Thermalization through Hagedorn States

Max Beitel, Kai Gallmeister, Carsten Greiner

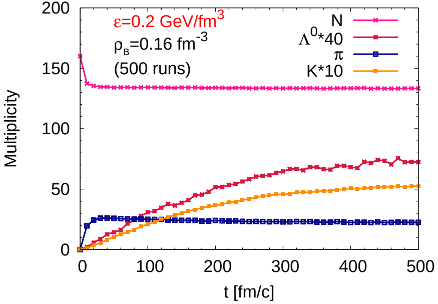
Institute for Theoretical Physics
Goethe University Frankfurt am Main

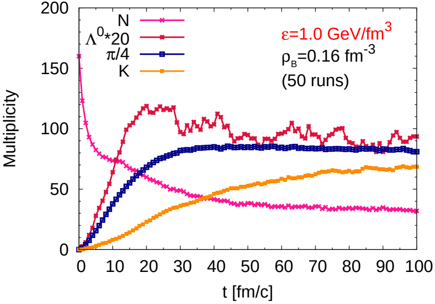
SQM 2013, Birmingham

26.07.2013

Problem: Chem. equilibration in UrQMD too slow

cf. M. Belkacem et al., PRC 58 (1998) 1727
see also E. Bratkovskaya et al., NPA 675 (2000) 661





- initial particles: 80 n + 80 p (uniformly distributed)
- string production disabled

Application of Hagedorn States (HS):

J. Noronha-Hostler, C. Greiner, I. A. Shovkovy., PRL 100 (2008) 252301

- SPS energies: strong increase of antiprotons/antihyperons through 'clustering' of mesons

$$n_1\pi + n_2K \leftrightarrow \bar{Y} + p$$

- chemical equilibration time of $t_{\mathrm{eq}} \approx 1 - 3\,\mathrm{fm}/c$
- RHIC energies: $t_{\mathrm{eq}} \sim 10\,\mathrm{fm}/c$ for antibaryons
- quick chemical equilibration mechanism through HS

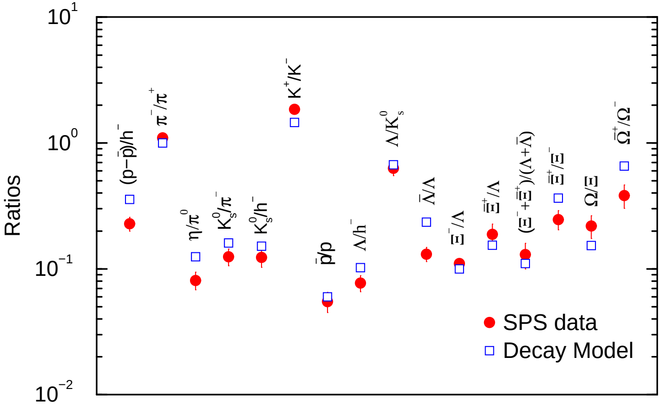
$$\left(n_1\pi + n_2K + n_3\bar{K} \leftrightarrow\right)\; HS \leftrightarrow \bar{B} + B + X$$

- dynamical evolution through set of coupled rate equations
- HS and pions in equilibrium $\rightarrow t_{\mathrm{eq}} \approx 5\,\mathrm{fm}/c$ for $B\bar{B}$-pairs

Application of Hagedorn States (HS) :

S. Pal, P. Danielewicz, Phys.Lett. B627 (2005) 55

- statistical model for decay and formation of HS
- Hagedorn Temperature of $T_H \simeq 170$ MeV
- subsequent decay of one single heavy resonance (HS)



Intention: UrQMD[1] and Hagedorn States (HS)

- UrQMD = Ultrarelativistic Quantum Molecular Dynamics
- microscopic transport model for p+p, p+N and A+A for Bevalac and SIS up to AGS, SPS and RHIC energies
- detailed balance
  - is enforced: meson-baryon, meson-meson, resonance-nucleon, resonance-resonance
  - is violated by string and some hadron decays ($\omega \to 3\pi$)
- for $\sqrt{s} \geq 2.5 - 10\,\mathrm{GeV}$: HS production replaces strings

Observables:

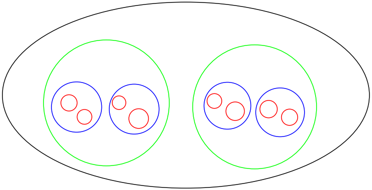
- particle multiplicities
- chemical equilibration times
- estimates for $\frac{\eta}{s}$ (shear viscosity over entropy density)
- ...

