Production of Strange, Non-strange particles and Hypernuclei in an Excluded-Volume model

Swatantra Kumar Tiwari

Department of Physics

Banaras Hindu University,

Varanasi, India

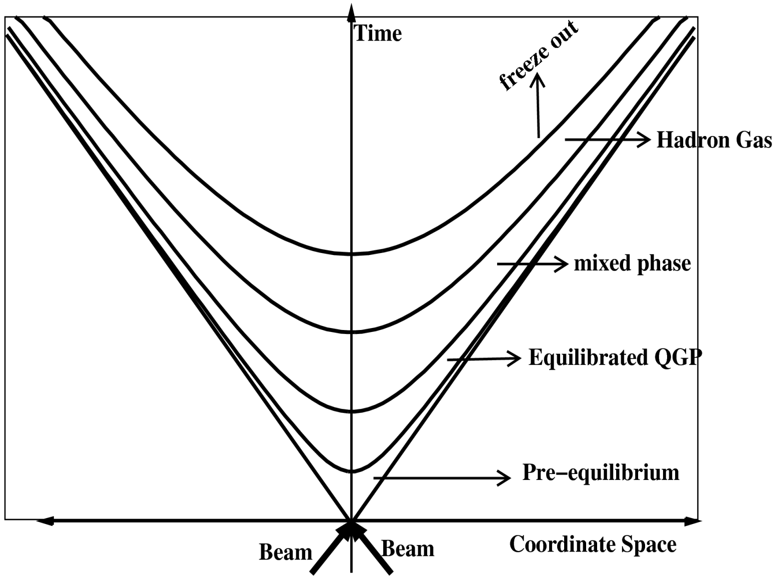
Work done in collaboration with Prof. C. P. Singh

Strangeness in Quark Matter (SQM)-2013, Birmingham, U.K.

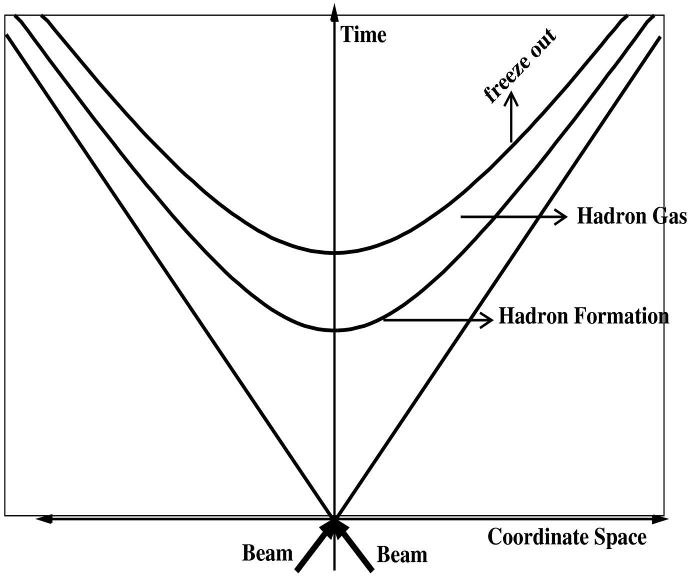
Outline

- ✓ Need for Hadron Gas (HG) Equation of State (EOS)-
  Statistical Thermal Models
- ✓ Geometrical Excluded-Volume Model
- ✓ Strange and Non-strange particles in Our Model
- ✓ Production of Light Nuclei, Hypernuclei and their Antinuclei
- ✓ Thermal Model + Flow
- ✓ Conclusions

Why we need HG EOS ?



QGP formation



No QGP formation

- Whether QGP or not, HG is always present
  - Chemical and thermal equilibrium achieved or not.

- To devise any unique signal for the formation of QGP phase we need to understand the behaviour and properties of hot, dense HG (because it forms the background).

Statistical Thermal Model

The basic condition to apply Statistical Thermal Model for any system is that the system should be in thermodynamical equilibrium.

Ideal HG EOS :

Ref.: Andronic et al., Nucl. Phys. A, 772, 167 (2006).

$$\left(n_i^0\right)_{mesonic} = \frac{g_i}{2\pi^2}\int_0^\infty k^2 dk \frac{1}{\exp\left[\frac{E_i \mp \mu_i}{T}\right]-1} \qquad \left(n_i^0\right)_{baryonic} = \frac{g_i}{2\pi^2}\int_0^\infty k^2 dk \frac{1}{\exp\left[\frac{E_i \mp \mu_i}{T}\right]+1}$$

Where $E_i = \sqrt{k^2 + m_i^2}$ is the energy, k is the momentum and g_i is the degeneracy factor of the ith particle.

Here $\mu = B \times \mu_B + S \times \mu_S$

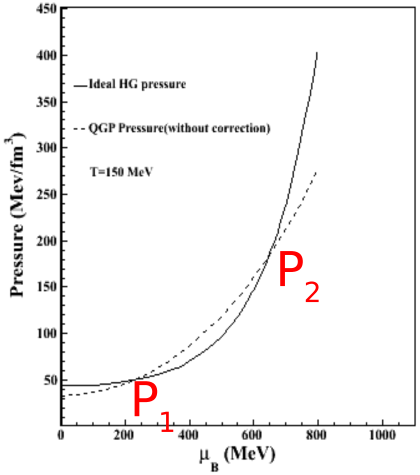
Inclusion of resonances- Attractive Forces (Welke, Venugopalan & Prakash, Phys. Lett. B 245, 137 (1990).)

Shortcomings of Ideal HG Model :

Construction of first-order Phase Transition

(Gibb's criteria) : $P_Q (T_C, \mu_C) = P_H (T_C, \mu_C)$

HG reappears as a stable phase after P_2 again.



Inclusion of Finite size particles- Repulsive Interactions-

→ Excluded-Volume Model

Hagedorn Model

Ref. : R. Hagedorn, Z. Phys. C 17, 265 (1983).

In the Hagedorn model it is assumed that the excluded-volume correction is proportional to the energy density ε^0 of the system of pointlike particles so that :

$$n_B = \frac{n_B^0}{1 + \frac{\varepsilon^0}{4B_0}} \qquad \varepsilon = \frac{\varepsilon^0}{1 + \frac{\varepsilon^0}{4B_0}}$$

Where B_0 is a bag constant .

Cleymans-Suhonen Model

Ref. : J. Cleymans and E. Suhonen, Z. Phys. C 37, 51 (1987).

Cleymans et al. adopted a hard sphere picture for a baryon.

Therefore, the volume for each baryon is : $V_i^0 = \frac{4\pi r^3}{3}$

No. density of baryon $n_i^{ex} = \frac{n_i^0}{1 + \sum_i n_i^0 V_i^0}$

Question of Thermodynamical consistency :

In all these models : $n \neq \left(\frac{\partial p}{\partial \mu}\right)_T, s \neq \left(\frac{\partial p}{\partial T}\right)_\mu$

Rischke, Gorenstein, Stocker and Greiner (RGSG) Model

Ref. : D. H. Rischke et al., Z. Phys. C 51, 485 (1991).

The question of thermodynamic consistency was first examined in detail by Rischke et al. They have proposed a thermodynamical consistent model.

$$Z_G^{ex}(T,\mu,V-V^0N)=\sum_{N=0}^{\infty}\exp(\mu N/T)Z_C(T,N,V-V^0N)\theta(V-V^0N)$$

After solving above equation using the Laplace transform one gets the following equation :

$$P^{ex}(T,\mu)=P^0(T,\tilde{\mu})$$

where $\tilde{\mu}=\mu-V^0P^{ex}(T,\mu)$

Features of Our EOS

- We assign an equal **hard-core volume** to each baryon to incorporate the repulsive interaction between baryons. Mesons, although they possess a small volume, can penetrate into each other.

- The attractive interaction between hadrons are realized by including **resonances** in our model upto 2.5 GeV/c^2.

- We use **full statistics** in grand canonical partition function in a **thermodynamically consistent** way so that our model works even at extreme temperature and/or densities.

- Numerical calculation indicates that the **causal behaviour** is fulfilled in our model even at extreme densities ($c_s^2 < 1$).

EOS for HG

Ref. : S. K. Tiwari, P. K. Srivastava, and C. P. Singh, Phys. Rev. C 85, 014908 (2012).

The Grand canonical partition function using full statistics and including excluded volume correction in a thermodynamically consistent manner is :

$$\ln Z_i^{ex} = \frac{g_i}{6\pi^2 T} \int_{V_i^0}^{V - \sum_j N_j V_j} dV \int_0^\infty \frac{k^4 dk}{\sqrt{k^2 + m_i^2}} \frac{1}{\left[\exp\left(\frac{E_i - \mu_i}{T}\right) + 1\right]}$$

Where g_i is the degeneracy factor of ith species of baryon, E is the energy of the particle V^0_i is the eigen volume of one ith species of baryon and $\sum_j N_j V_{j^0}$ is the total volume occupied .

We can write above equation as - $\ln Z_i^{ex} = V(1-\sum_j n_j^{ex} V_j^0) I_i \lambda_i$

where $$I_i = \frac{g_i}{6\pi^2 T}\int_0^\infty \frac{k^2 dk}{\sqrt{k^2+m_i^2}} \frac{1}{[\exp(\frac{E_i}{T})+\lambda_i]}$$

$\lambda_i = \exp(\pm\frac{\mu_i}{T})$ is the fugacity of the ith particle.

