Quarkyonic matter: theory and phenomenology

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What is "Quarkyonic matter"
Name introduced in L.MacLerran,R.Pisarski, NPA796 (2007) 83-100
300 Citations, 5 conferences, 1 wikipedia entry. So its a big deal! but
definitions found in the literature so far include, these and more...

- Coexistence between Confinement+pQCD (Mclerran,Pisarski,2007)
- Confinement+Chiral restoration (Fukushima,McLerran, 2008)
- Deconfinement+Chirally broken (Satz, Csernai,...)
- Chiral spiral inhomogeneities (Kojo,Pisarski,Tsvelik, 2009)
- Generic chirally inhomogeneous regions (Buballa et al)
- Condensation of "baryons" in 2-color QCD (Hands,Skullerud,Giudice)
All relevant for “high density low temperature” matter, produced in neutron stars or “low energy” uRHICs

- RHIC low energy scan
- SPS experiment NA61
- FAIR
- NICA
The issue: QCD at $\mu_Q \geq \Lambda_{QCD}, T < T_c$ is really not understood

**Hadronic or EFTs** ($\sigma$, NJL, PNJL etc): based under the assumption that $p_i - p_j \ll \Lambda_{\text{fundamental}}$

Only scale in QCD is $\Lambda_{\text{fundamental}} = \Lambda_{QCD}$, and $p_i - p_j \sim \mu_Q \sim \Lambda_{QCD}$

So EFT at $\mu_Q \simeq \Lambda_{QCD}$ means Taylor-expanding around 1! For operator $\hat{O}(x)$ (e.g. $q, P, ...$) Not guaranteed $\hat{O}^n \ll \hat{O}^{n-1}$ for any $n$

**Lattice QCD** has the sign problem, any expansion is good for $\mu_q \ll T$

**AdS/CFT** apart from the many unrealistic assumptions, classical Gauge dual depends on $N_c \rightarrow \infty$, on which more later

**Bottom-line:** wild speculations as reliable as “developed” models!
The only hierarchy that seems to be roughly correct is the large $N_c$ limit 

't Hooft, over 20 years ago, showed that provided a continuous limit exists where $N_c \to \infty$, $g_{YM} \to 0$, $g_{YM}^2 N_c \to \lambda$

Theory still strongly coupled and confining below $\Lambda_{QCD}$ but in this limit drastic simplifications are possible, as some observables $\sim N_c^2$, some $\sim N_c^0$ etc. Plugging in $N_c = 3 \to \mathcal{O}(10)$ hierarchy

- Quasi-particle picture of "light" mesons
- Quasi-classical structure of "heavy" baryons (Skyrme model)
- OZI rule, planar diagrams (strings, AdS/CFT) etc

all compatible with this hierarchy
The phase diagram: if deconfinement $\Leftrightarrow$ quark-hole loops “beat” gluon antiscreening...

If confinement set by coupling constant becoming “strong”, $N_c$ counting of $\beta$-function $\Rightarrow$ deconfinement line flattens, for deconfinement

$$\mu_Q \sim N_c^{-1/2} N_f^{-1/2} \Lambda_{QCD}$$

NB: Bag model intuition different, deconfinement $\mu_Q \sim \Lambda_{QCD}$. But, unless non-perturbative $N_c$ scaling of $\beta$-function different, $\sim \sqrt{N_c/N_f}$ baryons have to overlap for deconfinement!
The line separating "vacuum" from "dense nuclear matter" narrows, since baryon abundance in vacuum phase $\sim \exp(-N_c \Lambda_{QCD}/T) \to 0$.

Simply a consequence of baryons being heavy. Is this phase "interesting"?
If this picture is right, at $\mu_B \geq m_B$ something interesting happens: Phase confined but inter-quark distance in this phase $\sim N_c^{-1/3} \to 0$, asymptotic freedom in configuration space! Confined but quasi-free quarks below fermi surface and $P \sim N_c$ (quark-hole?) A new phase to look for at low energy, high density (Neutron stars, FAIR, NICA, etc.)
Can we exclude phase transitions in $N_f/N_c$?

In fact, phase transitions in $N_c$ are certain to happen!: Symmetry principles ($Z_N$) dictate that deconfinement is a phase transition, at $N_f \ll N_c$, Critical point in $N_f/N_c$, at $N_c \to \infty$, $\mu_B/N_c \sim \Lambda_{QCD}$, the ground state of nuclear matter is widely understood to be a crystal, not a liquid.
"quarkyonic matter" might be nuclear matter at $N_c \gg N_{\text{neighbours}}$. Or not as dependence on flavor, density not so clear. But $N_{\text{neighbors}}$ scaling motivates percolation.
You have a (regular or irregular) lattice of sites, which can be "on" and "off" (links "switched on", particles "in sites", etc), with probability $p$. Count adjacent sites $\langle N_{sites} \rangle$. When $p \simeq p_c$, $\langle N_{sites} \rangle \rightarrow \infty$

- second order transition ($\langle N_{sites} \rangle \equiv$ correlation), with critical behavior.