[1] S. A. Bass et al., Prog.Part.Nucl.Phys. 41 (1998) 225

Historical Background of Statistical Bootstrap Model

1965: Rolf Hagedorn postulates "Statistical Bootstrap Model"[2][3]

- fireballs and their constituents are the same
- nesting fireballs into each other leads to
  self-consistency condition (bootstrap-equation)



- solution: "Hagedorn Spectrum", exponentially rising

[2] R. Hagedorn, Nuovo Cim.Suppl. 3 (1965) 147-186
[3] S. Frautschi Phys. Rev. D (1971) 2821-2834

Covariant Formulation of Hagedorn Spectrum

- number of states of particle in (resting) volume V

$$\mathrm{d}N = \left(V \frac{m}{E}\right) \frac{\mathrm{d}^3 p}{(2\pi)^3} = 2V \frac{\mathrm{d}^4 p}{(2\pi)^4}\, m\, (2\pi)\delta\left(p^2 - m^2\right)$$

- include mass degeneration $\tilde{\tau}\left(m^2\right)$

$$\mathrm{d}N = 2V \frac{\mathrm{d}^4 p}{(2\pi)^4} m\, \mathrm{d}m^2\, \tilde{\tau}\left(m^2\right) (2\pi)\delta\left(p^2 - m^2\right)$$

- convolution of single state densities

$$\tilde{\tau}\left(m^2\right) = \frac{V}{m} \frac{1}{(2\pi)^3} \prod_{i=1}^{2} \int \mathrm{d}m_i^2 \tilde{\tau}\left(m_i^2\right) m_i \Phi_2\left(m\right)$$

Generalization for additional quantum numbers

- $\tilde{\tau}\left(m^2\right) = \tau(m)/(2m)$ and $V = 4\pi R^3/3$

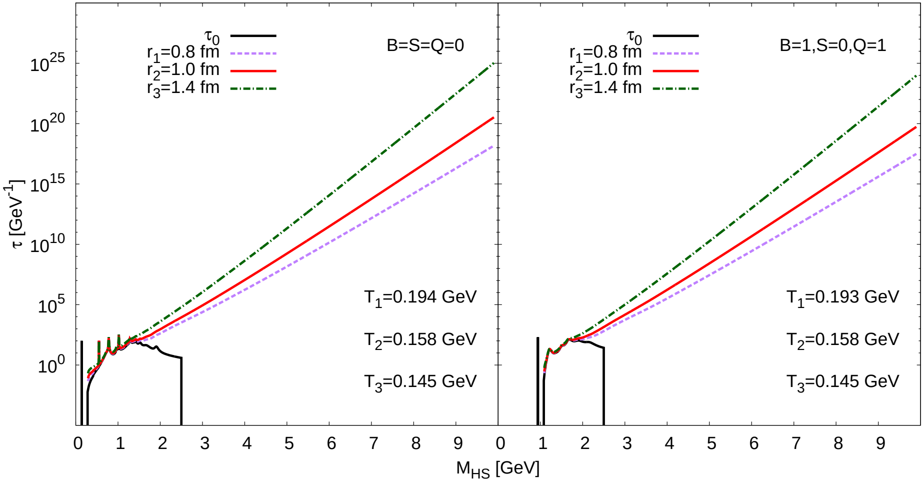
$$\tau\left(m\right) = \frac{R^3}{3\pi m}\iint \mathrm{d}m_1\mathrm{d}m_2\tau\left(m_1\right)\tau\left(m_2\right)m_1 m_2 p_{cm}$$

