LANDAU HYDRODYNAMICS FOR NON-CENTRAL HEAVY-ION COLLISIONS AND LONGITUDINAL SCALING OF ELLIPTIC FLOW

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Historical Remarks

- In 1950 Fermi suggests to apply thermodynamics to particle production in high-energy collisions.
- 1953 Landau improves the idea with the assumption, that system strongly interacts and particles are produced after the ideal hydrodynamic expansion.
- Landau's dN/dy distribution works well for RHIC and LHC energies.

Basic assumptions of Landau hydro

- Longitudinal and Transverse parts of hydrodynamic expansion are solved separately
- EoS of the matter is the ideal relativistic gas i.e. statistical equilibrium is reached
- Transverse expansion does not include initial flow and is pressure gradient driven

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (e + P)u^\mu u^\nu - P g^{\mu\nu}.$$

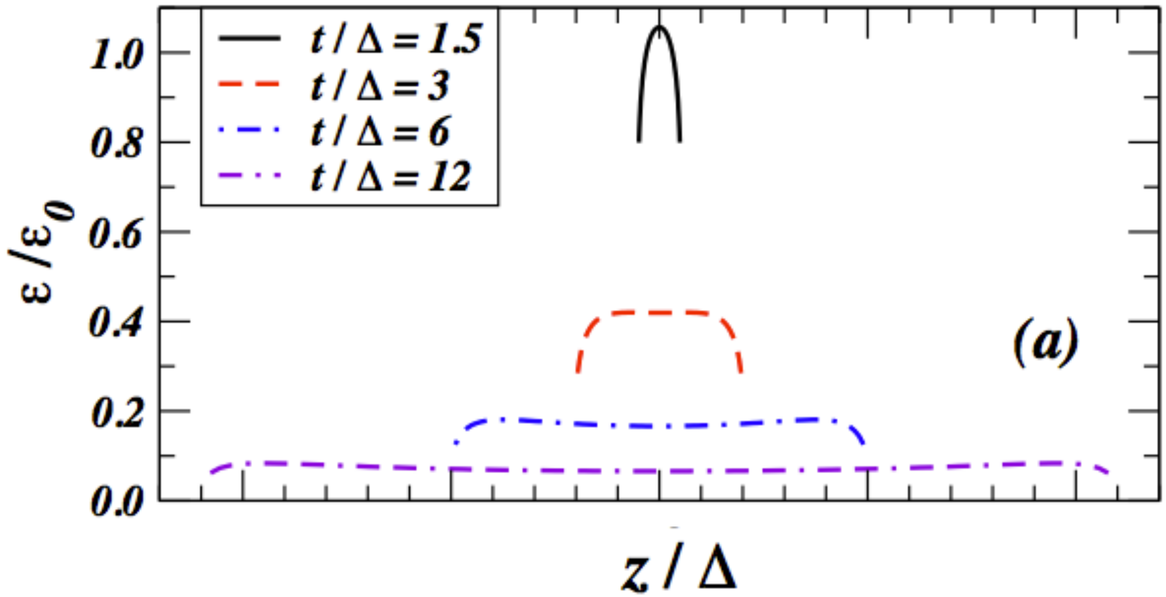
Longitudinal expansion

$$\frac{\partial T^{00}}{\partial t} + \frac{\partial T^{0z}}{\partial z} = 0,$$

$$\frac{\partial T^{0z}}{\partial t} + \frac{\partial T^{zz}}{\partial z} = 0.$$

$$u^0 = \cosh y,$$

$$u^z = \sinh y$$



$$y_+ = \ln((t+z)/\Delta),$$
$$y_- = \ln((t-z)/\Delta)$$

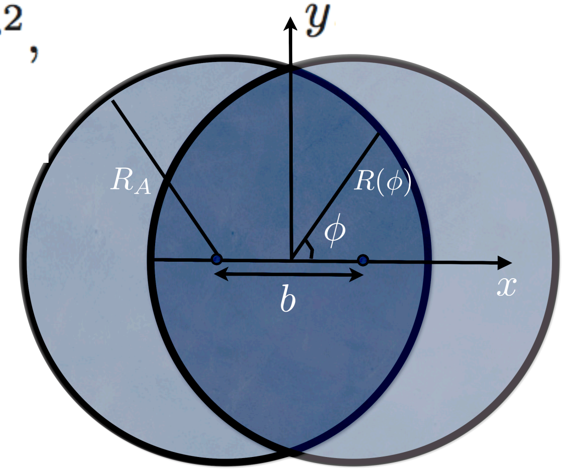
with EoS $P = e/3$

From C. Wong arXiv:0809.0517

$$e(y_+, y_-) = e_0 \exp\left[-4/3(y_+ + y_- - \sqrt{y_+ y_-})\right],$$
$$y(y_+, y_-) = (y_+ - y_-)/2,$$