(+) sign is used for particles and (-) sign is used for anti- particles.

n^{ex}_j is the number density of jth type of baryons after excluded volume correction.

Using the basic thermodynamical relation between number density and partition function we can get :

$$n_i^{ex} = (1-R) I_i \lambda_i - I_i \lambda_i^2 \frac{\partial R}{\partial \lambda_i} + \lambda_i^2 (1-R) I_i'$$

Where $R = \sum_i n_i^{ex} V_i^0$ is the fractional volume occupied. We can write R in an operator form as :

$$R = \hat{R} + \Omega R$$

where $\hat{R} = \dfrac{R^0}{1+R^0}$ with $R_i^0 = \sum n_i^0 V_i^0 + \sum I_i' V_i^0 \lambda_i^2$

and the operator $\Omega = -\dfrac{1}{1+R^0} \sum_i n_i^0 V_i^0 \lambda_i \dfrac{\partial}{\partial \lambda_i}$

Using Neumann iteration method, we get after truncation up to 2^{nd} order term :

$$R = \hat{R} + \Omega \hat{R} + \Omega^2 \hat{R}$$

By calculating R we can calculate n_i^{ex} .

After solving above equations and using basic thermodynamical relations we can get the pressure of baryons as follows :

$$P_B^{ex} = T(1-R)\sum_i I_i \lambda_i$$

Similarly we can calculate no. density of mesons by using following formula:

$$n_i^{meson} = \frac{g_i \lambda_i}{2\pi} \int_0^{\infty} k^2 \frac{1}{[\exp(\frac{E_i}{T}) - \lambda_i]} dk$$

The pressure due to mesons is given by :

$$P_i^{meson} = n_i^{meson} T$$

Total Number Density :

Total no. density of hadrons can be given as follows :

$$n_{HG} = n_B^{ex} + n_i^{meson}$$

Hadronic Pressure :

Pressure due to hadrons can be calculated by using the following formula :

$$P_{HG} = P_B^{ex} + P_i^{meson}$$

Thermodynamical and Statistical Consistency

In RGSG model, $\overline{N} = \langle N \rangle$ $\overline{E} = \langle E \rangle$

Ref. : M. I. Gorenstein, Phys. Rev. C 86, 044907 (2012).

In Our model, $Z^{ex}(T,V,\mu) = \sum_{N=0}^{\infty} \exp(\mu N/T) \frac{(V - V^0 \overline{N})^N}{N!} z^N$

Here $\overline{N}$ depends on the baryon chemical potential .

$$\overline{N} = \langle N \rangle - \frac{T}{Z^{ex}} \sum_{N=0}^{\infty} \exp(\mu N/T) \frac{(V - V^0 \overline{N})^{N-1}}{(N-1)!} z^N V^0 \frac{\partial \overline{N}}{\partial \mu}$$

Ref. : S. K. Tiwari, and C. P. Singh, arXiv: 1306.3291 [hep-ph].

Correction term

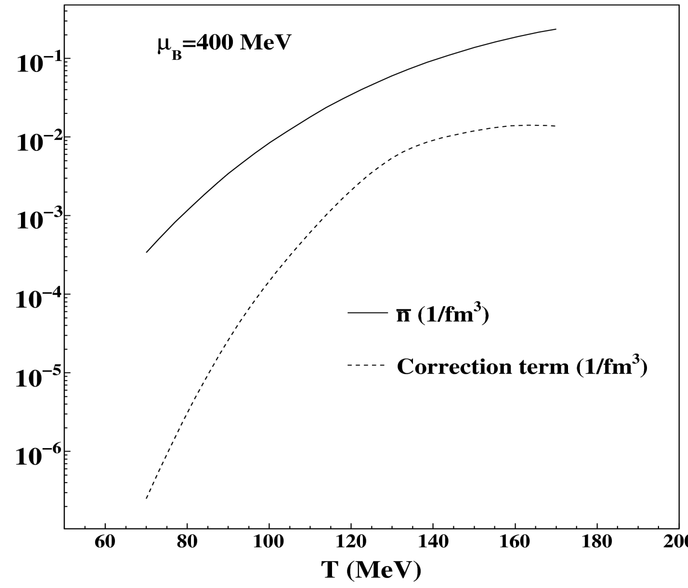


Fig : Variation of thermodynamical average of no. density of baryons and correction term with respect to temperature at constant baryon chemical potential =400 MeV .

Ref. : S. K. Tiwari et al., Phys. Rev. C 85, 014908 (2012); arXiv: 1306.3291 [hep-ph] .

Causality

If the medium transmits the information with the speed greater than the speed of light then the causality violates.

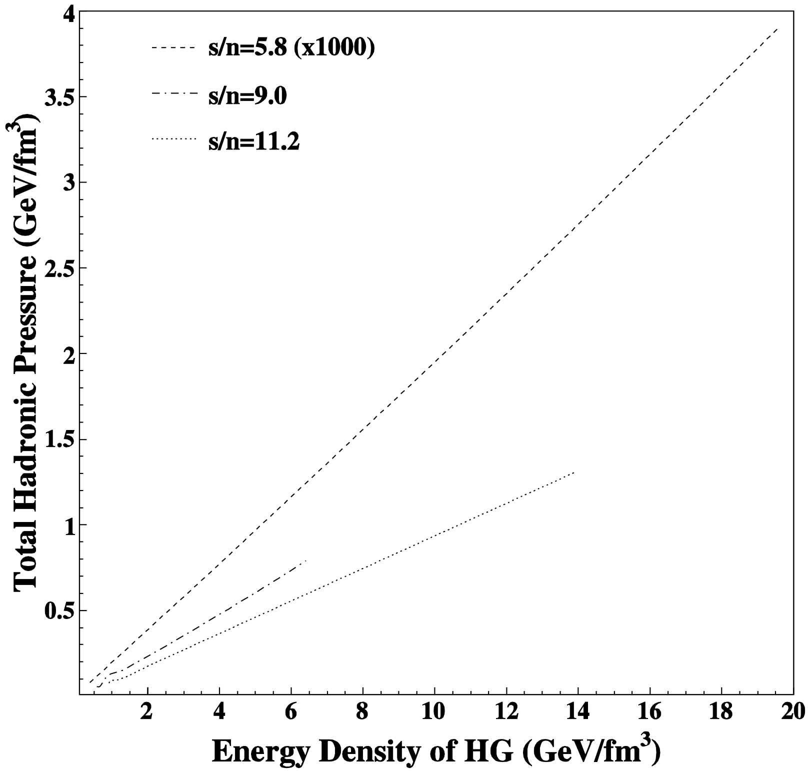


Fig : Variation of Hadronic Pressure with respect to energy density of hadrons at constant s/n

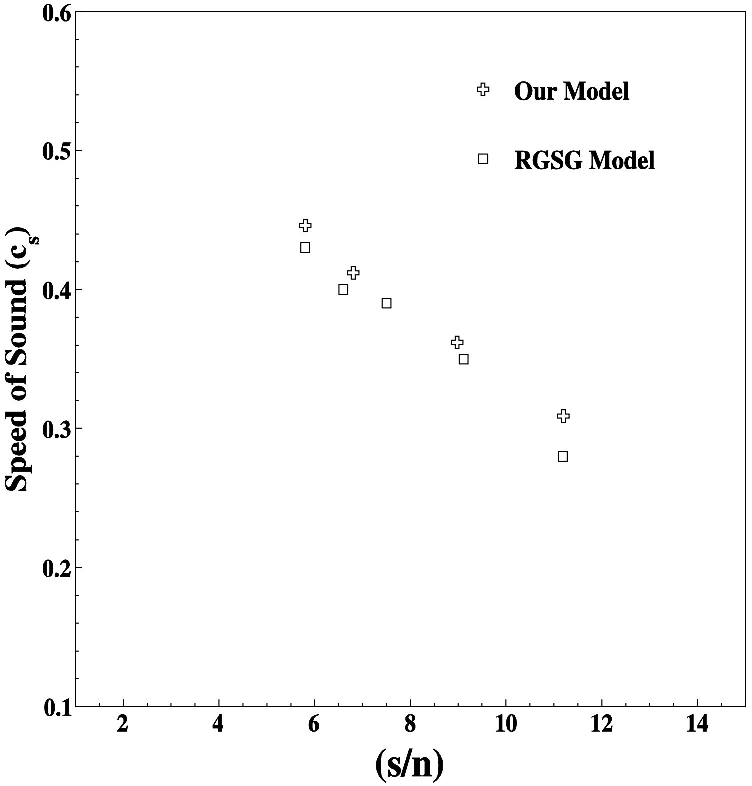


Fig : Variations of speed of sound with respect to s/n.

Chemical Freeze-out Parameters

TABLE I: Thermal parameters (T, μ_B) values obtained by fitting the experimental particle-ratios in different model calculations.

$\sqrt{s_{NN}}$(GeV)	IHG Model			RGSG Model			Cleymans-Suhonen Model			Our Model		
	T	μ_B	δ^2	T	μ_B	δ^2	T	μ_B	δ^2	T	μ_B	δ^2
2.70	60	740	0.85	60	740	0.75	70	753	1.19	70	760	1.15
3.32	80	670	0.89	78	680	0.34	89	686	0.75	90	670	0.45
3.84	100	645	0.50	86	640	0.90	101	639	0.37	100	640	0.34
4.32	101	590	0.70	100	590	0.98	109	600	0.17	105	600	0.23
8.76	140	380	0.45	145	406	0.62	144	386	0.05	140	360	0.25
12.3	148	300	0.31	150	298	0.71	153	300	0.03	150	276	0.20
17.3	160	255	0.25	160	240	0.62	158.6	228	0.63	155	206	0.27
130	172.3	35.53	0.10	165.5	38	0.54	165.8	35.84	0.15	163.5	32	0.05
200	172.3	23.53	0.065	165.5	25	0.60	165.9	23.5	0.10	164	20	0.05

Ref. : S. K. Tiwari, and C. P. Singh, arXiv: 1306.3291 [hep-ph].

