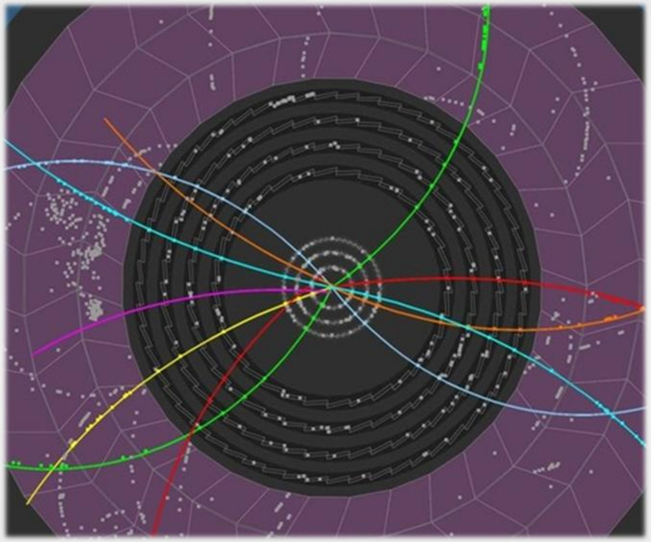
QUANTUM AND SEMI-CLASSICAL APPROACHES TO QUARKONIA SUPPRESSION



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<u>Advisor</u>: P.B. Gossiaux

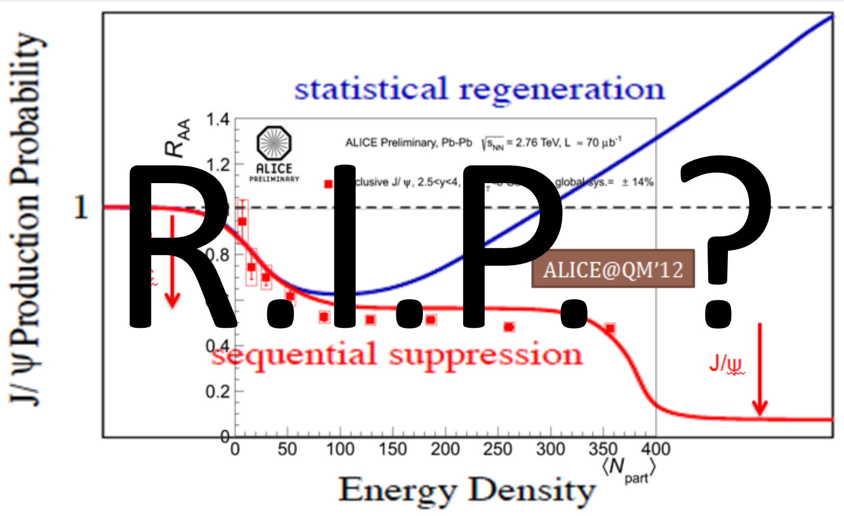
Introduction

Background?

- **Quarkonia suppression was predicted** by Matsui and Satz as a sign of Quark-Gluon Plasma production in heavy-ion collisions.
- Quarkonia suppression has been observed but is **still poorly understood**.

Project goal ?

- Describe the quarkonia suppression from **dynamical point of views** as an **alternative scheme to** sequential suppression, regeneration…

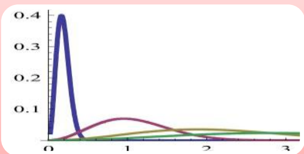


Plot from R. de Cassagnac's talk.

For now ?

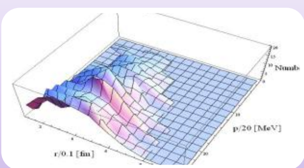
- **Study the wavefunction of a $Q\bar{Q}$ pair** in an isotropic QGP at thermal equilibrium through:

Quantum approach



- Non relativistic Schrödinger equation
- Thermalisation through additional terms in the Hamiltonian

Semi-classical approach



- Quantum Wigner distribution
- Classical, 1st order in $\hbar$, Wigner-Moyal equation

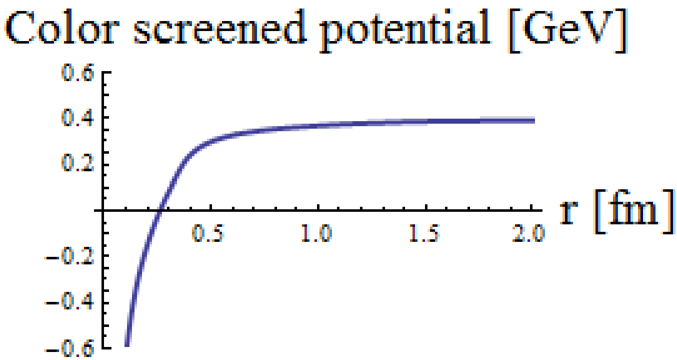
and **projection** onto the quarkonia S states.

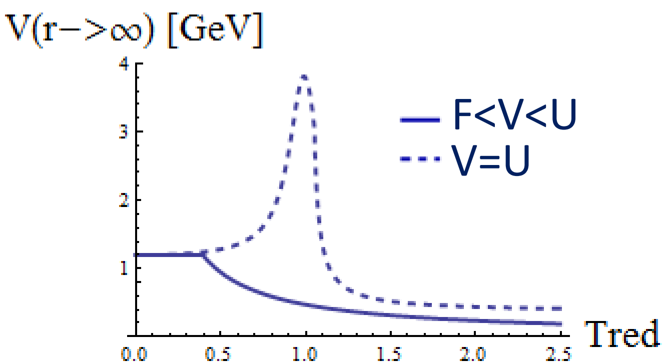
- **Results comparison to validate/unvalidate** Young and Shuryak* semi-classical approach to J/ψ survival with stochastic Langevin equation.
- **Quantum results** without thermalisation compared to data - Clues that **quantum thermalisation** will give interesting results.

* Phys.Rev.C79:034907,2009

The common tools

The color potentials V(Tred, r)





- "Weak potential F<V<U"

$$V_{\rm close}(r) = -\frac{\alpha}{r} + \sigma r - \frac{0.8\sigma}{m^2 r},$$

$$V_{\rm int}(r) = \frac{V0 + g_1(r-r_0) + g_2(r-r_0)^2}{1 + g_3(r-r_0) + g_4(r-r_0)^2},$$

$$V_{\rm far}(r) = V_\infty - \frac{4}{3}\frac{\alpha_1}{r} e^{-\sqrt{4\pi\ \tilde{\alpha}_1} Tr}$$

- "Strong potential V=U"

$$U = \left(-\frac{\alpha}{r} + \sigma r - \frac{0.8\sigma}{m^2 r}\right) \times e^{-(\mu r)^2} + V_0 \times \left(1 - e^{-(\mu r)^2}\right)$$

Evaluated by Mócsy & Petreczky* and Kaczmarek & Zantow**
from lQCD results and reparametrized by Gossiaux

 * Phys.Rev.D77:014501,2008 **arXiv:hep-lat/0512031v1

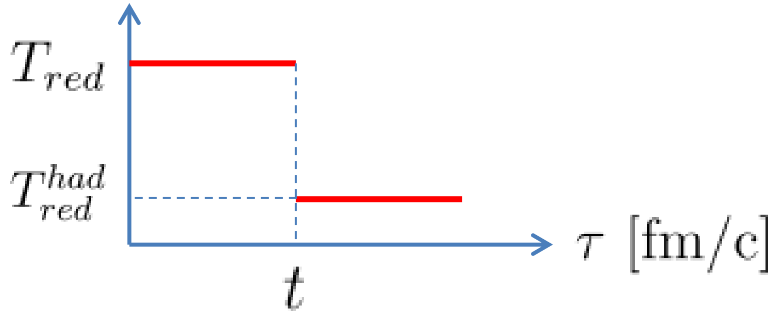
The common tools

The temperature scenarios

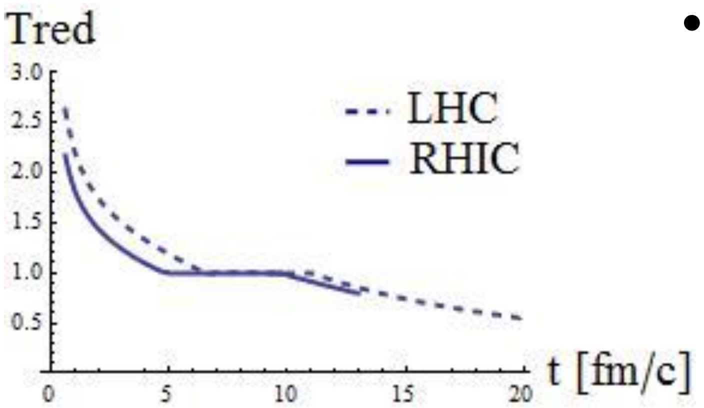
- At fixed temperatures

$T_{red} = T/T_c,$

where $T_c = 0.165\ \text{Gev}$



Instantaneous transition from QGP at T_{red} to hadronisation phase at $T_{red}^{had} \leq 0.4$



