Heavy flavor Suppression: Langevin vs Boltzmann

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Outline Of my talk

- Introduction
- Langevin Equation and the Thermalization Issue
- Boltzmann Equation and the Thermalization Issue
- Nuclear Suppression: Langevin vs Boltzmann
- Summary and outlook
Introduction

At very high temperature and density hadrons melt to a new phase of matter called Quark Gluon Plasma (QGP).

$\tau_{HQ} > \tau_{LQ}$, $\tau_{HQ} \sim (M/T) \tau_{LQ}$
Heavy flavor at RHIC

At RHIC energy heavy flavor suppression is similar to light flavor
Heavy Flavors at LHC

Again at LHC energy heavy flavor suppression is similar to light flavor

Is the momentum transfer really small!
Boltzmann Kinetic equation

\[
\left( \frac{\partial}{\partial t} + \frac{p}{E} \frac{\partial}{\partial x} + \mathbf{F} \cdot \frac{\partial}{\partial \mathbf{p}} \right) f(x, p, t) = \left( \frac{\partial f}{\partial t} \right)_{\text{col}}
\]

\[
R(p, t) = \left( \frac{\partial f}{\partial t} \right)_{\text{col}} = \int d^3k \left[ \omega(p + k, k)f(p + k) - \omega(p, k)f(p) \right]
\]

\[\omega(p, k) = g \int \frac{d^3q}{(2\pi)^3} f'(q) \nu_{q,p} \sigma_{p,q \rightarrow p-k,q+k} \]

is rate of collisions which change the momentum of the charmed quark from \( p \) to \( p-k \)

\[\omega(p + k, k)f(p + k) \approx \omega(p, k)f(p) + k \cdot \frac{\partial}{\partial p}(\omega f) + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f)\]

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(p)f + \frac{\partial}{\partial p_j} \left[ B_{ij}(p)f \right] \right]
\]

where we have defined the kernels

\[A_i = \int d^3k \omega(p, k)\]

\[\rightarrow \text{Drag Coefficient}\]

\[B_{ij} = \int d^3k \omega(p, k)\]

\[\rightarrow \text{Diffusion Coefficient}\]

B. Svetitsky PRD 37(1987)2484
It will interesting to study both the equation in a identical environment to ensure the validity of this assumption.
Langevin Equation

\[ dx_j = \frac{p_j}{E} \, dt \]
\[ dp_j = -\Gamma p_j \, dt + \sqrt{dt} C_{jk}(t, p + \xi dp) \rho_k \]

where \( \Gamma \) is the deterministic friction (drag) force
\( C_{ij} \) is stochastic force in terms of independent Gaussian-normal distributed random variable

\[ \rho = (\rho_x, \rho_y, \rho_z) \quad , \quad P(\rho) = \left( \frac{1}{2\pi} \right)^3 \exp\left( -\frac{\rho^2}{2} \right) \]

With \( \langle \rho_i(t)\rho_k(t') \rangle = \delta(t-t')\delta_{jk} \)
\( \xi = 0 \) the pre-point Ito interpretation of the momentum argument of the covariance matrix.

H. v. Hees and R. Rapp
arXiv:0903.1096
Langevin process defined like this is equivalent to the Fokker-Planck equation:

\[
\frac{\partial f}{\partial t} + \frac{p_j}{E} \frac{\partial f}{\partial x_j} = \frac{\partial}{\partial p_j} \left[ \left( p_j \Gamma - \xi C_{lk} \frac{\partial C_{jk}}{\partial p_l} \right) f \right] + \frac{1}{2} \frac{\partial^2}{\partial p_j \partial p_k} (C_{jl} C_{kl} f)
\]

the covariance matrix is related to the diffusion matrix by

\[
C_{jk} = \sqrt{2B_0 P_{jk}^\perp} + \sqrt{2B_1 P_{jk}^{\parallel}}
\]

and

\[
A_i = p_j \Gamma - \xi C_{lk} \frac{\partial C_{ij}}{\partial p_i}
\]

With

\[
B_0 = B_1 = D \quad \quad C_{jk} = \sqrt{2D(E)\delta_{jk}}
\]

Relativistic dissipation-fluctuation relation

\[
A(E)ET - D(E) + T(1 - \xi)D'(E) = 0
\]
For Collision Process the $A_i$ and $B_{ij}$ can be calculated as following:

\[
A_i = \frac{1}{2E_p} \int \frac{d^3q}{(2\pi)^3} \frac{d^3q'}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{2E_{p'}} \sum |M|^2 (2\pi)^4 \delta^4(\vec{p} + \vec{q} - \vec{p'} - \vec{q'}) f(q) \langle (p - p')_i \rangle = \langle \langle p - p' \rangle \rangle
\]

\[
B_{ij} = \frac{1}{2} \langle \langle (p - p')_i (p' - p)_j \rangle \rangle
\]

**Elastic processes**

- We have introduce a mass into the internal gluon propagator in the t and u-channel-exchange diagrams, to shield the infrared divergence.