- $p_c(1D) = 1, p_c(2D) \sim O(0.5), p_c(3D) \sim O(0.2)$ (depends on $N_{neighbors}$). So "small" $\sim N_c^{-1}$ correction could trigger it.
an EFT of $\mu_Q \sim \Lambda_{QCD}, N_c \gg 1$ matter

**Baryons** are heavy and immobile “background”

**Quarks** are delocalized, since $\rho_{baryon}^{-1/3} \leq R_{baryon}$ Such delocalization compatible with confinement

An immediate physical analogy: conductor in QED, with baryons playing the role of atoms.
Such a “conducting phase”, not predicted by any EFT, could be the “surprise” we were looking for.

But remember, conductor insulator phase transition is governed by number of electrons in the “conducting band”.

However, since Quark/baryon $\sim N_c$, conductor/insulator transition in full $T - \mu_Q - N_c$ space!
$N_c \leq N_c^{\text{crit}}$ Deconfinement happens below percolation, ie percolation transition does not exist separately from deconfinement

$N_c \geq N_c^{\text{crit}}$ Percolation, deconfinement separate (Quarkyonic phase?)
A sliver of $n - \rho - N_c = 3$ space which is percolating but confined seems to be there, but... depends a lot on $N_f = 2$ or 3.

\[
\frac{dS}{dV} = \frac{dP}{dT} = \frac{P + \rho - \mu n}{T}
\]

And any diagrams of this type will give $T\mu_B$ contributions to pressure, and hence $dS/dV$. So need theory with confinement but free quarks! Physical example: Electrons in a metal
pQCD but not quite: the role of baryons

Unlike pQCD, quarkyonic matter’s “vacuum” is a classical dense baryon state. Treating baryons as mean fields will give a momentum-dependent form factor

\[
F(k) \sim 
\]

\( F(k) \) gives the F.T. of the baryonic gluon content. For the equation of state, it should just be a \( O(1) \) normalization factor, but for scattering processes it is a qualitative difference from naive QCD. Spin-color-flavor separation can ensure color neutrality with quark-like degrees of freedom. Baryons motion doesn't influence quarks up to \( N_c^{-1} \) corrections.
NB: Quarks delocalized by **tunneling**, not confinement

Gluons, antiquarks still confined, **only** processes with outgoing quarks allowed!

In particular, $gg \rightarrow s\bar{s}$ not expected. If these are responsible for strangeness enhancement, rate is comparable to hadronic rate
From EoS to dynamics: An EFT of percolating matter

In percolation regime, asymptotically free quark wavefunctions of different baryons can superimpose across large distances.

Thus, even if \( E_{\text{state}} \sim 1/L_{\text{baryon}} \sim N_c^0 \ll N_c^{1/2} \Lambda_{\text{QCD}} \) \( |_{\text{deconfinement}} \) degrees of freedom quark-like, so \( P \sim N_c, s \sim N_c \) (In the same way electrons in a metal have a much lower energy than ionization).

Periodic wavefunctions \( \Rightarrow \) leading component always \( p \geq \Lambda_{\text{QCD}}^{-1} \)
Modeling quarkyonic matter for RHIC/NICA/FAIR

Quark dynamics: QED, QCD + form factor
\[ F(k) \]

Baryon dynamics: uRQMD

\[ \gamma \rightarrow q \rightarrow X = \Psi(k) \Psi(k') M_{qq \rightarrow X} \]

Where \( M_{qq \rightarrow X} \) is the pQCD matrix element

\[ \Psi(k) \sim \exp \left( \sum_i [ikx_{0i}] F(k) \sim \exp \left[ ikx_{0i} - \frac{k^2}{\Lambda_{QCD}} \right] \right) \]

\( F(k) \) is the quark function inside a “classical” proton potential well (\( \sim \) Gaussian) and \( x_{0i} \) are the baryon locations. The latter is given by uRQMD.
Photon production in this approach

As antiquarks, gluons suppressed leading channel is quark Brehmsstrahlung.

\[ M^2 = L^2(k_1, k_2 \rightarrow k_3, k_4, p) + L^2(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_4) \]

\[ L^2 = -\frac{1}{4} e^2 \lambda^2 N_c^{-2}(k_2 - k_4)^{-4} Tr \left[ k_4 \gamma^\sigma k_2 \gamma^\rho \right] Tr \left[ k_3 Z^\mu_\sigma k_1 Z^\rho_\mu \right] \]

\[ Z^\beta_\alpha = \gamma_\alpha(k_1 - p)^{-1} \gamma^\beta + \gamma^\beta(k_3 + p)^{-1} \gamma_\alpha \]
\[
\frac{dN_\gamma}{dy p_T d\phi} = \int \frac{d^4k_1}{k_1^0} \frac{d^4k_2}{k_2^0} \frac{d^4k_3}{k_3^0} \frac{d^4k_4}{k_4^0} (M(k_1, k_2 \rightarrow k_3, k_4, p) \Psi(k_1)\Psi(k_2))^2
\]

