First Principles Calculation of Finite Temperature Charmonium Potentials

Chris Allton (Swansea University)

First Principles Calculation of Finite Temperature Charmonium Potentials

Gert Aarts, CRA, Alessandro Amato, Wynne Evans, Pietro Giudice, Simon Hands, Aoife Kelly, Seyong Kim, Maria-Paola Lombardo, Sinead Ryan, Jon-Ivar Skullerud, Don Sinclair, Tim Harris

FASTSUM Collaboration

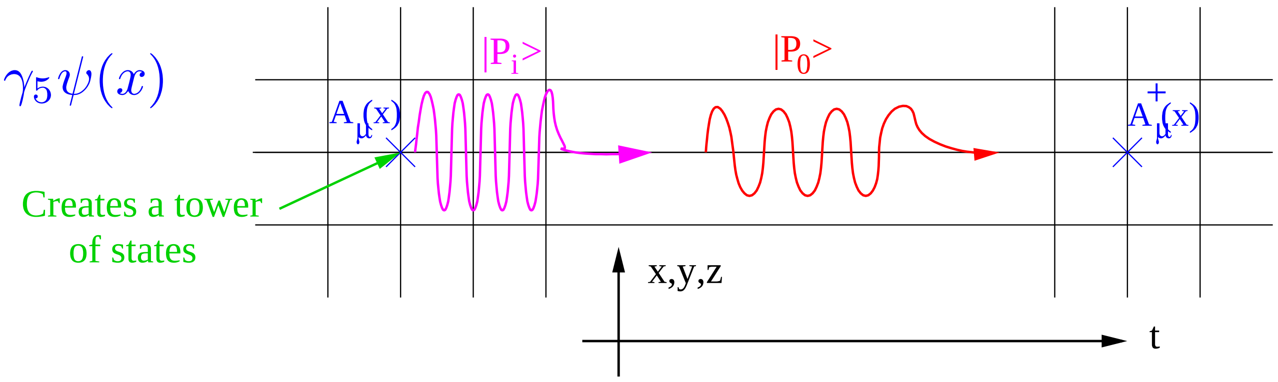
Particle Data Book

$\sim 1,500$ pages

zero pages on Quark-Gluon Plasma...

Studying a Meson via Lattice

$$A_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)$$



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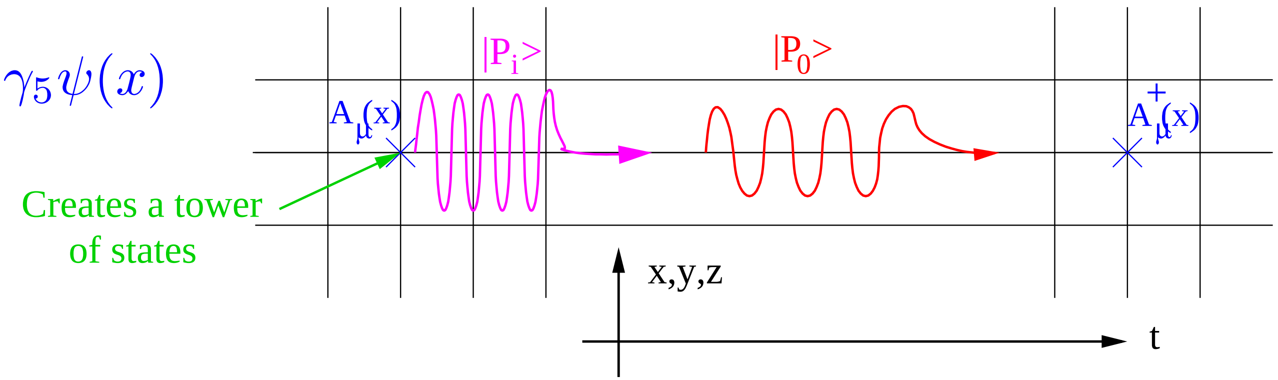

$$\langle\mathcal{O}\rangle = G_2(t) = \sum_{\{U\}}\sum_{\vec{x}}\langle 0|A_\mu(\vec{x},t)A_\mu^\dagger(\vec{0},0)|0\rangle$$

$$= \sum_{\{U\}}\sum_{\vec{x}}\sum_{i}\int\frac{d^3k}{2E}\langle 0|A_\mu(\vec{x},t)|P_i(\vec{k})\rangle\langle P_i(\vec{k})|A_\mu^\dagger(\vec{0},0)|0\rangle$$

$$= \sum_{\{U\}}\sum_{i}\frac{1}{2M_i}\langle 0|A_\mu(0)|P_i(0)\rangle\langle P_i(0)|A_\mu^\dagger(0)|0\rangle e^{-M_i t}$$

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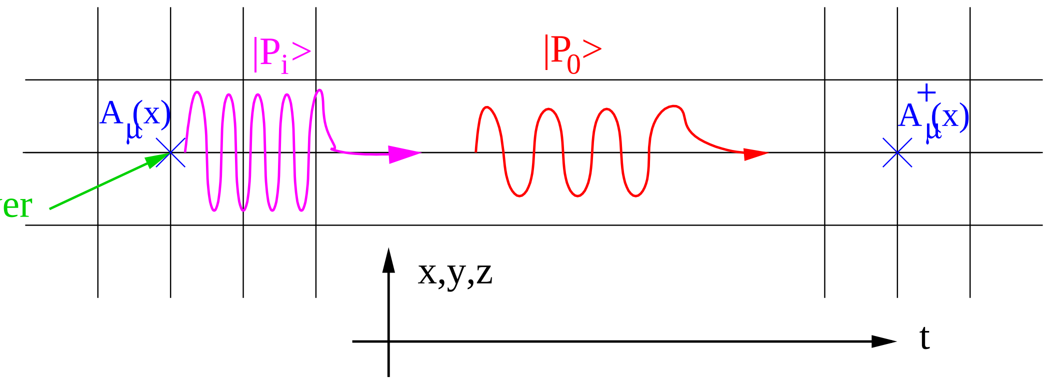

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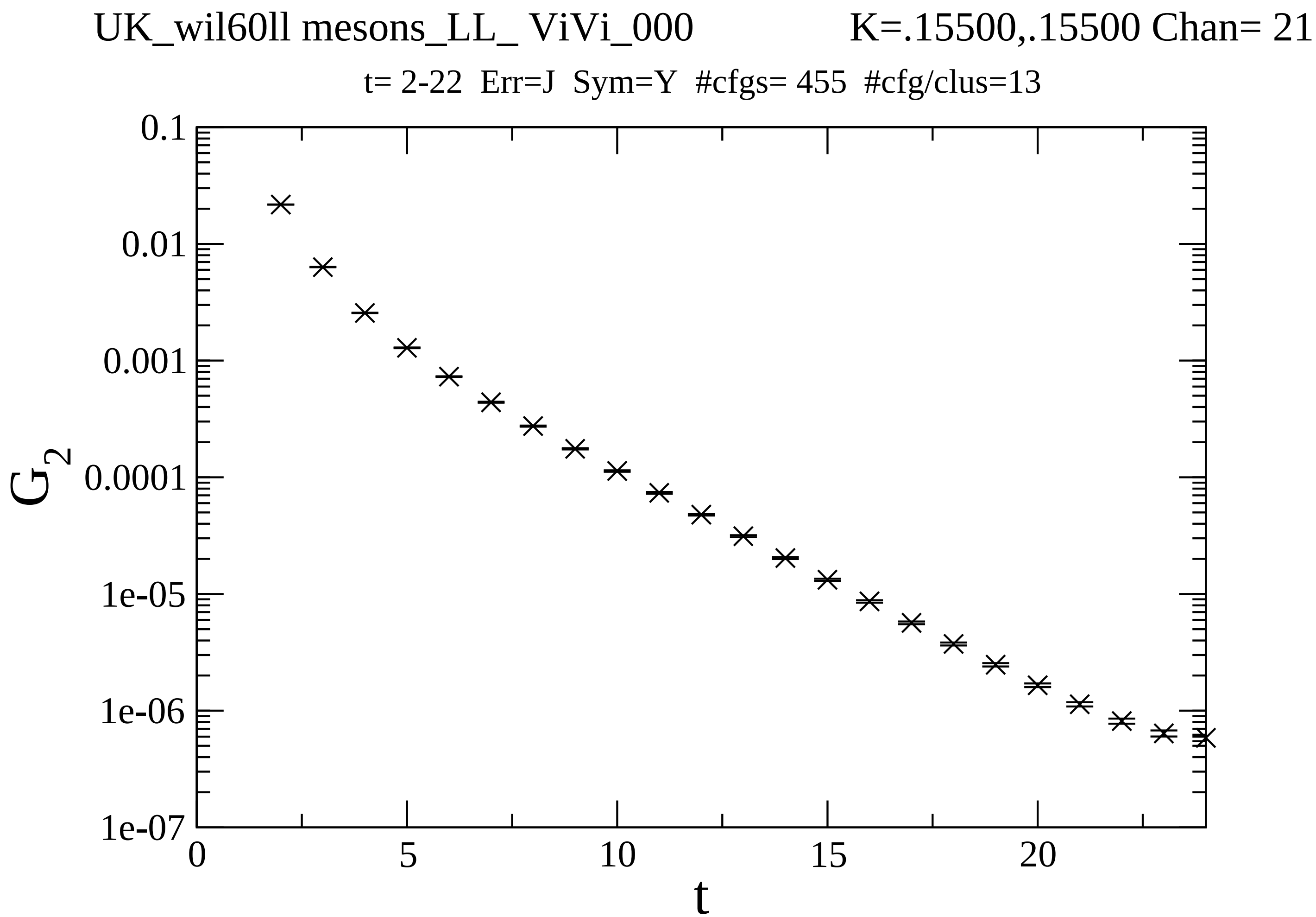
$$= \sum_{\{U\}}\sum_i\frac{1}{2M_i}\langle 0|A_\mu(0)|P_i(0)\rangle\langle P_i(0)|A_\mu^\dagger(0)|0\rangle e^{-M_i t}$$

t large: $\rightarrow \dfrac{|\langle 0|A_\mu(0)|P(0)\rangle|^2}{2M_0}\; e^{-M_0 t} \equiv Z\; e^{-M_0 t}$

Lightest state!

Example Correlation Function

$$G_2(t) = \sum_{\{U\}} \sum_{\vec{x}} \langle 0 | A_\mu(\vec{x}, t) A_\mu^\dagger(\vec{0}, 0) | 0 \rangle \quad \rightarrow \quad Z\, e^{-M_0 t}$$



Actions:

$$\mathcal{S}(\psi,\bar\psi,A)^{\textbf{cont}} = \int \mathcal{L}_E d^4x = \int \left\{ \bar\psi(x)(\not{D} + m)\psi(x) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right\} d^4x$$

$$\mathcal{S}(\psi,\bar\psi,A)^{\textbf{latt}} = \sum_x \mathcal{L}_E = \sum_x \bar\psi(x)(\not{\Delta} + m)\psi(x) + \sum_p U_p$$

Introduction to Lattice QCD

Actions:

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Expectation Values:

$$< \mathcal{O} >^{\textbf{cont}} = \frac{1}{Z} \int \mathcal{D}\psi\mathcal{D}\bar\psi\mathcal{D}A \, \mathcal{O}(\psi,\bar\psi,A) \, e^{-S(\psi,\bar\psi,A)^{\textbf{cont}}}$$

$$< \mathcal{O} >^{\textbf{latt}} = \frac{1}{Z} \sum_{U_{x\mu}} \mathcal{O}(\psi,\bar\psi,U) \, e^{-S(\psi,\bar\psi,U)^{\textbf{latt}}}$$

Introduction to Lattice QCD

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= “Boltzmann Weight”

Lattice calculations of QCD are analogous to Stat Mech calculations!

c.f. Ising Model

Extrapolations Required

Lattice simulations don't solve real QCD:

$$< \mathcal{O} >^{\textbf{latt}} = f(N_{\{U\}}, g_0, m, \mu, L, N_f)$$

$N_{\{U\}} = \mathcal{O}(100)$

$a \approx 0.1$ fm

$m_q \approx$ 50 MeV

$L \approx 2$ fm

$N_f =$ 0, 2 or 2+1

Extrapolations Required

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$$< \mathcal{O} >^{\textbf{latt}} = f(N_{\{U\}}, g_0, m, \mu, L, N_f)$$

$N_{\{U\}} = \mathcal{O}(100) \rightarrow \infty$

$a \approx 0.1$ fm $\rightarrow$ 0

$m_q \approx$ 50 MeV $\rightarrow$ few MeV

$L \approx 2$ fm $\rightarrow \infty$

$N_f =$ 0, 2 or 2+1 $\rightarrow$ "1+1+1+1+1+1"

Extrapolations Required

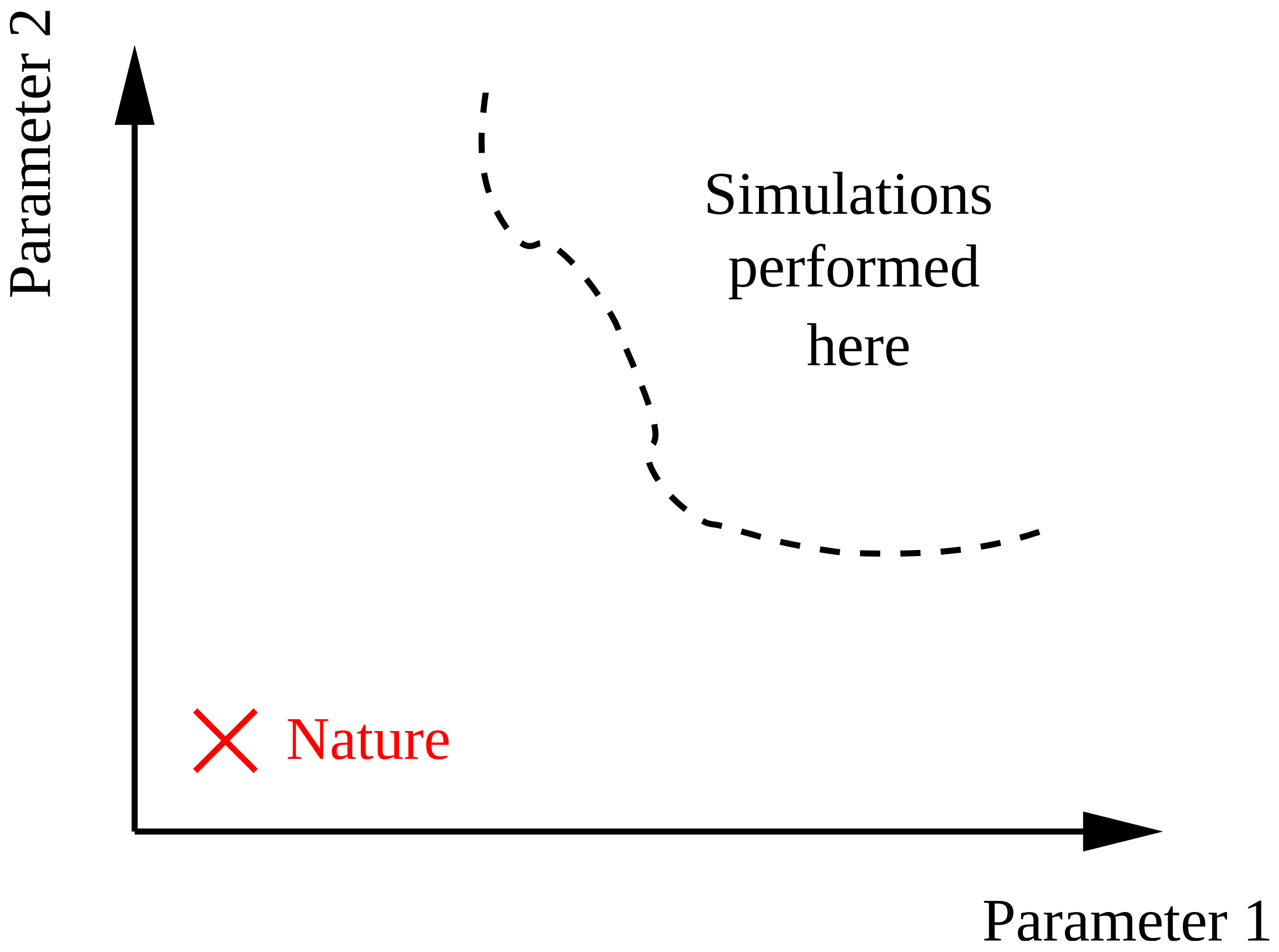
Lattice simulations don't solve real QCD:

$$< \mathcal{O} >^{\textbf{latt}} = f(N_{\{U\}}, g_0, m, \mu, L, N_f)$$

$N_{\{U\}} = \mathcal{O}(100)$	$\rightarrow$	∞	"trivial"
$a \approx 0.1$ fm	$\rightarrow$	0	RG
$m_q \approx$ 50 MeV	$\rightarrow$	few MeV	Chiral PT
$L \approx 2$ fm	$\rightarrow$	∞	Finite Size Scaling
$N_f =$ 0, 2 or 2+1	$\rightarrow$	"1+1+1+1+1+1"	Expensive