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Thermalization in Langevin approach in a static medium

1) Diffusion $D=$ Constant
   Drag $A = D/ET$ from FDT

2) Diffusion $D(p)$ and Drag $A(p)$ both from pQCD

Case: 1
1) $D=$ Constant
   $A = D/ ET$ from FDT

Due to the collision charm approaches to thermal equilibrium with the bulk

Bulk composed only by gluon in Thermal equilibrium at $T= 400$ MeV.
Case: 2

Diffusion coefficient: $D(p)$ pQCD
Drag coefficient: $A(p)$ pQCD

In this case we are away from thermalization.
Transport theory

\[ p^\mu \partial_\mu f(\mathbf{x}, p) = C_{22} \]

\[ C_{22} = \frac{1}{2E_1} \int \frac{d^3p_2}{(2\pi)^32E_2} \frac{1}{\nu} \int \frac{d^3p'_1}{(2\pi)^32E'_1} \frac{d^3p'_2}{(2\pi)^32E'_2} f_1'f_2' |\mathcal{M}_{1'2'\rightarrow 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2) \]

\[ 1 \int \frac{d^3p_2}{(2\pi)^32E_2} \frac{1}{\nu} \int \frac{d^3p'_1}{(2\pi)^32E'_1} \frac{d^3p'_2}{(2\pi)^32E'_2} f_1f_2 |\mathcal{M}_{12\rightarrow 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p'_1 - p'_2) \]

\[ \Delta t \rightarrow 0 \]

\[ \Delta^3 x \rightarrow 0 \]

Exact solution

Collision integral is solved with a local stochastic sampling

[ Z. Xhu, et al. PRC71(04)]
Greco et al PLB670, 325 (08)]

\[ P_{22} = \frac{\Delta N_{\text{coll}}^2}{\Delta N_1 \Delta N_2} = u_{\text{rel}} \sigma_{22} \frac{\Delta t}{\Delta^3 x} \]
Cross Section $gc \rightarrow gc$

The infrared singularity is regularized introducing a Debye-screening-mass $\mu_D$

$$m_D = \sqrt{4\pi\alpha_s T}$$

$$\sigma = \frac{1}{16\pi(s-M^2)^2} \int_0^{s-M^2} dt \sum |\mathcal{M}|^2$$

Charm evolution in a static medium

Simulations in which a particle ensemble in a box evolves dynamically
Bulk composed only by gluons in thermal equilibrium at $T=400$ MeV

C and Cbar are initially distributed: uniformly in r-space, while in p-space

Due to collisions charm approaches to thermal equilibrium with the bulk
Langevin vs Boltzmann

Case: 1 \( m_D = 0.83 \text{ GeV} \) (\( \sim gT, \text{ pQCD} \))

Case: 2 \( m_D = 3 \text{ GeV} \) (Isotropic)

Case: 3 \( m_D = 0.4 \text{ GeV} \) (Forward-backward peak)

We have scaled our interaction in such a way that our thermalization time is always same for all the three cases.
Ratio between Langevin and Boltzmann
At fixed time (pQCD)

$m_D = 0.83 \text{ GeV}$
T-matrix cross section are usually isotropic

Hees, Mannarelli, Greco, and R. Rapp
Ratio between Langevin and Boltzmann
At fixed time (FBP)

$m_D = 0.4 \text{ GeV}$
Nuclear Suppression: Langevin vs Boltzmann (pQCD)

Suppression is more in Langevin approach than Boltzmann

\[ R_{AA} = \frac{\left(\frac{dN}{d^3p}\right)_{\text{output}}}{\left(\frac{dN}{d^3p}\right)_{\text{input}}} \]

\[ m_D = 0.83 \text{ GeV} \]
Nuclear Suppression: Langevin vs Boltzmann
(Isotropic case)

Suppression is more in Langevin approach than Boltzmann
To have a phenomenological touch let put the RHIC data although our calculation is only for a box.
To compensate the difference in the RAA we need to reduce the diffusion coefficient around 30-40% which is the phenomenological interest.

Calculation in a realistic background is under progress.
Both Langevin and Boltzmann equation has been solved in a box for heavy quark propagating in a thermal bath composed of gluon at $T = 400$ MeV.

Boltzmann equation follow exact thermalization criteria.

In Langevin case suppression is stronger than the Boltzmann case by a factor around 2 for the isotropic.

To compensate the difference between the Langevin and Boltzmann we need to reduce the diffusion coefficients around 30-40%.

For the anisotropic (FBP) case Langevin dynamics is a good approximation.

It would be interesting to compare the $v2$ from both Langevin and Boltzmann side.

Calculations in a realistic background is under progress.
Thank You
Momentum transfer

Distribution of the squared momenta transfer $k^2$ for fixed momentum $P$ of the charm

$P = 1.5$ GeV  

$P = 5.0$ GeV

The momenta transfer of $gg\rightarrow gg$ and $gc\rightarrow gc$ are not so different.