Transverse expansion

$$T^{rr} = (e+P)(u^0)^2 v_r^2 + P, \quad T^{\phi\phi} = (e+P)(u^0)^2 v_\phi^2 + P/r^2,$$
$$T^{0r} = (e+P)(u^0)^2 v_r, \qquad T^{0\phi} = (e+P)(u^0)^2 v_\phi.$$



$$\frac{\partial T^{0r}}{\partial t} + \frac{\partial T^{rr}}{\partial r} = 0 \qquad v_\phi = 0$$

$$4e(u^0)^2 \frac{\partial v_r}{\partial t} + 4e(u^0)^2 \frac{\partial v_r^2}{\partial r} + \frac{\partial e}{\partial r} = 0.$$

Landau original case:

$$\partial v_r / \partial t = 2r(t)/t^2$$

$$\partial e/\partial r \approx -e/R_A$$

$$x(t) = \frac{t^2}{4a(u^0)^2} \qquad a = 2R_A$$

Modification for peripheral collisions:

$$\frac{\partial e}{\partial r} = \frac{e(r = R_\phi) - e(r=0)}{R_\phi}$$

$$f(R_\phi) = e(r = R_\phi)/e(r=0)$$

$$r(t) = \frac{(1 - f(R_\phi))t^2}{8(u^0)^2 R_\phi}$$

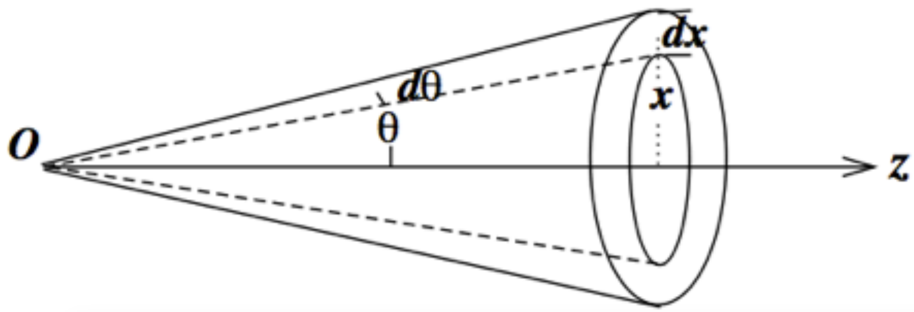
With the condition: b = 0 => original Landau

Conic flight stage

Connecting long and trans expansions

$$r(t_{FO}) = a = 2R_A$$

$$t_{FO} = 2\cosh y \sqrt{\frac{2aR_\phi}{(1 - f(R_\phi))}}$$



$$u^0 = \cosh y$$

energy and entropy fluxes stop changing in element $2\pi r dr$

$$dN/dy \propto \frac{dS}{dy} = c e_0^{3/4} \exp\left[-(y_+ + y_- - \sqrt{y_+ y_-})\right] \frac{t}{\cosh y}$$

For a fixed angle

For central collisions the result is:

$$dN/dy \propto \exp\sqrt{(y_{beam}^2 - y^2)}$$

For peripheral one needs:

$$\Delta(\phi) \quad f(R_\phi)$$

Let's plug in the Initial State

Simplest Wood-Saxon density distribution:

$$\rho_A(\mathbf{r}) = \frac{\rho_0}{1+\exp(\frac{\mathbf{r}-R_A}{d})} \qquad T_A(r,\phi) = T_A(x-b/2,y) = \int \mathrm{d}z\, \rho_A(\mathbf{r})$$

$$n_{WN}(r,\phi) = T_A(r,\phi)\left[1-\left(1-\frac{\sigma T_B(r,\phi)}{B}\right)^B\right] + T_B(r,\phi)\left[1-\left(1-\frac{\sigma T_A(r,\phi)}{A}\right)^A\right]$$