But . . . it is systematically improvable

Extrapolations Req'd II



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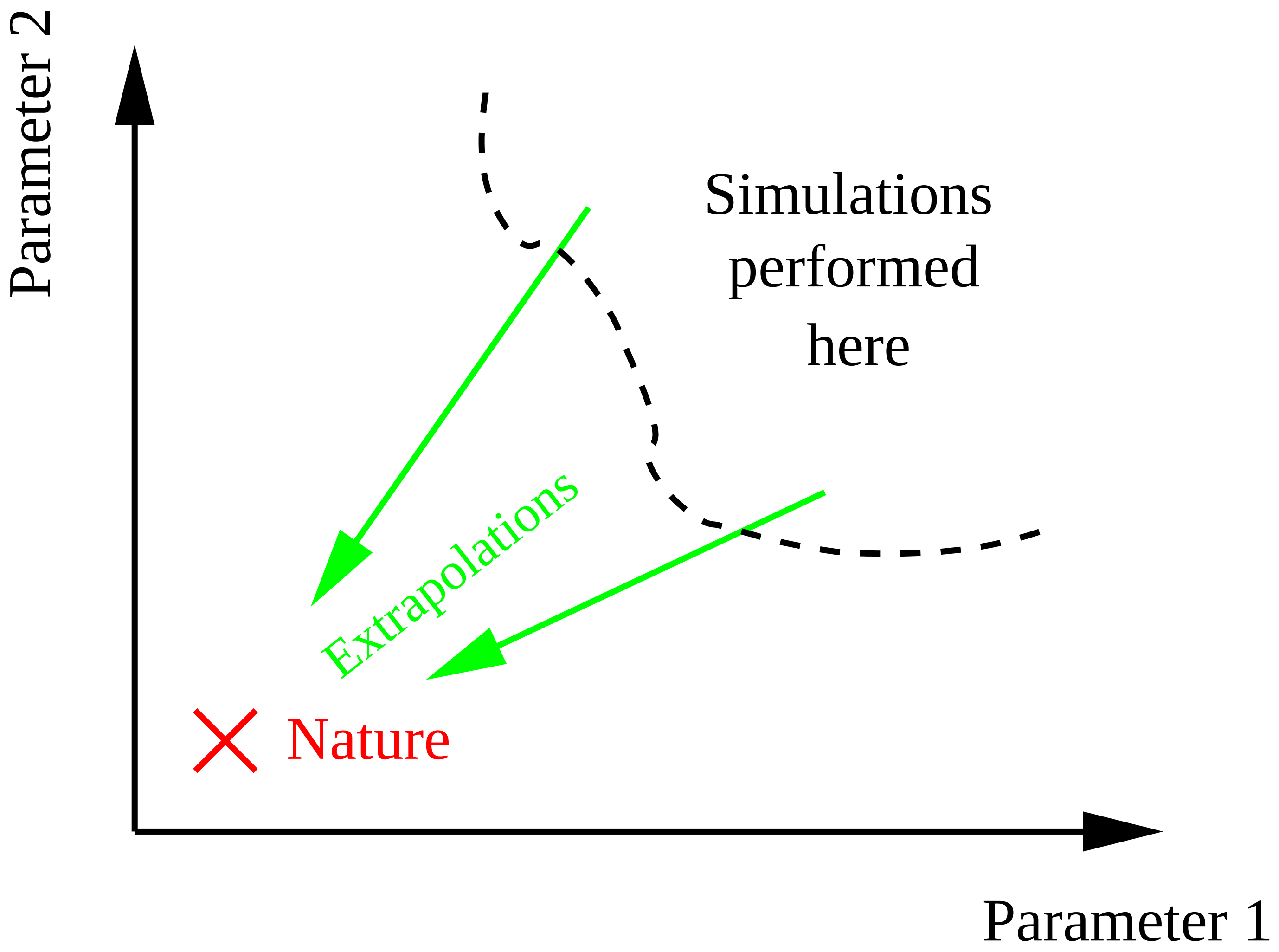

Lattice Approach to QCD phase diagram

- QCD is confining for $T, \mu \sim 0$

$\longrightarrow$ nonperturbative study required

lattice QCD

status:

- works well at $\mu = 0$
- progress for $\mu \lesssim T$, $T \sim T_c$
- standard approach breaks down at $\mu > 0$
  - "sign problem"

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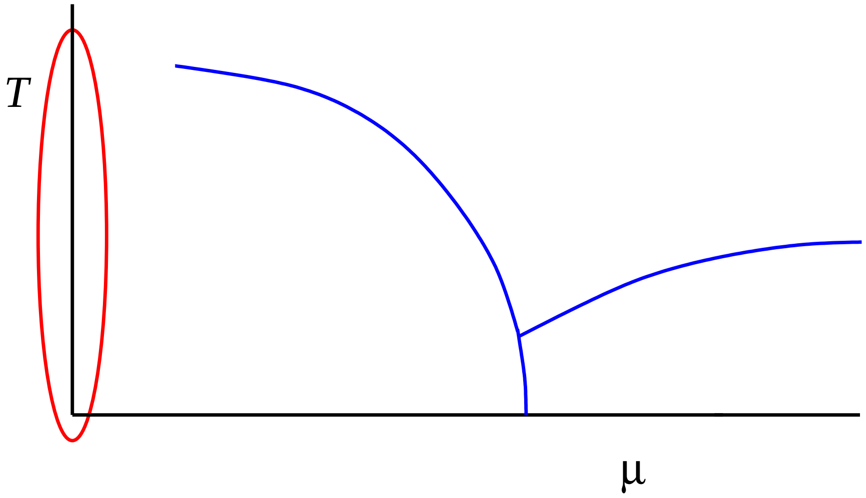

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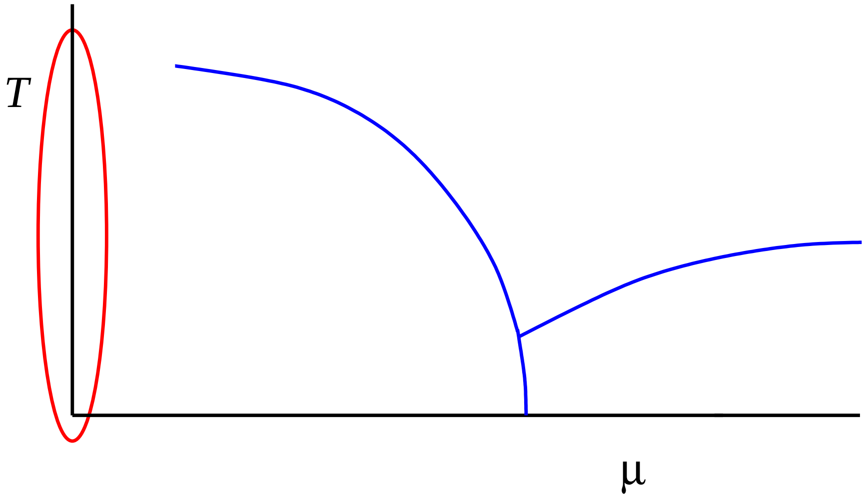

status:

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- progress for $\mu \lesssim T$, $T \sim T_c$
- standard approach breaks down at $\mu > 0$
  - “sign problem”

In this talk I’ll consider the $\mu = 0$ axis only.

Outline

Part A: Finite Temperature Charmonium Potentials

- Schrödinger Equation Approach
  - HAL QCD approach
- Extraction of Potential

Part B: Other Finite Temperature Lattice Results

- Bottomonium Spectral Functions
  - “Melting” of states
- Conductivity
  - Transport Coefficients on the Lattice

Lattice Parameters

1st Generation

$N_f = 2$ simulation

smaller volume

$a_s = 0.167$ fm, $a_t = 0.028$ fm

$\longrightarrow$ anisotropy = $a_s/a_t = 6$

N_s	N_τ	T(MeV)	T/T_c
12	20	368	1.68
12	24	306	1.40
12	28	263	1.20
12	32	230	1.05
12	80	90	0.42

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2nd Generation

$N_f = 2 + 1$ simulation

larger volume

$a_s = 0.1227(8)$ fm, $a_t = 0.0351(2)$ fm

$\longrightarrow$ anisotropy = $a_s/a_t = 3.5$

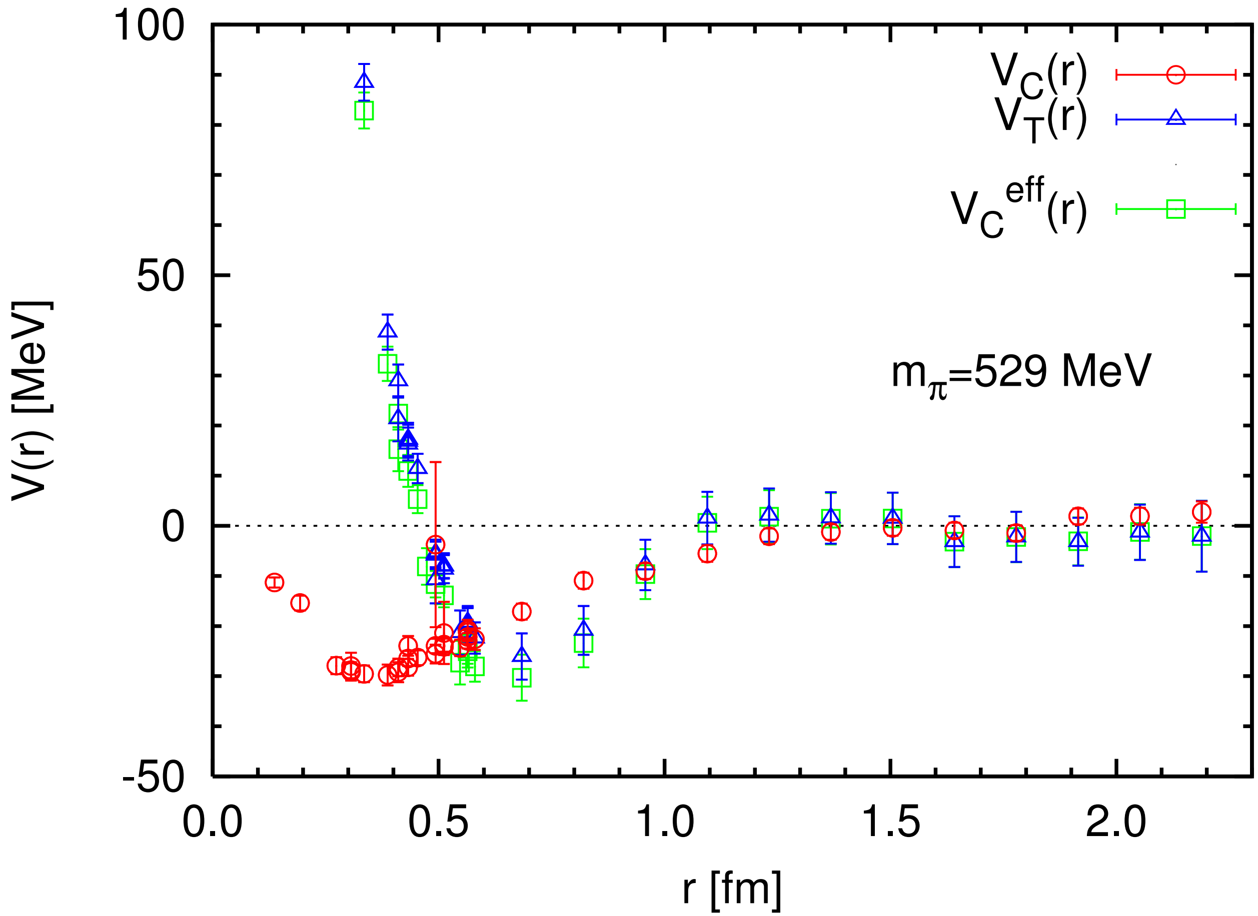
N_s	N_τ	T(MeV)	T/T_c
32	16	350	1.89
24	20	280	1.52
32	24	235	1.26
32	28	201	1.08
32	32	176	0.95
24	36	156	0.84
24	40	140	0.76
32	48	117	0.63

PART A

Finite Temperature Charmonium Potentials

Lattice goes Nuclear

N-N potential



HAL QCD Collaboration, Aoki, Doi, Hatsuda, Ikeda, Inoue, Ishii, Murano, Nemura, Sasaki

Schrödinger Equation Approach

Hatsuda, PoS CD09 (2009) 068 use the Schrödinger equation to "reverse engineer" the potential, $V(r)$, given the Nambu-Bethe-Salpeter wavefunction, $\psi(r)$:

$$\left(\frac{p^2}{2M} + \underset{\substack{\downarrow \\ \text{output}}}{V(r)}\right) \overset{\substack{\text{input} \\ \downarrow}}{\psi(r)} = \overset{\substack{\text{input} \\ \downarrow}}{E} \; \overset{\downarrow}{\psi(r)}$$

$\psi(r)$ is determined from a lattice simulation from correlators of *non-local* (point-split) operators, $J(x;\vec{r}) = q(x)\,\Gamma\,U(x, x+\vec{r})\overline{q}(x+\vec{r})$

$$C(\vec{r},t) = \sum_{\vec{x}} < J(0;\vec{r})\; J(x;\vec{r}) >$$

$$\longrightarrow \quad |\psi(r)|^2\; e^{-Et}$$

The Method (HAL QCD time-dependent)

Charm treated relativistically

Charmonium Operators: $J_\Gamma(x;\mathbf{r}) = q(x)\,\Gamma\,U(x,x+\mathbf{r})\,\overline{q}(x+\mathbf{r})$

Correlation F'ns: $C_\Gamma(\mathbf{r},\tau) = \sum_{\mathbf{x}} \langle J_\Gamma(\mathbf{x},\tau;\mathbf{r})\; J_\Gamma{}^\dagger(0;\mathbf{0})\rangle$

$$= \sum_j \frac{\psi_j(\mathbf{r})\psi_j^*(\mathbf{0})}{2E_j}\left(e^{-E_j\tau} + e^{-E_j(N_\tau-\tau)}\right)$$

Schrödinger Eq'n $\left[-\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + V_\Gamma(r)\right]\psi_j(r) = E_j\psi_j(r)$

Apply this to C_Γ :

$$\frac{\partial C_\Gamma(r,\tau)}{\partial\tau} = \sum_j \left(\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} - V_\Gamma(r)\right)\frac{\psi_j^*(0)\psi_j(r)}{2E_j}e^{-E_j\tau}$$

$$= \left(\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} - V_\Gamma(r)\right)C_\Gamma(r,\tau)$$

This gives an algebraic equation for $V_\Gamma(r,\text{“}\tau\text{”})$

Spin In/dependent Potential Definition

Decomposition:

$$V_\Gamma(r) = V_C(r) + \mathbf{s}_1 \cdot \mathbf{s}_2 \, V_S(r)$$

- Spin Independent (aka "Central"):

$$V_C \equiv V_{\text{Indept}} = \frac{1}{4}[V_{\text{PS}}(r) + 3V_{\text{V}}(r)]$$

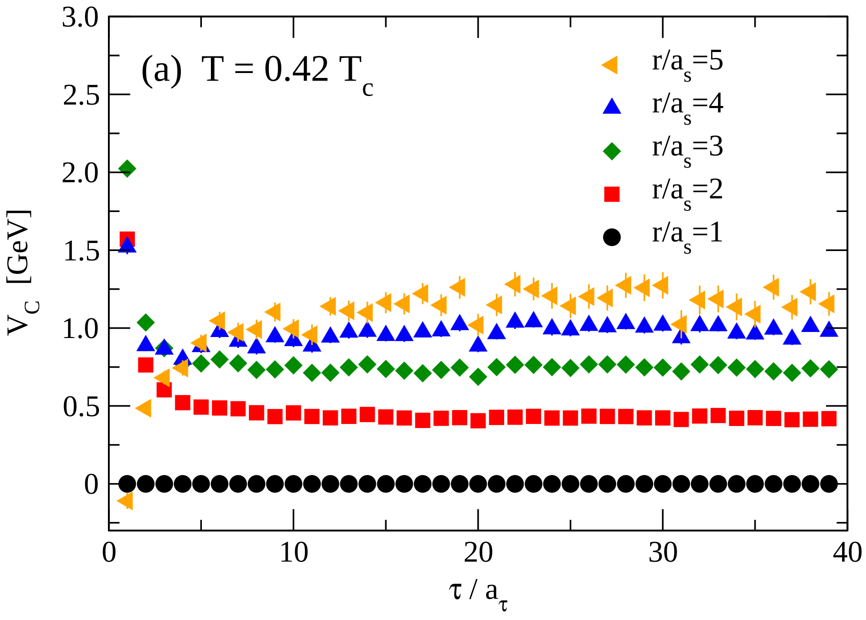
- Spin Dependent:

$$V_S \equiv V_{\text{Dept}} = [V_{\text{V}}(r) - V_{\text{PS}}(r)]$$

V (vector): $\Gamma = \sum_i \gamma_i$

PS (pseudoscalar): $\Gamma = \gamma_5$

$V_C(r,t)$ (1st Generation)



