

Quarkyonic matter:theory and phenomenology

Based on [PRL107:152301,2011](#) , 1204.3272 [Stefano Lottini](#) , [PRL111](#) ,
[012301](#) with [Sascha Vogel](#),[Bjorn Beauchle](#)

Also , 1006.2471 ([PRC](#)),with [Igor Mishustin](#) ,1105.0188 ([JHEP](#)) with
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What is "Quarkyonic matter"

Name introduced in [L.MacLerran,R.Pisarski, NPA796 \(2007\) 83-100](#)

300 Citations, 5 conferences, 1 wikipedia entry. So its a big deal! but definitions found in the literature so far include, [these and more...](#)

- Coexistence between Confinement+pQCD (Mclerran,Pisarski,2007)
- Confinement+Chiral restoration (Fukushima,McLerran, 2008)
- Deconfinement+Chirally broken (Satz, Csernai,...)
- Chiral spiral inhomogeneities (Kojo,Pisarski,Tsvetik, 2009)
- Generic chirally inhomogeneous regions (Buballa et al)
- Condensation of "baryons" in 2-color QCD (Hands,Skullerud,Giudice)

All relevant for “high density low temperature” matter, produced in neutron stars or “low energy” uRHICs

- RHIC low energy scan
- SPS experiment NA61
- FAIR
- NICA

The issue: QCD at $\mu_Q \geq \Lambda_{QCD}, T < T_c$ is really not understood

Hadronic or EFTs ($\sigma, NJL, PNJL$ etc): based under the assumption that $p_i - p_j \ll \Lambda_{fundamental}$
Only scale in QCD is $\Lambda_{fundamental} = \Lambda_{QCD}$, and $p_i - p_j \sim \mu_Q \sim \Lambda_{QCD}$

So EFT at $\mu_Q \simeq \Lambda_{QCD}$ means **Taylor-expanding around 1!** For operator $\hat{O}(x)$ (e.g. q, P, \dots) Not guaranteed $\hat{O}^n \ll \hat{O}^{n-1}$ for any n

Lattice QCD has the sign problem, any expansion is good for $\mu_q \ll T$

AdS/CFT apart from the many unrealistic assumptions, classical Gauge dual depends on $N_c \rightarrow \infty$, on which **more later**

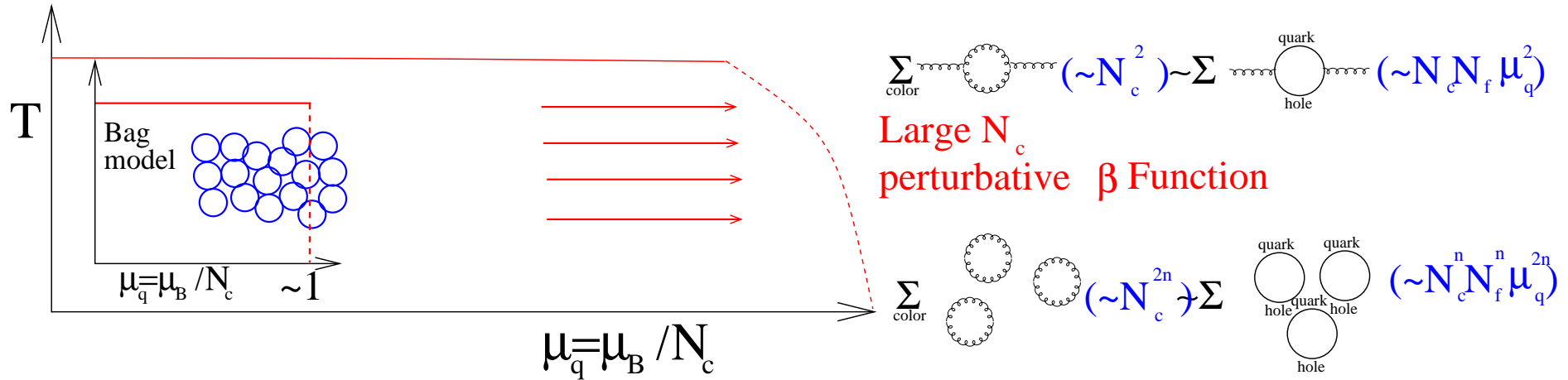
Bottom-line: wild speculations as reliable as “developed” models!

The only hierarchy that seems to be roughly correct is the large N_c limit
't Hooft, over 20 years ago, showed that provided a continuous limit exists
where $N_c \rightarrow \infty, g_{YM} \rightarrow 0, g_{YM}^2 N_c \rightarrow \lambda$
Theory still strongly coupled and confining below Λ_{QCD} **but** in this limit
drastic simplifications are possible, as some observables $\sim N_c^2$, some $\sim N_c^0$
etc. Plugging in $N_c = 3 \rightarrow \mathcal{O}(10)$ hierarchy

- Quasi-particle picture of “light” mesons
- Quasi-classical structure of “heavy” baryons (Skyrme model)
- OZI rule, planar diagrams (strings, AdS/CFT) etc

all compatible with this hierarchy

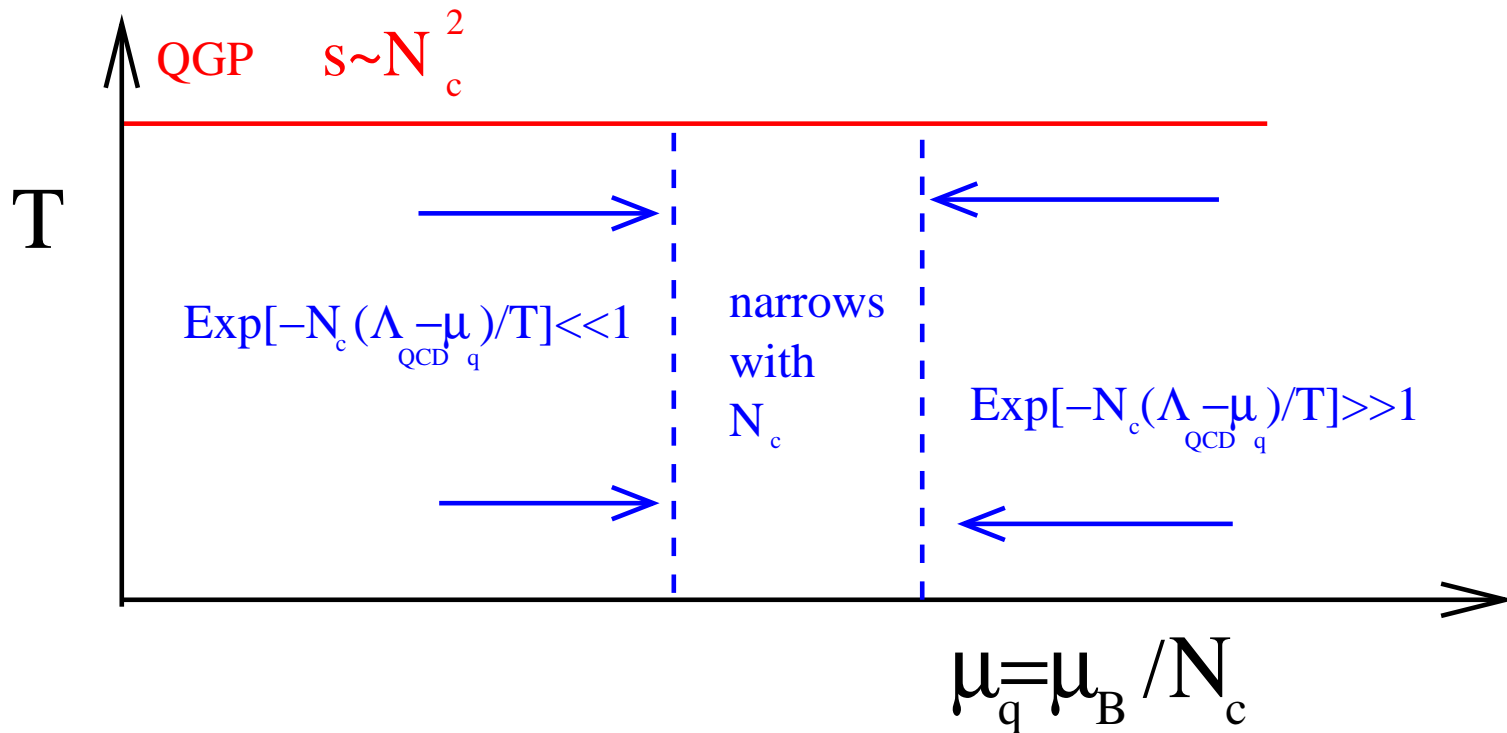
The phase diagram: if deconfinement \Leftrightarrow quark-hole loops “beat” gluon antiscreening...



