Transport coefficients from the Nambu-Jona-Lasinio model for $SU(3)_f$

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

FIAS Frankfurt Institute for Advanced Studies

HIC for FAIR Helmholtz International Center

Rudy Marty

SQM2013, Birmingham

in collaboration with:

E. Bratkovskaya, W. Cassing and J. Aichelin

Based on **arXiv:1305.7180 [hep-ph]**

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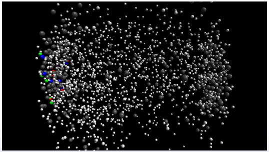

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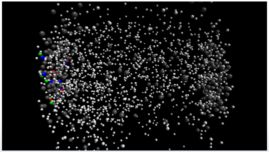

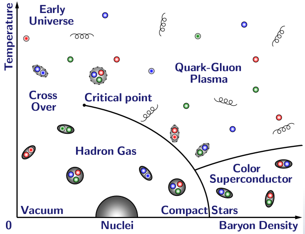

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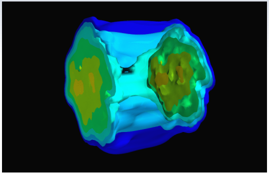

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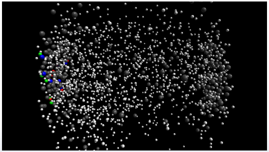
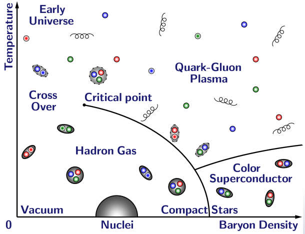
- For transport codes (compare in/out of equilibrium systems),
- For the phase diagram study of nuclear matter,
- As input for hydrodynamics calculations (η/s, κ),
- ...



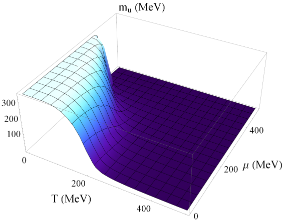


The Nambu-Jona-Lasinio model

Lagrangian:

$$\mathscr{L}_{NJL} = \bar{\psi}\left(i\partial\!\!\!/ - m_0\right)\psi + G\sum_{a=0}^{8}\left[\left(\bar{\psi}\lambda^a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda^a\psi\right)^2\right] - K\left[\det\bar{\psi}\left(1-\gamma_5\right)\psi + \det\bar{\psi}\left(1+\gamma_5\right)\psi\right]$$

- Chiral model for $q/\bar{q}$,
- QCD symmetries,
- hadrons construction,
- Finite (T, μ).



Quark mass:

$$m_i = m_{0i} - 4G\langle\langle\bar{\psi}_i\psi_i\rangle\rangle + 2K\langle\langle\bar{\psi}_j\psi_j\rangle\rangle\langle\langle\bar{\psi}_k\psi_k\rangle\rangle$$

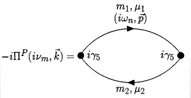
Chiral condensate:

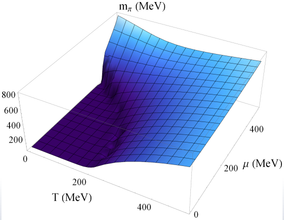
$$\langle\langle\bar{\psi}_i\psi_i\rangle\rangle = -2N_c\int_0^{\Lambda}\frac{d^3p}{(2\pi)^3}\frac{m_i}{E_{ip}}\left[1 - f_q - f_{\bar{q}}\right]$$

Meson mass:

$$\frac{-ig^2_{\pi q\bar{q}}}{k^2 - M^2} = \frac{2iG}{1 - 2G\,\Pi(k)}$$

Polar. loop:



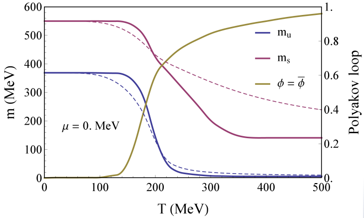


Based on **PRC 87, 034912 (2013)**

The Polyakov-NJL model

Lagrangian:

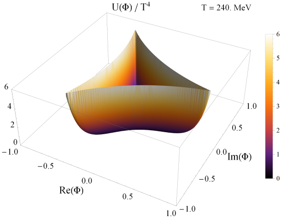
$$\mathscr{L}_{PNJL} = \bar{\psi}\left(i\not{D} - m_0\right)\psi + \mathcal{U}(\phi, \bar{\phi}, T) + \mu\bar{\psi}\gamma_0\psi$$
$$+\ G\sum_{a=0}^{8}\left[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2\right]$$
$$+\ K\left[\det\bar{\psi}\left(1-\gamma_5\right)\psi + \det\bar{\psi}\left(1+\gamma_5\right)\psi\right]$$



Based on **PRD 79, 116003 (2009)**

Modified dist.:

$$f_q \rightarrow f_q^{\Phi}(\mathbf{p}, T, \mu)$$
$$f_{\bar{q}} \rightarrow f_{\bar{q}}^{\Phi}(\mathbf{p}, T, \mu)$$

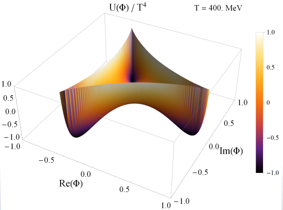


Modified chiral cond.:

$$\langle\langle\bar{\psi}\psi\rangle\rangle = -2N_c\int_0^{\Lambda}\frac{d^3p}{(2\pi)^3}\frac{m}{E_{\mathbf{p}}}\left[1 - f_q^{\Phi} - f_{\bar{q}}^{\Phi}\right]$$

Effective potential:

$$\frac{\mathcal{U}(\Phi, \bar{\Phi}, T)}{T^4} = -\frac{a(T)}{2}\bar{\Phi}\Phi + b(T)$$
$$\log[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) + 3(\bar{\Phi}\Phi)^2]$$



The Dynamical Quasi-Particle Model

Quasi-partons:

Masses:

$$M_g^2(T,\mu_q) = \frac{g^2}{6}\left(\left(N_c + \frac{N_f}{2}\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right)$$

$$M_{q/\bar{q}}^2(T,\mu_q) = \frac{N_c^2-1}{8N_c} g^2 \left(T^2 + \sum_q \frac{\mu_q^2}{\pi^2}\right)$$

Widths:

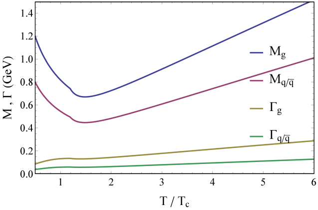
$$\Gamma_g(T) = \frac{1}{3} N_c \frac{g^2 T}{8\pi} \ln\left(\frac{2c}{g^2}+1\right),$$

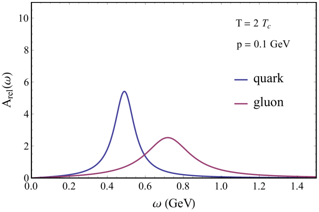
$$\Gamma_{q/\bar{q}}(T) = \frac{1}{3} \frac{N_c^2-1}{2N_c} \frac{g^2 T}{8\pi} \ln\left(\frac{2c}{g^2}+1\right),$$

Coupling constant:

$$g^2(T/T_c) = \frac{48\pi^2}{(11N_c - 2N_f)\ln[\lambda^2(T/T_c - T_s/T_c)^2]}$$

Based on EPJ ST 168, 3 (2009)





Off-shellness:

Breit-Wigner spectral function:

$$A(\omega,\mathbf{p}) = \frac{\Gamma}{E}\left(\frac{1}{(\omega-E)^2+\Gamma^2} - \frac{1}{(\omega+E)^2+\Gamma^2}\right)$$

with $E^2 = \mathbf{p}^2 + M^2 - \Gamma^2$ and

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, \omega A(\omega,\mathbf{p}) = \int_0^{\infty} \frac{d\omega}{2\pi}\, 2\omega A(\omega,\mathbf{p}) = 1$$