Fit/average $V_C(r, \text{“}\tau\text{”})$ over t to get $V_C(r)$

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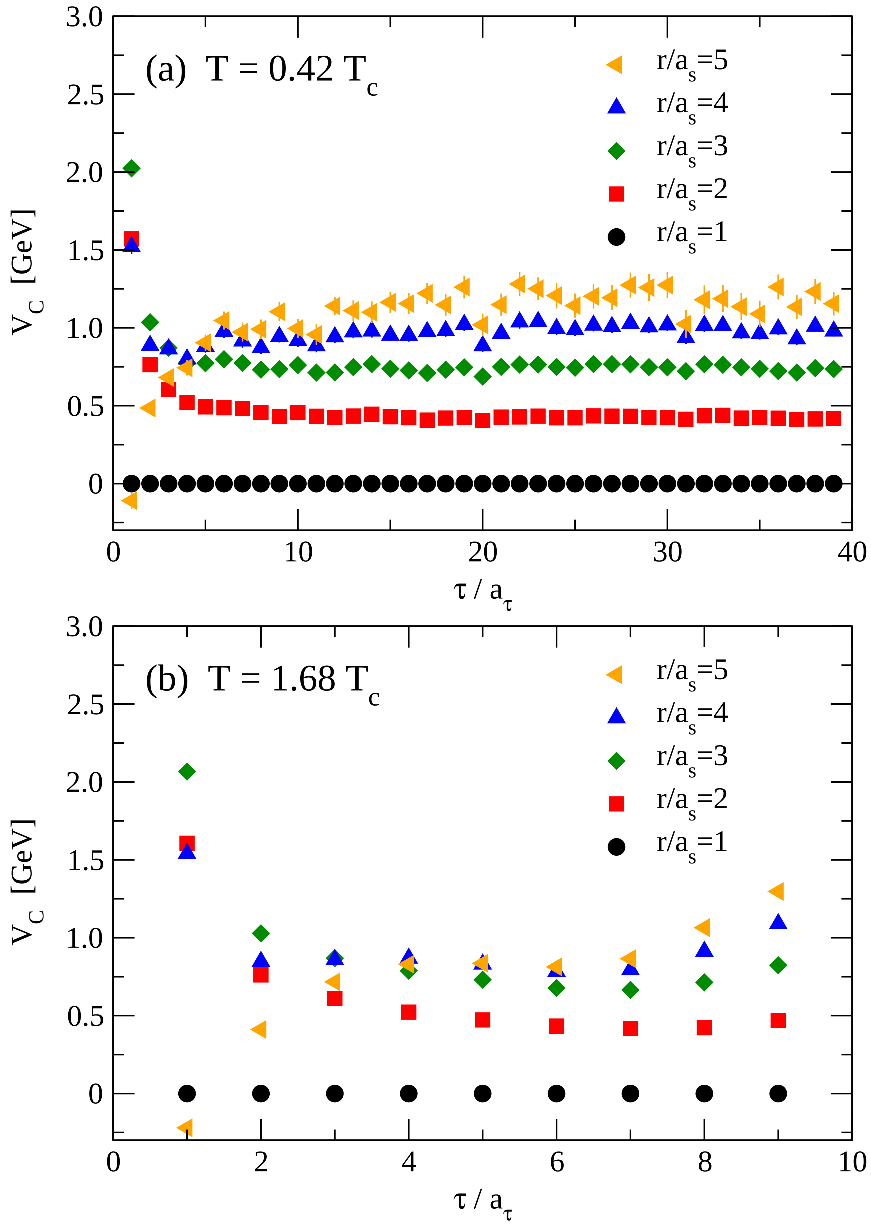

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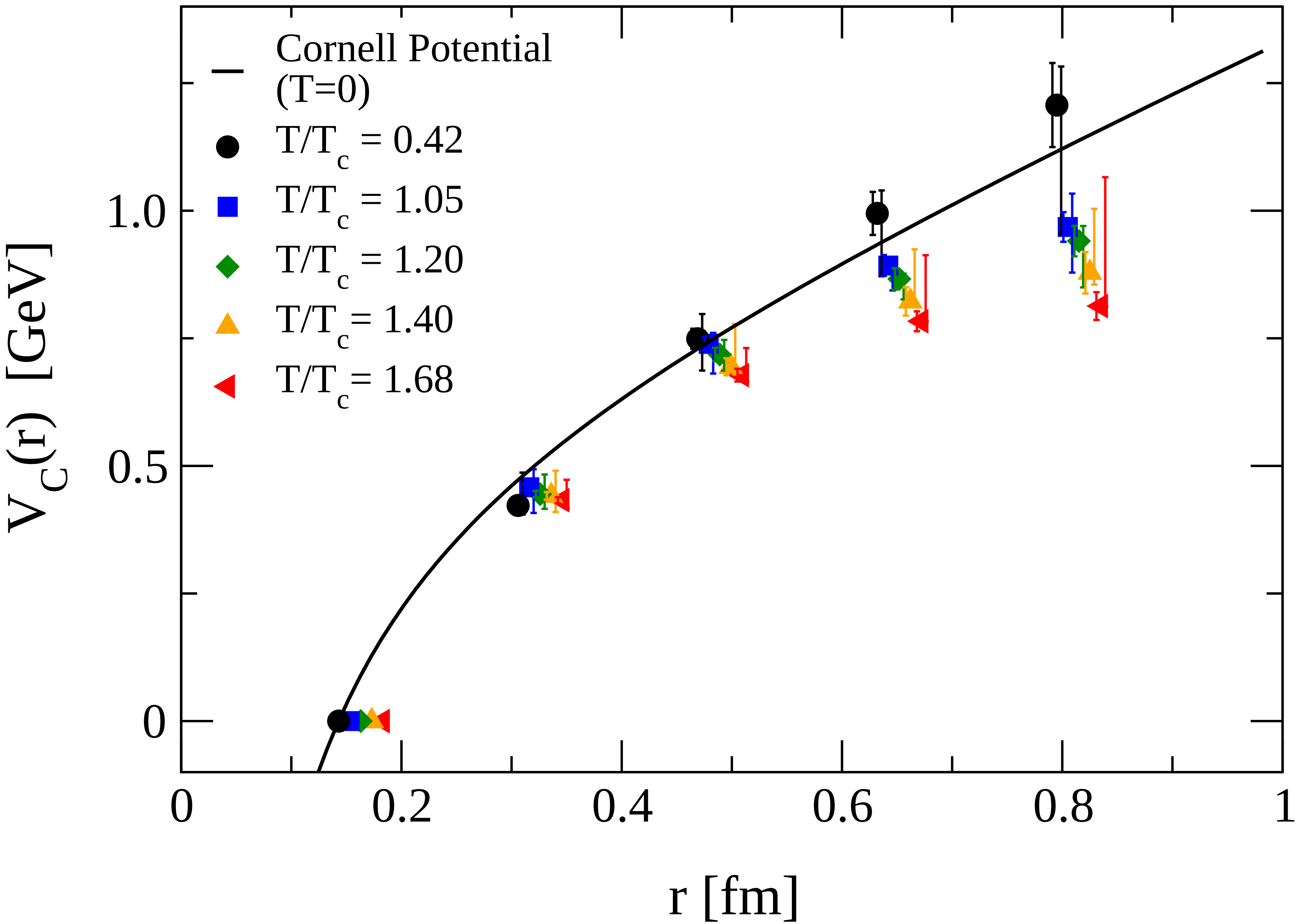
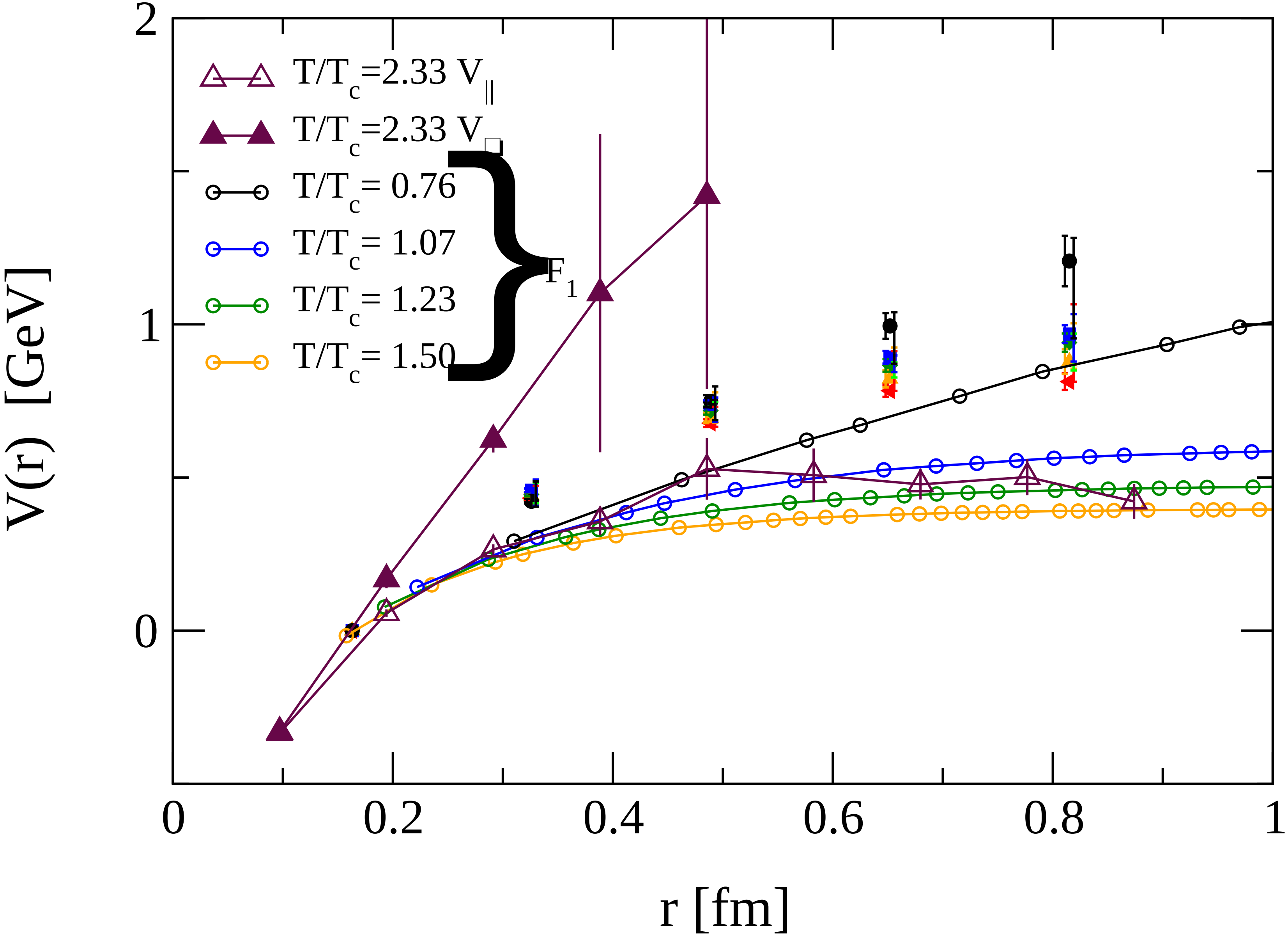
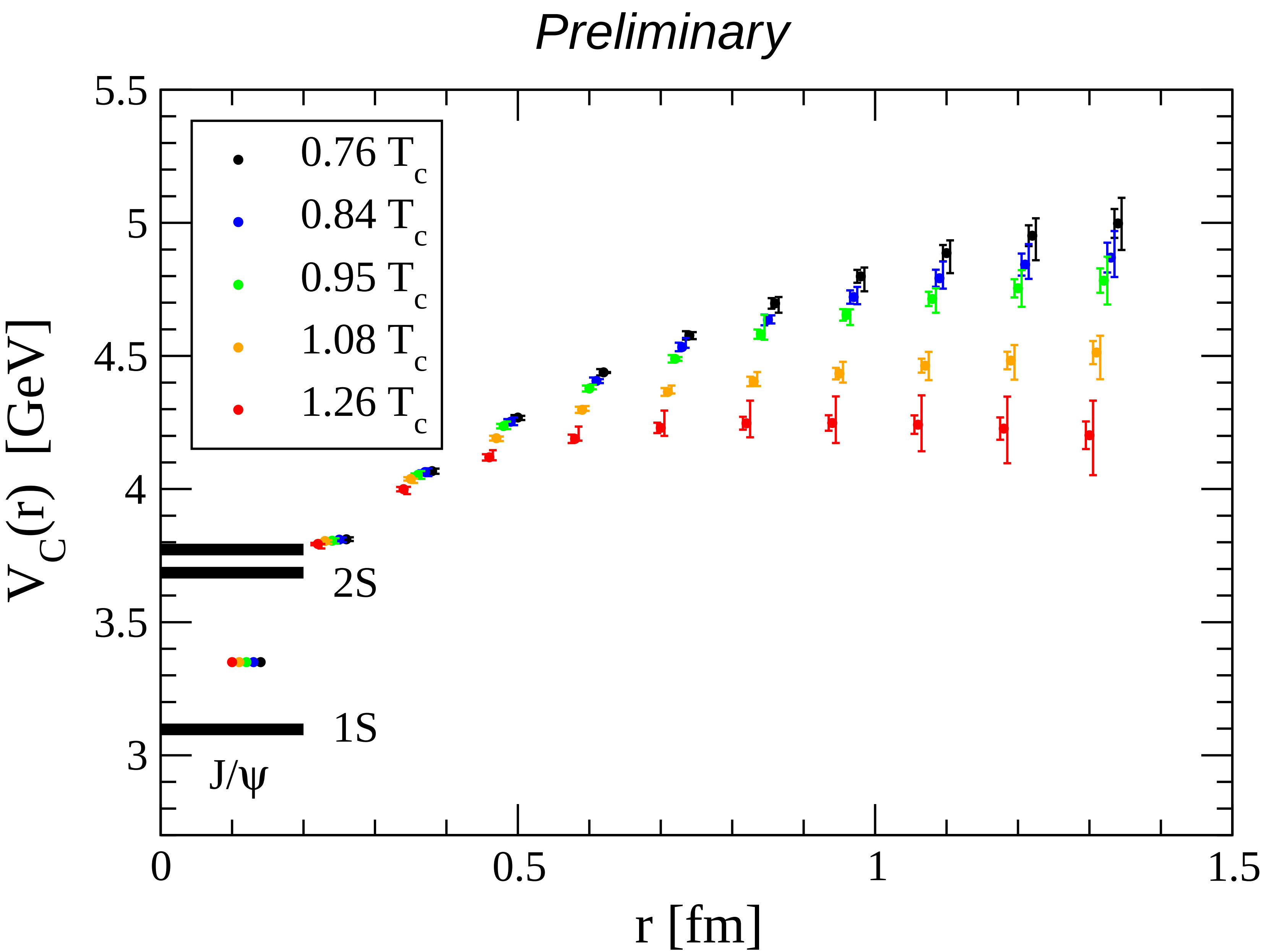
$V_C(r)$ (1st Generation)



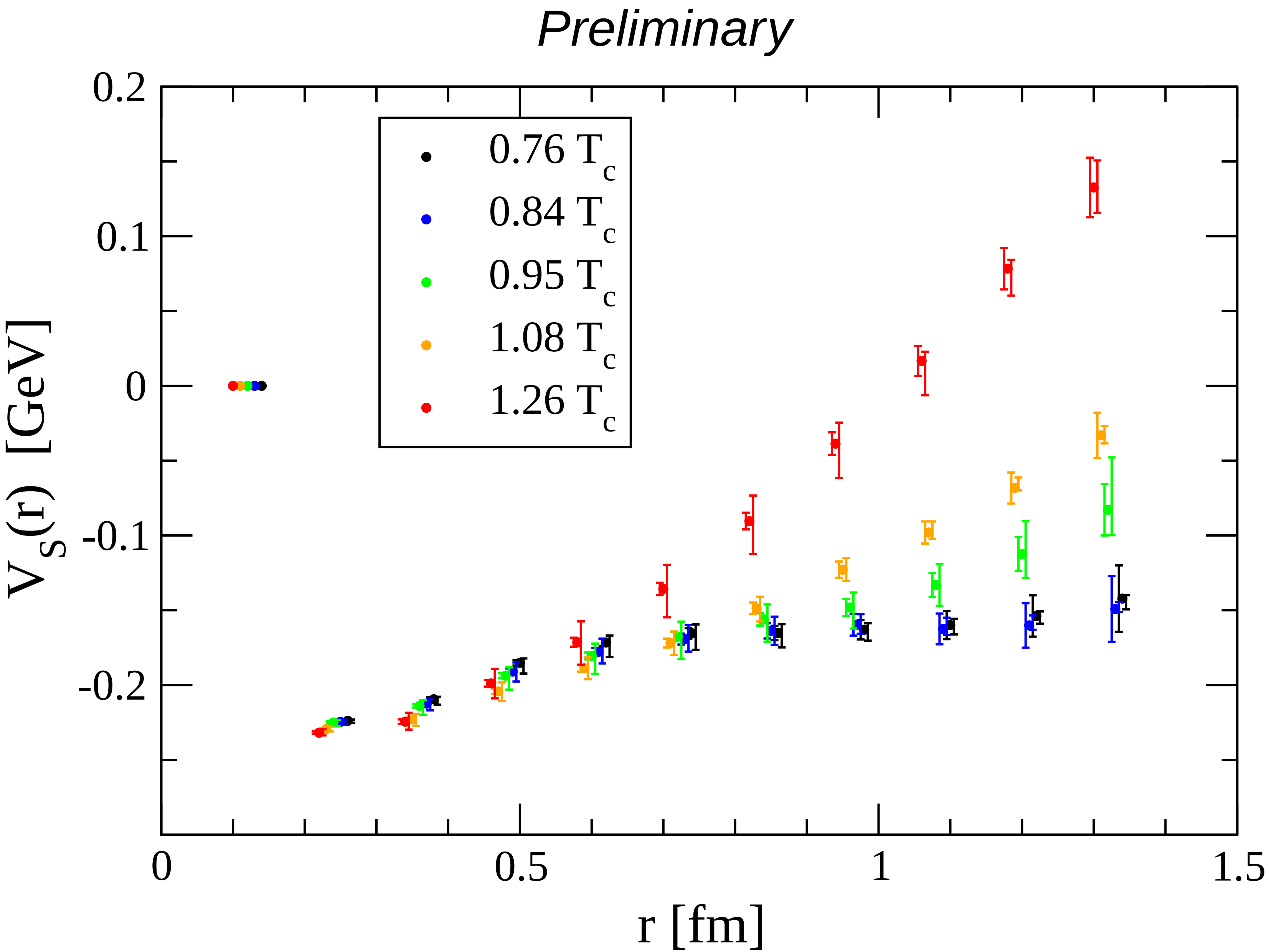
$V_S(r)$ plot comparison (1st Generation)



$V_C(r)$ (2nd Generation)



$V_S(r)$ (2nd Generation)



PART A: Summary & Future Plans

Calculated the inter-quark potential in charmonium at finite temperature.