- conserve baryon number B, strangeness S and electric charge Q $\left(\vec{C} = (B, S, Q)\right)$

$$\tau_{\vec{C}}\left(m\right) = \frac{R^3}{3\pi m}\sum_{\vec{C}_1,\vec{C}_2}\iint \mathrm{d}m_1\mathrm{d}m_2$$

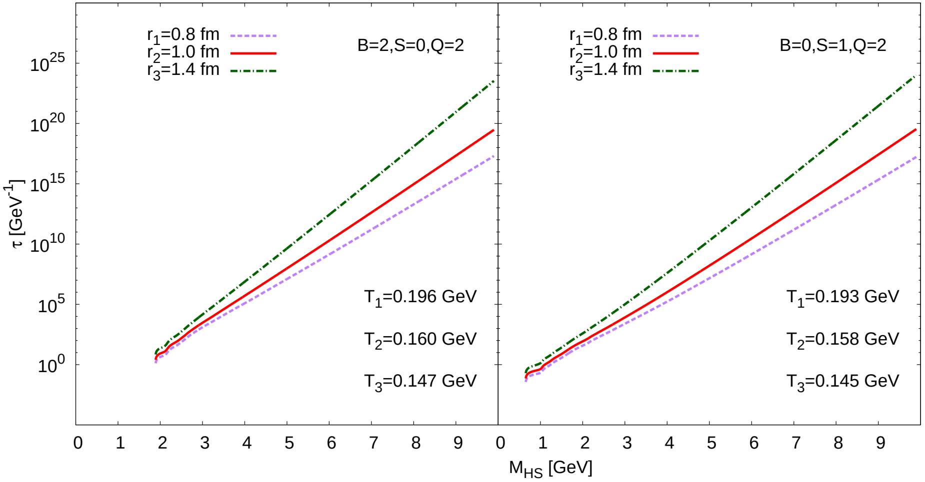
$$\times\,\tau_{\vec{C}_1}\left(m_1\right)\tau_{\vec{C}_2}\left(m_2\right)m_1 m_2 p_{cm}\delta^3\left(\vec{C} - \vec{C}_1 - \vec{C}_2\right)$$

Mesonic (left), baryonic (right) Hagedorn Spectra



fit function: $f(m) = cm^a \exp\left(\frac{m}{T}\right)$

Exotic Hagedorn Spectra



fit function: $f(m) = cm^a \exp\left(\frac{m}{T}\right)$

Cross Section and Decay Width

- cross section for creation of HS in binary collisions

$$\sigma = \frac{\pi \left|\mathcal{M}_c\right|^2}{4m^2 p_{cm}} \tau\left(m\right)$$

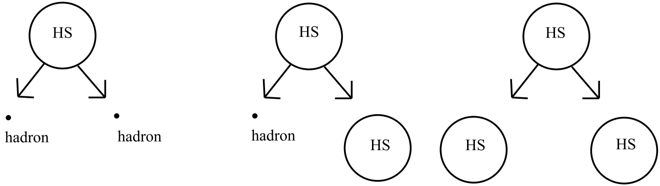
- decay width of HS in two particles only

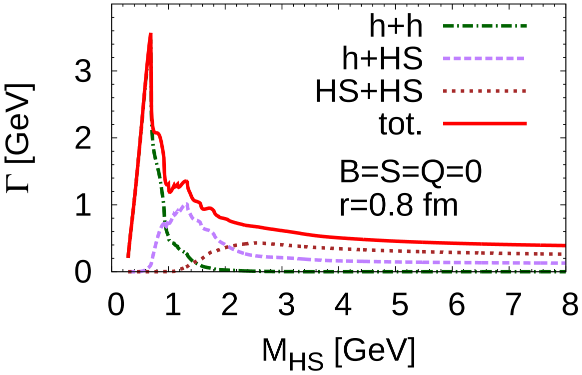
$$\Gamma = \frac{\left|\mathcal{M}_d\right|^2}{8\pi m^2} \iint \mathrm{d}m_1 \mathrm{d}m_2 \tau\left(m_1\right) \tau\left(m_2\right) p_{cm}$$

- detailed balance: $\left|\mathcal{M}_c\right|^2 = \left|\mathcal{M}_d\right|^2$