- Time dependent temperature
  - **Cooling** of the QGP over time by Kolb and Heinz* (hydrodynamic evolution and entropy conservation)
  - At LHC ($\sqrt{s_{NN}} = 2.76\ \text{TeV}$) and RHIC ($\sqrt{s_{NN}} = 200\ \text{GeV}$) energies

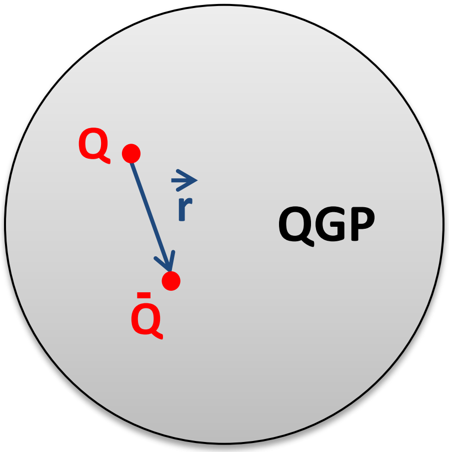
* arXiv:nucl-th/0305084v2

Quantum approach

- **Schrödinger equation for the $Q\bar{Q}$ pair evolution**

Where $$\widehat{H} = 2m_q - \frac{(\hbar c)^2}{m_q}\nabla^2 + V(r, T_{\text{red}})$$

$$\Psi_{Q\bar{Q}}(\mathbf{r}, t) = R_{Q\bar{Q}}(r, t) \times \cancel{Y_{Q\bar{Q}}(\theta, \phi)}$$



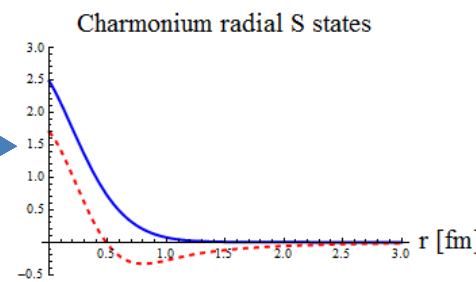
Initial wavefunction: $$R_{Q\bar{Q}}\,(r, t=0) = \left(\frac{1}{\pi a^2}\right)^{3/4} e^{-\frac{r^2}{2a^2}}$$

where $a_{c\bar{c}} = 0.165 \text{ fm}$ and $a_{b\bar{b}} = 0.045 \text{ fm}$

- **Projection onto the S states: the S weights**

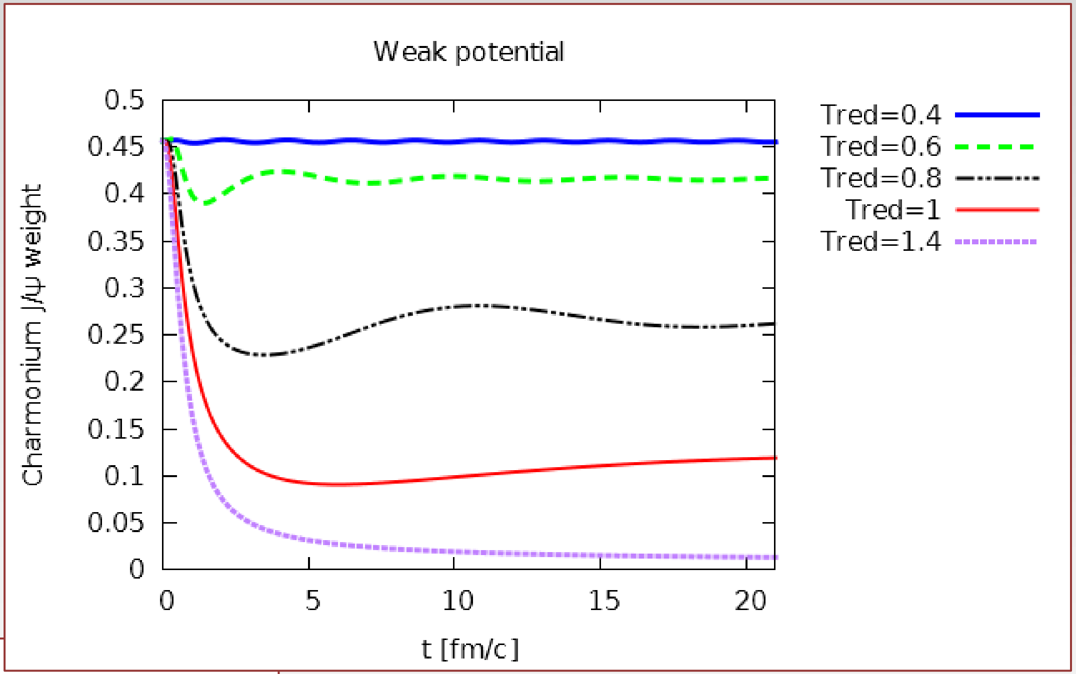
$$W_S(t) = \left(4\pi \,\text{Abs}\left[\int_0^{\infty} R_{Q\bar{Q}}\,(r, t, T_{red}) \times \underline{R_S(r, T_{red}^{had})}\; r^2\, dr\right]\right)^2$$

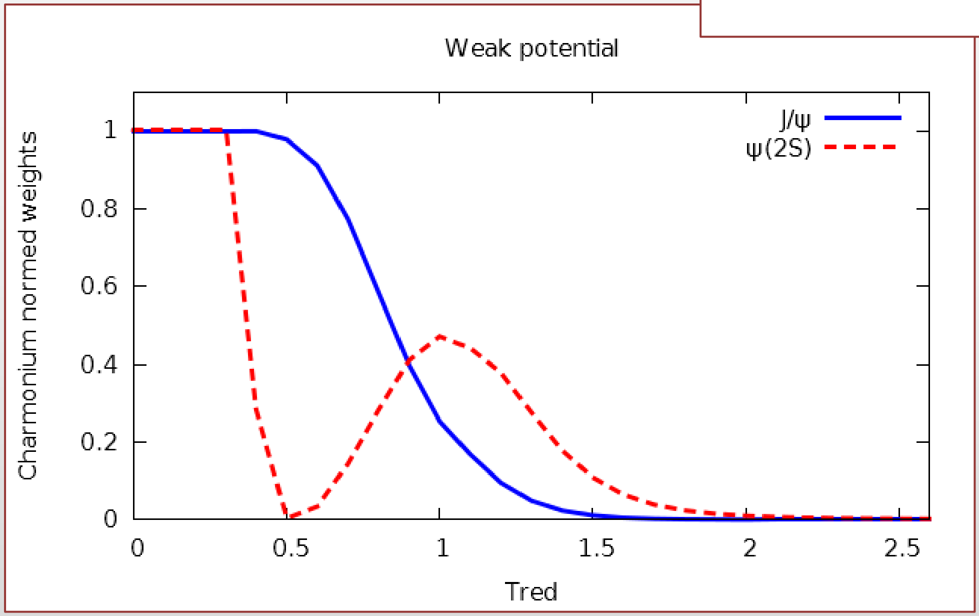
Radial eigenstates of the hamiltonian



Charmonia and weak color potential (F<V<U)