First time this was done with:

- relativisitc quarks rather than static quarks
  - No issue with Free Energy and the Entropy Term...
- finite temperature rather than $T = 0$

Future Plans:

- Study P-wave states
- Understand excited states
- Larger volumes
- Take continuum limit

Other Finite Temperature Lattice Results

- Bottomonium Spectral Functions
  - Spectral Functions
- Conductivity
  - Transport Coefficients on the Lattice

NRQCD

- An expansion in v/c valid as quark mass $M \to \infty$
  - applicable for b-quarks
- Heavy quark mass, $M > T$
- M factored out of energy scale: $\omega \to \omega - M$
- no periodicity in time
  - bottom quark is a **probe** of thermal media
  - simpler numerically to deal with correlation f'ns
- NRQCD formulism we use is correct to $\mathcal{O}(v^4)$

Zero temperature spectrum results

Aarts et al, Phys. Rev. Lett. **106** (2011) 061602, [arXiv:1010.3725]

state	$a_\tau \Delta E$	Mass (MeV)	Expt (MeV)
$1^1S_0(\eta_b)$	0.118(1)	9438(8)	9390.9(2.8)
$2^1S_0(\eta_b(2S))$	0.197(2)	10019(15)	-
$1^3S_1(\Upsilon)$	0.121(1)	9460*	9460.30(26)
$2^3S_1(\Upsilon')$	0.198(2)	10026(15)	10023.26(31)
$1^1P_1(h_b)$	0.178(2)	9879(15)	
$1^3P_0(\chi_{b0})$	0.175(4)	9857(29)	9859.44(42)(31)
$1^3P_1(\chi_{b1})$	0.176(3)	9864(22)	9892.78(26)(31)
$1^3P_2(\chi_{b2})$	0.182(3)	9908(22)	9912.21(26)(31)

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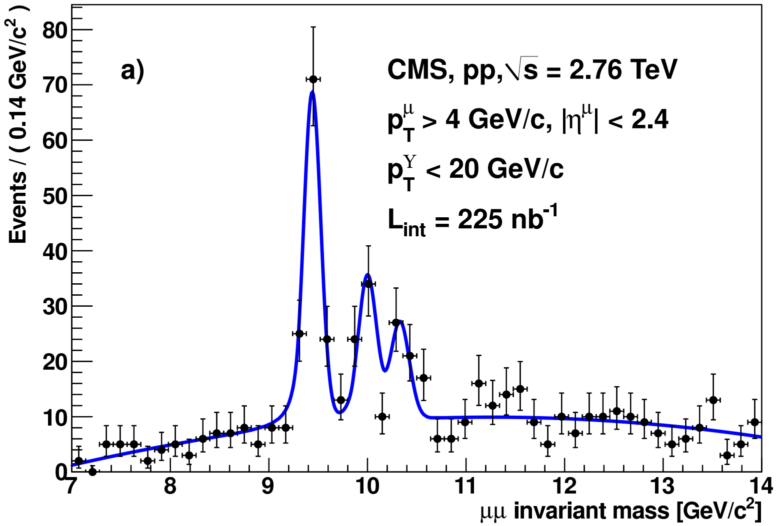
Belle Collaboration

CMS Υ Results arXiv:1105.4894



p-p collisions

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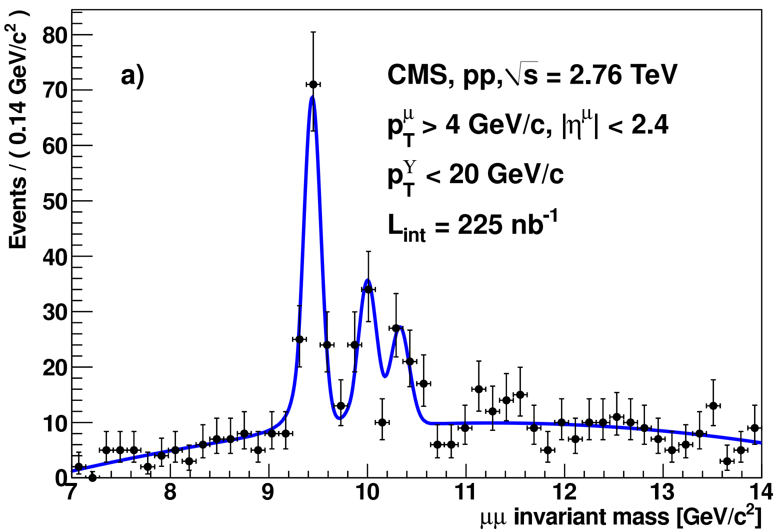

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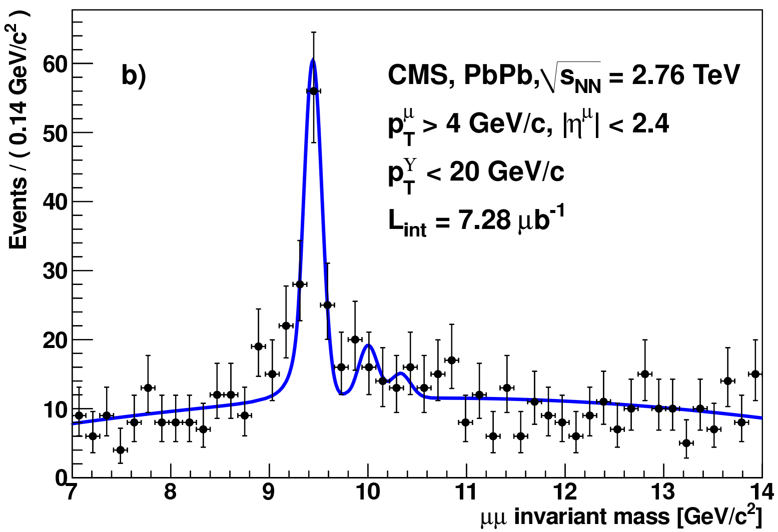

Pb-Pb collisions

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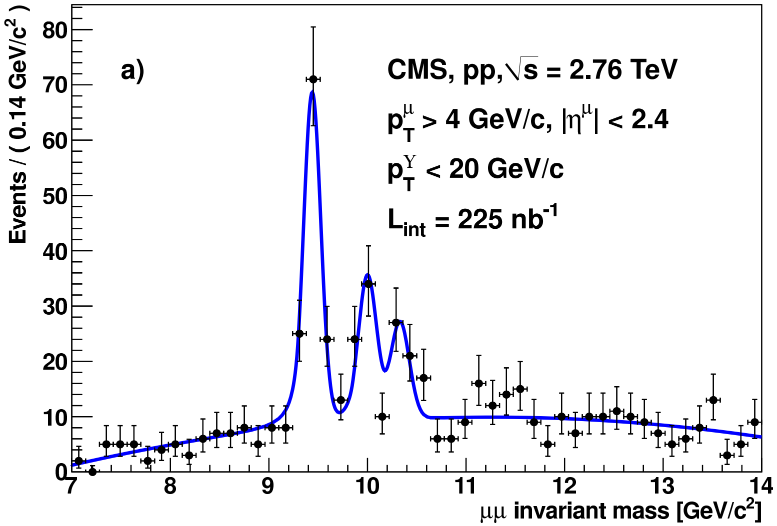
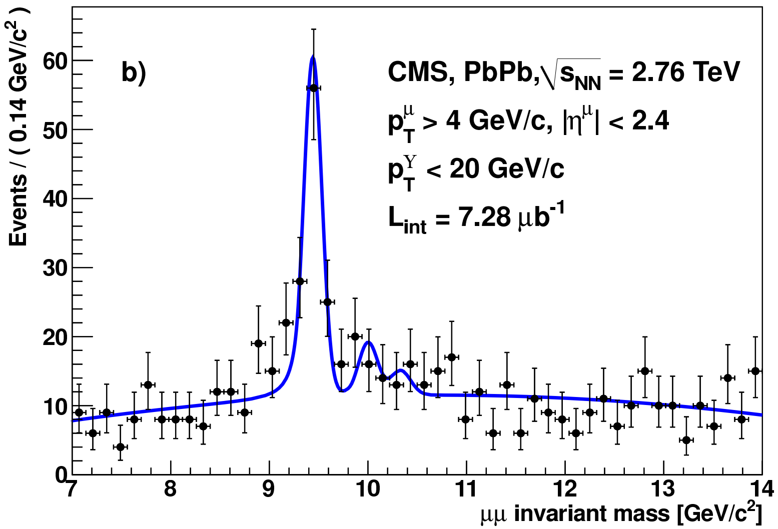


p-p collisions



Pb-Pb collisions

So $\Upsilon(1S)$ state survives, whereas $\Upsilon(2S, 3S)$ *melt...*

Melting pattern = thermometer of QGP

Maximum Entropy Method: Motivation

Do bound hadronic states persist into the "quark-gluon" plasma phase?
How can we extract transport coefficients?

- *Spectral functions* can answer this!

$$C(t,\vec{p}) \quad = \quad \int \rho(\omega,\vec{p})\; K(t,\omega)\; d\omega$$

$$\uparrow \qquad\qquad \downarrow \qquad \nwarrow$$

Euclidean Correlator	Spectral Function	(Lattice) Kernel

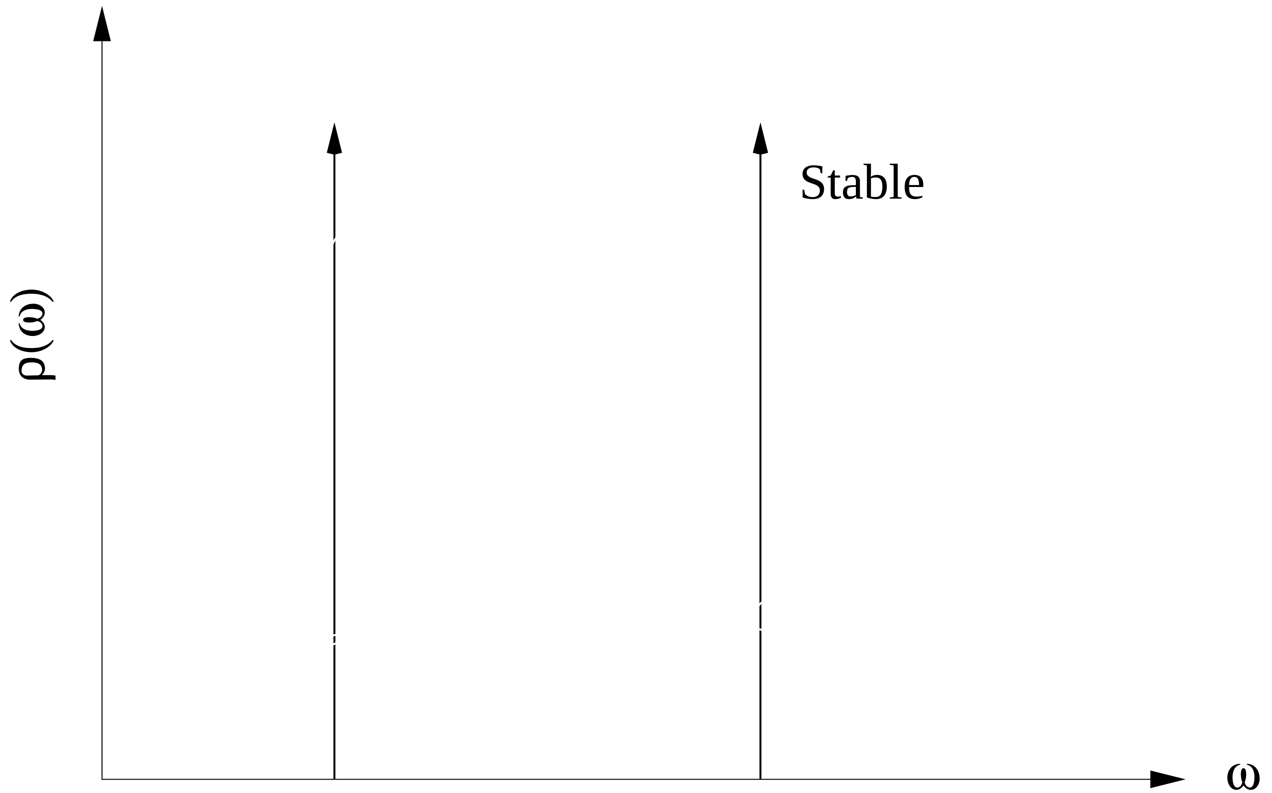
where the (lattice) Kernel is:

$$K(t,\omega) \quad = \quad \frac{\cosh[\omega(t-N_t/2)]}{\sinh[\omega/(2T)]}$$

$$\sim \quad \exp[-\omega t]$$

Example Spectral Functions

$$C(t) \sim \int \rho(\omega)\, e^{-\omega t}\, d\omega$$



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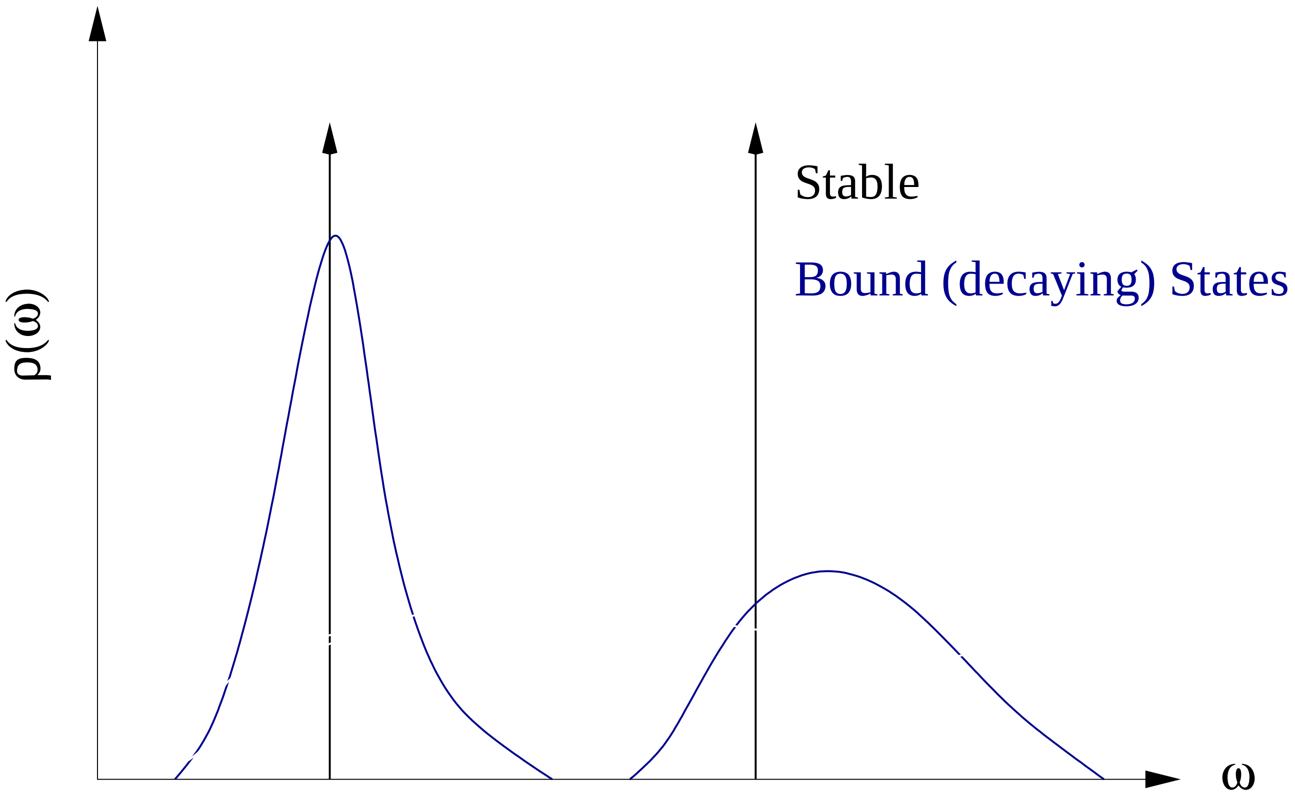