Equations of state I

For a non-interacting particle gas:

$$n(T,\mu) = g_g \int_0^\infty \frac{d^3p}{(2\pi)^3} f_g + \frac{g_q}{6} \int_0^\infty \frac{d^3p}{(2\pi)^3} \left[\sum_q^{u,d,s} f_q + \sum_{\bar{q}}^{\bar{u},\bar{d},\bar{s}} f_{\bar{q}} \right]$$

$$\varepsilon(T,\mu) = g_g \int_0^\infty \frac{d^3p}{(2\pi)^3} f_g E_g + \frac{g_q}{6} \int_0^\infty \frac{d^3p}{(2\pi)^3} \left[\sum_q^{u,d,s} f_q + \sum_{\bar{q}}^{\bar{u},\bar{d},\bar{s}} f_{\bar{q}} \right] E_q$$

$$P(T,\mu) = g_g \int_0^\infty \frac{d^3p}{(2\pi)^3} f_g \frac{\mathbf{p}^2}{3E_g} + \frac{g_q}{6} \int_0^\infty \frac{d^3p}{(2\pi)^3} \left[\sum_q^{u,d,s} f_q + \sum_{\bar{q}}^{\bar{u},\bar{d},\bar{s}} f_{\bar{q}} \right] \frac{\mathbf{p}^2}{3E_q}$$

$$s(T,\mu) = \frac{\varepsilon(T,\mu) + P(T,\mu) - \mu n_B(T,\mu)}{T}$$

with

$$n_B(T,\mu) = \frac{g_q}{6} \int_0^\infty \frac{d^3p}{(2\pi)^3} \left[\sum_q^{u,d,s} f_q - \sum_{\bar{q}}^{\bar{u},\bar{d},\bar{s}} f_{\bar{q}} \right]$$

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$$\varepsilon(T,\mu) = g_g \int_0^\infty \frac{d^3p}{(2\pi)^3} f_g E_g + \cdots \int_0^\infty \frac{d^3p}{(2\pi)^3} \left[\sum_q^{u,d,s} f_q + \sum_{\bar{q}}^{\bar{u},\bar{d},\bar{s}} f_{\bar{q}} \right] E_q$$

Attention !

For (P)NJL: no gluons

For PNJL: consider $f^\Phi(\mathbf{p},T,\mu)$ and $\mathcal{U}(\Phi,\bar{\Phi},T)/T^4$

For DQPM: consider spectral function $A(\omega,\mathbf{p})$

Notice that $P_{SB}(NJL) \neq P_{SB}(PNJL) = P_{SB}(DQPM)$

$$P(T,\mu) = g_g \int_0^\infty \frac{d^3p}{(2\pi)^3} f_g \frac{\cdots}{3E_g} + \frac{g_q}{6} \int_0^\infty \frac{d^3p}{(2\pi)^3} \left[\sum_q^{u,d,s} f_q + \sum_{\bar{q}}^{\bar{u},\bar{d},\bar{s}} f_{\bar{q}} \right] \frac{\mathbf{p}^2}{3E_q}$$

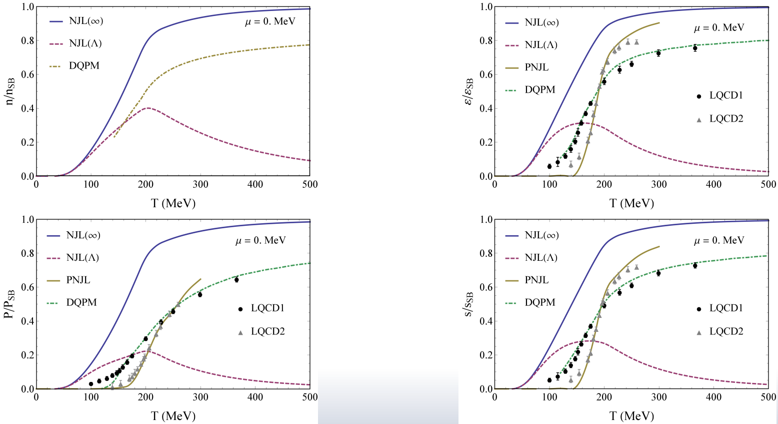
$$\cdots \frac{\cdots - P(T,\mu) - \mu n_B(T,\mu)}{T}$$

with

$$n_B(T,\mu) = \frac{g_q}{6} \int_0^\infty \frac{d^3p}{(2\pi)^3} \left[\sum_q^{u,d,s} f_q - \sum_{\bar{q}}^{\bar{u},\bar{d},\bar{s}} f_{\bar{q}} \right]$$