At fixed temperatures

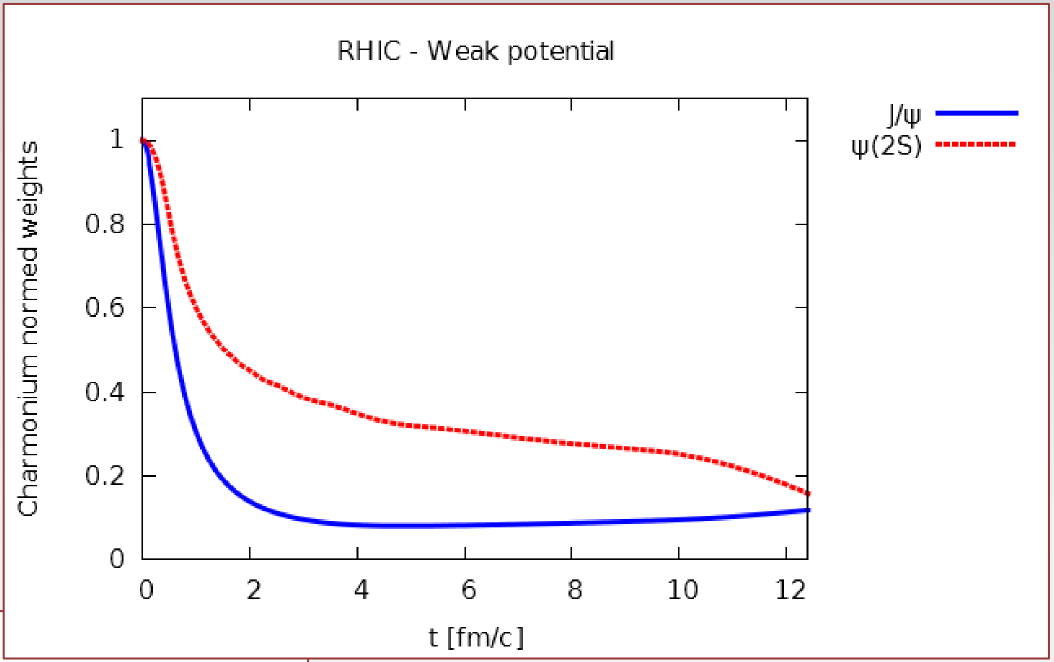


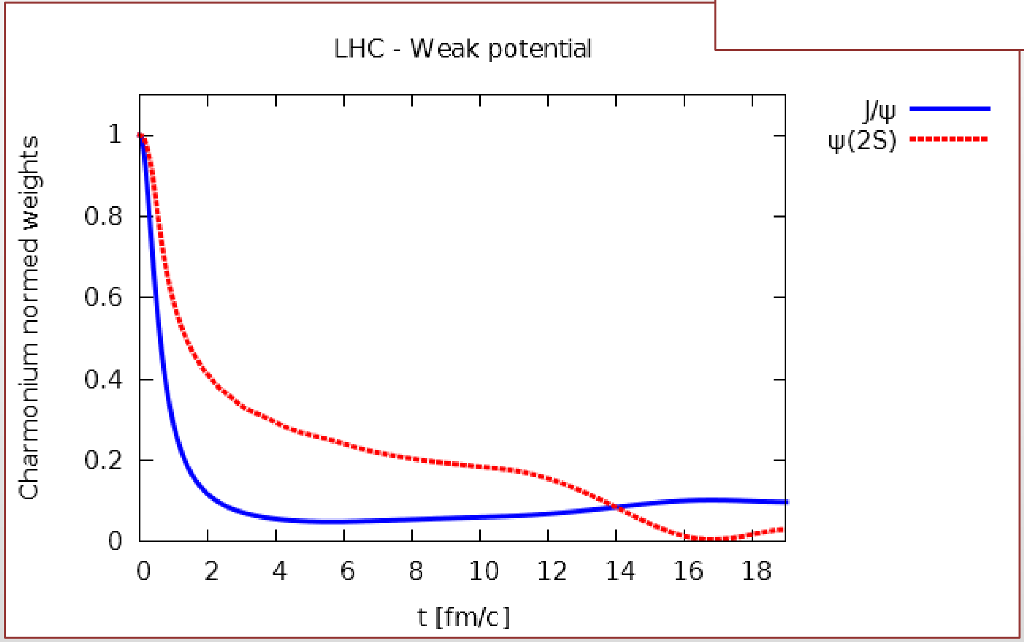


The normed weights at t->∞ function of the temperature

Charmonia and weak color potential (F<V<U)

RHIC temperature scenario

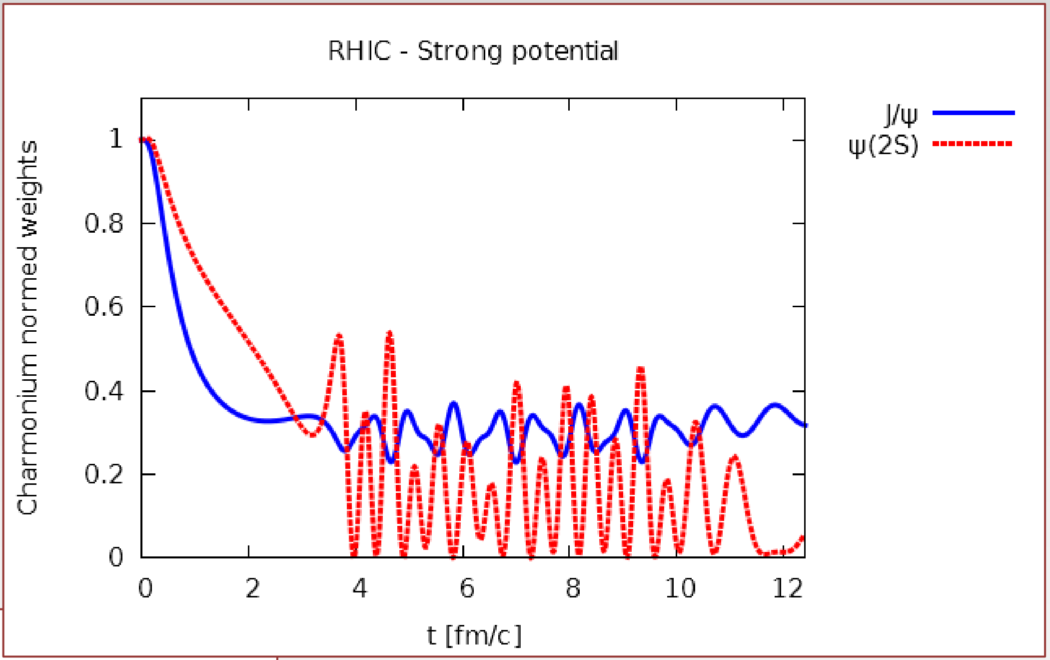


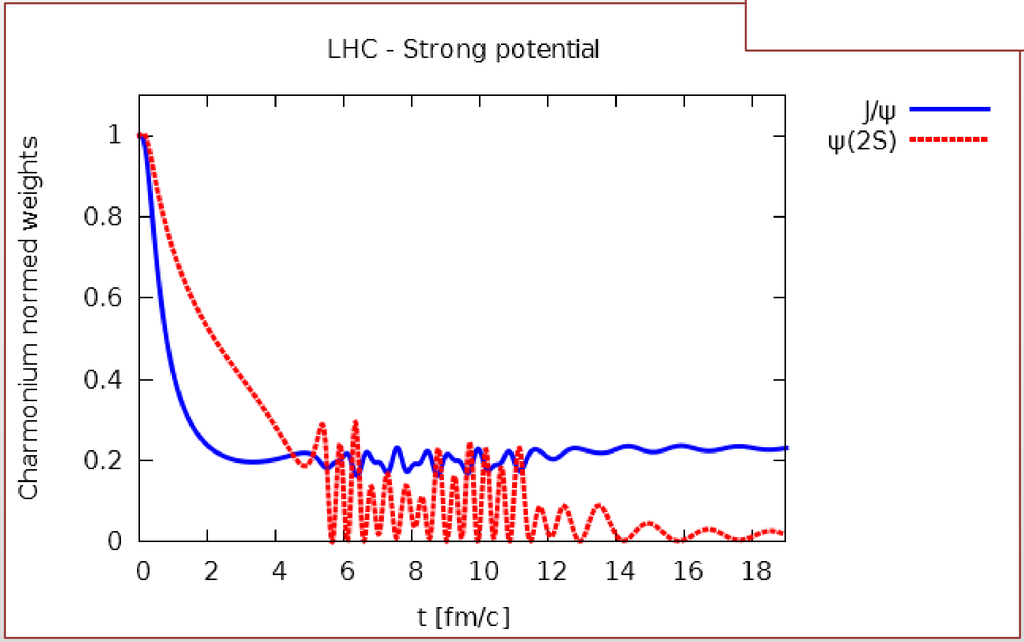


LHC temperature scenario

Charmonia and strong color potential (V=U)