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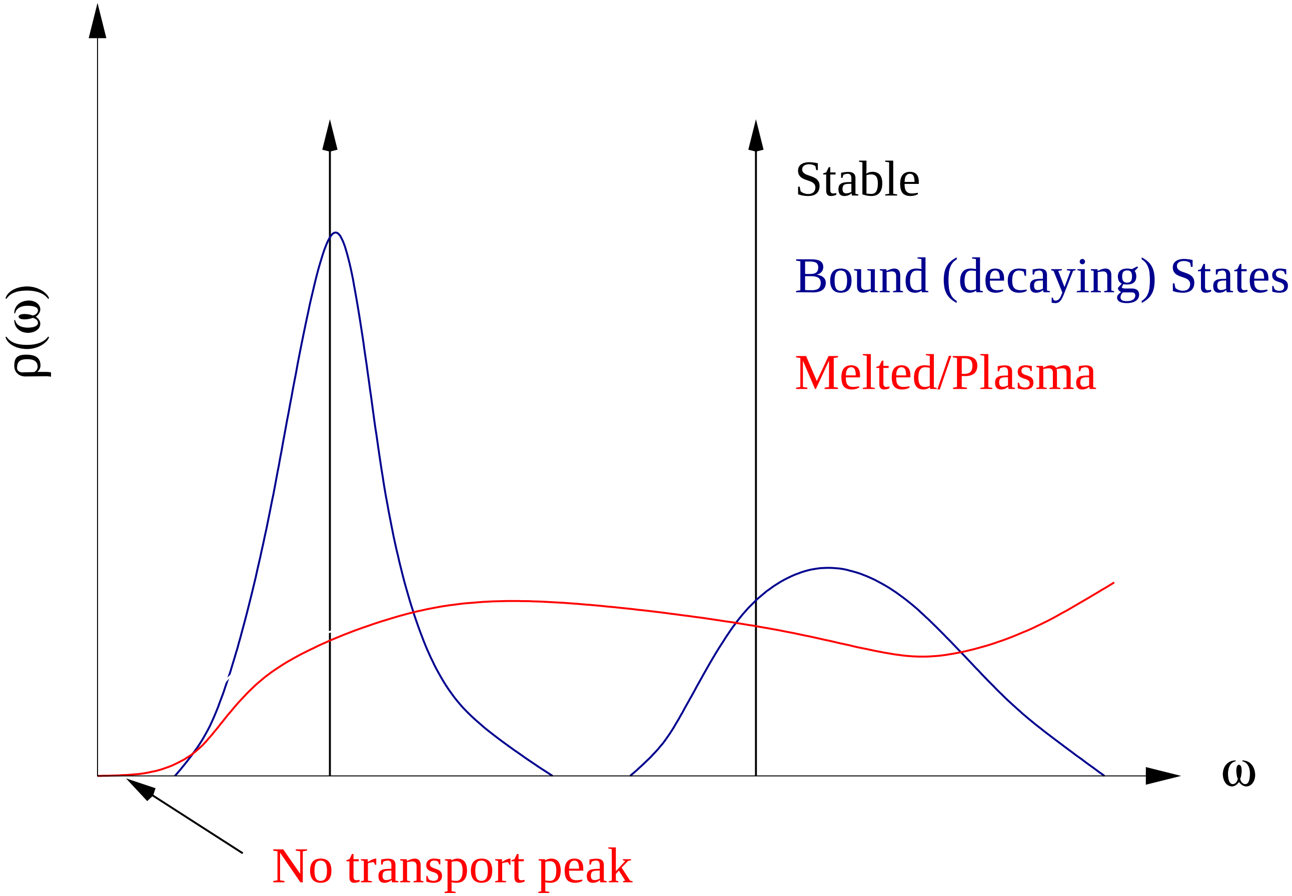

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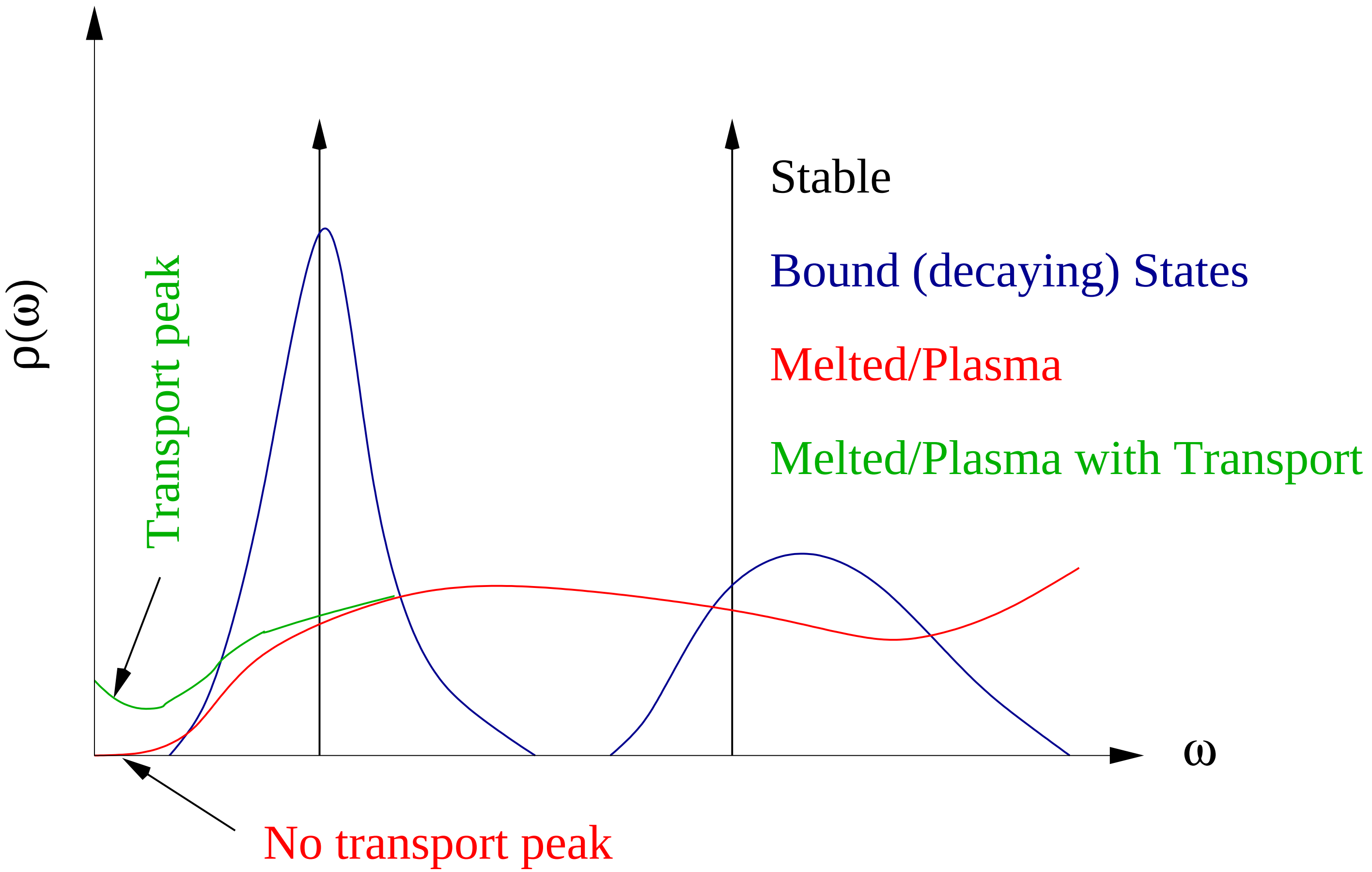


Spectral Functions via MEM

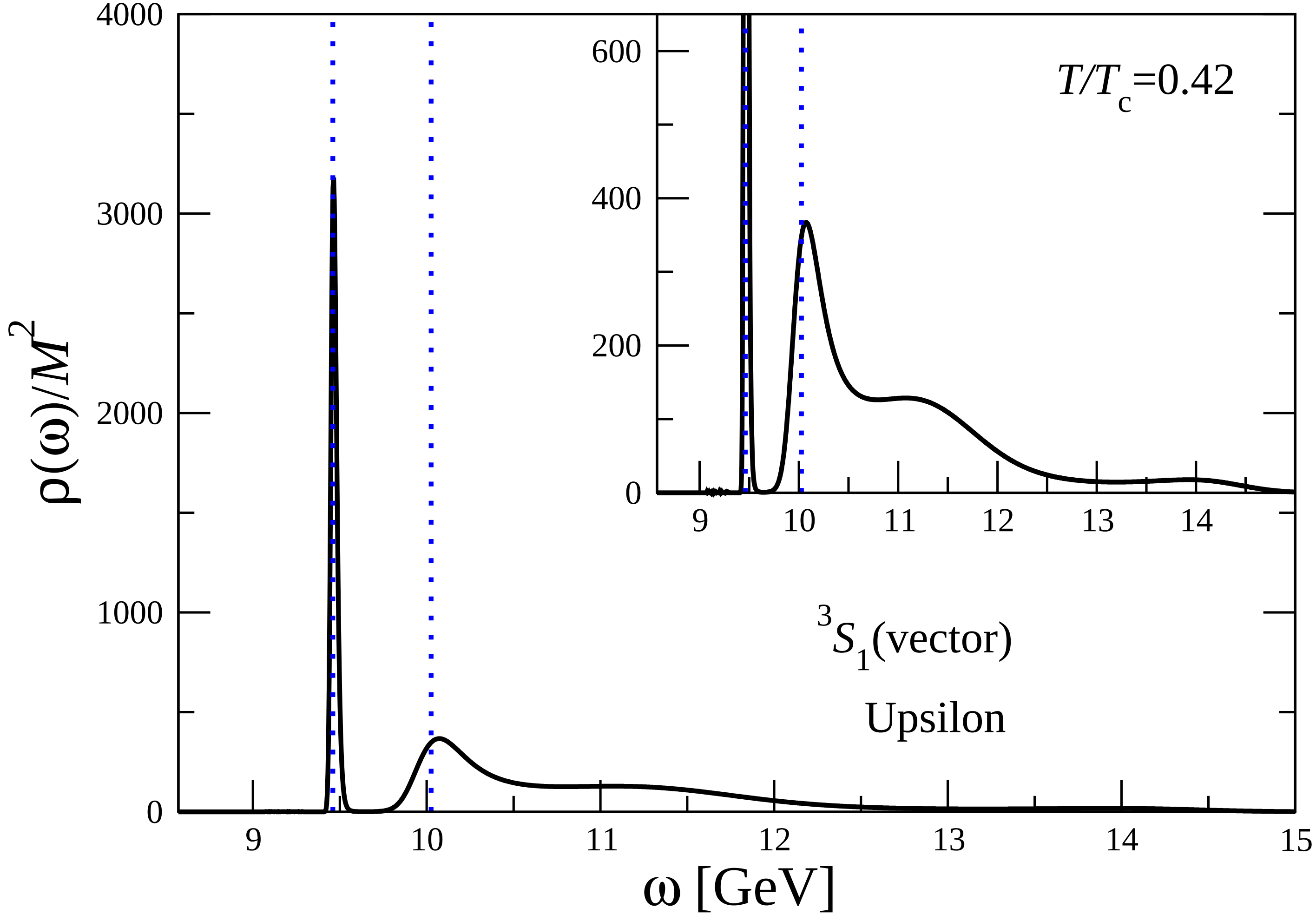
- Extraction of a spectral density from a lattice correlator is an ill-posed problem:
  - *Given* $C(t)$ *derive* $\rho(\omega)$
  - *More* ω *data points then* t *data points!*

- Requires the use of Bayesian analysis - Maximum Entropy Method (MEM)
  - Hatsuda, Asakawa et al
  - Commonly used in other areas...

- Need to check MEM output w.r.t. choice of:
  - Default model
  - Statistics
  - Energy range
  - Euclidean time range

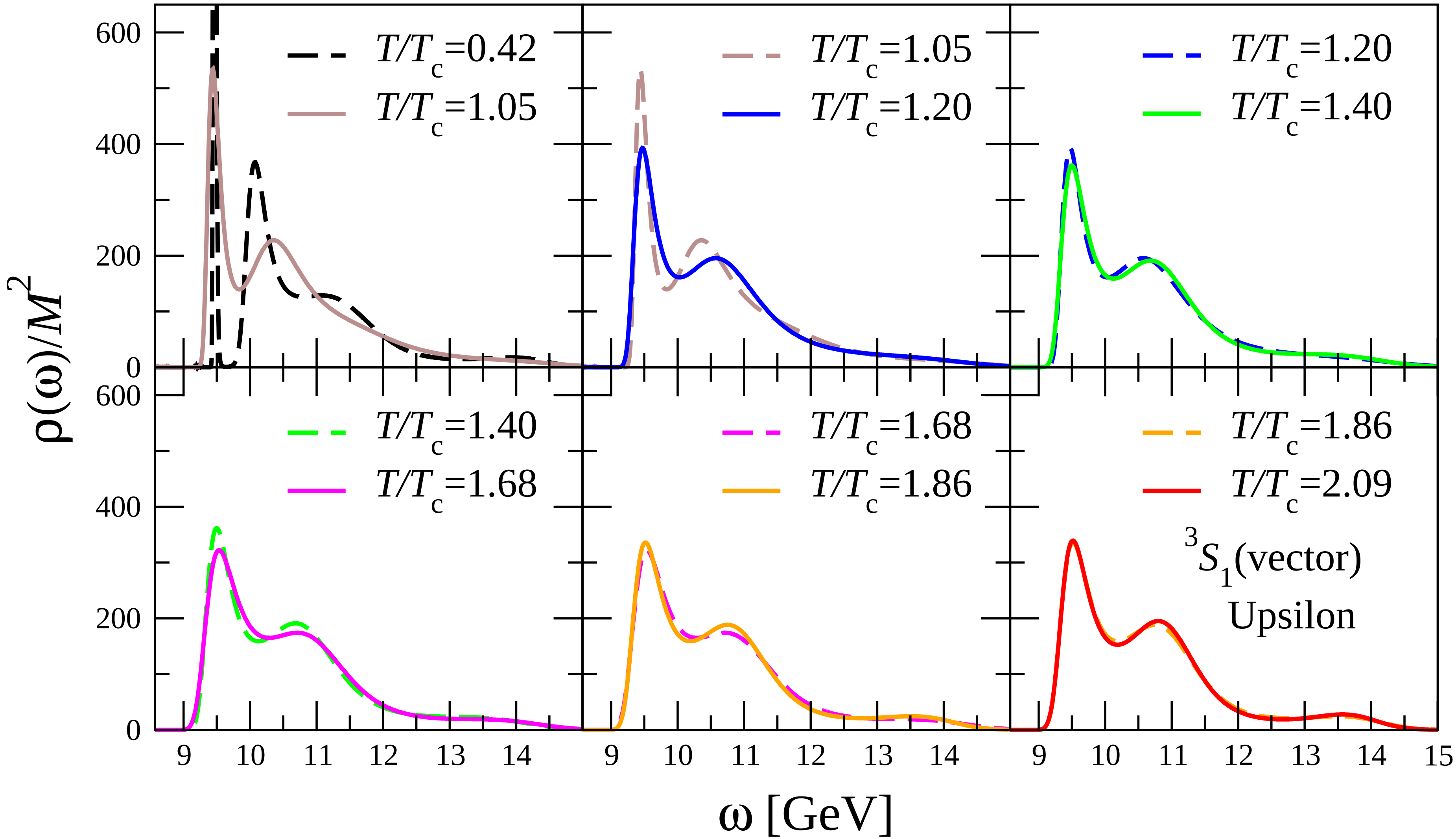
$T = 0$ spectral functions, $p = 0$ (1st Generation)

Aarts et al, JHEP **1111** (2011) 103 [arXiv:1109.4496]

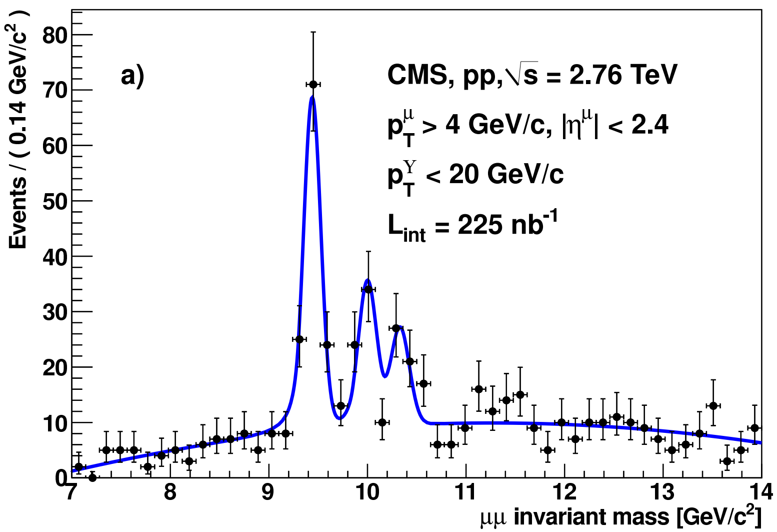


$T \neq 0$ spectral functions, $p = 0$ (1st Generation)

Aarts et al, JHEP **1111** (2011) 103 [arXiv:1109.4496]