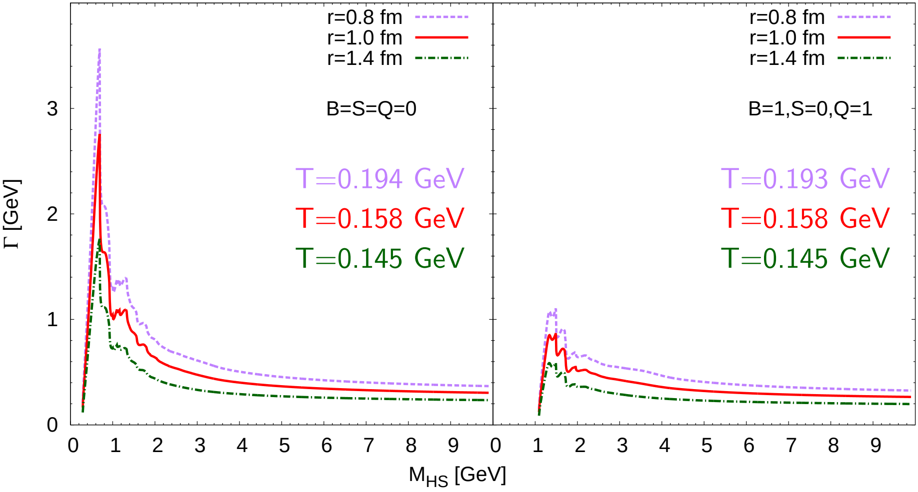
$$\Gamma\left(m\right) = \frac{\sigma}{2\pi^2 \tau\left(m\right)} \iint \mathrm{d}m_1 \mathrm{d}m_2 \tau\left(m_1\right) \tau\left(m_2\right) p_{cm}^2$$