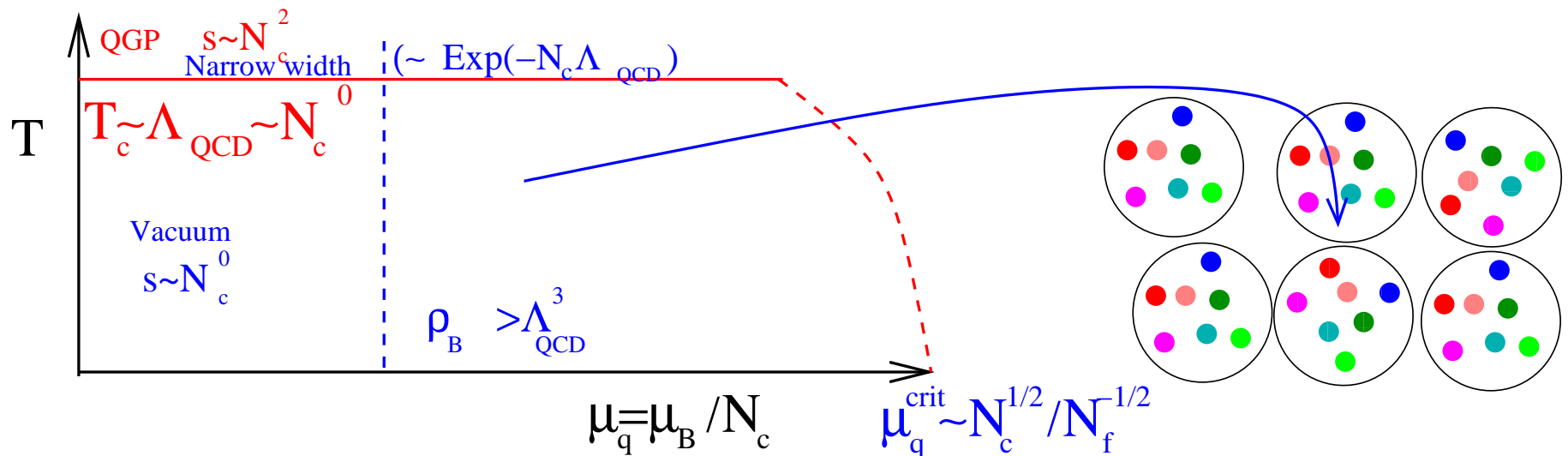
If confinement set by coupling constant becoming “strong” , N_c counting of β -function \Rightarrow deconfinement line flattens, for deconfinement

$$\mu_Q \sim N_c^{1/2} N_f^{-1/2} \Lambda_{QCD}$$

NB: Bag model intuition different , deconfinement $\mu_Q \sim \Lambda_{QCD}$. But, unless non-perturbative N_c scaling of β - function different, $\sim \sqrt{N_c/N_f}$ baryons have to overlap for deconfinement!



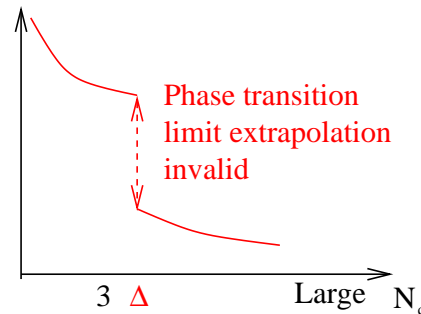
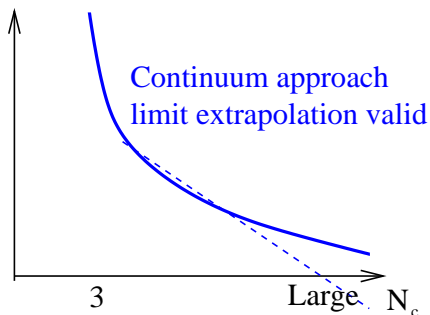
line separating "vacuum" from "dense nuclear matter" narrows , since
 baryon abundance in vacuum phase $\sim \exp(-N_c \Lambda_{\text{QCD}} / T) \rightarrow 0$
 Simply a consequence of baryons being heavy. Is this phase "interesting"?



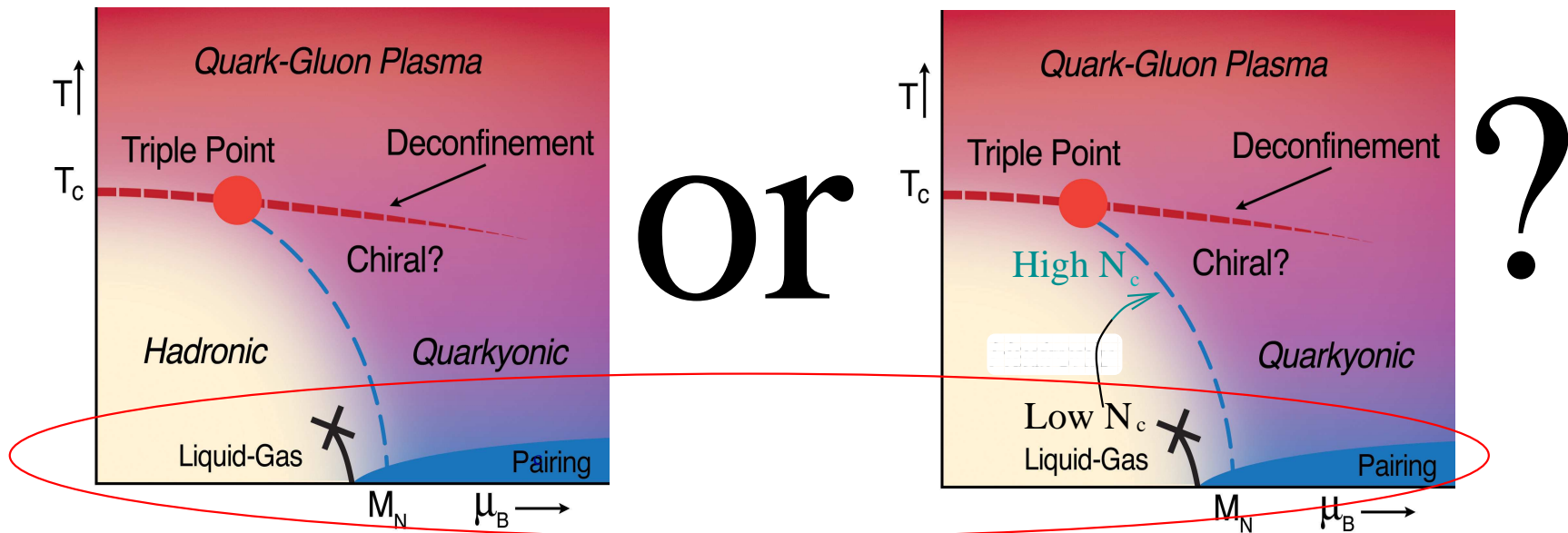
If this picture is right, at $\mu_B \geq m_B$ something interesting happens: Phase confined but inter-quark distance in this phase $\sim N_c^{-1/3} \rightarrow 0$, **asymptotic freedom in configuration space!** Confined but quasi-free quarks below fermi surface and $P \sim N_c$ (quark-hole?) A new phase to look for at low energy, high density (Neutron stars, FAIR, NICA, etc.)

