Gluon radiation by heavy quarks

Joerg Aichelin, and Pol Gossiaux, and Thierry Gousset (SubaTech)

July 23, 2013
Energy loss

- quark and gluon energy loss is a central topic in ultra-relativistic heavy ion collisions

- collisional and radiative

- light quark and heavy quark
Heavy quark transport

- understanding transport of heavy quarks in the medium produced in heavy ion collisions \(\rightarrow\) MC@HQ program

- collisional energy loss implemented using pQCD-type \(Q - q\) elastic scattering
Heavy quark transport

- understanding transport of heavy quarks in the medium produced in heavy ion collisions → MC@HQ program
- collisional energy loss implemented using pQCD-type $Q - q$ elastic scattering
- including radiation ↔ Gunion-Bertsch who proposed a pQCD model for light quark radiation phenomenology in high-energy collisions
Heavy quark transport

- understanding transport of heavy quarks in the medium produced in heavy ion collisions $\rightarrow$ MC@HQ program
- collisional energy loss implemented using pQCD-type $Q - q$ elastic scattering
- including radiation $\leftrightarrow$ Gunion-Bertsch who proposed a pQCD model for light quark radiation phenomenology in high-energy collisions

will consider relativistic HQ, but intermediate energy where coherence effects are not dominant

- extension of the Gunion-Bertsch model to heavy quarks
- investigating the influence of a finite energy
\[ \int x \frac{d\sigma}{dx} \, dx \]

Energy loss
HQ transport
\[ \int x \, d\sigma \]

Kinematics
High-energy
Screening
Hard scattering
Soft scattering
Finite energy

Outlook

\[ \int \frac{d\sigma}{dx} \, dx \sim \frac{1}{\rho E_{\text{beam}}} \frac{dE_{\text{rad}}}{dz} \]
$\int x \frac{d\sigma}{dx} \, dx$

Energy loss
HQ transport
$\int x \, d\sigma$
$x \, d\sigma / dx$
Kinematics
High-energy
Screening
Hard scattering
Soft scattering
Finite energy
Outlook

$E_c \sim 10 \text{ GeV}$
$E_b \sim 40 \text{ GeV}$
\[ \int x \frac{d\sigma}{d\sigma} \, dx \]

asymptotic behavior \( \propto \frac{1}{\mu m_Q} \)

- suppression of heavy quark radiation \( \propto \frac{1}{m_Q} \)
- sensitivity to the details of screening
Energy loss
HQ transport
\[ \int x d\sigma \]
\[ x \frac{d\sigma}{dx} \]
Kinematics
High-energy
Screening
Hard scattering
Soft scattering
Finite energy

Outlook

\[
\frac{1}{\mu m_Q} \leftrightarrow \int x \frac{d\sigma}{dx} \quad \text{receives contributions from both below} \]

\[
x_M = \mu / m_Q \quad \text{('hard' collisions) and above ('soft' collisions)}
\]
\[ x \frac{d\sigma}{dx} \]

\[ \frac{1}{\mu m_Q} \left\{ \int x \frac{d\sigma}{dx} \, dx \right\} \] receives contributions from both below and above (‘hard’ collisions) and above (‘soft’ collisions).
\[ \frac{1}{\mu m_Q} \left( x \frac{d\sigma}{dx} \right) \rightarrow \int x \frac{d\sigma}{dx} dx \text{ receives contributions from both below} \]

\[ x_M = \mu / m_Q \] (‘hard’ collisions) and above (‘soft’ collisions).
\[ x \frac{d\sigma}{dx} \]

\[ \frac{1}{\mu m_Q} \left\langle \int x \frac{d\sigma}{dx} \right\rangle \]

receives contributions from both below and above (‘hard’ collisions) and above (‘soft’ collisions).

\[ x_M = \frac{\mu}{m_Q} \]

\[ \sqrt{s} = \infty \]

\[ \sqrt{s} = 10 \text{ GeV} \]

\[ \sqrt{s} = 7 \text{ GeV} \]

\[ \sqrt{s} = 4 \text{ GeV} \]

\[ \sqrt{s} = 7 \text{ GeV} \]

\[ \sqrt{s} = 10 \text{ GeV} \]

\[ \sqrt{s} = \infty \]

\[ \sqrt{s} = 10 \text{ GeV} \]

\[ \sqrt{s} = 7 \text{ GeV} \]

\[ \sqrt{s} = 4 \text{ GeV} \]

→ some details on the origin of radiation
**Kinematics**

$p, q$ two light-like vectors such that $2p \cdot q = s - m_Q^2$

$Q: \quad P = p + \frac{m_Q^2}{s - m_Q^2} q$

$g: \quad k = x p + y q + \vec{k}_\perp$

$q: \quad$ momentum transfer $q - q' = \ell, t = \ell^2$
High-energy

At large $s$

$$\frac{d\sigma}{dxd^2k_\perp d^2\ell_\perp} \approx \frac{d\sigma_{el}}{d^2\ell_\perp} \times P_g(x, \vec{k}_\perp, \vec{\ell}_\perp)$$

high-energy is when $s - m_Q^2 \gg |t|, k_\perp^2$

where $s$-dependence disappears from the cross section (either differential or integrated)

- since $\frac{d\sigma_{el}}{d^2\ell_\perp} \propto \frac{1}{t^2}$ at large $|t|$