- Quarkyonic quark wavefunctions
  \[
  \Psi(k) \sim \exp \sum_i [ikx_{0i}] F(k) \sim \exp \left[ ikx_{0i} - \frac{k^2}{\Lambda_{QCD}} \right], \text{uRQMD} \Rightarrow x_{0i}
  \]

- “Boosted quarkyonic”: Same wavefunction as above boosted to flow of a “random” baryon: An upper limit to \( N_c^{-1} \) backreaction (effect of baryon flow on quark wavefunction)

- “QGP” quark wavefunctions
  \[
  \Psi(k)\Psi(k') = \delta(k' - k) \exp \left[ -k_\mu u^\mu / T \right]
  \]
Rapidity distribution

\[ y \]

\[ dN/\gamma (A.U.) \]

-5-4 -3 -2 0 1 2 3 4 5

Very little difference. **NB** “static baryon” approximation breaks down away from mid-rapidity
Quarkyonic wavefunction similar to\textit{cold} quark gluon plasma, unrealistic temperatures. NB: “boosted quarkyonic” increases flow, but still cold!
Random distribution of quark wavefunctions quenches total $v_2$ but produces big fluctuation in event and $p_T$: oscillation frequency $\sim p_T \rho_B^{-1/3}$
"pure" quarkyonic effect, it is due to sensitivity of quark wavefunctions to baryon location. signature?
Dileptons potentially more direct probe but more complicated

Both quarks and holes needed

Sensitivity to equilibration

\[
\frac{dN_{l^+l^-}}{d^3P_{l^+l^-}d^3M_{l^+l^-}} = \int \frac{d^4k_1}{k_1^0} \frac{d^4k_2}{k_2^0} \frac{d^4k_3}{k_3^0} \frac{d^4k_4}{k_4^0} \langle \mathcal{M} (k_1, k_2 \rightarrow k_3, k_4, p) \Psi(k_1)\Psi(k_2) \rangle^2 \times \\
\times \delta \left( (k_1 - k_2)_\mu (k_1 - k_2)^\mu - M_{l^+l^-}^2 \right) \delta \left( (k_1 + k_2)_\mu (k_1 + k_2)^\mu - P^2 \right)
\]

But, extending the idea used for photons

\( \tilde{F}(M^2) \) connects baryon distribution to \( M^2 \) dilepton spectrum
If baryons were regular (pasta phase?) one could observe bloch waves! ("upside down resonance"?)
Event by event fireball structure not regular, but Collective structures exist in events flow profile (radial, longitudinal flow) and baryons have repulsive potential, so structures in 3D dilepton spectral function $Q_{z,r,\phi}$ bound to exist!
Conclusions

• “naive” hadronic EFT unreliable for regime at $\mu_Q \simeq \Lambda_{QCD}$

• Large $N_c$ expansion tells us quark degrees of freedom could appear even at confinement!

• On the other hand, not at all clear $\simeq \infty$

• Phenomenology of quarkyonic matter needed.

The best physicist in the USSR is Yakov Frenkel, who uses in his papers only quadratic equations. I am slightly worse, I sometimes use differential equations.

L.D. Landau, quoted in
BULLETIN OF THE American Mathematical Society
Volume 43, Number 4, October 2006, Pages 563–565
Spare slides
Confinement and quasi-free quarks: spin-color-flavor separation?

Confinement remains, so regions above $\sim 1\, fm$ cannot be color charged. (Same problem at $T \geq T_c$, but correlations required to maintain confinement can be $\left( N_c^0 \Lambda^{-1}_{QCD} \right) \ll s \left( T \geq T_c \right) \sim N_c^2 T^3$.

Spin-color-flavor separation can achieve this and maintain $N_c, N_f$ scaling! Pisarski, McLerran, Kojo, NPA843 (2010) 37-58 and subsequent works: implement this by 1D WZW model.

$$S = S_{2N_f}^{WZW} [h_{\text{color}}] + S_{N_c}^{WZW} [h_{\text{flavor}}]$$

which generalized spin-charge separation to $SU(N_f), SU(N_c)$. Modifications to $S_{2N_f}$ could localize color, maintain $\sim N_c$ degeneracy.

“Naively” WZW incompatible with percolation (1D), but could work as EFT in percolation regime. Work in progress.