Quantity	$N_c \rightarrow \infty$	QCD
$E_{Nucleus}^{binding}$	$N_c \Lambda_{QCD}$	$\ll \Lambda_{QCD}, m_\pi$
$\Delta E_{spin-flip}$	$\sim \Lambda_{QCD}/N_c$	$\sim \Lambda_{QCD}$
Ground state	Crystal	Liquid

Can we exclude phase transitions in N_f/N_c ?

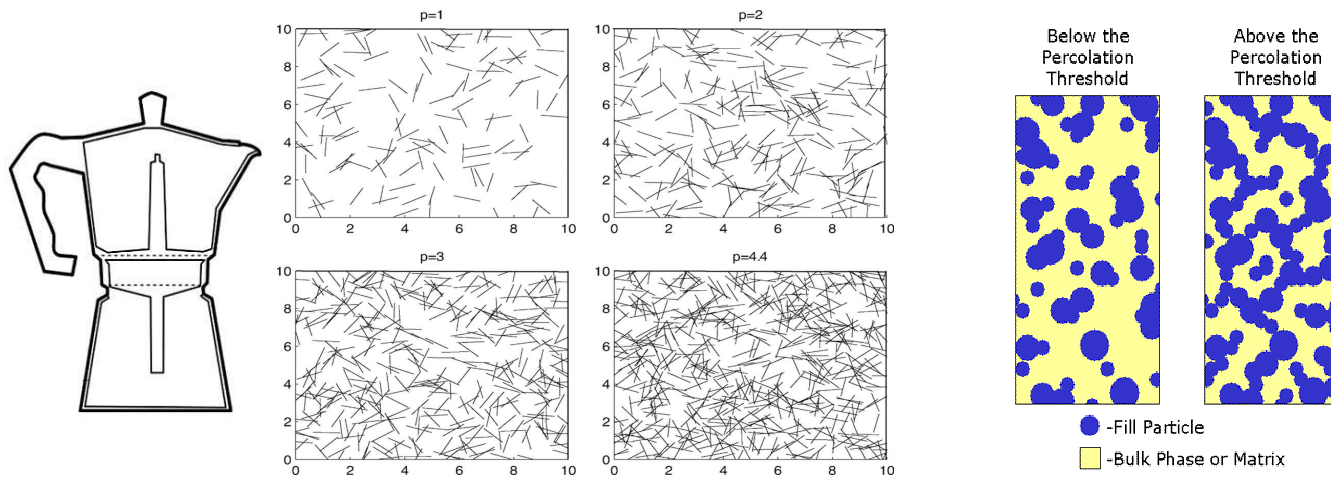


In fact, phase transitions in N_c are certain to happen! : Symmetry principles (Z_N) dictate that deconfinement is a phase transition, at $N_f \ll N_c$, Critical point in N_f/N_c , at $N_c \rightarrow \infty$, $\mu_B/N_c \sim \Lambda_{QCD}$, the ground state of nuclear matter is widely understood to be a crystal, not a liquid.



GT, I. Mishustin, PRC82 055202 “quarkyonic matter” might be nuclear matter at $N_c \gg N_{neighbours}$. **Or not** as dependence on flavor, density not so clear. **But** $N_{neighbours}$ scaling motivates percolation.

You have a (regular or irregular) lattice of sites, which can be "on" and "off" (links "switched on", particles "in sites", etc), with probability p . Count adjacent sites $\langle N_{sites} \rangle$. When $p \simeq p_c$, $\langle N_{sites} \rangle \rightarrow \infty$



- second order transition ($\langle N_{sites} \rangle \equiv$ correlation), with critical behavior.
- $p_c(1D) = 1, p_c(2D) \sim \mathcal{O}(0.5), p_c(3D) \sim \mathcal{O}(0.2)$ (depends on $N_{neighbors}$). So "small" $\sim N_c^{-1}$ correction could trigger it.

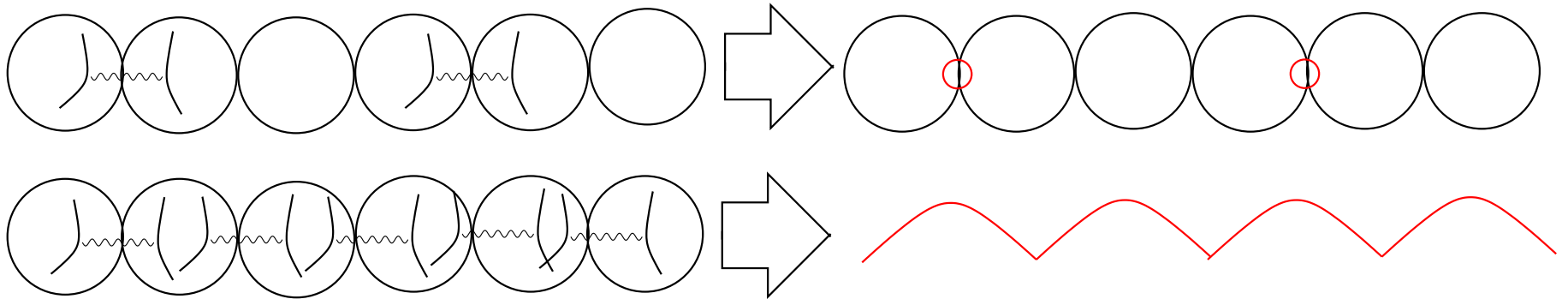
an EFT of $\mu_Q \sim \Lambda_{QCD}, N_c \gg 1$ matter

Baryons are heavy and immobile “background”

Quarks are delocalized, since $\rho_{baryon}^{-1/3} \leq R_{baryon}$ Such delocalization compatible with confinement

An immediate physical analogy: conductor in QED, with baryons playing the role of atoms.

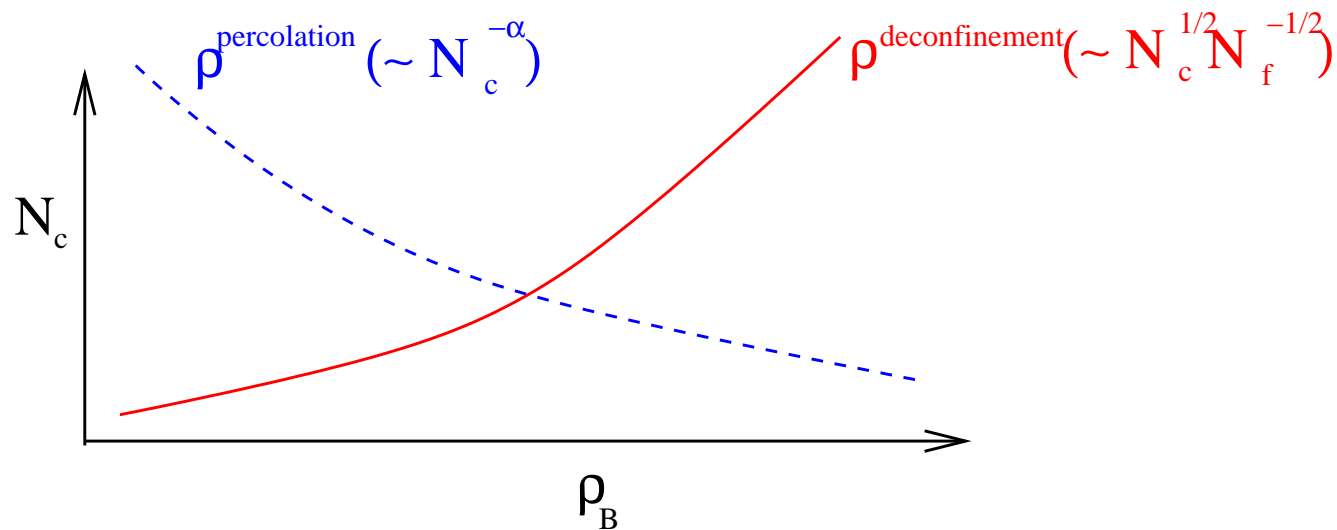
Such a “conducting phase”, not predicted by any EFT, could be the “surprise” we were looking for



But remember, conductor insulator phase transition is governed by number of electrons in the “conducting band”.