Hagedorn state decay modes

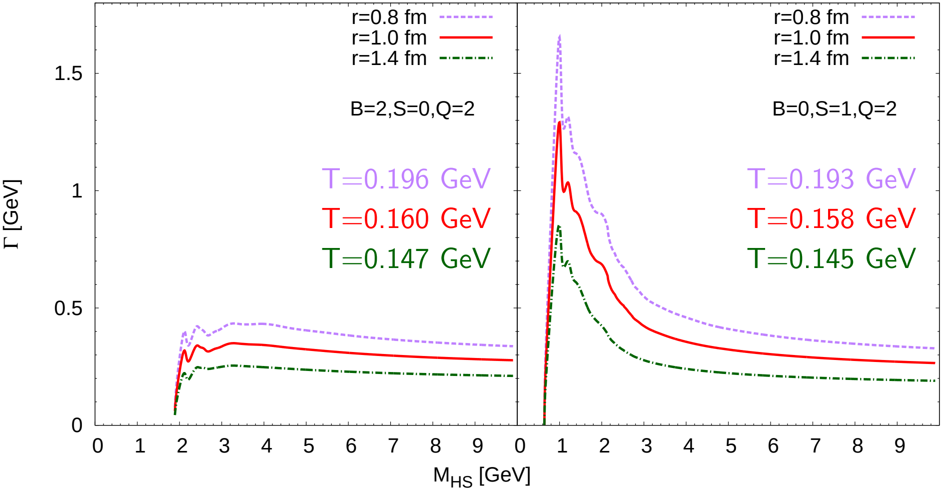




Decay Width for mesonic (left) and baryonic (right) HS

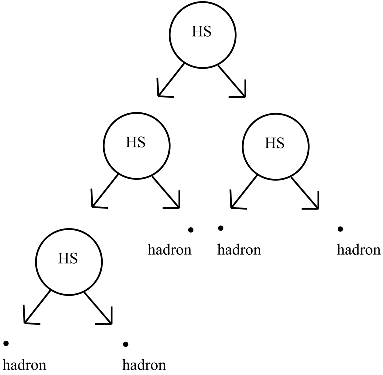


Decay Width for exotic HSs

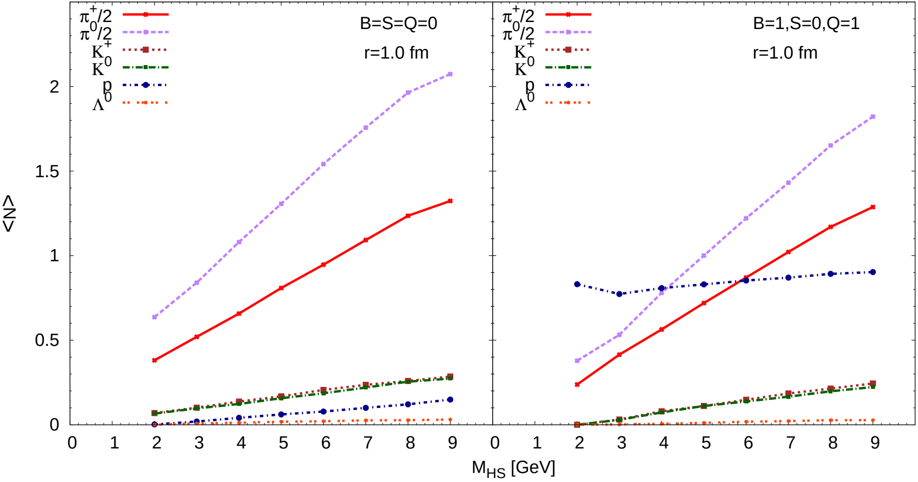


Hagedorn State Cascade Simulation

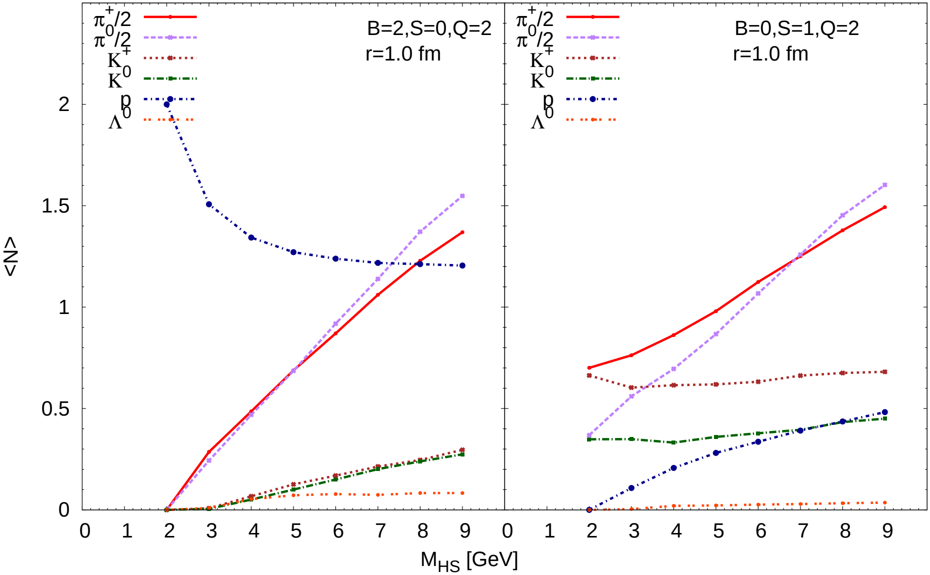
- **One** massive charged HS cascades down to hadrons (resonances)
- resonances further cascade down to stable hadrons (hadronic feeddown)



Multiplicities from mesonic (left) and baryonic (right) HS



Multiplicities from exotic HSs



Summary and Outlook

- in UrQMD not for all decays a back reaction is realized
- chem. equilibration times in UrQMD (box) too long
- HS as catalyst in rate equations leads to quick low chem. equilibration
- derivation of covariant formulated Hagedorn Spectrum $\tau(m)$
- presentation of full Hagedorn Spectra to show main properties
- presentation of full and detailed HS decay widths
- presentation of hadronic multiplicities from HS cascade simulations
- use of HS to lower chem. equilibration times in UrQMD
- impact of HS on η/s in UrQMD

Phase-Space Approach: S. Frautschi, 1971[4]