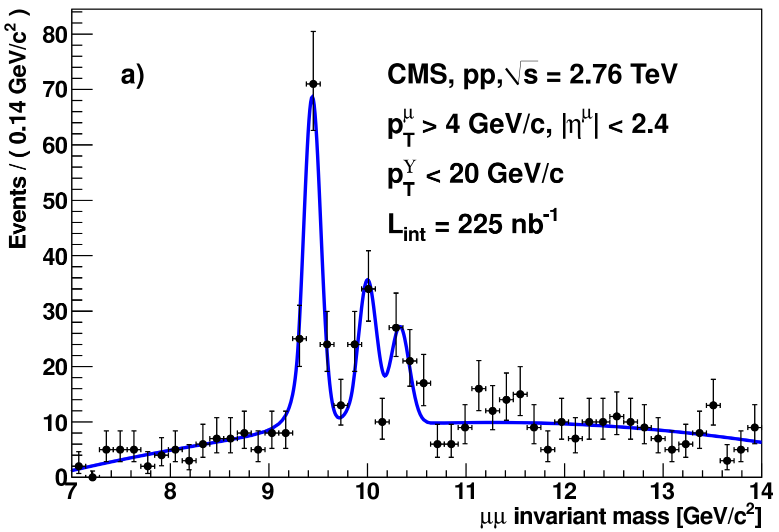
CMS Results arXiv:1105.4894



p-p collisions

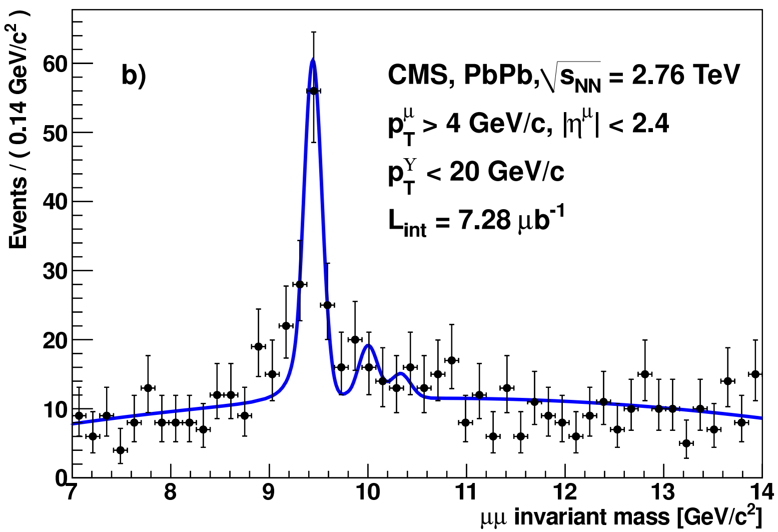
$$T = 0$$

CMS Results arXiv:1105.4894



p-p collisions

$$T = 0$$

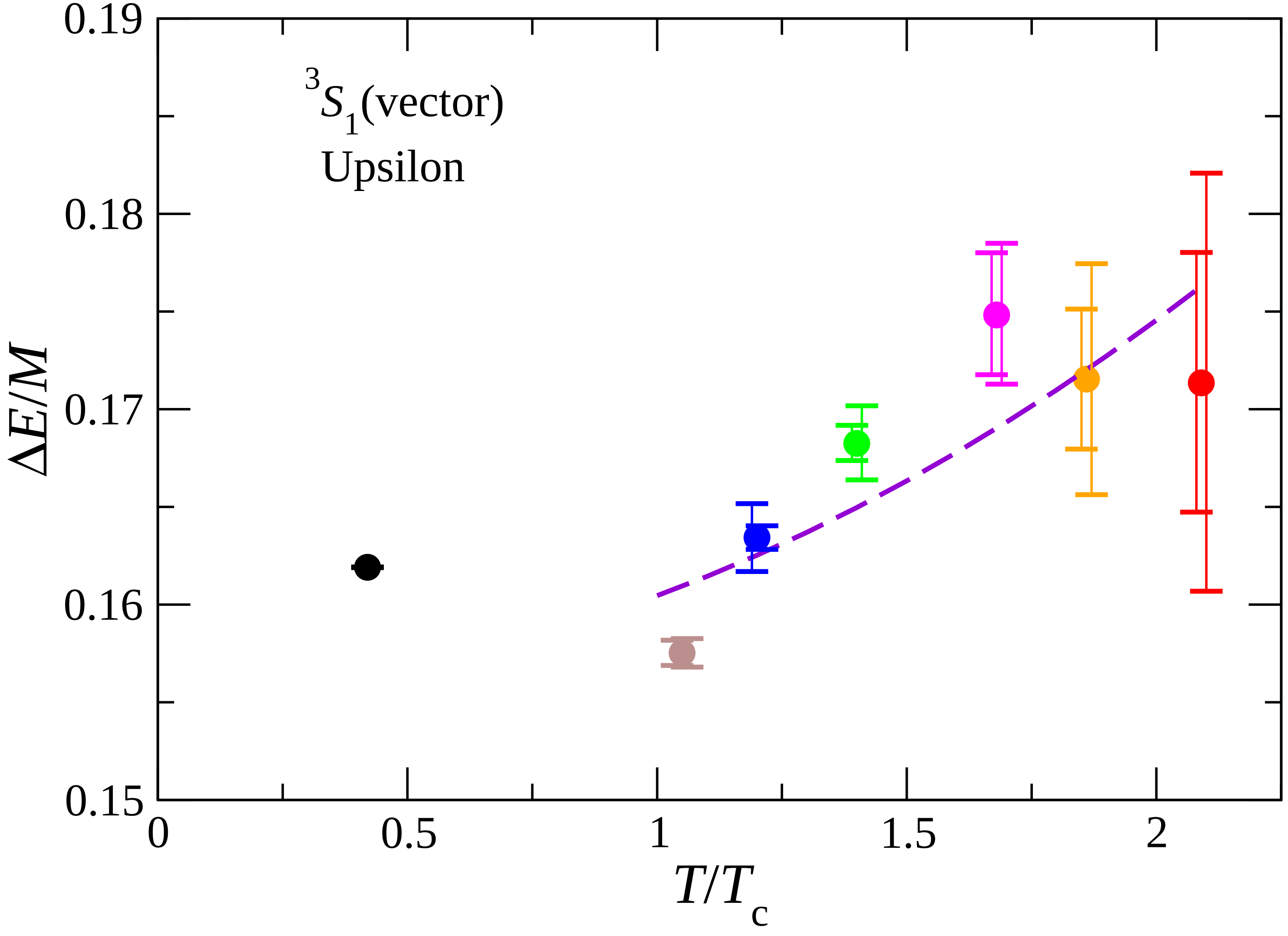


Pb-Pb collisions

$$T > T_c$$

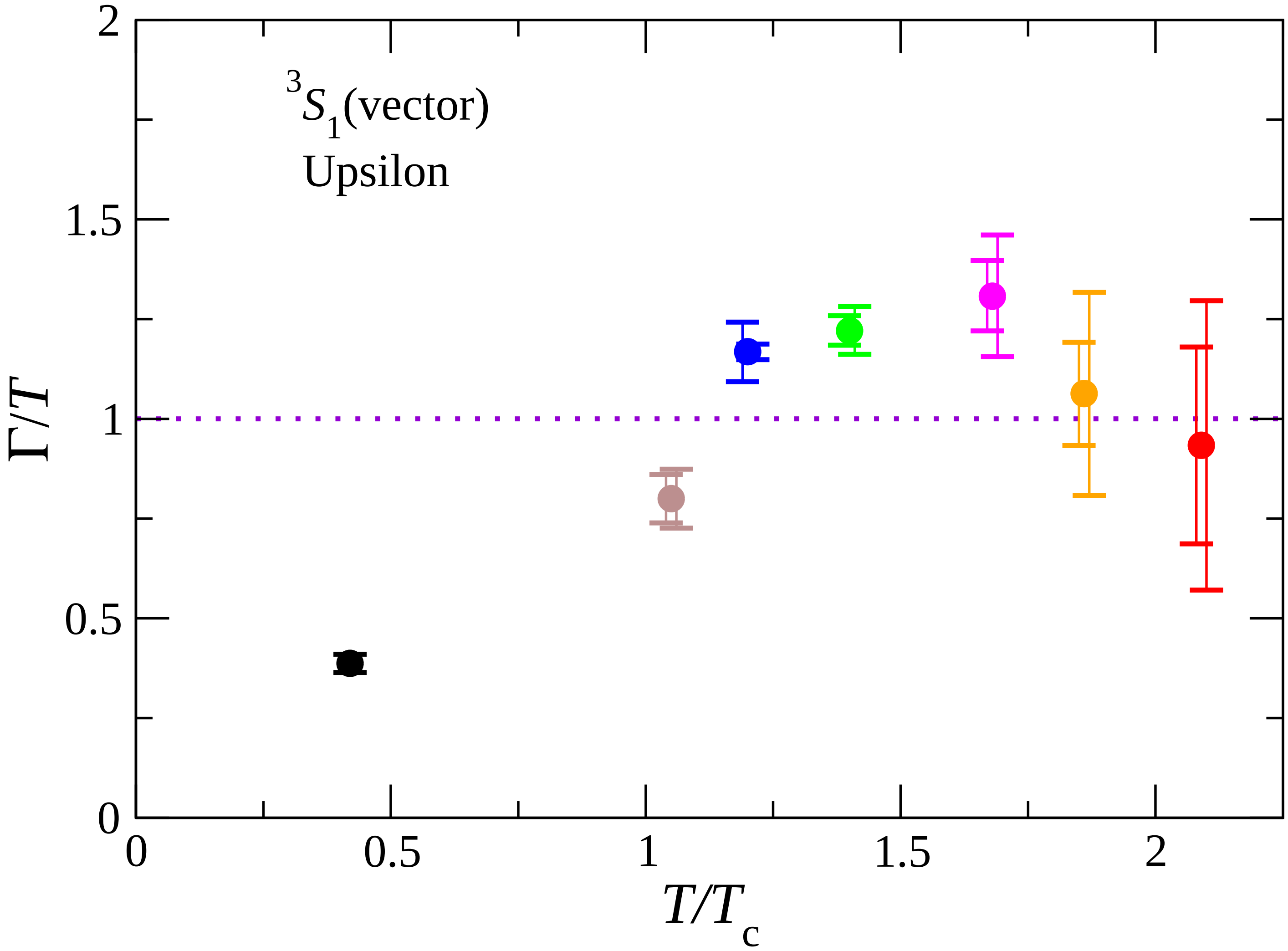
Mass for $T \neq 0$, $p = 0$ (1st Generation)

Aarts et al, JHEP **1111** (2011) 103 [arXiv:1109.4496]



Width for $T \neq 0$, $p = 0$ (1st Generation)

Aarts et al, JHEP **1111** (2011) 103 [arXiv:1109.4496]



Comparison with phenomenology

From Brambilla et al thermal contribution to the width is

$$\frac{\Gamma}{T} = \frac{1156}{81}\alpha_s^3 \simeq 14.27\alpha_s^3,$$

(at leading order in weak coupling and large mass expansion).

Our results $\longrightarrow \Gamma/T \sim 1$ so $\alpha_s \sim 0.4$.

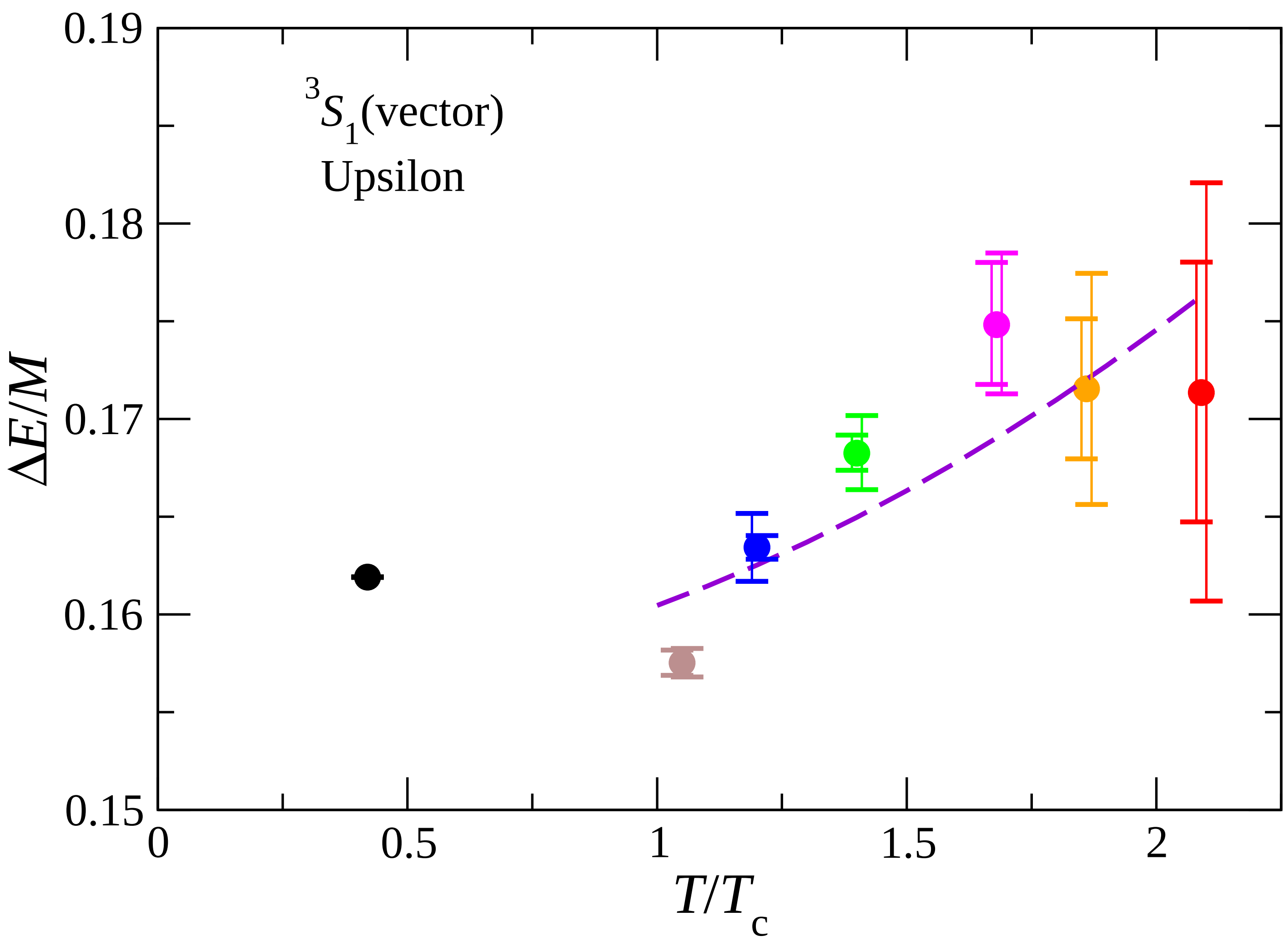
Also from Brambilla et al thermal contribution to the mass is

$$\delta E_{\rm thermal} = \frac{17\pi}{9}\alpha_s\frac{T^2}{M} \simeq 5.93\alpha_s\frac{T^2}{M}$$

(see dashed line)

Mass for $T \neq 0$, $p = 0$ (1st Generation)

Aarts et al, JHEP **1111** (2011) 103 [arXiv:1109.4496]



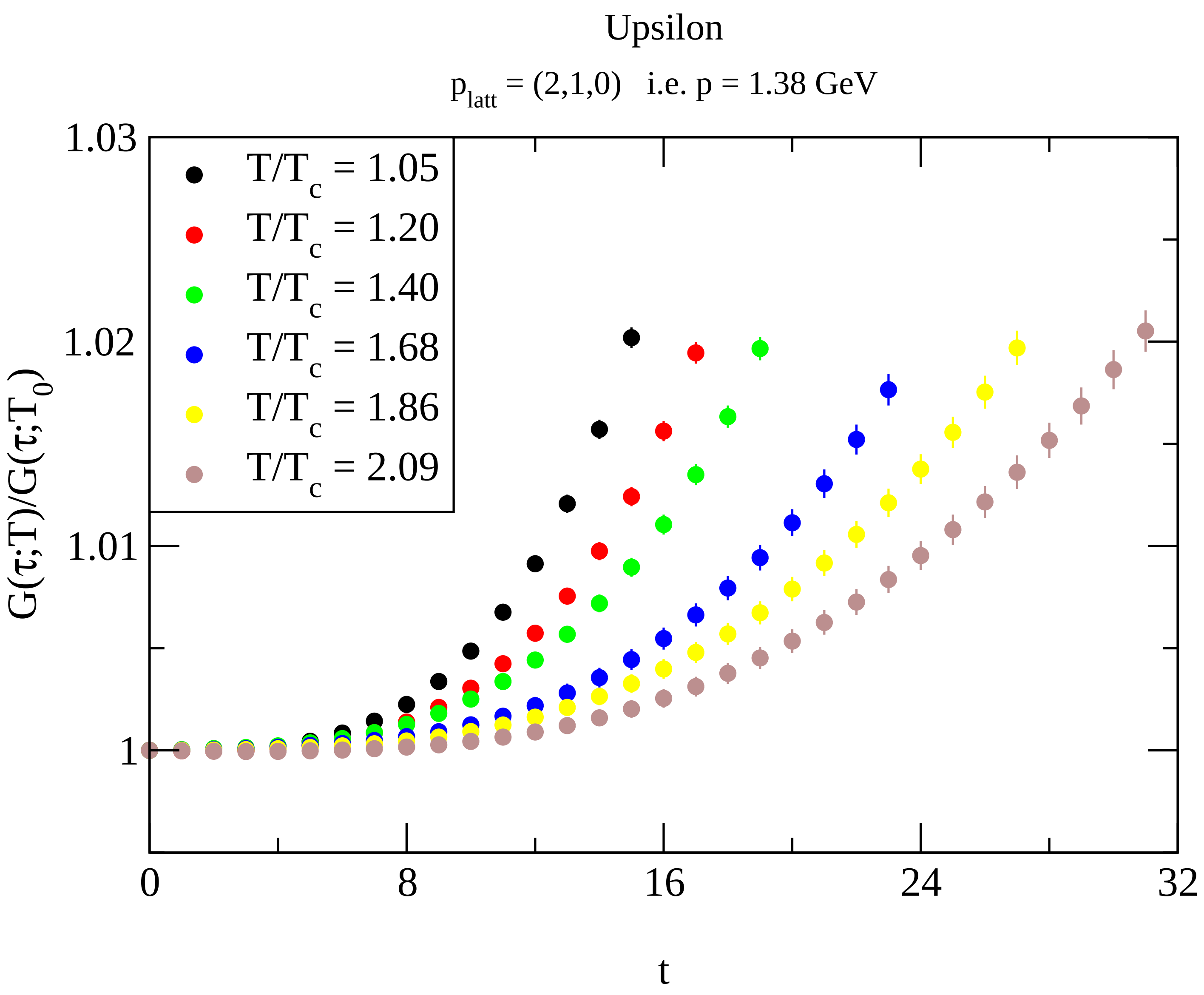
Recent results at $p \neq 0$ (1st Generation)

