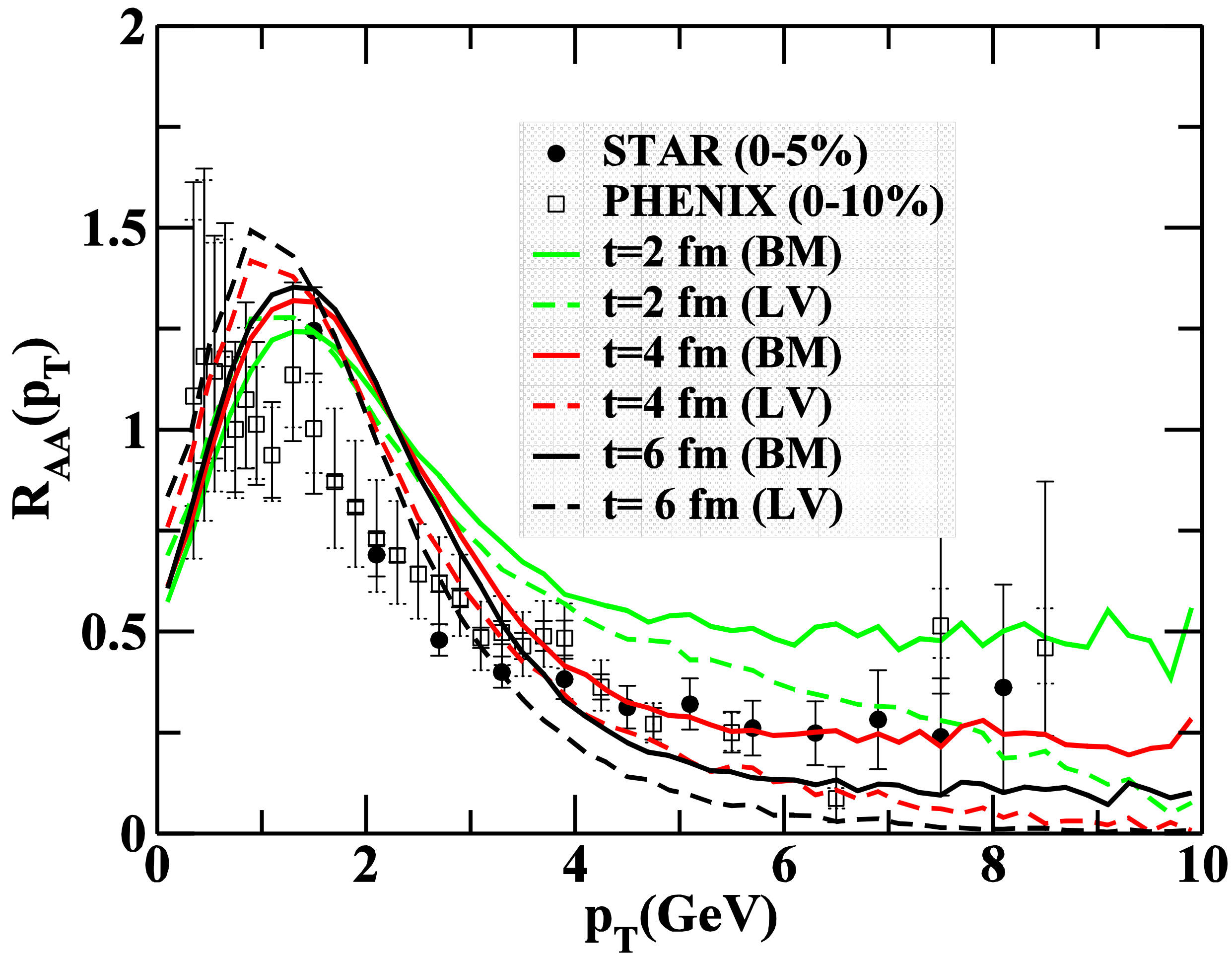
Suppression is more in Langevin approach than Boltzmann