- no use of temperature introduction needed (microcanonical approach)
- two particle system in rest frame of HS with volume V

$$\rho(m) = \frac{V}{(2\pi)^3} \frac{1}{2!} \prod_{i=1}^{2} \int \mathrm{d}m_i \, \rho(m_i) \times \int \mathrm{d}^3 p_i \, \delta\left(\sum_{i=1}^{2} E_i - m\right) \delta^{(3)}\left(\sum_{i=1}^{2} \vec{p}_i\right) \qquad (1)$$

- non-covariant formulation in rest frame of HS
- mass degeneration considered by integrals over m_i

[4] doi=10.1103/PhysRevD.3.2821

Non-Covariant Formulation of Hagedorn Spectrum

- indroduce (inverse) Lorentz factors $\gamma^{-1} = m/E$

$$\tau(m) = \frac{R^3}{3\pi m} \iint \mathrm{d}m_1 \mathrm{d}m_2 \tau(m_1)\, \tau(m_2) \times \gamma_1^{-1} \gamma_2^{-1} E_1 E_2 p_{cm} \tag{2}$$

- neglect relativistic effects $\gamma_i \approx 1$ [5]

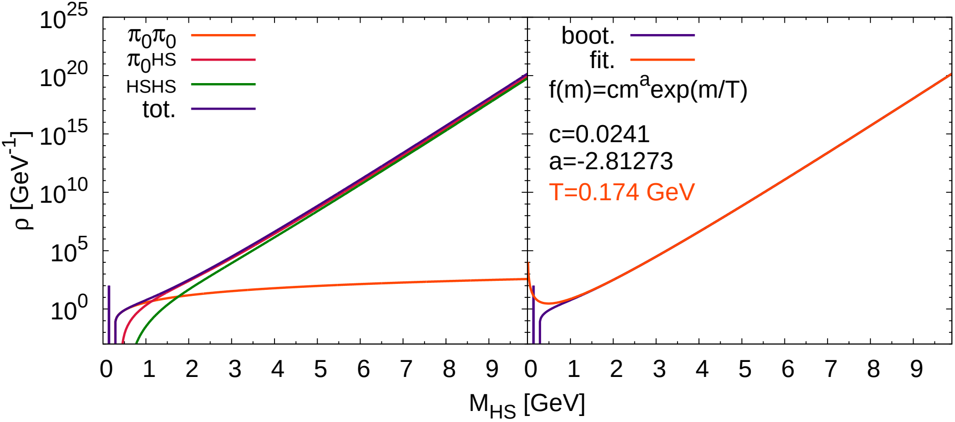
$$\rho(m) = \frac{R^3}{3\pi m} \iint \mathrm{d}m_1 \mathrm{d}m_2 \rho(m_1)\, \rho(m_2)\, E_1 E_2 p_{cm}$$

[5] S. C. Frautschi, Phys.Rev., D3:2821-2834, 1971

Toy Model: Hagedorn Spectrum

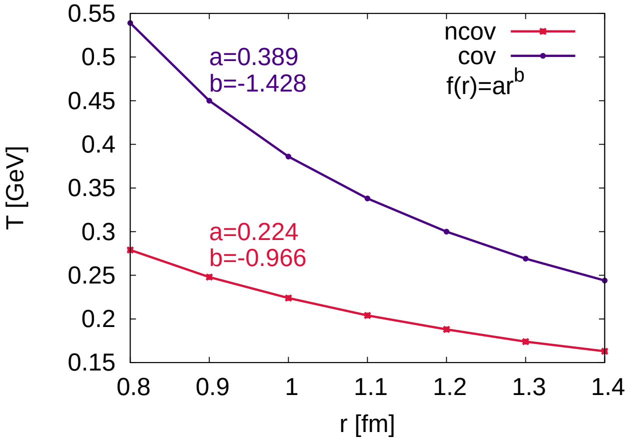
- π_0 as low mass input only

$$\rho\left(m\right)=\delta\left(m-m_{\pi_0}\right)$$



Volume Temperature Dependence

- fit Hagedorn temperatures for different radii



Toy Model Decay Width (Cov.)