LANDAU HYDRODYNAMICS FOR NON-CENTRAL HEAVY-ION COLLISIONS AND LONGITUDINAL SCALING OF ELLIPTIC FLOW

Karolis Tamosiunas
Institute of Theoretical Physics and Astronomy
Historical Remarks

- In 1950 Fermi suggests to apply thermodynamics to particle production in high-energy collisions.
- 1953 Landau improves the idea with the assumption, that system strongly interacts and particles are produced after the ideal hydrodynamic expansion.
- Landau’s dN/dy distribution works well for RHIC and LHC energies.
Basic assumptions of Landau hydro

- Longitudinal and Transverse parts of hydrodynamic expansion are solved separately
- EoS of the matter is the ideal relativistic gas i.e. statistical equilibrium is reached
- Transverse expansion does not include initial flow and is pressure gradient driven

\[ \partial_\mu T^{\mu\nu} = 0 \]
\[ T^{\mu\nu} = (e + P)u^\mu u^\nu - P g^{\mu\nu}. \]
Longitudinal expansion

\[ \frac{\partial T^{00}}{\partial t} + \frac{\partial T^{0z}}{\partial z} = 0, \]
\[ \frac{\partial T^{0z}}{\partial t} + \frac{\partial T^{zz}}{\partial z} = 0 \]

\[ u^0 = \cosh y, \]
\[ u^z = \sinh y \]

\[ y_+ = \ln((t + z)/\Delta), \]
\[ y_- = \ln((t - z)/\Delta) \]

with EoS \[ P = e/3 \]

\[ e(y_+, y_-) = e_0 \exp \left[ -\frac{4}{3}(y_+ + y_- - \sqrt{y_+y_-}) \right], \]
\[ y(y_+, y_-) = (y_+ - y_-)/2, \]

From C. Wong arXiv:0809.0517
Transverse expansion

\[ T^{rr} = (e+P)(u^0)^2 v_r^2 + P, \quad T^{\phi\phi} = (e+P)(u^0)^2 v_\phi^2 + P/r^2, \]
\[ T^{0r} = (e + P)(u^0)^2 v_r, \quad T^{0\phi} = (e + P)(u^0)^2 v_\phi. \]

\[ \frac{\partial T^{0r}}{\partial t} + \frac{\partial T^{rr}}{\partial r} = 0 \quad v_\phi = 0 \]

\[ 4e(u^0)^2 \frac{\partial v_r}{\partial t} + 4e(u^0)^2 \frac{\partial v_r^2}{\partial r} + \frac{\partial e}{\partial r} = 0. \]

Landau original case:
\[ \frac{\partial v_r}{\partial t} = \frac{2r(t)}{t^2} \]
\[ \frac{\partial e}{\partial r} \approx -\frac{e}{R_A} \]
\[ x(t) = \frac{t^2}{4a(u^0)^2} \quad a = 2R_A \]

Modification for peripheral collisions:
\[ \frac{\partial e}{\partial r} = \frac{e(r = R_\phi) - e(r = 0)}{R_\phi} \]
\[ f(R_\phi) = e(r = R_\phi)/e(r = 0) \]
\[ r(t) = \frac{(1 - f(R_\phi))t^2}{8(u^0)^2 R_\phi} \]

With the condition: \( b = 0 \Rightarrow \) original Landau
Conic flight stage

Connecting long and trans expansions

\[ r(t_{FO}) = a = 2R_A \]

\[ t_{FO} = 2 \cosh y \sqrt{\frac{2aR_\phi}{(1 - f(R_\phi))}} \]

\[ u^0 = \cosh y \]

Energy and entropy fluxes stop changing in element \( 2\pi r dr \)

\[
\frac{dN}{dy} \propto \frac{dS}{dy} = ce_0^{3/4} \exp \left[-(y_+ + y_- - \sqrt{y_+ y_-})\right] \frac{t}{\cosh y}
\]

For a fixed angle

For central collisions the result is:

\[
\frac{dN}{dy} \propto \exp \sqrt{(y_{beam}^2 - y^2)}
\]

For peripheral one needs:

\[ \Delta(\phi) f(R_\phi) \]
Let’s plug in the Initial State

Simplest Wood-Saxon density distribution:

\[
\rho_A(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R_A}{d}\right)} \quad T_A(r, \phi) = T_A(x - b/2, y) = \int dz \rho_A(r)
\]

\[
n_{WN}(r, \phi) = T_A(r, \phi) \left[1 - \left(1 - \frac{\sigma T_B(r, \phi)}{B}\right)^B\right] + T_B(r, \phi) \left[1 - \left(1 - \frac{\sigma T_A(r, \phi)}{A}\right)^A\right]
\]

Assuming, that \( e \) proportional to \( n \)

\[
e(r, \phi; b) \propto n_{WN}(r, \phi; b)
\]

For \( b=0 \) we get original Landau:

\[
f(0) = 0 \quad \text{and} \quad k=1
\]

\[
\Delta(\phi) = \kappa_\phi R_A/\gamma, \quad \text{where}
\]

\[
\kappa_\phi = \sqrt{n_{WN}(R_\phi, \phi; b)/\max(n_{WN}(R_\phi, \phi; b))}
\]
Results for $v_2$ at LHC and RHIC

\[ v_2(y) = \frac{\int d\phi (dN/d\phi dy) \cos(2\phi)}{\int d\phi (dN/d\phi dy)} \]
Longitudinal Scaling of $v_2$

$y_{beam} = \ln(\sqrt{s_{NN}/m_N})$

From ALICE: nucl-ex/1210.7095
Longitudinal Scaling

Getting $v_2$ and it’s longitudinal scaling without any free parameters!

$$v_2(y) = \frac{\int d\phi (dN/d\phi dy) \cos(2\phi)}{\int d\phi (dN/d\phi dy)}$$
Fitting initial energy density profile from data

experimentally known: \[ \frac{dN}{d\phi} \bigg|_{\eta} \quad \frac{dN}{d\eta} \bigg|_{\phi} \]

redefining

\[ t_{FO} = 2 \cosh y \sqrt{\frac{2aR_{\phi}}{(1 - f(R_{\phi}))}} \equiv g(\phi) \]

\[ f(R_{\phi}) = e(r = R_{\phi})/e(r = 0) \]

need to find by fitting: \( \Delta(\phi) \quad g(\phi) \)

event-by-event

\[ \frac{dS}{dy} = ce_0^{3/4} \exp \left[ - (y_+ + y_- - \sqrt{y_+y_-}) \right] \frac{t}{\cosh y} \]
Where we can use it?

- Calculating $v_n$ with more realistic (fluctuating) initial conditions, like MC Glauber or CGC
- In the MC Glauber model case for IC, one has to transform microscopic quantities to macroscopic – e
- Fitting raw experimental multiplicity data to obtain initial pressure gradients
Conclusions

• Very simple analytical description, which gives $v_2$ without any free parameters
• The possibility to probe initial state directly from experimental event-by-event data
• $\Rightarrow$ separate flow and investigate initial fluctuations.

Thank you