However , since Quark/baryon $\sim N_c$, conductor/insulator transition in full $T - \mu_Q - N_c$ space!

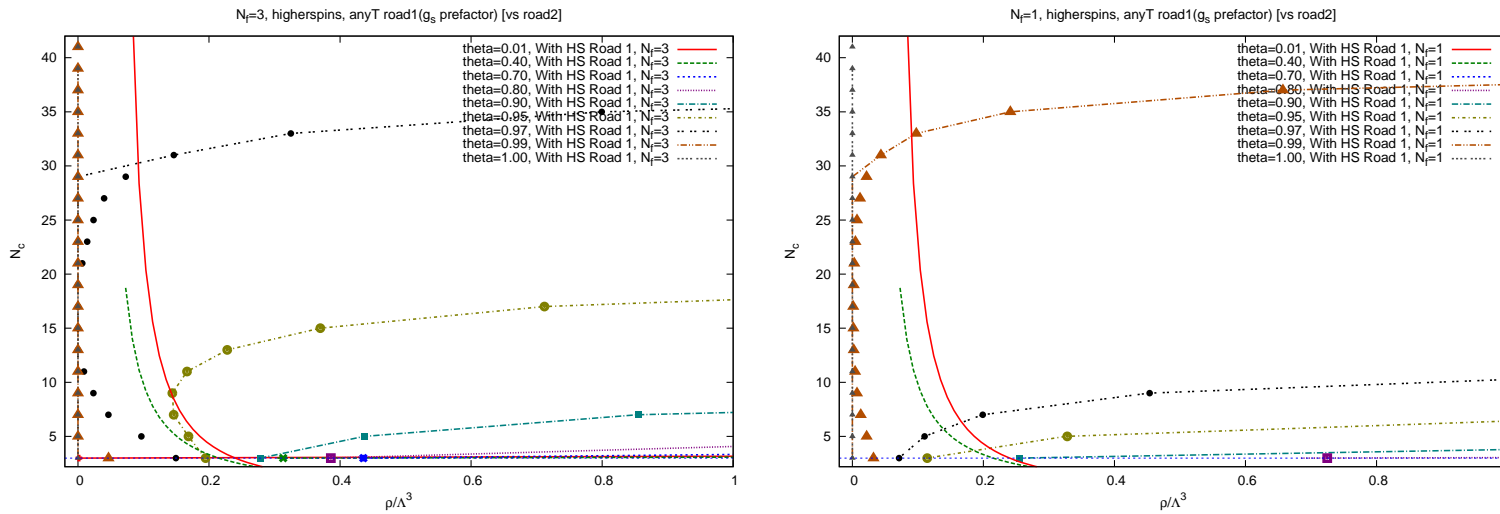
Percolation and deconfinement



$N_c \leq N_c^{\text{crit}}$ Deconfinement happens below percolation, ie percolation transition does not exist separately from deconfinement

$N_c \geq N_c^{\text{crit}}$ Percolation, deconfinement separate (Quarkyonic phase?)

Numerical simulation with percolation model

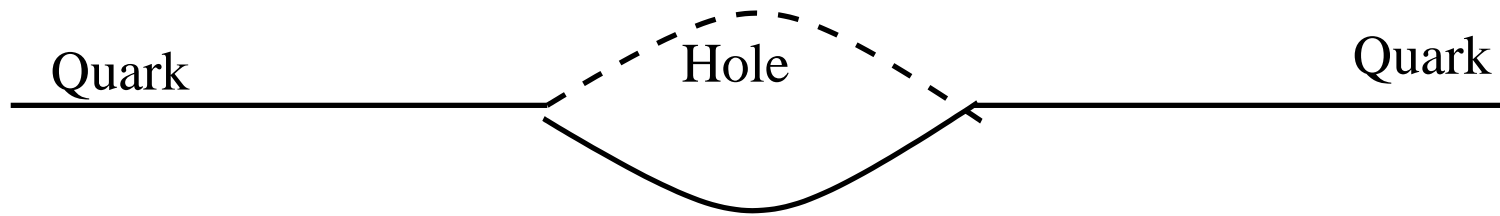


A sliver of $n - \rho - N_c = 3$ space which is percolating but confined seems to be there, **but...** depends a lot on $N_f = 2$ or 3.

Observing such a percolating phase: What does it look like?

How do confinement and free quarks coexist? McLerran, Pisarski, Kojo :
quark Fermi surface and baryonic excitations. But..

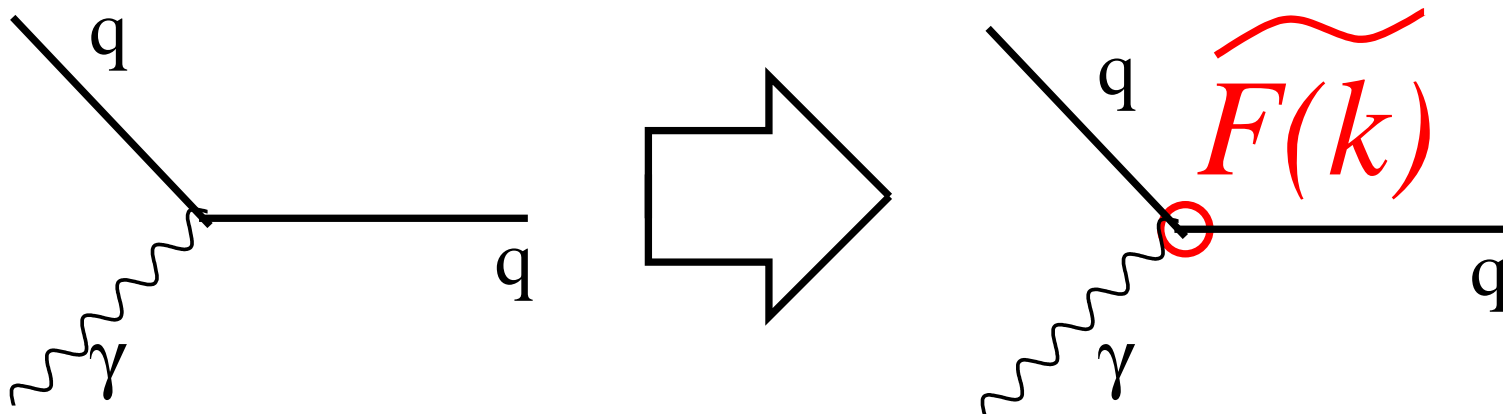
$$\frac{dS}{dV} = \frac{dP}{dT} = \frac{P + \rho - \mu n}{T}$$



And any diagrams of this type will give $T\mu_B$ contributions to pressure, and hence dS/dV . So need theory with confinement but free quarks! Physical example: Electrons in a metal

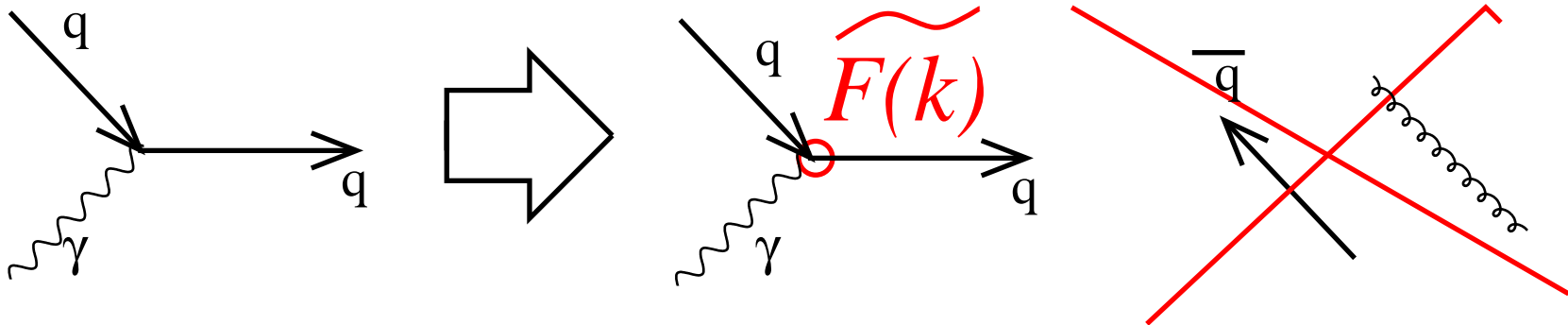
pQCD but not quite: the role of baryons

Unlike pQCD, quarkyonic matter's "vacuum" is a classical dense baryon state. Treating baryons as mean fields will give a momentum-dependent form factor