Why Strangeness is so Important ?

- Strangeness enhancement is an important signature of QGP formation .
  J. Rafelski and B. Müller, PRL48, 1066 (1982)
  P. Koch, B. Müller, and J. Rafelski, Phys. Rep. 142, 167 (1986)
  C. P. Singh, Phys. Rep. 236, 147 (1993); Int. J. Mod. Phys. A 7, 7185 (1992).

- The production threshold for the associated production of strangeness via strange- antistrange quark pairs is considerably smaller than the one for hadrons.

- Strange flavour is not present at the beginning of the heavy-ion collisions, it has to be produced in the reaction.

.

Strange and Non-strange Hadrons

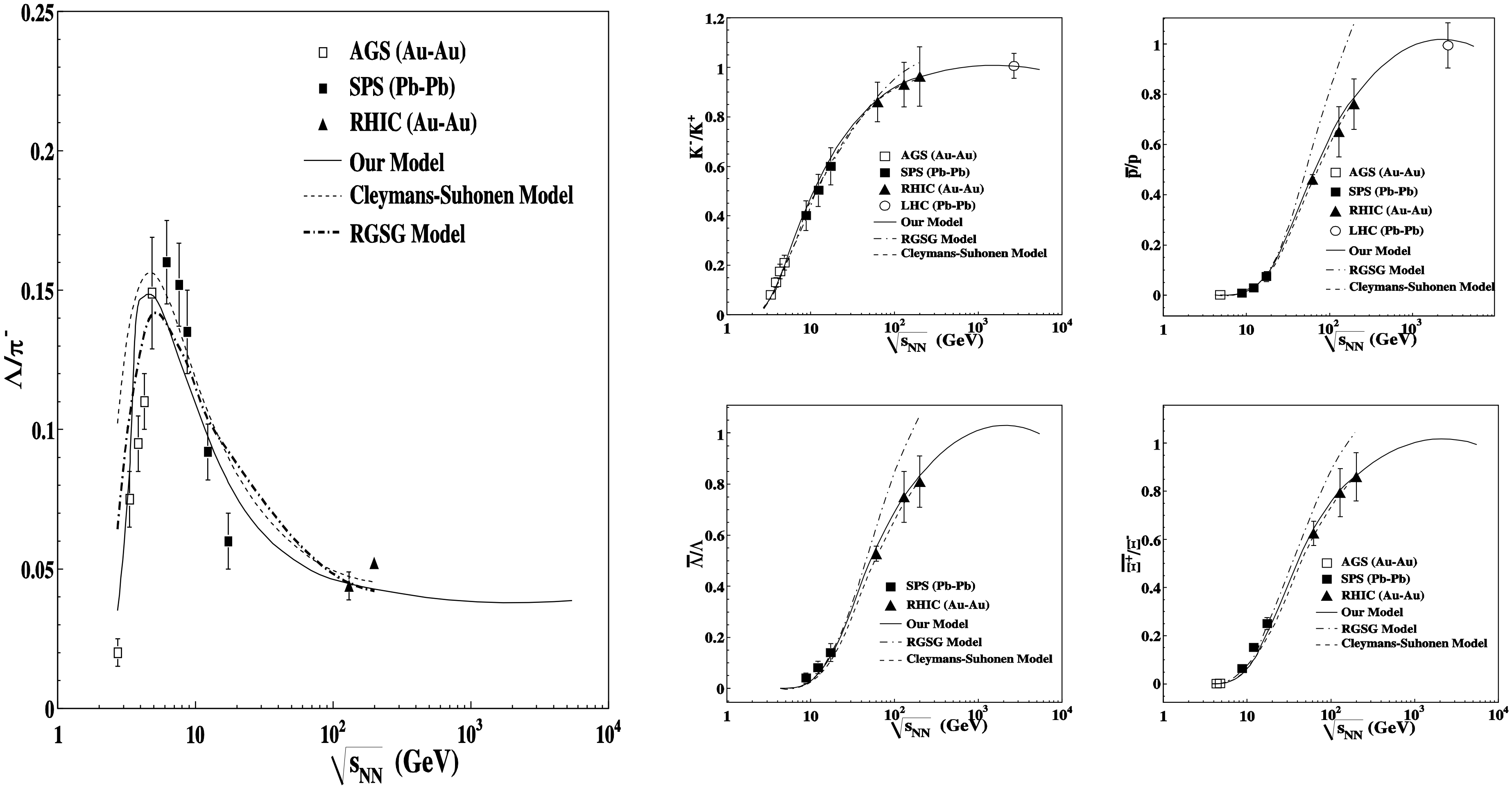


Fig. : Variation of hadrons ratios with respect to center-of-mass energies.

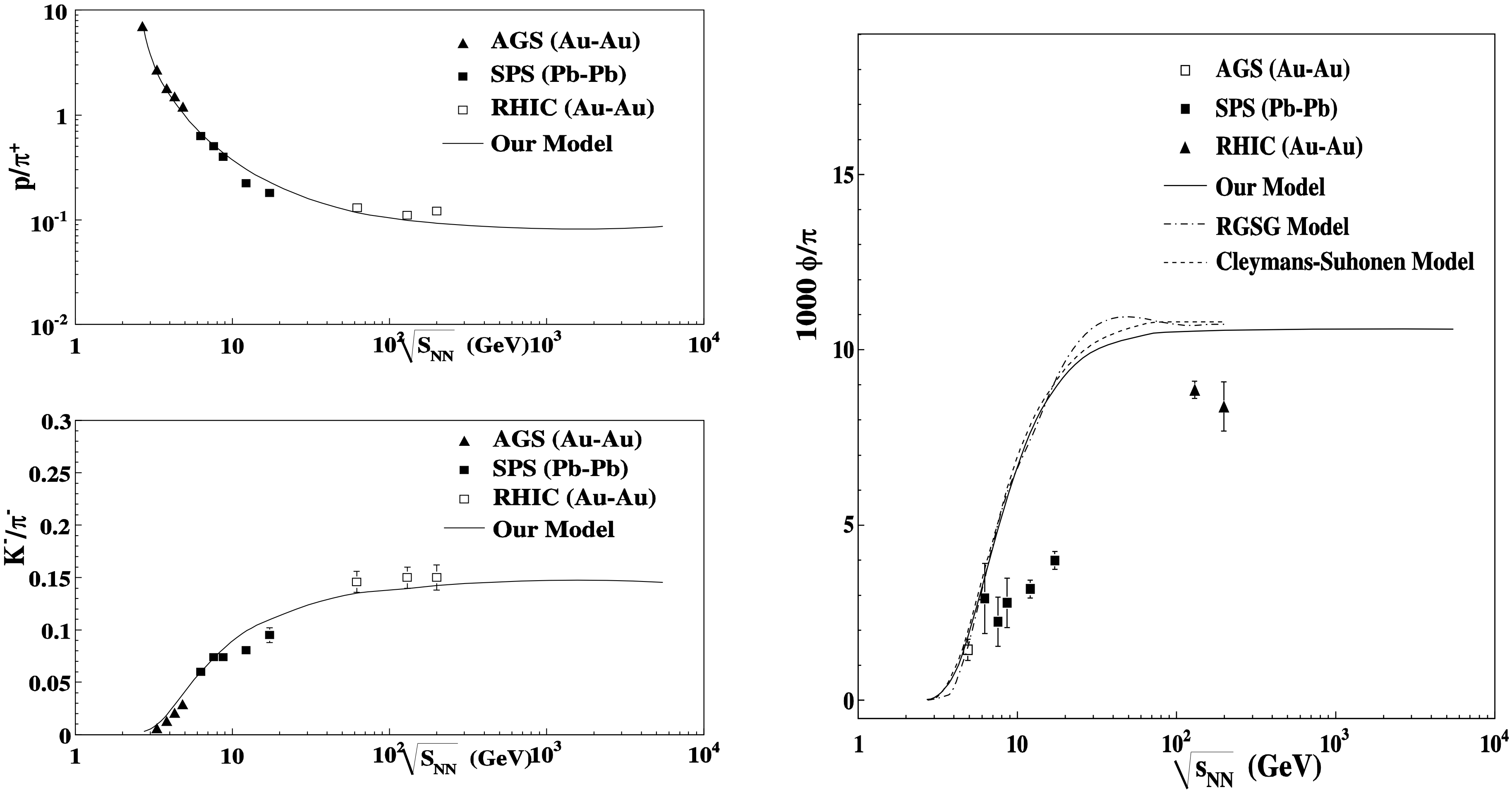


Fig. : Variation of various hadrons ratios with respect to center-of-mass energies.

Thermal model fails to describe the production of the particles with hidden-strange quarks e. g. phi meson.

Light Nuclei, Hypernuclei and their Anti-nuclei

- Ultra-relativistic heavy-ion collisions offer a best way to study the production of light nuclei, hypernuclei and their antinuclei.