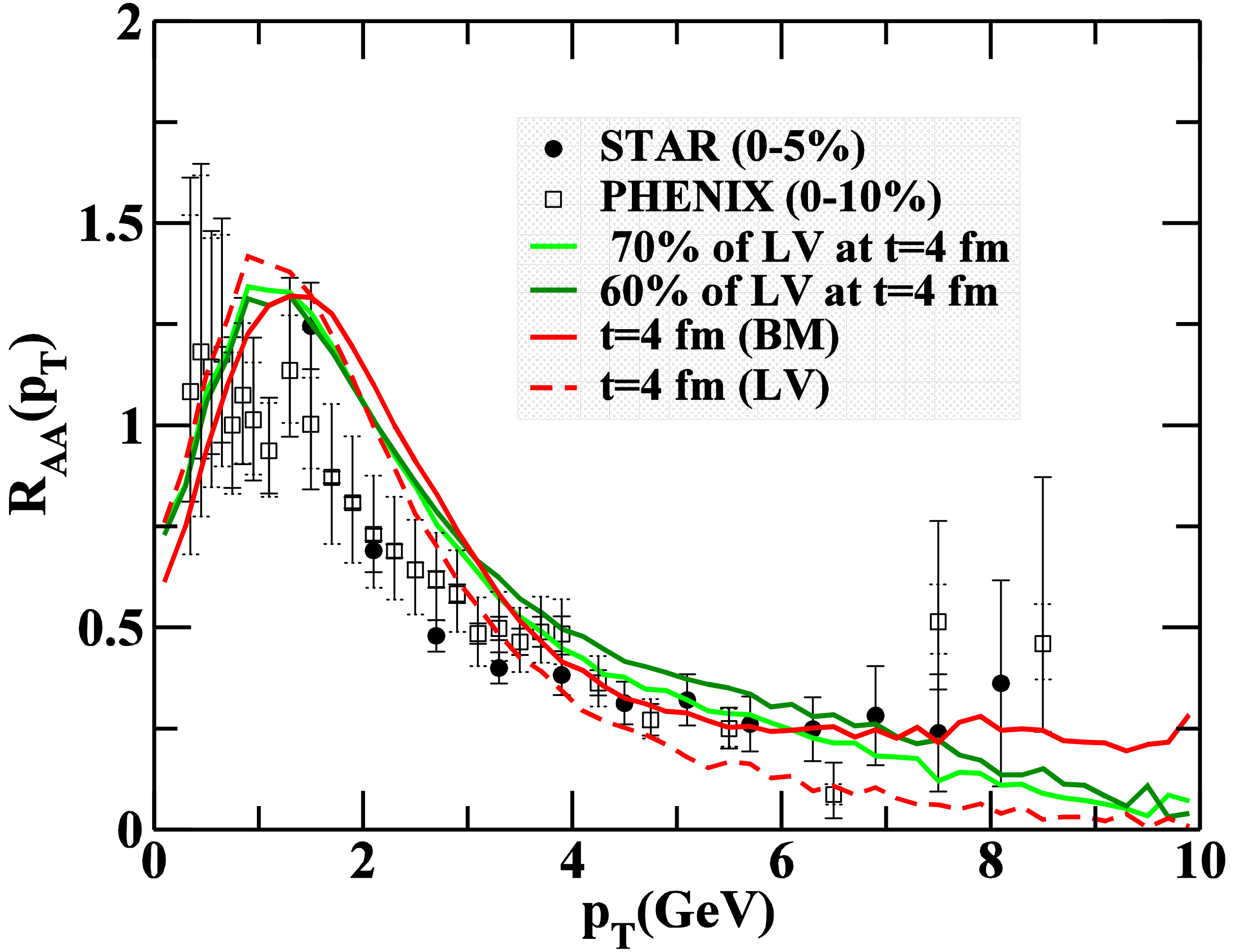
Nuclear Suppression: Langevin vs Boltzmann (Isotropic case)



Suppression is more in Langevin approach than Boltzmann

To have a phenomenological touch let put the RHIC data although our calculation is only for a box





To compensate the difference in the RAA we need to reduce the diffusion coefficient around 30-40 % which is the phenomenological interest