$F(k)$ gives the F.T. of the baryonic gluon content. For the equation of state, it should just be a $\mathcal{O}(1)$ normalization factor, but for scattering processes it is a qualitative difference from naive QCD. **Spin-color-flavor separation** can ensure color neutrality with quark-like degrees of freedom. **Baryons motion doesn't influence quarks up to N_c^{-1} corrections**

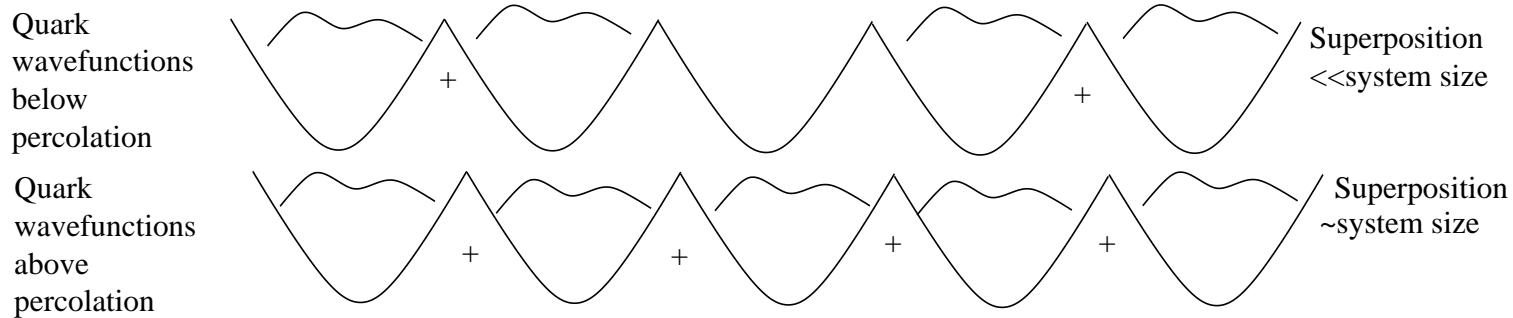
NB: Quarks delocalized by tunneling, not confinement



Gluons, antiquarks still confined, only processes with outgoing quarks allowed!

In particular, $gg \rightarrow s\bar{s}$ not expected. If these are responsible for strangeness enhancement, rate is comparable to hadronic rate

From EoS to dynamics: An EFT of percolating matter

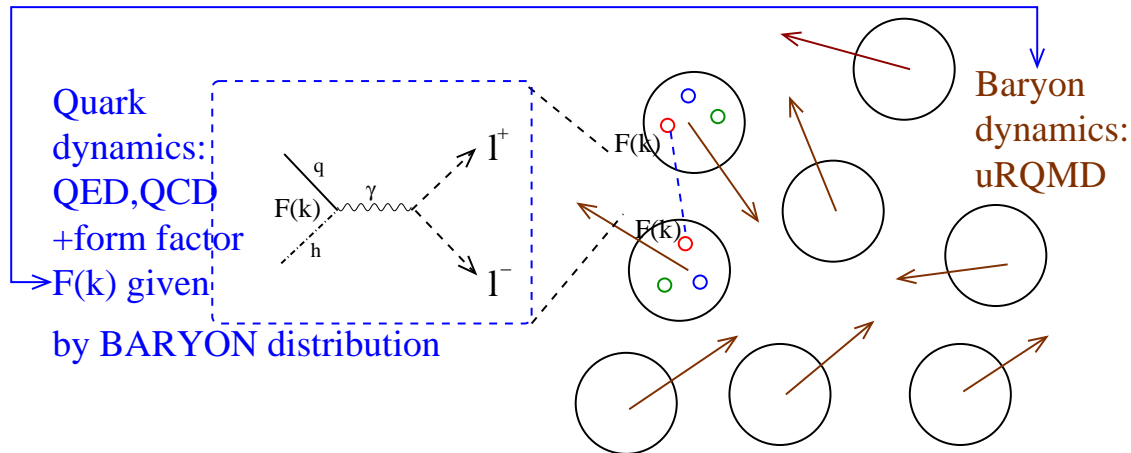


In percolation regime, asymptotically free quark wavefunctions of different baryons can superimpose across large distances.

Thus, even if $E_{state} \sim 1/L_{baryon} \sim N_c^0 \ll N_c^{1/2} \Lambda_{QCD} \Big|_{deconfinement}$ degrees of freedom quark-like, so $P \sim N_c, s \sim N_c$ (In the same way electrons in a metal have a much lower energy than ionization).

Periodic wavefunctions \Rightarrow leading component always $p \geq \Lambda_{QCD}^{-1}$

Modeling quarkyonic matter for RHIC/NICA/FAIR

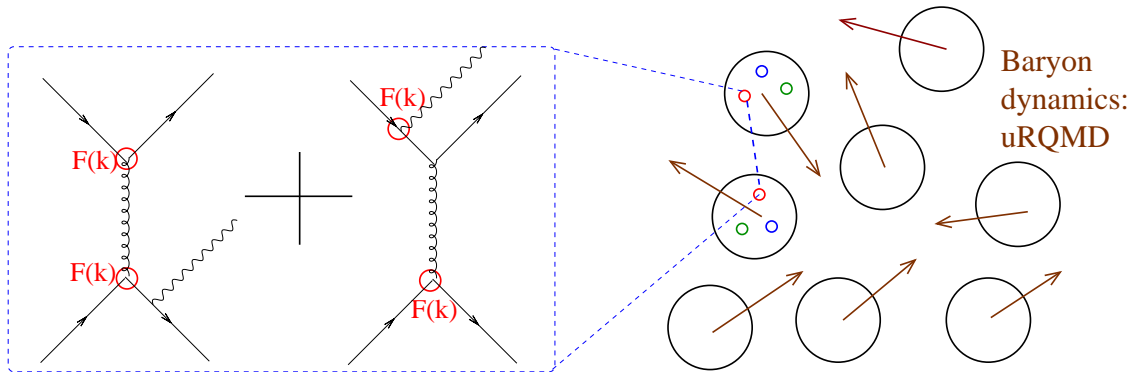


$R_{qq \rightarrow X} = \Psi(k)\Psi(k')M_{qq \rightarrow X}^2$ Where $M_{qq \rightarrow X}$ is the pQCD matrix element

$$\Psi(k) \sim \exp \sum_i [ikx_{0i}] F(k) \sim \exp \left[ikx_{0i} - \frac{k^2}{\Lambda_{QCD}} \right]$$

$F(k)$ is the quark function inside a “classical” proton potential well (\sim Gaussian) and x_{0i} are the baryon locations. The latter is given by uRQMD.