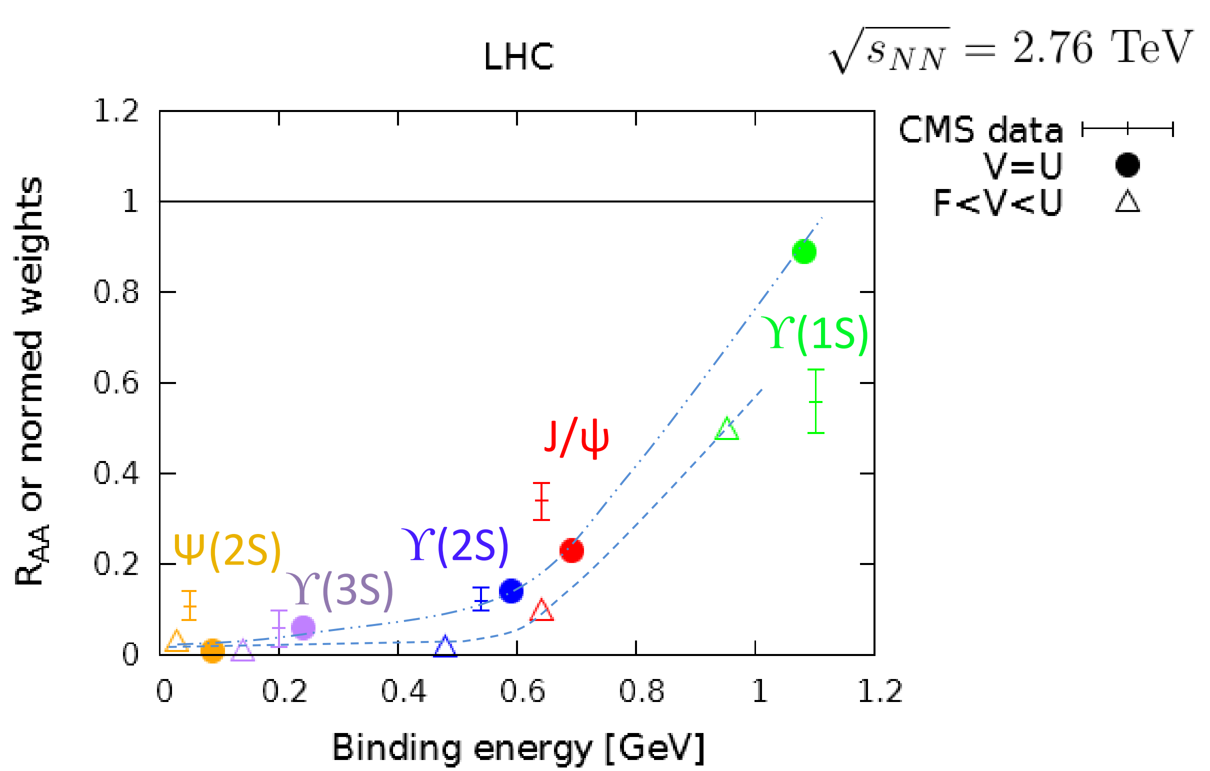
RHIC temperature scenario





LHC temperature scenario

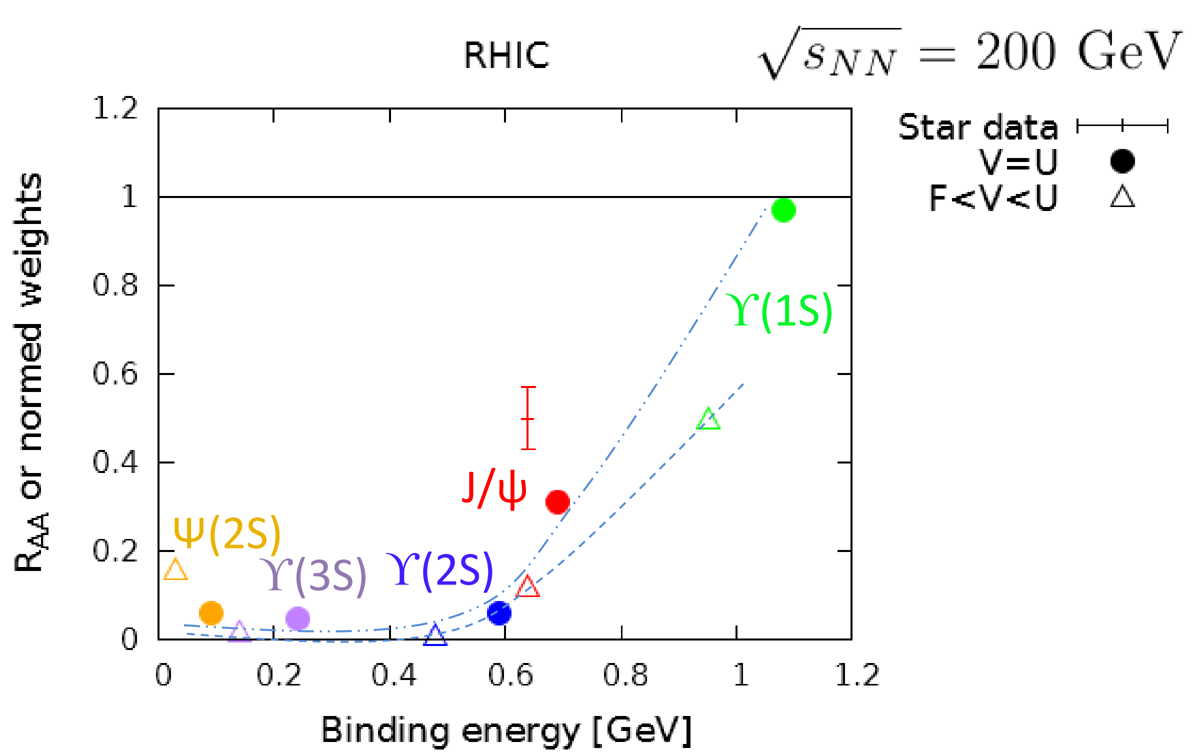
Sum up of LHC scenario results



- The results are quite relevant for such a simple scenario !
- J/ψ and ψ(2S) are underestimated and ϒ(1S) overestimated.
- The feed downs from exited states to lower states may change slightly these values…

See M. Jo's talk for data.

Sum up of RHIC scenario results



- Similar suppression tends obtained for both RHIC and LHC.
- Similarly to the data, these results exhibit less J/ψ suppression at RHIC than at LHC.
- ϒ(1S+2S+3S) suppression can be estimated with Star data to ~ 0.55±0.10,

we obtain ~ 0.48 for V=U and ~ 0.24 for F<V<U.

See D. Kikola's talk for data.

A taste of quantum thermalisation

Background?

- RHIC and LHC experimental results => quarkonia thermalise partially in the QGP
- But how to thermalise our wavefunction ? Quantum friction/stochastic effects have been a long standing problem because of their irreversible nature.

The open quantum approach: ✖
Considering the whole system, quarkonia and environment, the latter being finally integrating out

Y. Akamatsu [arXiv:1209.5068]
Laine et al. JHEP 0703 (2007) 054

2nd possible approach: ✓
Unravel the open quantum approach by using a stochastic operator and a dissipative non-linear potential

A. Rothkopf et al. Phys. Rev. D 85, 105011 (2012)
N. Borghini et al. Eur. Phys. J. C 72 (2012)
S. Garashchuk et al. Jou. of Chem. Phys. 138, 054107 (2013)

- **New Schrödinger equation**

$$\boxed{i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\widehat{H}(\mathbf{r}) - \underline{\mathbf{F}(t).\mathbf{r}} + \underline{A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big)}\right)\Psi_{Q\bar{Q}}(\mathbf{r},t)}$$

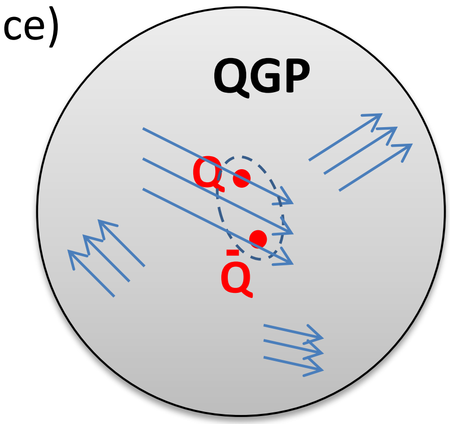
Where: $S(\mathbf{r},t) = \arg\big(\Psi_{Q\bar{Q}}(\mathbf{r},t)\big)$ and $\langle\mathbf{F}(t)\rangle = 0\,,\ \langle\mathbf{F}(t)\mathbf{F}(t')\rangle = \Gamma(t,t')$

How to build the stochastic operator ?