- and $P_g \propto \frac{1}{k_\perp^4}$ at large $k_\perp$
Debye screening effect

at small $t \to$ will use prescription

$$\frac{d\sigma_{el}}{d^2\ell_\perp} \propto \frac{1}{t^2} \to \frac{1}{(t - \mu^2)^2}$$

with $\mu$ related to $m_D \to$ natural scale for $\ell_\perp \sim \mu$
\[ \ell_{\perp} \gg x m_Q \quad \text{—— hard scattering regime} \]

\[ P_g \propto \left( \frac{\vec{k}_{\perp}}{\vec{k}_{\perp}^2 + x^2 m_Q^2} - \frac{\vec{k}_{\perp} - \vec{\ell}_{\perp}}{(\vec{k}_{\perp} - \vec{\ell}_{\perp})^2 + x^2 m_Q^2} \right)^2 \]
\[ \ell_\perp \gg x m_Q \quad \text{— hard scattering regime} \]

\[ P_g \propto \left( \frac{\vec{k}_\perp}{\vec{k}_\perp^2 + x^2 m_Q^2} - \frac{\vec{k}_\perp - \vec{\ell}_\perp}{(\vec{k}_\perp - \vec{\ell}_\perp)^2 + x^2 m_Q^2} \right)^2 \]

- Energy loss
- HQ transport
- Kinematics
- High-energy screening
- Hard scattering
- Soft scattering
- Finite energy
- Outlook
$\ell_\perp \gg x m_Q$ — hard scattering regime

\[
\int P_g \, d^2k_\perp \sim \ln \frac{\ell_\perp}{x m_Q}
\]

$\ell_\perp \rightarrow \mu : x < x_M \equiv \mu / m_Q$
\[ \ell_\perp \ll x m_Q \rightarrow \text{soft scattering regime} \]

Energy loss
HQ transport
\[ \int x \, d\sigma \]
 xd\sigma / dx
Kinematics
High-energy
Screening
Hard scattering
Soft scattering
Finite energy
Outlook

\[ \rightarrow \text{strong interference} \]
\( \ell_\perp \ll x m_Q \) — soft scattering regime

\[
P_g \propto \left( \frac{\vec{k}_\perp}{\vec{k}_\perp^2 + x^2 m_Q^2} - \frac{\vec{k}_\perp - \vec{\ell}_\perp}{(\vec{k}_\perp - \vec{\ell}_\perp)^2 + x^2 m_Q^2} \right)^2
\]
\[ \ell_\perp \ll x m_Q \quad \text{— soft scattering regime} \]

Energy loss
HQ transport
\[ \int x \, d\sigma \]
\[ x \, d\sigma / dx \]
Kinematics
High-energy
Screening
Hard scattering
Soft scattering
Finite energy
Outlook

\[ \int P_g \, d^2 k_\perp \sim \frac{\ell^2_\perp}{x^2 m_Q^2} \]
\[ \ell_\perp \to \mu : x > x_M \]
Finite energy

\[
\frac{d\sigma_{Qq\rightarrow Qgg}}{dx d^2k_\perp d^2\ell_\perp} = \frac{1}{2(s - m_Q^2)}|\mathcal{M}|^2 \frac{1}{4(2\pi)^5 \sqrt{\Delta}} \Theta(\Delta)
\]

\[
\mathcal{M} = g C_3 \left(-2 \frac{g^2 (s - m_Q^2)}{t^2}\right) \times \bar{\epsilon}_t \cdot \left(\frac{2(1 - x) - x'}{k_\perp^2 + x^2 m_Q^2} - \frac{2(1 - x - x')(k_\perp - \ell_\perp)}{(k_\perp - \ell_\perp)^2 + (x + x')^2 m_Q^2}\right)
\]
Finite energy

Energy loss
HQ transport
\[ \int x \, dx \, d\sigma \]
\[ x \, d\sigma / dx \]
Kinematics
High-energy
Screening
Hard scattering
Soft scattering
Finite energy

Outlook

\[
\frac{d\sigma^{Qq \to Qgg}}{dx \, d^2k_\perp \, d^2\ell_\perp} = \frac{1}{2(s - m_Q^2)} |M|^2 \frac{1}{4(2\pi)^5} \sqrt{\Delta} \Theta(\Delta)
\]

\[
M = g \, C_3 \left( \frac{-2 \, g^2 \, (s - m_Q^2)}{t^2} \right)
\times \vec{\epsilon}_t \cdot \left( \frac{(2(1 - x) - x') \, \vec{k}_\perp}{\vec{k}_\perp^2 + x^2 m_Q^2} - \frac{2(1 - x - x')(\vec{k}_\perp - \vec{\ell}_\perp)}{(\vec{k}_\perp - \vec{\ell}_\perp)^2 + (x + x')^2 m_Q^2} \right)
\]

\[ x' \equiv x'(x, \vec{k}_\perp, \vec{\ell}_\perp) \text{ momentum fraction of the outgoing light quark} \]
Finite energy

\[
\frac{d\sigma_{Qq\to Qgg}}{dx d^2k_\perp d^2\ell_\perp} = \frac{1}{2(s - m_Q^2)} |\mathcal{M}|^2 \frac{1}{4(2\pi)^5 \sqrt{\Delta}} \Theta(\Delta)
\]

\[\mathcal{M} = g C_3 \left( \frac{-2 g^2 (s - m_Q^2)}{t^2} \right)\]

\[
x' = x' (x, \vec{k}_\perp, \vec{\ell}_\perp) \text{ momentum fraction of the outgoing light quark}
\]

\[x' \to 0 \text{ at high-energy}\]
compact expression but numerical integration needed
Outlook

- Gluon radiation by HQ in the Bethe-Heitler regime, at large and intermediate energies

- Coherence effects to be investigated

→ embedding in MC@HQ generator → talk of Pol Gossiaux on friday afternoon