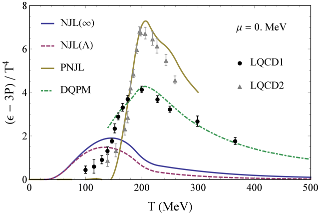
Equations of state II

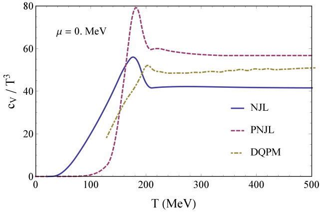
For a non-interacting particle gas:



Equations of state III

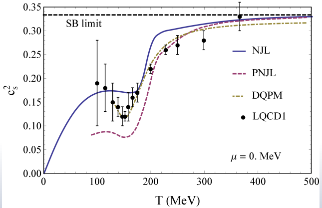
For a non-interacting particle gas:



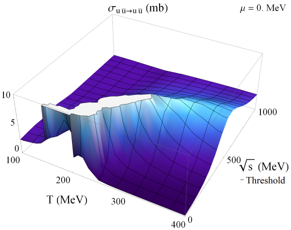


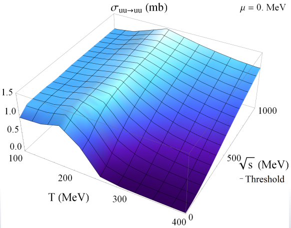
$$c_V = T\left(\frac{\partial s}{\partial T}\right)_V$$

$$c_s^2 = \left(\frac{\partial P}{\partial \varepsilon}\right)_{n_B}$$



Integrated cross sections





Transition rate:

$$\sigma(T,\mu) = \int ds\, \sigma(T,\mu,s)\ L(T,\mu,s)$$

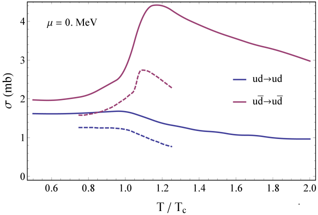
with the probability to find a $q\bar{q}$ (qq) pair with the energy $\sqrt{s} \overset{\text{cm}}{=} E_1 + E_2$ in the medium (T,μ):

$$L(T,\mu,s) = 2\sqrt{s}\ C(T,\mu) \times p_{\text{cm}}(s) \times v_{\text{rel}}(s)$$
$$\times f_q\,(E_1 - \mu) \times f_{\bar{q}(q)}\,(E_2 + (-)\mu)$$

while C is a normalization factor fixed by

$$C^{-1}(T,\mu) = \int ds\ L(T,\mu,s)$$

- $4\ \sigma(qq \to qq)$
- $4\ \sigma(q\bar{q} \to q\bar{q})$
- $3\ \sigma(q\bar{q} \to q'\bar{q}')$



Based on **PRD 51, 3728 (1995)**

Relaxation time

We compute transport coefficients using the Relaxation Time Approximation

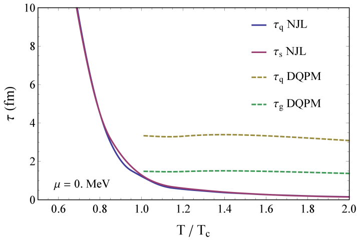
Relaxation time:

For the (P)NJL model it is the transition rate inverse, weighted by the particle density:

$$\tau_i^{-1}(T,\mu) = \sum_j n_j(T,\mu)\, \sigma_{ij}(T,\mu)$$

For the DQPM, it is directly proportional to the inverse of the width:

$$\tau_q^{-1}(T) = \Gamma_q(T) \quad \text{and} \quad \tau_g^{-1}(T) = \Gamma_g(T).$$



This quantity is related to the **mean free path** and therefore gives a good estimation of the thermalization time of the species.