Aarts et al, JHEP **1303** (2013) 084 [arXiv:1210.2903]

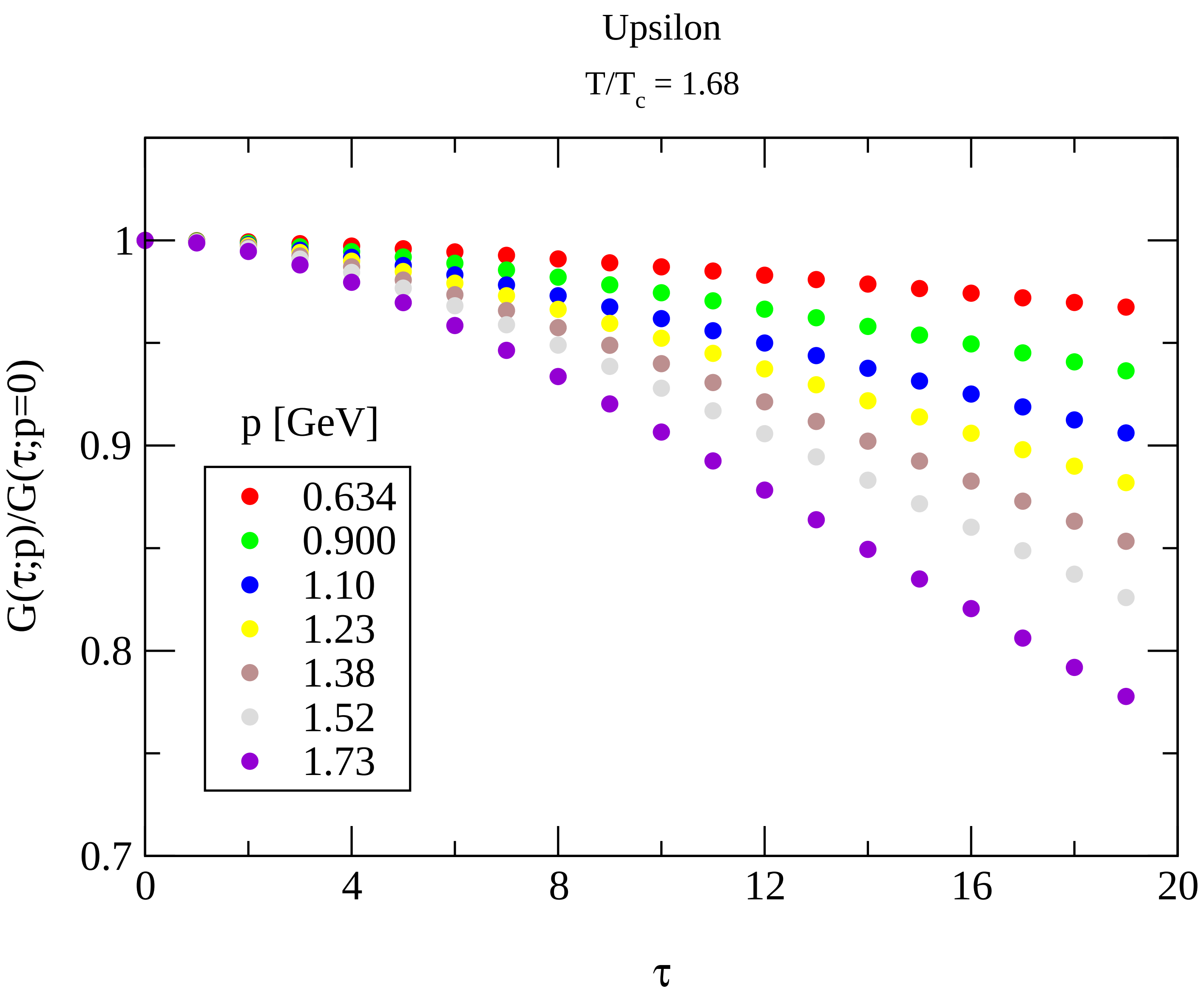
$p_{\text{latt}} = (1,0,0), \ldots (2,2,0)$

i.e. $p = \frac{2\pi p_{\text{latt}}}{L} = 0.634, \ldots 1.73$ GeV

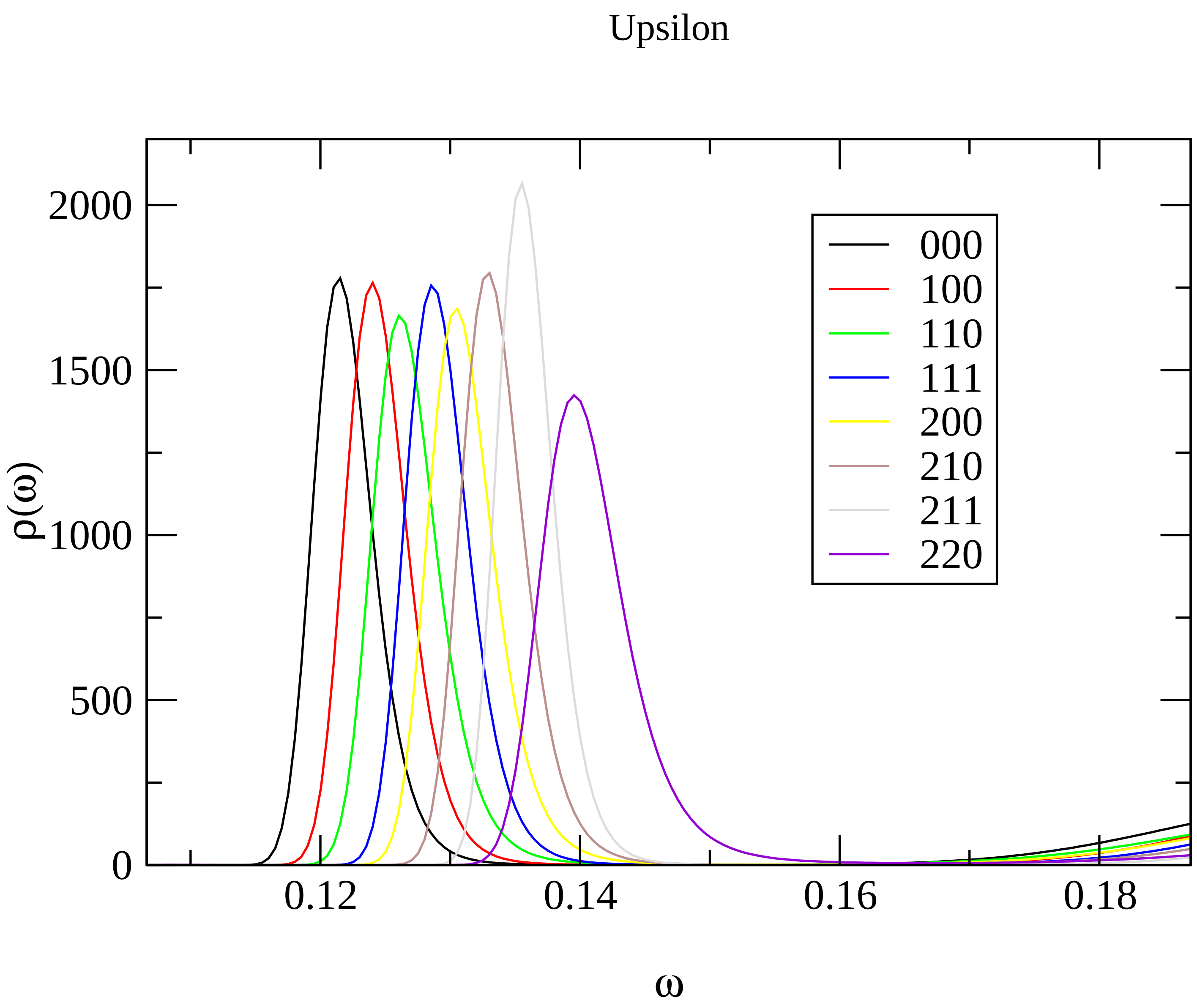
Variation with T, fixed $p \neq 0$ (1st Generation)



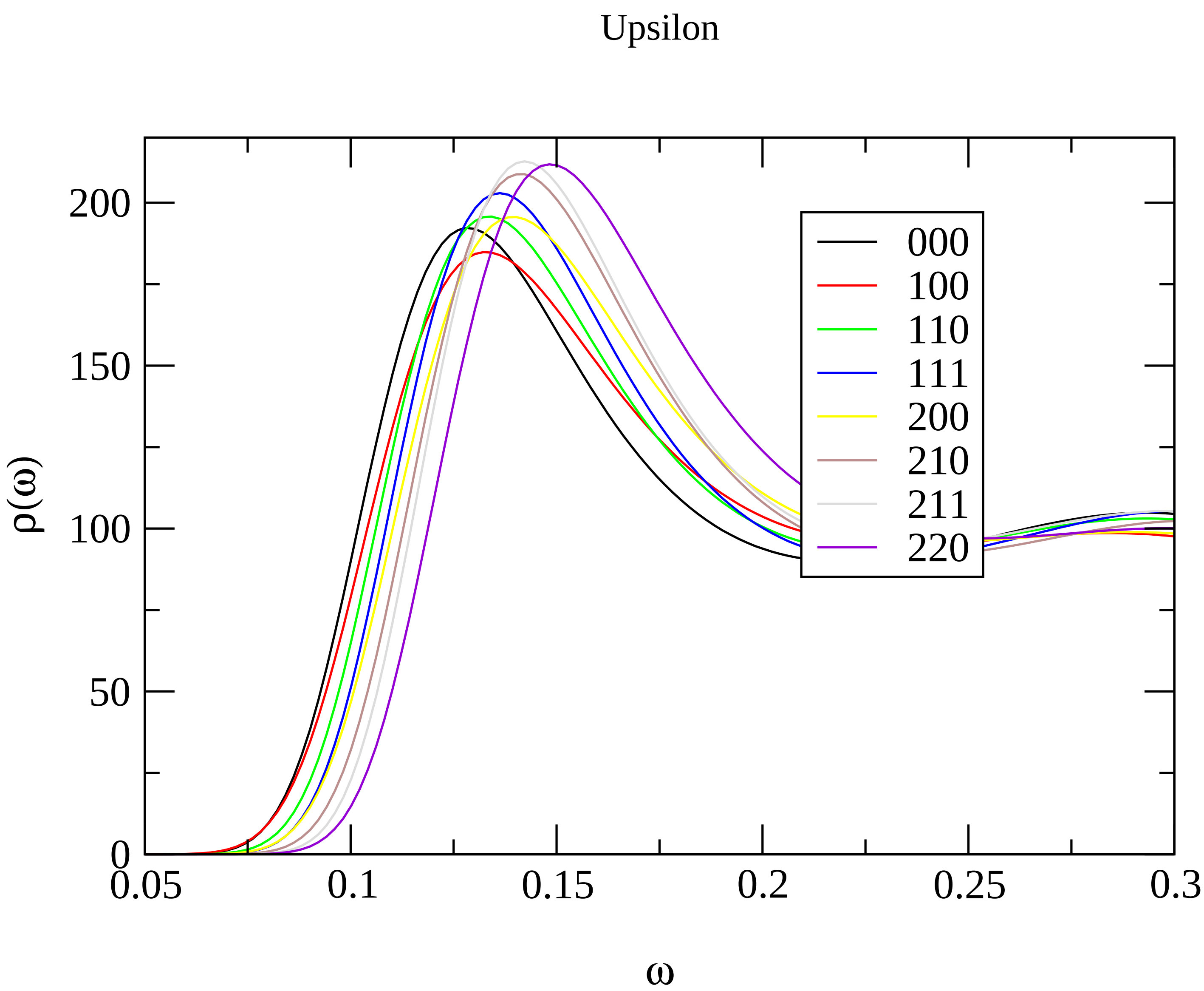
Variation with p, fixed T (1st Generation)



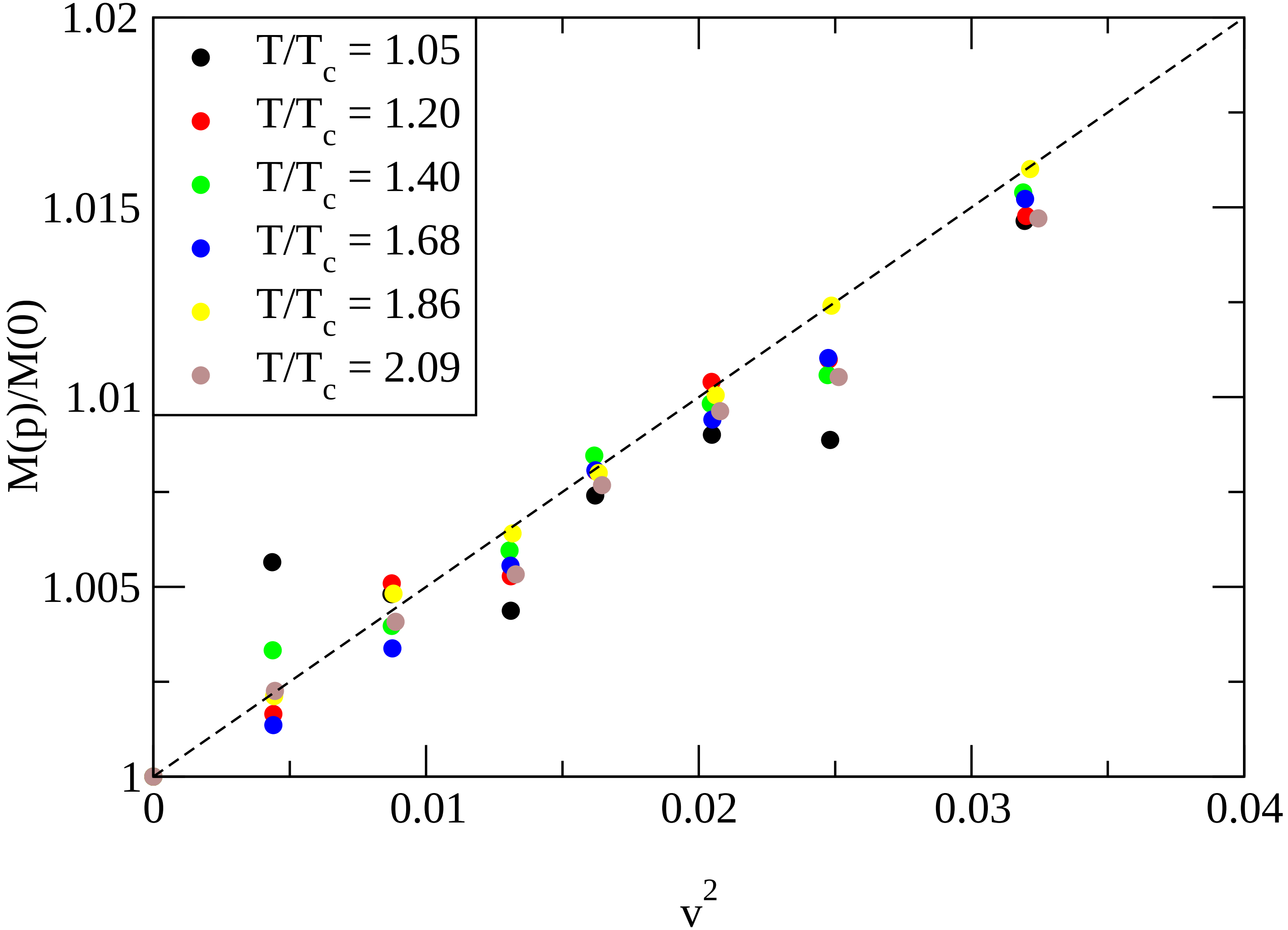
Spectral functions: $p \neq 0$, $T = 0$ (1st Generation)