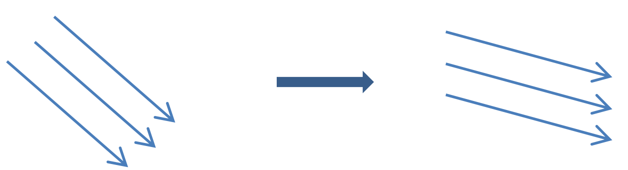
- **The hierarchy** $m \gg T \;\Rightarrow\; \sigma \ll \tau_{\mathrm{relax}}$ (adiabatic invariance)

  where ✓ σ is the quarkonia autocorrelation time with the gluonic fields (if $\sigma = 0$ the fluctuations are uncorrelated)

  ✓ τ_{relax} is the quarkonia relaxation time



- $\langle \mathbf{F}(t)\mathbf{F}(t') \rangle = \Gamma(t,t')$?



at t

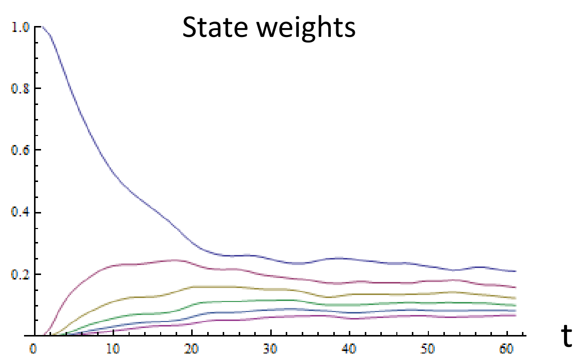
at t+Δt

$$\Gamma(t,t') = B\,\frac{e^{-\frac{(t-t')^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma} \xrightarrow[\sigma\to 0]{} B\,\delta(t-t')$$

- One has finally 3 parameters: A (the Drag coefficient), B (the diffusion coefficient) and σ .

First tests?

- Tested in an harmonic potential:



➞ One gets Boltzmann distributed state weights ! Independently of σ and with the Einstein relation B ≈ 2mTA between the diffusion coefficient and the Drag coefficient.

At a finite time:

➞ high pt => high velocity => smaller σ => more excited states => more suppression

➞ low pt => small velocity => higher σ => less excited states => less suppression (=> no need for regeneration !)

➞ **Will be generalized and used to our quarkonia thermalisation in the near future !**

Semi-classical approach

The"Quantum" **Wigner distribution of the cc pair**:

$$F\left(\vec{x},\vec{p},t\right)=\int e^{\frac{i\vec{p}\vec{y}}{\hbar}}\,\Psi^*\left(\vec{x}+\frac{\vec{y}}{2}\right)\Psi\left(\vec{x}-\frac{\vec{y}}{2}\right)d\vec{y}$$

… is **evolved** with the "classical", 1st order in ħ, Wigner-Moyal equation:

$$\left[\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\frac{\partial}{\partial\vec{x}}\right)-\frac{\partial}{\partial\vec{p}}\frac{\partial}{\partial\vec{x}}V\left(\vec{x}\right)\right]F\left(\vec{x},\vec{p},t\right)=0$$

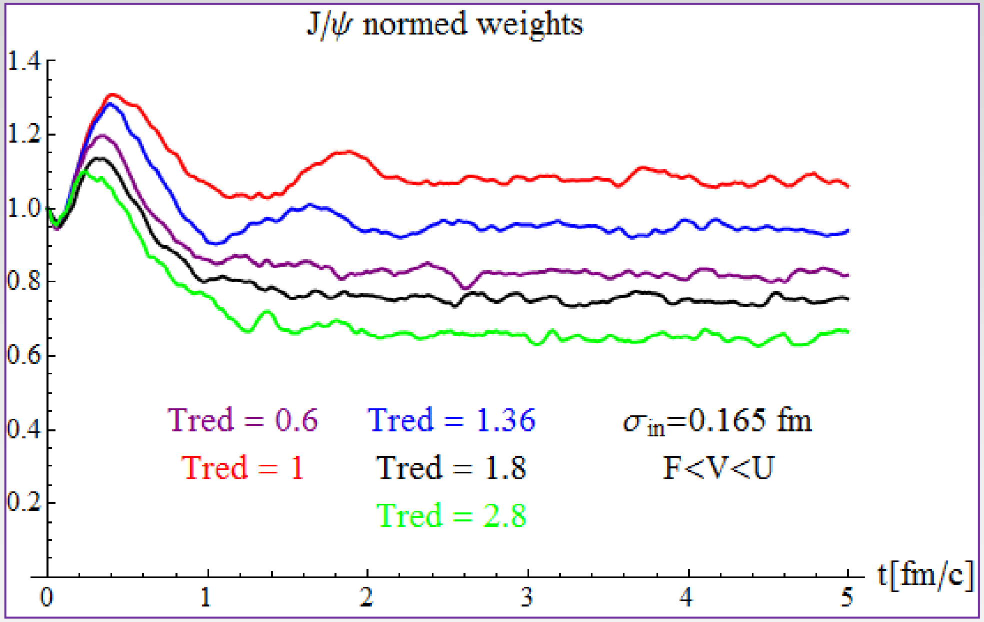
Finally the **projection** onto the J/ψ state is given by:

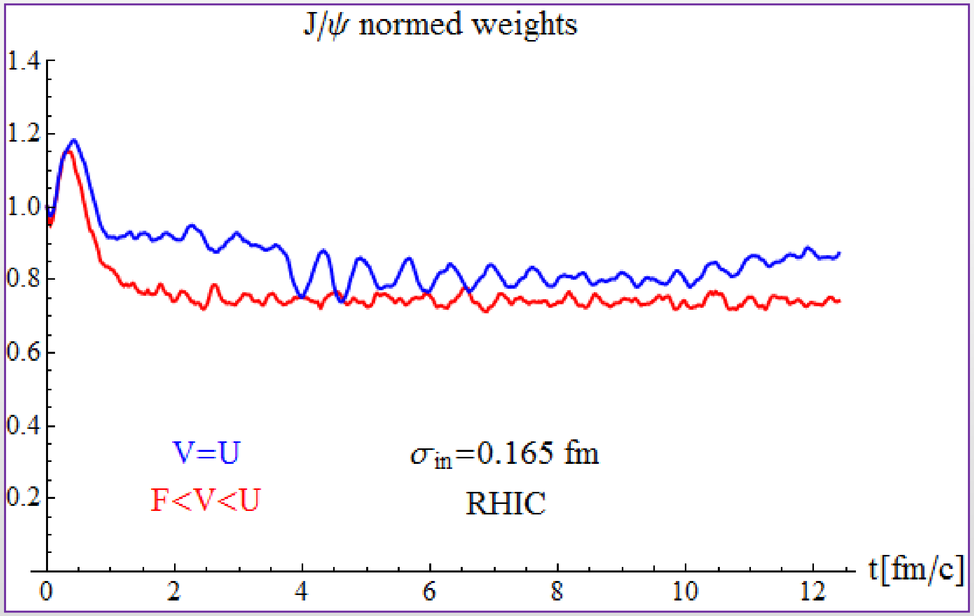
$$W_S(t)=\int F\left(\vec{r},\vec{p},t\right)F_S\left(\vec{r},\vec{p}\right)\frac{d^3\vec{p}d^3\vec{r}}{\left(\hbar c\right)^3}$$

But in practice: N test particles (initially distributed with the same gaussian distribution in (r, p) as in the quantum case), that evolve with Newton's laws, and give the J/ψ weight at t with:

$$W_S(t)=\frac{1}{N}\sum_{i=1}^{N}F_S\left(r_i(t),p_i(t)\right)$$

With the **weak** color potential (F<V<U) at **fixed temperatures**





RHIC temperature scenario

With additional stochastic and drag forces

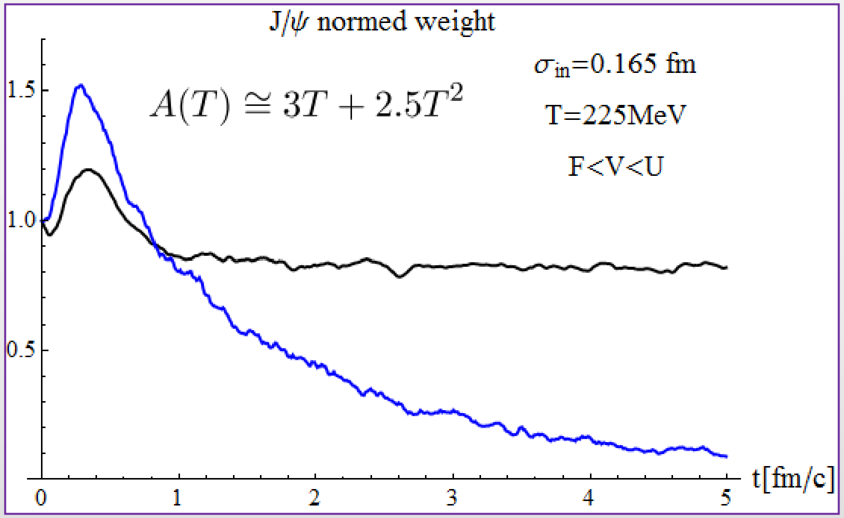
The Langevin forces are added in Newton's laws.