Calculation in a realistic background is under progress

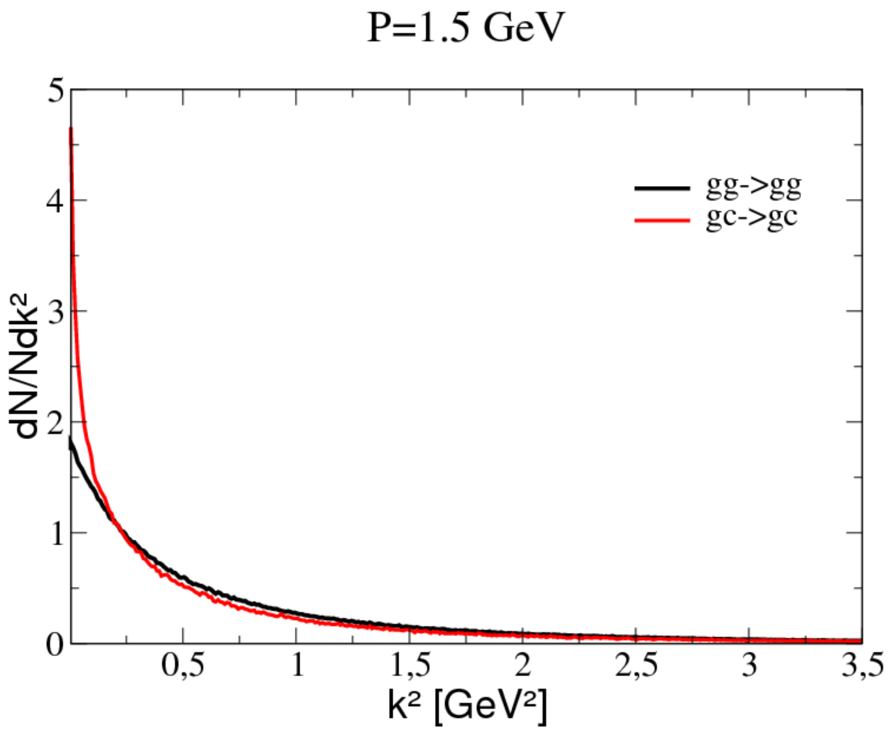
Summary & Outlook ……

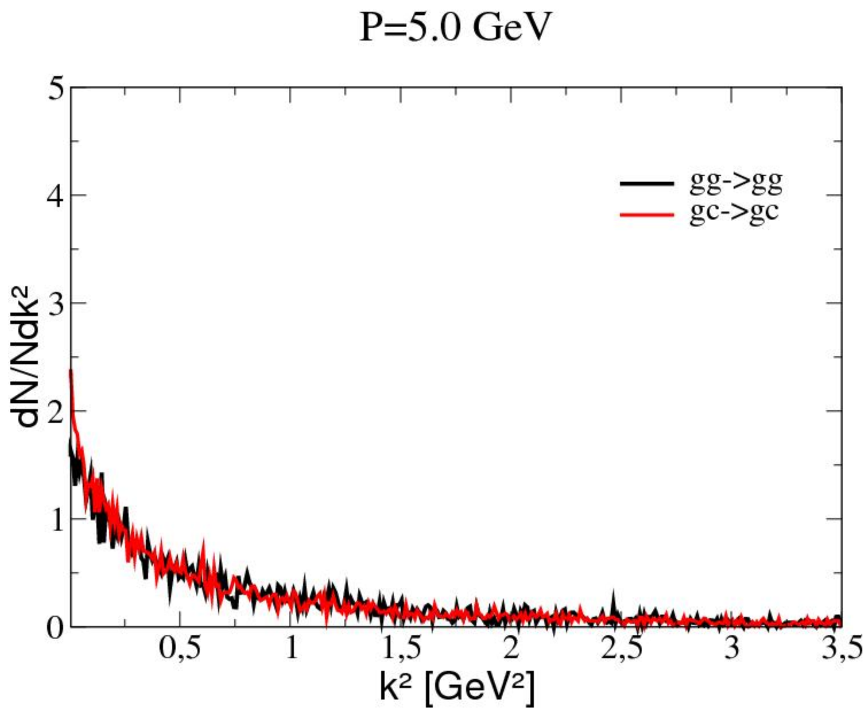
- Both Langevin and Boltzmann equation has been solved in a box for heavy quark propagating in a thermal bath composed of gluon at T= 400 MeV.
- Boltzmann equation follow exact thermalization criteria.
- In Langevin case suppression is stronger than the Boltzmann case by a factor around 2 for the isotropic.
- To compensate the difference between the Langevin and Boltzmann we need to reduce the diffusion coefficients around 30-40 %.
- For the anisotropic (FBP) case Langevin dynamics is a good approximation.
- It would be interesting to compare the v2 from both Langevin and Boltzmann side.
- Calculations in a realistic background is under progress.