- The production of light nuclei and hypernuclei is entirely based on the entropy conservation which constitute the basis for the thermal model analysis for the yields of these particles

- The analysis of the production of light nuclei, hypernuclei and their antinuclei throws light on the understanding of the creation of matter-antimatter asymmetry arising in the early universe and also the strength of nuclear interaction for the antinuclei.

- Such analysis hints about the degree of thermalization for heavy nuclei in the fireball created after the heavy-ion collisions.

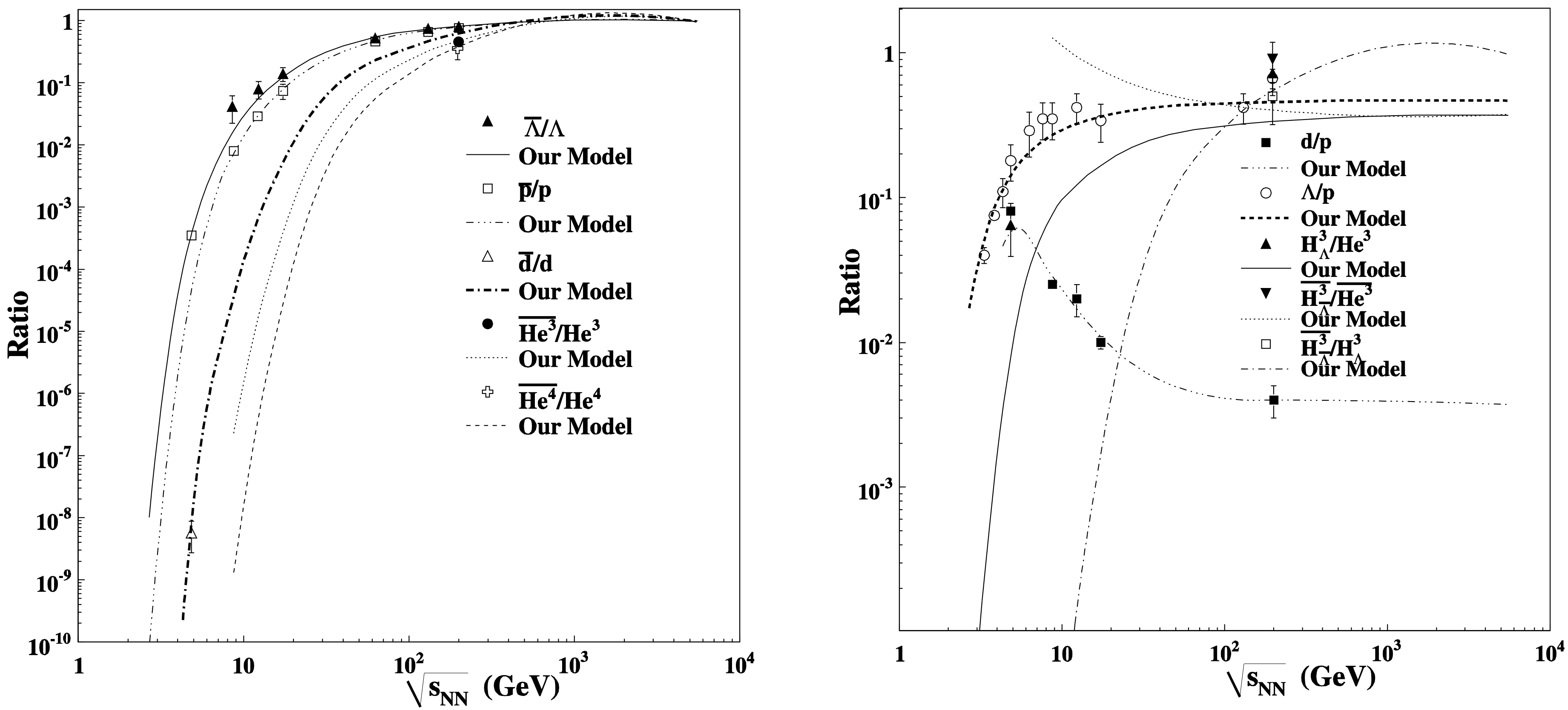


Fig. : center-of-mass energy dependence of various ratios of light nuclei, Hypernuclei and their antinuclei.

Ref. : S. K. Tiwari, P. K. Srivastava, and C. P. Singh, J. Phys. G 40, 045102 (2013).

Our model describes the experimental data on the ratio of light nuclei, hypernuclei and their antinuclei over a broad energy range from SIS energies to RHIC energy.

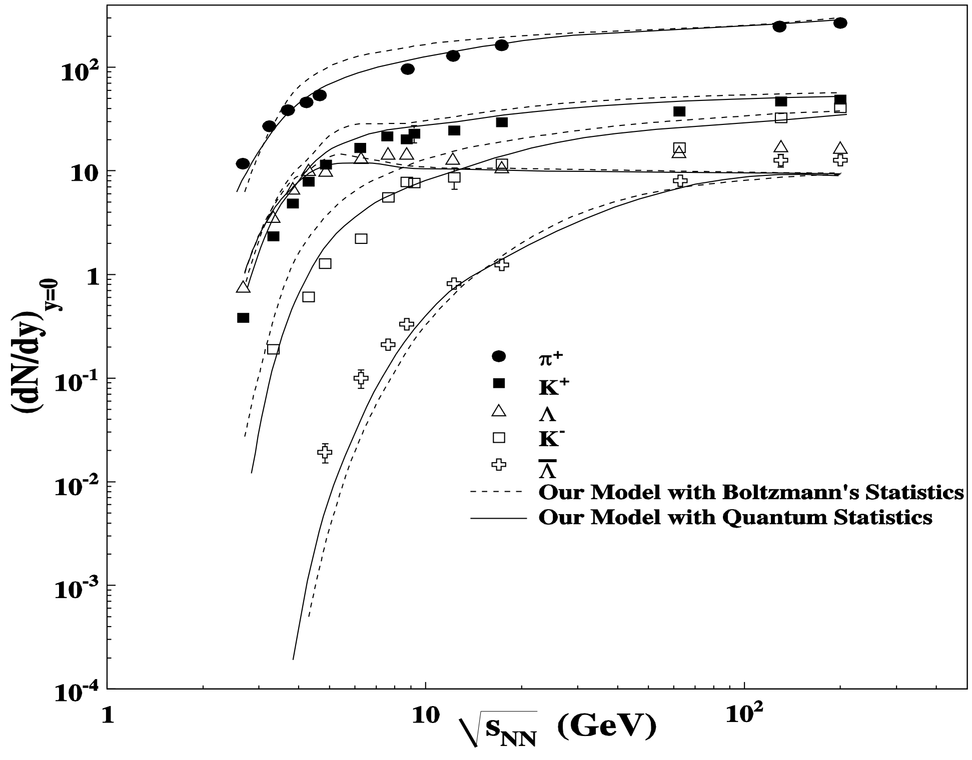
Fails in case of mixed ratios → New production mechanism needed which is not contained in the thermal approach

Rapidity Distributions

Thermal Model :

$$\left(\frac{dN_i}{dy}\right)_{th} = \frac{g_i V \lambda_i}{(2\pi^2)}\left[((1-R)-\lambda_i\frac{\partial R}{\partial \lambda_i})\int_0^\infty \frac{m_T^2 \cosh y dm_T}{\left[\exp\left(m_T \cosh y/T\right)+\lambda_i\right]} - \lambda_i(1-R)\int_0^\infty \frac{m_T^2 \cosh y dm_T}{\left[\exp\left(m_T \cosh y/T\right)+\lambda_i\right]^2}\right]$$

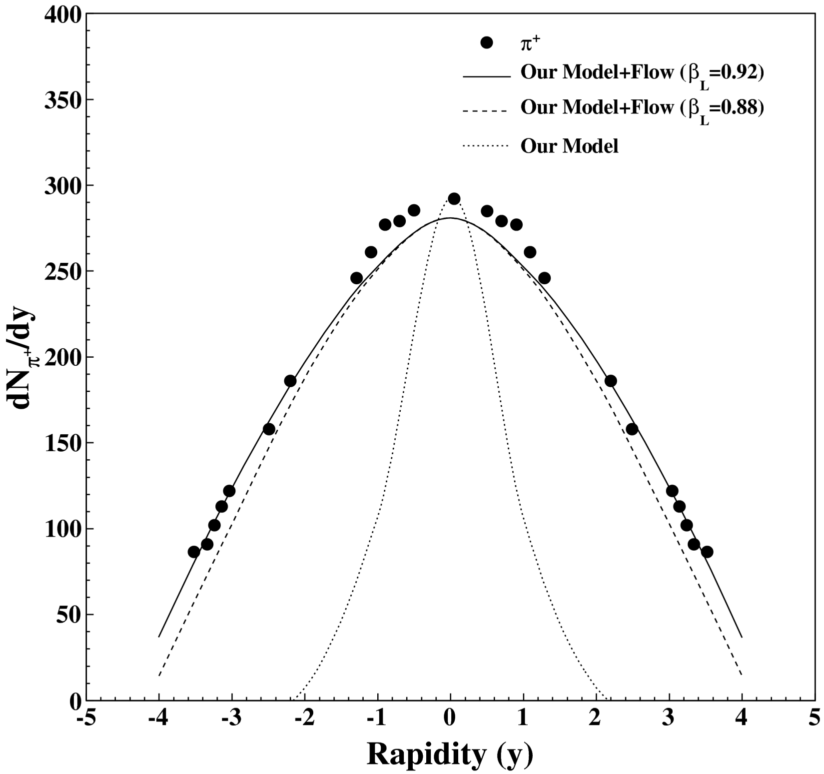
Ref. : S. K. Tiwari, P. K. Srivastava, and C. P. Singh, J. Phys. G 40, 045102 (2013).