$$\frac{d\vec{p}}{dt} = \vec{f}(\vec{p},\vec{x}) = -A(\vec{x})\,\vec{p} + \xi(\vec{x})$$

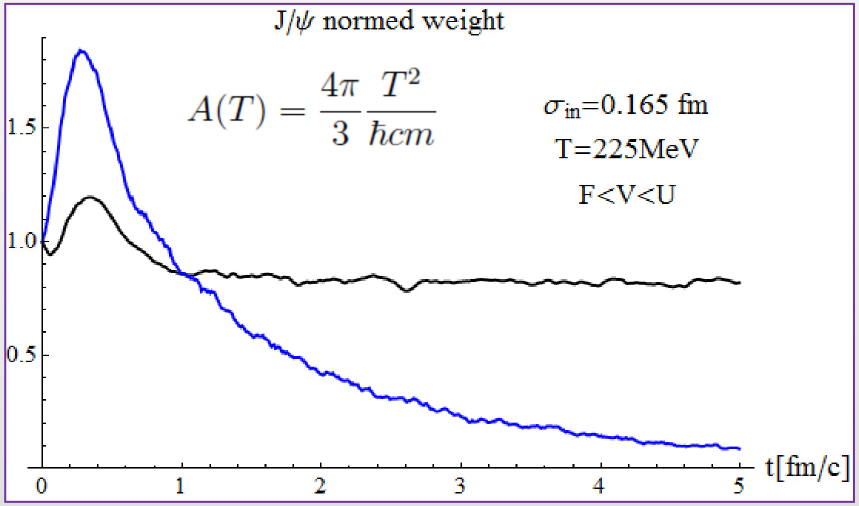
where $<\xi_i(t)\ \xi_j(t')> \ = \ \kappa\delta_{i,j}\delta(t-t')$

Einstein relation: $\frac{\kappa}{A} = 2mT$

with J. Aichelin and P.B. Gossiaux's A(T)



with C. Young and E. Shuryak's A(T)



Conclusion

Semi classical

- Results obtained at fixed temperatures, with RHIC and LHC cooling scenarios and with and without Langevin forces.
- Strong discrepancies between semi-classical and quantum results => Young and Shuryak's approach may not be relevant.

Quantum

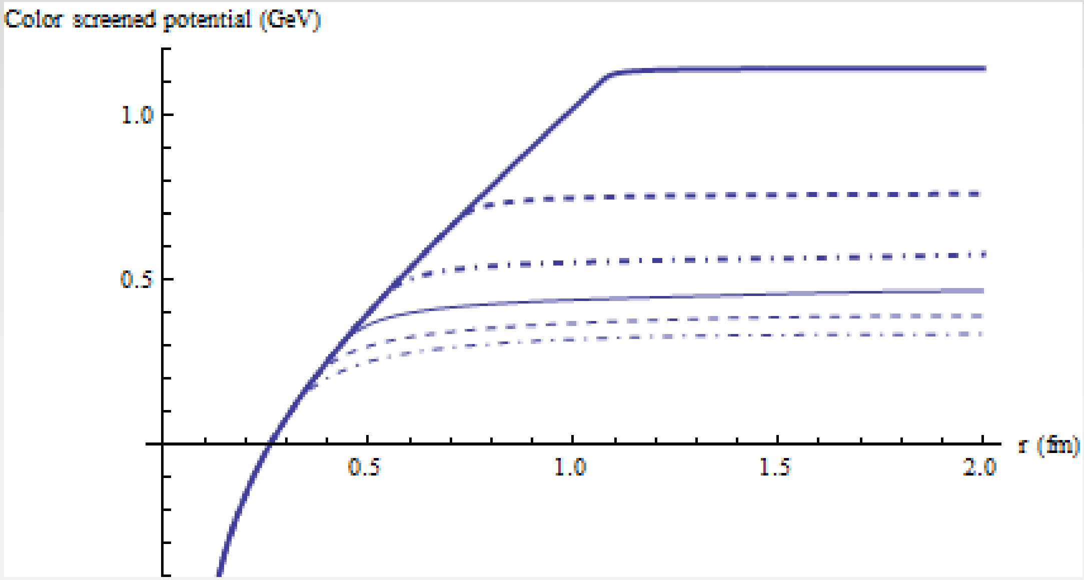
- Interesting results are obtained at fixed temperatures and with RHIC and LHC cooling scenarios.
- This dynamical approach ("continuous scenario") might replace the sequential suppression ("binary scenario").
- Will be further developed with more complex temperature scenarios.

Quantum thermalisation

- Theoretically developed, successfully tested with an harmonic potential, will be applied to the quarkonia in QGP case.
- Might be able to give alternative explanations to regeneration and suppression of the suppression.

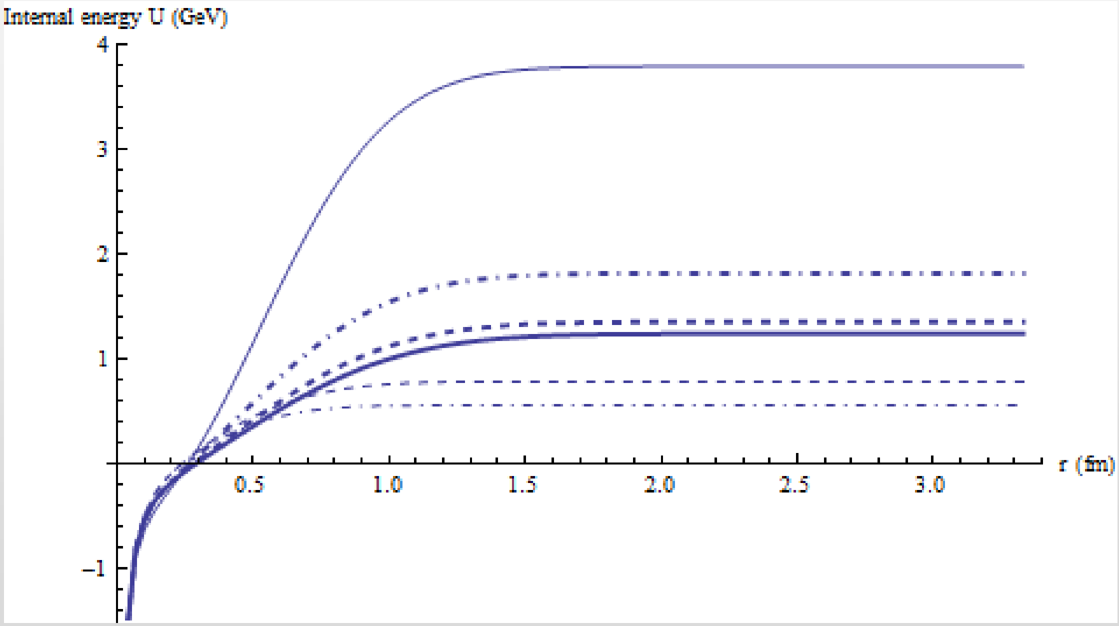
BACK UP SLIDES

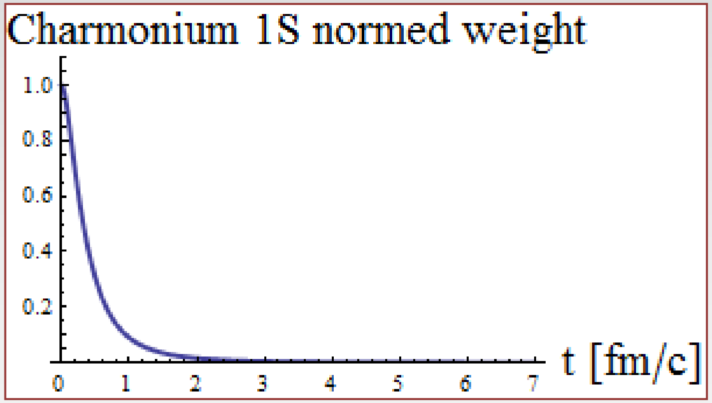
Some plots of the potentials



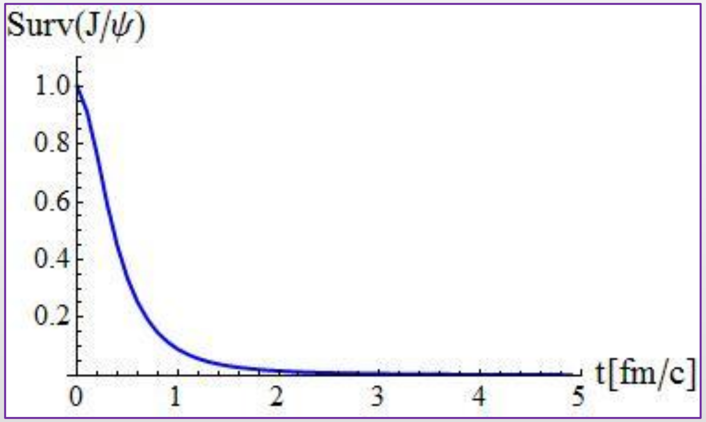
With weak potential F<V<U with Tred from 0.4 to 1.4