Based on **PRD 51, 3728 (1995)**

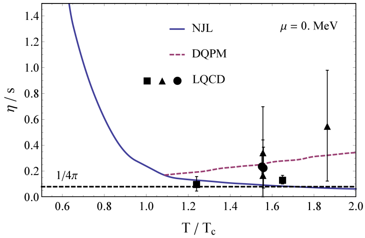
Viscosity

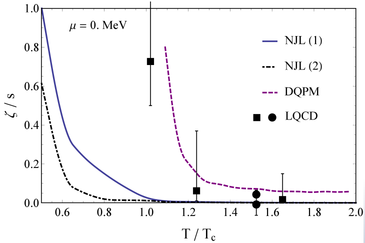
Shear viscosity:

$$\eta(T,\mu) = \frac{1}{15T} g_g \int_0^\infty \frac{d^3p}{(2\pi)^3} \tau_g f_g \frac{\mathbf{p}^4}{E_g^2} + \frac{1}{15T}\frac{g_q}{6} \int_0^\infty \frac{d^3p}{(2\pi)^3} \left[\sum_q^{u,d,s} \tau_q f_q + \sum_{\bar{q}}^{\bar{u},\bar{d},\bar{s}} \tau_q f_{\bar{q}} \right] \frac{\mathbf{p}^4}{E_q^2}$$

Bulk viscosity:

$$\zeta(T,\mu) = \frac{1}{9T} g_g \int_0^\infty \frac{d^3p}{(2\pi)^3} \tau_g f_g \frac{1}{E_g^2} \left[\mathbf{p}^2 - 3c_s^2 \left(E_g^2 - T^2 \frac{dm_g^2}{dT^2} \right) \right]^2 + \frac{1}{9T}\frac{g_q}{6} \int_0^\infty \frac{d^3p}{(2\pi)^3} \left[\sum_q^{u,d,s} \tau_q f_q + \sum_{\bar{q}}^{\bar{u},\bar{d},\bar{s}} \tau_q f_{\bar{q}} \right] \frac{1}{E_q^2} \left[\mathbf{p}^2 - 3c_s^2 \left(E_q^2 - T^2 \frac{dm_q^2}{dT^2} \right) \right]^2$$





Based on **PRC 83, 014906 (2011)**

Conductivity

Conductivity:

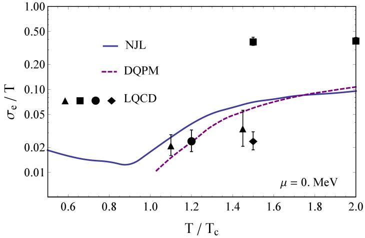
Electric conductivity:

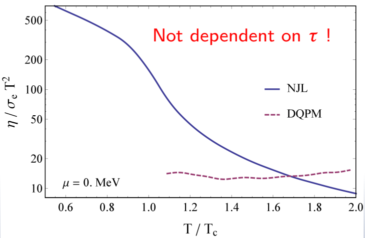
$$\sigma_e(T,\mu) = \sum_f \frac{e_f^2 \, n_f(T,\mu) \, \tau_f(T,\mu)}{m_f(T,\mu)}$$

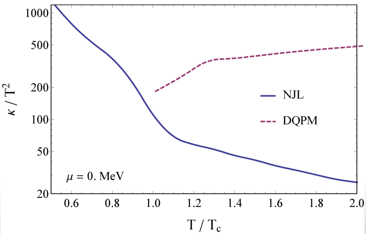
with $f = u, d, s, \bar{u}, \bar{d}, \bar{s}$, and the electric charge of quarks e_f

Heat conductivity:

$$\kappa(T,\mu) = \frac{1}{3} v_{\text{rel}} \; c_V(T,\mu) \; \sum_f \tau_f(T,\mu)$$







Conclusion

What we did:

- Complete calculations of equations of state and of a set of transport coefficients for several models for $0.5 < T/T_c < 2.0$,
- Despite the lack of confinement and gluons degrees of freedoms, the NJL model gives results not so far from QCD arround T_c,
- We have a better understanding of the range of validity of the NJL model,
- The DQPM compares well with lQCD for the observables studied.

To do list:

- Results for PNJL $SU(3)_f$,
- Finite chemical potential calculations,
- Include hadron interaction for NJL $(\sigma(qM \to qM))$,
- Calculate differential parton cross sections on the basis of the DQPM couplings,
- Try box calculations for NJL as it was did for DQPM (PRC 87, 064903 (2013)).

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THANK YOU FOR YOUR ATTENTION !

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