Spectral functions: $p \neq 0$, $T \neq 0$ (1st Generation)

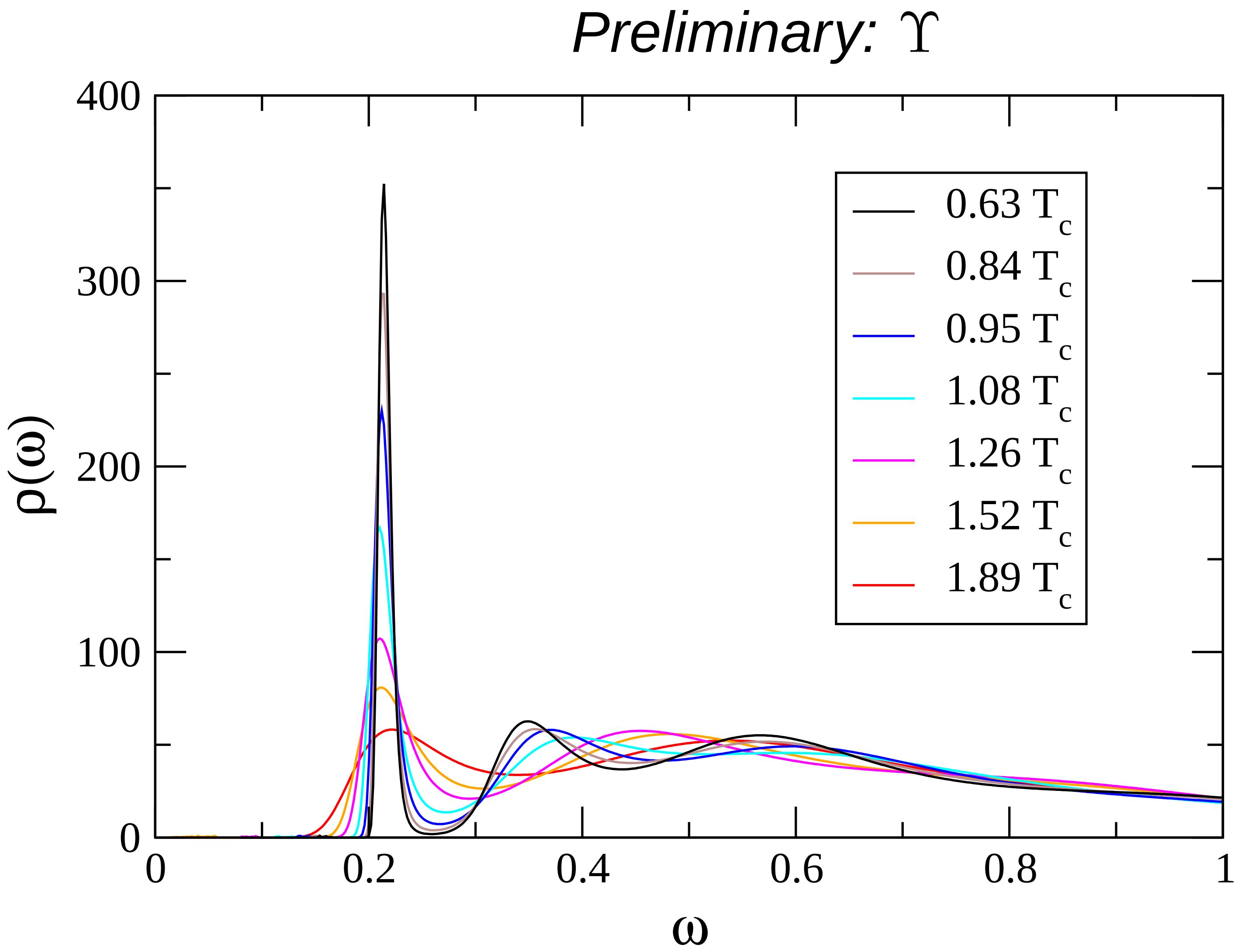


Energy as function of speed (1st Generation)

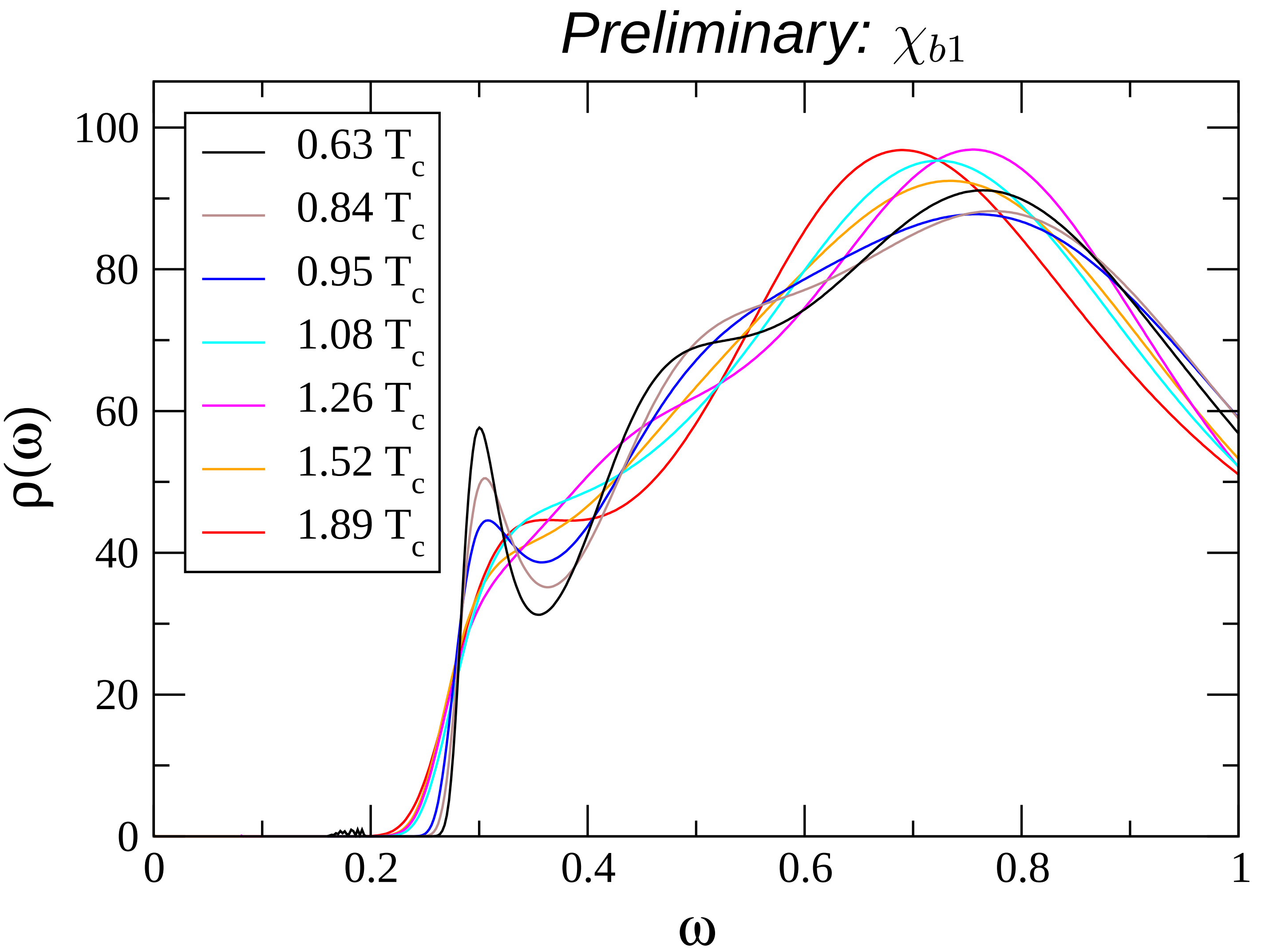


$$E(p)/M = 1 + \frac{1}{2}v^2$$

New, larger volume results (2nd Generation)



New, larger volume results (2nd Generation)



PART B

Other Finite Temperature Lattice Results

- Bottomonium Spectral Functions
  - Spectral Functions
- Conductivity
  - Transport Coefficients on the Lattice

Conductivity (Theory)

Electromagnetic current:

$$j_\mu^{\text{em}}(x) = e \sum_f q_f j_\mu^f(x)$$

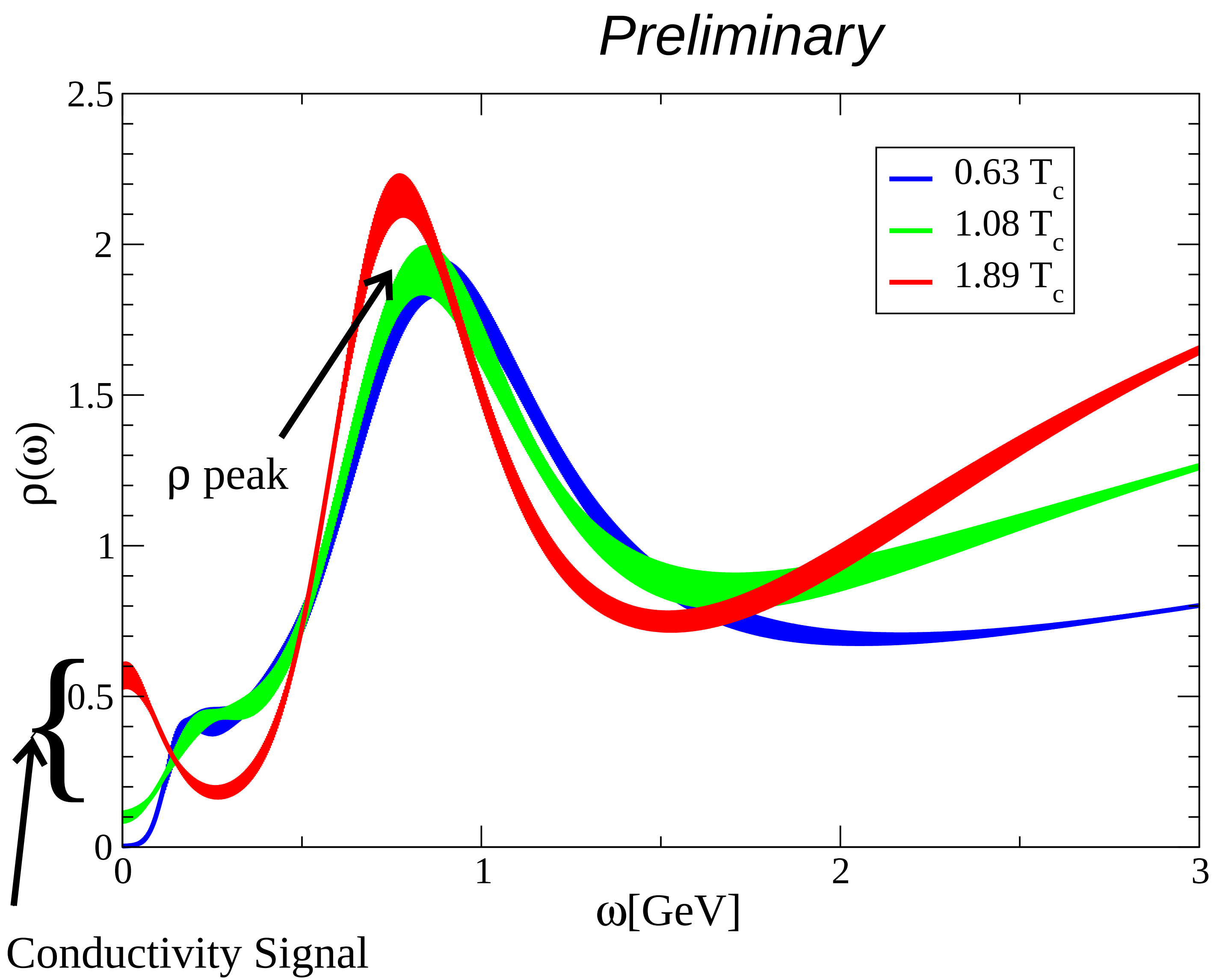
Correlation F'ns: $$G_{\mu\nu}(\tau, \mathbf{p}) = \int d^3x \, e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \langle \, j_\mu^{\text{em}}(0, \mathbf{x}) j_\nu^{\text{em}}(\tau, \mathbf{y})^\dagger \, \rangle$$

$$= \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \, \rho_{\mu\nu}(\omega, \mathbf{p})$$

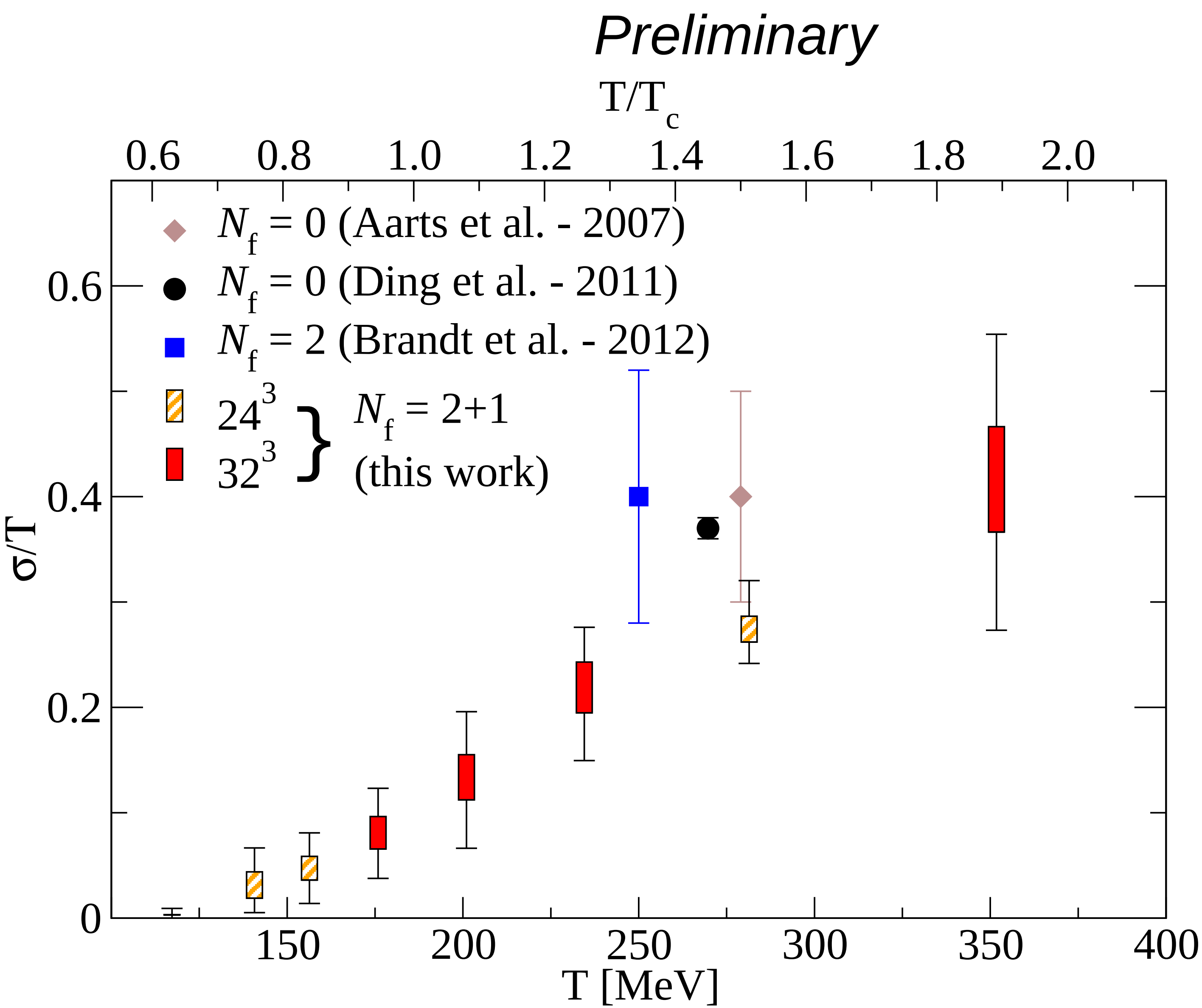
with kernel: $$K(\tau, \omega) = \frac{\cosh[\omega(\tau - /2T)]}{\sinh[\omega/2T]}$$

Conductivity: $$\frac{\sigma}{T} = \frac{1}{6T} \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}, \qquad \rho(\omega) = \sum_{i=1}^{3} \rho_{ii}(\omega)$$

Conductivity Signal



Temperature Dependence of Conductivity



PART B: Summary & Future Plans

Successfully calculated:

1. Bottomonium spectral functions at finite temperature.

- First time on the lattice
- s-wave (J/ψ and η_b) survive to large T
- p-wave (χ_{b1} at least) melts at $T \sim T_c$
- Observed momenta via spectral functions

2. Electrical Conductivity

- First time the temperature dependency has been uncovered on lattice
- Results compatible with previous determinations

Future Plans:

- Other transport coefficients?
- Take continuum limit

Particle Data Book

$\sim 1,500$ pages

zero pages on Quark-Gluon Plasma...

the end