With strong potential V=U with Tred from 0.4 to 1.4





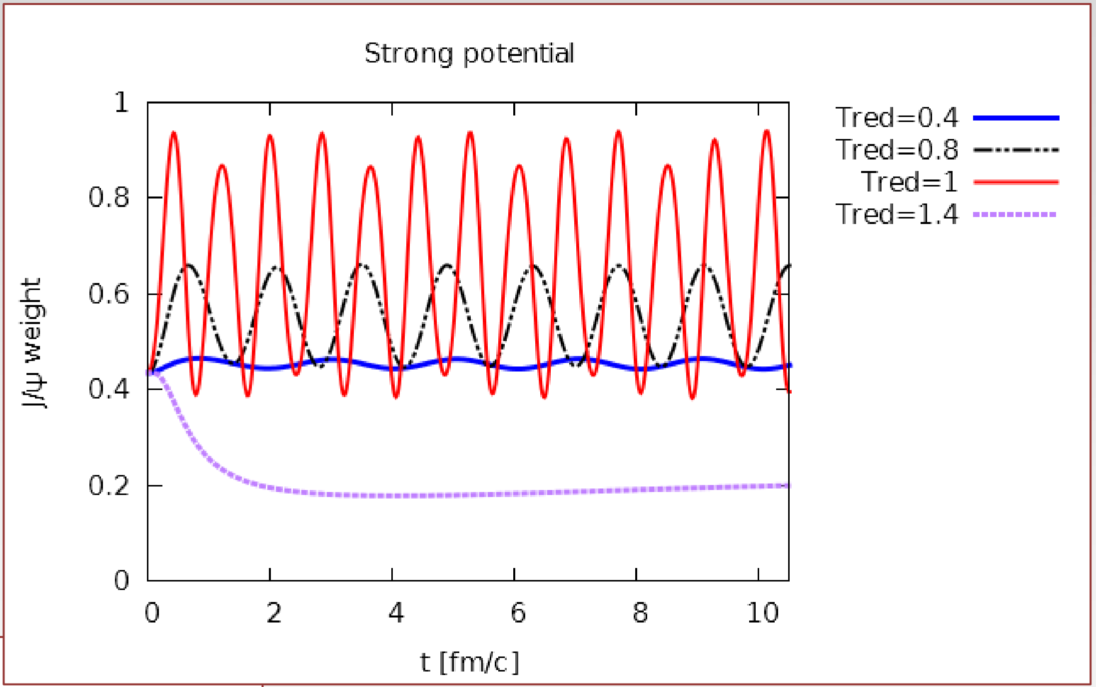
Quantum formalism with **no** color potential: V=0

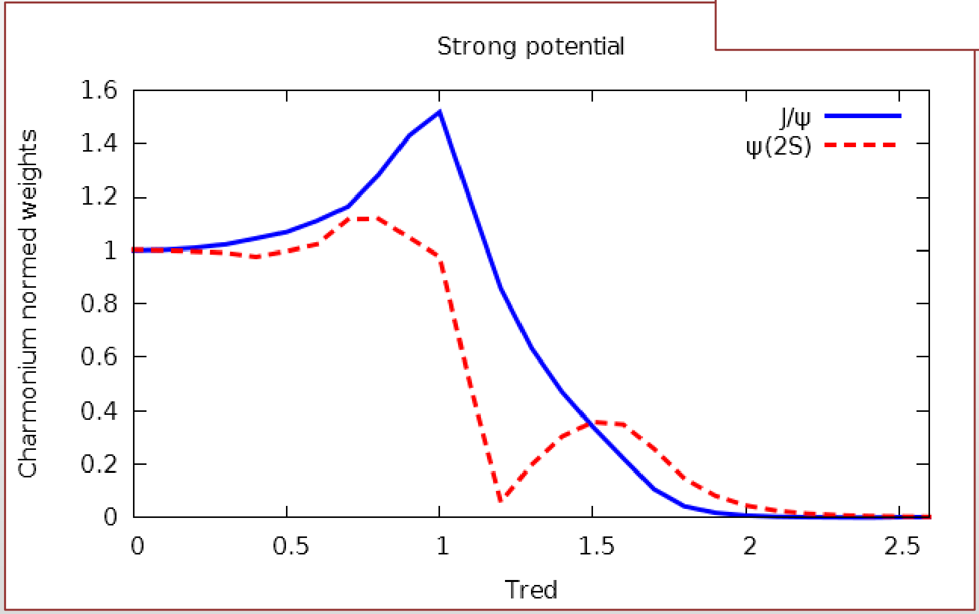


Semi-classical formalism with **no** color potential: V=0

Charmonia and strong color potential (V=U)

At fixed temperatures

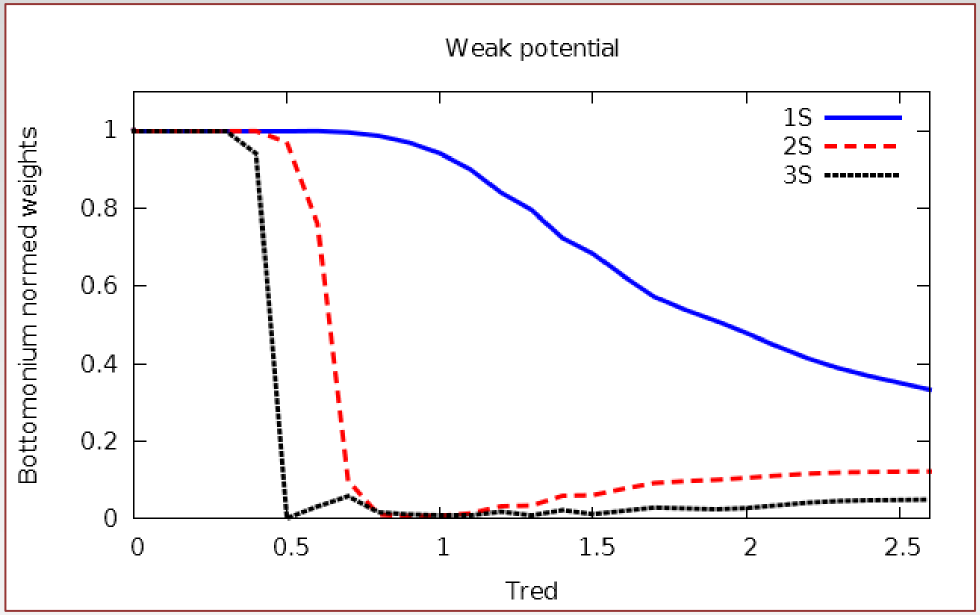




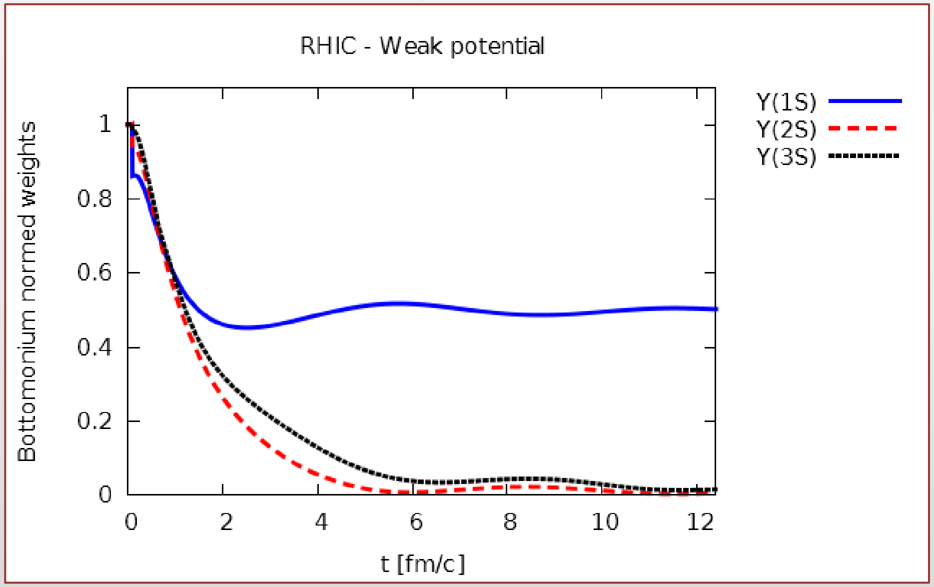
The normed weights at t->∞ function of the temperature