Assuming, that *e* proportional to *n*

$$e(r,\phi;b) \propto n_{WN}(r,\phi;b)$$

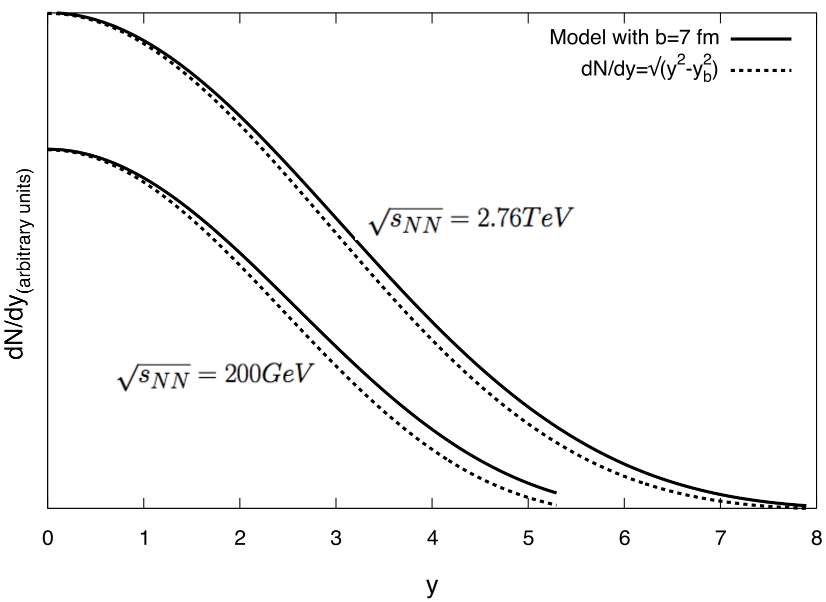
$$f(R_\phi) = \frac{n_{WN}(R_\phi,\phi;b) - \min(n_{WN}(R_\phi,\phi;b))}{n_{WN}(0,0;b)}$$

For b=0 we get original Landau: f=0 and k=1

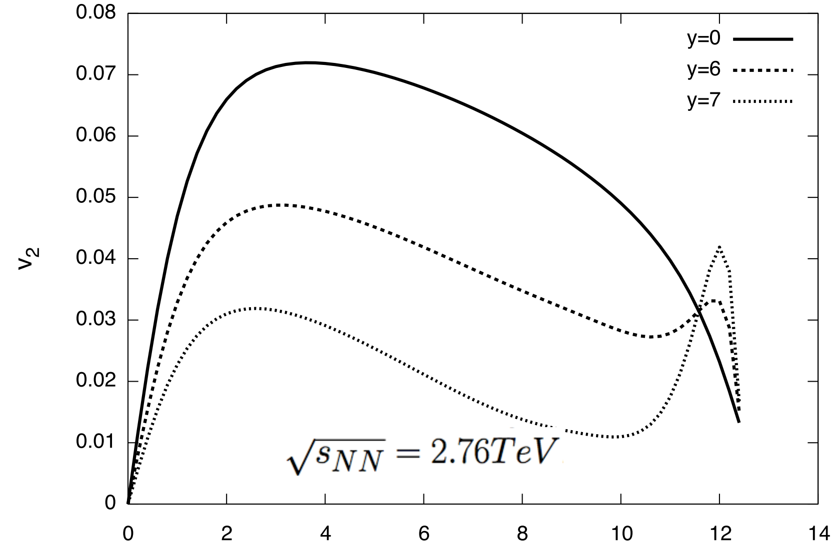
$\Delta(\phi) = \kappa_\phi R_A/\gamma,$ where