Photon production in this approach



As antiquarks, gluons suppressed leading channel is quark Brehmsstrahlung.

$$\mathcal{M}^2 = L^2(k_1, k_2 \rightarrow k_3, k_4, p) + L^2(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_4)$$

$$L^2 = -\frac{1}{4}e^2\lambda^2 N_c^{-2}(k_2 - k_4)^{-4} \text{Tr} [k_4 \gamma^\sigma k_2 \gamma_\rho] \text{Tr} [k_3 Z_\sigma^\mu k_1 Z_\mu^\rho]$$

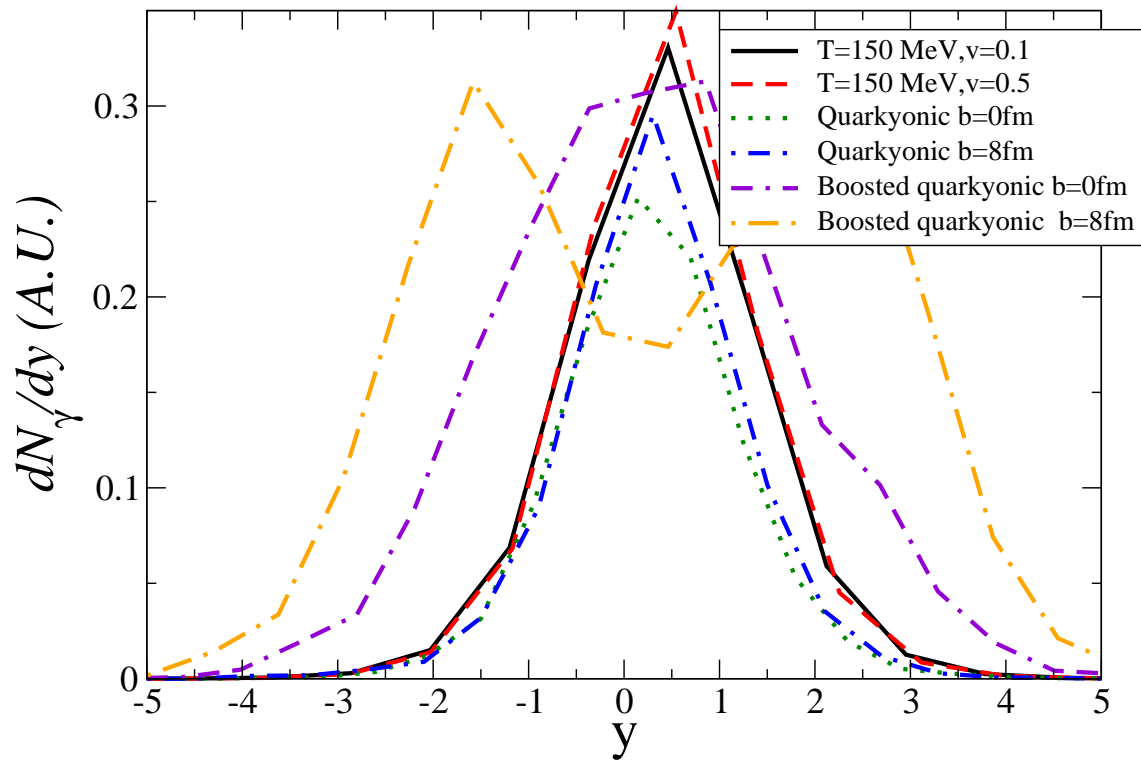
$$Z_\alpha^\beta = \gamma_\alpha (k_1 - p)^{-1} \gamma^\beta + \gamma^\beta (k_3 + p)^{-1} \gamma_\alpha$$

$$\frac{dN_\gamma}{dy p_T dp_T d\phi} = \int \frac{d^4 k_1}{k_1^0} \frac{d^4 k_2}{k_2^0} \frac{d^4 k_3}{k_3^0} \frac{d^4 k_4}{k_4^0} (\mathcal{M}(k_1, k_2 \rightarrow k_3, k_4, p) \Psi(k_1) \Psi(k_2))^2$$

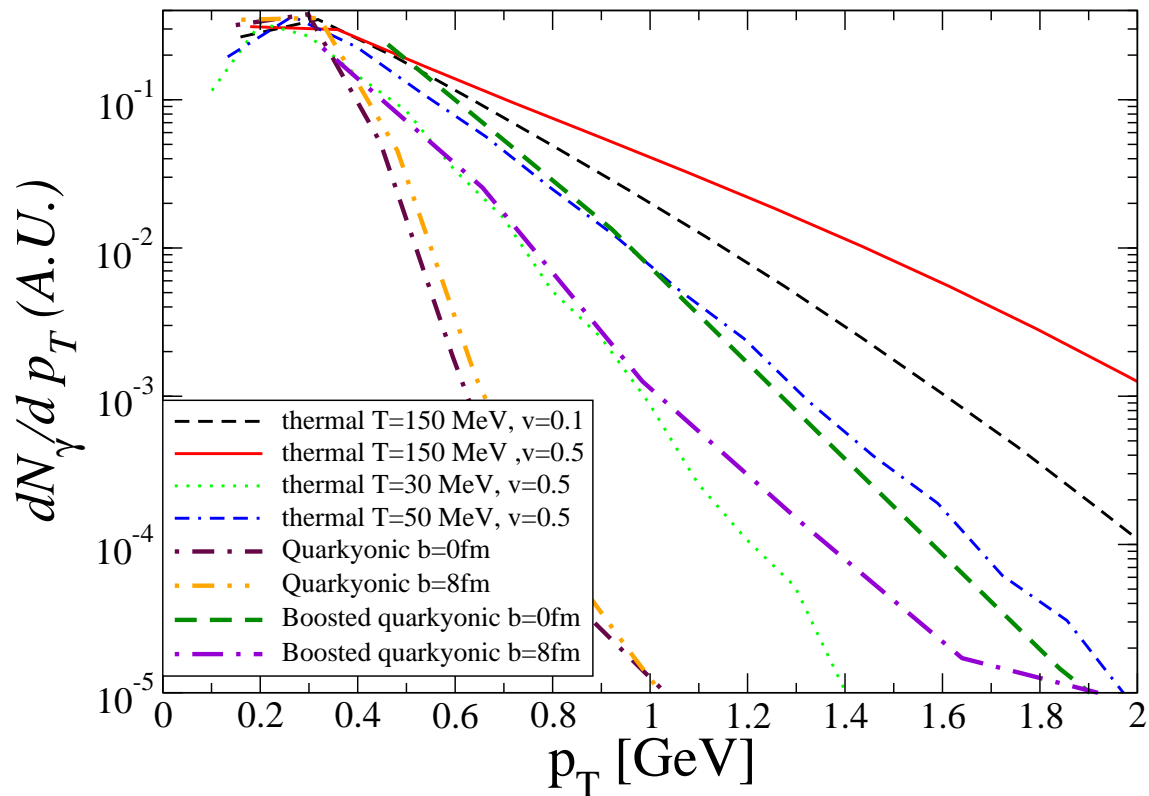
- Quarkyonic quark wavefunctions

$$\Psi(k) \sim \exp \sum_i [ikx_{0i}] F(k) \sim \exp \left[ikx_{0i} - \frac{k^2}{\Lambda_{QCD}} \right], uRQMD \Rightarrow x_{0i}$$