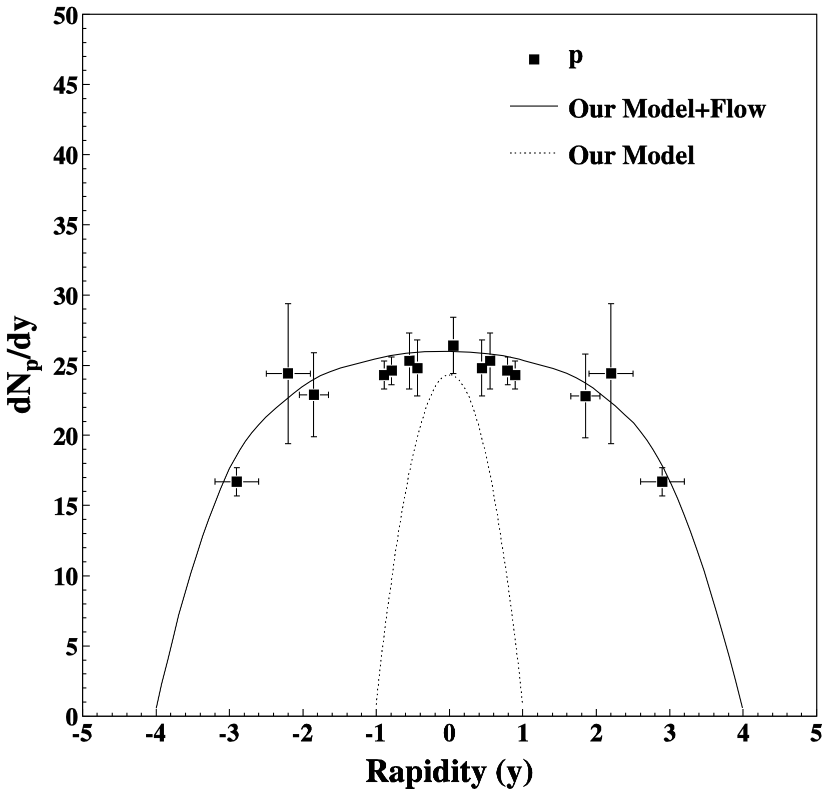


Thermal Model + Flow :

$$\frac{dN_i}{dy} = \int_{-\eta_{\max}}^{\eta_{\max}} \left(\frac{dN}{dy} \right)_{th} (y-\eta) d\eta \quad \text{where} \quad \langle \beta_L \rangle = \tanh(\eta_{\max}/2)$$

Ref. : S. K. Tiwari, P. K. Srivastava, and C. P. Singh, J. Phys. G 40, 045102 (2013).





Transverse Mass Spectra

Thermal Model :

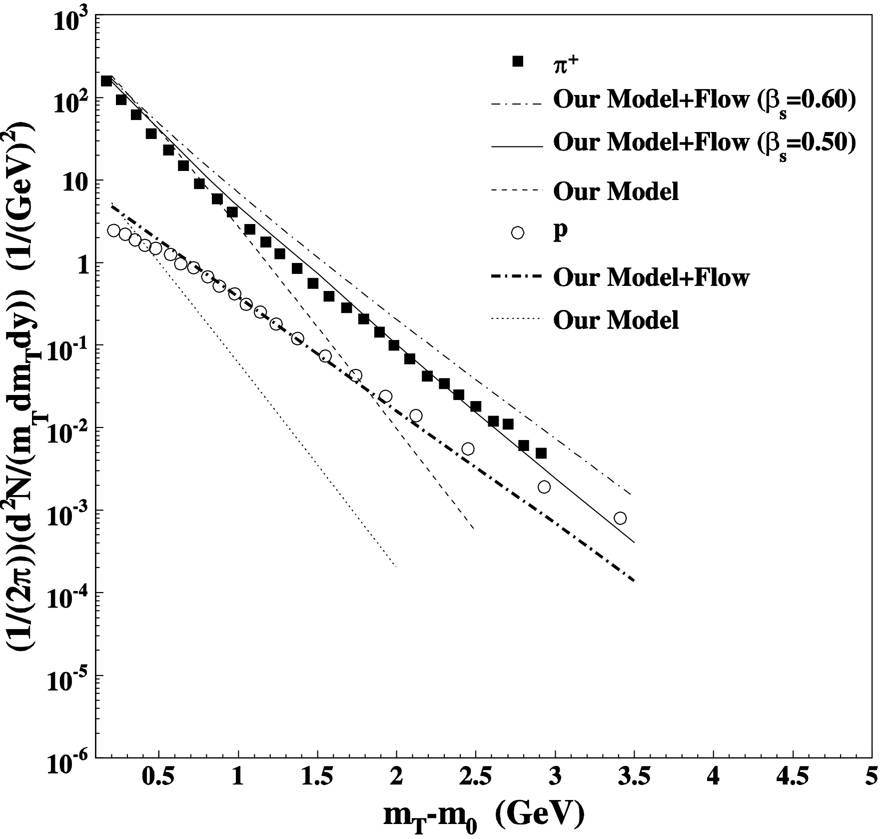
$$\frac{dN_i}{m_T\, dm_T} = \frac{g_i V \lambda_i}{(2\pi^2)} \left[(1-R) - \lambda_i\, \partial R / \partial \lambda_i \right] m_T K_1 \left(\frac{m_T}{T} \right)$$

Thermal Model + Flow :

$$\frac{dN_i}{m_T\, dm_T} = \frac{g_i V \lambda_i m_T}{(2\pi^2)} \left[(1-R) - \lambda_i\, \partial R / \partial \lambda_i \right] \int_0^{R_0} r\, dr\, K_1\left(m_T \cosh\rho / T \right) I_0\left(p_T \sinh\rho / T \right)$$

where $\rho = \tanh^{-1}\beta_r$ $\quad \beta_r = \beta_s (\xi)^n$ $\quad \xi = (r/R_0)$

Ref. : S. K. Tiwari, P. K. Srivastava, and C. P. Singh, J. Phys. G 40, 045102 (2013).



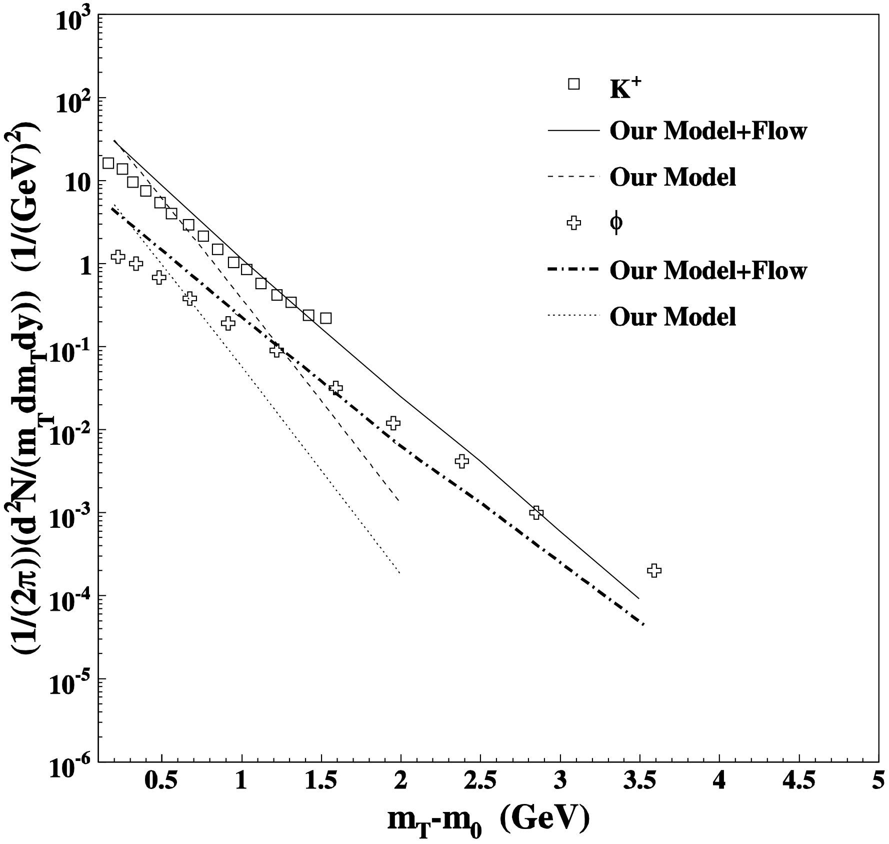
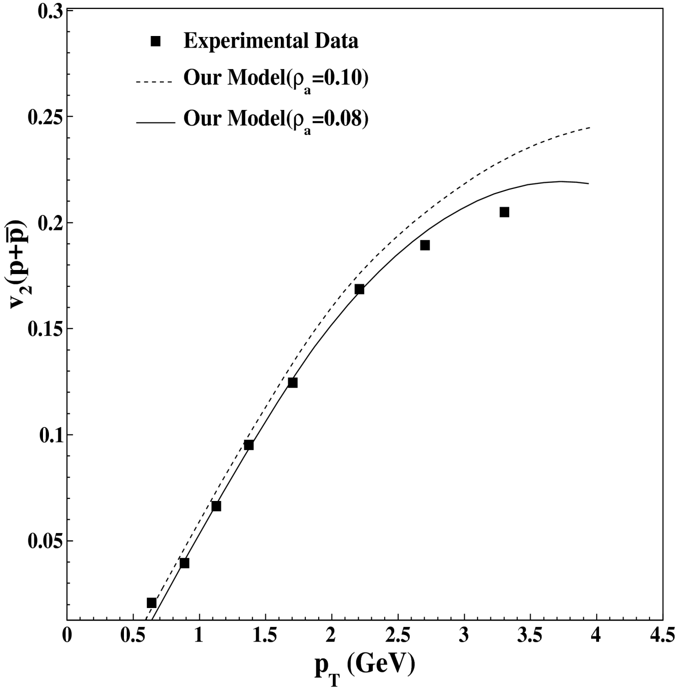


Fig. : Transverse mass spectra of various particles at centre-of-mass energy of 200 GeV.