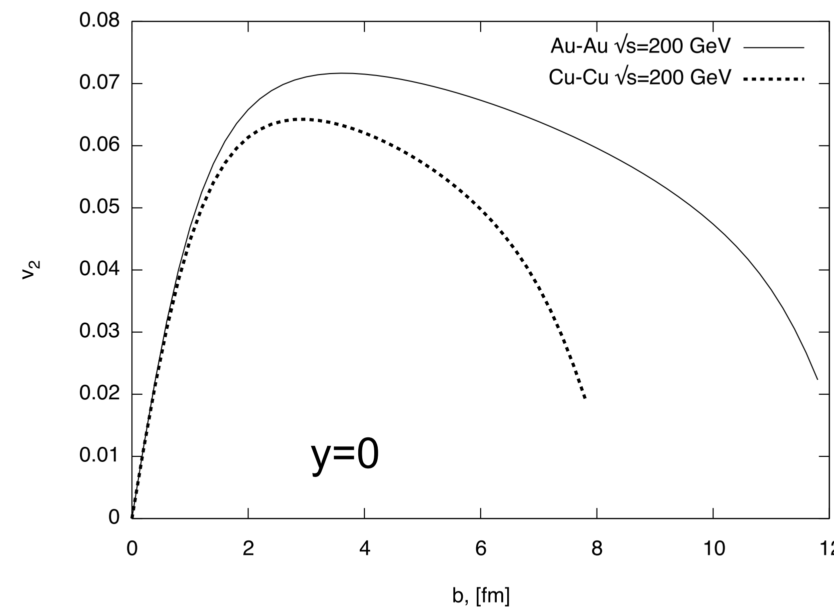
$$\kappa_\phi = \sqrt{n_{WN}\left(R_\phi,\phi;b\right)/\max\left(n_{WN}\left(R_\phi,\phi;b\right)\right)}$$

Results for v_2 at LHC and RHIC

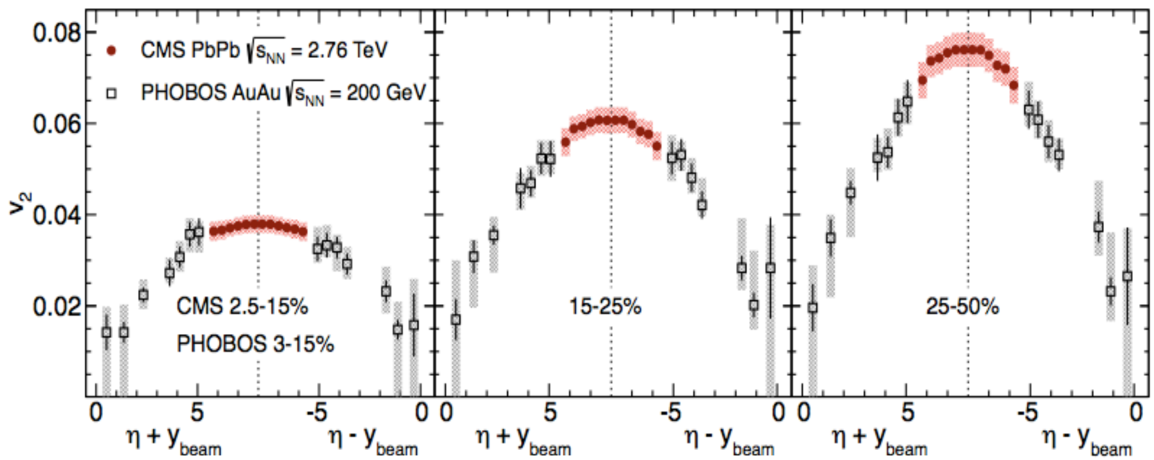


$$v_2(y) = \frac{\int d\phi (dN/d\phi dy) \cos(2\phi)}{\int d\phi (dN/d\phi dy)}$$