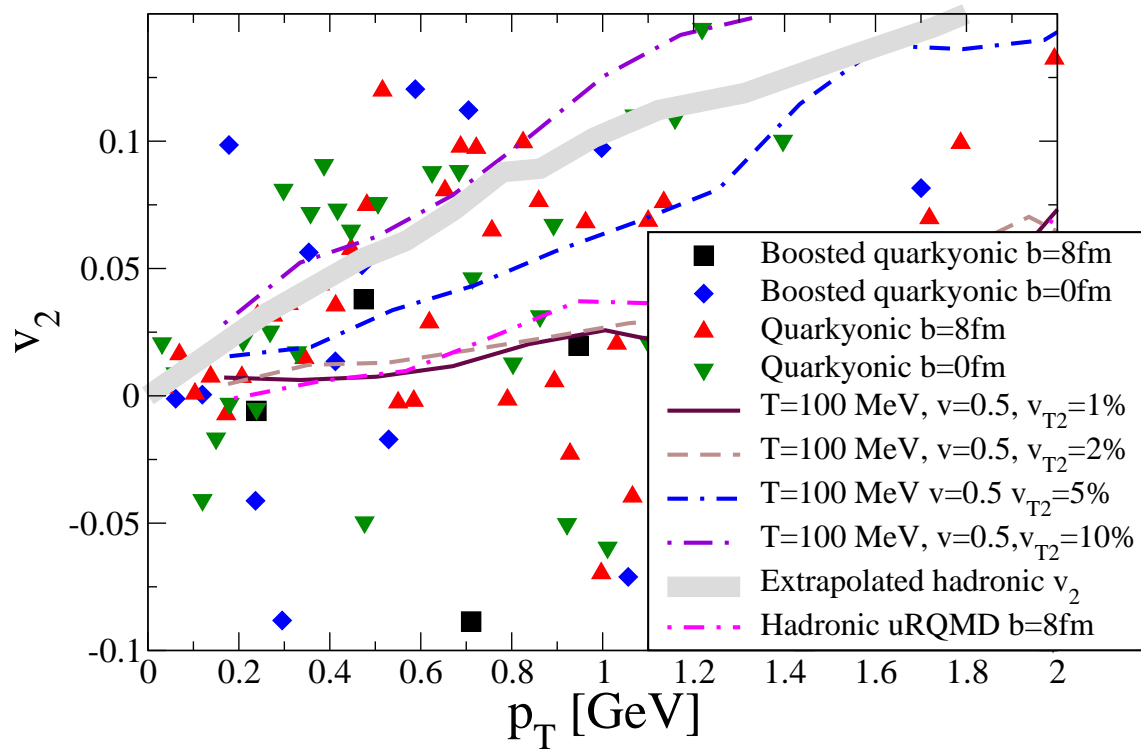
- “Boosted quarkyonic”: Same wavefunction as above boosted to flow of a “random” baryon: An upper limit to N_c^{-1} backreaction (effect of baryon flow on quark wavefunction)
- “QGP” quark wavefunctions $\Psi(k)\Psi(k') = \delta(k' - k) \exp[-k_\mu u^\mu / T]$



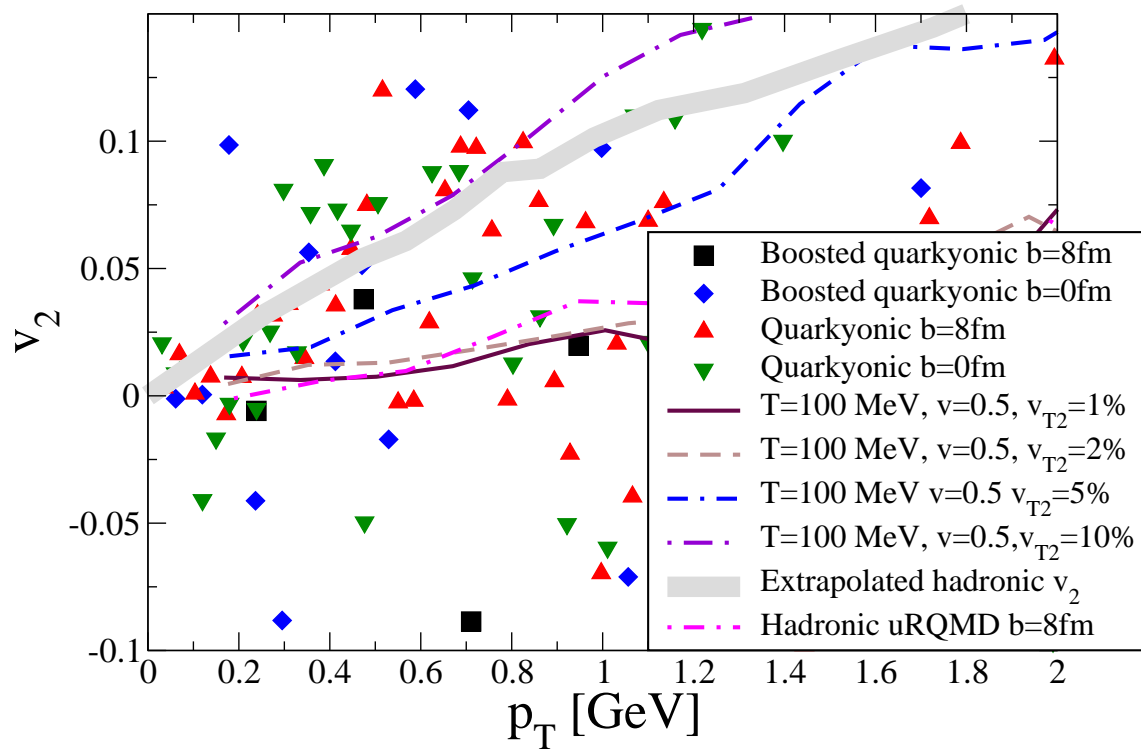
Very little difference. **NB** “static baryon” approximation breaks down away from mid-rapidity



Quarkyonic wavefunction similar to cold quark gluon plasma, unrealistic temperatures. NB: “boosted quarkyonic” increases flow, but still cold!

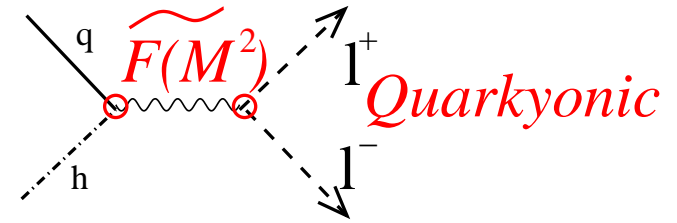
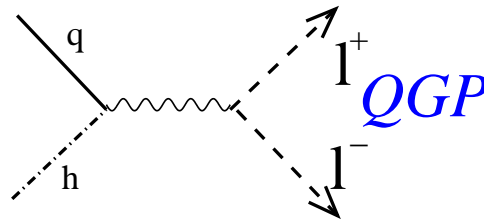
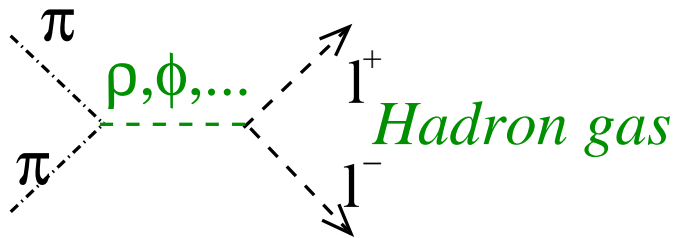


Random distribution of quark wavefunctions quenches total v_2 but produces big fluctuation in event **and** p_T : oscillation frequency $\sim p_T \rho_B^{-1/3}$



“pure” quarkyonic effect, it is due to sensitivity of quark wavefunctions to baryon location. signature?

dileptons potentially more direct probe but more complicated
 Both quarks and holes needed Sensitivity to equilibration