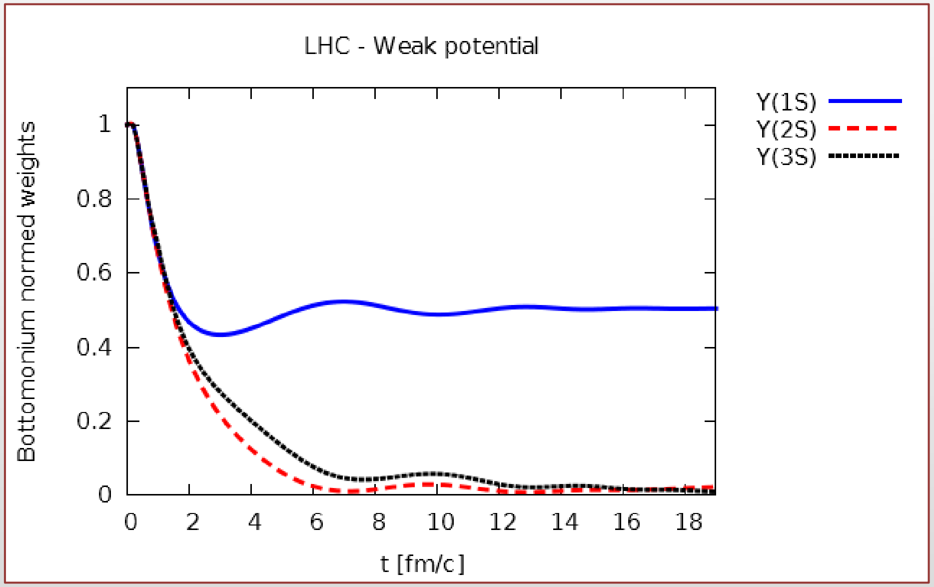
Bottomonia and weak color potential (F<V<U)

The normed weights at t->∞ function of the temperature



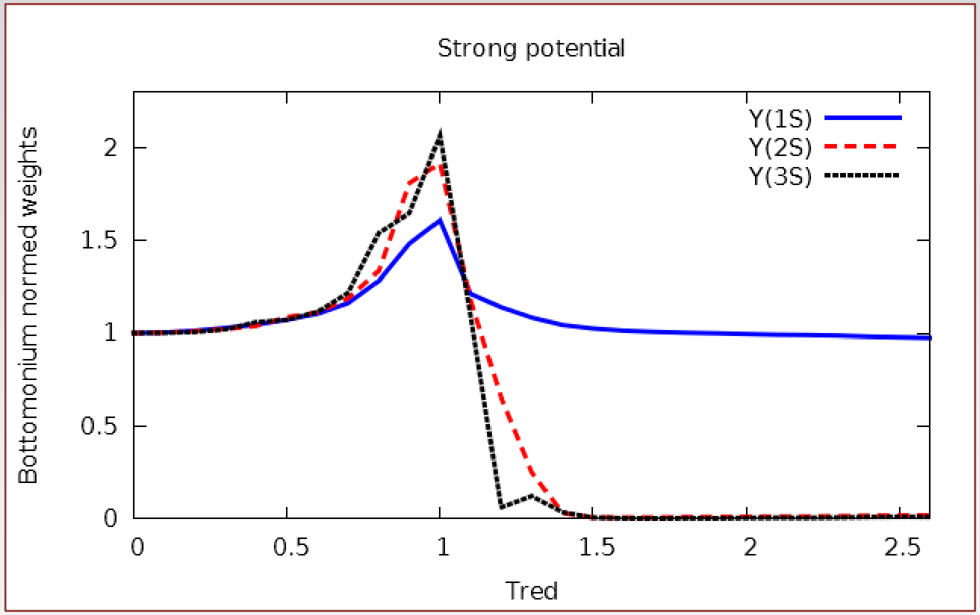
Temperature scenarios



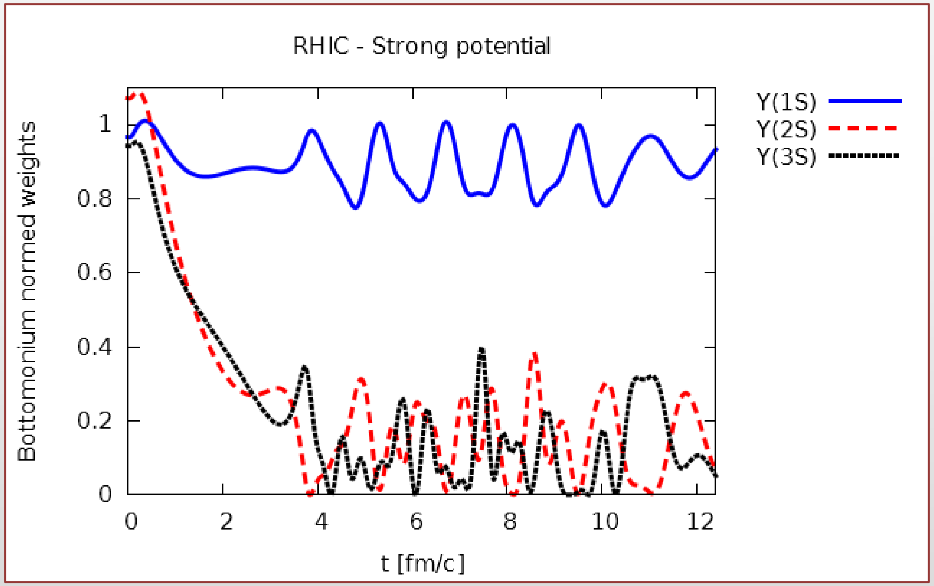


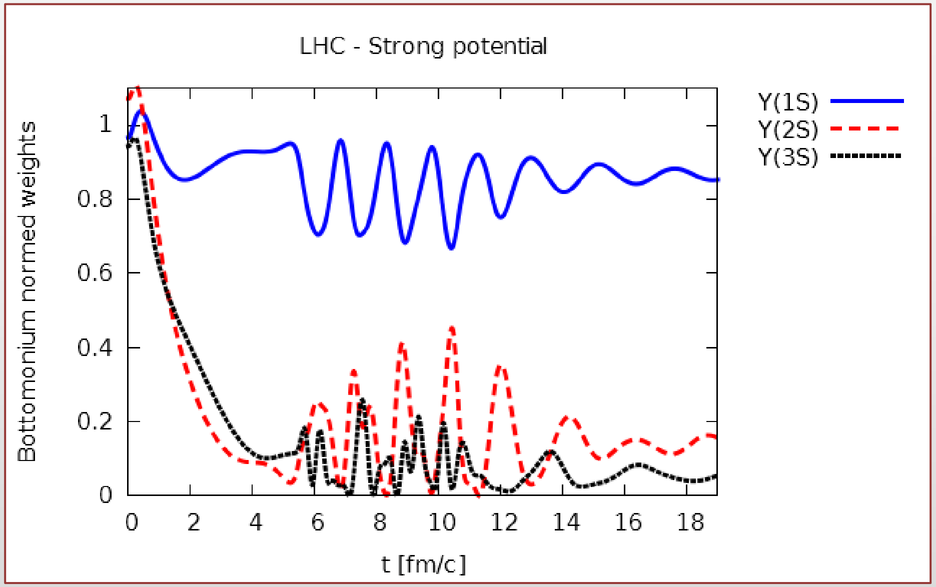
Bottomonia and strong color potential (V=U)

The normed weights at t->∞ function of the temperature



Temperature scenarios





Explain the evolution of the weights in the semi-classical case:
From observing the distribution of test particles in the phase space over time:

- ✓ *The increase of Surv(J/ψ)* for t < 0.5 fm/c **<=** some particles loose momentum while climbing the potential
- ✓ *And decrease of Surv(J/ψ)* for t < 1 fm/c **<=** the particles with sufficient momentum go out the « J/ψ zone » by climbing the potential
- ✓ *Finally Surv(J/ψ) remains constant* for t > 1 fm/c **<=** the latter particles reach the continuum