Ref. : S. K. Tiwari, P. K. Srivastava, and C. P. Singh, J. Phys. G 40, 045102 (2013).

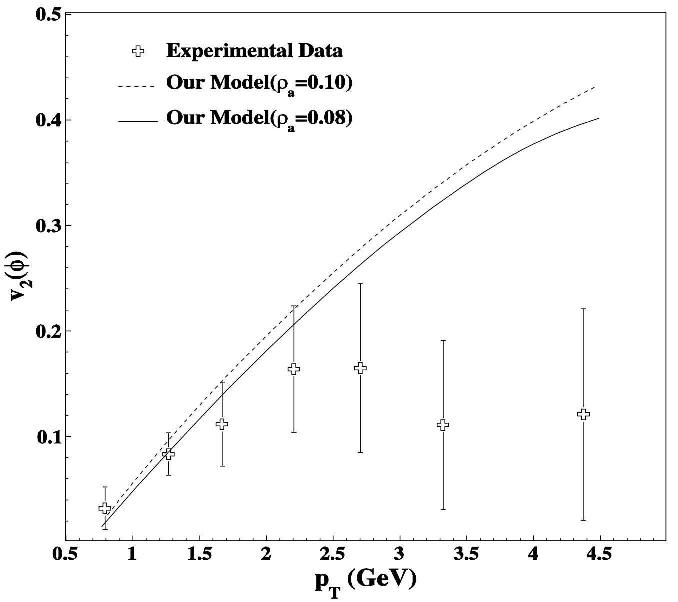


Elliptic Flow of Hadrons

$$v_2(p_T) = \frac{\int_0^{2\pi} I_2(\alpha(\phi))K_1(\beta(\phi))\cos(2\phi)d\phi}{\int_0^{2\pi} I_0(\alpha(\phi))K_1(\beta(\phi))d\phi}$$

where $\alpha(\phi) = \frac{p_T \sinh\rho(\phi)}{T}$ $\quad \beta(\phi) = \frac{m_T \cosh\rho(\phi)}{T}$

$$\rho = \rho_0 + \rho_a \cos(2\phi)$$



Ref. : P. Huovinen et al., Physics Lett. B 503, 58 (2001).

Fig. : Elliptic flow of hadrons with respect to transverse momentum at center-of-mass energy of 200 GeV.

Conclusions :

- We have proposed an (approximately thermodynamically consistent) excluded-volume model which works even at extreme values of temperature and baryon chemical potential where other excluded-volume models fail.

- Our model provides a suitable description of the experimental data on the midrapidity yields, rapidity as well as transverse mass spectra, elliptic flow, ratios etc. of various hadrons obtained at various centre-of-mass energies.

- Our model describes the production of strange particles and hypernuclei successfully but fails in case of particles with hidden-strange quark combinations which suggests a different kind of production mechanism for these particles (e.g. , quark coalescence model).

Our model provides a proper and more realistic EOS for a hot and dense HG.

Thank You

Back-up Slides

Backup :

● Space-Time Evolution of System in Ultra- Relativistic Heavy Ion Collision

Pre equilibrium stage At (z,t) =(0,0), nuclei collides and pass through each other, nucleons interact with each other.

Formation stage Quarks and gluons (qq,gg) are produced in the central region a large amount of energy is deposited.

Equilibration Due to parton interaction plasma evolves from formation stage to a thermalized QGP.

Hadronization Thermalized plasma expands and cools until hadronization takes place and mesons and baryons are created.

Freeze-out When temperature falls further, the hadrons no longer interact and they stream out of the collision region

towards the detectors.

Chemical freeze-out when inelastic scattering cease

Thermal or Kinetic freeze-out when elastic interactions cease

Energy Density of Hadrons :

$$\varepsilon_{HG} = \varepsilon_B^{ex} + \varepsilon^{meson}$$

Where $$\varepsilon_B^{ex} = \frac{T^2}{V}\frac{\partial \ln Z_i^{ex}}{\partial T} + \mu_i n_i^{ex}$$

and $$\varepsilon_i^{meson} = \frac{g_i \lambda_i}{2\pi}\int_0^\infty \sqrt{k^2+m^2}\,\frac{k^2 dk}{[\exp(\frac{E_i}{T}) - \lambda_i]}$$

Entropy Density of Hadrons :

$$S_{HG} = S_B^{ex} + S^{meson}$$

Where $$S_B^{ex} = \frac{\varepsilon_B^{ex} + P_B^{ex} - \mu_B n_B - \mu_s n_s}{T}$$

and $$S_i^{meson} = \frac{\varepsilon_i^{meson} + P_i^{meson} - \mu_s n_s}{T}$$

Cleymans-Suhonen Model :

Therefore, the volume for each baryon is :

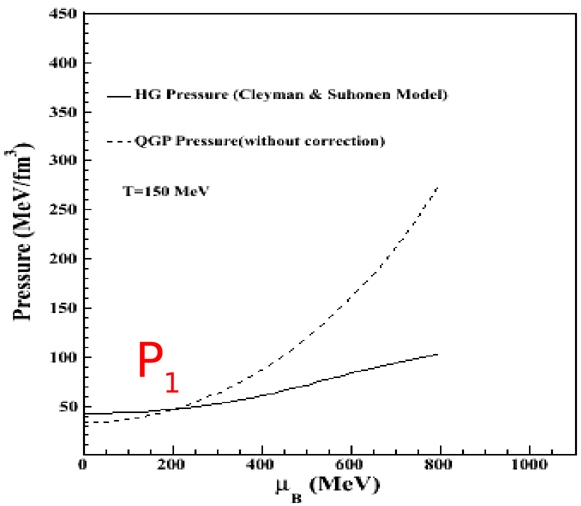
After the excluded volume correction the number density, pressure and the energy density for ith hadron is given as below :

For many species -->

$$p_i^{ex} = \frac{p_i^0}{1+\sum_i n_i^0 V_i^0}$$

$$\varepsilon_i^{ex} = \frac{\varepsilon_i^0}{1+\sum_i n_i^0 V_i^0}$$

here $n_i^0, p_i^0 and \varepsilon_i^0$ are the number density, pressure and energy density for pointlike particles respectively.



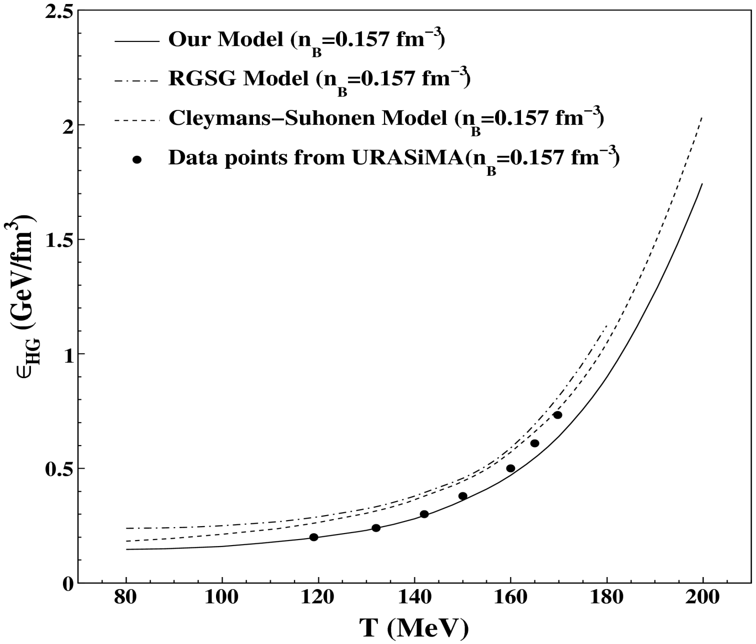
Solves the problem of stability of QGP phase at higher densities

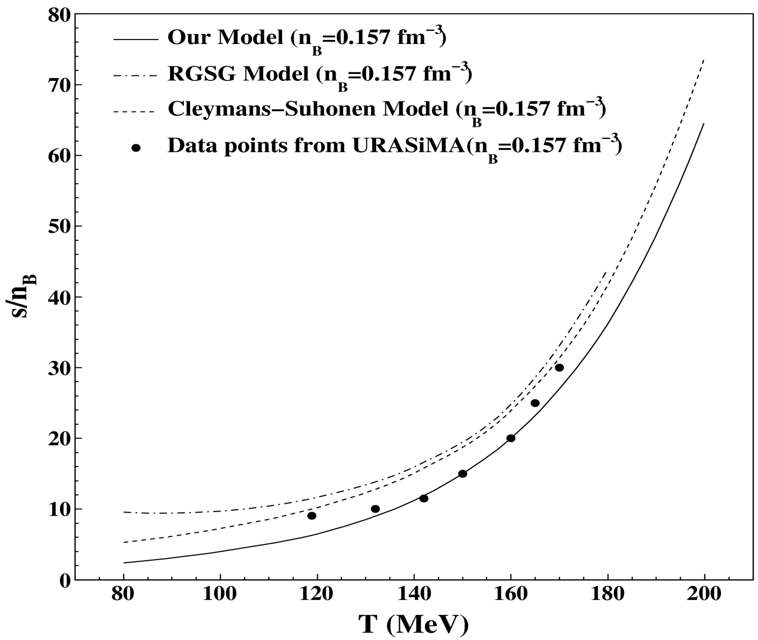
P_1 is the only one critical point.