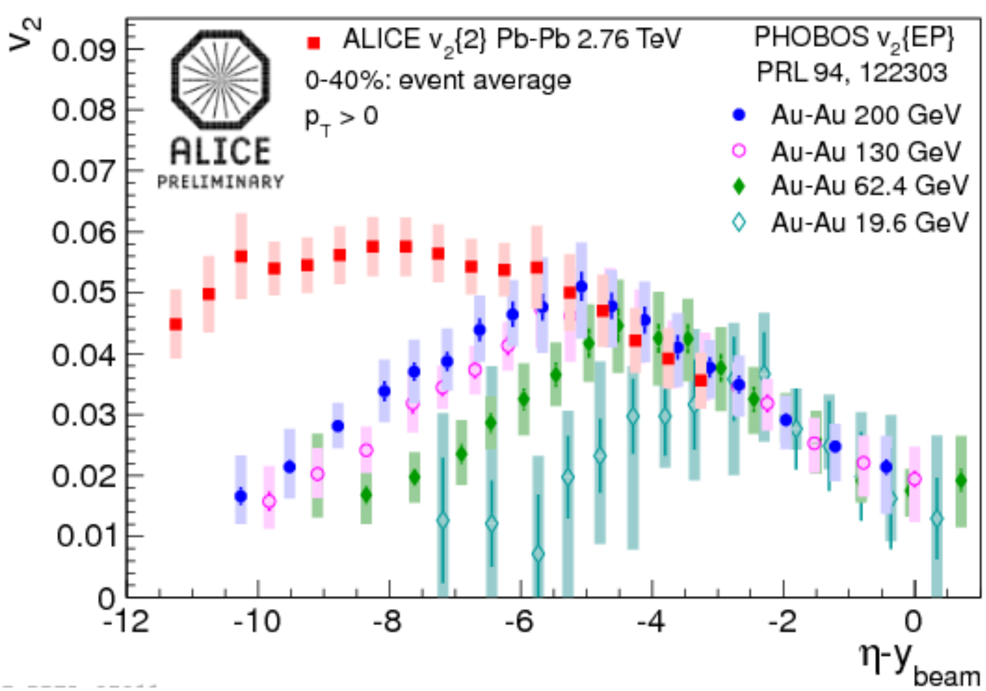




Longitudinal Scaling of v_2



$$y_{beam} = \ln(\sqrt{s_{NN}}/m_N)$$

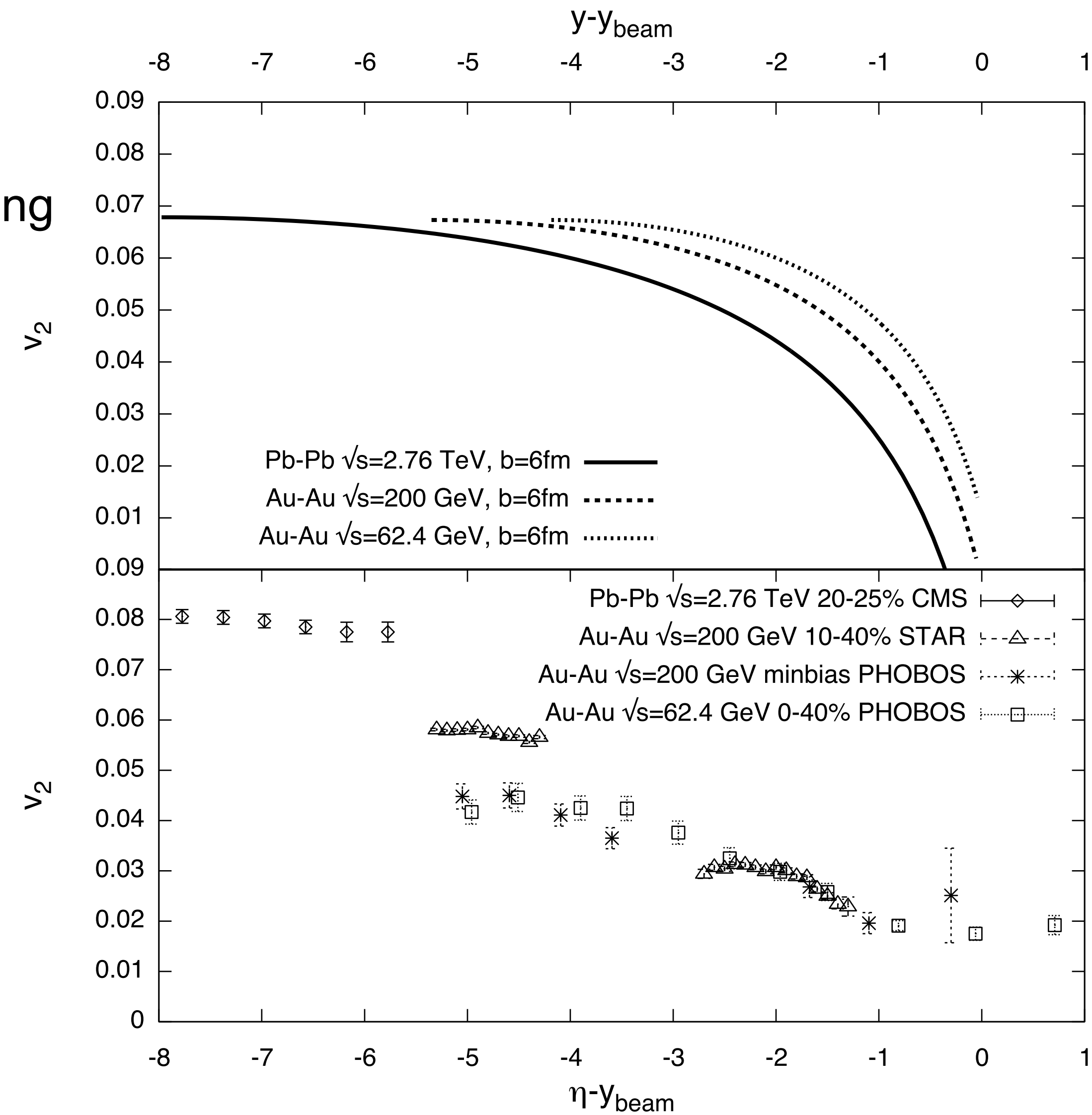


From ALICE: nucl-ex/1210.7095

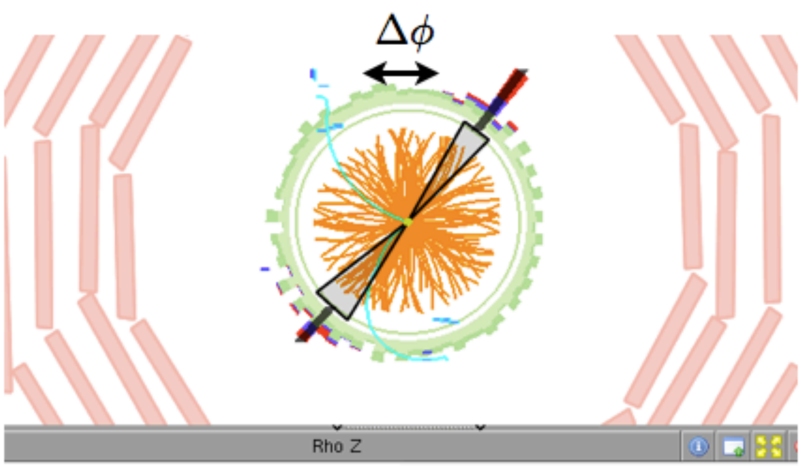
Longitudinal Scaling

Getting v_2 and it's longitudinal scaling without any free parameters!

$$v_2(y) = \frac{\int d\phi (dN/d\phi dy) \cos(2\phi)}{\int d\phi (dN/d\phi dy)}$$



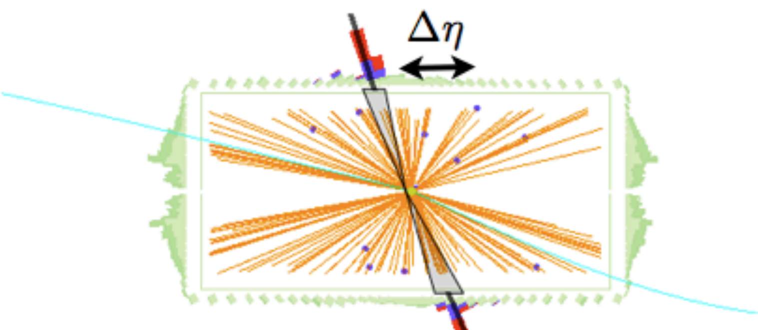
Fitting initial energy density profile from data



experimentally known: $\left.\frac{dN}{d\phi}\right|_{\eta}$ $\left.\frac{dN}{d\eta}\right|_{\phi}$

redefining

$$t_{FO} = 2\cosh y \boxed{\sqrt{\frac{2aR_\phi}{(1-f(R_\phi))}}} \equiv g(\phi)$$



$$f(R_\phi) = e(r = R_\phi)/e(r = 0)$$

need to find by fitting : $\Delta(\phi)$ $g(\phi)$

event-by-event

$$\frac{\mathrm{d}S}{\mathrm{d}y} = ce_0^{3/4} \exp\left[-(y_+ + y_- - \sqrt{y_+ y_-})\right] \frac{t}{\cosh y}$$

Where we can use it?

- Calculating v_n with more realistic (fluctuating) initial conditions, like MC Glauber or CGC
- In the MC Glauber model case for IC, one has to transform microscopic quantities to macroscopic – *e*
- Fitting raw experimental multiplicity data to obtain initial pressure gradients

Conclusions

- Very simple analytical description, which gives v_2 without any free parameters
- The possibility to probe initial state directly from experimental event-by-event data
- → separate flow and investigate initial fluctuations.

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Thank you