Thank You . . . .

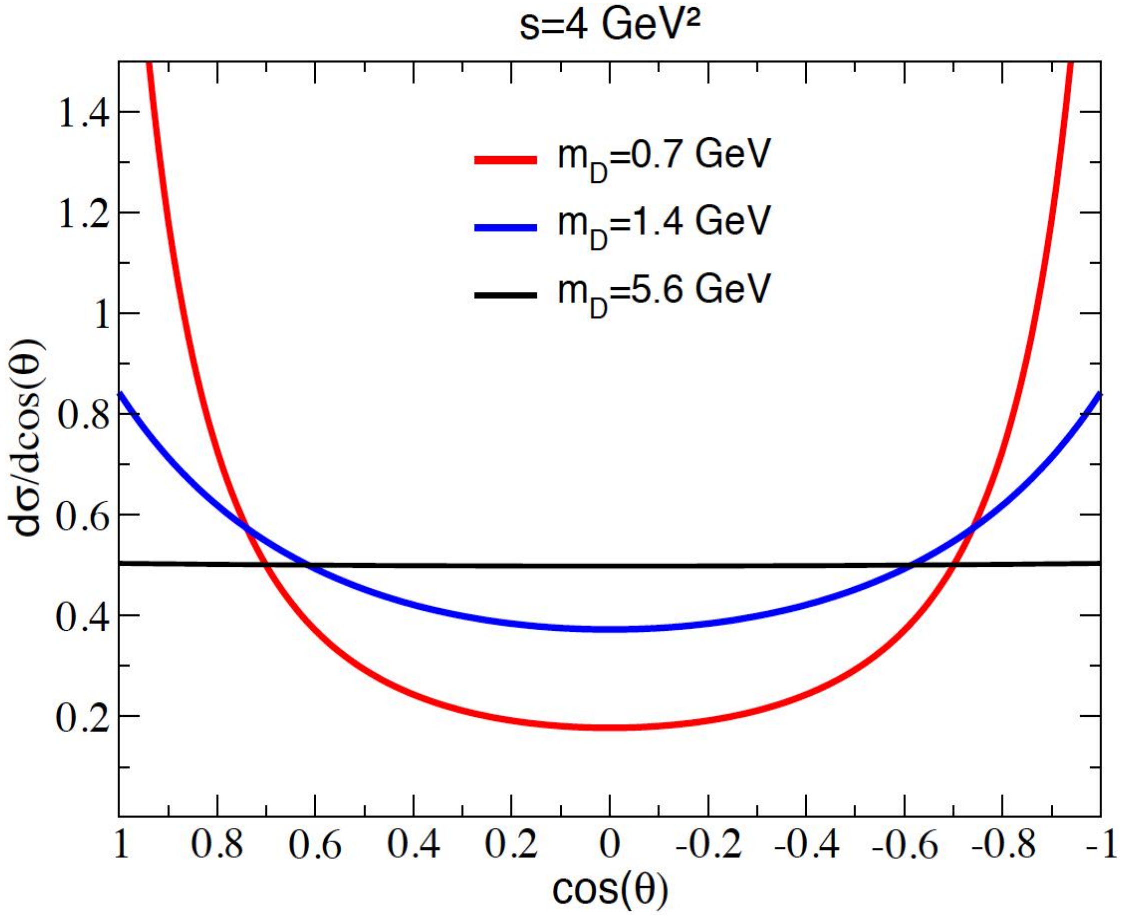
Momentum transfer

Distribution of the squared momenta transfer k^2 for fixed momentum P of the charm





The momenta transfer of gg->gg and gc-> gc are not so different



s=4 GeV²
m_D=0.7 GeV
m_D=1.4 GeV
m_D=5.6 GeV
dσ/dcos(θ)
cos(θ)
1.4
1.2
1
0.8
0.6
0.4
0.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1