Demerits of above model :

- The above model involves cumbersome, transcendental expressions.
- They use the available volume as $(V-v_0N)$ where N is a fixed number.
- This model fails ambiguously at $\mu = 0$.

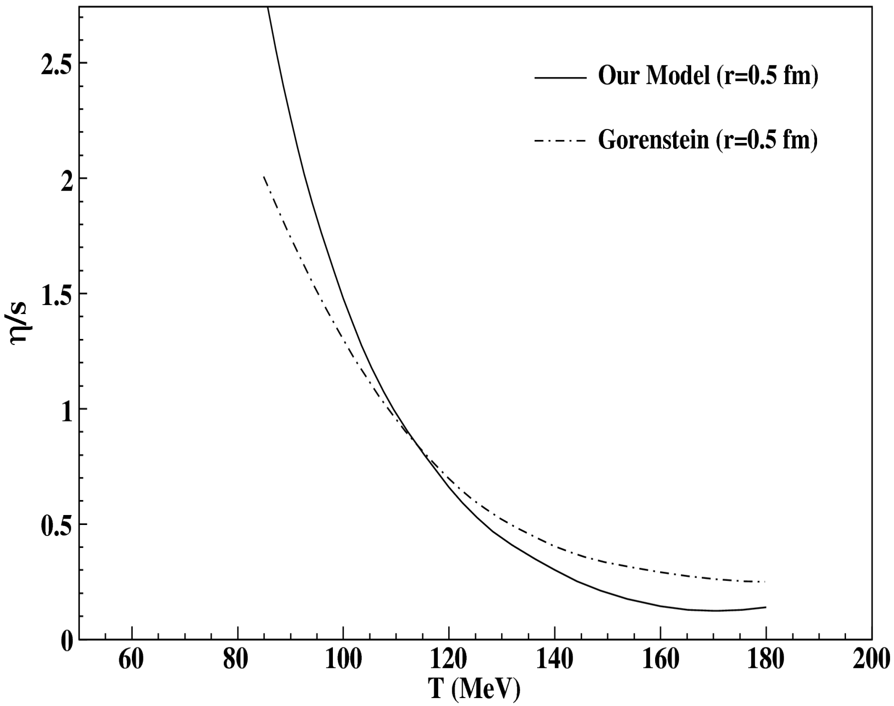
Thermodynamical Properties :





Ref. : S. K. Tiwari, and C. P. Singh, arXiv: 1306.3291 [hep-ph].

Transport Properties :



Shear viscosity :

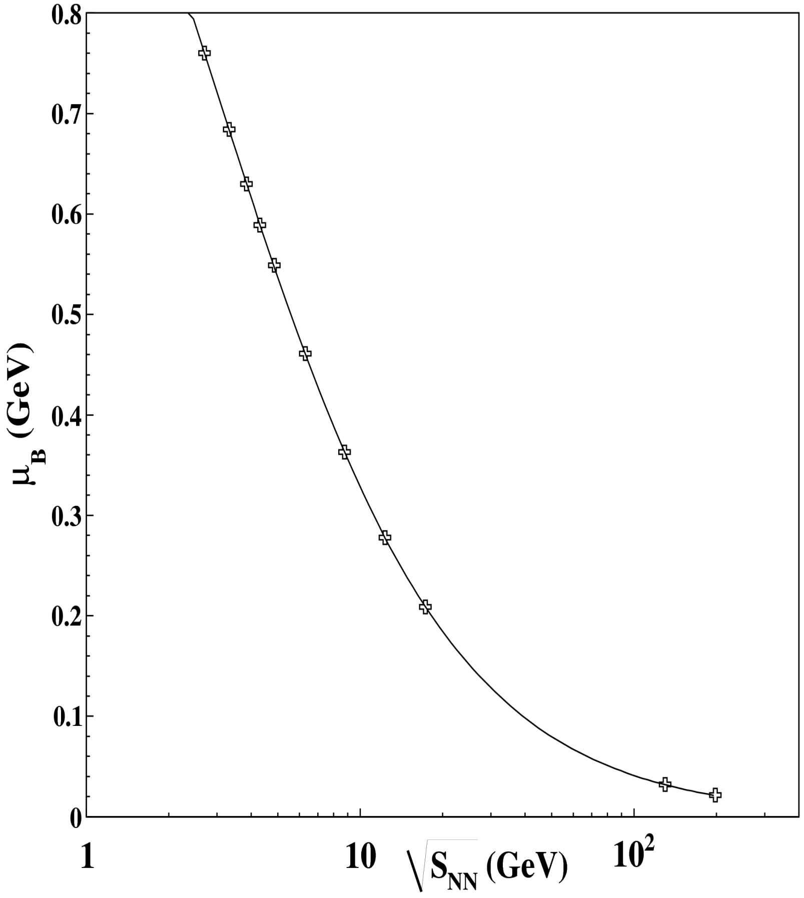
$$\eta = \frac{5}{64\sqrt{8}\, r^2} \sum_i \langle |\mathbf{p_i}| \rangle \times \frac{n_i}{n}$$

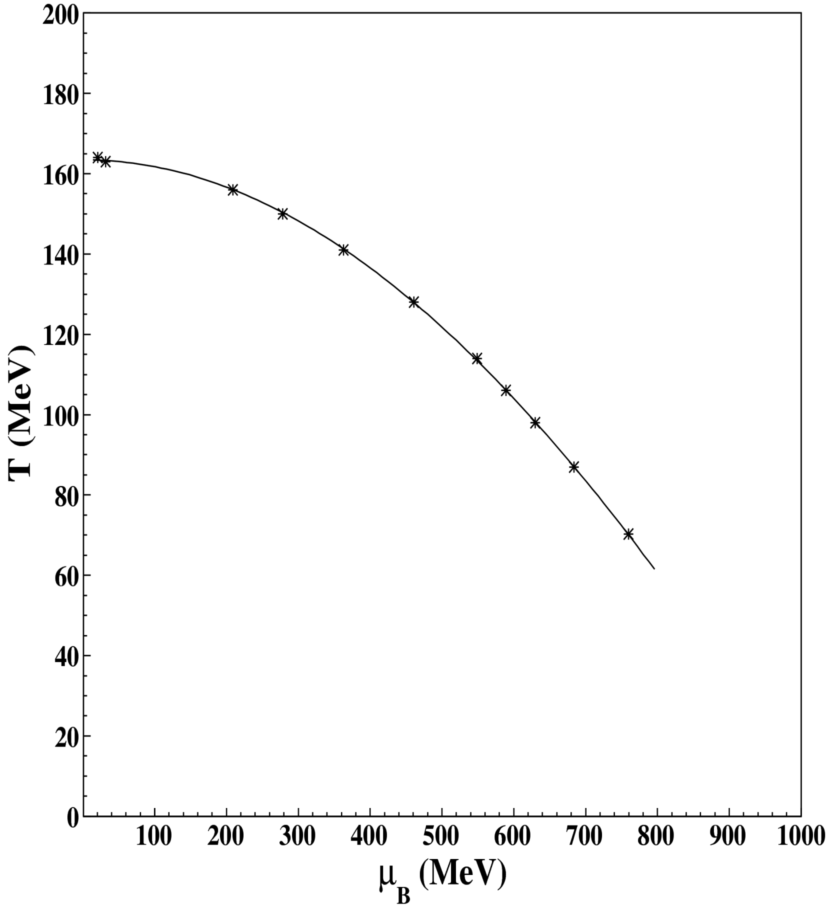
Where n is the total baryon density and

$$\langle |\mathbf{p}| \rangle = \frac{\int_0^\infty p^2\, dp\, p\, \mathbf{A}}{\int_0^\infty p^2\, dp\, \mathbf{A}}$$

A is the Fermi-Dirac distribution function of baryons and r is the hard-core radius.