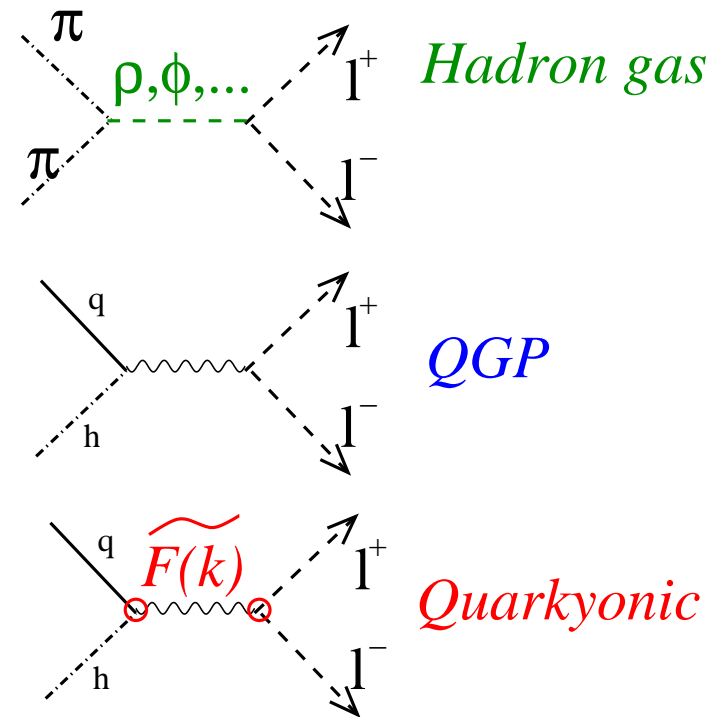
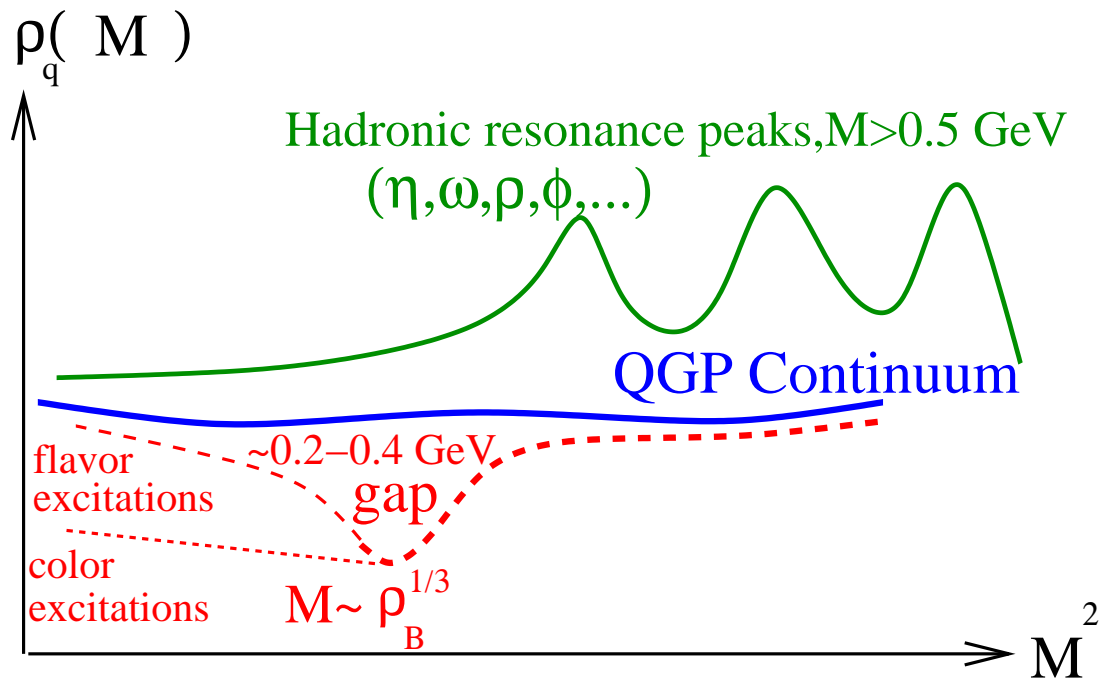


$$\frac{dN_{l+l-}}{d^3P_{l+l-}d^3M_{l+l-}} = \int \frac{d^4k_1}{k_1^0} \frac{d^4k_2}{k_2^0} \frac{d^4k_3}{k_3^0} \frac{d^4k_4}{k_4^0} (\mathcal{M}(k_1, k_2 \rightarrow k_3, k_4, p) \Psi(k_1) \Psi(k_2))^2 \times$$

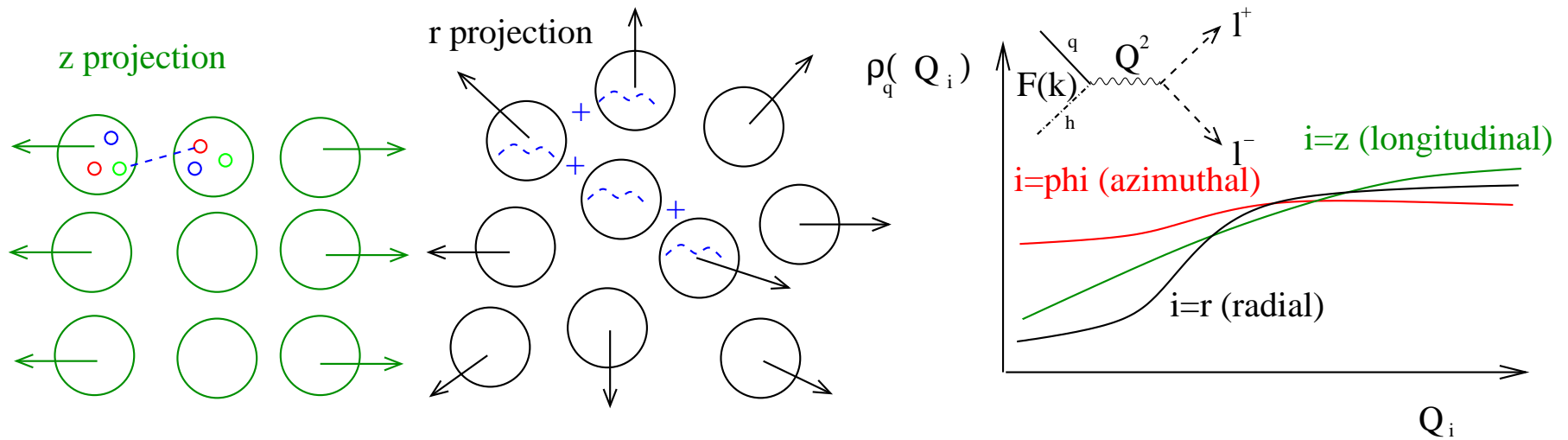
$$\times \delta((k_1 - k_2)_\mu (k_1 - k_2)^\mu - M_{l+l-}^2) \delta((k_1 + k_2)_\mu (k_1 + k_2)^\mu - P^2)$$

But , extending the idea used for photons

$\tilde{F}(M^2)$ connects baryon distribution to M^2 dilepton spectrum



If baryons were regular (pasta phase?) one could observe bloch waves!
 (“upside down resonance”?)



Event by event fireball structure not regular, but Collective structures exist in events flow profile (radial, longitudinal flow) and baryons have repulsive potential, so structures in 3D dilepton spectral function $Q_{z,r,\phi}$ bound to exist!

Conclusions

- “naive” hadronic EFT unreliable for regime at $\mu_Q \simeq \Lambda_{QCD}$
- Large N_c expansion tells us quark degrees of freedom could appear **even at confinement!**
- On the other hand, not at all clear $\simeq \infty$
- Phenomenology of quarkyonic matter needed.



The best physicist in the USSR is Yakov Frenkel, who uses in his papers only quadratic equations.

I am slightly worse, I sometimes use differential equations.

L.D.Landau, quoted in

BULLETIN OF THE American Mathematical Society
Volume 43, Number 4, October 2006, Pages 563–565

Spare slides

Confinement and quasi-free quarks: spin-color-flavor separation?

Confinement remains, so regions above $\sim 1 fm$ can-not be color charged.
(Same problem at $T \geq T_c$, but correlations required to maintain confinement can be $\left(N_c^0 \Lambda_{QCD}^{-1}\right) \ll s(T \geq T_c) \sim N_c^2 T^3$

Spin-color-flavor separation can achieve this and maintain N_c, N_f scaling!
Pisarski, McLerran, Kojo, NPA843 (2010) 37-58 and subsequent works:
implement this by 1D WZW model.

$$S = S_{2N_f}^{WZW} [h_{\text{color}}] + S_{N_c}^{WZW} [h_{\text{flavor}}]$$

which generalized spin-charge separation to $SU(N_f), SU(N_c)$.

Modifications to S_{2N_f} could localize color, maintain $\sim N_c$ degeneracy.

“Naively” WZW incompatible with percolation (1D), but could work as EFT in percolation regime. **Work in progress**.