Ref. S. K. Tiwari et al., Phys. Rev. C 85, 014908 (2012)





$$\mu_B = \frac{a}{(1 + b\sqrt{S_{NN}})}$$

a=1.482 ± 0.0037GeV

b=0.3517 ± 0.009 $(\mathrm{GeV})^{-1}$

$$T = c - d\,\mu_B^2 - e\,\mu_B^4$$

c=0.163 ± 0.0021 GeV

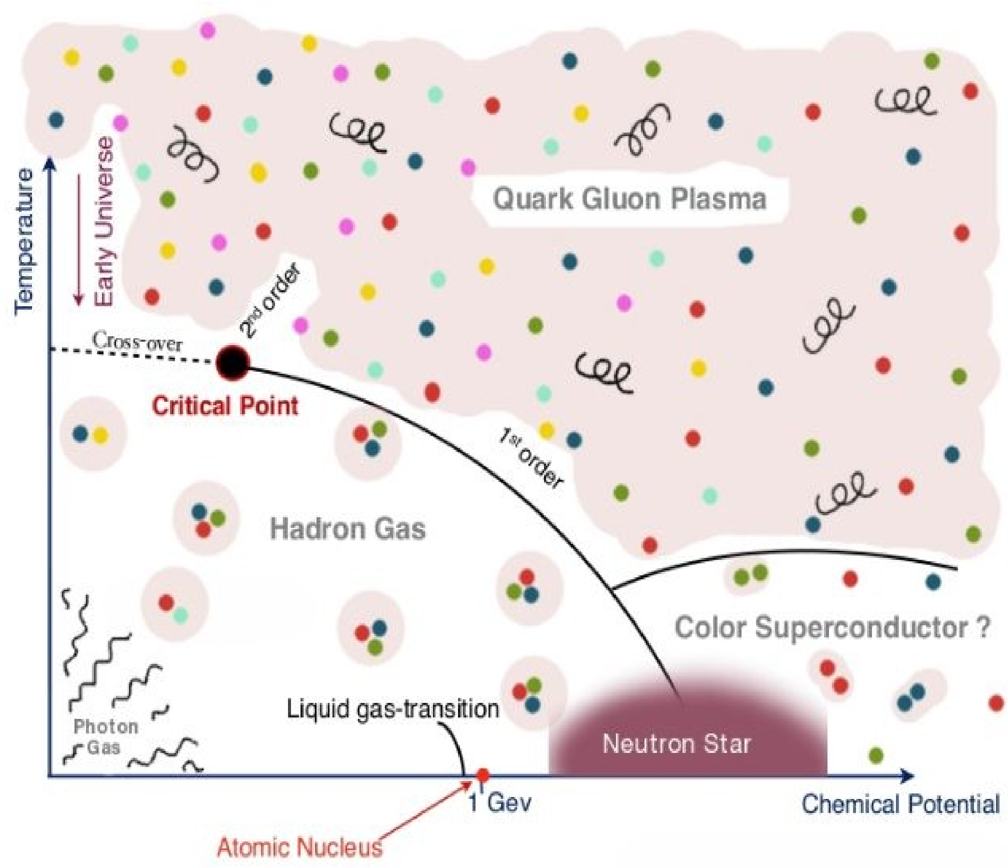
d=0.170 ± 0.02 $(\mathrm{GeV})^{-1}$

e=-0.015 ± 0.01$(\mathrm{GeV})^{-3}$

Introduction

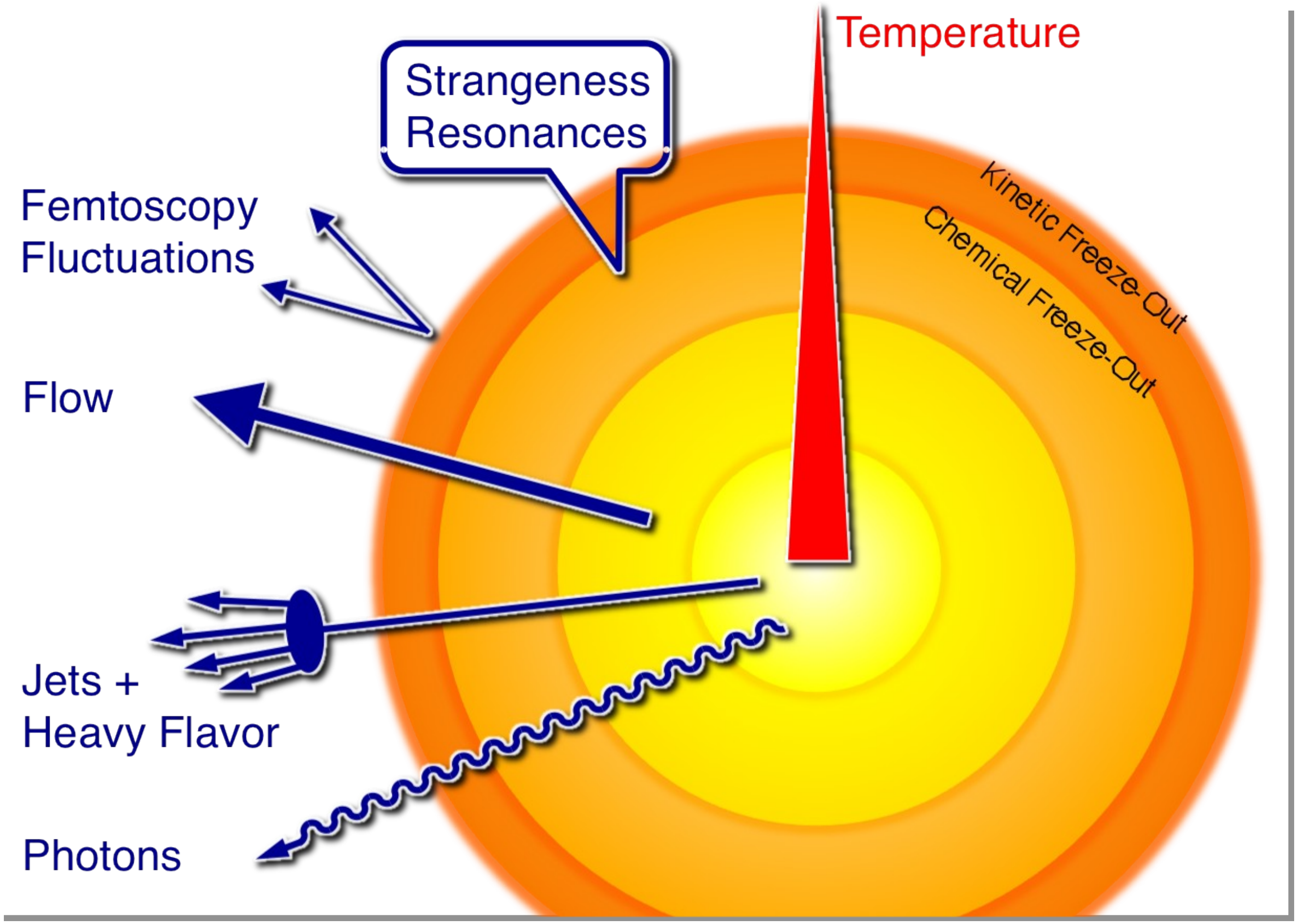
Quantum Chromodynamics (QCD), a theory of strong interaction predicts a phase transition from a hot, dense hadron gas (HG) to a deconfined and/or chiral symmetric phase of quarks and gluons called as Quark-Gluon Plasma (QGP).

The ultimate goal of ultra-relativistic heavy-ion collision programmes running at various places is to study the properties of QGP in laboratory.



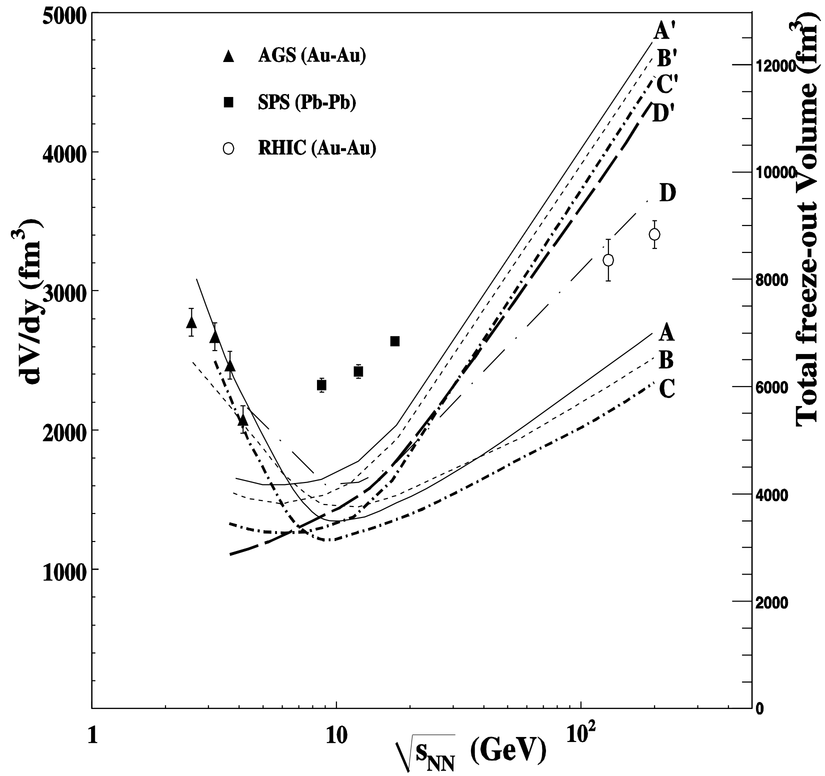
arXiv : 1304.1452

Observables



Taken from the presentation of C. Blume

No unambiguous signal for the formation of QGP observed yet (C. P. Singh Phys. Rep. 236, 147 (1993), A. Ranjan and V. Ravishankar Ind. J. Phys. 84, 11 (2010)) .
Phenomenological study required for the proposal of any unambiguous signal.



AGS (Au-Au)
SPS (Pb-Pb)
RHIC (Au-Au)
A'
B'
C'
D'
D
A
B
C
dV/dy (fm^3)
Total freeze-out Volume (fm^3)
$\sqrt{s_{NN}}$ (GeV)